# Performance Analysis of Greedy Subchannel Allocation Schemes for Adaptive OFDMA Systems

by

Nan Lv
B.Eng., XI'AN University of Technology, 2008

A Thesis Submitted in Partial Fulfillment of the
Requirements for the Degree of

MASTER OF APPLIED SCIENCE

in the Department of Electrical and Computer Engineering

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Supervisory Committee

Dr. H.-C. Yang, Supervisor
(Department of Electrical and Computer Engineering)

Dr. X. Dong, Departmental Member
(Department of Electrical and Computer Engineering)

**Supervisory Committee**

Dr. H.-C. Yang, Supervisor
(Department of Electrical and Computer Engineering)

Dr. X. Dong, Departmental Member
(Department of Electrical and Computer Engineering)

## ABSTRACT

Subchannel allocation schemes for OFDMA (orthogonal frequency division multiple access) systems with adaptive transmission are investigated. The analysis involves two scenarios: single-hop transmission and dual-hop relaying transmission. The overall objective is to utilize minimum system resource (in terms of subchannel) while satisfying scheduled users' data rate requirements under a certain error rate performance restriction. The ordered subchannel selection with adaptive modulation scheme is applied to both cases. In single-hop case, a low-complexity greedy subchannel allocation algorithm with two options is proposed, depending on whether a common modulation mode is applied on all selected subchannels or not. The resulting two options are termed as GS-FA (greedy selection with full adaptivity) and GS-LA (greedy selection with limited adaptivity), respectively. In dual-hop relaying case, two relaying schemes are considered: Amplified-and-Forward (AF) and Decode-and-Forward (DF). Based on this, four different combinations of relaying schemes and ordered subchannel selection with adaptive modulation scheme are investigated: DF with full adaptivity (DF-FA), AF with full adaptivity (AF-FA), AF with limited adaptivity (AF-LA), and AF with limited adaptivity and subchannel mapping (AF-LA with SCM). For both single-hop and dual-hop relaying case, performances of different proposed schemes are evaluated through mathematical analysis and compared with selected numerical results.

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# ACKNOWLEDGEMENTS

I would like to thank:

**my supervisor, Dr. H.-C. Yang,** for his mentoring, support, encouragement, and patience during my two-year study.

**Dr. X. Dong, Dr. K. Wu, Vicky Smith and Moneca Bracken,** for helping me a lot.

**my parents, Xiujuan Deng and Chuangmin Lv** for supporting and believing in me all the way.

# Chapter 1

# Introduction

Future wireless communication systems must efficiently utilize the limited radio spectrum to support the stringent Quality of Service (QoS) requirements of multimedia applications. OFDM (Orthogonal frequency division multiplexing) is an attractive transmission scheme because of its capability of operating effectively over broadband selective fading channels by dividing wideband channel into parallel narrowband flat fading channels [1]. OFDM also facilitates flexible resource allocation through its multiple access functionality, i.e., allocating different subchannels (groups of highly correlated subcarriers) to different users, resulting the so-called orthogonal frequency division multiple access (OFDMA). As such, OFDM/OFDMA transmission scheme has been included in several new wireless standards [2].

Effective resource allocation, in terms of subchannels, bits, and power, is critical for wireless systems to maximally benefit from OFDM and OFDMA [3]. Over the past two decades, significant research efforts have been devoted to this subject [4] [5]. Most of the previous works formulate the resource allocation process into optimization problems with different objectives and constraints [3]. While such approaches, when solution exists, can lead to the best performance, they usually entail relatively high computational complexity, which limits their application in real-world systems. In addition, solving the optimization problem usually requires the complete channel state information of all users at the base station, which mandates high feedback load, especially when the number of users or subchannels are large. As such, it is of great practical interest to develop a low-complexity resource allocation algorithm with low feedback overhead.

In chapter 2, we propose a low-complexity greedy subchannel allocation algorithm for adaptive OFDMA single-hop transmission systems. The objective is to satisfy the rate requirement of scheduled users with a minimum system resource, in terms of subchannels and feedback overhead, under a certain error rate performance restriction. Specifically, users are sequentially considered for subchannel allocation based on their known priority. For each user under consideration, the minimum number of the best subchannels are allocated to it such that its data rate requirement can be satisfied. The ordered subchannel selection with adaptive modulation scheme is applied to the proposed scheme, depending on whether a common modulation mode is applied on all selected subchannels or not. The resulting two options we present for the proposed scheme are termed as GS-FA (greedy selection with full adaptivity) and GS-LA (greedy selection with limited adaptivity), respectively. The idea of ordered subchannel selection was first proposed for OFDM system in [6] and later applied to OFDMA system in [7], where the average error rate performance was investigated. The schemes there, however, always select a fixed number of subchannels without taking users' rate requirement into account, and therefore lack flexibility and may lead to a waste of system resource. Contrasted to previous schemes, the proposed greedy subchannel allocation scheme may flexibly assign different number of subchannels to users depending on fading channel condition and users' rate requirements, aiming to provide same or better QoS but with less resource. In this work, performances of the proposed scheme are evaluated through accurate mathematical analysis. In particular, analytical expression for the outage probability, average number of selected subchannels and average achieved data rate are derived. Based on these, the associate tradeoff of performance versus feedback load is investigated. Selected numerical examples are presented to illustrate the mathematical formulism. In the end of this chapter, performance comparisons between two options of the proposed scheme are presented.

