THROUGHPUT OPTIMIZATION IN RETRANSMISSION BASED COMMUNICATION SYSTEMS

by

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M.Sc., Leningrad Electrical Institute of Communication, 1983

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of

DOCTOR OF PHILOSOPHY

in the Department of Electrical and Computer Engineering

We accept this dissertation as conforming to the required standard

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ABSTRACT

Optimization of throughput, in a channel perturbed by random and burst noise, has been extensively studied. Block coding has been chosen as the type of code used in forward error correction.

Constraints on the probability of packet loss and number of transmissions of a single packet have been chosen as the criteria for coding parameter selection for codes to be used in the forward path. Techniques for choosing the optimum code parameters in such situations have been described and throughput performance has been analyzed.

Information contents of feedback signals have been used to quantitively identify the requirements on average sample number for channel state inference. Different types of feedback signals have been analyzed and compared with respect to their information contents. Number of errors occuring in a packet has been shown to be the most informative.

Superiority of the proposed scheme, compared to the traditional acknowledgement or non-acknowledgement based schemes, has been proven by example for both gaussian and fading channels.

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Notation

Abbreviations

ACK	Acknowledgement
ARQ	Automatic Repeat Request
ASG	Average Sample Gain
ASN	Average Sample Number
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
CRC	Cyclic Redundancy Check
DMC	Discrete Memoryless Channel
DPSK	Differential Phase Shift Keying
DQPSK	Differential Quaternary Phase Shift Keying
FEC	Forward Error Correction
FSK	Frequency Shift Keying
GBN	Go-back-N
GF(q)	Galois Field of q elements
HARQ	Hybrid Automatic Repeat Request
MLE	Maximum Likelihood Estimate
NAK	Negative Acknowledgement
PDF	Probability Density Function
PER	Packet Error Rate
PSK	Phase Shift Keying Modulation
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
RS	Reed Solomon Code

RTD	Return Trip Delay
SAW	Stop & Wait
SLR	Sequential Likelihood Ratio
SNR	Signal to Noise Ratio
SR	Selective Repeat
UMVUE	Uniformly Minimum-Variance Unbiased Estimator
	Unbiased Estimator
VA	Viterbi Algorithm
c.d.f.	Cumulative Distribution Function
r.m.s.	Root Mean Square

Notations used

B	Bandwidth, Multipath Spread
C	Channel Capacity
C_{FB}	Channel Capacity with Feedback
D	Round Trip Delay, Statistical Data
T	Statistic
Th	Throughput
V	Score Function
θ	State of Nature
λ	Likelihood Ratio, Wavelength
μ	Population Mean
σ	Population Standard Deviation
σ^2	Population Variance
ν	Median
$\hat{\theta}$	Maximum Likelihood Estimate of θ
d	Sample Standard Deviation
f	Probability Density Function

l	Loss Function, $r(\theta, a)$
n	Packet Length
p	Probability
$\hat{p}$	Observed Proportion
p_b	Bit Error Probability
r	Regret, $r(\theta, a)$
s	Strategy, $s(Z)$
v	Mobile Speed

Acknowledgement

I would like to acknowledge all those who supported and encouraged me to produce this thesis. Deserving of special mention is my supervisor Dr. Vijay K. Bhargava. Without his guidance, inspiration and support this thesis would never have been possible.

I would like to thank my supervisory committee members, Dr. P. Agathoklis, Dr. M. Serra and Dr. W.-S. Lu for their special cooperation and encouragement.

I would also like to thank the members of digital communication group, for encouragement and help with the proof reading of the thesis.

I am grateful to Natural Science and Engineering Research Council of Canada for funding me with the postgraduate scholarship, and to the University of Victoria for the President's Research Scholarship.

Dedication

To

Ornob,

Usama,

Noel

and

Nargis

Chapter 1

Introduction

1.1 Introduction

Error control coding is an area of increasing importance in digital communications. Communication system designers must cope with the ever-increasing demand for digital communication services. The data link protocol used for providing an error-free link between two communication nodes must use sophisticated error control techniques. Automatic Repeat Request (ARQ) schemes improve the reliability of data transmitted, by repeating the same information a number of times. In very poor channel conditions, simple ARQ schemes cannot cope with the channel degradation. Hybrid Automatic Repeat Request (HARQ) schemes make use of both error detection and error correction coding to improve throughput and undetected error probabilities.

In future, demand for mobile communication will become very high. Therefore, future use of allocated spectrum should be as efficient as possible. Sophisticated cell design, channel allocation, and bandwidth efficient coding, will all be integrated into the system design. Transmission of delay limited data with maximum throughput efficiency is a requirement for efficient and reliable data transmission. Only with proper error correction coding, and retransmission strategy, the goal can be achieved.

In a mobile environment, the transmission between the base station and the mobile terminal is impaired by signal shadowing and multipath fading. This causes the transmission quality to vary with time. This puts stronger demands on the error control process which cannot be effectively satisfied with non-adaptive techniques. All error control techniques require transmission of extra symbols along with the information symbols. Amount of redundancy in the transmission increases with the increase of channel symbol error probability. In a non-adaptive system, during good channel states, the redundancy will be wasted. This will reduce the system throughput. Whereas, if the system is adaptive and the error control scheme adaptively matches the channel condition, no redundancy will be wasted and the system will have the maximum throughput possible under the given channel condition.

For error correction in the transmitted block, both convolutional and block codes have been used successfully. Both types of codes can be chosen to match the channel state. Reliability can be achieved by retransmission of a block when the receiver fails to recover the transmitted symbols. Code combining can further enhance the reliability and throughput performance of the system.

1.2 Previous Results

Adaptive error control techniques in communication have been studied in [91], [90], [89], [95], [34], [74], [94], [64], [43], [92], [82], [76], [86], [55], [54] [98]. In all the studies, it was shown that, system throughput of an adaptive scheme is never worse than a non-adaptive one.

Mandelbaum in [65] introduced the idea of adaptive feedback coding scheme using incremental redundancy.

Idea of using sequential likelihood ratio testing for channel state inference was introduced in [75]. It was shown to be effective means to estimate channel state with any predefined amount of reliability.

Optimal packet length for Automatic Repeat Request (ARQ) schemes was studied in [3]. In [25], optimality was analyzed for Stop-and-Wait schemes. The mathematical model was based on the work of W. W. Chu [16].

Code combining scheme has been studied in [13], [12], [49], [48]. In these schemes, previously transmitted but unsuccessful packets are not discarded. All the replicas of a packet are combined to form a codeword with more error correction capability.

Concatenated coding schemes have been used with Reed-Solomon (RS) outer code and convolutional or block code as inner code [46], [21], [32]. Several features of RS codes make them suitable for use as the outer code. Among them are, the maximum distance separability, availability over a wide range of block lengths, symbol sizes and code lengths and existence of efficient decoding algorithm for both errors and erasures.

In [15], trellis code has been used as the inner code.

1.3 Objectives

The main objective of this work is to explore means of improving the throughput performance of a retransmission based communication system. The fields of investigation are:

- selection of optimum code and its parameters,
- use of code combining for better throughput performance,
- selection of type of feedback signals,
- use of concatenated coding, and
- finding means of reduction of sample number for the purpose of state estimation in adaptive error control coding schemes.

The motif of this research is to find ways to improve the performance of error control schemes compared to the work already done and mentioned above.

1.4 Outline of The Thesis

This thesis is divided into six chapters including the Introduction and Conclusions.

The second chapter describes the fundamental concepts of error control in communication systems. It introduces to the idea of hybrid and adaptive error control schemes.

The third chapter describes different error control schemes and analyzes performance of different schemes in gaussian as well as Rayleigh environments. Extensive simulation of receiver buffer is performed and the effect of limited size buffer is analyzed. A novel concatenated code combining scheme is proposed and its performance is analyzed.

The fourth chapter introduces the idea of information content of different kinds of feedback signals and compares their performance with regard to the achievable throughput in different environments. Different channel state inference methods are presented here.

The fifth chapter presents performance of adaptive error control methods in both fading and random noise channels. Throughput performances of traditional acknowledgement (ACK) based feedback signals and error count based signals are compared. Error count based schemes are shown to be superior to the ACK based schemes.

The sixth chapter concludes the thesis with a short summary of the results of the work presented in the thesis.

Chapter 2

Fundamentals of Error Control in Communication

In the fundamental sense, communication involves implicitly the transmission of *information* from one point to another. This transmission of information can take various forms. But irrespective of the particular form, there are three basic elements in every communication system, namely, the *transmitter*, the *channel* and the *receiver*. The purpose of the transmitter is to transfer the signal generated by the information *source* into a form suitable for transmission over the *channel*. While propagating through the channel, the signal is distorted due to channel nonlinearity and noise and interference signals are added to it. The *receiver* reconstructs the received signal and delivers it to the *destination*. The basic system is shown in fig. 2.1. A communication system

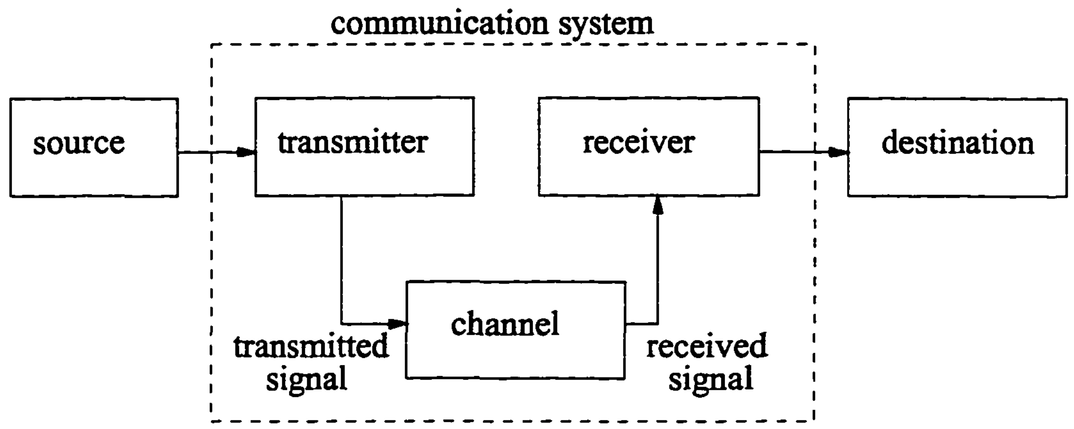


Figure 2.1. *Elements of communication system.*

can roughly be classified into three main categories: discrete, analog and mixed. In a discrete system both the message and signal are sequences of discrete symbols. In analog systems, both are continuous functions of time, and in mixed system both types exist. In analog systems, no significant effort is made by the designer to tailor the signal waveform to suit the channel. In discrete systems, a finite set of waveforms, matched to the channel characteristics, are used for transmission of source generated signals. The use of digital communications provides the capability of both *efficient* and *reliable* information transmission. Therefore, from now on, our discussion will focus mainly on discrete systems.

2.1 Discrete Noiseless Channel

A discrete source generates a message symbol by symbol. The generation of successive symbols depends, in general, on preceding choices as well as the particular symbols in question [79]. Therefore, a discrete source can be represented by a stochastic process. This type of discrete stochastic process is also known as Markov Process and can be represented graphically by a number of states $S_j (j = 1, 2, \cdots, n)$ and a set of transition probabilities, $p_{ij} (i = 1, \cdots, n; j = 1, \cdots, n)$, among the states.

Similar to the discrete source, a channel can also be represented by a set of input symbols, a set of output symbols and a set of transition probabilities between the input and the output symbols. A discrete memoryless channel (DMC) is a discrete channel in which each symbol of the output sequence is statistically dependent only on the corresponding symbol of the input sequence [81]. A DMC is specified by its set of transition probabilities $P(j|k) (k \leq r,\ j \leq s)$, where $P(j|k)$ is the probability of receiving symbol j when k was transmitted. If corresponding to the set of input symbols $(x_1, x_2, \cdots, x_N)$, the set of output symbols is $(y_1, y_2, \cdots, y_N)$, then,

$$Pr(\mathbf{y}|\mathbf{x}) = \prod_{n=1}^{N} P(y_n|x_n). \tag{2.1}$$

In [66], a DMC is defined as a channel that accepts every given interval of time, one of r input symbols and in response expels one of s output symbols, where r and s are both finite numbers. The block diagram of a DMC with r inputs and s outputs is shown in fig. 2.2.

When the number of channel symbols is two and the probability of correct reception of a symbol is the same for both of them, the channel is called a Binary Symmetric Channel (BSC). A BSC is represented by a single parameter p, the transition probability. This is a nonnegative probability that the output will be different from the input . The transition probability diagram of a BSC is shown in fig. 2.3.

When the numbers r and s are not too big they are represented by a diagram similar to the one given by fig. 2.4. The output is governed by a matrix of transition probabilities, with elements $P(y|x)$, where x and y are the input and the output symbol respectively. Symmetric channels are special cases of a DMC. A q-ary symmetric channel has the property that

$$p(j\,|\,i) = \begin{cases} p & i = j \\ \frac{1-p}{q-1} & i \neq j. \end{cases} \tag{2.2}$$

2.2 Some Channel Models

A communication channel can be modeled as a linear filter [52]. The specification of the channel reduces to the specification of the mean and correlation function of either the impulse response or the received process conditioned upon the transmitted waveform. A channel is said to be a fading channel when the impulse response is randomly time-varying. A few of the channel response models are described below.

2.2.1 Scattering Function Model

In mobile, satellite, and other radio communications, the signal transmitted by the transmitter might reach the receiver via more than a single path. A scattering function

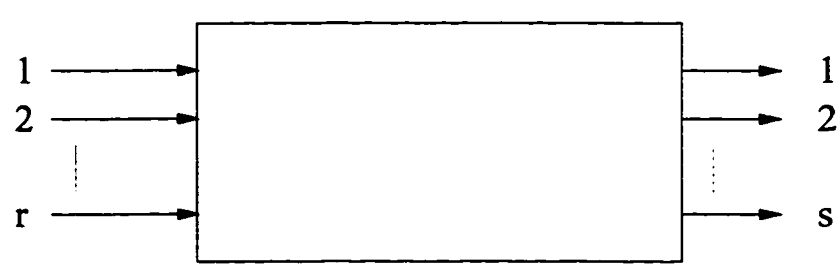


Figure 2.2. *Discrete memoryless channel (DMC).*

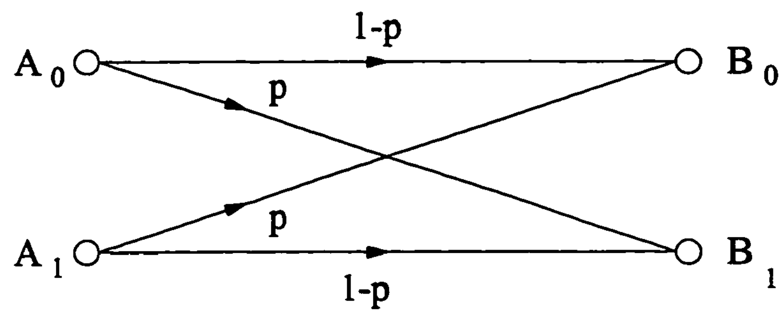


Figure 2.3. *Binary symmetric channel.*

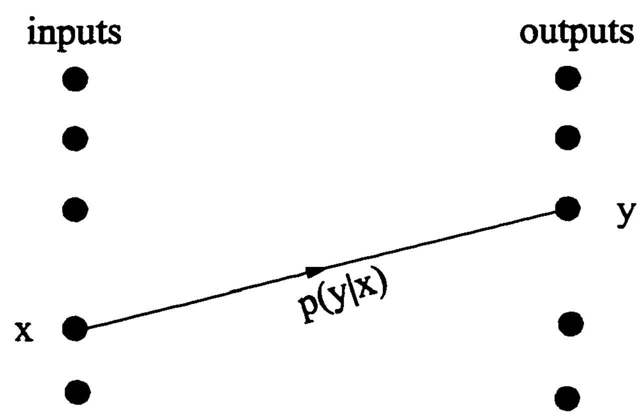


Figure 2.4. *DMC represented by transition probabilities.*

model suits such channels very well. The transmitted signal scattered by different scatterers reach the receiver at different time instants. Each scatterer is described by its reflection energy cross-section, ρ_i^2, and propagation time delay, $T_i(t)$. We denote the channel input and output waveforms by $s(t)$ and $y(t)$, respectively. If $u(t)$ is the complex envelope of $s(t)$, then,

$$s(t) = Re[u(t) \exp\ j\omega_0 t]. \tag{2.3}$$

The average scatterer cross-section is defined as,

$$\bar{\sigma}(r, f) = \sum_i \overline{\rho_i^2}. \tag{2.4}$$

We characterize scattering functions by their Doppler Spread, B, Multi-path Spread.

L, and Total Spread, S, defined as [52],

$$B = \left[\int \sigma^2(f)\, df\right]^{-1}, \tag{2.5}$$

$$L = \left[\int \sigma^2(r)\, dr\right]^{-1}, \tag{2.6}$$

and

$$S = \left[\int\int \sigma^2(r,f)\, dr\, df\right]^{-1}, \tag{2.7}$$

where

$$\sigma(r) = \int \sigma(r,f)\, df. \tag{2.8}$$

The value of $\sigma(r,f)$ may be regarded as a fraction of the scatterer cross section contributed by scatterers in the vicinity of range delay r and Doppler shift f. Thus $\sigma(r)$ is the fraction of cross section contributed by scatterers in the vicinity of r, regardless of their Doppler shifts. Similarly, $\sigma(f)$ is the fraction of the scatterer cross section contributed by scatterers whose Doppler shift is in the vicinity of f, regardless of their range delays. $\sigma(r)$ is called the *delay scattering function* and $\sigma(f)$ the *frequency scattering function.*

The quantity B provides a rough measure of the frequency interval over which the Doppler shifts of the scatterers are spread, whereas L provides a measure of the interval over which their range delays are spread. These quantities are also called *frequency dispersion* and *time dispersion* of the channel.

The duration T, and bandwidth W, of the transmitted waveform are defined as follows [52]:

$$T = \left[\int |u(t)|^4\, dt\right]^{-1}, \tag{2.9}$$

and

$$W = \left[\int |U(f)|^4\, df\right]^{-1}, \tag{2.10}$$

where $U(f)$ is the Fourier transform of $u(t)$, that is,

$$U(f) = \int u(t)\, \exp(-j2\pi ft)\, dt. \tag{2.11}$$

In almost all realistic situations, neither the frequency dispersion B, nor the time dispersion L, vanishes. But there arise situations when one or both of them may be considered *zero* for all practical purposes.

A channel whose scattering function is a unit impulse is a non-dispersive channel; that is,

$$\sigma(r, f) = \delta(r)\, \delta(f). \tag{2.12}$$

This channel is sometimes called *flat-flat fading channel.* This channel is, in essence, random-phase Rayleigh fading channel. The attenuation and phase of the channel output signal are statistically independent random variables that are not functions of time.

A channel is said to dispersive only in time if its scattering function is impulsive in frequency, that is, if

$$\sigma(r, f) = \delta(f)\, \sigma(r). \tag{2.13}$$

Because, the output of these channels do not vary in time, they are also called *time-flat fading channels.* The effect of time dispersion is the attenuation of certain frequency components of the transmitted waveform. This type of fading is called *frequency selective fading.*

We say that a channel is dispersive only in frequency if its scattering function is of the form

$$\sigma(r, f) = \delta(r)\, \sigma(f). \tag{2.14}$$

This kind of channel selectively alters certain time segments of the transmitted waveform. Hence frequency dispersion is sometimes called *time selective fading.*

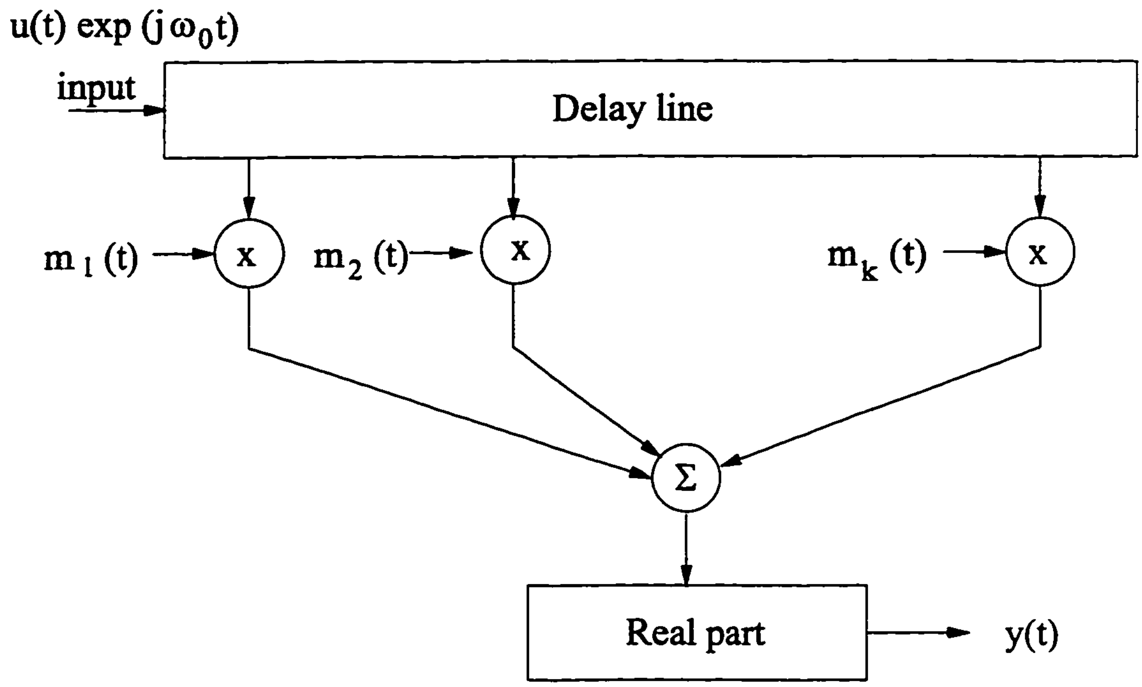


Figure 2.5. *Tapped delay line model of fading channels.*

Channels that are dispersive in both time and frequency are called *doubly dispersive.* Such channels exhibit both time-selective and frequency-selective fading.

2.2.2 Delay-line Model of Fading Channels

In the delay-line model, the received process is expressed in the form

$$y(t) = Re\left[\sum_{i}^{k} m_i(t)\ u(t-\Delta_i)\ \exp\ j\omega_0(t-\Delta_i)\right], \tag{2.15}$$

where $m_i(t)$ are complex functions. Thus the channel may be represented by a tapped delay line as shown in fig. 2.5. As illustrated in the figure, the input is fed into the delay line and the waveform from each tap is multiplied by the complex tap-gain function $m_i(t)$. The resulting waveforms are added and the real part of the result is extracted to obtain the channel output waveform. For fading dispersive channels these tap gain functions are assumed to be zero-mean Gaussian random processes.

As a result, statistical properties of $y(t)$ are completely specified by the correlation function

$$R_y(t,\tau) = \overline{y(t)\, y(\tau)}. \tag{2.16}$$

2.2.3 Statistical Model of Fading Channels

There are several probability distributions that can be considered to model the statistical characteristics of a fading channel. They all depend on the particular situations concerned. Presence of a direct path from the transmitter to the receiver, shadowing due to trees and foliage, and number of scatterers around the receiver all contribute to different statistical models of the fading channel. The frequency-non-selective channel results in multiplicative distortion of the transmitted signal $s(t)$. It is assumed that the multiplicative process remains constant during at least one signaling interval. Therefore, the received equivalent low-pass signal in one signaling interval can be represented by

$$r(t) = \alpha\, e^{-j\phi}\, s(t) + z(t), \quad 0 \le t \le T, \tag{2.17}$$

where $z(t)$ represents the complex-valued gaussian noise process. Different fading models exist which all depend on the distribution of α. We shall discuss here the most commonly used models:

Rayleigh Channel: When there are a large number of scatterers in the channel that contribute to the signal at the receiver, application of the central limit theorem leads to a Gaussian process model for the channel impulse response [33]. If the process is zero-mean, the envelope of the channel response at any time has a Rayleigh probability distribution and the phase is uniformly distributed in the interval $(0, 2\pi)$. Examples of this type of channel is the ionospheric and tropospheric channel and urban mobile channel. In the urban channel, the direct path from the transmitter to the receiver is blocked by man made structures. The Rayleigh probability density function (PDF) is given by

$$p(r) = \frac{2r}{\sigma^2} \exp\left\{-\frac{r^2}{\sigma^2}\right\} \quad \text{for } 0 \le r \le +\infty \tag{2.18}$$

which has the corresponding cumulative distribution function (c.d.f.)

$$p(r \le \mathcal{R}) = \int_0^{\mathcal{R}} p(r)\, dr = 1 - \exp\left\{-\frac{\mathcal{R}^2}{\sigma^2}\right\}. \tag{2.19}$$

This distribution has an r.m.s. value of $\sqrt{r^2} = \sigma$, a mean value $\langle r\rangle_{av} = \frac{\sqrt{\pi}}{2} = 0.886\sigma$ and the most probable value $\frac{\sigma}{\sqrt{2}} = 0.707\sigma$

The normalized correlation coefficient of the signal envelopes at two points separated by a distance, ξ, is given by

$$\rho(\xi) \cong J_0^2(k\xi), \tag{2.20}$$

where $J_0()$ is the zeroth order Bessel function of the first kind. The fraction of time the signal strength remains below a certain level $\mathcal{R}$ is represented by the c.d.f. of the signal strength. The average number of times the signal level crosses a particular level $\mathcal{R}$ is a function of $\mathcal{R}$, speed of the moving vehicle, v, and the carrier frequency of the signal. The level crossing rate for a Rayleigh density function is given by [56]

$$n(\mathcal{R}) = n_0\, n_{\mathcal{R}} \tag{2.21}$$

$$= \frac{\beta v}{\sqrt{2\pi}}\, e^{(-\mathcal{R}^2)}, \tag{2.22}$$

where $\mathcal{R}$ is the envelope of signal strength E w.r.t. its r.m.s. value, i.e. $\mathcal{R} = \frac{r}{\sqrt{r^2}}$. Average duration of fades can be computed from the relation

$$\bar{t}(\mathcal{R}) = \frac{c.p.d.}{n(\mathcal{R})} = \frac{P(r \le \mathcal{R})}{n(\mathcal{R})}. \tag{2.23}$$

With a change of variable to γ_b, the signal-to-noise ratio, for a Rayleigh channel,

$$p(\gamma_b) = \frac{1}{\bar{\gamma}_b}\, e^{-\gamma_b/\bar{\gamma}_b}, \quad \gamma_b \ge 0, \tag{2.24}$$

where $\bar{\gamma}_b$ is the average signal-to-noise ratio, defined as

$$\bar{\gamma}_b = \frac{E_b}{N_0}\, E(\alpha^2). \tag{2.25}$$

Nakagami Channel: Another statistical model for the envelope of the channel response is the Nakagami-m distribution [72]. This distribution has two parameters that can be matched to the fading channel statistics. Since this distribution involves two parameters, this is more flexible in matching the observed signal statistics compared to the Rayleigh fading model which is described by a single parameter. Suzuki [85] has shown that Nakagami-m distribution is the best fit for data signals received in urban radio multi-path channels. Random variable γ has the PDF

$$p(\gamma) = \frac{m^m}{\Gamma(m)\,\bar{\gamma}^m}\,\gamma^{m-1}\,e^{-m\gamma/\bar{\gamma}}, \tag{2.26}$$

where $\bar{\gamma} = E(\alpha^2)\mathcal{E}/N_0$.
$m = 1$ corresponds to Rayleigh Fading and as $m \to \infty$, the channel assumes AWGN character.
Rice Channel: The Rice distribution is also a two-parameter distribution. It expressed as

$$p_R(r) = \frac{r}{\sigma^2}\,e^{-(r^2+s^2)/2\,\sigma^2}\,I_0\Big(\frac{rs}{\sigma^2}\Big), \quad r \geq 0, \tag{2.27}$$

where $I_0()$ is modified Bessel function of zeroth order. This model is suitable for Line of Sight (LOS) communication systems where the direct signal is corrupted with multi-path components arising from secondary reflections. This is typical in aeronautical [5], maritime [42], and land mobile satellite communications [44], [45], [88], [26].

Log-normal Channel: When the direct component is shadowed, the received SNR can be characterized by a log-normal density function given by

$$p(\gamma) = \frac{1}{\sqrt{2\,\pi\,d_0}\,\gamma}\,\exp\Big[-\frac{(\ln\gamma - \mu_0)^2}{2\,d_0}\Big], \tag{2.28}$$

where μ_0 and d_0 are the mean and variance of $\ln\gamma$ respectively.

There are more complicated models of fading channels. Suzuki [85] postulated that in mobile communications the statistical behavior of the narrow-band signal envelope, r, may be approximately described through a mixture of probability density function (PDF) in the form

$$P_r(r) = \int_0^\infty p_r(r|S)\,p_s(S)\,dS, \tag{2.29}$$

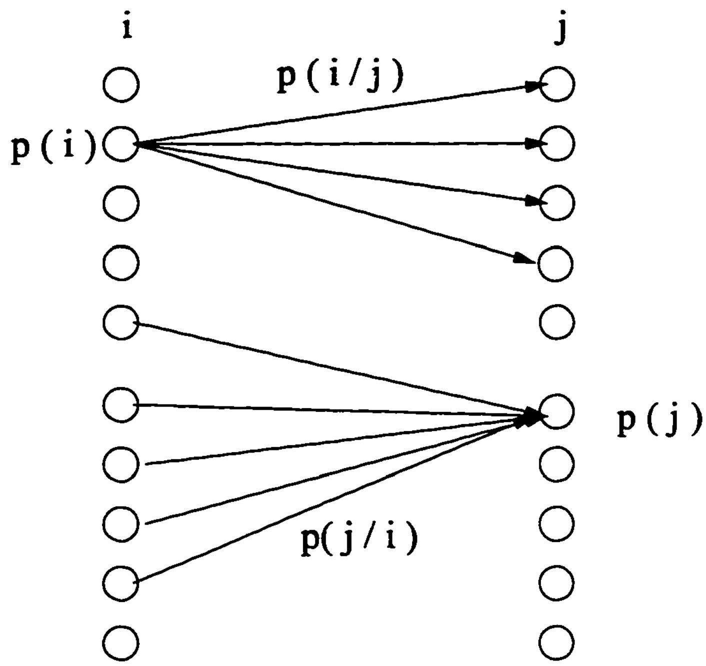


Figure 2.6. *Transition diagram for discrete noisy channel.*

where $p_s(S)$ is the lognormal PDF of S and $P_r(r|S)$ is the PDF of r conditioned on S. Lorenz [62] proved that a Rayleigh-lognormal model is suitable for macro-cellular propagation. In micro-cellular environments and for mobile satellite communications the presence of a direct component is very probable. So the Rayleigh distribution is replaced by Rice distribution. Loo [60] proposed a mixed model in which a lognormal component and a Rayleigh component are additive. More recently, a Rice-Log-normal model for the envelope, r, has been introduced [18]. Generalized Rice-lognormal channel model has been introduced in [87]. This model is a combination of the models in [60] and [18].

2.3 Capacity of Communication Channels

Capacity of a channel is by definition the maximum error-free information rate that can be transmitted through it. The channel of any practical communication system imposes limitations on this rate because of noise. The amount of information for discrete noisy channels can be defined with the help of fig. 2.6. The average mutual information is defined as

$$I = \sum_i \sum_j p(j/i) \log_2 \frac{p(j/i)}{p(i)\, p(j)}. \tag{2.30}$$

The above expression can be maximized w.r.t. $p(i)$ and the resulting value, the capacity, is obtained. For q-ary symmetric channels this capacity becomes

$$C = \log_2 q + \sum_j p(j/i) \log_2 p(j/i), \tag{2.31}$$

in bits per symbol. For BSC this relation reduces to

$$C = 1 + p \log_2 p + q \log_2 q. \tag{2.32}$$

If the channel has a capacity C, it is possible to send information through the channel at the rate C or lower with as small a frequency of errors or equivocation as desired [79].

