Physical Layer Network Coding for the Multi-way Relay Channel

by

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B.Sc., Ferdowsi University of Mashhad, 2006

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ABSTRACT

Wireless networks have received considerable attention recently due to the high user demand for wireless services and the emergence of new applications. This thesis focuses on the problem of information dissemination in a class of wireless networks known as the multi-way relay channel. Physical layer network coding is considered to increase the throughput in these networks. First, an algorithm is proposed that increases the full data exchange throughput by 33% compared to traditional routing. This gain arises from providing common knowledge to users and exploiting this knowledge to restrain some users from transmitting. Second, for complex field network coding, a transmission scheme is designed that ensures the receipt of a QAM constellation at the relay. This requires precoding the user symbols to make all possible combinations distinguishable at the relay. Using this approach, the throughput of data exchange is 1/2 symbol per user per channel use. The error performance of both schemes is derived analytically for AWGN channels.

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Chapter 1

Introduction

With the emergence of bandwidth hungry applications such as streaming video, increasing the throughput of wireless networks is a serious challenge. As an example, starting from 2008, Cisco has predicted an annual growth of 100 percent in cellular data traffic, which is expected to continue indefinitely [1]. Studies show that the capacity of wireless networks is much higher than previously believed [2]. To harness the available capacity of these networks, new techniques must be developed and deployed. Network coding, which was originally introduced to improve throughput in multicast scenarios in wired networks, is a promising technique with the potential to enhance the throughput of wireless networks.

In this chapter, the origins of network coding are explored, and the application of network coding in the two-way relay channel (TWRC) is reviewed. The multi-way relay channel, which is a natural generalization of the TWRC, is introduced and finally the contributions of this thesis are summarized.

1.1 Origins of Network Coding

In 1948, in his classic paper Claude Shannon formulated the fundamental limits of reliable communication. In particular, he determined the channel capacity below which reliable communication is possible. This formulation pertains to point to point communication where a single source wishes to communicate with a single receiver. Since the early days of information theory, researchers were interested in extending Shannon's results to scenarios more general than the point to point case. Shannon himself was a pioneer in this endeavor, working on two-way communication channels

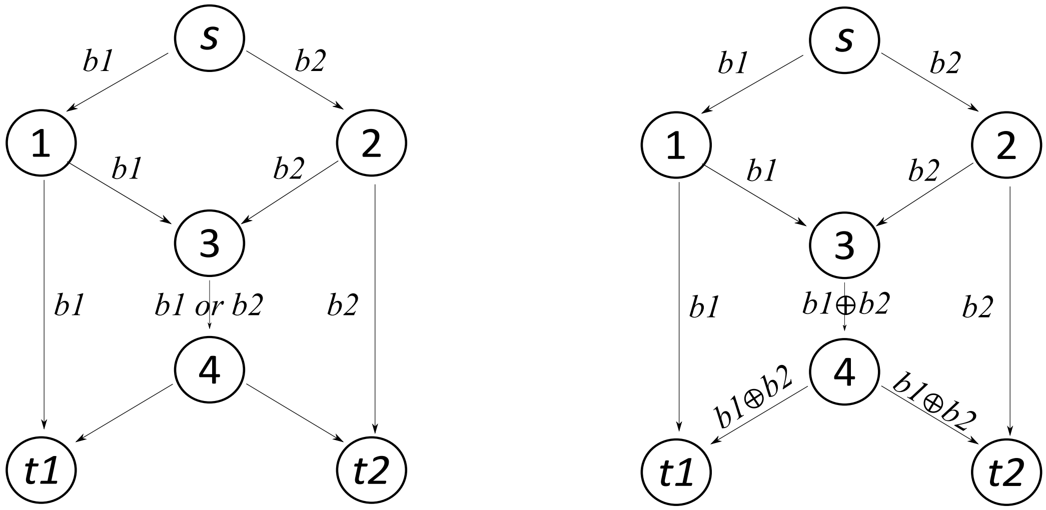
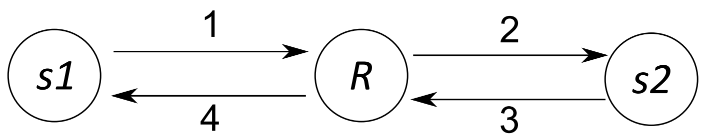


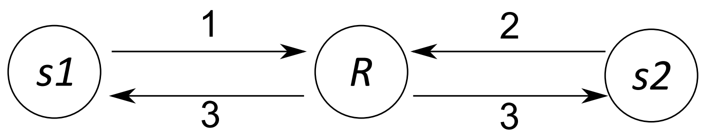
Figure 1.1: The butterfly network model.

[3], and proving the max-flow min-cut theorem for unicast flows among others [4]. Other notable early achievements in network information theory are Ahlswede's explicit formulation of the capacity region of the multiple access channel [5], and Cover's work on the broadcast and relay channels [6] [7]. As another advance in characterizing the capacity of communication networks, in a seminal paper [8], Ahlswede et al. showed that the capacity of multicast flows can only be achieved by applying network coding. Simply put, their work shows that, unlike an unalterable physical commodity, information can be replicated or coded at intermediate nodes of network to increase the network throughput.

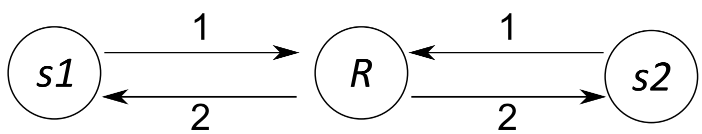
The butterfly network in Fig. 1.1 is a standard example used to illustrate the need for network coding. This network has two terminal nodes $t1$ and $t2$. Suppose the communication scenario is for the source to send two bits $b1$ and $b2$ to both terminals. In Fig. 1.1 it is obvious that the link between nodes 3 and 4 is the bottleneck, i.e., two channel uses are required to transmit $b1$ and $b2$ from node 3 to node 4. Alternatively, if $b1 \oplus b2$ is sent through node 3 to node 4 and subsequently to the end terminals, $t1$ and $t2$ can decode both bits by XORing the received transmission from node 4 with that of nodes 1 and 2, respectively. Thus employing network coding here saves one channel use and it turns out that the multicast capacity is achieved for this specific setting. Ahlswede et al. proved that by allowing network coding at intermediate nodes, the capacity of multicast flows can always be achieved.



(a) plain routing: 4 transmissions required



(b) Conventional network coding: 3 transmissions required



(c) physical-layer network coding: 2 transmissions required

Figure 1.2: Transmission schemes for the two-way relay channel.

1.2 Network Coding in Wireless Networks

Network coding for wireless networks has mainly been studied in the context of the two-way relay channel. In the TWRC, as depicted in Fig. 1.2, two sources wish to exchange information through an intermediate relay. In the traditional wireless routing approach, 4 transmissions are required for the two users to exchange information. If network coding at the relay is performed, i.e., the sources transmit their data to the relay and the relay transmits back a coded combination of the source data, the data exchange can be accomplished in 3 transmissions which translates into a 33% throughput gain. In [9], the results of the first testbed deployment of wireless network coding were reported. These results show a large increase in network throughput. The gains vary from a few percent to several times depending on the traffic pattern, congestion level, and transport protocol.

The wireless medium has two unique features that must be considered when designing communication protocols. First, communications is inherently broadcast, meaning that signals from a transmitter can be heard at multiple nodes. Second, a node can receive the superposition of signals from multiple transmitters. Although these features generally make wireless communications challenging, they can also be exploited to advantage.

Analog network coding (ANC) [10] and physical layer network coding (PNC) [11] were introduced in the context of the two-way relay channel to exploit the broadcast and superposition features of the wireless medium. Both techniques rely on the fact that the superposition principle holds for electromagnetic waves, i.e. when two or more EM waves are traveling through the same space, the net amplitude will be the sum of the individual wave amplitudes. This property can be regarded as a network coding operation that naturally occurs in the wireless environment. In both ANC and PNC, the two sources transmit their data to the relay simultaneously. The network coding operation is performed in the air, and the relay receives the sum of the transmitted signals. The relay then transmits this signal back to the users. The users can decode the messages of one another by removing their own message from the received signal. Thus data exchange can be completed in 2 transmissions. The difference between ANC and PNC is that in the former, the relay simply amplifies and forwards the received sum signals, whereas in PNC the relay performs a decode and forward operation. Throughout this thesis, the decode and forward operation is adopted for the relay.

The potential benefits of physical layer network coding for the two-way relay channel have been studied from several perspectives. From an information theoretic point of view, it has been shown that assuming an additive white Gaussian (AGWN) channel, PNC used in conjunction with a lattice code can achieve rates within 1/2 bit of the cut-set outer bound in a TWRC [12]. Communication theoretic aspects of PNC such as channel coding [13], detection issues at the relay [14], synchronization [15], and error performance of PNC in fading channels [16], have also been studied extensively. When extending PNC beyond the TWRC, networking is a major concern. Issues such as distributed MAC protocol design [17] and asymptotic network performance [18], have been studied in this regard. An extensive examination of the state of the art in physical layer network coding is given in [17]. Finally, it should be noted that PNC has been prototyped on a GNU software defined radio testbed to demonstrate its practicality [19].

1.3 The Multi-way Relay Channel

The multi-way relay channel (MWRC) is a natural generalization of the two-way relay channel. the MWRC consists of multiple users intending to exchange information through an intermediate relay. This channel was introduced in [20], where information theoretic aspects were considered and bounds on the capacity region for amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) relaying were derived. Interest in the MWRC arises from the fact that this channel can model a variety of communication scenarios. For example, in a wireless sensor network, temperature sensors may exchange local temperature measurements between themselves, aided by a relay. As another example, multiple ad-hoc networks with nodes distributed and a single communication satellite acting as the enabler of communication among them, can be modeled as a multi-way relay channel. Furthermore, as with the two-way relay channel, the MWRC can be considered as a new fundamental building block for general multicast communication.