In addition, one important challenge for the next generation wireless systems is to provide universal coverage for high data rate (HDR) services, which has attracted much attention in recent years [8] [9]. In order to provide HDR coverage, a large amount of base stations (BSs) have to be deployed to cover the whole cell for a fixed bandwidth and power in traditional cellular networks [10]. Relaying system, received great interests in recent years, can provide HDR coverage at low cost [11]. Furthermore, multihop relaying communication systems can bring additional benefits, such as combating shadowing in high radio frequencies and enhancement of capacity in cellular network [12]. Meanwhile, OFDMA scheme with advantage of inherent robustness against ISI (Intersymbol Interference) and selective fading can provide high spectral efficiency [13]. Therefore, the combination of OFDMA and relaying systems would be a promising solution for the next generation wireless communications. The concept of multihop relay has already been introduced to the OFDMA based standard, e.g., 802.16j [14] [15] [16].

In chapter 3, we investigate subchannel allocation schemes for dual-hop relaying transmission systems with adaptive OFDMA. We consider both Amplified-and-Forward (AF) and Decode-and-Forward (DF) relaying strategies. Again, the ordered subchannel selection with adaptive modulation scheme is applied to both cases. Depending on the used relaying schemes and the adaptation options, we arrive at four schemes, which are DF with full adaptivity (DF-FA), AF with full adaptivity (AF-FA), AF with limited adaptivity (AF-LA), and AF with limited adaptivity and subchannel mapping (AF-LA with SCM). The objective is still to achieve fair scheduling while satisfying the data rate requirement and error rate performance restriction of scheduled users with a minimum system resource in terms of subchannels. Specifically, users are sequentially considered for subchannel allocation based on their known priority. For each user, the minimum number of the best subchannels are allocated to satisfy its rate requirement. In this work, performances of the proposed schemes are evaluated through mathematical analysis. In particular, analytical expression for the average number of selected subchannels, average achieved data rate and outage probability are derived. Selected numerical examples are presented to illustrate the mathematical formulism. Performance comparisons among different schemes are given in the end of the chapter .

In chapter 4, we provide a brief review of the work presented in this thesis and an outlook of future research.

# Chapter 2

# Greedy Subchannel Allocation Scheme for Single-Hop Scenario

## 2.1 Introduction

Researchers have been dedicating to seek solutions to achieve better performances with effective utilization of limited radio spectrum for years. OFDM/OFDMA can bring benefits to resource allocation due to its flexibility and robustness to ISI. However, for OFDM/OFDMA systems, effective resource allocation with optimal solution often leads to optimization problem with high computational complexity. Therefore, resource allocation algorithm for OFAM/OFDMA systems with low complexity is of great interest.

In this chapter, we propose a low-complexity greedy subchannel allocation algorithm for adaptive OFDMA single-hop transmission systems. The objective is to achieve fair scheduling while satisfying the rate requirement of scheduled users with a minimum system resource, in terms of subchannels and feedback overhead, under a certain error rate performance restriction. To simplify the bit and power loading process and minimize the feedback load, constant-power variable-rate adaptive modulation scheme is applied on the selected subchannels [17]. In particular, the modulation mode is determined based on the instantaneous fading channel gain of the selected subchannels, to guarantee acceptable error rate performance of the scheduled users. Note that with the proposed scheme, each user needs only to feed back the index and the modulation modes of its selected subchannels, which incur much lower feedback load than feeding back the channel quality of each subchannel.

Two different options are presented for the proposed scheme, depending on whether the selected subchannels can use different modulation modes or not. Intuitively, allowing different modulation modes to be used on different selected subchannels will lead to a better performance, i.e., achieving higher data rate with less subchannels. But as compensation, the option allowing different modulation modes to be used on different selected subchannels will incur higher feedback load than the other option which uses the same modulation mode on all selected subchannels. It is due to the reason that the modulation mode of each selected subchannel needs to be fed back to the base station for the former option whereas only one modulation mode for all selected subchannels is fed back to the base station for the latter option.

In the following sections, performances of the proposed scheme with both options are evaluated through accurate mathematical analysis. In particular, analytical expression for the outage probability, average number of selected subchannels and average achieved data rate are derived. Based on these, the associate tradeoff of performance versus feedback load is investigated. Selected numerical examples are given to illustrate the mathematical formulism. In the end, performance comparisons between two options are presented.

## 2.2 System and Channel Models

A single-cell downlink OFDMA system is considered, where there is one base station (BS) and multiple users (M users). The system model is shown in Figure 2.1. Subchannel allocation and data transmission happens on a frame-by-frame basis. During each transmission frame, the whole bandwidth in the frequency domain is divided into total $N$ subchannels and each of the subchannels consists a fixed number of highly correlated subcarriers, which are affected similarly by the fading channel. We assume that different subchannels experience independent identically distributed (i.i.d.) Rayleigh fading. As such, the probability density function (PDF) of the