2.3.1 Capacity of Gaussian Channel

For a Gaussian memoryless channel of nominal bandwidth B Hz., received signal power P, and noise spectral density N_0, the channel capacity is given by

$$C = B \log_2\left(1 + \frac{P}{N_0 B}\right). \tag{2.33}$$

Whenever the information rate R is less than C, then some coding- modulation-demodulation-decoding scheme exists which yields an arbitrarily small error probability. If R is greater than C, then regardless of the coding or modulation scheme, the error probability will be greater than zero. For an infinite bandwidth the asymptotic value of the capacity is

$$C_\infty = \lim_{B\to\infty} B \log_2\left(1 + \frac{P}{N_0 B}\right) = \frac{P/N_0}{\ln 2}. \tag{2.34}$$

From eqn. 2.33, E_b/N_0 can be expressed as a function of C/B.

$$\frac{E_b}{N_0} = \frac{1}{C/B}(2^{C/B} - 1). \tag{2.35}$$

2.3.2 Capacity of a Rayleigh Channel

In a Rayleigh channel the SNR does not remain constant. It varies with time according to Rayleigh density function given in eqn. 2.18. Probability density function can also be expressed as

$$p(\gamma) = \frac{1}{\Gamma} e^{-\gamma/\Gamma}, \tag{2.36}$$

where Γ is the average value of γ. The average value of channel capacity is obtained as [57]

$$\langle \hat{C} \rangle = \int_0^\infty B \log_2(1+\gamma).\frac{1}{\gamma} e^{-\gamma/\Gamma}\, d\gamma. \tag{2.37}$$

The solution to eqn. 2.37 is obtained as

$$\langle \hat{C} \rangle = -B \ \log_2 e.\ e^{-1/\Gamma}\ Ei(-1/\gamma), \tag{2.38}$$

where $Ei(x)$ is the exponential integral function and can be expressed in following two forms

$$Ei(-x) = E + \ln(x) + \sum_{k=1}^{\infty} \frac{(-x)^k}{k.k!} \tag{2.39}$$

$$= e^{-x}.\sum_{k=1}^{n} (-1)^k.\frac{(k-1)!}{x^k} + R_n, \tag{2.40}$$

where $x > 0$ and E is the Euler constant ($E = 0.5772157$) and R_n is a remainder term. In case where $\Gamma > 2$, expression for capacity becomes

$$\langle \hat{C} \rangle = B \ \log_2 e.e^{-1/\gamma}\Big(-E + \ln \Gamma + \frac{1}{\Gamma}\Big). \tag{2.41}$$

2.3.3 Capacity of Channels with Feedback

In his classic paper on communication theory Shannon examined the effects of feedback on the capacity of a unidirectional system according to the scheme of figure 2.7. The feedback path is assumed to be error free. It can be recalled that error-free information can be transferred over a noisy channel at a rate lower than the channel capacity. The capacity can be achieved if optimum coding is used. This is accomplished at the cost of certain amount of redundancy in the coding. The redundancy has to be introduced

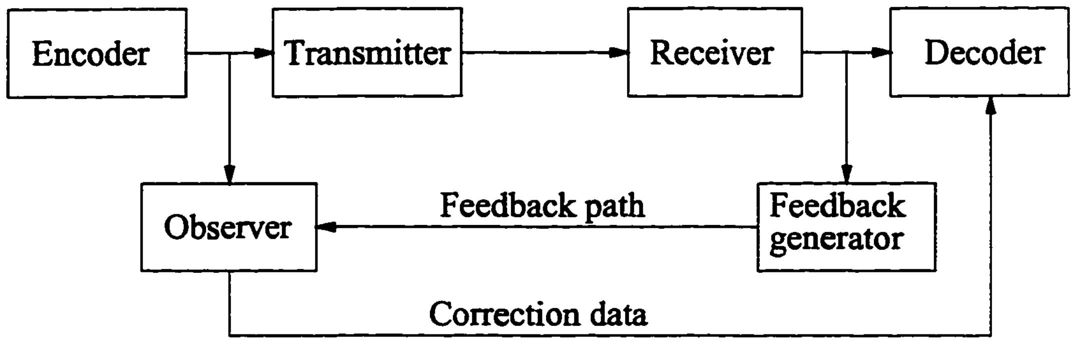


Figure 2.7. *Shannon's scheme of feedback correction system.*

in the proper way to combat the particular noise structure. It may be in the source coding or in the channel coding. Ideal coding involves the introduction of just the right amount of redundancy. Excess redundancy will reduce the system throughput. An infinitely large amount of delay is required to achieve this goal.

The addition of a feedback path helps to achieve the channel capacity with a finite delay. In [80] Shannon proved that in a memoryless discrete channel with feedback, the forward capacity is the same as the capacity with feedback. That means that feedback cannot increase capacity of a memoryless channel. Zero error capacity of a channel, C_0, is defined as the least upper bound of rates for block codes with no errors. Zero error capacity C_{0F} of a feedback channel is defined in a similar way. If all the input symbols are not adjacent to one another, it is possible to transmit information through the channel at non-zero rate with zero error probability. In the same paper cited above, Shannon proved that for channels having positive zero error capacity C_0, complete feedback may improve the channel capacity.

Shannon, in [79], mentioned that capacity of a channel with memory will be increased by the addition of feedback. In communication channels where errors tend to occur in bursts, the channel is said to exhibit *memory*. It has been shown that a discrete memoryless process has maximum entropy among the class of binary stochastic processes with error rate p and arbitrary memory length. That is, the entropy of binary channel with memory is upper bounded by

$$H \leq H_0, \tag{2.42}$$

where H_0 is the entropy of the DMC channel. So the measure of memory can be defined as the ratio

$$\theta = \frac{H_0 - H}{H_0}. \tag{2.43}$$

The channel capacity of the channel can now be expressed as

$$C = C_0 + \theta H_0. \tag{2.44}$$

In [19], a new upper bound on the feedback capacity was given. It was proved that the feedback capacity for an additive gaussian noise channel with memory is bounded by

$$C_{FB} \leq C + \frac{1}{2}. \tag{2.45}$$

One example of the channel with memory is the additive colored gaussian noise channel with an average power limitation on the transmitted signal. It has been proved in [31] that the capacity of such a channel is upper bounded by

$$C_{FB} \leq 2C, \tag{2.46}$$

where C_{FB} is the capacity with feedback. As the noise tends to white noise, the feedback channel capacity approaches that of a nonfeedback channel.

2.4 Error Control Schemes in Communication

Due to the addition of noise and interference signals, signal transmitted through a channel is distorted; and as a result, the receiver may not always recover the transmitted message correctly. There always exists a certain probability of incorrect reception. *Reliability* is a measure of correctness of the received data. It is quantitatively measured by $P(E)$, the probability that received data is accepted but is in error. There are numerous methods to reduce this error probability: error-detection coding, error-correction coding, iterative coding, automatic repeat requests (ARQ), erasure decoding in combination with ARQ and other combinations of the above.

2.4.1 Iterative Coding

The simplest iterative coding scheme involves the repetition of transmitted symbols. In erasure decoding, if the received symbol does not meet the threshold limit, it is discarded. If the probability of erasure is p, the probability that a symbol will be erased in all the n times it is transmitted, is p^n. Whatever the value of p may be, the probability of incorrect reception can be made arbitrarily small by properly choosing n. In a majority count decoding, a binary symbol is transmitted an odd number of times and the majority counted symbol is accepted. Probability of incorrect reception in this case is

$$P_e = \sum_{i=(n+1)/2}^{n} \binom{n}{i} p^i \,(1-p)^{n-i}. \tag{2.47}$$

As the value of n increases, P_e decreases, improving the reliability of reception.

In practice iterative decoding is performed on blocks of symbols called packets. Multiple copies of the same packet are transmitted over the channel and the receiver uses majority count decoding and extracts the transmitted information. Though iterative methods are very simple to implement, the major drawback with them is that channel throughput is reduced drastically and efficiency is far below the channel capacity.

2.4.2 Forward Error Correction

In forward error correction (FEC), the transmitter takes the user data, adds some redundancy and transmits both the user data and the redundancy through the channel. The data corrupted by channel noise is received by the receiver and is processed by the decoder to extract correct user data from it. The redundant symbols help to locate the positions in the packet where an error has occurred and correct them within a certain limit. A *block code* with M codewords of length N is a mapping from a set of M *source messages* onto a set of M codewords where each codeword is a sequence of M symbols from the channel input symbol set. A decoding operation is a mapping from a set of output sequences of length N into the symbols $1, 2, \cdots, M$, the input symbol set. If this reverse mapping does not produce the input message, it is said that a decoding error has occurred. The minimum number of symbols required to construct M different messages is $\lceil \log_2 M \rceil$. The generic name of the $N - \lceil \log_2 M \rceil$ symbols are parity symbols. Because of errors introduced by the transmission channel, some of the symbols in a codeword will be changed. If this number does not exceed a certain limit, defined by the particular code, the decoder can map the received symbols of a codeword onto the transmitted message correctly. When the number of errors occurring in a packet exceeds this number, the decoder might be able to flag the block as uncorrectable or it can find a codeword other than the transmitted one. The first event is called the *decoder failure* and the second event *decoder error* or *undetected error*. The minimum distance of a code is the minimum number of symbol locations any two codeword differ. If the minimum distance of a code is d, the code can correct a maximum of $\lfloor \frac{d-1}{2} \rfloor$ symbols. Performance of a code is measured by the *coding gain*, defined as the difference expressed in **dB** in the required E_b/N_0 for a given error performance, between the ideal PSK and the particular coding scheme [7].

If the number of input symbols is 2, the code is called a binary code. Most error correcting codes are of this type. The best known binary codes for memoryless channels are Bose-Choudhury-Hocquenghem (BCH) codes. These are cyclic codes whose symbols are elements from Galois Files $GF(q)$. When $q = 2$, BCH code is called binary BCH codes. For any integers m and t, there exists a binary BCH code of length $2^m - 1$ which has no greater than mt parity symbols and which corrects all combination of t or fewer errors in a codeword. Parameters of BCH codes of length $255, 127$ and 63 are given in tables D.1, D.2 and D.3 respectively.

Non-binary codes has symbols from $GF(q)$. Among all the non-binary codes used in practice, Reed-Solomon (RS) codes are the most prominent. A RS code of length n with k information symbols in a block has a minimum distance $d = n - k + 1$. When used in a channel for which all error patterns of the same weight have equal probability of occurrence, it can correct a maximum of $\lfloor \frac{n-k}{2} \rfloor$ symbol errors. Decoder error probabilities for Reed-Solomon codes obey the following relations [67].

$$P_E(u) = 0, \qquad \text{for } u \leq d - t - 1 \tag{2.48a}$$

$$\leq (q-1)^{-r} \sum_{s=d-u}^{t} \binom{n}{s} (q-1)^s, \qquad \text{for } d - t \leq u \leq d - 1 \tag{2.48b}$$

$$\leq Q', \qquad \text{for } u \geq d. \tag{2.48c}$$

where

$$Q' = (q-1)^{-r} V_n(t). \tag{2.49}$$

The term $V_n(t)$ denotes the volume of a Hamming sphere of radius t and expressed as

$$V_n(t) = \sum_{s=0}^{t} \binom{n}{s} (q-1)^s. \tag{2.50}$$

A slightly weaker but simpler bound for all $u \geq d - t$ is given by

$$P_E(u) \leq \begin{cases} \frac{1}{(q-1)^{r-2}} + \frac{1}{(q-1)^r}, & \text{if} \quad t = 1 \\ \frac{1}{(q-1)^{r-2t}} \cdot \frac{1}{t!}, & \text{if} \quad t \geq 2. \end{cases} \tag{2.51}$$

Decoded bit error probability for RS codes is given by [39]

$$P_b \leq \frac{2^{k-1}}{2^k - 1} \sum_{j=t+1}^{n} \left(\frac{j+t}{n}\right) \binom{n}{j} p_s^j (1-p_s)^{n-j}, \tag{2.52}$$

where p_s is the channel symbol error probability. The symbol error probability may be the error probability of an actual non-binary channel or may be the probability of one or more binary errors in an m-bit word on a binary channel. In a binary channel, the symbol error probability is given by

$$p_s = \sum_{e=1}^{m} \binom{m}{e} p^e (1-p)^{m-e}. \tag{2.53}$$

Throughput efficiency of a system can be defined as the ratio of the correctly delivered symbols at the receiver to the total number of symbols transmitted by the transmitter. For an (n, k) blockcode with an overhead of h symbols per packet, this quantity can be expressed as

$$Th = S \cdot \frac{k}{n+h}, \tag{2.54}$$

where S, the probability of correct reception for a channel, depends on the type of channel in question. For a Gaussian channel with symbol error probability p, this probability is

$$S = \sum_{i=0}^{t} \binom{n}{i} p^i (1-p)^{n-i}. \tag{2.55}$$

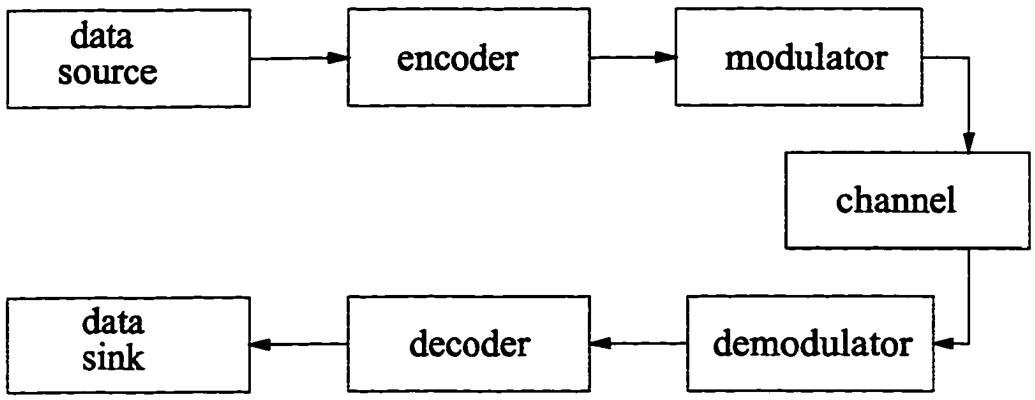


Figure 2.8. *Forward error-correction scheme.*

The block diagram of a digital communication system using FEC is shown in figure 2.8. A FEC scheme provides constant throughput efficiency, set by the code rate, regardless of the channel conditions. Because the decoded word must be delivered to the user regardless of whether it is correct or not, the system reliability can not be guaranteed if the channel degrades.

2.4.3 Pure ARQ Schemes

Automatic Repeat Request (ARQ) schemes can provide high reliability in exchange for less throughput and higher delay in delivering data to the user. The fundamental idea of a plain error control scheme with ARQ is that the receiver detects erroneous data blocks and via the feedback channel requests a retransmission of those blocks. The most important advantage of ARQ is that the delivered data has predictable quality [6]. The disadvantage of the system is that throughput depends on the channel condition.

Depending on the mode of retransmission, ARQ protocols are basically of three types: the stop-and-wait ARQ, the go-back-N ARQ, and the selective-repeat ARQ. In SAW ARQ, the transmitter stops at the end of each transmission and waits for a reply from the receiver. The receiver may answer with an ACK, indicating that the message has been successfully received or with a NAK, indicating that the message has not been correctly received. In the Go-back-N scheme, the receiver discards all the packets arriving after an unsuccessful reception of an earlier packet and the transmitter also resends all these packets along with the unsuccessful packet. In the selective-repeat scheme (SR), only those packets which are unsuccessful, are retransmitted. The receiver stores all the correctly received packets until they can be delivered to the user in sequence with earlier packets. This scheme is most effective in the sense of throughput efficiency. This is achieved at the expense of complicated buffer management at the receiver.

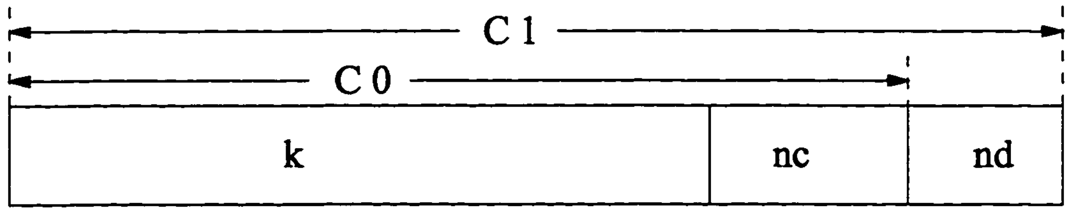
The advantage of a Go-back-N scheme is that the blocks do not have to be individually labeled and that algorithms are simpler than a comparable SR scheme [6]. There are several disadvantages of the scheme:

- In order to make sure that an erroneous block is repeated, the maximum round-trip delay on the link and the maximum range is limited. The delay cannot exceed the time it takes to transmit N blocks. This restriction creates problem in satellite communication.
- The scheme sends more data than necessary and thus throughput performance is degraded compared to the SR scheme.

These schemes are described in more detail in [58] and [17]. Because of simple error detection in these schemes, they are not very suitable for mobile data communication with medium to high bit rates.

In all ARQ schemes discussed in this thesis, it is assumed the feedback channel is error-free. In the feedback channel two types of errors can occur; an acceptance can be interpreted as a rejection ($A \rightarrow R$), or vice versa ($R \rightarrow A$) [77]. Because of the lower required information rate, the feedback channel can be made very reliable. When fading or severe noise bursts are present in the reverse channel, it is not always possible to simultaneously reduce both types of errors much below the limit defined by the interruptions.

An ($R \rightarrow A$) error may result in the loss of an entire codeword or codegroup. If the transmitter uses a sufficiently asymmetric decision to interpret the feedback signal, the ($R \rightarrow A$) error can be reduced sufficiently in exchange for increased ($A \rightarrow R$) error.



k - information symbols
nc - symbols for error correction
nd - symbols for error detection

Figure 2.9. *Hybrid ARQ scheme.*

2.4.4 Hybrid ARQ Schemes

These schemes are combinations of both the forward-error-correction and error-detection. They were first proposed by Wozencraft and Horstein in 1960 [96]. They are implemented by correcting a limited number of frequently occurring error patterns and requesting a retransmission when a more complicated or less frequent error pattern is encountered. This way the probability of retransmission is reduced. And the throughput over that of the ARQ scheme alone is improved. Retransmission of unreliable packets increases the system reliability beyond that of the FEC system. Thus, positive features of both the schemes are combined. Any of the retransmission scheme can be used with this scheme.

There are two basic approaches for implementing a hybrid ARQ system. The first scheme employs two codes, one for error detection purpose only and the other for error correction. Alternatively, a single code can be used for both error correction and detection. But in all practical systems, a strong error detection code is used for higher system reliability. The most widely used error checking codes are CRC codes. A high rate error-detection code in conjunction with an error-correcting code is shown in fig. 2.9.

To a block of k information bits, nc parity bits are added to form a codeword of error-correcting code $C0$. To these $k + nc$ bits, nd bits of error-detecting code $C1$ are added. The packet for transmission is formed by adding extra header and control bits to these encoded bits. The receiver first decodes the received block by trying to correct any error which might have been introduced by the channel. Then the packet is checked for any residual error. In case of no error remaining, the packet is accepted and an ACK is sent, otherwise, a NAK is sent to the transmitter.

The type of hybrid ARQ scheme described above is called type-I hybrid automatic repeat request (HARQ) scheme to differentiate it from another type of more sophisticated scheme called type-II HARQ scheme. In the HARQ-II schemes, the FEC parity bits are not sent with the message and error detecting parity bits. The transmitter alternates between message bits along with error-detecting bits on one transmission and only FEC parity bits on the next. If the first transmission is error-free, parity bits are never sent. And if the code is invertible [13], any error-free copy of the parity bits can recover message bits. If neither transmission is error-free, bits from two transmissions are combined and decoded for error correction and can deliver the message, provided the error-correcting capability of the code is not exceeded. This scheme provides better throughput efficiency compared to type-I scheme, but is more complicated to implement. Further discussion on this issue is presented in subsection 3.3.2.

The second approach is implemented by using reliability information from the decoder/demodulator to either accept a packet or discard it. The overall number of redundant bits required by this method is less than the number required by HARQ-I scheme, and is achieved at the expense of decoder complexity. In this dissertation we shall focus on the first approach.

In hybrid ARQ schemes, the choice of RS codes is very common due to the following reasons:

- The RS codes can provide very long blocklengths which is required for averaging channel errors over long periods of time.

- The RS codes are symbol based. The encoder and decoder operates on bytes rather than on bits. This reduces complexity of the encoder and decoder compared to binary codes.

- The RS codes have a very low decoding error rate. In most cases, RS codes can detect uncorrectable errors. This information can be used in ARQ purposes.

- The RS codes are good at burst correction. In multi-bit modulation schemes, their performance is better than for pure random noise.

In an RS coding scheme, if the probability of successful reception of a packet is S, the average number of transmissions needed for a packet to be successfully received can be obtained from the following relations [97]:

$$N_{av} = \frac{1}{S} \qquad \text{selective-repeat system} \tag{2.56}$$

$$= \frac{S}{S+(1-S)N} \qquad \text{Go-back-N system} \tag{2.57}$$

$$= \frac{S}{1+D\,\tau/n} \qquad \text{stop and wait system} \tag{2.58}$$

where D is the idle time from the end of transmission of one code vector to the beginning of transmission of the next and τ is the signaling rate of the transmitter in bits per second. Throughput is obtained by multiplying N_{av} with code rate k/n.

In the above derivations it is assumed that block failure probabilities are independent of each other. More general derivations are presented in chapter 3. Optimum performance of hybrid ARQ systems are analyzed in chapter 3. One example showing the effect of code rate on system throughput is given in fig. 2.10.

2.4.5 Adaptive Error Control Schemes

For a slowly varying channel, throughput optimization can be done by adaptively changing the code parameters to match the channel conditions. In chapter 5 an in-depth analysis of these schemes is given.

2.5 Interleaving

In a coding scheme, redundancy is used to combat noise. In order to ensure that most of the time the noise affects only a small portion of the codeword, the noise has to be averaged over a long period of time. To achieve this, the codeword must

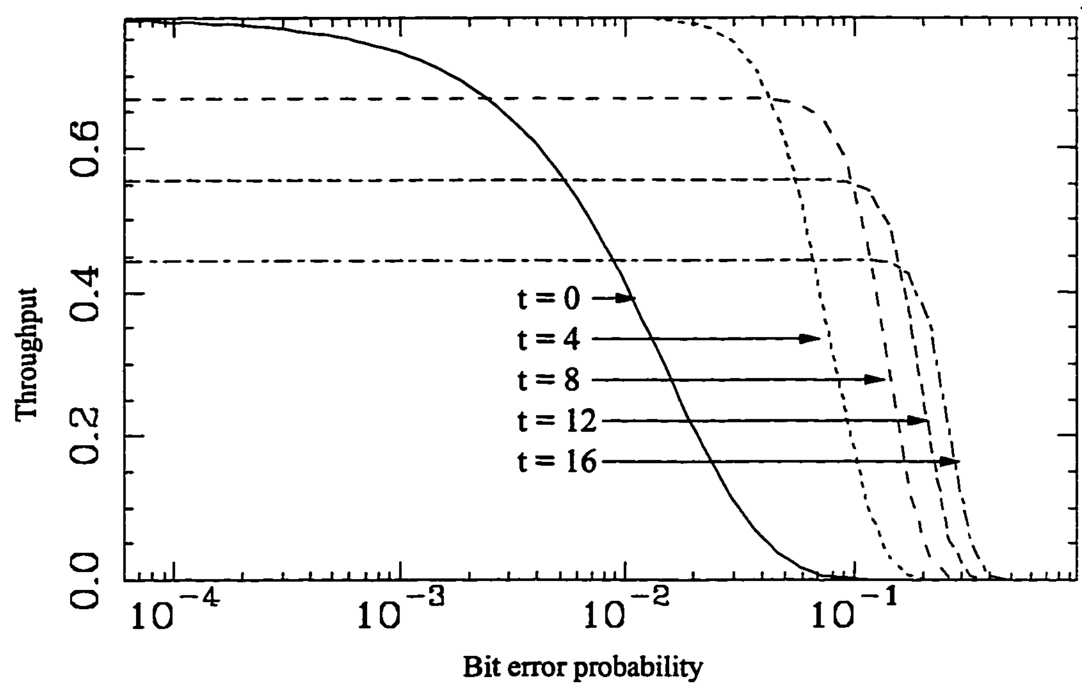


Figure 2.10. *Throughput of RS* (64, k) *for different symbol error rates*

have a very large block length. Practically, this is not possible due to the decoding complexity of very large blocklength codes. However, an alternative method to achieve this is the use of interleaving. By interleaving, burst errors are dispersed over a wider range and appear random. This way the errors are distributed over a span of several blocks and thus the probability of successful decoding might increase if the average bit-error-rates do not exceed the error-correcting capability of the code used. However, in information theoretic sense, the channel becomes worse with respect to the information capacity due to interleaving, this is a practical necessity in many applications. System delay constraints preclude interleaving to a depth so great that channel noise becomes completely random. The correlation among successive errors is the deciding factor of the depth of interleaving.

Several types of interleaving design exist. The block diagram of an interleaver is shown in fig. 2.11. The encoded data are reordered by the interleaver and transmitted over the channel. At the receiver, the deinterleaver puts the data in proper sequence and passes it to the decoder. As a result of interleaving/deinterleaving, error bursts are spread out in time and errors in a codeword appear to be independent. A block

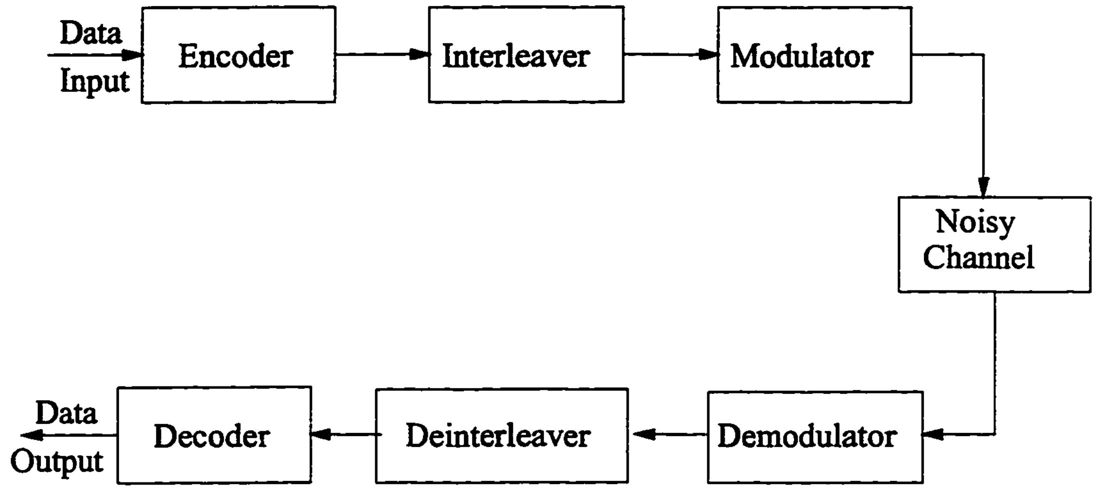


Figure 2.11. *Block diagram of system using interleaving.*

interleaver formats the encoded data in a rectangular array of m rows and n columns, m, being called the interleaving depth. Usually a row is formed by a codeword. At the transmitter, bits are read-out column-wise, whereas at the receiver deinterleaver it is read-out row-wise. As a result, a burst of length $l = m\ b$ is broken up into m bursts of length b each. The block diagram of the interleaving system is shown in fig. 2.11 and a 5x9 block interleaver is shown in fig. 2.12. The numbering corresponds to the sequence the bits are read out from the encoder.

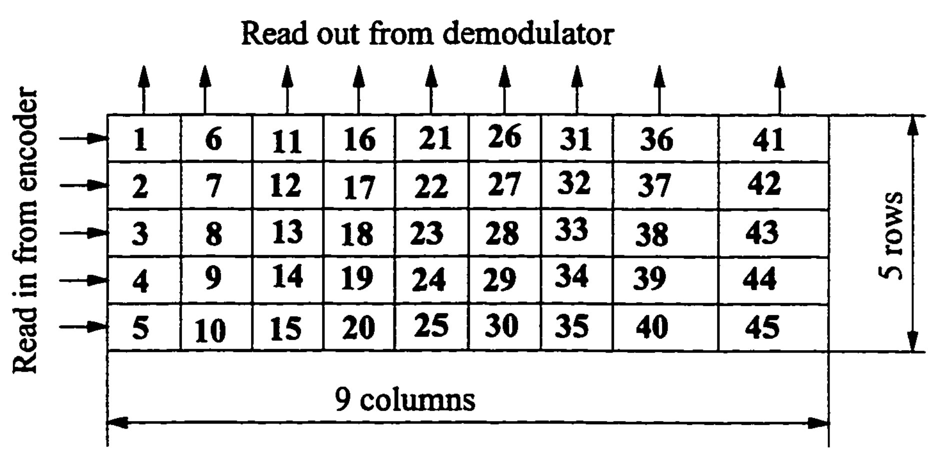


Figure 2.12. *A block interleaver.*

2.6 Summary

In this chapter we have described fundamentals that will be used in subsequent chapters of this thesis. The capacity of discrete as well as analog channels have been introduced in section 2.3. Different channel models have been discussed in section 2.2. Different error control schemes and coding techniques for retransmission based communication systems have been introduced in section 2.4. Block interleaving technique has been described in section 2.5.

Chapter 3

Performance of Error Control Schemes in Noisy Environment

In this chapter we analyze the performance of different error control schemes with the objective of optimizing their throughput performance.

3.1 Simple ARQ schemes

The throughput of a simple ARQ error control scheme depends on the size of the packet, the channel BER and the amount of overhead bits in a packet. It is assumed that error process in any transmitted block is independent of any other block. For SR schemes, assuming that messages are single block, the throughput is given by eqn. 3.1.

$$Th = \frac{n-h}{n}\,(1-p)^n, \tag{3.1}$$

where h is the number of overhead bits in a packet of length n with a channel BER p. The optimum value of the packet length can be obtained from the above relation by differentiating Th w.r.t. n [78], [69], [16] and then choosing the integer closest to the result which provides the best throughput. Differentiation of the equation w.r.t. n yields

$$\frac{d\,Th}{d\,n} = \frac{n-h}{n}\,(1-p)^n\,\log_e(1-p) + (1-p)^n\,\frac{h}{n^2}. \tag{3.2}$$

By equating $\frac{dT}{dn}$ to zero, the value of n is obtained as follows:

$$n = \frac{h\,\log_e(1-p) - \sqrt{h^2\,\log_e^2(1-p) - 4\,h\,\log_e(1-p)}}{2\,\log_e(1-p)}. \tag{3.3}$$

Table 3.1 gives the maximum throughput obtainable with plain ARQ systems for different BER's. Fig. 3.1 gives the optimum packet length and the average number of transmissions corresponding to maximum throughput.