The communication theoretic aspects of the MWRC have also been studied, with several protocols introduced and compared. In [21], the case is considered where N users wish to broadcast their information through a relay. it is shown that the throughput of plain routing, conventional network coding and PNC is $\frac{1}{2N}$, $\frac{1}{2N-1}$, and $\frac{1}{2N-2}$ symbol per source node per channel use (sym/S/CU), respectively, when source nodes are unable to hear nodes other than the relay. The conclusion of their work is that as the number of source nodes increases, the performance gains of PNC over plain routing diminish. In another approach introduced in [22] called complex field network coding (CFNC), the authors devise a transmission scheme that can be applied to the MWRC which achieves a throughput of $\frac{1}{2}$ sym/S/CU. The key idea of CFNC is to precode the user symbols such that any combination of user symbols will be distinguishable at the relay. The main drawback of this technique is that as the number of users increases (e.g. $N \geq 4$), the performance deteriorates dramatically, thus limiting its applicability.

1.4 Organization of the Thesis

This thesis considers the design of communication protocols for the multi-way relay channel (MWRC). the organisation of the thesis is as follows.

Chapter 2 considers the problem of full data exchange in a multi-way relay channel. It is shown that a throughput of $\frac{1}{1.5N}$ symbol per node per channel use can be achieved using binary signaling, which is a 33% increase compared to plain routing. This is accomplished by providing common knowledge to all nodes and exploiting this knowledge to restrain some nodes from transmitting. The error performance of the proposed algorithm is evaluated analytically.

Chapter 3 presents a new design of QAM constellations for multi-way relay channels which is based on complex field network coding. Precoding vectors are introduced that guarantee that every superimposed combination of user symbols at the relay is distinguishable. The performance in AWGN channels is derived. The algorithm presented allows users to employ different signal constellations, and also has the flexibility to allow users to join or leave the network at any time.

Chapter 4 contains some concluding remarks and suggestions for future work.

Chapter 2

Physical-Layer Network Coding in Multi-way Relay Channels with Binary Signaling

In this chapter, the problem of full data exchange in multi-way relay channels is considered. It is shown that a throughput of $\frac{1}{1.5N}$ symbol per node per channel use can be achieved using binary signaling, which is a 33% increase compared to traditional plain routing. This is accomplished by providing common knowledge to all nodes and exploiting this knowledge to restrain some nodes from transmitting. The error performance of the proposed algorithm is evaluated analytically. First, the network model and notation are introduced. Then the algorithm is described and the throughput of the system analyzed in Section 2.2. The performance is evaluated in Section 2.3. The results of this chapter were published in [23].

2.1 The Network Model

As shown in Fig. 2.1, we consider a multi-way relay channel which has N source nodes, $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N$, and one relay. Information theoretic aspects of this model were studied in [20]. We assume full data exchange in which every node must receive the messages of all other source nodes. As in [20], we assume there is no direct link between any two source nodes, so the relay is the enabler of communications. As with all cooperative relay networks, time synchronization is required. This can be achieved via techniques originally developed for MIMO systems [24, Ch. 11]. Furthermore, the

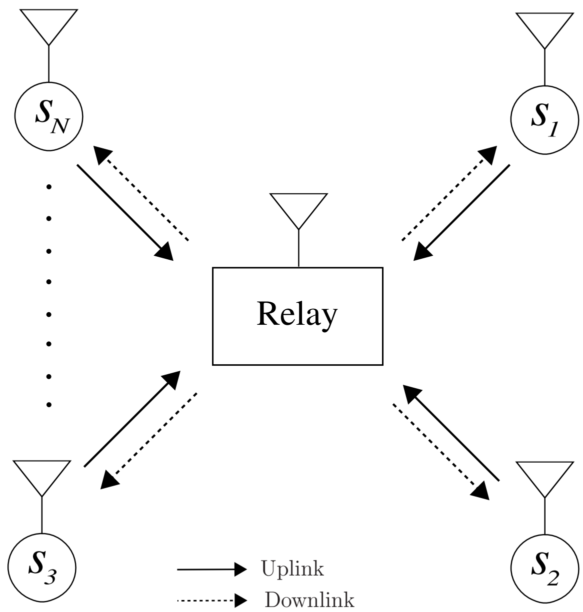


Figure 2.1: The communication network model.

transmissions are half-duplex.

2.2 Algorithm Description and Throughput Analysis

With plain routing, $2N$ channel uses (CUs) are required for full data exchange between N nodes. In this case, the throughput is $\frac{1}{2N}$ sym/S/CU. Here we propose a network coding scheme that improves the throughput by at least 33%, i.e. $\frac{1}{1.5N}$ sym/S/CU, if binary signaling is used. Our transmission scheme consists of three main steps:

Step 1: All nodes transmit their BPSK symbols to the relay in the same time slot. Due to the network coding operation that naturally occurs in the air, the relay receives the superimposed electromagnetic waves, i.e., the sum of the symbols.

Step 2: The relay broadcasts the received sum back to the nodes. At this stage each node will know the exact number of nodes that have sent 1, and thus also the number that have sent -1.

Step 3: By exploiting this common information (the received sum signal from the relay), only some of the nodes, called minority nodes, send their symbols to the relay for broadcasting. The goal of this step is to identify these minority nodes to all source nodes. This is accomplished by a divide-and-conquer method in which nodes are successively divided into smaller groups over a number of rounds. The details of this step are later illustrated with examples.

Note that for the case $N = 2$, transmission can be completed after Step 2 by using self information as in [11]. Since we are considering binary signaling for each of the N nodes, $N + 1$ different sum values can be received by the relay in Step 1. If the number of nodes sending 1 (-1) is less than the number of nodes sending -1 (1), those nodes are said to be 'in minority'. If the number of nodes sending 1 and -1 are the same, those nodes sending -1 are chosen to be in minority. By the end of Steps 1 and 2, each node has the following information: **a)** whether it is a minority node or not, **b)** the number of minority nodes.

In Step 3, the objective is to identify the minority nodes, thus making available the symbol of each node to every other node. To achieve this, the nodes are divided into two approximately equal groups. If there are M nodes, the two groups are $\mathbb{G}_1 = \{S_1, \ldots, S_{\lfloor M/2 \rfloor}\}$ and $\mathbb{G}_2 = \{S_{\lfloor M/2 \rfloor + 1}, \ldots, S_M\}$. The minority nodes in $\mathbb{G}_1$ transmit 1 and the minority nodes in $\mathbb{G}_2$ transmit -1 to the relay simultaneously. The relay then broadcasts the sum back. In this manner, the number of minority nodes in each group is known. By successively repeating this procedure, the minority nodes can be identified. This method is illustrated below for the two and three node cases. These cases will serve as the basis for the general throughput analysis for N nodes.

2.2.1 Two Nodes

With two nodes, after the first two transmissions in Steps 1 and 2, if both nodes had sent the same symbols, there are no minority nodes, and the communication is complete, i.e., both nodes know the information symbol of the other node and Step 3 is not required. This is shown in Fig. 2.2 (a). The case of two nodes having different symbols is shown in Fig. 2.2 (b) with the minority node colored. To identify the minority node, the two nodes are grouped into $\mathbb{G}_1 = \{S_1\}$ and $\mathbb{G}_2 = \{S_2\}$. If the minority node is in $\mathbb{G}_1$, it sends 1 and if it is in $\mathbb{G}_2$ it sends -1. The relay broadcasts this information to both nodes. Thus in this case two transmissions are needed to identify the minority node in Step 3.

Table 2.1 shows the transmission cases for two nodes. The cases in which the received sum at the relay in Step 1 is -2 or 2 have no minority node, and the probability of this occurring is $\binom{2}{0}/2^2$. The case where the received sum at the relay in Step 1 is

Figure 2.2: Node grouping in Step 3 for the two node case.

0 has one minority node, and the corresponding probability of occurrence is $\binom{2}{1}/2^2$. Including the two transmissions needed in Steps 1 and 2, the average number of channel uses is

$$C(N) = 2 + \frac{1}{4}\left[\binom{2}{0}\times 0 + \binom{2}{1}\times 2 + \binom{2}{0}\times 0\right] = 3. \quad (2.1)$$

Compared to plain routing where 4 channel uses are required, the information exchange is done in $0.75 \times 4 = 3$ channel uses on average, giving a 33% increase in throughput.

For the special case of two nodes, if the self information is considered as in [11], the information exchange can be completed in just 2 channel uses. The above two node solution is presented for illustration purposes, and more importantly to develop a general solution for an arbitrary number of nodes.