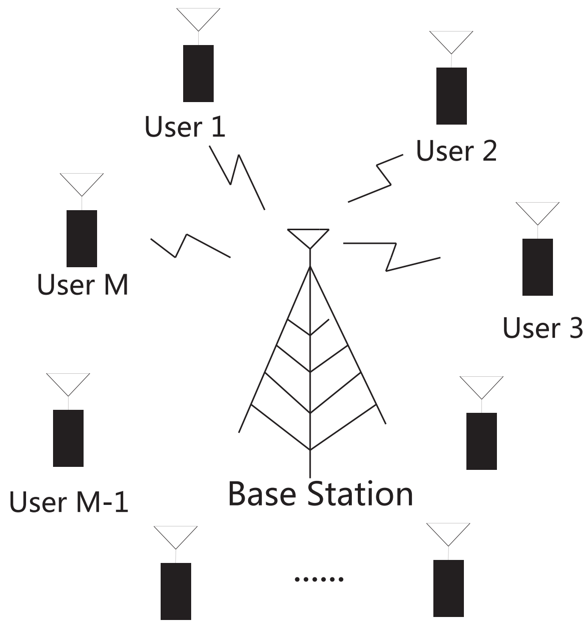


Figure 2.1: System Model

instantaneous SNR on different subchannels are commonly given by

$$p(\gamma) = \frac{1}{\bar{\gamma}} \exp(-\frac{\gamma}{\bar{\gamma}}), \tag{2.1}$$

where $\bar{\gamma}$ is the average SNR per subchannel.

## 2.3 Mode of Operation

The goal of the proposed scheme is to allocate the minimum number of the best subchannels to the scheduled users to satisfy their data rate requirements while meeting the instantaneous error rate requirement. During each transmission frame, a group of users are considered sequentially according to their predetermined priority. Without loss of generality, the focus is on the subchannel allocation of the first user of a group with data rate requirement of $R_T$ bits per OFDM symbol, while noting that the remaining users in the group will be served in a similar fashion with less available subchannels. After proper channel estimation, the user ranks the subchannels in a descending order according to their SNR values and determine the lowest modulation mode that each subchannel can support. Consequently, the minimum supported rate of the subchannels are $R_1 \geq R_2 \geq R_3 \geq \cdots \geq R_N$. Two subchannel selection options are considered for the proposed scheme, depending on whether the same modulation mode is used on all selected subchannels or not.

The first option allows the selected subchannels to use different modulation modes and, as such, is termed as greedy selection with full adaptivity (GS-FA). With GS-FA, the user will select the minimum number of best subchannels whenever the total data rate with selected subchannels become greater than or equal to the target rate $R_T$, i.e., $N_s$ subchannels where $N_s$ satisfies $\sum_{j=1}^{N_s-1} R_j < R_T$ but $\sum_{j=1}^{N_s} R_j \geq R_T$. After the selection, the user informs the transmitter the indexes of those selected subchannels and the corresponding modes to use through feedback process. The second option requires all selected subchannels to use the same modulation mode, thus is termed as greedy selection with limited adaptivity (GS-LA). Specifically, the common modulation mode is determined based on the selected subchannel with the smallest SNR to ensure the error rate performance on all selected subchannels are satisfactory. Again, the user selects the minimum number of best subchannels to satisfy the rate requirement. But the total data rate of $j$ selected subchannels becomes $j \cdot R_j$. Therefore, with GS-LA, $Ns$ best subchannels are selected when $(N_s - 1)R_{N_s-1} < R_T$ but $N_s R_{N_s} \geq R_T$. For both options, if the target rate still can not be achieved by using all $N$ subchannels, no transmission occurs and the user will be announced an outage. The condition of subchannels will be re-estimated and the subchannels will be re-ordered in the next transmission frame where a new group of users will be served in the same method as previous served group. In summary, the condition of subchannels is estimated and the subchannels are ordered every transmission frame to serve different groups of users.

Intuitively, we can see that GS-FA enjoys more flexibility and is expected to provide better performance. On the other hand, GS-LA has lower complexity as the same modulation mode is used on all selected subchannels. Furthermore, GS-LA incurs lower feedback load as it only needs to feed back a single index for the selected mode. Specifically, for both GS-FA and GS-LA options, at least 1 bit is needed to indicate whether the target rate is achieved or not (whether there will be a transmission or not). If the target rate is achieved, the user will feed back $N$ bits to indicate the subchannels selection result. Then, the user feeds back additional $N_s \log_2 M$ bits for the mode indexes of the selected subchannels for GS-FA option whereas the mode selection entails only $\log_2 M$ bits for GS-LA option. For both options, we can see that the feedback load is related to the total number of subchannels ($N$) in the system. To be specific, with an increasing number of subchannels, the feedback load will increase as well. In the following section, this tradeoff of performance versus feedback load will be quantified through accurate statistical analysis.

## 2.4 Performance Analysis

In this section, the performance and complexity of the proposed scheme with both options are quantified. Specifically, the focus is on the performance measures of outage probability, i.e., the probability of no transmission, average achieved data rate, and average number of selected subchannels. The complexity in terms of average feedback load is also quantified.

For both options, adaptive modulation is applied to each subchannel. It is assumed that there are $M$ different modulation modes, denoted by $\mathcal{M}_i$, where $i = 1, 2, \cdots, M$. The $i$th mode $\mathcal{M}_i$ can support a transmission rate of $R_i$ bits/symbol per subchannel. The mode selection is based on the instantaneous qualities of subchannels, characterized by their signal-to-noise ratio (SNR). In particular, the SNR value range is divided into intervals, defined by the thresholds $\gamma_{T_i}$, where $0 < \gamma_{T_1} < \gamma_{T_2} < \gamma_{T_3} < \ldots < \gamma_{T_M} < \gamma_{T_{M+1}} = \infty$. When the SNR is in $[\gamma_{T_i}, \gamma_{T_{i+1}}]$, the modulation mode $\mathcal{M}_i$ is selected. When the SNR is in $[0, \gamma_{T_1}]$, no transmission occurs. The threshold values are chosen to satisfy a target instantaneous error rate requirement. We use bit error rate (BER) as the measure of error rate requirement [1] in this thesis.