Table 3.1. *Maximum throughput table for ARQ schemes (overhead = 48 bits).*

BER	Optimum Packet Length	Average Number of Transmissions	Throughput
1e-05	2216	1.02241	0.956898
1.58e-05	1765	1.02837	0.945969
2.51e-05	1407	1.03597	0.932344
3.98e-05	1123	1.04572	0.915403
6.31e-05	897	1.05823	0.894406
0.0001	718	1.07444	0.868493
0.000158	575	1.09542	0.836685
0.000251	462	1.12307	0.797907
0.000398	373	1.16012	0.751055
0.000631	301	1.20922	0.6951
0.001	245	1.27778	0.629281
0.00158	200	1.37332	0.553404
0.00251	165	1.51434	0.468249
0.00398	137	1.72719	0.376123
0.00631	115	2.07073	0.281355
0.01	98	2.67763	0.190543
0.0158	84	3.82659	0.111998

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0.0251	74	6.57017	0.053477
0.0398	66	14.6032	0.018676
0.0631	61	53.2834	0.003999
0.1	57	405.674	0.000389

An approximate expression for the optimum packet length can be obtained from eqn. 3.3 by expanding the logarithmic expression. The equation found is given by

$$n = \frac{h}{2}\left(1 + \frac{2}{\sqrt{hp}} + \frac{\sqrt{hp}}{4}\right). \tag{3.4}$$

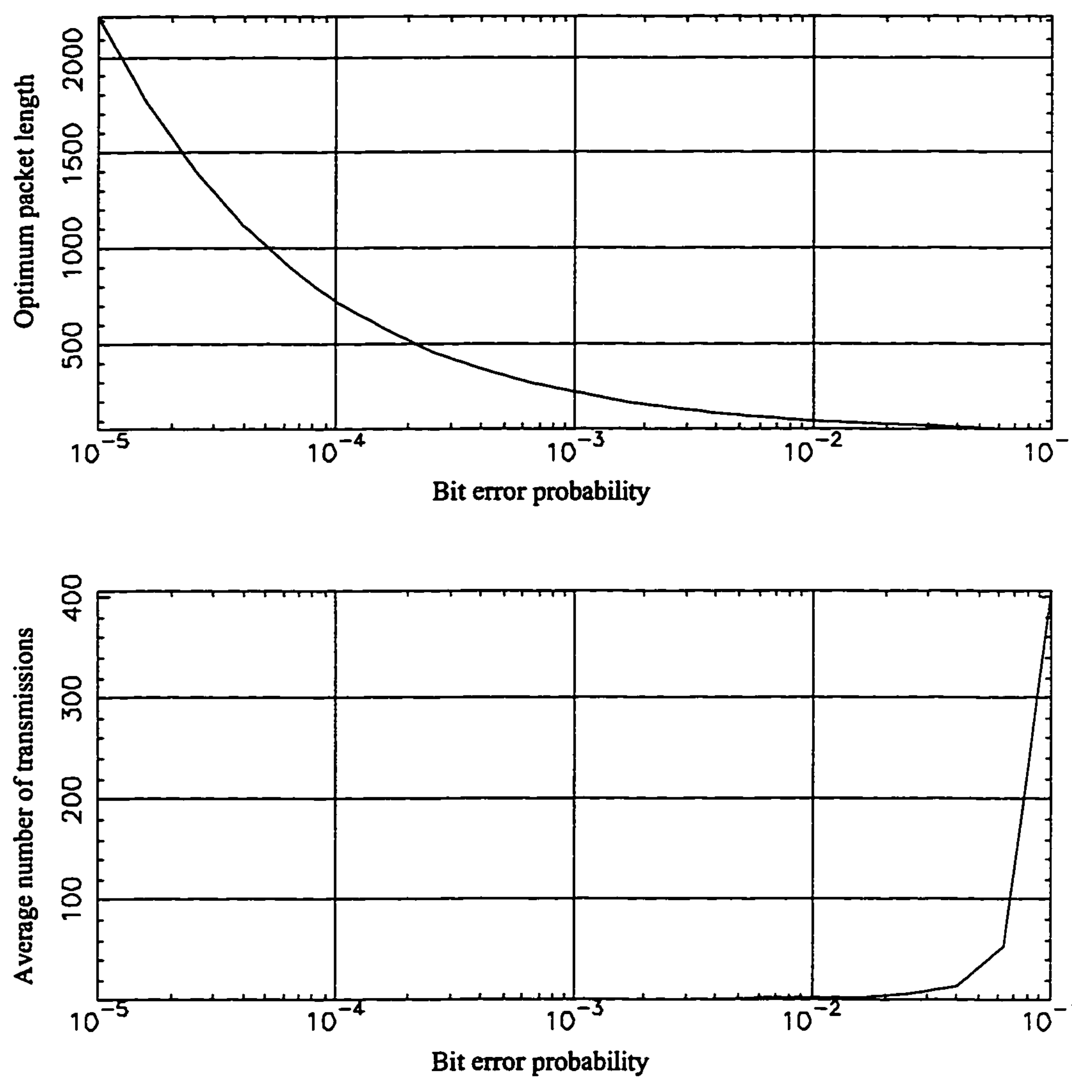

Figure 3.1. *Optimum packet length with plain ARQ schemes.*

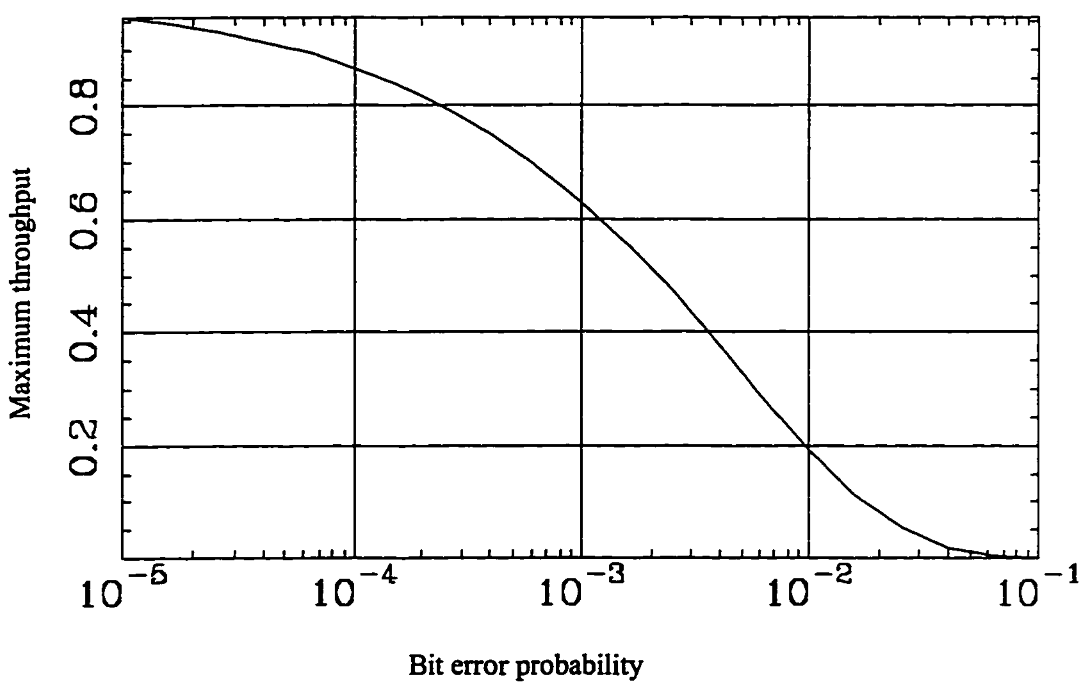

Figure 3.2. *Maximum throughput values for plain ARQ systems.*

3.1.1 HARQ-I Scheme in Gaussian Environment

In practical communication systems, the number of retransmissions must be limited due to delay constraints or limited length of receiver buffers. We shall analyze cases, when number of transmissions is unlimited and when there is a limit on the number of transmissions. In both cases, lower limit on successful reception after allowed number of transmissions exists. When there is no limitation on the number of transmissions, a packet can be successfully transmitted by a sufficiently large number of transmissions. In restricted environment, use of a low rate code increases the probability of success and by using sufficiently low rate code, a packet can be successfully transmitted within the limit imposed on the number of transmissions.

3.1.1.1 Unlimited Number of Transmissions

We assume the probability of an undetected error in a packet by the receiver to be negligible. This assumption is justified in the sense that undetected error probability P_u is of the order of 2^{-b} where b is the number of parity bits in the CRC code. The probability, P_r, that a block of length n and error correcting capability of t symbols in the block will be rejected by the receiver is given by

$$P_r = \sum_{i=t+1}^{n} \binom{n}{i} p^i \, (1-p)^{n-i} \tag{3.5}$$

$$= 1 - S, \tag{3.6}$$

where S is the probability of successful reception of a packet and expressed as

$$S = \sum_{i=0}^{t} \binom{n}{i} p^i \, (1-p)^{n-i}. \tag{3.7}$$

Average number of transmissions required for the packet to be successfully received by the receiver is

$$N_{av} = \frac{1}{(1-P_r)} = \frac{1}{S}. \tag{3.8}$$

Hence the throughput of the scheme can be expressed as

$$\begin{aligned} Th &= \frac{km}{nm+h+b} \frac{1}{N_{av}} \\ &= \frac{km}{nm+h+b} S. \end{aligned} \tag{3.9}$$

As with the case of plain ARQ, there exist optimum values for n and t which provide maximum throughput. This is not evident from the above equation and the derivation of optimum values is not as easy as in the case of ARQ without error correction. We have found the optimum values of n and t through computer search and the results are given below. The maximum throughput values for channel bit error probabilities $p = 0.001, p = 0.01$ and $p = 0.1$ for a Gaussian channel are shown in table 3.2, table 3.3 and table 3.4 respectively.

Table 3.2. *Throughput table for BER* = 0.001.

n	t(max)	throughput(max)
31	0	0.38
63	0	0.533
125	1	0.677
250	1	0.791
500	2	0.869
1000	4	0.917
5000	12	0.965
10000	20	0.973
20000	35	0.978
30000	49	0.981
40000	62	0.982
50000	75	0.983
60000	87	0.983
70000	92	0.98
80000	91	0.887
90000	89	0.48
100000	88	0.122

As seen from the table, the value of packet length for which the maximum throughput occurs is rather very high. For a channel BER of 0.01, it is around 10000 and for BER of 0.1, it is about 2000. As BER goes below 0.01, optimum packet length becomes greater than 10000 bits.

Table 3.3. *Throughput table for BER* = 0.01.

n	t(max)	throughput(max)
31	2	0.316
63	3	0.457
125	4	0.59
250	7	0.698
500	11	0.778
1000	19	0.827
2000	32	0.859
3000	46	0.871
4000	58	0.879
5000	70	0.884
6000	83	0.887
7000	95	0.890
8000	106	0.892
9000	118	0.893
9050	119	0.893
10000	130	0.895
10050	130	0.895
11000	131	0.879
12000	128	0.690
13000	126	0.322
14000	124	0.0725
15000	122	0.0075

The derivation of the maximum throughput is based on the sphere packing bound [71], given by the equation,

$$\sum_{i=0}^{t} \binom{n}{i} \leq 2^{n-k}. \tag{3.10}$$

From the above bound, the minimum number of parity symbols required to correct t errors in a block of length n is found to be

$$\begin{aligned} n-k &\geq \log_2\Big(\sum_{i=0}^{t}\binom{n}{i}\Big) \\ &= \lceil \log_2\Big(\sum_{i=0}^{t}\binom{n}{i}\Big)\rceil. \end{aligned} \tag{3.11}$$

Table 3.4. *Throughput table for BER* $= 0.1$.

n	t(max)	throughput(max)
31	5	0.153
63	10	0.228
125	17	0.3
250	33	0.362
500	62	0.411
1000	119	0.445
1900	219	0.469
1950	224	0.469
2000	228	0.470
2050	225	0.455
2100	222	0.404
2200	217	0.214
2500	205	0.00054
3000	190	0.2.69e-13

A typical three-dimensional plot of throughput as function of blocklength n and error-correcting capability t is shown in fig. 3.3.

It is interesting to compare the maximum throughput obtained with the channel capacity at the corresponding channel BER's. Table 3.5 gives a comparison of channel capacity and throughput obtained by HARQ systems for BER's $0.01, 0.1$ and 0.001.

As seen from the table, the channel capacity can almost be achieved with hybrid ARQ schemes which is not possible with pure ARQ schemes at high channel BER. With HARQ schemes, the average number of transmissions of a packet is considerably

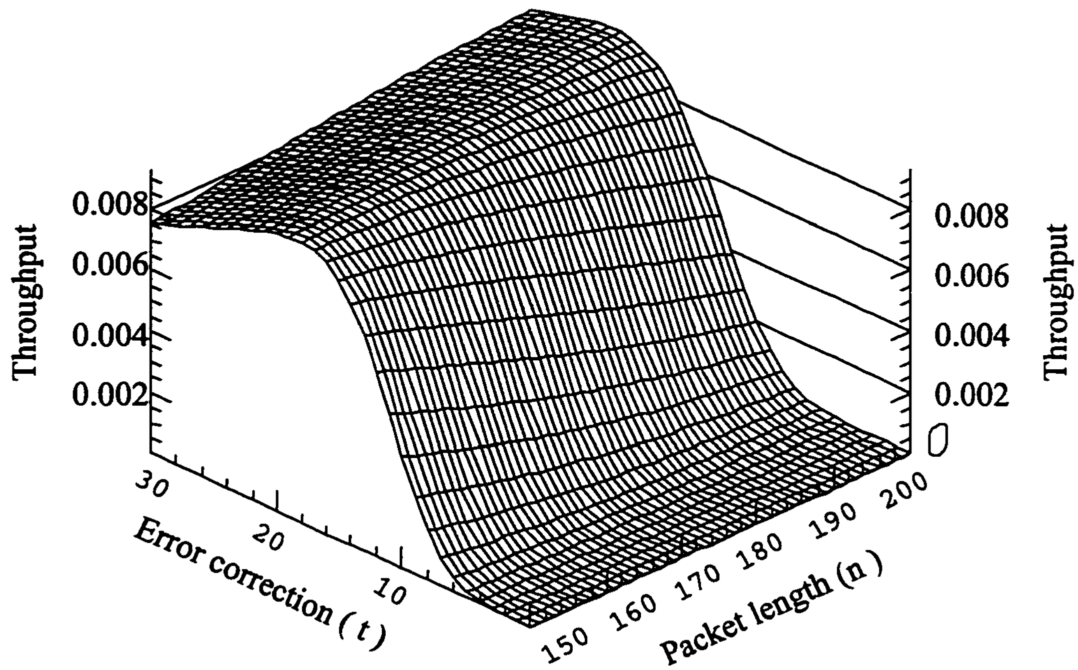

Figure 3.3. *Throughput for different n and t.*

smaller than pure ARQ schemes. This is evident from the value in table 3.1 and the data presented in table 3.5.

Table 3.5. *Comparison of throughput with channel capacity.*

BER	Throughput		Capacity
	overhead = 48 bits	no overhead	
0.1	0.470	0.481	0.531
0.01	0.895	0.899	0.919
0.001	0.983	0.984	0.989

3.1.1.2 Transmission under constraints

When there is no limit on the maximum number of transmissions, the receiver buffer requirement might exceed feasible dimensions. If the buffer is limited, packets received successfully might be rejected because of lack of space to store them. This will seriously reduce the throughput efficiency of the system. If the number of transmissions is also limited, there will be no guarantee that lower limit on the success probability will be achieved. To guarantee this probability of minimum packet loss, the system should be designed with these constraints in mind.

Let N_{max} be the maximum allowable number of transmissions of a packet and P_l be the maximum packet loss probability. This means that the probability of successful reception of a packet will be at least $1 - P_l$. To achieve this limit, let us assume that the error correcting capability of the code must not be less than t. The probability of a retransmission request is denoted by P_r and is given by eqn. 3.5. Packet loss occurs when a packet is unsuccessful after N_{max} transmissions. Probability of this event is

$$P_l = P_r^{N_{max}}. \tag{3.12}$$

So, the error correction capability of the code used should be such that the retransmission probability is lower than $P_l^{1/Nmax}$. This implies that

$$\sum_{i=0}^{t} \binom{n}{i} p^i (1-p)^{n-i} > 1 - P_l^{(1/Nmax)}. \tag{3.13}$$

From the above implicit relationship, t can only be solved numerically or graphically. Table 3.6 shows the required success probability for each transmission for different packet loss probabilities and maximum allowed number of transmissions.

Table 3.6. *Lower bounds on success probability.*

P_l	Maximum transmissions				
	1	2	3	4	5
0.0001	0.9999	0.99	0.953584	0.9	0.841511
0.000158	0.999842	0.987411	0.945883	0.887798	0.82622
0.000251	0.999749	0.984151	0.936904	0.874107	0.809454
0.000398	0.999602	0.980047	0.926436	0.858746	0.79107
0.000631	0.999369	0.974881	0.91423	0.841511	0.770913
0.001	0.999	0.968377	0.9	0.822172	0.748811
0.00158	0.998415	0.960189	0.883409	0.800474	0.724577

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0.00251	0.997488	0.949881	0.864064	0.776128	0.698005
0.00398	0.996019	0.936904	0.841511	0.748811	0.668869
0.00631	0.99369	0.920567	0.815215	0.718162	0.636922
0.01	0.99	0.9	0.784557	0.683772	0.601893
0.0158	0.984151	0.874107	0.748811	0.645187	0.563484
0.0251	0.974881	0.841511	0.707136	0.601893	0.52137
0.0398	0.960189	0.800474	0.658545	0.553316	0.475193
0.0631	0.936904	0.748811	0.601893	0.498813	0.42456
0.1	0.9	0.683772	0.535841	0.437659	0.369043

The minimum successful transmission probabilities required to satisfy the constraint on number of transmissions and packet loss probability, are plotted in fig. 3.4. Table 3.7

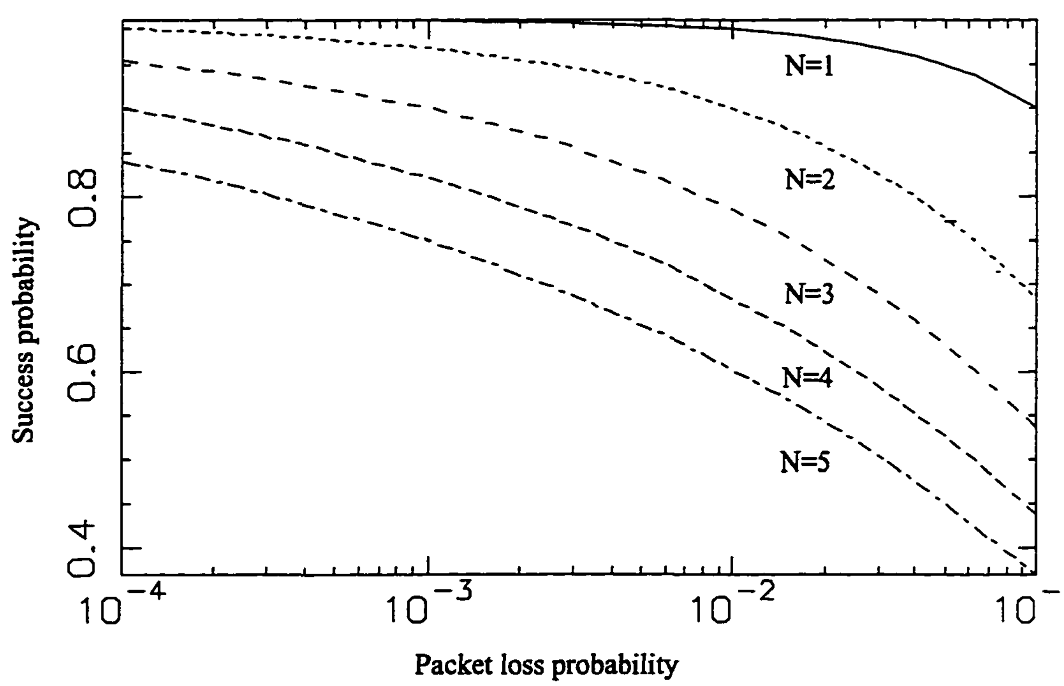

Figure 3.4. *Success probabilities in each transmission.*

gives the minimum values of required error correcting capabilities of the transmitted codeword for channel symbol error rate of 0.01. These minimum values may not be the optimum t's, but they give the lower bound on error correction so that the required packet loss probability constraint is not violated. Table 3.8 gives the minimum error correction capabilities for channel symbol error rate of 0.05.

Table 3.7. *Required error correction for* $p = 0.01$

P_l	Maximum transmissions				
	1	2	3	4	5
0.0001	10	7	5	5	4
0.000158	10	7	5	5	4
0.000251	10	6	5	4	4
0.000398	9	6	5	4	4
0.000631	9	6	5	4	4
0.001	9	6	5	4	4
0.00158	8	6	4	4	3
0.00251	8	5	4	4	3
0.00398	8	5	4	4	3
0.00631	7	5	4	3	3
0.01	7	5	4	3	3
0.0158	6	4	4	3	3
0.0251	6	4	3	3	2
0.0398	6	4	3	3	2
0.0631	5	4	3	2	2
0.1	5	3	3	2	2

Table 3.8. *Required error correction for* $p = 0.05$

P_l	Maximum transmissions				
	1	2	3	4	5
0.0001	27	22	19	17	16
0.000158	27	21	19	17	16
0.000251	26	21	18	17	16

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0.000398	26	20	18	17	16
0.000631	25	20	18	16	15
0.001	25	20	17	16	15
0.00158	24	19	17	16	15
0.00251	24	19	17	15	14
0.00398	23	18	16	15	14
0.00631	22	18	16	15	14
0.01	22	17	15	14	14
0.0158	21	17	15	14	13
0.0251	20	16	15	14	13
0.0398	19	16	14	13	12
0.0631	18	15	14	13	12
0.1	17	14	13	12	12

When a code is used whose failure probability is given by P_r, the average number of retransmissions is given by

$$\begin{aligned} N_{retran} &= P_r(1-P_r) + 2P_r^2(1-P_r) + \cdots + (N-2)\,P_r^{N-2}(1-P_r) + (N-1)\,P_r^{N-1} \\ &= \frac{P_r - P_r^N}{1-P_r}. \end{aligned} \tag{3.14}$$

The average number of transmissions is

$$N_{av} = 1 + N_{retran} = \frac{1-P_r^N}{1-P_r}. \tag{3.15}$$

Throughput of the system is given by

$$Th = \frac{k}{n+h+b}\,\frac{1}{N_{av}}. \tag{3.16}$$

For RS codes $k = n - 2\,t$ and for binary codes $n - k$ should satisfy the Hamming bound described earlier.

The throughput of the system using BCH codes of length 255 and different code rates is given in table 3.9 and table 3.10. The parameters k and t for different codes are given in table D.1 and are obtained from [70]. A system can be designed with the restriction on the average number of transmissions of a packet, rather than the maximum number. Requirements on success probabilities for this case will not be as severe as the above case. If the required average number of transmissions is given by N, the lower bound on the success probability will be $1/N$.

Table 3.9. *Throughput of truncated system* ($p = 0.01$).

P_l	Maximum transmissions				
	1	2	3	4	5
0.0001	0.564	0.63	0.673	0.673	0.673
0.000158	0.564	0.63	0.673	0.673	0.673
0.000251	0.564	0.654	0.673	0.673	0.673
0.000398	0.591	0.654	0.673	0.673	0.673
0.000631	0.591	0.654	0.673	0.673	0.673
0.001	0.591	0.654	0.673	0.673	0.673
0.00158	0.617	0.654	0.673	0.673	0.673
0.00251	0.617	0.673	0.673	0.673	0.673
0.00398	0.617	0.673	0.673	0.673	0.673
0.00631	0.63	0.673	0.673	0.673	0.673
0.01	0.63	0.673	0.673	0.673	0.673
0.0158	0.654	0.673	0.673	0.673	0.673
0.0251	0.654	0.673	0.673	0.673	0.673
0.0398	0.654	0.673	0.673	0.673	0.673
0.0631	0.673	0.673	0.673	0.673	0.673
0.1	0.673	0.673	0.673	0.673	0.673

Table 3.10. *Throughput of truncated system* ($p = 0.05$).

P_l	Maximum transmissions				
	1	2	3	4	5
0.0001	0.234	0.326	0.376	0.393	0.393
0.000158	0.234	0.351	0.376	0.393	0.393

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0.000251	0.261	0.351	0.393	0.393	0.393
0.000398	0.261	0.351	0.393	0.393	0.393
0.000631	0.287	0.351	0.393	0.393	0.393
0.001	0.287	0.351	0.393	0.393	0.393
0.00158	0.287	0.376	0.393	0.393	0.393
0.00251	0.287	0.376	0.393	0.393	0.393
0.00398	0.3	0.393	0.393	0.393	0.393
0.00631	0.326	0.393	0.393	0.393	0.393
0.01	0.326	0.393	0.393	0.393	0.393
0.0158	0.351	0.393	0.393	0.393	0.393
0.0251	0.351	0.393	0.393	0.393	0.393
0.0398	0.376	0.393	0.393	0.393	0.393
0.0631	0.393	0.393	0.393	0.393	0.393
0.1	0.393	0.393	0.393	0.393	0.393

When a restriction on the average number of transmissions is imposed on the system design, the packet loss probability must also be defined. Because, it is impossible to achieve 100% success probability in a single transmission. With defined packet loss probability P_l, the success probability for a single transmission must not be less than $(1 - P_l)/N$. To achieve this bound, the error correcting capability, t, must satisfy the relation

$$\sum_{i=0}^{t} \binom{n}{i} p^i (1-p)^{n-i} \geq \frac{(1-P_l)}{N}. \tag{3.17}$$

3.1.2 Delay Analysis

Delay associated with the transmission of a message includes the queuing delay at the transmitter while the packets wait to be serviced (transmitted), decoding delay at the receiver and re-sequencing delay at the receiver while the packet waits at the receiver buffer while earlier packets are delivered to the user or data sink.

In the SR retransmission scheme new packets are transmitted continuously as long as no NAK exists. Only those packets, which have been rejected by the receiver, are retransmitted. But the packets though correctly received at the receiver, wait at the receiver buffer until all the packets with smaller sequence numbers are delivered. The probability of r retransmissions of a packet, for it to be received error-free, is given by the equation,

$$P\{X = 1 + rD\} = (1 - p)p^r, \quad \text{r=0,1, \dots .} \tag{3.18}$$

D is the round trip delay (RTD). Assuming that a message contains N packets, the delays for each packet, till it is received correctly by the receiver, can be written as follows. Let k_i denote the number of retransmissions required for the i th packet. The round-trip delay (RTD) is the time difference, from the transmission of a packet, to the reception of the feedback signal. This is conveniently measured by the units of packets which could be transmitted during this time interval. Because a packet cannot be delivered if earlier packet is still being in service or in receiver buffer,

$$\begin{aligned}
d1 &= 1 + k_1 D \\
d2 &= \text{Max}\{d_1, (1 + k_2 D)\} \\
d3 &= \text{Max}\{d_2, (1 + k_3 D)\} \\
&\vdots \\
d_N &= \text{Max}\{d_{N-1}, (1 + k_N D)\} \\
&= \text{Max}\{(1 + k_1 D), (1 + k_2 D), \cdots, (1 + k_N D)\} \\
&= (1 + k_m D)
\end{aligned}$$

where k_m is the $\text{Max}(k_1, k_2, \cdots, k_n)$.

Now the probability of k_m being equal to r is given by

$$Pr.\{k_m = r\} = N(1-p)\,p^r \left[\sum_{i=0}^{r}(1-p)p^i\right]^{N-1}$$
$$= N\,p^r\,\frac{(1-p^{r+1})^{N-1}}{(1-r)^{N-3}} \tag{3.19}$$

In the derivation of the above relation it is assumed that any of the N packets has the retransmission number r and other packets have this number no greater than that. The whole message transmission is delayed by the amount $d_N - 1$ in excess of the normal transmission time, i.e. without any retransmission of packets. This is the re-sequencing delay at the receiver buffer.

3.1.3 Simulation of Buffer Statistics

The buffer requirement at the receiver is a function of the round-trip delay and the probability of successful decoding of a packet. If round-trip delay is D packets and success probability is S, the maximum buffer size is $r(D-1)+1$ and the probability of its occurrence $(1-S)^r$. For SR protocols buffer studies have been done in [68]. We have simulated buffering for SR protocols with different channel bit error rates. In practice, buffer size is limited. Invariably, some packets are lost when buffer overflow occurs due to channel degradation. Fig. 3.5 shows percentage of packets lost due to buffer overflow. Retransmission delay was considered to be 4 times the packet transmission time. It is considered to be fixed. Simulation has been carried out for 100000 packets. The packet size for all of them were taken as 127 symbols and three different BER chosen were $0.001, 0.01$ and 0.05.

When the buffer dynamics is considered, channel BER, packetlength and error correcting capability of the code used can all be replaced by the probability of successful decoding of a packet. The retransmission delay and this success probability are the only two parameters required to calculate the buffer requirements. If simulation is done with only these parameters, the results will be generally applicable. Fig. 3.6 and 3.7 give the plots of packet loss for different success probabilities. The retransmission delays are taken to be 4 and 8 packet transmission times. The data from which above plots were made are obtained by simulation and are given in appendix E in tables E.1 and E.2 respectively.

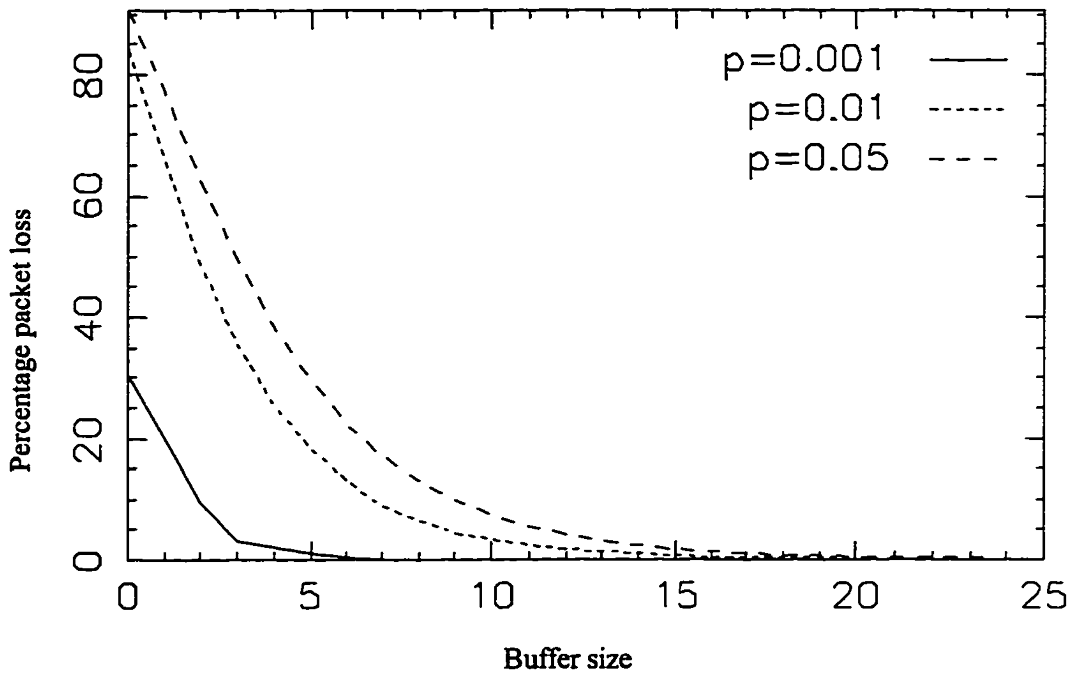


Figure 3.5. *Packet loss probabilities for different buffer sizes.*

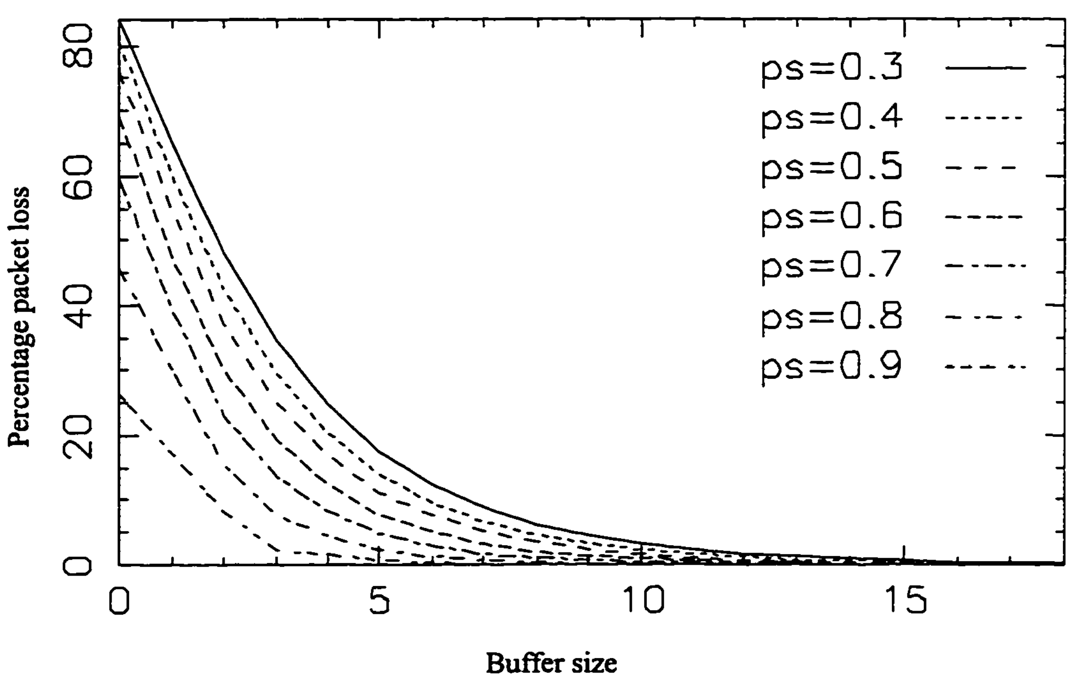


Figure 3.6. *Packet loss probabilities for different buffer sizes* $(D = 4)$.