Table 2.1: Transmission Cases for Two Nodes

Sum	Symbols	Prob.	# Channel uses
-2	two (-1) and zero (1)	$\binom{2}{0}/2^2$	2 + 0
0	one (-1) and one (1)	$\binom{2}{1}/2^2$	2 + 2
2	zero (-1) and two (1)	$\binom{2}{0}/2^2$	2 + 0

2.2.2 Three Nodes

With three nodes, as in the previous case, if there are no minority nodes the two channel uses in Steps 1 and 2 are sufficient to complete the information exchange. The only other possibility is having one minority node, and this case is shown in Fig. 2.3. To identify the minority node in Step 3, the three nodes are grouped into $\mathbb{G}_1 = \{S_1\}$ and $\mathbb{G}_2 = \{S_2, S_3\}$. As in the two node case, if the minority node is in $\mathbb{G}_1$, it sends 1 to the relay and the relay broadcasts it back. After these two transmissions,

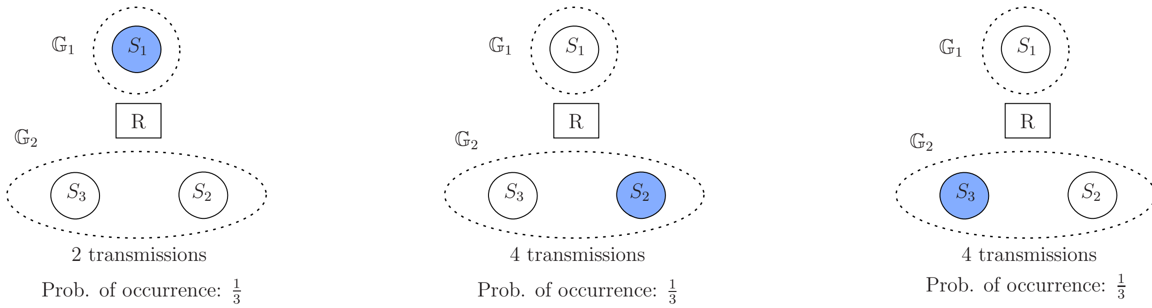


Figure 2.3: Node grouping in Step 3 for the three node case when there is a minority node.

the minority node is identified. If the minority is in $\mathbb{G}_2$ it sends -1 to the relay for broadcasting. In this case, the minority node cannot be identified but it is clear to all nodes that $\mathbb{G}_1$ does not contain the minority node. Therefore $\mathbb{G}_2$ is divided into two groups to identify the minority node. This leads to two more transmissions giving four in total in Step 3.

The probability of having the minority node in $\mathbb{G}_1$ and $\mathbb{G}_2$ is $\frac{1}{3}$ and $\frac{2}{3}$, respectively. Thus the average number of channel uses in Step 3 is $(\frac{1}{3} \times 2) + (\frac{2}{3} \times 4) = 3.33$.

Including the two transmissions in Steps 1 and 2, and noting the symmetry of the cases in Table 2.2, the average number of channel uses is

$$C(N) = 2 + 2 \times \frac{1}{8}\left[\binom{3}{0} \times 0 + \binom{3}{1} \times 3.33\right] = 4.5. \tag{2.2}$$

Thus the information exchange is completed in 4.5 channel uses on average which is 75% of the six channel uses required for plain routing. This is a 33% increase in throughput.

Table 2.2: Transmission Cases for Three Nodes

Sum	Symbols	Prob.	# Channel uses
-3	three (-1) and zero (1)	$\binom{3}{0}/2^3$	2 + 0
-1	two (-1) and one (1)	$\binom{3}{1}/2^3$	2 + 3.33
1	one (-1) and two (1)	$\binom{3}{1}/2^3$	2 + 3.33
3	zero (-1) and three (1)	$\binom{3}{0}/2^3$	2 + 0

2.2.3 Four Nodes

When four nodes are communicating and all nodes transmit the same symbol, the information exchange is complete after Steps 1 and 2. The other possibilities are one or two minority nodes. These cases are shown in Fig. 2.4. The nodes are divided into two groups $\mathbb{G}_1 = \{S_1, S_2\}$ and $\mathbb{G}_2 = \{S_3, S_4\}$ for Step 3. As indicated previously, the minority nodes in $\mathbb{G}_1$ and $\mathbb{G}_2$ send 1 and -1, respectively, and the relay transmits the sum to all nodes. When there is only one minority node ($\mathcal{S}_1$ in Fig. 2.4(a)), all nodes acquire knowledge that $\mathcal{S}_3$ and $\mathcal{S}_4$ are not minority nodes. The next smaller groups are $\mathbb{G}_1 = \{S_1\}$ and $\mathbb{G}_2 = \{S_2\}$, and the minority is identified with two more transmissions. Thus there are four transmissions in Step 3. When there are two minority nodes, if they are in the same group ($\mathcal{S}_1$ and $\mathcal{S}_2$ in Fig. 2.4), two transmissions are adequate to identify both nodes. If on the other hand the minority nodes are in different groups ($\mathcal{S}_1$ and $\mathcal{S}_3$ in Fig. 2.4), for each group two separate transmissions are required to identify the minority nodes. Therefore, in this case six transmissions are needed in Step 3.

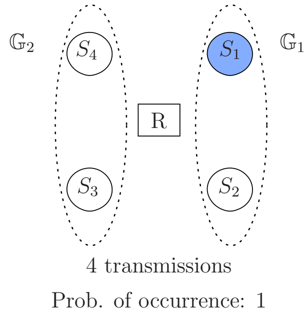
The probability of two minority nodes in one group is $\frac{1}{3}$, and the probability of two minority nodes in different groups is $\frac{2}{3}$. Consequently, when there are two minority nodes, the average number of channel uses in Step 3 is $(\frac{1}{3} \times 2) + (\frac{2}{3} \times 6) = 4.66$, as shown in Table 2.3. Including the two transmissions needed in Steps 1 and 2, and noting the symmetry in Table 2.3, the average number of channel uses is

$$
\begin{aligned}
C(N) = & 2 + 2 \times \frac{1}{16}\left[\binom{4}{0} \times 0 + \binom{4}{1} \times 4\right] \\
& + \frac{1}{16}\left[\binom{4}{2} \times 4.66\right] = 5.75.
\end{aligned}
\tag{2.3}
$$

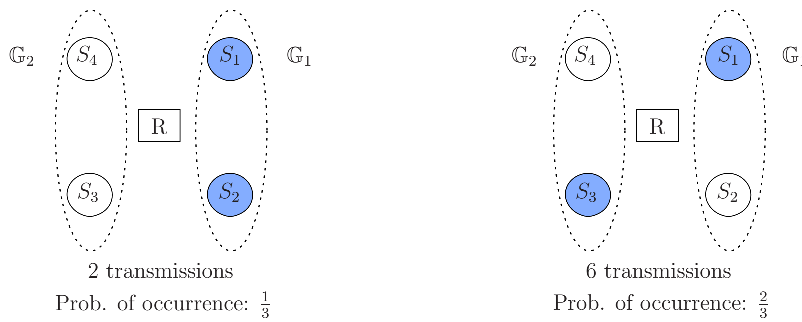
In comparison to plain routing, where 8 channel uses are required, the information exchange is done in $0.719 \times 8 = 5.75$ channel uses on average, a 39% increase in throughput.

2.2.4 The General N Node Case

In the general case of N nodes operating in a multi-way relay channel, by induction on N it can be shown that the required number of channel uses is at most 0.75 times that of plain routing. The throughput gain will therefore not be less than 33%. The



(a) One minority node



(b) Two minority nodes

Figure 2.4: Node grouping for Step 3 with four nodes. The minority nodes are colored.

results for $N = 2$ and $N = 3$ given previously are used as the base for a proof by induction. In the inductive step, we assume N nodes require at most $0.75 \times 2N$ channel uses, and then prove that $N + 2$ nodes will need at most $0.75 \times 2(N + 2)$ channel uses, i.e., a throughput gain no worse than 33%.

Proof. After Steps 1 and 2, and determining the total number of minority nodes, the $N + 2$ nodes are divided into groups of two and N nodes. The minority nodes that are placed in the two node group transmit 1, and the minority nodes that are placed

Table 2.3: Transmission Cases for Four Nodes

Sum	Symbols	Prob.	# Channel uses
-4	four (-1) and zero (1)	$\binom{4}{0}/2^4$	2 + 0
-2	three (-1) and one (1)	$\binom{4}{1}/2^4$	2 + 4
0	two (-1) and two (1)	$\binom{4}{2}/2^4$	2 + 4.66
2	one (-1) and three (1)	$\binom{4}{1}/2^4$	2 + 4
4	zero (-1) and four (1)	$\binom{4}{0}/2^4$	2 + 0

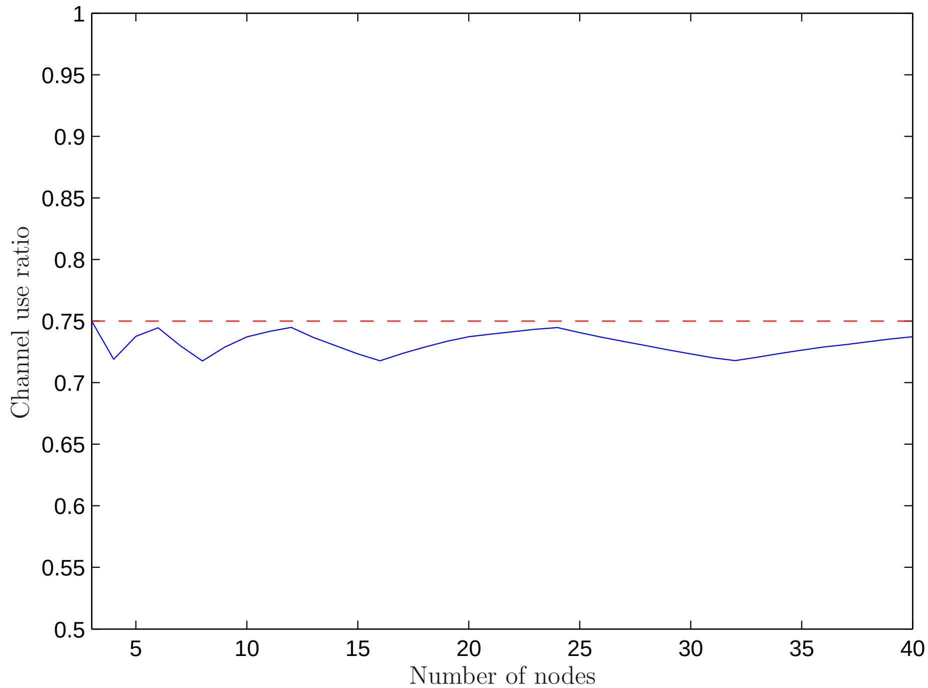


Figure 2.5: The ratio of the number of channel uses with the proposed algorithm to that with plain routing (solid line) is always less than or equal to 0.75 (dashed line).

in the N node group transmit -1. At this stage, the number of minority nodes in each group is known. If there are no or two minority nodes in the two node group, they are determined at this stage. If there is one minority node in the two node group, another two transmissions are needed. Therefore on average one channel use is required for the two node group in Step 3. Since we have assumed the N node group can complete transmissions in $0.75 \times 2N$ channel uses, $N + 2$ nodes require $2 + 1 + 0.75 \times 2N = 0.75 \times 2(N + 2)$ channel uses. Thus $N + 2$ nodes require on average at most 0.75 of channels uses needed with plain routing. $\square$

Fig. 2.5 shows the channel use ratio of the proposed algorithm to plain routing for up to 40 nodes. Each point was generated by averaging the number of channel uses over 6×10^5 runs. The ratio is always less that 0.75, confirming the result above.