Applying the fading channel model in previous section, we can calculate the probability that the SNR is in the $i$th interval $\Pr[\gamma_{T_i} < \gamma < \gamma_{T_{i+1}}]$ and, as such, mode $\mathcal{M}_i$ can be used as

$$P_i = \int_{\gamma_{T_i}}^{\gamma_{T_{i+1}}} p(\gamma)\, \mathrm{d}\gamma = \exp(-\frac{\gamma_{T_i}}{\bar{\gamma}}) - \exp(-\frac{\gamma_{T_{i+1}}}{\bar{\gamma}}). \tag{2.2}$$

The probability that a subchannel can not support $\mathcal{M}_i$ can also be determined as

$$P_i^- = \Pr[\gamma < \gamma_{T_i}] = 1 - \exp(-\frac{\gamma_{T_i}}{\bar{\gamma}}), \quad i = 1, 2, \ldots, M. \tag{2.3}$$

### 2.4.1 GS-FA option

To derive the analytical expression of the performance measures of interest for GS-FA option, the total probability theorem is applied and mutually exclusive events are considered, based on the lowest order modulation mode used on the selected subchannels.

In particular, the case that the rate requirement can be satisfied by only using the highest modulation mode $\mathcal{M}_M$, i.e., when more than $\lceil \frac{R_T}{R_M} \rceil$ subchannels can support the highest modulation mode $\mathcal{M}_M$, the lowest order mode used is $\mathcal{M}_M$. The probability of this event, denoted as $P_{\mathcal{M}_M^+}$, can be written as

$$P_{\mathcal{M}_M^+} = \sum_{j_M = \lceil \frac{R_T}{R_M} \rceil}^{N} \binom{N}{j_M} P_M^{j_M} \cdot P_M^{-(N-j_M)}, \tag{2.4}$$

where $j_M$ is the number of subchannels which can support mode $\mathcal{M}_M$.
In this case, the number of selected subchannels is $\lceil \frac{R_T}{R_M} \rceil$ and the achieved data rate is $R_M \lceil \frac{R_T}{R_M} \rceil$. Based on these, the average number of selected subchannels and the average achieved data rate, denoted as $\bar{N}_M$ and $\bar{R}_M$, can be derived as

$$\bar{N}_M = P_{\mathcal{M}_M}^+ \cdot \lceil \frac{R_T}{R_M} \rceil \tag{2.5}$$

and

$$\bar{R}_M = P_{\mathcal{M}_M}^+ \cdot \ R_M \lceil \frac{R_T}{R_M} \rceil, \tag{2.6}$$

respectively.

The case that the rate requirement can not be satisfied by only using the highest modulation mode $\mathcal{M}_M$, but can be satisfied by using the highest modulation mode $\mathcal{M}_M$ and the second highest modulation mode $\mathcal{M}_{M-1}$ together, i.e., when $j_M$ subchannels can support $\mathcal{M}_M$, where $0 \leq j_M < \lceil \frac{R_T}{R_M} \rceil$ and more than$\lceil \frac{R_T - R_M j_M}{R_{M-1}} \rceil$ subchannels can support $\mathcal{M}_{M-1}$, the second highest order modulation mode $\mathcal{M}_{M-1}$ is the lowest mode used. The probability of such event will be a function of $j_M$ and is given by

$$P_{\mathcal{M}_{M-1}^+}(j_M) = \binom{N}{j_M} P_M^{j_M} \cdot P_M^{-(N-j_M)} \sum_{j_{M-1}=\lceil \frac{R_T - R_M j_M}{R_{M-1}} \rceil}^{N-j_M} \binom{N-j_M}{j_{M-1}} (\frac{P_{M-1}}{P_M^-})^{j_{M-1}} \cdot (\frac{P_{M-1}^-}{P_M^-})^{(N-j_M-j_{M-1})}. \tag{2.7}$$

Note that the ratio $\frac{P_{M-1}}{P_M^-}$ is the conditional probability that a subchannel can support $\mathcal{M}_{M-1}$ given that it can not support $\mathcal{M}_M$. In this case, the number of selected subchannels and the achieved data rate, conditioned on $j_M$, are $j_M + \lceil \frac{R_T - R_M j_M}{R_{M-1}} \rceil$ and $R_M j_M + R_{M-1} \lceil \frac{R_T - R_M j_M}{R_{M-1}} \rceil$, respectively. Based on these, the average number of selected subchannels and the average achieved data rate, denoted as $\bar{N}_{M-1}$ and $\bar{R}_{M-1}$, can be derived as

$$\bar{N}_{M-1} = \sum_{j_M=0}^{\lceil \frac{R_T}{R_M} \rceil - 1} P^+_{\mathcal{M}_{M-1}}(j_M) \cdot (j_M + \lceil \frac{R_T - R_M j_M}{R_{M-1}} \rceil) \tag{2.8}$$

and

$$\bar{R}_{M-1} = \sum_{j_M=0}^{\lceil \frac{R_T}{R_M} \rceil - 1} P^+_{\mathcal{M}_{M-1}}(j_M) \cdot (R_M j_M + R_{M-1} \lceil \frac{R_T - R_M j_M}{R_{M-1}} \rceil), \tag{2.9}$$

respectively.