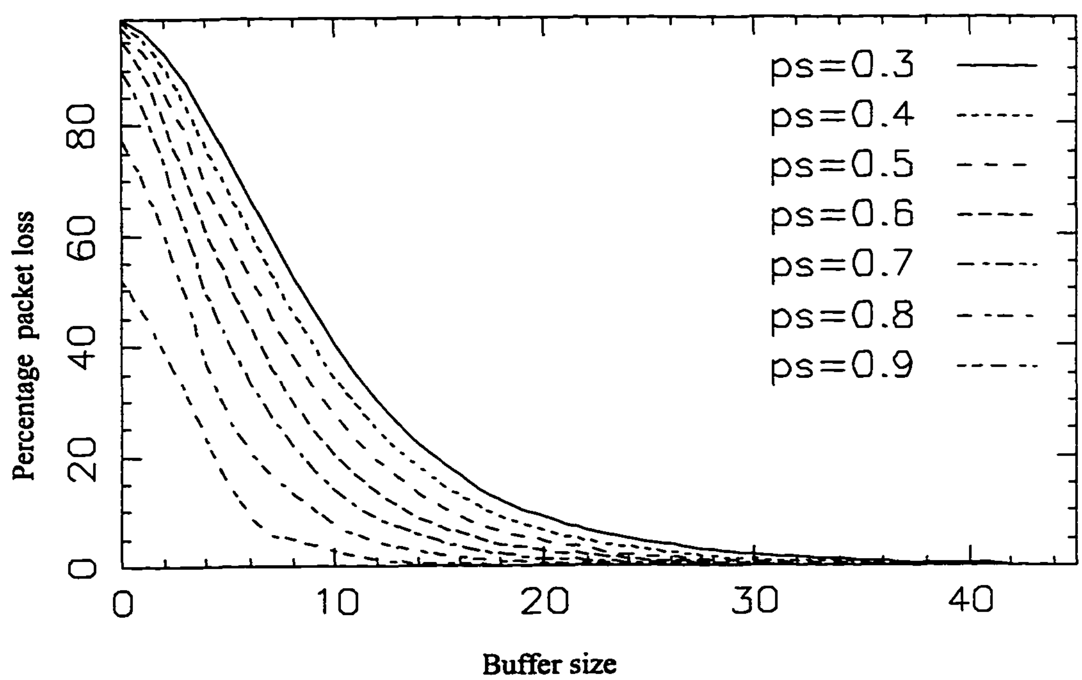


Figure 3.7. *Packet loss probabilities for different buffer sizes* ($D = 8$).

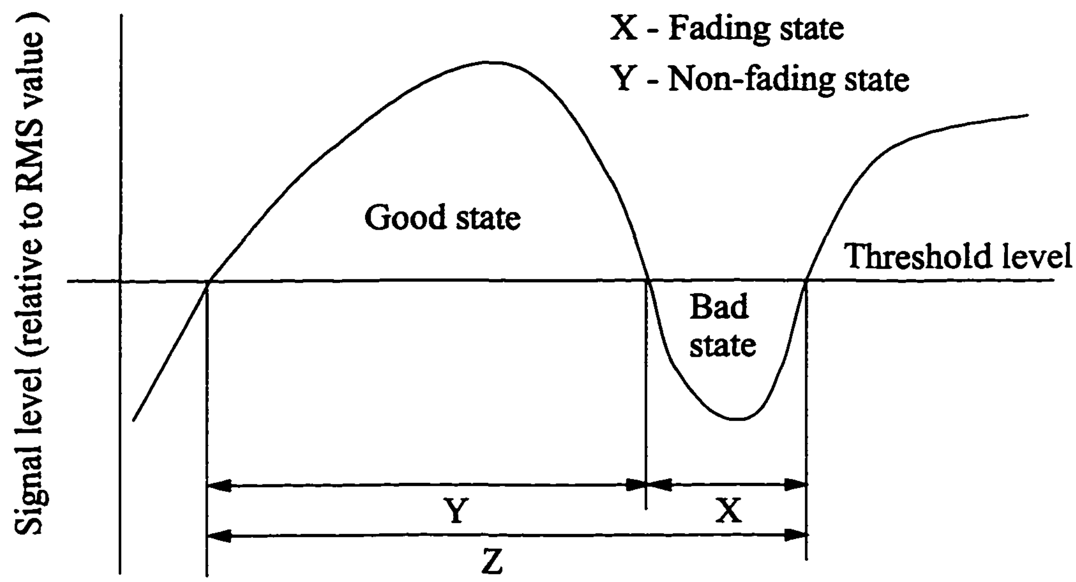


Figure 3.8. *Two-state fading channel.*

3.2 ARQ Scheme in Rayleigh Fading Environment

Rayleigh fading is caused by the multi-path arrivals of signals to the receiver and has been described in chapter 2. To consider the performance of block codes in a Rayleigh fading environment, we assume that errors are caused mainly due to fading, i.e. errors from other sources are not considered. Optimal packet length in fading channels has been analyzed for stop-and-wait protocols in [25]. We shall analyze the optimality for Selective-Repeat protocols.

As in [25], it is assumed that at any instant the channel is in one of two possible states: the received signal is above a threshold level or it is below the threshold. These situations are shown in fig. 3.8.

The probability of a packet being successfully received can be expressed as [25]

$$P_s = \frac{\bar{Y}}{(\bar{X}+\bar{Y})} Prob.[Y > T_d], \tag{3.20}$$

where T_d is the duration of a packet in seconds, $\bar{Y}$ and $\bar{X}$ are the average values of fade-free and inter-fade intervals. The density function of variable Y can be written as

$$f(Y) = \frac{1}{\bar{Y}}\, e^{-(Y/\bar{Y})}\, dY. \tag{3.21}$$

The above formula is based on [23] and [24]. For Rayleigh channels, the average fade duration and level crossing rates are given by eqn. 2.23 and eqn. 2.22 respectively. From these equations and eqn. 3.20 and eqn. 3.21, the following relations are obtained:

$$P[Y \leq T_d] = 1 - e^{-(T_d/\bar{Y})} \tag{3.22}$$

$$S = \frac{\bar{Y}}{\bar{Z}}\, e^{-(t/\bar{Y})} \tag{3.23}$$

$$= \exp\left[-(\rho + f_D\sqrt{2\,\pi\rho}\,T_d)\right], \tag{3.24}$$

where f_D is the Doppler frequency v/λ_c (v= vehicle speed, λ_c = carrier wavelength) and ρ is the ratio of the threshold power level and the average received power level. The throughput of the system with the SR scheme will be given by

$$Th = \frac{n-h}{n}\, \exp\left[-(\rho + f_D\sqrt{2\,\pi\rho}\,T_d)\right]. \tag{3.25}$$

Solving this equation for maximum throughput, the optimum value of n is obtained as,

$$n_{opt} = \frac{1}{2}\left(h + \sqrt{h^2 + \frac{4\,h}{k}}\right), \tag{3.26}$$

where $k = \frac{f_D\sqrt{2\,\pi\rho}}{R}$ and R is the data rate. The optimum value of the packet length is a function of ρ, R, λ_c and v. Dependence on the last three parameters can be combined to form a single parameter $W = \frac{\lambda_c R}{v}$. This way the equation will assume a more general character. We note that the optimum packet length is inversely proportional to the term $\frac{\lambda_c R}{V}$, which is the number of bits transmitted while the mobile unit moves by a distance of a single wavelength. Figure 3.9 shows the variation of n_{opt} with ρ and W. It gives a general idea about the variation of the optimum packet length with threshold level and other parameters.

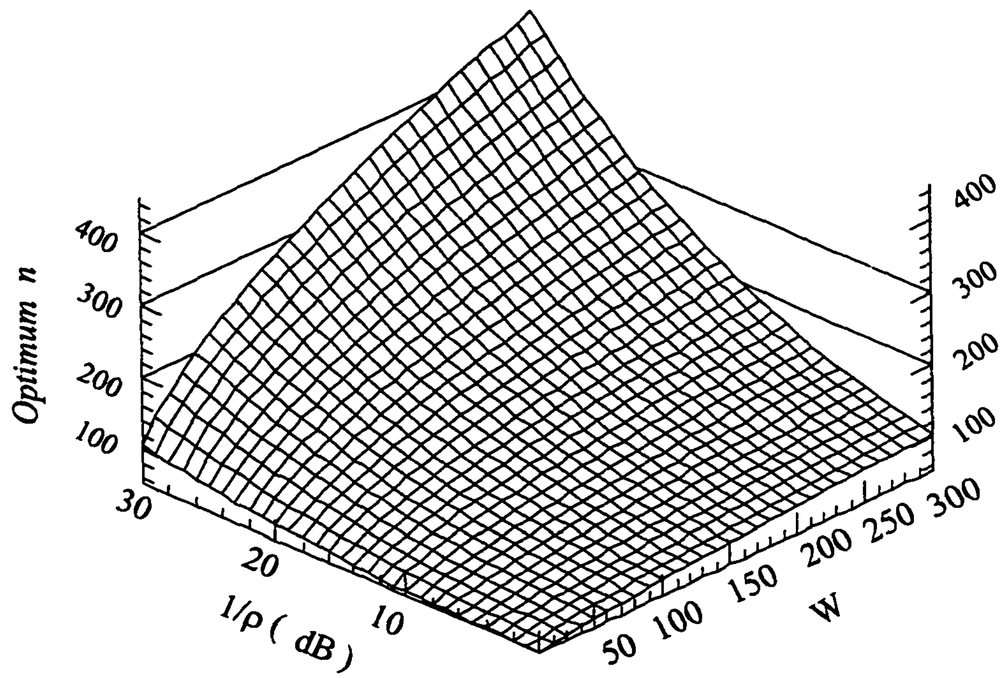


Figure 3.9. *Optimum packet length in Rayleigh environment.*

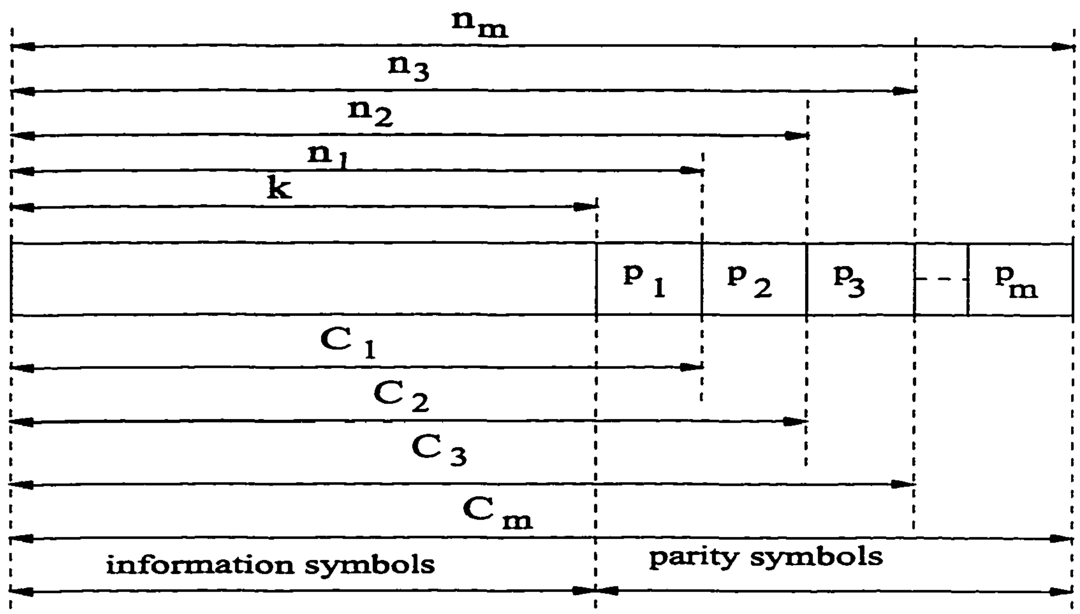


Figure 3.10. *Incremental redundancy HARQ scheme.*

3.3 HARQ-II scheme

The concept of HARQ -II scheme has been presented in chapter 2. Here we analyze some type-II schemes and discuss their suitability of being used in noisy channels. Any hybrid ARQ scheme in which the probability of successful decoding of a packet varies from transmission to transmission, can be called a type-II scheme. The idea of incremental redundant codes with variable packet length was first proposed by Davida and Reddy [27] for binary block codes. Later Mandelbaum [65] proposed punctured maximum distance separable (MDS) codes for transmitting redundancy in incremental steps by using the MDS property of Reed-Solomon codes. These schemes provide check symbols to the decoder only if they are required. The general scheme of using Incremental Redundancy Codes in a hybrid ARQ scheme is shown in fig. 3.10.

Mandelbaum [65] proposed a feedback decision scheme in which a punctured codeword is initially transmitted. If an uncorrectable error is detected, the receiver signals the transmitter to send another increment of redundancy. The procedure continues if the combined word is still uncorrectable. An (n, k) linear block code of length n symbols having k information symbols is assumed. The $n-k$ parity symbols are broken into s subblocks. Subblocks are so chosen that $t_i < t_{i+1}$ for $i = 1, 2, \cdots, u$ where t_i is the error correction capability of the codeword formed by $k + c_1 + c_2 + \cdots + c_i$ symbols. Punctured Reed-Solomon codes are very suitable candidates for this scheme. The encoding method proposed by Mandelbaum uses the Chinese Remainder Theorem [83].

We discuss two incremental redundancy coding schemes with fixed packet lengths.

3.3.1 Incremental Redundancy Codes(fixed packet length)

In most communication systems, the packet length is usually constant. This reduces the burden of synchronization and eases buffer management. Incremental redundancy codes with fixed packet size works as follows: The message is divided into smaller packets of same length and transmitted over the channel in sequence. The first packet contains all the information symbols and optionally some parity symbols. Other packets carry only parity symbols. The number of information symbols in a codeword must not be greater than the length of the packets. It is assumed here that the code used is maximum distance separable. Let us assume that the length of each packet is n. So, if the original message is subdivided into M packets, the length of the original message is $M\,n$. After reception of the first packet, the receiver erases $(M-1)n$ unreceived symbols and tries to decode the packet. If the number of errors in the sub-packet is less than the error correcting capability, if any, it is successfully decoded. Otherwise, it is stored in the receiver buffer and NAK is fed back through the reverse channel. After reception of the second subpacket, receiver again tries to decode it. If successful, it is delivered to the user or stored in the re-sequencing buffer. If decoding is failed, the symbols are combined with the previously received first subpacket and remaining $(M-2)\,n$ yet unreceived symbols are erased. Then it tries to decode again. Similar stand-alone decoding of subpackets and combining with previously received ones are continued until the packet is correctly decoded. In the $M+1$ th transmission, the first subpacket is retransmitted and the previously transmitted first subpacket is erased from the receiver buffer. The receiver from now on tries to decode the whole packet after receiving each subpacket and erasing the previously received one. The scheme is illustrated in fig. 3.11.

Let $F(i)$ be the probability of correct reception of a codeword in fewer than i

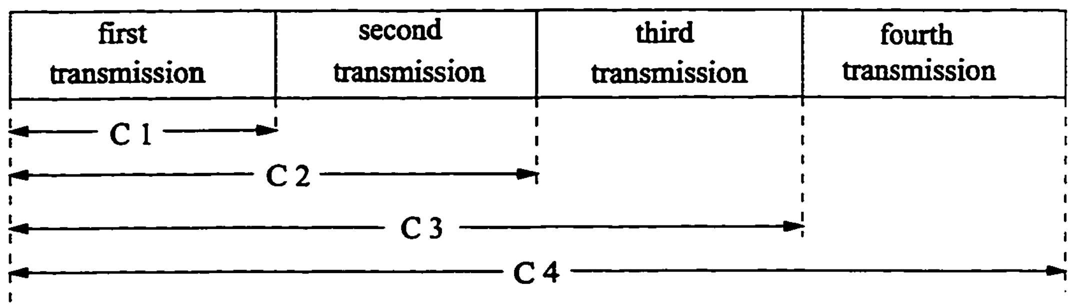


Figure 3.11. *Fixed packet length incremental redundancy scheme.*

transmissions. Then $1 - F(i)$ is the probability that the i th transmission occurs. The expected number of transmissions required to deliver a packet will be

$$E(H) = \sum_{i=1}^{\infty} [1 - F(i)] \tag{3.27}$$

$$= 1 + \sum_{i=2}^{\infty} [1 - F(i)] \tag{3.28}$$

$$= 1 + \sum_{i=2}^{\infty} \prod_{j=1}^{i-1} P(j). \tag{3.29}$$

If $P(j)$ is the probability of failure on the j th transmission,

$$1 - F(i) = \prod_{j=1}^{i-1} P(j). \tag{3.30}$$

For the above scheme, the transmission failure probability for different transmissions will be different. For the first transmission, the failure probability will be given by

$$P(1) = 1 - \sum_{i=0}^{t_1} \binom{n}{i} p^i (1-p)^{n-i}. \tag{3.31}$$

If we define $f(i)$ as the probability of i errors occurring in a packet of length n, and denote the number of errors in the i th transmission as e_i, the probability can be written as

$$1 - F(2) = P(1) = \sum_{e1 > t_1} f(e_1). \tag{3.32}$$

The second transmission fails when both, the stand-alone decoding of the second packet and decoding of combined first and the second packet, fail. The failure probability of stand-alone decoding is same as $P(1)$. When two packets are combined there joint error correcting capability is now t_2. The first packet, because it was unsuccessful, has more than t_1 errors in it. If the total number of errors in the first and the second packet exceeds t_2, the transmission will fail. So $P(2)$ can be expressed by

$$1 - F(3) = P(1)\,P(2) = \sum_{\substack{e_1>t_1, e_2>t_2\\ e_1+e_2>t_2}} f(e_1)\,f(e_2). \tag{3.33}$$

Similarly, the failure probability of the third transmission is given by

$$1 - F(4) = P(1)P(2)P(3) = \sum_{\substack{e_1>t_1, e_2>t_2, e_3>t3\\ e_1+e_2>t_2\\ e_1+e_2+e_3>t_3}} f(e_1)\,f(e_2)\,f(e_3). \tag{3.34}$$

The probability of the M th transmission failing is given by

$$1 - F(M+1) = \prod_{j=1}^{M} P(j) = \sum_{\substack{e_1>t_1,\cdots,e_M>t_1\\ e_1+e_2>t_2\\ \cdots\\ e_1+e_2+\cdots+e_M>t_M}} f(e_1)\,f(e_2)\cdots f(e_M). \tag{3.35}$$

In the scheme described above, the optimum packet combination strategy will combine all the received subpackets so far in all the possible combinations. If r subpackets have been received, the total number of combinations will be $2^r - 1$. It will be straightforward to generalize the above expressions for all the combinations included.

In our analysis, we restrict number of transmissions to M, corresponding to the transmission of M subpackets, the average number of transmissions is given by

$$E_M(H) = 1 + \sum_{j=2}^{M+1} [1 - F(j)]. \tag{3.36}$$

Then, the throughput can approximately be expressed as

$$Th = \frac{n}{n+h+b}\,\frac{1}{E_M(H)}\,F(M+1). \tag{3.37}$$

Example: We demonstrate the performance of the scheme with an example. RS code of length 64 with symbols from $GF(2^6)$ is used. We assume that t_1 is zero. A block of 16 symbols is encoded using the code (64, 16). Four packets of length 16 are formed from the symbols of a codeword. They are transmitted successively in the scheme described above.

Table 3.11. *Throughput of code combining scheme.*

p	F(2)	F(3)	F(4)	F(5)	E(H)	Throughput
0.025	0.667	1	1	1	1.33	0.5
0.05	0.44	1	1	1	1.56	0.427
0.1	0.185	0.997	1	1	1.82	0.367
0.2	0.0281	0.825	0.991	0.99	2.151	0.305
0.4	0.000282	0.0576	0.22	0.22	3.72	0.0395

For this code,

$$t_1 = 0; \quad t_2 = \frac{n}{2}; \quad t_3 = n \quad \text{and} \quad t_4 = \frac{3n}{2}. \tag{3.38}$$

Table 3.11 gives the success probabilities at different channel bit error rates. Improvement in the success probabilities for lower number of transmissions can be improved by making t_1 more than zero.

3.3.2 Concatenated Code Combining Scheme

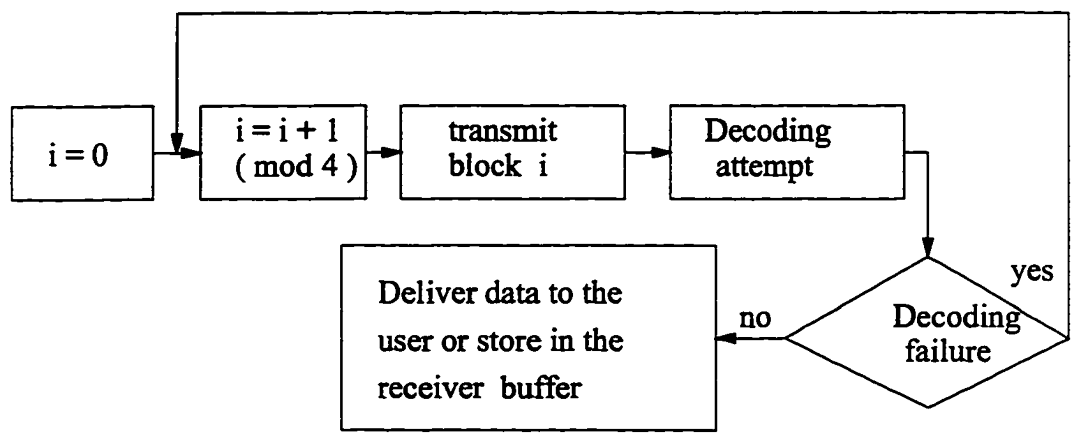
A concatenated coding scheme improves the throughput performance, particularly in fading environment. Incremental redundancy retransmission schemes improve the throughput by sending redundancy only when it is needed. Here we propose a concatenated coding scheme with incremental redundancy and analyze its performance. This is a concatenated coding scheme with an outer Reed-Solomon code and a half-rate invertible inner block code. The scheme retains all the benefits of a concatenated coding scheme with a possible reduction in the transmission of parity bits thus allowing greater throughput. The scheme is shown in fig. 3.13. There, k information symbols from $GF(2^m)$ are encoded with a Reed-Solomon $(2k, k)$ encoder. Each of these $2k$ symbols are again encoded using a rate $1/2$ code. Thus, from k information symbols, four packets, with k symbols in each of them, are created. Each packet is again encoded with CRC codes for error checking.

On the first transmission, only the first block with information symbols and CRC bits are sent over the channel. On detection of an error, the receiver transmits a NAK. When the transmitter receives a NAK, it sends the first parity block. The receiver tries to decode the codeword by erasing symbols from the first transmission. If it fails, it combines the symbols from the first and the second transmission and tries to decode the codeword. If this fails, transmitter sends the first set of inner parity symbols. Upon receiving these symbols, the decoder now tries to decode k codewords of the inner code. If all the symbols can not be corrected, these symbols are combined with the symbols from the second transmission and decoding attempt is tried. In the fourth transmission, the transmitter sends the second set of parity symbols. Similarly, the decoder now tries to decode the symbols from second transmission combined with symbols from the fourth transmission (parity symbols). It again combines both sets of inner code decoded symbols and tries the final decoding attempt.

If more than four transmissions are permitted, the cycle again continues. On subsequent transmissions, corresponding previously transmitted symbols are erased from the receiver buffer. Operation of the transmission and decoding procedures are shown using a flowchart in fig. 3.12.

For convenience, we denote the four parts of the codeword as block I, II, III and IV respectively (as in the above figure). A non-binary code can be used as the inner block code. If the symbol length is even and equals $2m$, a RS code with symbols from $GF(2^m)$ can be used. The resulting $(4, 2)$ code will correct one m bit symbol in 4 symbols in the inner codeword. The ultimate residual error probabilities will depend on the error generation process. It is not possible to give a general formula for the error probability for all situations. The derivation of error probabilities for a gaussian noise channel with bit error probability p_b is given below.

The transmission procedure is similar to the one discussed in subsection 3.3.1. Here we define $F(i)$ to be the probability of correct reception of a codeword in fewer than i transmissions. Equations 3.32- 3.35 will all be valid in this case. But because of inner decoder, the inequality constraints will be different. New equations in our case will be expressed as,



Procedures inside the block 'Decoding attempt' are as follows:

i = 1 - Attempt to decode. Check for error,

i = 2 - Attempt to decode. On failure combine block II with block I and attempt to decode again.

i = 3 - Symbols from block I and III are used for decoding with the inner code. Form decoded block I' (estimate of block I). On failure combine with block II and attemp to decode again.

i = 4 - Symbols from block II and IV are used for decoding with the inner code. Form block II' (estimate of block II). On failure, combine blocks I' and II' and attempt to decode again.

Figure 3.12. *Flowchart of incremental redundancy with concatenated coding scheme.*

First Transmission		Second Transmission	
K	CRC	K	CRC
Third Transmission		Fourth Transmission	
K	CRC	K	CRC

Figure 3.13. *Concatenated retransmission scheme.*

$$1 - F(2) = P(1) = \sum_{e1>0} f(e_1). \tag{3.39}$$

$$1 - F(3) = P(1)\, P(2) = \sum_{\substack{e_1>0, e_2>0 \\ e_1+e_2>n/2}} f(e_1)\, f(e_2). \tag{3.40}$$

$$1 - F(4) = P(1)P(2)P(3) = \sum_{\substack{e_1>0, e_2>0, e_3>0; e_{r(13)}>0 \\ e_1+e_2>n/2; e_2+e_{r(13)}>n/2}} f(e_1)\, f(e_2)\, f(e_3). \tag{3.41}$$

The probability of a 4 th transmission failure is given by

$$1 - F(5) = \prod_{j=1}^{4} P(j) = \sum_{\substack{e_1>0; e_2>0; e_{r(13)}>0; e_{r(24)}>0 \\ e_1+e_2>n/2; e_{r(13)}+e2>n/2; e_{r(13)}+e_{r(24)}>n/2}} f(e_1)\, f(e_2)\, f(e_3)\, f(e_4),$$

where $e_{r(13)}$ is the residual error after combining the first and third packet and $e_{r(24)}$ is the residual error after combining the second and the fourth packet. When non-binary inner code is used, and the symbols of the outer code has even number of bits and which equal $2m$, the codeword consists of 4 m-bit symbols. Because of 2 parity symbols, only a single m bit symbol can be corrected. The probability that decoding will be successful is the probability that either no error will occur or error will be concentrated only in a single symbol. If the BER is p_b, the probability of an inner symbol error is

$$p_{is} = 1 - (1 - p_b)^m. \tag{3.42}$$

Hence the probability of unsuccessful inner decoding is

$$p_r = 1 - (1 - p_{is})^4 - 4p_{is}(1 - p_{is})^3. \tag{3.43}$$

Probability of residual error $f(e_r)$ is given by

$$f(e_r) = \binom{n}{e_r} p_r^{e_r}(1 - p_r)^{n-e_r}. \tag{3.44}$$

Figure 3.14 shows the residual error for different channel error rates.