2.3 Performance Analysis

We now analyze the error performance of our proposed scheme. The channels between the source nodes and relay are assumed to be additive white Gaussian noise (AWGN)

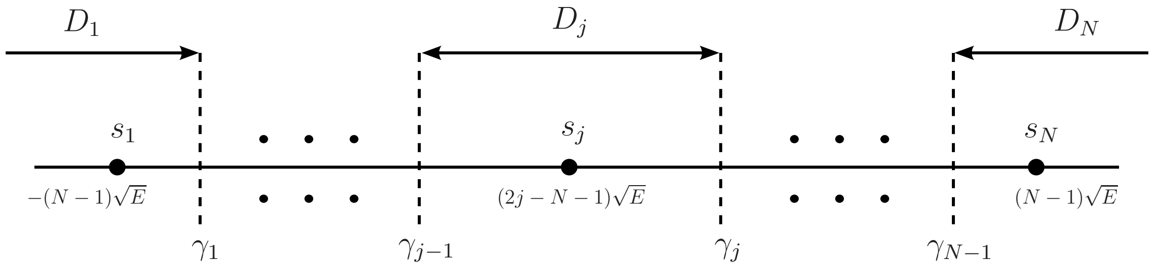


Figure 2.6: Decision regions for an N-PAM signal constellation.

with power spectral density (PSD) $N_0/2$, and the channels have the same coefficients and are symmetric. If this is not the case, pre-equalization can be performed before transmission by providing channel state information at the transmitter (CSIT) as in [25]. The algorithm has three steps with error probabilities P_{e1}, P_{e2} and P_{e3}, respectively. The probability that a node receives the symbol of at least one other node in error is

$$P_e = 1 - (1 - P_{e1})(1 - P_{e2})(1 - P_{e3}). \tag{2.4}$$

For N nodes, the probability of error in Step 1 (the uplink step), P_{e1}, is the error probability of $(N+1)$-PAM modulation with unequal symbol probabilities, which is always lower than $(N+1)$-PAM modulation with equiprobable symbols. To prove this, we derive the symbol error rate (SER) of N-PAM modulation with unequal symbol probabilities. The signal space is shown in Fig. 2.6 where E is the BPSK symbol energy and symbol s_j has probability p_j for $j = 1, \ldots, N$.

Applying the maximum a posteriori probability (MAP) rule [26, Ch. 4], the optimal decision region boundaries are

$$\gamma_j = \frac{-N_0}{4\sqrt{E}} \ln \frac{p_{j+1}}{p_j} - (N - 2j)\sqrt{E} \quad 1 \leqslant j \leqslant N - 1. \tag{2.5}$$

When the received signal is between γ_{j-1} and γ_j, $j = 1, \ldots N$, we declare it to be s_j, where $\gamma_0 = -\infty$ and $\gamma_N = \infty$. To calculate the SER, note that there are $N - 2$ inner points and 2 outer points in the constellation. The error probabilities for the outer and inner points are

$$\begin{aligned}P_{e,outer} =& p_1 \left[\Pr\left(n < -(N-1)\sqrt{E} - \gamma_1\right)\right] \\ &+ p_N \left[\Pr\left(n > (N-1)\sqrt{E} - \gamma_{N-1}\right)\right] \\ =& p_1 \cdot Q\left(\frac{(N-1)\sqrt{E} + \gamma_1}{\sqrt{N_0/2}}\right) \\ &+ p_N \cdot Q\left(\frac{(N-1)\sqrt{E} - \gamma_{N-1}}{\sqrt{N_0/2}}\right), \end{aligned} \tag{2.6}$$

and

$$\begin{aligned}P_{e,inner} =& \sum_{j=2}^{N-1} p_j \left[\Pr\left(n < \gamma_{j-1} - (2j-N-1)\sqrt{E}\right)\right. \\ &\left. + \Pr\left(n > \gamma_j - (2j-N-1)\sqrt{E}\right)\right] \\ =& \sum_{j=2}^{N-1} p_j \left[Q\left(\frac{-\gamma_{j-1} + (2j-N-1)\sqrt{E}}{\sqrt{N_0/2}}\right)\right. \\ &\left. + Q\left(\frac{\gamma_j - (2j-N-1)\sqrt{E}}{\sqrt{N_0/2}}\right)\right], \end{aligned} \tag{2.7}$$

respectively, where n corresponds to the AWGN with variance $N_0/2$, and $Q(x) = (1/\sqrt{2\pi})\int_x^\infty \exp(-t^2/2)dt$. Therefore $P_{e,1} = P_{e,outer} + P_{e,inner}$. In the proposed algorithm, the probability distribution of the symbols is $p_j = \binom{N-1}{j-1}/2^{N-1}$ for $j = 1, \ldots, N$.

For Step 2 (the downlink step), since each user receives the same signal as the relay in Step 1, the probability of error P_{e2} is the same as in Step 1. Therefore (2.4) can be approximated as $P_e = 2P_{e1} + P_{e3}$ if higher order terms are ignored. In Step 3, only the minority nodes (at most half of the total number of nodes) will transmit, thus the error probability will be much lower than in the previous steps and can be ignored. This was confirmed analytically for three values of N as shown in Fig. 2.7, where the signal to noise ratio (SNR) is the ratio of the average bit energy to N_0. Consequently, the error performance is well approximated by $P_e = 2P_{e1}$. Fig. 2.8 shows this approximation for various numbers of nodes. For comparison purposes, the SER with equiprobable PAM and BPSK modulation is also shown.

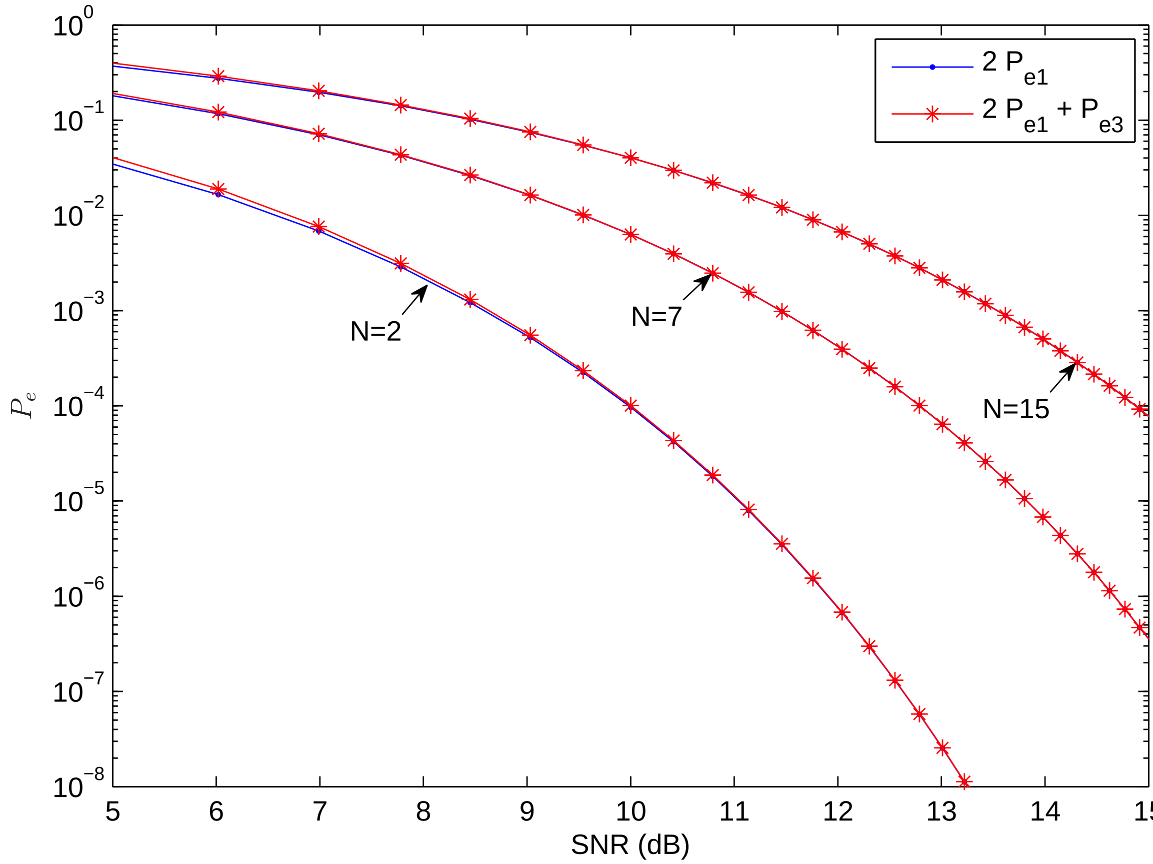


Figure 2.7: The error probability P_e with and without Step 3.

2.4 Chapter Summary

In this chapter, full data exchange in a multi-way relay channel was considered. An algorithm was proposed which provides a throughput of $\frac{1}{1.5N}$ sym/S/CU, a 33% improvement over plain routing. This shows that physical-layer network coding can also be beneficial in systems with more than two source nodes. Besides having low complexity, this algorithm can easily be scaled to higher numbers of nodes. It can also be employed with QPSK modulation, which also provides a 33% gain. This is achieved by separately (and concurrently) dealing with the in-phase and quadrature components of QPSK symbols. For higher order modulations, since it is not straightforward to define minority nodes as done in this chapter, the proposed approach is not directly applicable.