In general, the probability that mode $\mathcal{M}_i$ is the lowest order mode used given that $j_M$ subchannels support $\mathcal{M}_M$, $j_{M-1}$ subchannels support $\mathcal{M}_{M-1}$, $\cdots$, $j_{i+1}$ subchannels support $\mathcal{M}_{i+1}$, denoted as $P_{\mathcal{M}_i^+}(j_M, j_{M-1}, \ldots, j_{i+1})$, can be calculated as

$$P_{\mathcal{M}_i^+}(j_M, j_{M-1}, \ldots, j_{i+1}) = \binom{N}{j_M} P_M^{j_M} \cdot P_M^{-\,(N-j_M)}$$
$$\prod_{l=i+1}^{M-1} \binom{N - \sum_{k=l+1}^{M} j_k}{j_l} \left(\frac{P_l}{P_{l+1}^-}\right)^{j_l} \cdot \left(\frac{P_l^-}{P_{l+1}^-}\right)^{(N-\sum_{l=i+1}^{M} j_l)}$$
$$\sum_{j_i=\lceil \frac{R_T-\sum_{m=i+1}^{M} R_m j_m}{R_i} \rceil}^{N-\sum_{m=i+1}^{M} j_m} \binom{N - \sum_{m=i+1}^{M} j_m}{j_i}$$
$$\left(\frac{P_i}{P_{i+1}^-}\right)^{j_i} \cdot \left(\frac{P_i^-}{P_{i+1}^-}\right)^{(N-\sum_{m=i}^{M} j_m)}. \tag{2.10}$$

Note that $j_m$, $m = M, M-1, \cdots, i+1$, satisfy $0 < j_m < \lceil \frac{R_T - \sum_{l=m+1}^{M} R_l j_l}{R_m} \rceil$.

Based on the analysis above, performances of the proposed subchannel allocation scheme can be evaluated as follows. First of all, the outage probability, denoted by $P_{\text{out}}$, can be determined as

$$P_{\text{out}} = 1 - \sum_{i=1}^{M} \sum_{j_M=0}^{\lceil \frac{R_T}{R_M} \rceil - 1} \sum_{j_{M-1}=0}^{\lceil \frac{R_T - R_M j_M}{R_{M-1}} \rceil - 1} \cdots$$
$$\sum_{j_{i+1}=0}^{\lceil \frac{R_T-\sum_{l=i+2}^{M} R_l j_l}{R_{i+1}} \rceil - 1} P_{\mathcal{M}_i^+}(j_M, j_{M-1}, \ldots, j_{i+1}). \tag{2.11}$$

Furthermore, the average number of selected subchannels and the average achieved data rate can be calculated as

$$\bar{N}_A = \sum_{i=1}^{M} \sum_{j_M=0}^{\lceil \frac{R_T}{R_M} \rceil - 1} \sum_{j_{M-1}=0}^{\lceil \frac{R_T - R_M j_M}{R_{M-1}} \rceil - 1} \cdots \sum_{j_{i+1}=0}^{\lceil \frac{R_T-\sum_{l=i+2}^{M} R_l j_l}{R_{i+1}} \rceil - 1}$$
$$P_{\mathcal{M}_i^+}(j_M, j_{M-1}, \ldots, j_{i+1}) \cdot \left(\sum_{l=i+1}^{M} j_l + \lceil \frac{R_T - \sum_{l=i+1}^{M} R_l j_l}{R_i} \rceil\right) \tag{2.12}$$

and

$$\bar{R}_A = \sum_{i=1}^{M} \sum_{j_M=0}^{\lceil \frac{R_T}{R_M} \rceil - 1} \sum_{j_{M-1}=0}^{\lceil \frac{R_T - R_M j_M}{R_{M-1}} \rceil - 1} \cdots \sum_{j_{i+1}=0}^{\lceil \frac{R_T - \sum_{l=i+2}^{M} R_l j_l}{R_{i+1}} \rceil - 1}$$

$$P_{\mathcal{M}_i^+}(j_M, j_{M-1}, \ldots, j_{i+1}) \cdot (\sum_{l=i+1}^{M} R_l j_l + R_i \lceil \frac{R_T - \sum_{l=i+1}^{M} R_l j_l}{R_i} \rceil), \quad (2.13)$$

respectively.

Finally, the average feedback load includes 1 bit to indicate whether there is a transmission or not, $N$ bits for subchannel allocation result and $\bar{N}_A \cdot \log_2 M$ bits of indexes of the modulation modes for selected subchannels ($\bar{N}_A$ subchannels with $\log_2 M$ bits for each selected subchannel). In addition, the average feedback load is affected by the probability of transmission and therefore it can be calculated as

$$\bar{N}_F = (N + \bar{N}_A \cdot \log_2 M)(1 - P_{\text{out}}) + 1. \quad (2.14)$$

### 2.4.2 GS-LA option

With GS-LA option, all selected subchannels will use the same modulation mode. We again apply the total probability theorem and consider the events of using the $i$th modulation mode to achieve the data rate requirement.