Above expressions for the residual error probabilities are true when nothing is known a priori. When first and the third packets are combined, it is already known that number of errors in the first and the third packets are e_1 and e_2 respectively. Therefore, the probability of a symbol error is $\frac{e_1}{n}$ for the first packet, and $\frac{e_3}{n}$ for the third. If A and B are two symbols in a particular location in first and the third packets respectively, four different situations may arise:

case	description	probability
(1)	A is in error and B is error free	$\frac{e_1}{n}(1 - \frac{e_3}{n})$
(2)	A is in error and B is in error.	$\frac{e_1 e_3}{n^2}$
(3)	A is error free and B is error free	$(1 - \frac{e_1}{n})(1 - \frac{e_3}{n})$
(4)	A is error free and B is in error	$(1 - \frac{e_1}{n})\frac{e_3}{n}$

Probability of the residual error after inner decoder will be

$$P_r = \begin{cases} p_{is}^2 & \text{case}(1) \\ 1 - 2p_{is}^2(1 - p_{is})^2 & \text{case}(2) \\ 0 & \text{case}(3) \\ p_{is}^2 & \text{case}(4). \end{cases} \tag{3.45}$$

Now, average probability of the residual error is given by

$$\begin{aligned} p_r &= \frac{e_1}{n}\left(1 - \frac{e_3}{n}\right) p_{is}^2 + \frac{e_1\, e_3}{n^2}\, 1 - 2p_{is}^2(1 - p_{is})^2 + \left(1 - \frac{e_1}{n}\right)\left(\frac{e_3}{n}\right) p_{is}^2 \\ &= \frac{e_1 e_3}{n^2} + p_{is}^2\left(\frac{e_1 + e_3}{n} - \frac{4e_1e_3}{n^2}\right) + \frac{2e_1e_3}{n^2}\, p_{is}^4 \end{aligned} \tag{3.46}$$

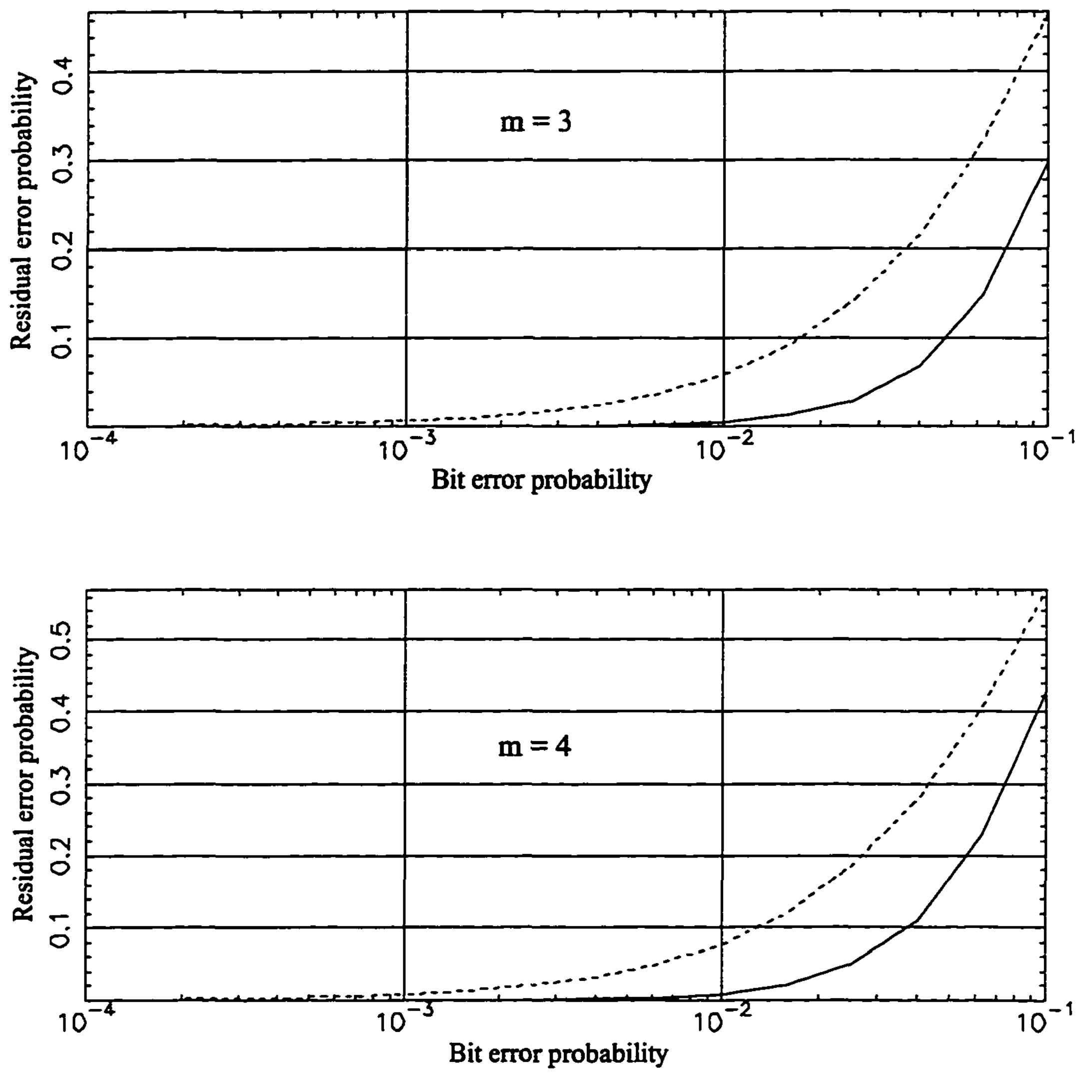


Figure 3.14. *Residual error for different channel BER.*

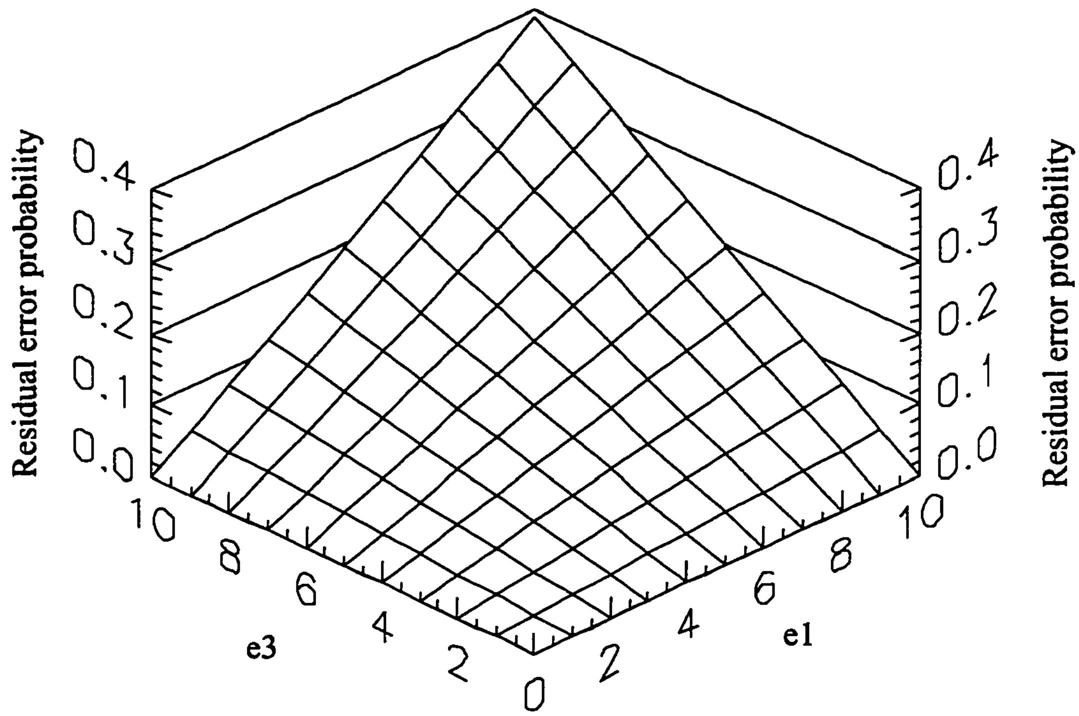


Figure 3.15. *Residual error for different channel BER* $(p = 0.01)$.

Hence, $f(e_{r(13)})$ will be given by the equation

$$f(e_{r(13)}) = \binom{n}{e_{r(13)}} p_r^{e_{r(13)}} (1 - p_r)^{n - e_{r(13)}}. \tag{3.47}$$

When the second and the fourth packets are combined, similar equation is obtained.

$$f(e_{r(24)}) = \binom{n}{e_{r(24)}} p_r^{e_{r(24)}} (1 - p_r)^{n - e_{r(24)}}. \tag{3.48}$$

In figure 3.15, the residual error probability is plotted for different e_1 and e_3. Table 3.12 gives these residual error probabilities for channel bit error probability 0.05.

In gaussian channels binary inner codes will outperform non-binary codes. But when inner codes correct burst errors, non-binary code like the one we used in our example, will perform better. Throughput for different channel BER is plotted in table 3.13.

Table 3.12. *Residual error probabilities for gaussian channel.*

e3	Residual error probabilities								
	el=0	el=1	el=2	el=3	el=4	el=5	el=6	el=7	el=8
0	0	0.0013	0.0025	0.0038	0.0051	0.0064	0.0076	0.0089	0.01
1	0.00127	0.0061	0.011	0.0159	0.0207	0.0256	0.0304	0.0353	0.04
2	0.00254	0.011	0.0195	0.0279	0.0364	0.0448	0.0533	0.0617	0.07
3	0.00381	0.0159	0.0279	0.04	0.052	0.064	0.0761	0.0881	0.1
4	0.00509	0.0207	0.0364	0.052	0.0676	0.0833	0.0989	0.115	0.13
5	0.00636	0.0256	0.0448	0.064	0.0833	0.103	0.122	0.141	0.16
6	0.00763	0.0304	0.0533	0.0761	0.0989	0.122	0.145	0.167	0.19
7	0.0089	0.0353	0.0617	0.0881	0.115	0.141	0.167	0.194	0.22
8	0.0102	0.0402	0.0702	0.1	0.13	0.16	0.19	0.22	0.25

Table 3.13. *Throughput of code combining with concatenation scheme.*

p	F(2)	F(3)	F(4)	F(5)	E(H)	Throughput
0.025	0.667	1	1	1	1.33	0.5
0.05	0.44	1	1	1	1.56	0.427
0.1	0.185	0.997	1	1	1.82	0.367
0.2	0.0281	0.825	1	0.963	2.15	0.299
0.4	0.000282	0.0576	0.995	0.061	2.95	0.0138

3.4 Summary

In this chapter, performance of different error control schemes in feedback communication system has been analyzed. In section 3.1, optimum packet length for maximum throughput has been found by computer search, and an empirical formula, based on the results of the search, has been derived.

It has been shown that the optimum packet lengths, which we found, give throughput very close to the channel capacity. The closeness to channel capacity is more pronounced in hybrid error control systems. For hybrid systems, the code parameters, namely the packet length and error correction capability of the code, have been found by computer search.

It has been shown in section 3.3, that throughput can be very close to the channel capacity only when there is no limit on the number of transmissions of a packet. When a limit is set, the throughput is reduced due to the excess amount of redundancy to be used to ensure reliability.

Two type-II hybrid schemes have been introduced in section 3.3. These schemes use a limited number of transmissions. A high degree of reliability can be achieved by these schemes, without much reduction in throughput.

Simulation has been performed to characterize buffer requirements by the receiver for storing correctly received packets. When the buffer is limited, overflow may occur in bad channel conditions. Packet loss probabilities due to limitation of buffer has been analyzed and the results have been tabulated in appendix E.

Chapter 4

Channel State Inference

Statistical inference is the method of drawing conclusions from statistical evidence, the data. These conclusions are concerned with the *states of nature*, which regulate the generation of data and give rise to the particular data at hand. Inference is never definitive with certainty and is always associated with some amount of risk. It is quite reasonable to assume that the generated data contain information about the state in question. The state is often described by a probability model that defines the probability distribution of data to be used in the inference process. An individual state is denoted by θ and the set of all probable states is denoted by Θ. In the situations that we are concerned with, θ's are real-valued parameters of a probability distribution.

The process of gathering data from the results of an experiment of chance is called *sampling*. The results are called *observations* and the collection of observations is called a *sample*. The term *statistic* denotes a descriptive measure computed from the observations in a sample. It is a function of the observations of a sample. The term applies to the relationship between independent and dependent variables - or to the random variable defined by the functional relation. If $f(x_1, x_2, \cdots, x_n)$ is a possible sample point, the functional relationship

$$Y = t(x_1, x_2, \cdots, x_n)$$

provides a transformation or mapping from the space of all sample points to the space of values of the function. The mapping induces a probability distribution and defines a random variable

$$Y = t(x_1, x_2, \cdots, x_n) = t(X).$$

A statistic, being a function on the space of values of the data Z, defines a *partition* of that space into mutually disjoint sets. A set is determined by a particular value of the function. Distinct values of the statistic define distinct partition sets. A partition Π_1 is said to be the *reduction* of a partition Π_2, if each partition set of Π_1 is precisely the union of the sets of Π_2. In such case the statistic T_1 that defines Π_1 must be a function of any statistic T_2 that defines Π_2. This function has no unique inverse unless the two partitions are exactly the same. As an example, we can cite the case of ARQ protocols. Let the statistic T_1 be defined as the value 1 or 0 depending on the successful or unsuccessful reception of a transmitted packet, and T_2 be defined as the number of actual errors occurring in a packet. In this case Π_1 is a reduction of Π_2. When the union of all the partition sets defined by a statistic form a universal set, and for all the members of a partition, if the value of the statistic is the same, the statistic is called *sufficient*. If T is a sufficient statistic, then by restricting attention to the procedures based on T, no information is discarded or overlooked. For example, for the statistic defined as the number of symbol errors in a packet, it does not matter which locations are erroneous. It conveys no new information as far as the channel symbol error probability is concerned. As a second example, for ACK/NAK based system, the actual number of errors in a packet is immaterial. The only concern is whether the value is 0 or greater than 0. A statistic is called *minimal sufficient*, when any reduction of it makes it non-sufficient.

Example: Consider the Bernoulli family with probability function

$$f(x;p) = p^x(1-p)^{1-x}, \quad x = 0, 1.$$

The above probability may be considered as the probability of a symbol in a packet being in error ($x = 0$). $f(x;p)$ denotes the probability that an observed symbol is error-free. For a sample X of independent observations, the joint probability function at x is

$$f(X;p) = p^{\sum x_i}(1-p)^{n-\sum x_i}. \tag{4.1}$$

Above probability is the probability of number of erroneous symbols in a received packet. For sample Y, the probability is

$$f(Y;p) = p^{\sum y_i}(1-p)^{n-\sum y_i}. \tag{4.2}$$

Ratio of the above functions expressed by

$$\frac{f(X;p)}{f(Y;p)} = \left(\frac{p}{1-p}\right)^{\sum x_i - \sum y_i} \tag{4.3}$$

is independent of p if and only if $\sum x_i = \sum y_i$. The sum $X_1 + X_2 \cdots + X_n$, i.e. the number of errors in a packet, forms a minimal sufficient statistic.

There are two approaches to determining the state of nature when there are many possible states. In *hypothesis testing*, one guesses or hypothesizes that θ has a certain value or lies within certain range of values and then uses the sample data to test the tenability of the hypothesis. In *estimation*, one takes a more direct approach of letting the sample data suggest the value of θ or the range within which its value lies. An *estimate* is a decision of the state of the channel based solely upon the available evidence which is in some way related to the state of nature. We denote by $\theta_j, \quad j = 1, 2, \cdots, M$, states of the channel. If the decision states a single value that the states are supposed to have, that value is called a *point estimate*. And if θ_j's are supposed to have values somewhere within an specified interval, the estimate is called an *interval estimate*. When the samples are small, it is seldom possible to estimate convincingly and within narrow limits what the value of θ is; but it is possible to reject a bad region convincingly. If the decision maker is more interested in the consequences of actions because of different states rather than the actual value of the state parameter itself, hypothesis testing is often more satisfactory approach than estimation. We study here both estimation and hypothesis testing procedures to infer the nature of state of the transmission medium.

Before proceeding further to evaluate different inference procedures, the idea of information content of a sample is introduced in the next section.

4.1 Information Content of a Sample

Inference about the state of nature requires sampling of data. Amount of data required for this purpose will vary among different statistics. The idea of information content of a sample was introduced in [20]. Before this idea is presented, the idea of *likelihood* is required. Suppose the generation of data Z is described by a model given by a probability function or density function $f(Z;\theta)$. For a given Z, the likelihood function $L(\theta; Z)$ is any function of θ proportional to $f(Z;\theta)$. This function describes the likelihood or probability of the value Z. Logarithm of likelihood function is an additive function over independent data. That is, if X and Y are independent,

$$L(\theta; X, Y) = L_1(\theta; X)L_2(\theta; X). \tag{4.4}$$

And so,

$$\log\ L(\theta; X, Y) = \log L_1(\theta; X) + \log L_2(\theta; X). \tag{4.5}$$

The log-likelihood function is called the *support* of θ. The differentiation of the *support* defines a generalized statistic called the *score*:

$$V = v(X;\theta) \equiv \frac{\delta}{\delta\theta}\log L(\theta; X) = \frac{L'(\theta)}{L(\theta)}. \tag{4.6}$$

Statistic V is sufficient, i.e., knowing its value is equivalent to knowing the value of the likelihood. The variance of the score function is called the *information* or *Fisher Information* in the experiment that yields the sample X.

$$I_x(\theta) = \mathrm{var}(V) = E\left\{\left[\frac{\delta}{\delta\theta}\log f(X;\theta)\right]^2\right\}. \tag{4.7}$$

$I_x(\theta)$ can also be represented alternately as

$$I_x(\theta) = -E\left(\frac{\delta V}{\delta\theta}\right). \tag{4.8}$$

Bernoulli, binomial, geometric, negative binomial, Poisson, negative exponential, normal, Gamma and Rayleigh density functions are all special cases of a broad class of functions called exponential family of functions. They are also called to be in Kooperman-Darmois form or Pitman-Kooperman form. Because this was first introduced in literature by R. A. Fisher [36], this is also called Fisher-Kooperman-Pitman-Darmois form (FKPD). For these functions the probability density function is expressible in the general form

$$f(x;\theta) = B(\theta)h(x)\ exp[Q(\theta)R(x)]. \tag{4.9}$$

For this family of distributions the statistic $t(X) = \sum R(X_i)$ is minimal sufficient. The following relations hold for members of this family [59].

$$E[R(X)] = -\frac{B'}{BQ'} \tag{4.10}$$

$$I_x(\theta) = (B'/B)^2 - B''/B + (Q''/Q')(B'/B). \tag{4.11}$$

Table 4.1 gives the values of B, h, Q and R for different distributions.

Table 4.1. *Exponential family of distributions.*

name	$f(x;\theta)$	$B(\theta)$	$Q(\theta)$	$R(x)$	$h(x)$
Bernoulli	$p^x(1-p)^{1-x}$	$1-p$	$\log\frac{p}{1-p}$	x	1
Binomial	$\binom{n}{x}p^x(1-p)^{n-x}$	$(1-p)^n$	$\log\frac{p}{1-p}$	x	$\binom{n}{x}$
Geometric	$p(1-p)^x$	p	$\log(1-p)$	x	1
Negative Binomial	$\binom{r+x-1}{x}p^r(1-p)^{x-1}$	$\frac{p^r}{1-p}$	$\log(1-p)$	x	$\binom{r+x-1}{x}$

Information contents of samples with above density functions are derived in Appendix A. If $T = t(X)$ is a statistic based on a sample X is considered to be an estimator for the parameter θ, the following relation will always hold:

$$\mathrm{var}(T) \geq \frac{1}{I(\theta)}. \tag{4.12}$$

This inequality is known as *Cramér-Rao Inequality.* This gives rise to the definition of efficiency of an estimator. It is ratio of var(T) and $I(\theta)$. Estimator, which has the highest possible ratio is called efficient estimator. in [20] an efficient estimator is defined to be the one with efficiency of unity. But there might be cases when the most efficient estimator cannot attain this limit [59]. As the sample size increases, the limiting value of efficiency is called the *asymptotic efficiency.*

4.2 State Estimation

The estimation problem can be generally stated as follows: Let $\{P_\theta\}$, $\theta \in \Theta$, be the class of possible underlying distributions for random data vector $\mathbf{X}$. We want to get value of θ by observing the value of $\mathbf{X}$. An estimation procedure, or *estimator*, is some systematic way of using the value of $\mathbf{X}$ to get a value of the parameter [10]. Formally, an estimator is defined as any function $\phi(\mathbf{X})$ of $\mathbf{X}$ taking values in the parameter space Θ. The definition of several terms which will come again and again in our discussion is given below:

Loss function: A loss function associates a numerical value of inaccuracy or loss to two members $\hat{\theta}$ and θ, where θ is the true state of nature and $\hat{\theta}$, the estimate. This function is denoted by $L(\hat{\theta},\ \theta)$. Loss function can be defined as desired by the statistician. Several generally used definitions are presented below

$$L(\hat{\theta},\ \theta) = |\hat{\theta} - \theta| \tag{4.13}$$

$$L(\hat{\theta},\ \theta) = \frac{|\hat{\theta} - \theta|}{\theta} \tag{4.14}$$

$$|\hat{\theta} - \theta| = (\hat{\theta} - \theta)^2. \tag{4.15}$$

The last definition is used in most situations because of the following reasons:

a: It is easiest to compute.

b: If using squared error loss, one estimator $\hat{\theta_1}$ is found to be superior to another estimator $\hat{\theta_2}$, then usually $\hat{\theta_1}$ will be better than $\hat{\theta_2}$ using any other loss function.

Risk: - Suppose that loss function $L(\hat{\theta},\ \theta)$ is specified. Then to measure how good an estimate is, its expected value is computed to find the expected loss. This expected loss value is called the *risk of the estimator*, denoted by $E_\theta\ [L(\hat{\theta},\ \theta)]$.

Biasness: If an estimator $\phi(\hat{\theta})$ has the property that $E_\theta[\hat{\theta}] = \theta$, it is called *unbiased*, otherwise it is called biased, with the bias given by

$$b(\theta) = E_\theta[\hat{\theta}] - \theta. \tag{4.16}$$

Consistency: If $\hat{\theta_n}$ is a sequence of estimators of θ defined for every sample size, then the estimation is consistent if the risk

$$E_\theta[(\hat{\theta_n} - \theta)^2] \to 0 \tag{4.17}$$

as $n \to \infty$, for all $\theta \in \Theta$.

UMVUE: T is defined to be uniformly minimum-variance unbiased estimator of $\tau(\theta)$ iff

(*a*) T is unbiased

(*b*) $\text{var}(T) \leq \text{var}(T_1)$ for any other unbiased estimator $T_1 = f(X_1, X_2, \cdots, X_n)$ of $\tau(\theta)$ for all $\theta \leq \Omega$.

To obtain a point estimate of θ, the decision maker must determine the most probable state θ_x on the basis of the available information and then estimate that $\theta = \theta_x$. Thus he estimates θ according to the following rule:

- the value yielding the largest $P(\theta_j|D)$,

- largest $P(\theta_j)$ if there are no data,
- largest $P(D|\theta_j)$ if there are data but no $P(\theta_j)$'s,

The first estimate is called the *conditionally most probable state*, the second is called the *most probable state* and the third one is called the *Maximum likelihood* estimate. If the decision maker wishes to make the estimate to be almost certainly correct, he should take the sample discriminating enough to make one $P(\theta|D)$ almost equal to unity or to make the largest $P(D|\theta_j)$ huge relative to its nearest rival. Such discrimination can generally be achieved by taking a sufficiently large number of sample observations [9].

4.2.1 Most Probable State Estimate

If the decision maker is unable to obtain any sample information about the state of nature on the occasion in question, and knows only the unconditional state probabilities $P(\theta_j)$'s, his best estimate of θ under these circumstances is the state having the maximum $P(\theta_j)$. This may be highly unconvincing in circumstances when there are more than two states and $P(\theta = \theta_x)$ is far smaller than $P(\theta \neq \theta_x)$, the probability that nature is not in the estimated state. Analogous statements are also true for the other two methods. But in those methods the decision maker can make the maximum probability convincingly large(close to 1.0) by taking sufficiently large sample of data.

4.2.2 Conditionally Most Probable State Estimate

On the basis of some data D, relevant to the occasion in question, the decision maker knows the values $P(\theta_j|D)$ for all the states. He estimates the state to be the one which corresponds to the maximum of $P(\theta_j|D)$. Three different situations might arise:

1. $P(\theta_j|D)$'s are known directly: If the maximum of the $P(\theta_j|D)$'s is $P(\theta_x|D)$, he estimates that θ is in the state θ_x.

2. $P(\theta_j|D)$'s obtained by simple application of Bayes' rule: The decision knows $P(\theta_j)$'s and $P(D|\theta_j)$'s based upon some relevant data. Required $P(\theta_j|D)$'s can be calculated using the formula

$$P(\theta_j|D) = \frac{P(D|\theta_j)P(\theta_j)}{\sum_j P(D|\theta_j)P(\theta_j)} \tag{4.18}$$

and then estimates θ to be in the state for which $P(\theta_j|D)$ is maximum.

3. $P(\theta_j|D)$'s obtained by Generalized Bayesian method: When the relevant Data consists of several sets of data $d_1, d_2, \cdots, d_k$, it is more convenient to have the Bayes' formula in a form involving d_i's rather than D. It is assumed that under each possible state of nature θ_j, the d_j's are all *mutually independent*. The $P(\theta_j|D)$'s may be obtained by using either of the two following formulas:

$$P(\theta_j|d_1\&d_2\&\cdots\&d_k) = \frac{P(d_1|\theta_j)\ P(d_2|\theta_j)\cdots P(d_k|\theta_j)}{\sum_j P(d_1|\theta_j)\ P(d_2|\theta_j)\cdots P(d_k|\theta_j)} \tag{4.19a}$$

$$= \frac{P(d_k|\theta_j P(\theta_j|d_1\&d_2\&\cdots\&d_{k-1})}{\sum_j P(d_k|\theta_j P(\theta_j|d_1\&d_2\&\cdots\&d_{k-1})}. \tag{4.19b}$$

The first equation is suitable for the situation when the decision maker has all sets of data in hand and wishes to calculate the conditional state probabilities. If data arrive sequentially, the decision maker does not know beforehand how many samples he will need ultimately. To obtain an up-to-date estimate always in hand, the second equation should be utilized. In that case, decision maker can always revise his previously calculated conditional state probabilities on the basis of the set of data acquired since then. From Formula 4.19b it is seen that $P(\theta_j|d_1\&d_2)$ can be obtained by revising already obtained $P(\theta_j|d_1)$'s on the basis of set of data d_2 and similarly $P(\theta_j|d_1\&d_2\&d_3)$'s can be obtained by revising $P(\theta_j|d_1\&d_2)$'s on the basis of subsequent set of data d_3 etc.

Thus if the decision maker has obtained K sets of data $d_1, d_2, \cdots, d_k$ that are mutually independent under each θ_j, and if he either knows all of the $P(\theta_j)$'s and $P(d_i|\theta_j)$'s as required by eqn. 4.19a or knows all of the $P(\theta_j|d_1\&d_2\&\cdots\&d_c)$'s and all of the $P(d_i|\theta_j)$'s for $i > c$, as required by eqn. 4.19b, he simply calculates $P(\theta_j|d_1\&d_2\&\cdots\&d_k)$ for each θ_j and estimates the actual state of nature to be the θ_j that maximizes the later probability.

4.2.3 Maximum Likelihood Estimate

When the decision maker has some data D and knows $P(D|\theta_j)$'s but does not have information about $P(\theta_j)$'s or $P(\theta_j|D)$'s, then he has little choice but to make a maximum likelihood estimate. This is done by estimating θ to have the value for which $P(D|\theta_j)$ is largest. By doing this he is assuming that all the $P(\theta_j)$'s have the same value, which, of course, is unlikely to be the case. This will introduce error in the estimation process. But if D is based on a sufficiently large or discriminating sample and the states differ appreciably from each other, the largest $P(D|\theta_j)$ will greatly exceed its closest rival, and the maximum likelihood estimate will almost certainly be correct even if the implicit assumption is badly in error.

Let $y_1, \cdots, y_N$ be N random variables with joint probability density function $f(y_1, \cdots, y_N; \theta_1, \cdots, \theta_p)$ which depend on parameters θ_i. Likelihood function $L(\theta; Y)$ is defined as

$$L(\theta; Y) = f(Y; \theta) \tag{4.20}$$

and the *maximum likelihood estimator (MLE)* of θ is defined as the vector $\hat{\theta}$ such that

$$L(\hat{\theta}; Y) \geq L(\theta; Y) \quad \text{for all } \theta \in \Theta. \tag{4.21}$$

Because of the invariance property of MLE, defined later in this section, if $l(\hat{\theta}; Y) = \log(\theta; Y)$ is the *log-likelihood function*, then $\hat{\theta}$ is the MLE if

$$l(\hat{\theta}; Y) \geq l(\theta; Y) \quad \text{for all } \theta \in \Theta. \tag{4.22}$$

The most convenient way to obtain the maximum likelihood estimator is to examine all the local maxima of $l(\hat{\theta}; Y)$ [28]. Local maxima points are the following points:

1. the solutions of

$$\frac{\delta l(\hat{\theta}; Y)}{\delta \theta_j} = 0, \quad j = 1, \cdots, p \tag{4.23}$$

such that θ belongs to Θ and the matrix of the second derivatives

$$\frac{\delta^2 l(\hat{\theta}; Y)}{\delta \theta_j \, \delta \theta_k} \tag{4.24}$$

is negative definite; and

2. any value $\hat{\theta}$ at the edges of the parameter space θ which correspond to maxima of $l(\hat{\theta}; Y)$. The value $\hat{\theta}$ giving the largest value of the local maxima is the maximum likelihood estimator.

Some of the desirable qualities of MLE are pointed out below:

1. If $\hat{\theta}$ is the maximum likelihood estimator of a parameter θ, then the maximum likelihood estimation of $h(\theta)$, any function of θ, is the function $h(\hat{\theta})$ of $\hat{\theta}$. This property of MLE is called the *invariance property* of an estimator.

2. The maximum likelihood method is consistent.

3. Maximum likelihood method is asymptotically as good as or better than any other estimation method.

4.2.3.1 Maximum Likelihood Estimation for ACK/NAK based system

The feedback signal for these system is just the ACK or the NAK. Suppose, in a test of N samples, success occured a times, and failure, b times. The likelihood function L can be written as

$$L = \frac{N!}{a!\, b!}\, p^a\, (1-p)^b \qquad N = a + b. \tag{4.25}$$

Logarithm of likelihood function is

$$\log L = \mathit{const.} + \ a\ \log\ p + b\ \log\ (1-p). \tag{4.26}$$

Maximum value of $\log L$ can be determined from the solution of the equation

$$\frac{d\ \log L}{dp} = \frac{a}{p} - \frac{b}{1-p} = 0. \tag{4.27}$$

The observed proportion of the sample, $p = a/(a+b)$, is the maximum likelihood estimate of p, the packet error probability.

4.2.3.2 MLE of Errors in a Packet

We assume that each packet contains n symbols and we examine a sample of N feedback signals. Probability of x_i errors in a packet is given by

$$p(x_i) = \binom{n}{x_i}\, p^{x_i}\, (1-p)^{(n-x_i)}. \tag{4.28}$$

So, the likelihood function for the estimation will be

$$L = \prod_{i=1}^{N} \binom{n}{x_i}\, p^{x_i}\, (1-p)^{n-x_i}. \tag{4.29}$$

Log-likelihood function is written as

$$\log L = \text{const.} + \sum x_i \cdot \log p + \sum (n - x_i) \cdot \log\, (1-p). \tag{4.30}$$

Maximum value of $\log L$ is decided from the solution of the equation

$$\frac{d \log L}{dp} = \frac{\sum x_i}{p} - \frac{\sum (n - x_i)}{1-p} = 0. \tag{4.31}$$

The above equation yields the result $p = \sum x_i/Nn$, i.e. average number of errors in a packet as the maximum likelihood estimator of p, the probability of symbol error.

It can easily be shown that sample mean is the most efficient unbiased linear combination of the observations in a random sample. Thus, in the above case the MLE, i.e. the sample mean is the most efficient estimator. Let Y denote the sample sum in a sequence of n independent Bernoulli trials. This is equivalent to a symbol in a packet of length n being in error. The statistic $T = Y/n = \hat{X}$ is unbiased in estimating p, the symbol error probability. The variance, which is also the mean square error is

$$E\left(\frac{Y}{n} - p\right)^2 = \text{var}\left(\frac{Y}{n}\right) = \frac{p(1-p)}{n}. \tag{4.32}$$

By definition, efficiency of the estimate is

$$e(T) = \frac{1/I(\theta)}{\text{var}\,(T)} = 1 \qquad \text{(see appendix A)}. \tag{4.33}$$

which proves the efficiency of the estimator from the information content viewpoint.

4.2.3.3 MLE of Errors and ACK/NAK combined

Error detection capability of any code used in communication can be made very high with the addition of a small amount of redundancy. But error counting capability is costlier. In an RS code, to detect an additional error, one extra overhead symbol is necessary. This can reduce the throughput by a great extent. Because the probability of higher error is low, much of the redundancy can be reduced by accepting a limited number of error counting/ detecting/correcting capability. When a decoder is unable to decode a codeword, it sends a NAK. A NAK for the estimator means the number of errors in the packet is anywhere between $t+1$ and n. So, from the point of view of the information content of the feedback signal, a NAK contains less information than signal with information about the actual number of errors.