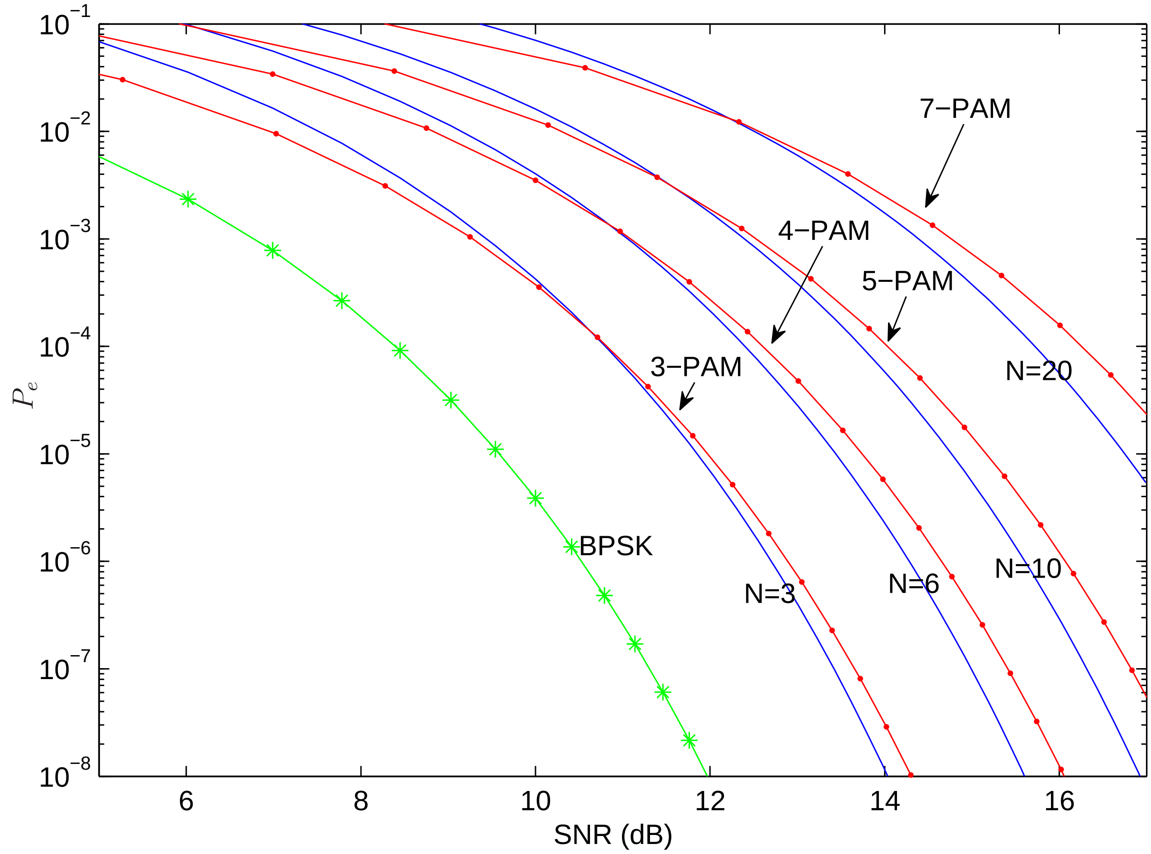


Figure 2.8: The performance of the proposed algorithm, N-PAM and BPSK for different numbers of nodes.

Chapter 3

QAM Constellation Design for Complex Field Network Coding in Multi-way Relay Channels

The algorithm proposed in the previous chapter achieves a 33% gain over plain routing. To increase this gain, in this chapter we consider the design of a transmission scheme based on complex field network coding (CFNC) [22]. CFNC provides a throughput of 1/2 (Sym/U/CU) for full data exchange in a multi-way relay channel. This gain is achieved by using precoding vectors to separate the different combinations of user symbols in the signal space, thereby making them distinguishable at the relay. The focus in this chapter is on the design of precoding vectors for a multi-way relay channel such that rectangular QAM signal constellations are received at the relay. By imposing the condition that QAM constellations are received at the relay, the performance of the CFNC-based system will be superior to that of [22] in AWGN channels, because in the work presented here, the minimum Euclidean distance between constellation points will be smaller. The precoding vectors introduced here have the flexibility to accommodate users employing different signal constellations, and also allow users to join or leave the network at any time. The results of this chapter appear in [27].

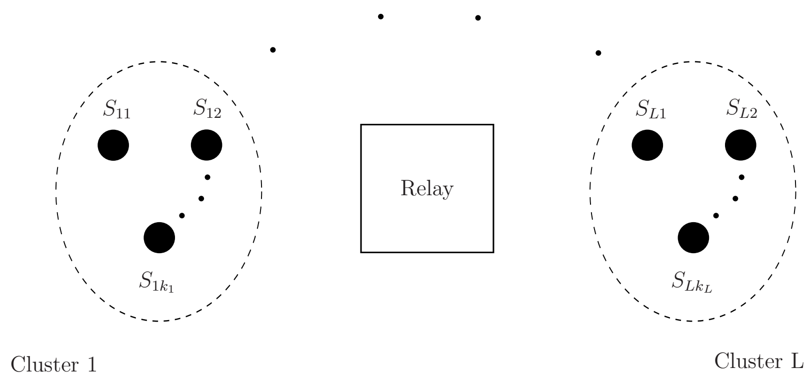


Figure 3.1: The multi-way relay channel model with L clusters.

3.1 System Model

We consider a multi-way relay channel as shown in Fig. 3.1 in which communications between users is enabled through a relay node. There are L clusters of users each containing k_l users $l = 1, \ldots, L$. Each user is interested in the information of the users in its own cluster. Note that the locations of the users in a cluster is arbitrary and the grouping in Fig. 3.1 is only a possible representation. It is assumed that the users cannot overhear transmissions from other users. The transmissions are half-duplex, i.e., communication cannot occur simultaneously in both directions. As with all cooperative relay networks, time synchronization is required. This can be achieved via techniques originally developed for MIMO systems [24, Ch. 11].

3.2 Full Data Exchange Algorithm

Here we focus on information exchange between users in a single cluster. We suppose there are K users in this cluster, each using M-QAM modulation for transmission. All users transmit their symbols s_i simultaneously. Due to the superposition of electromagnetic waves, the relay receives the sum of the transmitted signals. The relay broadcasts a signal to the users using the decode-and-forward (DAF) protocol. To ensure unique decodability of the symbols of every user and achieve a throughput of 1/2 (Sym/U/CU), i.e., one uplink and one downlink transmission, the relay must be able to distinguish between the M^K possible constellation points. This can be achieved by multiplying the vector $S = (s_1, \ldots, s_K)^T$ by a suitable precoding vector $\Phi^T = (\phi_1, \ldots, \phi_K)$ where the elements of S are the symbols of the users. The precoding vector has a major impact on decoding performance at the relay and thus the overall symbol error probability (SEP). One approach to designing the precoding vector is based on algebraic number theory [22]. In this approach, no structure is imposed on the M^K constellation points at the relay. This may lead to small distances between constellation points resulting in performance degradation. Here we propose a precoding vector that preserves the minimum distance between constellation points at the relay.

We first assume the K users transmit BPSK symbols. The generalization to M-QAM will be presented later. If the 2^K different combinations of symbols from the K users are the rows of a matrix A, this will be a $2^K \times K$ matrix containing (1)s and (-1)s. The set of all possible received points at the relay is a $2^K \times 1$ vector b given by

$$b_{(2^K \times 1)} = A_{(2^K \times K)} \Phi_{(K \times 1)} \tag{3.1}$$

where b is predetermined according to the constellation desired at the relay. Equation (3.1) is an overdetermined system of linear equations which may not have a solution in general, but it has a unique solution in our case since we always have $\text{rank}(A) = K$. The only case where a unique solution exists is when $\text{rank}(A) = \text{rank}([A\ b])$, where $[A\ b]$ is the augmented matrix obtained by appending the columns of A and b. This occurs when b represents the points of a rectangular 2^K-QAM constellation.

The average transmitted power of a rectangular QAM constellation is only slightly greater than that of an optimal QAM constellation and it is easier to demodulate [26, Ch. 4]. Thus the constellation received at the relay is assumed to be rectangular M-QAM with $M = 2^K$, and $2^K = 2^{K_1} \times 2^{K_2}$ where K_1 and K_2 are the number of inphase and quadrature components, respectively. If b represents a 2^K-QAM constellation, it can be shown that a vector Φ always exists that satisfies (3.1).

Proposition 1: *For K users each employing BPSK, the precoding vector $\Phi = (1, \ldots, 2^{K_1-1}, j, \ldots, j2^{K_2-1})^T$ satisfies $b = A\Phi$ where b represents the points of a rectangular 2^K-QAM constellation and $K_1 + K_2 = K$.*

Proof. Since a rectangular QAM constellation can be constructed using two PAM signal sets, it is sufficient to consider the real elements of Φ. The proof is by induction. Consider the case with 2 users, so that

$$A = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}^T,$$

and $\Phi = (1,\, 2)^T$, which gives $b = (-3,\, -1,\, 1,\, 3)^T$, thus the proposition holds. In the inductive step we assume the proposition holds for $\Phi = (1, \ldots, 2^n)^T$, i.e., $b = (L\ R)^T$ with a distance of 2 between adjacent points where $L = (-2^{n+1} + 1, \ldots, -1)$ and $R = (1, \ldots, 2^{n+1} - 1)$ are the negative and positive parts of b, respectively . If 2^{n+1} is added to Φ, i.e., now $\Phi = (1, \ldots, 2^n, 2^{n+1})^T$, the newly generated vector $\hat{b}$ can be written as

$\hat{b} = (L_{out},\, L_{in},\, R_{in},\, R_{out})^T$ with

$$\begin{aligned} R_{out} &= 2^{n+1} \cdot\!+ R \\ R_{in} &= 2^{n+1} \cdot\!+ L \\ L_{in} &= -2^{n+1} \cdot\!+ R \\ L_{out} &= -2^{n+1} \cdot\!+ L \end{aligned} \tag{3.2}$$

where $\cdot+$ is element-wise addition. It can be readily verified that $R_{in} = R$, $L_{in} = L$, R_{out} is the shifted version of R by 2^{n+1}, and L_{out} is the shifted version of L by -2^{n+1}. Since none of these four parts overlap, the distance between adjacent points is preserved in the new vector $\hat{b}$. □

Consider a K user relay system with each user transmitting information using BPSK, and a precoding vector Φ constructed according to Proposition 1. The signal received by the relay is from a 2^K-QAM constellation from which the symbols of each user can be derived. The relay broadcasts the decoded constellation point to the users. By decoding the signal sent from the relay, each user can obtain the information of all other users.