Specifically, the highest modulation mode $\mathcal{M}_M$ will be used on all selected subchannels to achieve the rate requirement if more than $\lceil \frac{R_T}{R_M} \rceil$ subchannels can support $\mathcal{M}_M$, the probability of which is given by

$$P_{\mathcal{M}_M} = \sum_{j_M=\lceil \frac{R_T}{R_M} \rceil}^{N} \binom{N}{j_M} P_M^{j_M} \cdot P_M^{-(N-j_M)}. \quad (2.15)$$

In this case, the number of selected subchannels and the achieved data rate are $\lceil \frac{R_T}{R_M} \rceil$ and $R_M \lceil \frac{R_T}{R_M} \rceil$, respectively. Based on these, the average number of selected subchannels and the average achieved data rate, denoted as $\bar{N}_M$ and $\bar{R}_M$, can be derived as

$$\bar{N}_M = P_{\mathcal{M}_M} \cdot \lceil \frac{R_T}{R_M} \rceil \tag{2.16}$$

and

$$\bar{R}_M = P_{\mathcal{M}_M} \cdot \; R_M \lceil \frac{R_T}{R_M} \rceil, \tag{2.17}$$

respectively.

For the case that the rate requirement can not be satisfied by using the highest modulation mode $\mathcal{M}_M$ but can be satisfied by applying the second highest modulation mode $\mathcal{M}_{M-1}$ on all selected subchannels (including subchannels which can support $\mathcal{M}_M$ and subchannels which can support $\mathcal{M}_{M-1}$ ), i.e., $j_M$ subchannels can support $\mathcal{M}_M$, where $0 \leq j_M < \lceil \frac{R_T}{R_M} \rceil$, but more than $\lceil \frac{R_T - R_{M-1} j_M}{R_{M-1}} \rceil$ subchannels can support $\mathcal{M}_{M-1}$, $\mathcal{M}_{M-1}$ is used on all selected subchannels to achieve the data rate requirement. We derive the probability of such case, which depends on $j_M$, as

$$P_{\mathcal{M}_{M-1}}(j_M) = \binom{N}{j_M} \; P_M^{j_M} \cdot {P_M^-}^{(N-j_M)} \sum_{j_{M-1} = \lceil \frac{R_T - R_{M-1} j_M}{R_{M-1}} \rceil}^{N-j_M} \binom{N-j_M}{j_{M-1}} (\frac{P_{M-1}}{P_M^-})^{j_{M-1}} \cdot \; (\frac{P_{M-1}^-}{P_M^-})^{(N-j_M-j_{M-1})}. \tag{2.18}$$

In this case, the number of selected subchannels is $j_M + \lceil \frac{R_T - R_{M-1} j_M}{R_{M-1}} \rceil$, and the achieved data rate is $R_{M-1}(j_M + \lceil \frac{R_T - R_{M-1} j_M}{R_{M-1}} \rceil)$. Based on these, the average number of selected subchannels and the average achieved data rate, denoted by $\bar{N}_{M-1}$ and $\bar{R}_{M-1}$, are

$$\bar{N}_{M-1} = \sum_{j_M=0}^{\lceil \frac{R_T}{R_M} \rceil - 1} P_{\mathcal{M}_{M-1}}(j_M) \cdot (j_M + \lceil \frac{R_T - R_{M-1} j_M}{R_{M-1}} \rceil) \tag{2.19}$$

and

$$\bar{R}_{M-1} = \sum_{j_M=0}^{\lceil \frac{R_T}{R_M} \rceil - 1} P_{\mathcal{M}_{M-1}}(j_M) \cdot \; R_{M-1}(j_M + \lceil \frac{R_T - R_{M-1} j_M}{R_{M-1}} \rceil), \tag{2.20}$$

respectively.

In general, probability that the $i$th mode $\mathcal{M}_i$ is used on all selected subchannels to achieve data rate requirement, denoted by $P_{\mathcal{M}_i}(j_M, j_{M-1}, \ldots, j_{i+1})$, is given as: (Given that $j_M$ subchannels can support $\mathcal{M}_M$, $j_{M-1}$ subchannels can support $\mathcal{M}_{M-1}$, $\cdots$, $j_{i+1}$ subchannels can support $\mathcal{M}_{i+1}$)

$$P_{\mathcal{M}_i}(j_M, j_{M-1}, \ldots, j_{i+1}) = \binom{N}{j_M} P_M^{j_M} \cdot P_M^{-\,(N-j_M)}$$
$$\prod_{l=i+1}^{M-1} \binom{N-\sum_{k=l+1}^{M} j_k}{j_l} (\frac{P_l}{P_{l+1}^-})^{j_l} \cdot (\frac{P_l^-}{P_{l+1}^-})^{(N-\sum_{l=i+1}^{M} j_l)}$$
$$\sum_{j_i=\lceil \frac{R_T - R_i \sum_{m=i+1}^{M} j_m}{R_i} \rceil}^{N-\sum_{m=i+1}^{M} j_m} \binom{N-\sum_{m=i+1}^{M} j_m}{j_i} (\frac{P_i}{P_{i+1}^-})^{j_i} \cdot (\frac{P_i^-}{P_{i+1}^-})^{(N-\sum_{m=i}^{M} j_m)}. \quad (2.21)$$

Note that $j_m$, $m = M, M-1, \cdots, i+1$, satisfy $0 < j_m < \lceil \frac{R_T - R_m \sum_{l=m+1}^{M} j_l}{R_m} \rceil$.