The statistic now loses its minimal sufficiency and efficiency. Because the original statistic was minimal sufficient, any reduction of data from it will render it minimal insufficient. The question is, what will now be the maximum likelihood estimate? The probability of receiving a NAK is given by

$$P_r = \sum_{i=t+1}^{n} \binom{n}{i} p^i (1-p)^{n-i}. \tag{4.34}$$

We assume that a sample of N packets are observed by the transmitter. Let j be the number of unsuccessful packets. Error counts in other packets are given by $x_1, x_2, \cdots, x_{N-j}$. The likelihood function, its logarithm and differentiation w.r.t. p are given by

$$L = p^{\sum_{N-j} x_i} \cdot (1-p)^{\sum_{N-j}(n-x_i)} \cdot P_r^j \tag{4.35}$$

$$\log(L) = \sum_{N-j} x_i \log p + \sum_{N-j}(n-x_i) \cdot \log(1-p) + j \log(P_r) \tag{4.36}$$

$$\frac{\partial \log(L)}{\partial p} = \frac{\sum_{N-j} x_i}{p} - \frac{\sum_{N-j}(n-x_i)}{1-p} + j \frac{P_r'}{P_r}. \tag{4.37}$$

The value of P_r can be written as [50]

$$P_r = \frac{1}{B(t+1, n-t)} \int_0^p x^t (1-x)^{n-t-1} \, dx. \tag{4.38}$$

Differentiation of P_r w.r.t. p yields

$$P_r' = \frac{1}{B(t+1, n-t)} p^t (1-p)^{n-t-1}. \tag{4.39}$$

So the third term in the equation 4.36 can be written as

$$\frac{P_r'}{P_r} = \frac{p^t (1-p)^{n-t-1}}{\int_0^p x^t (1-x)^{n-t-1} \, dx}. \tag{4.40}$$

Maximum value of the function in the numerator occurs at $x = t/n - 1$ and the value of the function at $p = 0$ is 0. When the value of p is much less than $t/n - 1$, the function can be approximated by a straight line in the range $[0, p]$. So the term under integration will equal

$$\int_0^p x^t (1-x)^{n-t-1} \, dx = \frac{1}{2} \cdot p \cdot p^t (1-p)^{n-t-1}. \tag{4.41}$$

and the third term of equation 4.36 becomes equal to $j \cdot 2/p$. By equating eqn. 4.36 to zero, we get

$$\sum_{N-j} x_i (1-p) - \sum_{N-j} (n-x_i) p + 2j(1-p) = 0. \tag{4.42}$$

The estimate of $\hat{p}$ is thus obtained from the above equation as

$$\frac{\sum_{N-j} x_i + 2\,j}{(N-j)\cdot n + 2j}. \tag{4.43}$$

4.2.3.4 MLE of the Median

The median is a special case of a larger class of statistics called the percentiles. The c.d.f of the largest observation $X_{(n)}$ in a random sample of size n from a population with c.d.f. $F(x)$ can be written as

$$\begin{aligned} P(X_{(n)} \leq y) &= P(X_1 \leq y \cdots, \text{and} X_n \leq y) \\ &= P(X_1 \leq y) \cdots P(X_n \leq y) \\ &= [F(y)]^n. \end{aligned} \tag{4.44}$$

The c.d.f of the k th smallest observation in terms of the population c.d.fwill be given by

$$\begin{aligned} P\big[X_{(k)} \leq y\big] &= P(k \text{ or more of the } n \text{ observations are} \leq y) \\ &= \sum_{j=k}^{n} \binom{n}{j} [F(y)]^j [1 - F(y)]^{n-j}. \end{aligned}$$

The probability density function is given by [59]

$$f_{X_{(k)}}(y) = n\, f(y) \binom{n-1}{k-1} [F(y)]^{k-1} [1 - F(y)]^{n-k}. \tag{4.45}$$

When the percentile is uniquely defined, the asymptotic distribution of the percentile is normal with mean $\bar{X}$ and variance

$$\text{var } X_p = \frac{1}{[f(x_p)]^2} \frac{p(1-p)}{n}. \tag{4.46}$$

Median is the percentile with $p = 0.5$. So, for the median, the variance will be

$$\text{var } X_{0.5} = \frac{1}{4\,n\,[f(x_p)]^2}. \tag{4.47}$$

4.3 Sample Number Requirements for Estimation Problems

To come to a decision about the state of nature, the decision maker (adaptive encoder in our case), relies on the available feedback information from the receiver. Average sample number (ASN) requirement will be a function of the accuracy of the estimate required, and the information content of the feedback information. Feedback information returned by the receiver may be of several types:

- ACK/NAK signal
- ACK/NAK with number of symbol errors in a packet
- ACK/NAK with number of symbol errors as well as bit errors in a received packet.
- percent of time received signal power remains below a defined level.

What type of feedback is provided by the receiver depends on the feasibility of implementation of the protocol and requirements by the adaptive encoder. The estimator may use the feedback signals like ACK/NAK signals directly or it might use some other statistics derived from these signals like number of transmissions required for correct reception of a packet. If the number of errors are received as feedback signal, the estimator may use them directly or it can use derived statistics like the *mode* or *median* of number of errors. In the following subsections we analyze the sample requirements of these estimation procedures.

4.3.1 Simple ACK/NAK Based Estimation

Binomial density function is represented by the formula

$$f(x,p) = \binom{n}{x} p^x \, (1-p)^{n-x}; \quad x = 0, 1, 2, \cdots, N, \; 0 \leq p \leq 1 \tag{4.48}$$

For binomial distributions the following relations are valid:

(1) f is complete w.r.t. p.
(2) $T = \sum X_i$ is complete and sufficient for p.
(3) $\frac{T}{n}$ is UMVUE for $\mu = Np$.
$\frac{T}{Nn}$ is UMVUE for p.

In packet communications, the probability of ACK obeys binomial distribution law if the error generating process in neighboring packets are independent. In a gaussian channel without memory this condition is automatically satisfied. In fading channels if sufficient interleaving is assumed, which is usually the case, independence of neighboring packets is also guaranteed. The probability of receiving an ACK or NAK will be given by

$$Prob.(ACK) = \sum_{i=0}^{t} \binom{n}{i} p^i (1-p)^{n-i}, \tag{4.49}$$

and

$$Prob.(NAK) = 1 - \sum_{i=0}^{t} \binom{n}{i} p^i (1-p)^{n-i}, \tag{4.50}$$

where t is the error correcting capability of the code used, if any. For an RS code $t = \lfloor \frac{n-k}{2} \rfloor$. So the number of samples required to make decision will depend on the one which is lesser than the other.

In decision making problems the stopping rule which is often applied is to wait till the event with lesser probability of occurrence happens a pre-defined number of times. This number is often taken as 10 [47]. This provides a confidence interval of about $(2\,\hat{p}, 0.5\,\hat{p})$, considered to be a reasonable uncertainty. Therefore, higher probability of occurrence helps to make early decisions. With a certain fixed confidence interval the sample requirement can be considered to vary inversely with the probability of the event.

In very good or very bad channel conditions, above strategy will make it almost impossible to make any decision for a sufficiently long time. Table 4.2 and table 4.3 give some sample numbers for RS code based encoding system with block lengths of 64 and 256 respectively.

Table 4.2. *ASN for ACK/NAK based system* ($n = 64$).

Symbol Error Probability	Average Sample Number
	continued on next page

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	t=0	t=2	t=4
0.0001	1568	2.41e+08	1.33e+14
0.0002	787	3.03e+07	4.14e+12
0.0004	396	3.82e+06	1.31e+11
0.0008	201	4.86e+05	4.16e+09
0.0016	103	6.3e+04	1.35e+08
0.0032	54	8.48e+03	4.57e+06
0.0064	30	1.22e+03	1.67e+05
0.0128	23	204	7.14e+03
0.0256	53	45	416
0.0512	289	28	44
0.1024	1.01e+04	292	50
0.2048	2.34e+07	1.55e+05	6.4e+03
0.4096	4.43e+15	4.36e+12	2.73e+10
0.8192	3.46e+48	8.31e+43	1.27e+40

As we see from the table, sample numbers are extremely high for low and high error probabilities. All the subsequent estimation procedures are compared with above ACK/NAK based estimation in regards to the sample number requirements.

Table 4.3. *ASN for ACK/NAK based system* ($n = 256$).

Symbol Error Probability	Average Sample Number		
	t=0	t=2	t=4
0.0001	396	3.69e+06	1.16e+11
0.0002	201	4.7e+05	3.7e+09
0.0004	103	6.1e+04	1.21e+08
0.0008	54	8.22e+03	4.09e+06
0.0016	30	1.19e+03	1.51e+05
0.0032	23	201	6.57e+03

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0.0064	52	45	394
0.0128	271	28	44
0.0256	7.64e+03	253	47
0.0512	6.97e+06	6.35e+04	3.44e+03
0.1024	1.03e+13	2.25e+10	3e+08
0.2048	3.01e+26	1.35e+23	3.67e+20
0.4096	3.86e+59	2.43e+55	9.32e+51
0.8192	1.44e+191	2.14e+185	1.94e+180

4.3.2 Estimation based on Number of Errors in Transmitted Packet

As seen from the tables, simple ACK/NAK fails most of the time as far as channel state estimation is concerned. A better method would be sending the number of erroneous symbols in the received packet. This, of course, assumes that in some of the observed packets, the number of errors occurring is greater than zero and less than the error handling capability of the code used. Information about number of errors is obtained from the decoder. In case of RS codes, the degree of the error polynomial gives this number.

If it is assumed that symbol error occurs independently of each other, errors in a packet will be binomially distributed. In a packet of length n and symbol-error-probability p, the probability of r errors in a packet is given by

$$Prob.(X = r) = \binom{n}{r} p^r \, (1-p)^{n-r}. \tag{4.51}$$

The average number of errors in a packet and its variance are given by,

$$E_{av} = np \tag{4.52}$$

$$Var(E) = np(1-p). \tag{4.53}$$

In the case of RS codes, through the feedback channel, the number of errors or NAK's can be sent to the transmitter. In most HARQ systems the error correction capability of the code is higher than the most probable number of errors in a packet. So when a NAK is received, the most probable number of errors in that packet is $t+1$. And average number of errors as estimated by the transmitter will be given by

$$E_{av} = \sum_{r=0}^{t} r \binom{n}{r} p^r (1-p)^{n-r} + (r+1) \left(1 - \sum_{i=0}^{t} \binom{n}{r} p^r (1-p)^{n-r}\right). \qquad (4.54)$$

In Table 4.4 actual and estimated values of the average is shown. It is assumed that sufficient number of samples has been taken.

Table 4.4. *Average symbol error estimate* ($n = 256$).

Symbol Error Probability	Actual	Estimated				
		t=2	t=6	t=10	t=14	t=18
0.0001	0.0256	0.0256	0.0256	0.0256	0.0256	0.0256
0.0002	0.0512	0.0512	0.0512	0.0512	0.0512	0.0512
0.0004	0.1020	0.1020	0.1020	0.1020	0.1020	0.1020
0.0008	0.2050	0.2050	0.2050	0.2050	0.2050	0.2050
0.0016	0.4100	0.4090	0.4100	0.4100	0.4100	0.4100
0.0032	0.8190	0.8080	0.8190	0.8190	0.8190	0.8190
0.0064	1.6400	1.5200	1.6400	1.6400	1.6400	1.6400
0.0128	3.2800	2.4400	3.2500	3.2800	3.2800	3.2800
0.0256	6.5500	2.9500	5.7500	6.4900	6.5500	6.5500
0.0512	13.1000	3.0000	6.9700	10.4000	12.4000	13.0000
0.1024	26.2000	3.0000	7.0000	11.0000	15.0000	18.9000
0.2048	52.4000	3.0000	7.0000	11.0000	15.0000	19.0000
0.4096	104.8000	3.0000	7.0000	11.0000	15.0000	19.0000
0.8192	210.0000	3.0000	7.0000	11.0000	15.0000	19.0000

It is seen from the table that estimated value differs from the actual one only for higher error rates and for low error correcting codes. For low symbol-error-rates and higher error correcting codes, the difference is negligible.

The information content of an ACK/NAK signal and a feedback signal with actual number of errors in the packet has been derived in appendix A. They are reproduced

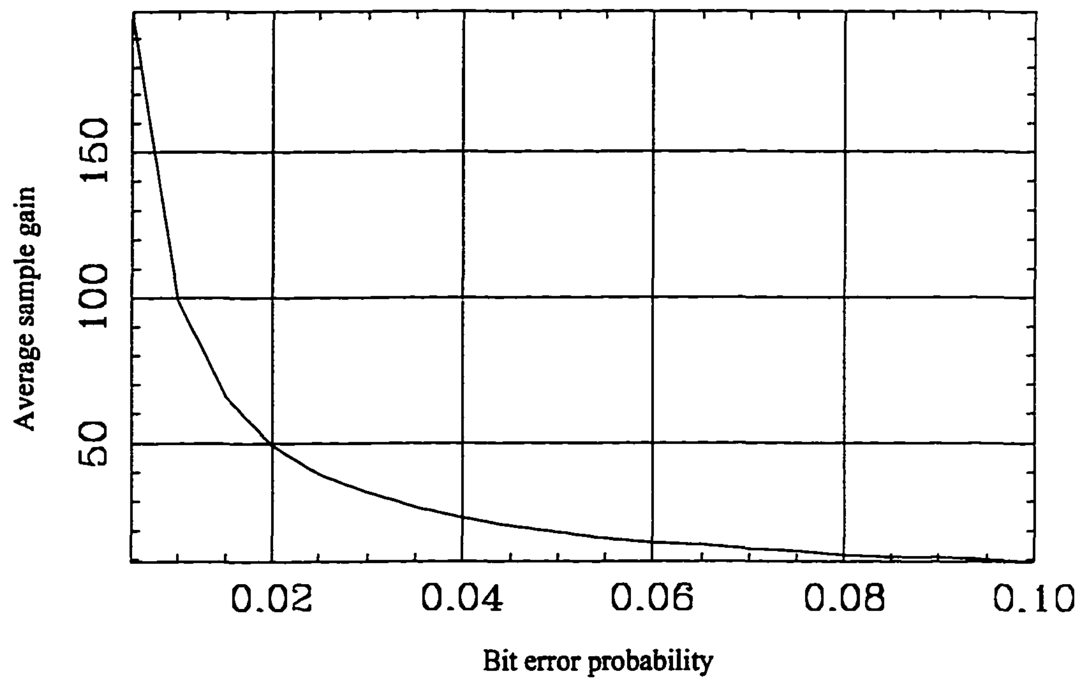


Figure 4.1. *Average sample gain for fading channels.*

here

$$
\begin{aligned}
I_X &= \frac{n}{(1-p)^2} && \text{ACK/NAK signal} \\
&= \frac{n}{p(1-p)} && \text{error information.} \qquad (4.55)
\end{aligned}
$$

We define *average sample gain* (ASG) as the reduction in requirement of average sample number for achieving a particular confidence level. The ASG in the above case is $\frac{(1-p)}{p}$. The gain for different symbol error probabilities are shown in fig. 4.1.

4.3.3 Estimation Based on Other Parameters

There are numerous other parameters which can be used in channel state estimation. They may be useful in certain situations because of the ease of their use in estimating the channel state. Several of them are discussed here:

median: The median is the value of the middle unit when a set of numbers are arranged in a line in order of increasing value. If N is odd, median is the middle unit and it is the midpoint between the values of the two mid-most units when N is even. In most error correcting systems, the value of t, the error-correcting-capacity of the code, is higher than the average number of errors in the packet. So the estimation of channel state can be done by calculating the median of the number of errors occurring in a packet. For this procedure, the exact number of errors in a packet, while receiving a NAK, is not required. The value of median, in this case, can be found without approximation or guessing.

mode: Though this is the crudest index of location, it can be helpful in certain situations. It has been used for channel state estimation in [97]. It is the value that occurs with the greatest frequency. For binomial probability distribution, this value is

$$\text{mode} = \lfloor (n+1)\, p \rfloor \tag{4.56}$$

This method will also be useful as in the previous case with the median for counting errors while NAK is received.

retransmission count: This statistic is derived from the received ACK/NAK and may be obtained as a by-product of the storage process of transmitted packets in the transmission buffer. A transmission count of packets can easily be stored along with other information and in practice, it is used in many cases.

4.4 Hypothesis Testing

A problem in which one of two actions must be chosen is said to be a problem in hypothesis testing. The least complicated hypothesis testing problems involve only two states of nature θ_1 and θ_2 or many states of nature which can be divided into two classes. Let θ_1 belongs to the set $\mathcal{N}_\infty$ and θ_2, to the set $\mathcal{N}_\in$. Two actions are defined: action a_1, optimal for states belonging to $\mathcal{N}_\infty$ and action a_2 optimal for states belonging to $\mathcal{N}_\in$. For problems in testing hypothesis, a *test* is defined to be the strategy chosen. If the data Z is observed, $f(Z|\theta)$ is called the *likelihood* of θ.

4.4.1 Likelihood Ratio Test

The likelihood ratio is defined by

$$\lambda(Z) = \frac{f(Z|\theta_1)}{f(Z|\theta_2)}. \tag{4.57}$$

A test is a likelihood-ratio test if there is a number k such that the test leads to

$$\begin{array}{rll} a_1 & \text{if} & \lambda(Z) > k \\ a_2 & \text{if} & \lambda(Z) < k \\ a_1 \text{ or } a_2 & \text{if} & \lambda(Z) = k \end{array}$$

For the distribution of number of errors in a packet, which follows the binomial probability law, the likelihood ratio test is reduced to the form

$$\begin{array}{rll} \text{Accept } H_1 & \text{if} & \hat{p} > k' \\ \text{Reject } H_1 & \text{if} & \hat{p} < k' \\ \text{Take either action} & \text{if} & \hat{p} = k' \end{array}$$

where k' is given by right hand side of eqn. C.3. As an example, let p_1 be 0.3 and p_2 be equal to 0.1 . Different values of k' for $n = 32, 64, 128$ and 256, are plotted in fig. 4.2.

4.4.1.1 Sample Number Analysis

For likelihood ratio tests, average sample number (ASN) is a function of the error probabilities ϵ_1 and ϵ_2 defined as

$$\epsilon_1 = P\{\hat{p} < k'|p_1\} \tag{4.58}$$

$$\epsilon_2 = P\{\hat{p} \geq k'|p_2\}. \tag{4.59}$$

When sample number is large, $\hat{p}$ is approximately normally distributed with mean p and standard deviation $\sqrt{p(1-p)/n}$. When the error probabilities are given, two numbers s_1 and s_2 can be found such that ϵ_1 is the area under the normal cdffrom $-\infty$ to k', and ϵ_2 is the area under the normal cdffrom k' to ∞ . These parameters are shown in fig. 4.3. We get the following relations:

$$p_1 - k' = s_1\sqrt{p_1(1-p_1)/n}$$
$$k' - p_2 = s_2\sqrt{p_2(1-p_2)/n}.$$

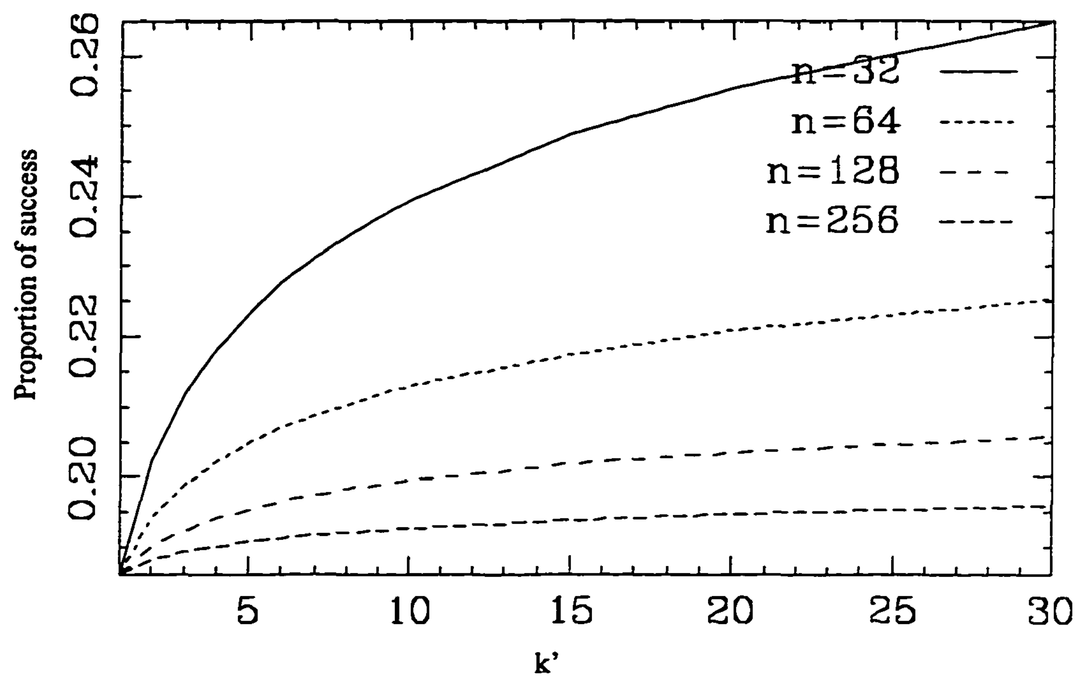


Figure 4.2. *Success proportions for different sample numbers.*

Solving the above two equations, the value of k' is obtained as

$$k' = \frac{p_1 + p_2 \cdot \frac{s_1}{s_2} \cdot \sqrt{\frac{p_1(1-p_1)}{p_2(1-p_2)}}}{1 + \frac{s_1}{s_2} \cdot \sqrt{\frac{p_1(1-p_1)}{p_2(1-p_2)}}}. \tag{4.60}$$

and n is given by

$$n = \frac{s_1^2 p_1(1-p_1)}{(p_1 - k')^2} = \frac{s_1^2 p_1(1-p_1)}{\left(\frac{(p_1-p_2)\frac{s_1}{s_2}\sqrt{\frac{p_1(1-p_1)}{p_2(1-p_2)}}}{1+\frac{s_1}{s_2}\sqrt{\frac{p_1(1-p_1)}{p_2(1-p_2)}}}\right)^2} \tag{4.61}$$

or

$$n = \frac{s_2^2 p_2(1-p_2)}{(k' - p_2)^2} = \frac{s_2^2 p_2(1-p_2)}{\left(\frac{p_1-p_2}{1+\frac{s_1}{s_2}\sqrt{\frac{p_1(1-p_1)}{p_2(1-p_2)}}}\right)^2}. \tag{4.62}$$

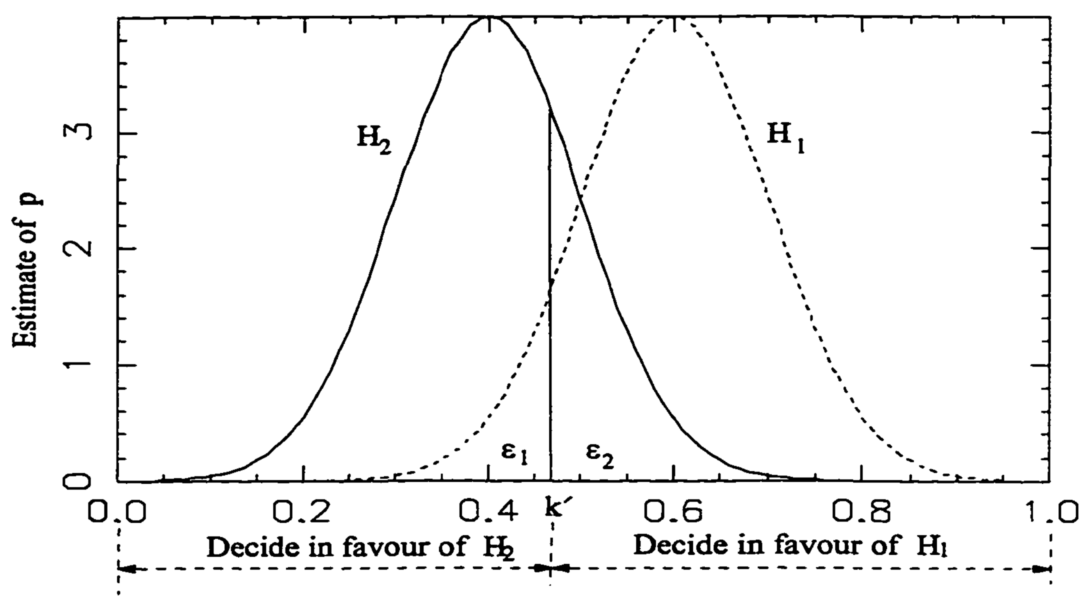


Figure 4.3. *Success proportions under different hypothesis.*

4.4.2 Sequential Likelihood-ratio Tests

A strategy which involves the design of an experiment must always consider the number of observations to take. In general, it is possible to decide after each observation, whether to continue taking further observations. If the decision is taken to make no more observations, a decision has to be taken about the next action. A plan which provides the rules for making these decisions after each observation is called a *sequential strategy* [14]. Let X_i denote the outcome of the i th experiment having the density function $f(x|\theta)$. The likelihood ratio based on the first n observations $Z_n = (X_1, X_2, \cdots, X_n)$ by

$$\Lambda_n = \frac{f(X_1|\theta_1)f(X_2|\theta_1)\cdots f(X_n|\theta_1)}{f(X_1|\theta_2)f(X_2|\theta_2)\cdots f(X_n|\theta_2)}. \tag{4.63}$$

A test is said to be a *sequential likelihood-ratio test* if there are two numbers A and B and after the n th observation, the test calls for:

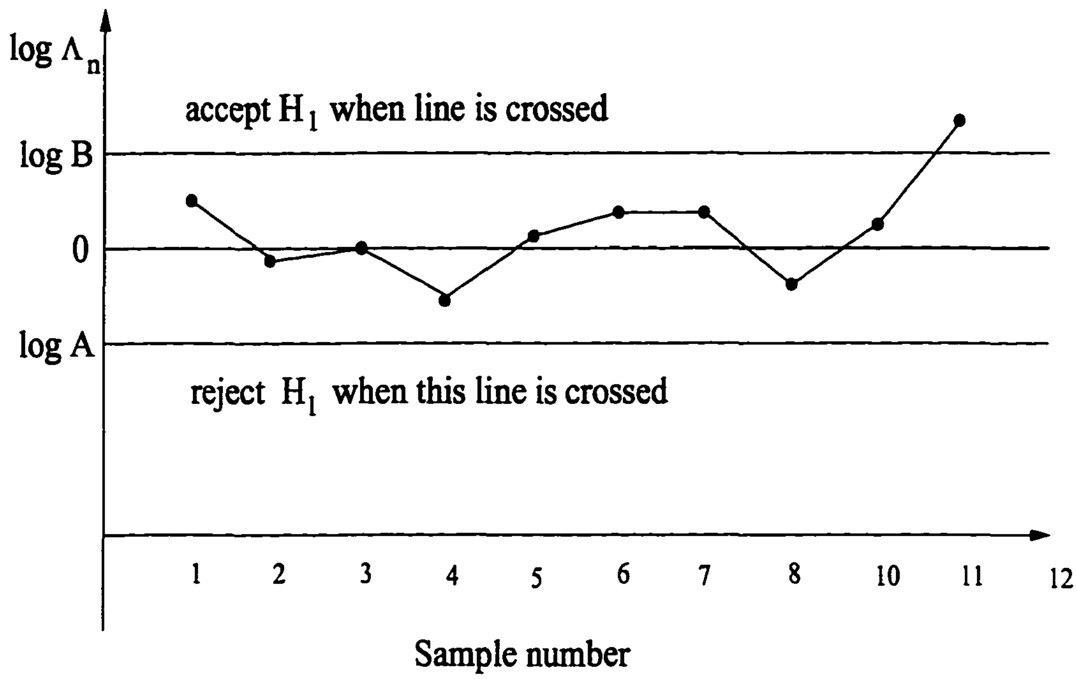


Figure 4.4. *Sequential likelihood ratio tests.*

Another observation	when $B > \Lambda_n > A$
Acceptance of H_1	when $B < \Lambda_n$
Rejection of H_1	when $A > \Lambda_n$

where $H_1 : \theta = \theta_1$ and $H_2 : \theta = \theta_2$. The inequality for continued sampling can be written in the form

$$\log\ A < \log\ \Lambda_n < \log\ B \tag{4.64}$$

The concept is illustrated in fig. 4.4. The decision is made whenever Λ_n crosses the thresholds log A or log B. The relationship between the threshold values and the proportion of packet failures are derived in appendix C.

4.4.2.1 Sample Number Analysis

Let $\epsilon_1 = P\{\text{accept } H_2|H_1\}$ and $1 - \beta = P\{\text{reject } H_1|H_2\}$. For sequential likelihood-ratio tests discussed above, the values of A and B are found from the error probabilities

ϵ_1 and β [35].

$$A \geq \frac{\epsilon_1}{1-\beta}$$
$$B \leq \frac{1-\epsilon_1}{\beta}. \tag{4.65}$$

Wald's approximations [93] consist of replacing above inequalities with equalities. So we have

$$A = \frac{\epsilon_1}{1-\beta}$$
$$B = \frac{1-\epsilon_1}{\beta}. \tag{4.66}$$

The required sample size will be given by [59]

$$E(N) = \frac{E(\log \Lambda_n)}{E(Z)}. \tag{4.67}$$

When the channel state is θ,

$$E_\theta(\log \Lambda_N) \doteq \log\ A\ \pi(\theta) + \log\ B\ [1 - \pi(\theta)], \tag{4.68}$$

where $\pi(\theta)$ is the power function or the probability that given test rejects H_1, when the state is actually θ. $E(Z)$ is given by

$$E(Z) = p \log(p_1/p_2) + (1-p)\log[(1-p_1)/(1-p_2)], \tag{4.69}$$

where p is given by the equation

$$p = \frac{1 - [(1-p_1)/(1-p_2)]^j}{(p_1/p_2)^j - [(1-p_1)/(1-p_2)]^j}. \tag{4.70}$$

As $j \to 0$, this becomes

$$p = \frac{-\log[(1-p_1)/(1-p_2)]}{\log(p_1/p_2) - \log[(1-p_1)/(1-p_2)]}. \tag{4.71}$$

In terms of j, the power function is given as

$$\pi(\theta) = \frac{B^j - 1}{B^j - A^j}. \tag{4.72}$$

As $j \to 0$,

$$\pi(\theta) = \frac{\log\, B}{\log\, B - \log\, A} \tag{4.73}$$

Now, the expression can be expressed as

$$E(N|\theta) = \frac{1}{E(Z|\theta)}\{\log\, A\; \pi(\theta) + \log\, B\; [1 - \pi(\theta)]\} \tag{4.74}$$

Although finite, the required N can be very large in a single experiment. In practice, a bound n_0 is established beforehand such that if no decision is yet done by the n_0 th stage, sampling is stopped and H_1 is accepted if $\Lambda_{n_0} > 1$ and rejected otherwise. The effect of such modification alters the error size of the test slightly if n_0 is large [93]. The decision line will be altered for this restricted case. Figure 4.5 gives this altered boundary.

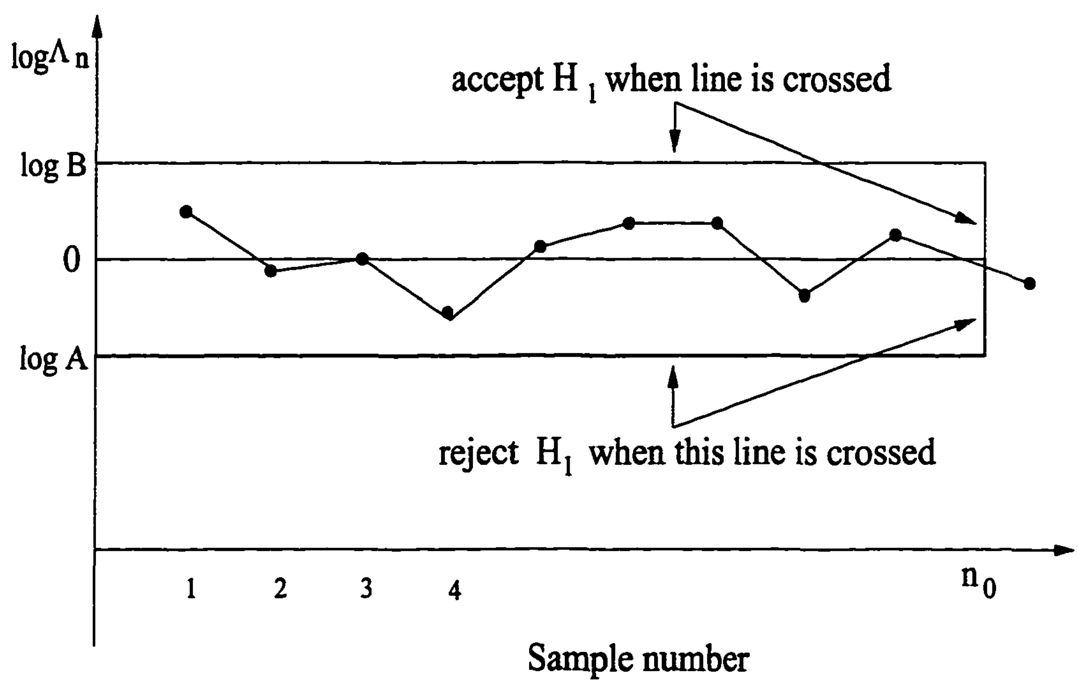


Figure 4.5. *Bounded hypothesis testing(adapted from[59]).*

4.5 Summary

The main contribution of this chapter has been the introduction of the information content of a sample in ARQ based communications. Section 4.1 introduces the idea of using this criteria for the purpose of the comparison of different sampling techniques with regard to the ASN requirement to achieve a particular probability of the risk function.