3.2.1 Higher Order Modulation

In this section, the solution for BPSK user modulation is generalized to M-QAM modulation. This can be accomplished by first generalizing BPSK to QPSK and M-PAM separately, and combining the results to give M-QAM.

Suppose K users use QPSK modulation to communicate. The relay must then distinguish between 4^K different combinations of user symbols. In this case, the number of different combinations is equivalent to that for $2K$ users communicating with BPSK. For these $2K$ users, the matrix $A = (a_1, \ldots, a_{2K})$ has size $2^{2K} \times 2K$ where a_i for $i = 1, \ldots, 2K$ are the columns of A, and the precoding vector $\Phi = (1, \ldots, 2^{K-1}, j, \ldots, j2^{K-1})^T$ has size $2K \times 1$. Combining pairs of columns of A gives a matrix $A_Q = (a_{1q}, \ldots, a_{Kq})$ where $a_{iq} = a_{2i-1} + ja_{2i}$ for $i = 1, \ldots, K$ are the columns of A_Q. The rows of A_Q contain all combinations of K-user QPSK symbols. The corresponding precoding vector Φ_Q for K users employing QPSK can be obtained by choosing the first K elements of Φ. Consequently $A_Q\,\Phi_Q$ generates 4^K distinguishable points in a square 4^K-QAM constellation.

The following propositions provide the precoding vectors for the cases where users employ M-PAM and M-QAM modulation.

Proposition 2: *For K users each employing M-PAM, the precoding vector $\Phi = (1, M, \ldots, M^{K_1-1}, j, jM, \ldots, jM^{K_2-1})^T$ satisfies $b = A\Phi$ where $K_1 + K_2 = K$ and b represents the points of a rectangular $(M^{K_1} \times M^{K_2})$-QAM constellation.*

Proof. See Appendix 3.A. □

Proposition 3: *For K users each employing M-QAM where $M = M_1 \times M_2$ and $(M_1 < M_2)$, the precoding vector $\Phi = (1, M_2, \ldots, M_2^{K-1})^T$ satisfies $b = A\Phi$ where b represents the points of a rectangular $(M_1^K \times M_2^K)$-QAM constellation.*

Proof. Using an approach similar to that for extending BPSK to QPSK, it is straightforward to generalize PAM to QAM modulation. □

3.3 Performance Analysis

In this section, the performance of the K-user one relay system with full data exchange is evaluated. It is assumed that white Gaussian noise with variance $\frac{N_0}{2}$ is also present at the relay in the uplink and at each user receiver in the downlink. We first evaluate the case where four users, each using BPSK, exchange data through a relay, and then generalize the results to K users, each using M-QAM.

3.3.1 Four Users Using BPSK

For four users using BPSK with $\Phi = (1,\, 2,\, j,\, 2j)^T$, the signal space at the relay is shown in Fig. 3.2.

The relay may decode the received symbol or simply amplify and forward it back to the users. Here we focus on the former approach. With decode-and-forward (DF), the signal transmitted by the relay depends on the received constellation point.

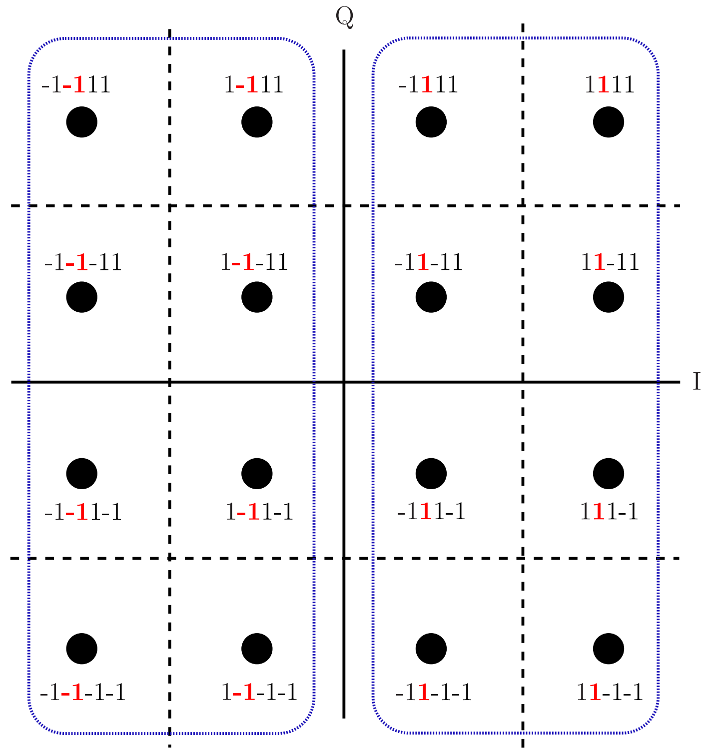


Figure 3.2: The signal space and the corresponding symbol map at the relay for 4-user BPSK.

Therefore a symbol error occurs if there is an error in the uplink or an error in the downlink when the uplink signal is correct (ignoring the case that an error in the uplink is followed by an error in the downlink that corrects the uplink error). Thus the probability that a node receives the symbol of at least one other node in error is $P_{e,tot} = P_{e,U} + (1 - P_{e,U})P_{e,D} \simeq P_{e,U} + P_{e,D}$ where $P_{e,U}$ and $P_{e,D}$ are SEPs of the uplink and downlink, respectively. Since the constellation at the relay and at each node is the same, $P_{e,U} = P_{e,D}$ assuming symmetric channels. With four users, $P_{e,D} = P_{e,16-QAM}$ and therefore $P_{e,tot} \simeq 2P_{e,16-QAM}$. To obtain the symbol error probabilities (SEPs), a nearest-neighbor approximation is employed which gives [26, Ch. 4]

$$P_{e,16-QAM} \approx 3Q\left(\sqrt{\frac{4E_b}{5N_0}}\right),$$

where $Q(x) = (1/\sqrt{2\pi})\int_x^\infty \exp(-t^2/2)dt$ and E_b is the energy per bit. The nearest neighbours of a given constellation point are defined as the points with minimum Euclidean distance from that point.

To determine the probability that a node receives the symbol from one of the other nodes in error, consider Fig. 3.2. Without loss of generality, consider the second user symbol (second from the left shown in bold). This symbol is received in error only if the signal value crosses the boundary between the two regions (the Q axis). There are 4 nearest-neighbor symbol pairs at this boundary. The total number of nearest-neighbor symbol pairs in this constellation is 24, so the probability that the other nodes receive the symbol of the second user erroneously is $P_{e,s_2} = \frac{4}{24} \times P_{e,tot}$. A similar analysis gives $P_{e,s_1} = P_{e,s_3} = \frac{1}{2} P_{e,tot}$ and $P_{e,s_4} = P_{e,s_2} = \frac{1}{6} P_{e,tot}$, where $P_{e,tot} \simeq 2P_{e,16-QAM}$ and P_{e,s_1}, P_{e,s_3} and P_{e,s_4} are the first, third and fourth user SEPs, respectively, at the other nodes. Note that users which are assigned larger precoding values (users 2 and 4 in this example), experience lower symbol error probabilities than the other users.

3.3.2 K Users Each Employing M-PAM

In the general case where K users each employing M-PAM modulation are communicating through a relay, if the precoding vector is chosen to be $\Phi = (1, M, \ldots, M^{K_1-1}, j, jM, \ldots, jM^{K_2-1})^T$ where $K_1 + K_2 = K$. A symbol from an $(M^{K_1} \times M^{K_2})$-QAM constellation is then received at the relay, and this symbol is transmitted back to the users. To determine the symbol error probability of user $i \in \{1, \ldots, K_1\}$ at the other nodes, the number of nearest-neighbor symbol pairs and the number of nearest-neighbor symbol pairs in which the symbol of user i changes, must to be calculated. In an $(M^{K_1} \times M^{K_2})$-QAM constellation, there are $(M^{K_2} - 1)M^{K_1}$ and $(M^{K_1} - 1)M^{K_2}$ nearest neighbor pairs in the inphase and quadrature directions, respectively, giving a total of $2M^K - M^{K_1} - M^{K_2}$ symbol pairs. Let S_i, $i \in \{1, \ldots, K_1\}$, be the symbol of the ith user with real precoding coefficient M^{i-1}, and S'_l, $l \in \{1, \ldots, K_2\}$, be the symbol of the lth user with imaginary precoding coefficient jM^{l-1}. Symbol S_i only changes every M^{i-1} symbol points in the inphase direction, therefore there are only $M^{K_2} \lfloor \frac{M^{K_1}-1}{M^{i-1}} \rfloor$ nearest-neighbor symbol changes in this direction. Similarly, for symbol S'_l there are $M^{K_1} \lfloor \frac{M^{K_2}-1}{M^{l-1}} \rfloor$ nearest neighbor symbol changes in the quadrature direction. Consequently, the symbol error probability for the ith user with real coefficient M^{i-1} at the other nodes is

$$P_{e,s_i} = \frac{M^{K_2} \left\lfloor \frac{M^{K_1}-1}{M^{i-1}} \right\rfloor}{2M^K - M^{K_1} - M^{K_2}} P_{e,tot}, \quad i = 1, \ldots, K_1, \tag{3.3}$$

and the symbol error probability for the lth user with imaginary coefficient M^{l-1} at the other nodes is

$$P_{e,s'_l} = \frac{M^{K_1}\left\lfloor \frac{M^{K_2}-1}{M^{l-1}} \right\rfloor}{2M^K - M^{K_1} - M^{K_2}} P_{e,tot}, \quad l = 1, \ldots, K_2, \tag{3.4}$$

where $P_{e,tot} \approx 2P_{e,(M^{K_1}\times M^{K_2})-QAM}$ and

$$P_{e,(M^{K_1}\times M^{K_2})-QAM} = \frac{N_{d_{min}}}{M^K} Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right),$$

where d_{min} is the minimum distance between the signal points and $N_{d_{min}}$ is the total number of nearest neighbors of all signal points in the constellation [26, Ch. 4].