Based on the analysis above, the outage probability expression can be obtained as

$$P_{\text{out}} = 1 - \sum_{i=1}^{M} \sum_{\substack{0 \le j_k \le \lceil \frac{R_T - R_k \sum_{m=k+1}^{M} j_m}{R_k} \rceil - 1 \\ i+1 \le k \le M}} P_{\mathcal{M}_i}(j_M, j_{M-1}, \ldots, j_{i+1}). \quad (2.22)$$

In addition, the average number of selected subchannels and the average achieved data rate can be calculated as

$$\bar{N}_A = \sum_{i=1}^{M} \sum_{\substack{0 \le j_k \le \lceil \frac{R_T - R_k \sum_{m=k+1}^{M} j_m}{R_k} \rceil - 1 \\ i+1 \le k \le M}} P_{\mathcal{M}_i}(j_M, j_{M-1}, \ldots, j_{i+1}) \cdot$$
$$(\sum_{l=i+1}^{M} j_l + \lceil \frac{R_T - R_i \sum_{l=i+1}^{M} j_l}{R_i} \rceil) \quad (2.23)$$

and

$$\bar{R}_A = \sum_{i=1}^{M} \sum_{\substack{0 \le j_k \le \lceil \frac{R_T - R_k \sum_{m=k+1}^{M} j_m}{R_k} \rceil - 1 \\ i+1 \le k \le M}} P_{\mathcal{M}_i}(j_M, j_{M-1}, \ldots, j_{i+1}) \cdot$$
$$R_i(\sum_{l=i+1}^{M} j_l + \lceil \frac{R_T - R_i \sum_{l=i+1}^{M} j_l}{R_i} \rceil), \tag{2.24}$$

respectively.

The average feedback load can be quantized as follows. Since all selected subchannels should use the same modulation mode, the feedback load only contains 1 bit transmission indicator, $N$ bits for subchannel allocation results and $\log_2 M$ bits of index of the modulation mode for all selected subchannels. Again the average feedback load is affected by the probability of transmission and can be written as

$$\bar{N}_F = (N + \log_2 M)(1 - P_{\text{out}}) + 1. \tag{2.25}$$

## 2.5 Numerical Results

In this section, we illustrate the analytical results with some selected numerical examples. In these examples, it is assumed that the following four modulation schemes are used: 32QAM, 16QAM, 8QAM and QPSK, with data rate of $5, 4, 3$ and 2 bits/symbol for uncoded case. The target error rate is set as $BER = 10^{-4}$. Correspondingly, the thresholds are 15.30 dB, 12.21 dB, 11.30 dB and 8.34 dB [18], respectively. Without loss of generality, the total number of subchannels is set to be $N = 10$.

In Figure 2.2 and Figure 2.3, we compare the outage probability of the two options of the proposed subchannel allocation algorithm. We can observe that GS-LA has higher outage probability than GS-FA over low average SNR region for both $R_T = 5$ bits/symbol and $R_T = 10$ bits/symbol cases. The reason is that GS-FA enjoys more flexibility. For example, with the rate requirement of $R_T = 5$ bits/symbol, if there

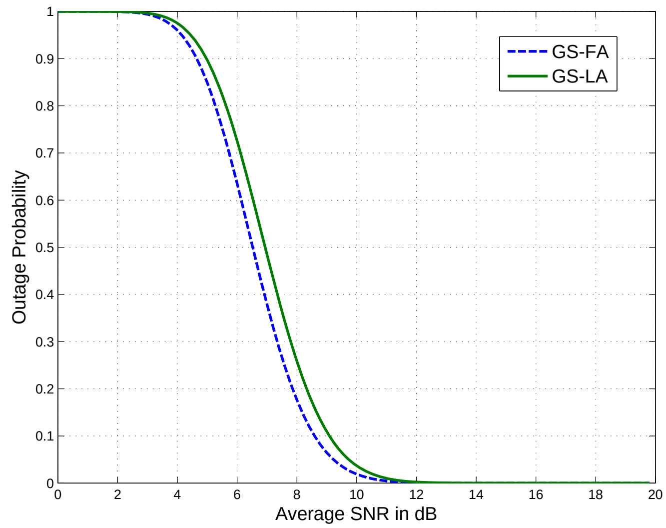


Figure 2.2: Outage Performance with $R_T = 5$ bits/symbol for both options

is only one subchannel with its highest supportive modulation mode of 8QAM, one subchannel with its highest supportive modulation mode of QPSK and all other subchannels can not even support QPSK out of total 10 subchannels over some low SNR region, GS-FA can flexibly use 8QAM and QPSK on corresponding subchannels to achieve the rate requirement whereas GS-LA will fail to transmit by applying QPSK on those two subchannels. This performance gap is obvious only over low average SNR region and disappears with increasing average SNR for both $R_T = 5$ bits/symbol and $R_T = 10$ bits/symbol cases. We can also observe that the performance gap of the two options is smaller in $R_T = 5$ bits/symbol case than in $R_T = 10$ bits/symbol case. In addition, larger SNR region experiences outage in $R_T = 10$ bits/symbol case than in $R_T = 5$ bits/symbol case. It is because that with the higher rate requirement for $R_T = 10$ bits/symbol case, it is more likely that the rate requirement can not be satisfied over low average SNR region, which leads to more outage events.