In section 4.2, different channel state estimation techniques have been discussed. Emphasis has been given on maximum likelihood estimation techniques. MLE estimates for different sampling procedures have been derived.

Hypothesis testing procedures have been discussed in section 4.4. Expressions for ASN requirements as functions of error probabilities, have been derived.

A comparative study of sample requirements for estimation problems has been done in section 4.3.

Chapter 5

Application of State Inference to Adaptive Error Control Schemes

An adaptive system is one that adjusts or adapts to changing conditions. We define 'adaptive' as 'modification to meet new condition'. Since the new condition has to be learned, an implicit or explicit learning feature is an essential part of any adaptive system. A performance monitoring feature is also a characteristic of an adaptive system. The system continuously or after every discrete interval monitors its own performance and when new condition arise which degrades the system performance, the system learns these new conditions and changes its parameters accordingly. An ARQ system has the monitoring feature but is not adaptive due to the lack of a learning process.

The introduction of adaptivity in a communication system means that measurement of signal and channel parameters is now part of the system design. The goodness of measurement is relevant insofar as it relates to system performance. During the design phase, the signal or channel conditions are unknown. This makes the design process very difficult. Things become easier when certain constraints like maximum delay, maximum packet length etc. are imposed on it. In most cases, the design criteria is the optimization of throughput efficiency under the given constraints. In chapter 3 this problem of optimization was addressed for a Gaussian channel and for a given channel BER. In this chapter we shall study the optimization procedures in a time-varying channel.

In order to reduce the number of retransmissions in fixed-rate ARQ/FEC schemes, relatively low rate codes are often used. If a fixed-rate scheme is used, on a time-varying channel, it will be very inefficient during periods when the channel is reliable.

On time-varying channels it is therefore desirable to use adaptive ARQ/FEC schemes that adjust channel code rate to match the current channel conditions. Examples of time-varying channels are mobile channel, HF channel, tropospheric channel and channels affected by rain fading. For the scheme to work it is required to construct a family of codes with decreasing code rate where all the code symbols of a higher rate code must be embedded in the lower rate codes. There are three different approaches to do this. First, a known low-rate parent code can be successively punctured to obtain higher rate codes. Second, starting from a high rate code, additional check symbols can be successively added to form lower rate code. Finally, completely different coding schemes can be used for different channel conditions.

Cygan and Offer [22] proposed a method of constructing a family of codes starting from higher rate codes while several other authors [27], [65], used punctured codes. Fukasawa [38] used rate 1/2 diffused code and 3 out of 5 voting majority-logic-decoding code in different channel conditions to improve throughput of the system. Sato in [76] used 4 different modes of transmission. Codes used for different modes were: plain ARQ, BCH(31,26,3)+ARQ, diffused code + ARQ and 3 – out-of-5 Majority decoding + ARQ. An FEC scheme for matching code to the prevailing channel conditions was reported in [13]. The method is based on convolutional codes with Viterbi decoding and consists of combining noisy packets to obtain a packet with a code rate low enough to achieve the specified error rate. Other adaptive decoding schemes are reported in [84], [92], [64], [49]. Hybrid ARQ schemes based on convolutional codes with sequential decoding on a memoryless channel were reported in [30], [29]. A type-II hybrid ARQ scheme formed by concatenation of convolutional codes was presented in [63].

Punctured convolutional codes has the privilege of availability of wide range of code rates without changing the basic codec structure. This has motivated use of this code with soft-decision Viterbi decoding [11], [99], [100], [41].

A generic adaptive ARQ scheme is shown in fig. 5.1. *Incoming message buffer* stores the incoming message till they are encoded for transmission. *Transmitter buffer* stores the packets after transmission till they are positively acknowledged by the receiver. *Input selector* decides whether to transmit a new packet or packet previously transmitted. If NAK for any packet is arrived, that packet is retransmitted from the

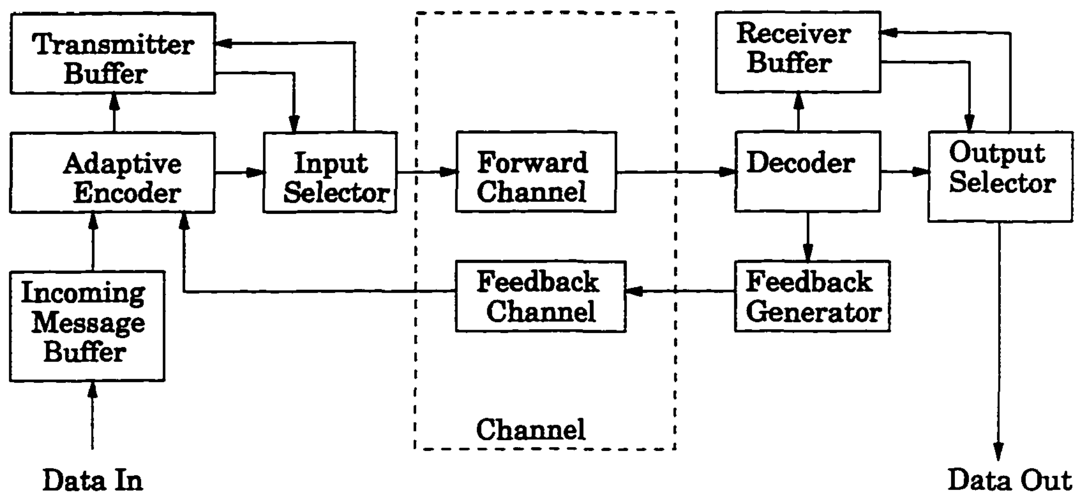


Figure 5.1. *Adaptive coding scheme for fading channels.*

buffer instead of a new packet. The *feedback generator* generates feedback information regarding the status of the received packet. Type of the feedback signal depends on the protocol used. *Receiver buffer* stores all the correctly received packets which are not in sequence, i.e. when some earlier packet is still not received correctly. The *adaptive encoder* analyzes the information received through the feedback channel and decides the code to be used in subsequent transmissions.

5.1 System Model

Many different models have been proposed to characterize propagation in different mobile channels. Most of them represent digital error sequences in real digital communication links [53], [73], [91], [61], [60]. Error sequences in these channels are characterized by Markov chains. The channel we are going to use is also modeled by a finite state Markov chain. Each state is considered to be stationary with constant parameters. The channel is considered to be in one of M states at any instant of time. The transition matrix, whose elements are the probabilities of transitions from one state to another, is given by the matrix Π, expressed as

$$\Pi = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1M} \\ p_{21} & p_{22} & \cdots & p_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ p_{M1} & p_{M2} & \cdots & p_{MM} \end{bmatrix} \tag{5.1}$$

where p_{ij} is defined as the probability of transition from state i to state j. Sums of elements of any row of the matrix is unity. The matrix $\Pi(n)$ is defined as

$$\Pi(n) = \begin{bmatrix} p_{11}(n) & p_{12}(n) & \cdots & p_{1M}(n) \\ p_{21}(n) & p_{22}(n) & \cdots & p_{2M}(n) \\ \cdots & \cdots & \cdots & \cdots \\ p_{M1}(n) & p_{M2}(n) & \cdots & p_{MM}(n) \end{bmatrix}, \tag{5.2}$$

whose element $p_{ij}(n)$ is the probability of transition from state i to state j in n steps. $\Pi(n)$ satisfies the matrix equation

$$\Pi(n) = \Pi^n \tag{5.3}$$

A vector $\pi'(0)$ is defined as

$$\pi'(0) = \begin{bmatrix} p_1(0) & p_2(0) & \cdots & p_M(0)] \end{bmatrix} \tag{5.4}$$

whose elements are the probabilities of different states at the beginning of the process. Probabilities of achieving different states after n steps is then given by $\pi'(n) = \pi'(0)Pi(n)$. As n approaches infinity, Π^n approaches the matrix all of whose rows are same and is the stationary state probability vector $\mathbf{W}$, given by

$$\mathbf{W} = \begin{bmatrix} w_1 & w_2 & \cdots & w_M \end{bmatrix}. \tag{5.5}$$

The stationary state probability vector can be found from the equation

$$\mathbf{W} \cdot \Pi = \mathbf{W}. \tag{5.6}$$

It can be proved that the solution of the above equation is any row of the matrix $P(n)$ when $n \to \infty$. Dwell time in channel state S_i is defined as the number of steps the Markov chain stays in state S_i. In our case, this may be the number of packets transmitted during the channels stay in state S_i. This can be obtained from the transition matrix as

$$D_i = \sum_{j=1}^{\infty} i\, p_{ii}^{j-1}(1-p_{ii}) = \frac{1}{1-p_{ii}}. \tag{5.7}$$

Throughput matrix $\mathbf{T}$ is defined as the matrix

$$\mathbf{T} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & \cdots & T_{1M} \\ T_{21} & T_{22} & T_{23} & \cdots & T_{2M} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ T_{M1} & T_{M2} & T_{M3} & \cdots & T_{MM} \end{bmatrix} \tag{5.8}$$

where T_{ij} is the throughput of the system when channel is in state S_i, and code C_j is used for encoding. The maximum throughput is obtained when channel state and the code used match, i.e., code C_i is used when channel is in state S_i. The maximum throughput will be given by

$$T_{ideal} = \mathbf{W} \cdot [T_{11}, T_{22}, \cdots, T_{MM}]' \tag{5.9}$$

During channel state estimation process, errors occur mainly due to insufficient number of samples. As has been shown in chapter 4, the estimation error can be reduced to any desired level by allowing the sample number to be greater than certain lower limit which is a function of the desired accuracy. We define the matrix A as

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1M} \\ a_{21} & a_{22} & \cdots & a_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ a_{M1} & a_{M2} & \cdots & a_{MM}, \end{bmatrix}$$

where a_{ij} is the probability that we accept the channel to be in state j while the actual channel state is i. This matrix can be called the **acceptance matrix**. In case of correct channel estimation, the acceptance matrix reduces to identity matrix $I(M)$. Considering the situation, when estimation error might occur, the throughput o f the system will be

$$T_{real} = \sum_{i=1}^{M} w_i \begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{iM} \end{bmatrix} \begin{bmatrix} T_{i1} & T_{i2} & \cdots & T_{iM} \end{bmatrix}'. \tag{5.10}$$

During the channel state estimation stage, the transmitter uses the code which it used in the previous channel state. It keeps on using the code unless it detects the change of state. If $p(j|i)$ denotes the probability that previous channel state was i, when the present state is j, then

$$p(j|i) = \frac{p_{ij}}{\sum_{1=1}^{M} p_{ij}}. \tag{5.11}$$

In the beginning of state j, the transmitter continues using the code C_i, till a decision about the channel state is reached. If the ASN for code C_i is denoted by ASN(i), average throughput at state j will be

$$E[T_j] = \frac{D_j - ASN(i)}{D_j} T_{jj} + \frac{ASN(i)}{D_j} \sum_{i=1}^{M} p(j|i)\, T_{ji}. \tag{5.12}$$

Hence the system throughput is given by

$$T_{system} = w_j \left(\sum_{i=1}^{M} \frac{D_j - ASN(i)}{D_j} p(j|i)\, T_{jj} + \sum_{i=1}^{M} \left(\frac{ASN(i)}{D_j} \sum_{i=1}^{M} p(j|i)\, T_{ji} \right) \right). \tag{5.13}$$

5.2 Adaptive Scheme Based on Simple ARQ

When channel condition changes, the probability of packet error also changes accordingly. For a particular channel condition packet error rate (PER) is a function of packet length. There exists an optimum packet length for which the throughput will be maximum. By measuring channel BER, the packet length can be so adjusted that it matches the BER. Such an adaptive method was proposed in [3]. Optimum packet lengths for different BER's have been found in the chapter 3. By measuring the channel BER by any suitable method, the codelength can be changed adaptively to achieve near optimum throughput. The code parameter can be chosen by using the formula given by 3.3.

Example: We consider a two state channel described in fig. 5.2. The transition probabilities and bit error rates for the states are shown in the figure. Optimum blocklengths for the two states are found from eqn. 3.4, and are 210 and 143 respectively for the states $S1$ and $S2$. Stationary state probabilities for the two states are

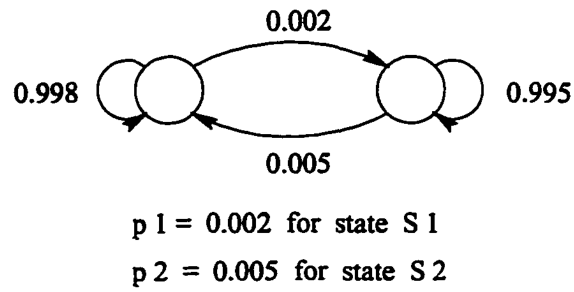


Figure 5.2. *Two-state channel example.*

found from the state transition matrix

$$\Pi = \begin{bmatrix} 0.998 & 0.002 \\ 0.005 & 0.995, \end{bmatrix} \tag{5.14}$$

and are given by the vector $\mathbf{W}$, which, for a transition matrix of dimension 2×2, is given by

$$\mathbf{W} = \left[\frac{1 - p_{22}}{2 - p_{11} - p_{22}} \quad \frac{1 - p_{11}}{2 - p_{11} - p_{22}}\right]. \tag{5.15}$$

For our example,

$$\mathbf{W} = [\,0.714 \quad 0.286\,] \tag{5.16}$$

We define a matrix of failure probabilities,

$$F = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}, \tag{5.17}$$

where f_{ij} is the probability of failure when channel is in state i and used packet length is optimum for state j. For the present example,

$$F = \begin{bmatrix} 0.343 & 0.25 \\ 0.65 & 0.513, \end{bmatrix} \tag{5.18}$$

In the maximum likelihood ratio test, the threshold value for the proportion of failure is a function of the failure probabilities and error tolerance. Table 5.1 gives the threshold values and ASN for packet lengths 210 and 143. In an ideal situation, the packet length chosen always matches the channel state. For an ideal case, the average throughput will be

$$T_{ideal} = w_1\, T_{11} + w_2\, T_{22} \tag{5.19}$$
$$= 0.714 \cdot 0.507 + 0.286 \cdot 0.324 \tag{5.20}$$
$$= 0.455 \tag{5.21}$$

Table 5.1. *ASN and threshold values for different error tolerances.*

ϵ	n=210		n=143	
	k'	ASN	k'	ASN
0.1	0.496	15.7	0.372	20.6
0.05	0.496	25.9	0.372	34
0.04	0.496	29.3	0.372	38.5
0.03	0.496	33.8	0.372	44.4
0.02	0.496	40.4	0.372	53
0.01	0.496	51.8	0.372	67.9
0.005	0.496	63.4	0.372	83.3
0.001	0.496	89	0.372	117

Table 5.2. *Throughput for different error tolerances.*

ϵ	0.1	0.05	0.04	0.03	0.02	0.01	0.005	0.001
Th	0.451	0.448	0.448	0.447	0.445	0.442	0.44	0.433

The throughput matrix is given by

$$T = \begin{bmatrix} 0.507 & 0.499 \\ 0.269 & 0.324 \end{bmatrix} \tag{5.22}$$

The dwell times in the two states are given by

$$D = [500 \quad 200] \tag{5.23}$$

In real situations, because of estimation delay, the throughput will be given by eqn. 5.13. The throughput for our example will be function of the error tolerance. For different values of ϵ, the throughput is given in the table 5.2.

As we see from the table, the throughput of the adaptive system is very close to that in the ideal case. Generally, throughput of the adaptive system will vary with the chosen error tolerance. For the example we have chosen, the throughput does not vary appreciably with the amount of error tolerance.

5.3 Adaptive Scheme Based on HARQ

5.3.1 Adaptive Scheme with State Estimation in Gaussian Environment

In a Gaussian environment, when the channel varies slowly, every state can be modeled as BSC described by a single parameter, the transition probability. When channel BER varies over a wide range but the rate is slow, adaptive scheme will give better throughput than non-adaptive single coding scheme.

Table 5.3. *BCH codes in an adaptive system.*

Code	n	k	t	Range
I	255	231	3	< 0.00851
II	255	191	8	$0.00851 - 0.0276$
III	255	139	15	$0.0276 - 0.0556$
IV	255	91	25	$0.0556 - 0.0998$
V	255	47	42	≥ 0.0998

To see how variable code rate affects the throughput, in the following analysis we use a hypothetical channel where five different codes with different error-correcting capabilities are used. The parameters of the codes are given in table. 5.3. Throughput curves of these codes for channel BER ranging from 10^{-4} to 10^{-1} are shown in fig. 5.3. As we move from low BER to higher value, crossover points are encountered after which

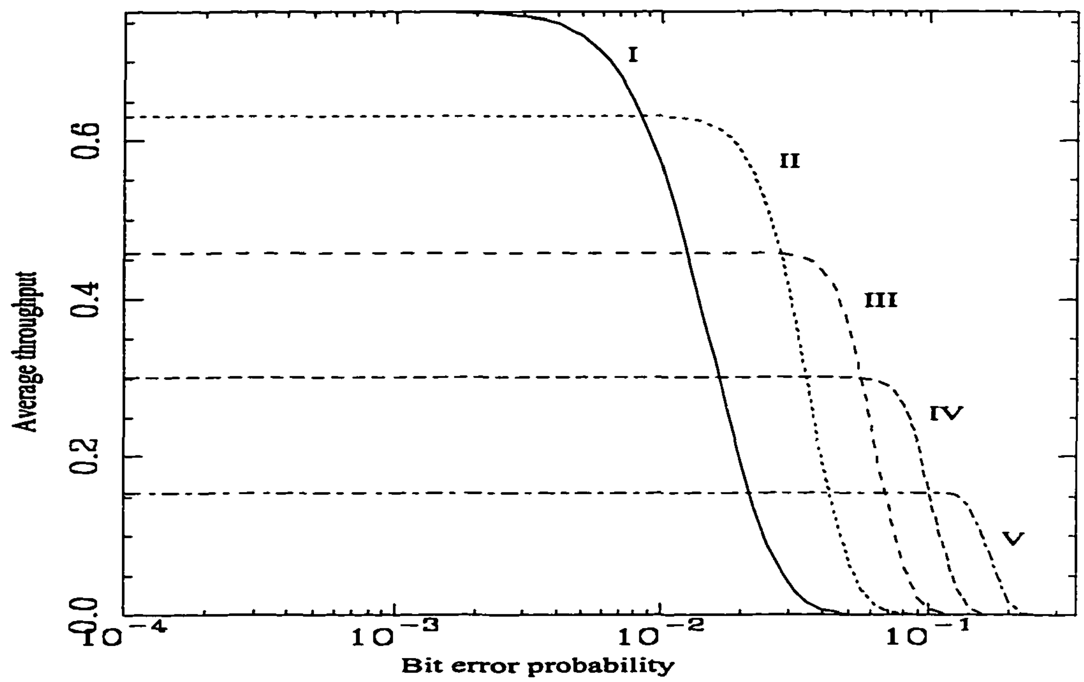


Figure 5.3. *Throughput for different BCH codes.*

next higher rate code performs better than other codes. Every code has a region where it outperforms others. The crossover points are called **threshold points**. Threshold points are the BER's where two adjacent codes perform equally. Analytically they are calculated from above consideration. Table. 5.3 also gives these points for the codes chosen above. In any adaptive system design if the FEC codes are given, the threshold values can be calculated and the system can be designed to use them as criteria for decisions about which operating state should be selected. The problem can also be solved the reverse way. If the average BER's of the states are given, optimum code can be selected by simple computer search as was done in chapter 3.

5.3.2 Adaptive Scheme with Sequential Likelihood-Ratio Testing

As in the example in section 5.2, we take the two-state example with the state parameters shown in fig. 5.4. Only difference in this example from the previous one is in the bit error rates at the states $S1$ and $S2$. The block length is taken as 210. Error

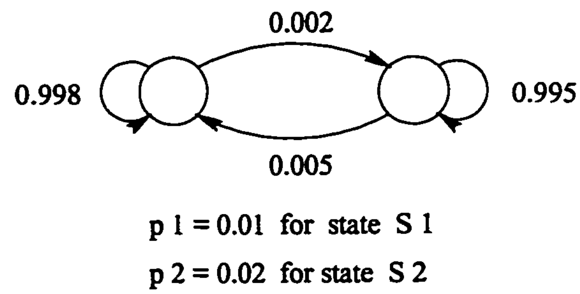


Figure 5.4. *Two-state adaptive system example.*

correcting capability of the code used varies from 0 to 7.

Table 5.4. *ASN for sequential likelihood ratio (SLR) tests.*

α	Average Sample Number							
	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7
0.001	195	75	51.2	47.3	53.8	72.5	113	200
0.002	158	60.7	41.4	38.3	43.6	58.6	91.3	162
0.005	115	44.1	30.1	27.8	31.6	42.6	66.2	118
0.01	86.4	33.2	22.7	20.9	23.8	32.1	49.9	88.6
0.02	61.9	23.8	16.3	15	17.1	23	35.8	63.6
0.03	49.4	19	13	12	13.6	18.4	28.6	50.7
0.04	41.3	15.9	10.8	10	11.4	15.3	23.9	42.4
0.05	35.5	13.6	9.3	8.59	9.78	13.2	20.5	36.4
0.1	19.7	7.59	5.18	4.79	5.45	7.33	11.4	20.3

Table 5.4 gives the ASN values for different error tolerances and error correcting capabilities. A three-dimensional plot of the sample numbers is given in fig. 5.5.

Table 5.5. *ASN for SLR tests (proposed scheme).*

α	0.001	0.002	0.005	0.01	0.02	0.03	0.04	0.05	0.1
ASN	33	27	19	15	11	9	7	6	4

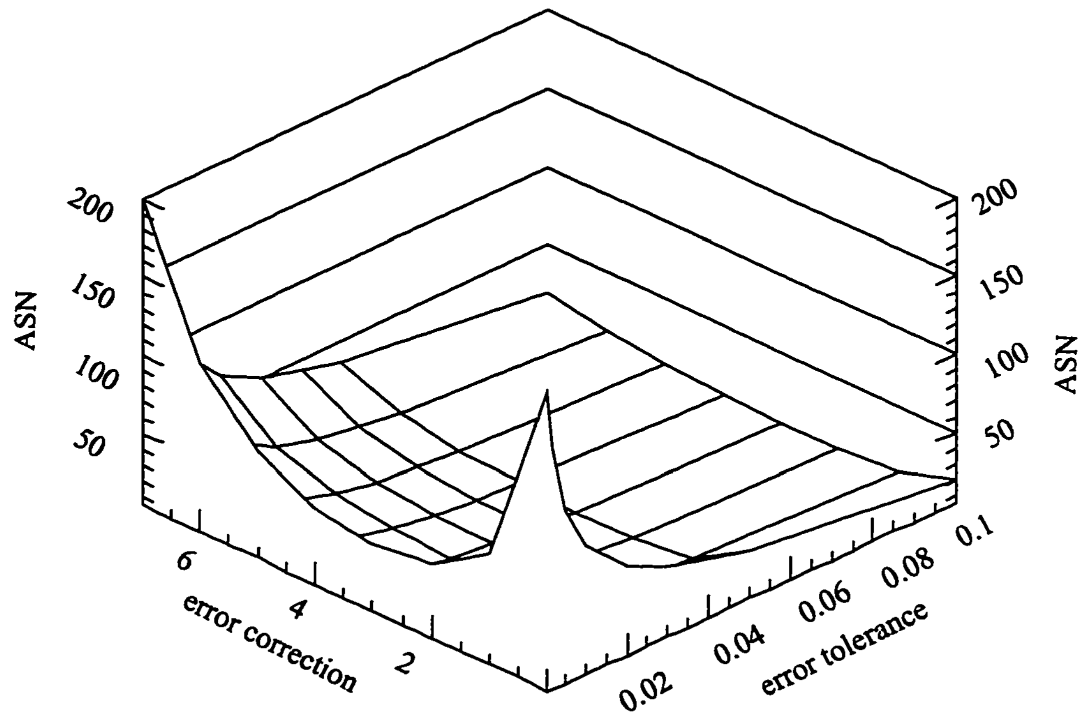


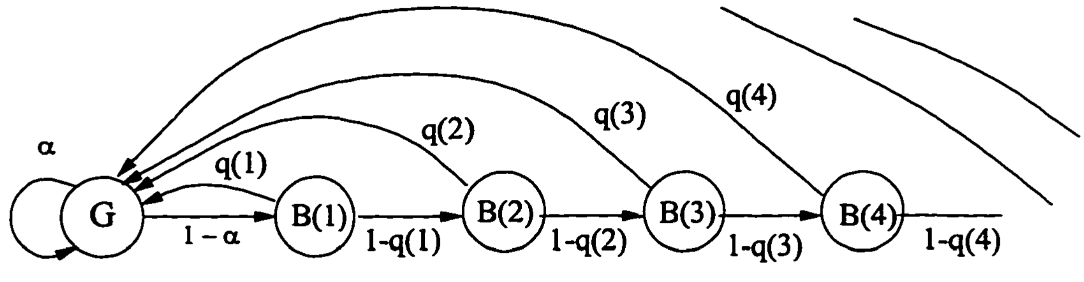
Figure 5.5. *ASN values for different α and t.*

The ASN plotted in the fig. 5.5 or shown in the table 5.4 corresponds to the ACK/NAK based schemes. For the scheme, where number of symbol errors are used as feedback signal, ASN is given by table 5.5.

The advantage of the error count based system is obvious from the tables 5.4 and 5.5. The previous method greatly reduces the number of samples required to estimate channel state. It is also independent of the code used for error correction. By allowing a quicker estimate of the channel state, it improves the throughput compared to the ACK/NAK based system.

5.3.3 Simulation of Error Generation in Rayleigh environment

Studies in the past has examined error statistics for various types of transmission channels, including conventional telephone circuits, terrestrial radio and satellite links. One common aspect of all these channels is that errors in digital links occur in bursts



$$P(i) = \frac{\lambda^{i} e^{-\lambda}}{(1 - e^{-\lambda})\, i!}$$

$$q(1) = p(1) \;;\; (1-q(1)) * (1-q(2)) * \cdots\cdots (1-q(j-1)) * q(j) = p(j)$$

Figure 5.6. *Error generation for fading channels.*

or clusters. By far the simplest model proposed so far which takes bursts of error into account was the one proposed by Gilbert [40]. Error bursts and error-gaps are random and both of them are exponentially distributed. This model is very simple to analyze but in most cases falls short of real channel conditions. Fritchman [37] investigated characteristics of a very general finite-state Markov chain model with N states partitioned into two groups, A of k error-free states and, B of $N-k$ error states. This model has been most widely used in practice to model the experimental data to match with the model by changing the number and values of the parameters. Some experimental evidence [8] suggests that the burst lengths in some cases are Poisson distributed while the error-free gaps are geometrically distributed. For generating errors in a particular state we have adopted the model shown in fig. 5.6.

This model is based on an infinite-state Markov chain called a *slowly spreading chain of the first kind* [51] and is a special case of the *compound infinite-state model* [1]. State G is the **good state** and states $B1, B2, B3, \cdots$ are error states. It is assumed that no error occurs in state G and all the bits in error states are in error, their number being 1 in state $B(1)$, 2 in state $B(2)$, 3 in state $B(3)$ and so on. For Poisson process, probability of r errors is given by

$$P_r = \frac{\lambda^r}{r!} e^{-\lambda} \tag{5.24}$$

where λ is the average value of r. Let α be the probability of transition from state

G to itself. All the transition probabilities are shown in fig. 5.6. Similar models with Poisson distribution of burst length has been described in [2], [4]. The model is described by two parameters, the minimum required for describing any burst statistics. Because the number of errors as 'zero' should be discarded, the density function will be described by a truncated Poisson distribution where the range of the independent variable will be from 1 to ∞. So,

$$P_r = \frac{\lambda^r}{r!}\, e^{-\lambda} \quad r \geq 1 \tag{5.25}$$

$$= \frac{1}{1-e^{-\lambda}}\, \frac{\lambda^r}{r!}\, e^{-\lambda}. \tag{5.26}$$

Average value of r will now become

$$\lambda' = \frac{\lambda}{1-e^{-\lambda}}. \tag{5.27}$$

Probability of gap length being r is given by

$$G_r = \alpha^{r-1}\,(1-\alpha). \tag{5.28}$$

Average gap length, obtained from above equation is given by,

$$m_g = \frac{1}{1-\alpha}. \tag{5.29}$$

These two parameters λ' and α define the model completely. To generate error sequences, only the average burst length and gap length are sufficient. For a Rayleigh channel average fade duration is obtained from eqn. 2.23. Average gap length is obtained by calculating the level-crossing-rate from eqn. 2.22 and subtracting the average burst length from the inverse of the LCR. Final expressions for average gap length and average burst length will be

$$\bar{Y} = \frac{1}{\sqrt{2\,\pi\,\rho}}\, \frac{\lambda}{V}\, R, \tag{5.30}$$

$$\bar{X} = \frac{exp(\rho)-1}{\sqrt{2\,\pi\,\rho}}\, \frac{\lambda}{V}\, R, \tag{5.31}$$

where R is the data transmission rate in bits/sec. Then average gap length and burst length are functions of threshold level, mobile speed, carrier wavelength and data transmission rate. Their ratio is given by

$$\frac{\text{Average burst length}}{\text{Average gap length}} = \frac{\bar{X}}{\bar{Y}} = \exp(\rho) - 1, \tag{5.32}$$

depends on the threshold level only. Average bit error rate can be expressed as the ratio of the average burst length and sum of average burst length and average gap lengths and is expressed as

$$BER = \frac{\exp(\rho) - 1}{\exp(\rho)} = 1 - \exp(-\rho). \tag{5.33}$$

This BER is plotted in fig. 5.7.

5.3.3.1 Simulation Results

Computer simulations were performed to test the performance of the adaptive model. Channel errors were generated according to the model presented in this chapter. We chose RS codes for the analysis because of reasons cited earlier.

It was seen earlier, that the average burst and gap lengths were functions of the receiver threshold level, mobile speed, carrier frequency, and data transmission rate. The quantity $\frac{\lambda R}{V}$, gives the number of data bits transmitted during the movement of the mobile by a single wavelength. We combine these three parameters into a single parameter d. This will simplify the analysis and reduce unnecessary complication in the dependence on these parameters. For carrier frequency of 900 MHz; the value of d for different mobile speed and data rates, is given in table 5.6. To find out symbol error probabilities at different threshold level and for different values of d, 100000 bits were generated and proportion of symbols in error was found out.

Table 5.6. *Values of d for different data rates and velocities.*

Data Rate	Vehicle Speed (km/hour)				
(kB/sec)	5	10	20	30	60
4	864	432	216	144	72
8	1728	864	432	288	144
16	3456	1728	864	576	288
32	6912	3456	1728	1152	576
64	13824	6912	3456	2304	1152

Symbols from $GF(2^m)$ for $m = 6$ were used. Average burst and gap (error-free) lengths are given in tables 5.7 and 5.8. As seen from table 5.6, d varies from 72 to 13824 for the mobile velocity and data range cited above.