For K users each employing M-QAM with $M = M_1 \times M_2$ and $(M_1 < M_2)$, the precoding vector $\Phi = (1, M_2, \ldots, M_2^{K-1})^T$ is assigned. Using the same arguments as above, the total number of nearest-neighbor symbol pairs in an $(M_1^K \times M_2^K)$-QAM constellation is given by $2M^K - M_1^K - M_2^K$. A change in symbol S_i with precoding value M_2^{i-1}, will result in a symbol at least M_2^{i-1} constellation points away from S_i either in the inphase or quadrature directions. Therefore a symbol change occurs in $M_2^K \lfloor \frac{M_1^K-1}{M_2^{i-1}} \rfloor$ places in the inphase direction and $M_1^K \lfloor \frac{M_2^K-1}{M_2^{i-1}} \rfloor$ places in the quadrature direction. As a result, the symbol error probability of each user at the other nodes is given by

$$P_{e,s_i} = \frac{M_2^K\left\lfloor \frac{M_1^K-1}{M_2^{i-1}} \right\rfloor + M_1^K\left\lfloor \frac{M_2^K-1}{M_2^{i-1}} \right\rfloor}{2M^K - M_1^K - M_2^K} P_{e,tot}, \quad i = 1, \ldots, K, \tag{3.5}$$

where $P_{e,tot} \approx 2P_{e,M^K-QAM}$.

Fig. 3.3 presents the symbol error probability from (3.3) and (3.4) for 4 users using BPSK and 4-PAM constellations. This shows that in both cases, users with larger precoding values experience lower error probabilities. Fig. 3.4 depicts the SEP of each user for the 4-user, 1-relay case when the users employ QPSK. In Fig. 3.5, the SEP from (3.5) is shown for an increasing number of users. For clarity, only users with the worst and best SEPs are shown in each case.

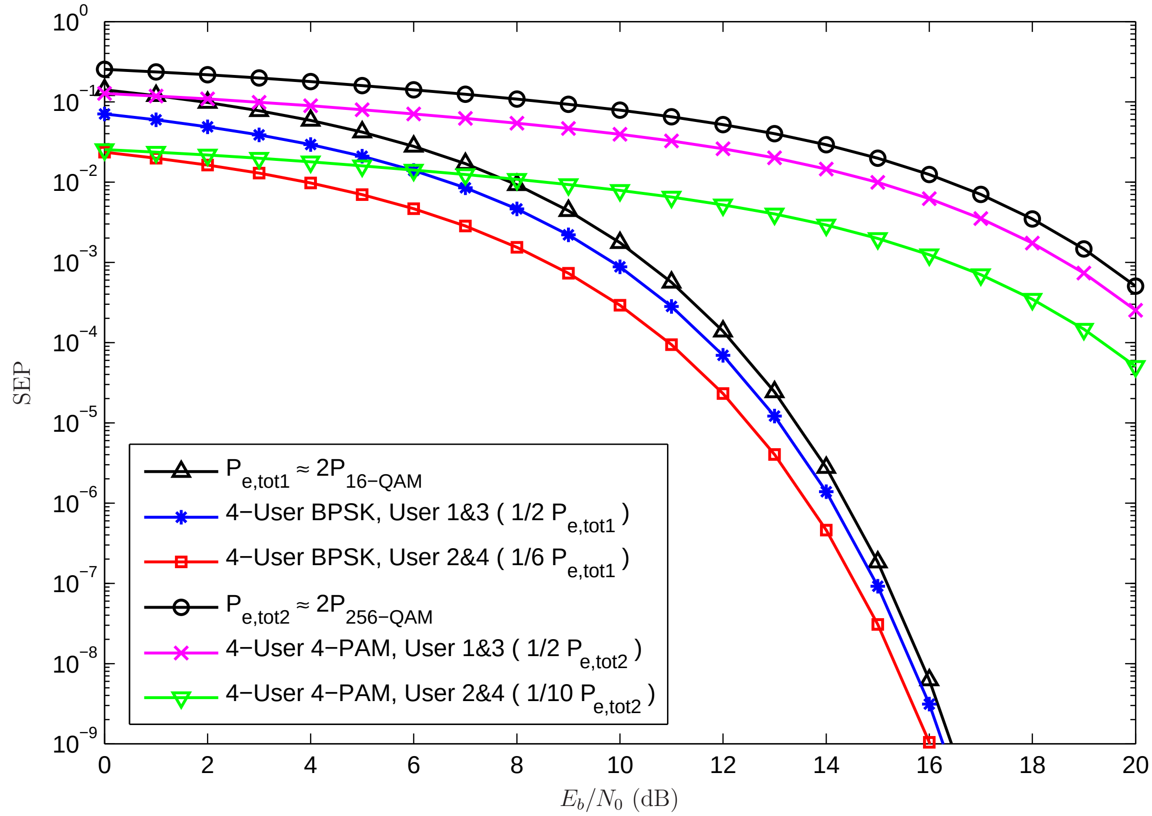


Figure 3.3: SEP for four users employing BPSK and 4-PAM constellations with one relay.

3.4 Multiple Clusters

Consider the system model in Fig. 3.1, where L clusters each have K_l nodes. Each cluster can be assigned a specific precoding vector and the clusters can sequentially communicate with the relay. In this way the throughput of each cluster is 1/2 sym/U/CU and the throughput of all clusters is $1/2L$ sym/U/CU.

In the proposed algorithm, it is not necessary that all users use the same constellation. For example, in a 3-user, 1-relay scenario, users 1 and 2 can employ BPSK while user 3 employs QPSK. With the appropriate precoding vector $\Phi = (1, j, 2)^T$, a square 16-QAM constellation similar to that in Fig. 3.2 will be received at the relay. Following the same procedure as in Section 3.3, the SEPs of users 1, 2 and 3 are $\frac{1}{2}P_{e,tot}$, $\frac{1}{2}P_{e,tot}$ and $\frac{1}{3}P_{e,tot}$, respectively, where $P_{e,tot} \approx 2P_{e,16-QAM}$. In the general case, a suitable precoding vector can be found to form a unique constellation point for every combination of user symbols.

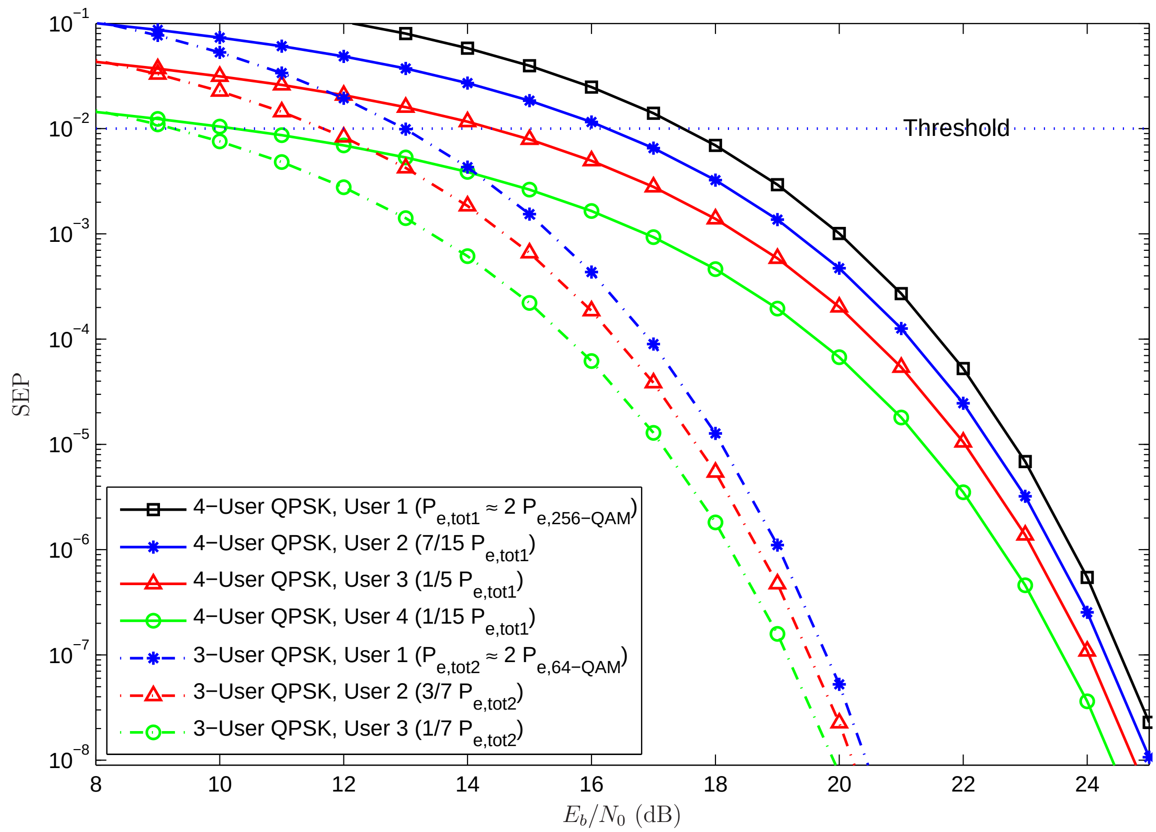


Figure 3.4: SEP for three and four users employing QPSK with one relay. The acceptable SEP threshold for remaining in the network is also shown.