Proportion of symbol errors are given in table 5.9 for symbol size of 6. Average symbol error rate for fading channels is greater than average BER. With the increase of the average burst and gap lengths, symbol error rate approaches the channel BER [56]. The same statement is also true for packet error rates. Packet error rate approaches the channel bit error rate when average burst and gap lengths are much higher than the packet length.

Table 5.7. *Average burst length.*

d	ρ(dB)				
	0	-5	-10	-15	-20
72	49.36	19	9.553	5.19	2.887
144	98.71	38	19.11	10.38	5.774
216	148.1	57	28.66	15.57	8.66
288	197.4	76	38.21	20.76	11.55
432	296.1	114	57.32	31.14	17.32
576	394.8	152	76.42	41.52	23.09
864	592.3	228	114.6	62.27	34.64
1152	789.7	304	152.8	83.03	46.19
1728	1185	456	229.3	124.5	69.28
2304	1579	608	305.7	166.1	92.38
3456	2369	912	458.5	249.1	138.6
6912	4738	1824	917.1	498.2	277.1
13824	9476	3648	1834	996.4	554.3

Table 5.8. *Average gap length.*

d	ρ(dB)				
	0	-5	-10	-15	-20
72	28.724	51.079	90.833	161.53	287.24

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144	57.448	102.16	181.67	323.05	574.48
216	86.172	153.24	272.5	484.58	861.72
288	114.9	204.32	363.33	646.1	1149
432	172.34	306.47	545	969.16	1723.4
576	229.79	408.63	726.66	1292.2	2297.9
864	344.69	612.95	1090	1938.3	3446.9
1152	459.58	817.26	1453.3	2584.4	4595.8
1728	689.37	1225.9	2180	3876.6	6893.7
2304	919.16	1634.5	2906.6	5168.8	9191.6
3456	1378.7	2451.8	4360	7753.3	13787
6912	2757.5	4903.6	8719.9	15507	27575
13824	5515	9807.2	17440	31013	55150

There does not exist, as in case of a gaussian channel, a simple relationship between the symbol or packet error rates and channel bit error rate. These error rates will depend on the burst characteristics in a complicated way. Except for these extreme cases, the symbol error rate will generally increase with the increase of symbol size. Similar condition will also be applicable for packet error rates. The lower bound of these error rates will be given by the average BER, given by equation 5.33. According to our model, average bit error rate is function of the threshold only. It is interesting to compare the symbol error probabilities with average bit error rates at the thresholds considered above (see table. 5.10).

Table 5.9. *Proportion of errors* ($m = 6$).

d	ρ(dB)				
	0	-5	-10	-15	-20
72	0.679	0.318	0.132	0.0727	0.0292
144	0.64	0.329	0.109	0.054	0.0209
216	0.658	0.266	0.0999	0.0321	0.0161

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288	0.657	0.297	0.111	0.0366	0.0121
432	0.637	0.291	0.099	0.0365	0.0109
576	0.658	0.306	0.099	0.0287	0.0121
864	0.634	0.253	0.0978	0.0364	0.0145
1152	0.633	0.307	0.102	0.0311	0.0122
1728	0.63	0.287	0.0843	0.0325	0.00955
2304	0.623	0.243	0.0991	0.0312	0.0106
3456	0.585	0.271	0.0962	0.0294	0.0125
6912	0.627	0.297	0.108	0.0321	0.0115
13824	0.606	0.269	0.0831	0.0297	0.0105

Table 5.10. *Average lower symbol error rate bound.*

ρ	0	-5	-10	-15	-20
p	0.632	0.271	0.0952	0.0311	0.00995

In a Rayleigh fading environment, two channel states will differ in average SNR and mobile speed. Here we compare performance of the adaptive scheme for a Rayleigh channel with two states, having threshold levels $\rho = -5$ dB and $\rho = -10$ dB.

Table 5.11. *Optimum code parameters for RS codes.*

p	n	t	Th
0.3	69	23	0.183
0.1	76	11	0.585

Average symbol error rates have been found, for these threshold levels, by computer search. For symbol of length 8 bits, they are 0.3 and 0.1 respectively. For the above error rates, optimum RS code parameters were found by computer search and are given in table 5.11. When the channel is perfectly interleaved, i.e., symbol errors in a packet seem to be independent of each other, the throughput can be calculated by usual method given by equation 3.1.

For the calculation of ASN required for state inference, we assume that the channel has been interleaved to a degree such that the symbol errors can be considered to be independent. If the channel is not interleaved, then we can use above result for ASN. We have to multiply the result obtained for perfectly interleaved channel by the required interleaving factor. Table 5.12 gives the ASN values for a fading channel where required interleaving factor or depth is considered to be 20. A practical way of choosing the interleaving depth would be a number which is about one order higher than the ratio of the sum of burst and gap lengths to the packet length.

Table 5.12. *ASN for Rayleigh channels.*

α	Average Sample Number			
	ACK Based		Error Count Based	
	State 1	State 2	State 1	State 2
0.001	264	72.1	51	46
0.002	214	58.4	41	37
0.005	155	42.4	30	27
0.01	117	31.9	23	21
0.02	83.8	22.9	16	15
0.03	66.8	18.3	13	12
0.04	55.9	15.3	11	10
0.05	48	13.1	10	9
0.1	26.7	7.3	6	5

The superiority of proposed error count scheme for adaptive error control scheme is evident from the table 5.12. Reduction of ASN translates directly to the improvement of throughput. The actual throughput will depend on the Dwell times in the states.

The above analysis can be extended for systems with more than two states.

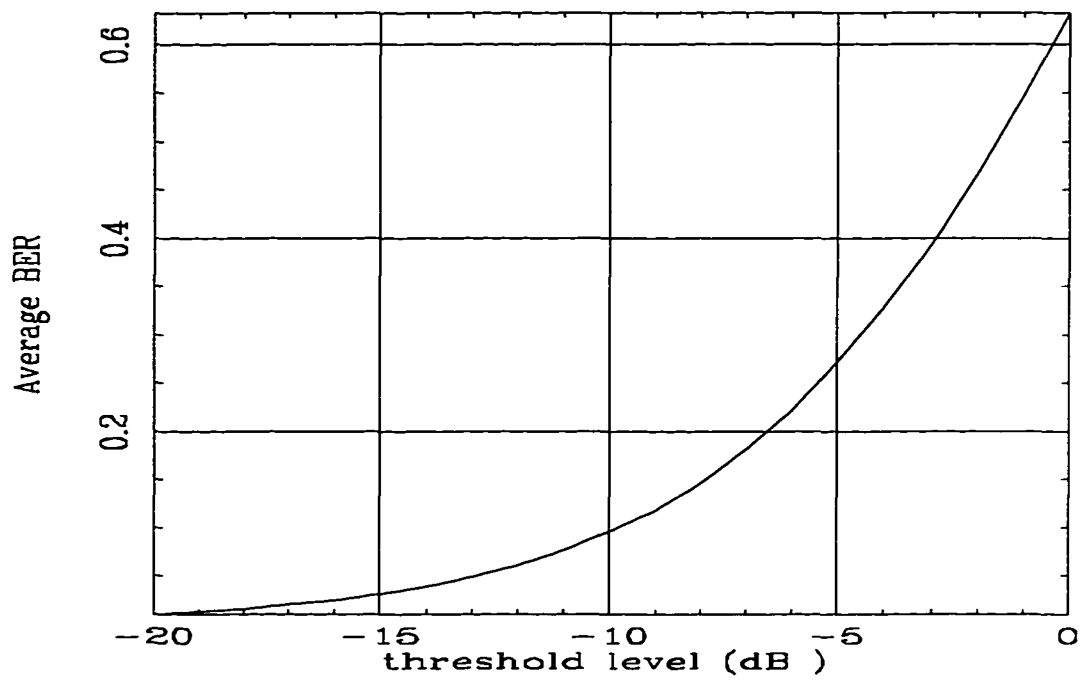

Figure 5.7. *Error generation for fading channels.*

5.4 Summary

Using the scheme developed in chapter 4, adaptive error control scheme has been analyzed in different environments.

A new error generating scheme, for fading channels, has been proposed. This model generates Poisson distributed bursts and geometrically distributed gap lengths for fading channels.

By using a two-state channel, it has been shown, in subsection 5.3.2, that adaptive scheme outperforms non-adaptive error control schemes in respect to throughput performance.

Maximum likelihood ratio testing for channel state inference has been shown to improve system throughput while maintaining reliability constraint.

Sequential likelihood ratio testing has been analyzed and proposed error count scheme has been compared to the traditional ACK/NAK based system. Proposed scheme has been shown to be much superior.

Chapter 6

Conclusions

In this chapter, we present the contributions of this thesis, and conclude with some citation of future work.

6.1 Summary of Results

For noisy channels, simple ARQ error control techniques, without any optimization, are not sufficient to provide high speed reliable communication. When dealing with constraints, like the limitation on the number of transmissions of a packet and minimum reliability, sophisticated hybrid error control techniques must be considered. Our goal is to obtain the maximum overall throughput in the present channel condition, while at the same time, satisfying the constraints imposed on the system.

In a stationary channel, the system throughput is affected by the blocklength and error correcting capacity of the code used in the forward error correction scheme. We have shown by extensive computer search that optimum throughput is obtained by a unique set of code parameters. Both the simple and the hybrid Automatic Repeat Request schemes have been investigated. Optimum block length in case of hybrid system is found to be much larger than in case of simple repeat request schemes.

Code combining techniques have been proven to be very effective in attaining the required reliability with a limited number of retransmissions of a packet. Two different combining techniques have been presented.

Adaptive error control scheme is effective for a slowly varying channel. The channel, at any instant, can be considered to be in one of a set of stationary states. As in case of the stationary channel, optimum code parameters can be found for each of the states. The factor having a negative effect on the throughput, in this case, is the time required to estimate the channel state , or the time to detect the change of stated. The lower is this time, the greater is the throughput. Use of the number of errors in the received packet reduces the time required to estimate the channel state. Quantitative analysis of the time requirement has been performed and it has been compared to the traditional acknowledgment based system.

6.2 Further Work

The channel state inference or estimation process is dependent on the error tolerance, which has to be defined before the estimation process. The ultimate system throughput will depend on this parameter. We have not carried out any analysis of the effect of this parameter on the throughput. This can be further investigated.

In chapter 3, we have analyzed two type-II HARQ schemes with fixed packet lengths. This was done considering the difficulty of synchronization in mobile communications when variable length packet is used. Variable length packets should give better throughput performance compared to the fixed length systems. In future, when synchronization technique improves, investigations in this direction will be worth doing.

While considering error processes in Rayleigh environment, we considered only burst errors. In dealing with real channels, error due to random noise has to be taken into account. Further studies can be done with the incorporation of random noise effects on the performance.

Investigation of performance of the adaptive system has not been carried out for Rician channels and Rician channel with shadowing. These channels are used for modeling mobile satellite channels. Throughput performance of these channels should be investigated.

We have used two-state channels in the examples we provided while analyzing adaptive error control schemes. Analysis for multi-state channels can be further pursued.

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Appendix A

Fisher Information of Feedback signal

Fisher information in a sample observation has been defined and discussed in chapter 4. Here we calculate Fisher information contents for a few relevant sampling techniques.

A.1 Information in the ACK signal (no error correction)

If the feedback signal is simple ACK in a plain ARQ system, the probability function can be written as

$$f(i;p) = (1-p)^n \tag{A.1}$$

The score V is given by

$$\begin{aligned} V &= \frac{\delta}{\delta p}(\log f(i;,p)) \\ &= \frac{f'(i;p)}{f(i;p)} \\ &= -\frac{n(1-p)^{n-1}}{(1-p)^n} \\ &= -\frac{n}{1-p} \end{aligned}$$

and the information is given by

$$
\begin{aligned}
I_X(p) &= -E\frac{\delta V}{\delta p} \\
&= E\left[\frac{n}{(1-p)^2}\right] \\
&= \frac{n}{(1-p)^2}
\end{aligned}
\tag{A.2}
$$

A.2 Information in Error Number Information

When the feedback signal is the number of errors in a packet, the probability of i errors in a packet is

$$
f(i;p) = C\binom{n}{i}\, p^i\, (1-p)^{n-i} \tag{A.3}
$$

where C is a value independent of p. The score V is

$$
\begin{aligned}
V &= \frac{\delta}{\delta p}(\log f(i;,p)) \\
&= \frac{f'(i;p)}{f(i;p)} \\
&= \frac{i-np}{p(1-p)}
\end{aligned}
\tag{A.4}
$$

Its derivative w.r.t p is:

$$
\frac{\delta V}{\delta p} = \frac{2ip - i - np^2}{p^2(1-p)^2} \tag{A.5}
$$

And the information content of the feedback signal is:

$$
\begin{aligned}
I_X(p) &= -E\frac{\delta V}{\delta p} \\
&= E\left(\frac{np^2 + i - 2ip}{p^2(1-p)^2}\right) \\
&= \frac{np^2 + E(i) - 2pE(i)}{p^2(1-p)^2} \\
&= \frac{n}{p(1-p)}
\end{aligned}
\tag{A.6}
$$

Alternate derivation: Alternatively the information amount can be derived from the general formula for exponential family of density functions given by eq˙ 4.10. From table 4.1, we find the following parameters:

$$B = (1-p)^n; \quad Q = \log_e \frac{p}{1-p}; \quad R = x; \quad h = \binom{n}{x} \tag{A.7}$$

So,

$$\begin{aligned}
B' &= n(1-p)^{n-1} \\
B'' &= n(n-1)(1-p)^{n-2} \\
\frac{B'}{B} &= \frac{n}{1-p} \\
\frac{B''}{B} &= \frac{n(n-1)}{(1-p)^2} \\
Q' &= \frac{1}{p(1-p)} \\
Q'' &= \frac{1-2p}{p^2(1-p)^2} \\
\frac{Q''}{Q'} &= \frac{1-2p}{p(1-p)}
\end{aligned}$$

And the information content can be written as

$$\begin{aligned}
I_X(p) &= \frac{n^2}{(1-p)^2} - \frac{n(n-1)}{(1-p)^2} + \frac{1-2p}{p(1-p)}\frac{n}{1-p} \\
&= \frac{n}{p(1-p)}
\end{aligned} \tag{A.8}$$

which is same as was obtained by direct computation.

Appendix B

Error Rates in Fading Channels

Fading channels are characterized by variations of signal amplitude levels over time. Depending on the rapidity of variation, they are broadly classified into three categories: slowly varying, moderately varying and fast varying. There are no hard and fast rule about which channels will fall in which category, but we shall define slow fading channel as the one whose signal strength remains constant during the transmission of a code block and fast fading channel as the channel whose signal strength remains constant over the duration of a bit but uncorrelated during successive transmissions of bits. Moderately fading channel is somewhat between the above two extremes. Bit error rates in communication channels depends on the noise power level as well as the type of modulation-demodulation used. For Gaussian channels BER's for several modulation schemes are given by eqns. B.1-B.7.

For a Rayleigh fading channel the SNR density function is given by eqn. 2.18. Average BER for this channel for different modulation schemes are given by eqns.B.9-B.12.

BER for gaussian channels for different modulation schemes:

$$P_E = \frac{1}{2}\,\mathrm{erfc}\Big(\sqrt{\frac{E_b}{N_0}}\Big) \qquad \text{Coherent Binary PSK} \qquad \text{(B.1)}$$

$$= \frac{1}{2}\,\mathrm{erfc}\Big(\sqrt{\frac{E_b}{2\,N_0}}\Big) \qquad \text{Coherent Binary FSK} \qquad \text{(B.2)}$$

$$= \frac{1}{2}\,\mathrm{erfc}\Big(\sqrt{\frac{E_b}{N_0}}\Big) \qquad \text{Coherent QPSK} \qquad \text{(B.3)}$$

$$= \frac{1}{2}\,\mathrm{erfc}\Big(\sqrt{\frac{E_b}{N_0}}\Big) \qquad \text{Coherent MSK} \qquad \text{(B.4)}$$

$$= \frac{1}{2}\,\exp\Big(-\frac{E_b}{2\,N_0}\Big) \qquad \text{Non-coherent Binary FSK} \qquad \text{(B.5)}$$

$$= \frac{1}{2}\,\exp\Big(-\frac{E_b}{N_0}\Big) \qquad \text{DPSK} \qquad \text{(B.6)}$$

$$\simeq \mathrm{erfc}\left(\sqrt{\frac{E}{N_0}}\sin\big(\frac{\pi}{M}\big)\right) \qquad \text{coherent M-ary PSK} \qquad \text{(B.7)}$$

Average BER for Rayleigh channels is given by

$$P_2 = \int_0^\infty P_E\, p(\gamma_b)\, d\,\gamma_b. \qquad \text{(B.8)}$$

For different modulation schemes they are as follows:

$$P_2 = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right) \qquad \text{binary PSK} \qquad \text{(B.9)}$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_b}{2+\bar{\gamma}_b}}\right) \qquad \text{binary FSK} \qquad \text{(B.10)}$$

$$= \frac{1}{2(1+\bar{\gamma}_b)} \qquad \text{binary DPSK} \qquad \text{(B.11)}$$

$$= \frac{1}{2+\bar{\gamma}_b} \qquad \text{non-coherent orthogonal FSK} \qquad \text{(B.12)}$$

Appendix C

Likelihood Tests for Binomial Distributions

C.1 Likelihood Ratio Test

In testing the hypothesis $H_1 : p = p_1$ versus $H_2 : p = p_2$, if in an observation of n samples, m is the number of failures,

$$f(\mathbf{Z}|p) = p^m(1-p)^{n-m}, \tag{C.1}$$

where m symbols in a packet of length n are in error. Likelihood ratio is given by

$$\begin{aligned} \Lambda(\mathbf{Z}) &= \frac{p_1^m(1-p_1)^{n-m}}{p_2^m(1-p_2)^{n-m}} \\ &= \left(\frac{1-p_1}{1-p_2}\right)^n \left(\frac{p_1/(1-p_1)}{p_2/(1-p_2)}\right)^m \end{aligned} \tag{C.2}$$

So, $\lambda(\mathbf{Z}) > k$ implies

$$\begin{aligned} \left[\frac{p_1(1-p_2)}{p_2(1-p_1)}\right]^m &> k\left(\frac{1-p_2}{1-p_1}\right)^n \\ m\ \log\left[\frac{p_1(1-p_2)}{p_2(1-p_1)}\right] &> \log\ k + n\ \log\left(\frac{1-p_2}{1-p_1}\right) \\ ma + mb &> \log\ k + nb \\ \frac{m}{n} &> \frac{1}{a+b}\left(b + \frac{\log k}{n}\right) \end{aligned} \tag{C.3}$$

where $a = \log(p_1/p_2)$ and $b = \log((1-p_2)/(1-p_1))$.

C.2 Sequential Likelihood Ratio Test

For sequential likelihood-ratio tests for binomial distribution, the likelihood ratio is given by

$$\Lambda_n(\mathbf{Z_n}) = \left(\frac{1-p_1}{1-p_2}\right)^n \left(\frac{p_1/(1-p_1)}{p_2/(1-p_2)}\right)^{n\hat{p_n}} \tag{C.4}$$

where $\hat{p_n}$ is the proportion of errors in a sample of size n. The relation

$\log\ A < \log\ \Lambda_n(\mathbf{Z_n}) < \log\ B$

implies [14]

$$\log\ A < n\log\Big(\frac{1-p_1}{1-p_2}\Big) + n\hat{p_n}\log\Big(\frac{p_1/(1-p_1)}{p_2/(1-p_2)}\Big) < \log\ B. \tag{C.5}$$

After some manipulations, we get

$$a - \frac{b_2}{n} < \hat{p_n} < a + \frac{b_1}{n}, \tag{C.6}$$

where a, b_1 and b_2 are given by the equations

$$a = \frac{\log[(1-p_2)/(1-p_1)]}{\log\left(\frac{p_1/(1-p_1)}{p_2/(1-p_2)}\right)} \tag{C.7}$$

$$b_1 = \frac{\log\ B}{\log\left(\frac{p_1/(1-p_1)}{p_2/(1-p_2)}\right)} \tag{C.8}$$

$$b_2 = -\frac{\log\ A}{\log\left(\frac{p_1/(1-p_1)}{p_2/(1-p_2)}\right)} \tag{C.9}$$

Appendix D

BCH code parameters

Table D.1. *BCH code parameters for* $n = 256$.

n=255					
k	t	k	t	k	t
247	1	163	12	79	27
239	2	155	13	71	29
231	3	147	14	63	30
223	4	139	15	55	31
215	5	131	18	47	42
207	6	123	19	45	43
199	7	115	21	37	45
191	8	107	22	29	47
187	9	99	23	21	55
179	10	91	25	13	59
171	11	87	26	9	63

Table D.2. *BCH code parameters for* $n = 127$.

n=127					
k	t	k	t	k	t
k	t	k	t	k	t
120	1	78	7	36	15
113	2	71	9	29	21
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106	3	64	10	22	23
99	4	57	11	15	27
92	5	50	13	8	31
85	6	43	14		

Table D.3. *BCH code parameters for* $n = 63$.

n=63					
k	t	k	t	k	t
57	1	36	5	16	11
51	2	30	6	10	13
45	3	24	7	7	15
39	4	18	10		

Appendix E

Packet Loss Probabilities due to Buffer Overflow

Table E.1. *Packet loss percentage for RTD =4.*

Buffer	Success Probability						
size	p=0.3	p=0.4	p=0.5	p=0.6	p=0.7	p=0.8	p=0.9
0	84.008	80.530	76.149	69.072	59.580	45.585	26.3004
1	64.962	59.907	54.641	47.363	39.364	29.681	17.2139
2	48.067	42.568	36.993	30.194	23.087	15.423	8.1777
3	34.656	29.519	24.953	19.213	13.581	7.6032	2.3629
4	24.669	20.397	16.708	12.223	8.1446	4.4534	1.4397
5	17.574	14.080	11.169	7.7152	4.7328	2.2187	0.6442
6	12.461	9.6033	7.5200	4.9407	2.7908	1.1221	0.1997
7	8.7837	6.6132	5.0082	3.1302	1.6943	0.6462	0.1124
8	6.1428	4.5835	3.3418	1.9712	1.0083	0.3207	0.0458
9	4.3578	3.2480	2.2328	1.2843	0.5908	0.1535	0.0116
10	3.0651	2.2660	1.4997	0.8176	0.3468	0.0703	0.0044
11	2.1582	1.6078	1.0116	0.5033	0.2153	0.0288	0.0017
12	1.5367	1.1430	0.6722	0.3119	0.1342	0.0128	0.0008
13	1.1356	0.7909	0.4726	0.1889	0.0832	0.0056	0.0000
14	0.8156	0.5378	0.3293	0.1307	0.0524	0.0016	0.0000
15	0.5818	0.3667	0.2365	0.0893	0.0349	0.0000	0.0000
16	0.4248	0.2511	0.1651	0.0605	0.0251	0.0000	0.0000

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17	0.2927	0.1829	0.1112	0.0413	0.0223	0.0000	0.0000
18	0.2086	0.1287	0.0808	0.0299	0.0195	0.0000	0.0000
19	0.1450	0.0884	0.0628	0.0197	0.0174	0.0000	0.0000
20	0.1074	0.0585	0.0439	0.0113	0.0132	0.0000	0.0000
21	0.0864	0.0390	0.0344	0.0065	0.0111	0.0000	0.0000
22	0.0612	0.0302	0.0249	0.0018	0.0090	0.0000	0.0000
23	0.0408	0.0191	0.0139	0.0000	0.0062	0.0000	0.0000
24	0.0249	0.0123	0.0099	0.0000	0.0048	0.0000	0.0000
25	0.0177	0.0095	0.0064	0.0000	0.0013	0.0000	0.0000
26	0.0123	0.0071	0.0054	0.0000	0.0000	0.0000	0.0000
27	0.0084	0.0019	0.0034	0.0000	0.0000	0.0000	0.0000
28	0.0036	0.0011	0.0029	0.0000	0.0000	0.0000	0.0000
29	0.0030	0.0000	0.0014	0.0000	0.0000	0.0000	0.0000
30	0.0021	0.0000	0.0004	0.0000	0.0000	0.0000	0.0000
31	0.0018	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
32	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
33	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table E.2. *Packet loss percentage for RTD =8.*

Buffer	Success Probability						
size	p=0.3	p=0.4	p=0.5	p=0.6	p=0.7	p=0.8	p=0.9
0	99.2020	98.6998	97.625	95.3243	89.989	77.515	51.861
1	96.9494	95.3939	93.194	89.3926	83.041	70.105	45.860
2	93.0202	90.2177	86.289	80.6082	72.882	60.447	39.042
3	87.5049	83.4854	78.087	70.5155	61.108	48.889	31.418
4	81.0511	75.9621	69.352	60.6174	49.751	37.098	23.177
5	74.0449	68.0214	60.699	51.6411	40.673	27.132	15.066
6	66.6358	60.2445	52.536	43.3411	33.324	20.542	8.8121
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7	59.4179	52.7061	44.996	36.0619	27.129	16.457	5.7283
8	52.4220	45.8235	38.229	29.7266	21.789	13.061	4.7160
9	46.0774	39.6091	32.341	24.4778	17.246	9.9987	3.7118
10	40.3058	33.9664	27.224	20.0340	13.672	7.4472	2.7382
11	34.9237	29.0753	22.826	16.3700	10.953	5.5204	1.8338
12	30.0878	24.8440	19.138	13.3176	8.7895	4.1575	1.1302
13	25.8719	21.1397	15.952	10.7730	6.9839	3.2529	0.6739
14	22.2011	17.9171	13.224	8.7950	5.5077	2.5538	0.4643
15	19.0356	15.1008	10.987	7.1969	4.3699	1.9652	0.3689
16	16.3183	12.7608	9.1003	5.8867	3.4321	1.4709	0.2762
17	13.9571	10.7706	7.5541	4.8051	2.7091	1.1173	0.1907
18	11.9802	9.10010	6.2700	3.9161	2.1230	0.8254	0.1250
19	10.2872	7.65206	5.2428	3.2022	1.6524	0.6422	0.0737
20	8.81320	6.38542	4.3565	2.6047	1.2797	0.4951	0.0476
21	7.60062	5.41301	3.6088	2.1098	0.9937	0.3719	0.0360
22	6.54475	4.57138	2.9920	1.7186	0.7650	0.2759	0.0261
23	5.64170	3.83460	2.4325	1.3857	0.5964	0.2015	0.0189
24	4.82660	3.22500	2.0203	1.1265	0.4552	0.1511	0.0108
25	4.15471	2.68398	1.6749	0.9106	0.3503	0.1103	0.0063
26	3.56629	2.22788	1.3799	0.7270	0.2741	0.0807	0.0018
27	3.04991	1.84394	1.1279	0.5704	0.2265	0.0623	0.0000
28	2.61340	1.56127	0.9302	0.4481	0.1874	0.0511	0.0000
29	2.22251	1.26982	0.7486	0.3551	0.1447	0.0415	0.0000
30	1.91539	1.06689	0.6063	0.2909	0.1153	0.0303	0.0000
31	1.63980	0.91738	0.5055	0.2261	0.0874	0.0255	0.0000
32	1.38731	0.79140	0.4052	0.1709	0.0643	0.0207	0.0000
33	1.16995	0.66900	0.3388	0.1295	0.0503	0.0176	0.0000
34	0.98591	0.57611	0.2670	0.0953	0.0370	0.0136	0.0000

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35	0.83400	0.47763	0.2205	0.0653	0.0293	0.0104	0.0000
36	0.68389	0.40347	0.1751	0.0425	0.0237	0.0088	0.0000
37	0.54039	0.34685	0.1432	0.0257	0.0188	0.0072	0.0000
38	0.43892	0.29662	0.1172	0.0167	0.0132	0.0056	0.0000
39	0.36747	0.26153	0.0938	0.0095	0.0076	0.0024	0.0000
40	0.29332	0.22605	0.0693	0.0059	0.0062	0.0016	0.0000
41	0.23777	0.19415	0.0553	0.0041	0.0041	0.0008	0.0000
42	0.18824	0.15628	0.0444	0.0029	0.0034	0.0000	0.0000
43	0.15281	0.13435	0.0314	0.0011	0.0020	0.0000	0.0000
44	0.12249	0.11282	0.0259	0.0000	0.0013	0.0000	0.0000
45	0.10688	0.09129	0.0224	0.0000	0.000	0.0000	0.0000
46	0.08677	0.06897	0.0184	0.0000	0.000	0.0000	0.0000
47	0.07206	0.05700	0.0149	0.0000	0.000	0.0000	0.0000
48	0.05555	0.04744	0.0129	0.0000	0.000	0.0000	0.0000
49	0.03994	0.03826	0.0114	0.0000	0.000	0.0000	0.0000
50	0.03184	0.03228	0.0104	0.0000	0.000	0.0000	0.0000
51	0.02523	0.02710	0.0094	0.0000	0.000	0.0000	0.0000
52	0.02103	0.02192	0.0084	0.0000	0.000	0.0000	0.0000
53	0.01712	0.01515	0.0074	0.0000	0.000	0.0000	0.0000
54	0.01292	0.01316	0.0044	0.0000	0.000	0.0000	0.0000
55	0.00901	0.00757	0.0034	0.0000	0.000	0.0000	0.0000
56	0.00661	0.00598	0.0024	0.0000	0.000	0.0000	0.0000
57	0.00631	0.00159	0.0019	0.0000	0.000	0.0000	0.0000
58	0.00571	0.00119	0.0009	0.0000	0.000	0.0000	0.0000
59	0.00541	0.00039	0.0000	0.0000	0.000	0.0000	0.0000
60	0.00391	0.00000	0.0000	0.0000	0.000	0.0000	0.0000
61	0.00211	0.00000	0.0000	0.0000	0.000	0.0000	0.0000
62	0.00091	0.00000	0.0000	0.0000	0.000	0.0000	0.0000

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63	0.00061	0.00000	0.0000	0.0000	0.000	0.0000	0.0000
64	0.00000	0.00000	0.0000	0.0000	0.000	0.0000	0.0000

Table E.3. *Packet loss percentage for different BER* $(n = 127, D = 4)$.

Buffer	BER		
size	0.001	0.01	0.05
0	30.52809	84.57563	90.626801
1	19.95387	65.73104	76.995422
2	9.560989	48.98923	62.492546
3	3.217514	35.49818	49.472591
4	1.932266	25.44634	38.293148
5	0.874138	18.18991	29.34074
6	0.296661	12.89109	22.37384
7	0.158455	9.063835	17.00758
8	0.068665	6.422241	12.93535
9	0.021126	4.573517	9.795090
10	0.010559	3.277008	7.369034
11	0.003517	2.353760	5.554634
12	0.000877	1.680656	4.242508
13	0.000000	1.224197	3.193123
14	0.000000	0.876343	2.437172
15	0.000000	0.637085	1.827744
16	0.000000	0.451988	1.389496
17	0.000000	0.326080	1.036133
18	0.000000	0.243164	0.786087
19	0.000000	0.175880	0.600037
20	0.000000	0.119484	0.458366
21	0.000000	0.074539	0.355408

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22	0.000000	0.047180	0.273964
23	0.000000	0.031548	0.211166
24	0.000000	0.016472	0.163460
25	0.000000	0.007820	0.128914
26	0.000000	0.004189	0.113205
27	0.000000	0.002792	0.093658
28	0.000000	0.001678	0.073036
29	0.000000	0.000565	0.057755
30	0.000000	0.000000	0.040970
31	0.000000	0.000000	0.021118
32	0.000000	0.000000	0.015167
33	0.000000	0.000000	0.010834
34	0.000000	0.000000	0.006790
35	0.000000	0.000000	0.004875
36	0.000000	0.000000	0.001404
37	0.000000	0.000000	0.000977
38	0.000000	0.000000	0.000389
39	0.000000	0.000000	0.000000