The algorithm offers flexibility in the sense that users can join or leave the network at any time. Considering the 4-user, 1-relay SEPs shown in Fig. 3.4, and suppose the maximum acceptable SEP for communications is 10^{-2}. This imposes a threshold signal to noise ratio (SNR) for each user, e.g., for user 1 this value is 17.5 dB. If the SNR falls below this threshold, user 1 should leave the network. By changing the precoding values from $\Phi = (1, 2, 4, 8)$ to $\Phi' = (0, 1, 2, 4)$ (a zero coefficient denotes that the corresponding user has left the network), the performance of the remaining three users is improved as shown in Fig. 3.4.

3.5 Chapter Summary

In this chapter, a multi-way relay channel in which clusters of users exchange data has been considered. Using complex field network coding (CFNC), a throughput of

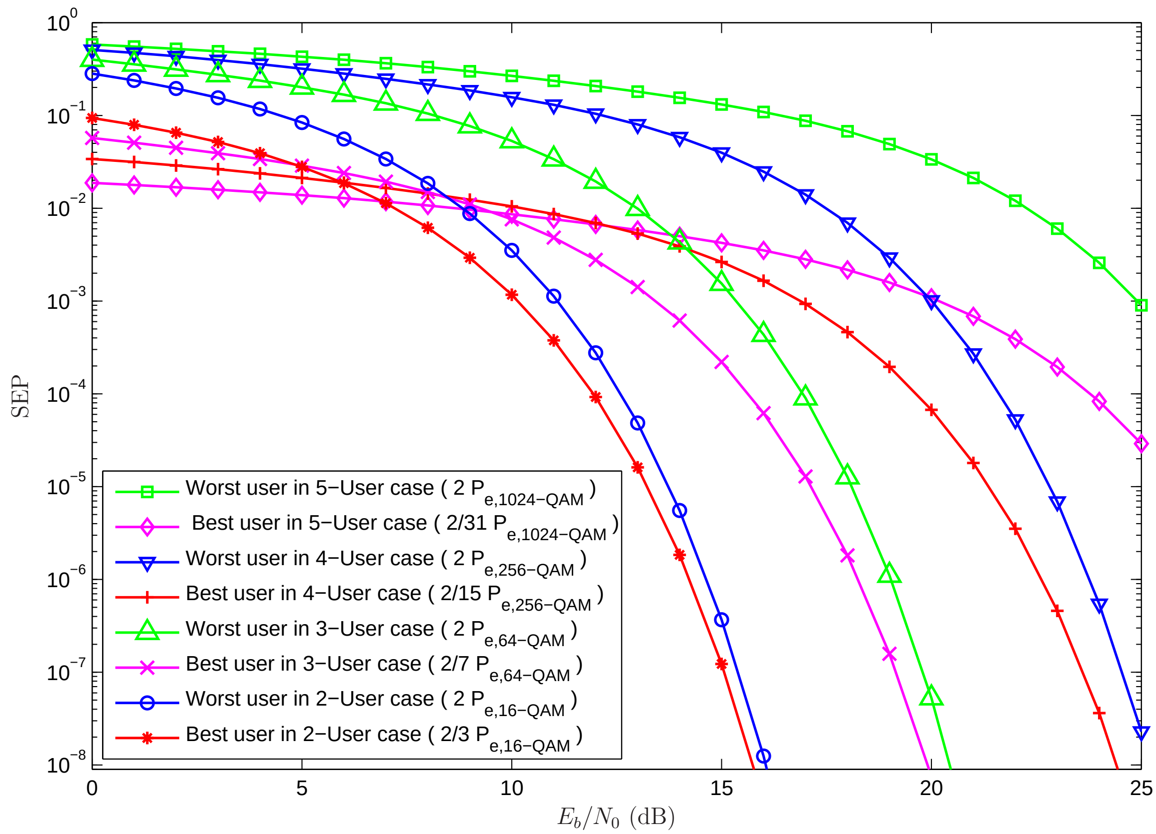


Figure 3.5: SEP for multiple users each employing QPSK with one relay. Only the best and worst user SEPs are shown.

1/2 sym/U/CU can be achieved in each cluster. To implement CFNC, precoding must be employed at each user. The average transmit power of a rectangular QAM constellation is only slightly greater than that of an optimal M-ary QAM constellation, and the corresponding signals are easier to demodulate. Thus a precoding vector was developed to allow a rectangular QAM constellation to be employed by the users in the uplink and at the relay in the downlink. The proposed algorithm for precoding vector design allows users to employ different constellations, and also to join or leave the network as necessary. The performance in AWGN channels was derived. Downlink performance was improved by exploiting user self information and grouping users. It was shown that users with larger precoding values have lower symbol error probabilities at the other nodes.

3.A Proof of Proposition 2

Proof. The proof is by induction. Since a rectangular QAM constellation can be constructed using two PAM signal sets, it is sufficient to consider only the real elements of Φ. We begin with the proof for the 2-user M-PAM case. With two users, each using M-PAM, the precoding vector will be $\Phi = (1, M)^T$ and M^2 points are generated at the relay in the range $-(M^2 - 1)$ to $+(M^2 - 1)$ which has length $2(M^2 - 1)$. We show that employing this vector Φ, the M^2 points will be distinct and equally spaced by a distance of 2. Consider two arbitrarily generated points at the relay, $a_1 + Mb_1$ and $a_2 + Mb_2$, where a_1, a_2, b_1 and b_2 are M-PAM symbols. If these points are equal, then $a_1 + Mb_1 = a_2 + Mb_2$ and rearranging gives $a_1 - a_2 = M(b_2 - b_1)$. Ignoring the trivial solution, since the minimum value of $M(b_2 - b_1)$, which is $2M$, is greater than the maximum value of $a_1 - a_2$, which is $2(M - 1)$, equality cannot occur and therefore the M^2 points are distinct. The distance between the two points is $d = (a_1 - a_2) + M(b_1 - b_2)$. Clearly the minimum value of d is 2, and with M^2 points equally spaced by the minimum distance 2, their range is $2(M^2 - 1)$. This completes the proof for the 2 user case.

As the induction hypothesis, suppose that N users employ M-PAM with the real part of the precoding vector given by $\Phi = (1, M, \ldots, M^{N-1})^T$. Thus there are M^N distinct equally spaced points with distance 2. Adding the $(N + 1)$th user with coefficient M^N increases the number of points to M^{N+1}. Each point in the new constellation can be expressed as $a_i + M^N b_i$ where a_i is a point in the previous constellation and b_i is an M-PAM symbol. Again suppose $a_1 + M^N b_1$, and $a_2 + M^N b_2$ are two arbitrarily generated points in the new constellation. If these points are equal then $a_1 - a_2 = M^N(b_2 - b_1)$. Ignoring the trivial solution, since the minimum value of $M^N(b_2 - b_1)$, which is $2M^N$, is greater than the maximum value of $a_1 - a_2$, which is $2(M^N - 1)$, equality cannot occur and therefore the M^{N+1} points are distinct. The distance between the two points is $d = (a_1 - a_2) + M^N(b_1 - b_2)$. Since we have assumed the N-user M-PAM constellation is equally spaced by 2, the minimum value of d is 2. The M^{N+1} points equally spaced by a distance of 2 cover a range of length $2(M^{N+1} - 1)$, which completes the proof of the induction step. □

Chapter 4

Conclusions and Future Work

4.1 Contributions

The focus of this thesis was on the design of transmission schemes that increase the data exchange throughput in a multi-way relay channel. In the second chapter, an algorithm was designed for full data exchange using binary signaling in a multi-way relay channel. This algorithm provides a throughput gain of 33% over plain routing. This approach is also applicable to users employing QPSK modulation. The results presented show that physical layer network coding can be beneficial in relay channels with more than two users.

In the third chapter, the goal was to increase the modest 33% gain of the approach in Chapter 2 by using the concept of complex field network coding. The main idea in complex field network coding is to make sure all possible combination of user symbols are distinguishable at the relay. To achieve this, symbol precoding must be performed by each user. Due to the desirable properties of QAM constellations, such as ease of decoding and near optimal performance in AWGN channels, precoding vectors were designed such that a QAM constellation is received by the relay. The full data exchange throughput of this scheme is 1/2 sym/U/CU. This increase in throughput is achieved at the expense of system complexity.

4.2 Future Work

The idea of restraining some nodes from transmitting and disseminating information by use of common knowledge is introduced in this work. This approach can be applied to other relay communication scenarios. The performance analysis of the proposed schemes was confined to AWGN channels. It is important to evaluate the error performance for fading channels. Introducing diversity to improve the performance in this case is also an interesting subject for future research. It is important to observe that in both proposed methods the relay does not need to decode the user symbols individually, i.e., decoding the sum of the user symbols is sufficient. In the context of relay communications, the relay decoding a function of the transmitted symbols is called compute-and-forward (CF). The information theoretic properties of CF have recently been analysed by Nazer and Gastpar [28] for the multiple access channel. It is shown that CF has a higher capacity then with the decode-and-forward approach. It will therefore be interesting to study the information theoretic aspects of the multi-way relay channel when the relay performs compute-and-forward.

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