In Figure 2.4 and Figure 2.5, the average number of selected subchannels for both options are compared. Firstly, we can see that over high average SNR region, the number of selected subchannels with both options eventually converge to 1 and 2 for $R_T = 5$ bits/symbol and $R_T = 10$ bits/symbol case, respectively as expected. It is because that a single subchannel using 32 QAM will satisfy the rate requirement of $R_T = 5$ bits/symbol and using 2 such subchannels can satisfy the rate requirement of $R_T = 10$ bits/symbol. Secondly, we can see that for $R_T = 5$ bits/symbol case, when

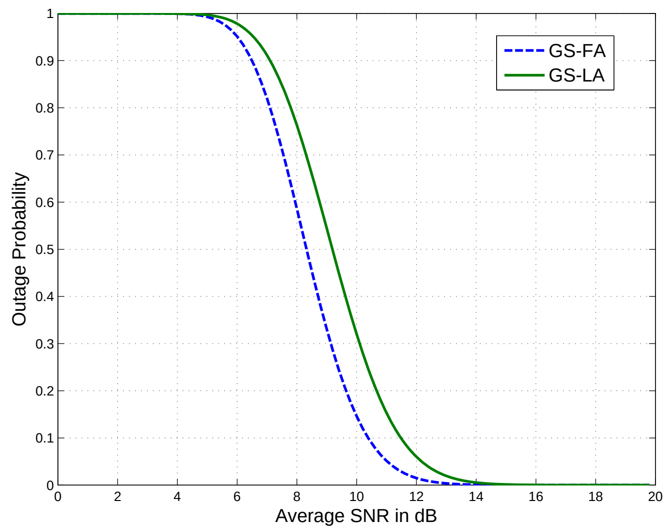


Figure 2.3: Outage Performance with $R_T = 10$ bits/symbol for both options

the average SNR is in the medium range of 7dB-11dB, GS-FA uses less subchannels than GS-LA due to its flexibility of allowing different modulation modes to be used on different subchannels. Similarly, GS-FA uses less subchannels than GS-LA over the medium average SNR region of 10dB-14dB for $R_T = 10$ bits/symbol case. Finally, GS-FA option leads to more selected subchannels over low average SNR region for both $R_T = 5$ bits/symbol case and $R_T = 10$ bits/symbol case. This is because that GS-LA option leads to more frequent outage event (has higher outage probability) over that average SNR region, where transmission does not occur.

The performance benefits of GS-FA option can also be observed in Figure 2.6 and Figure 2.7, where the average achieved data rate is plotted as a function of average SNR for both options. We can observe that GS-FA option always achieves higher rate than GS-LA option. Although both options demonstrate a similar behavior as the average SNR increases in $R_T = 5$ bits/symbol case, the performance gap is very obvious in $R_T = 10$ bits/symbol case. It worths noting that in medium average SNR range, both options may lead to a higher rate than the target as expected for both $R_T = 5$ bits/symbol and $R_T = 10$ bits/symbol cases. Take $R_T = 5$ bits/symbol case for example. If 16 QAM is the highest supportive modulation mode among all subchannels over medium average SNR region, at least 2 subchannels are needed to satisfy the rate requirement, which may lead to an achieved data rate of 8 bits (higher than the data rate requirement).

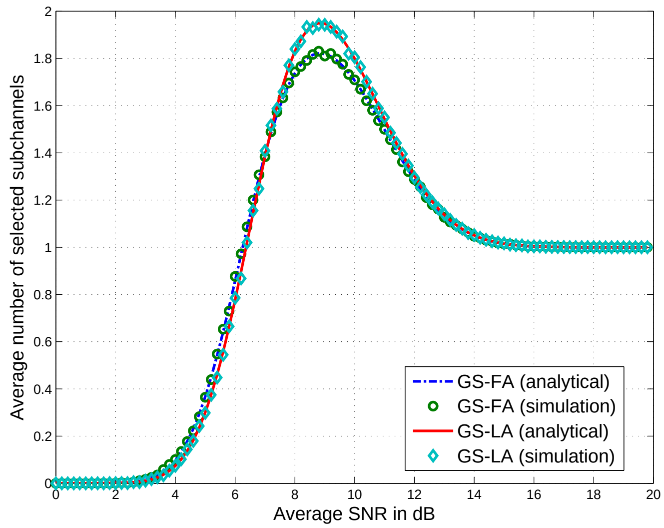


Figure 2.4: Average number of selected subchannels with $R_T = 5$ bits/symbol for both options

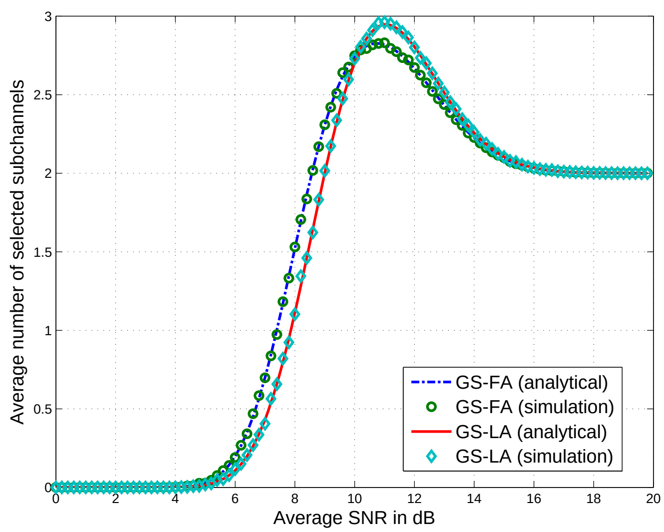


Figure 2.5: Average number of selected subchannels with $R_T = 10$ bits/symbol for both options

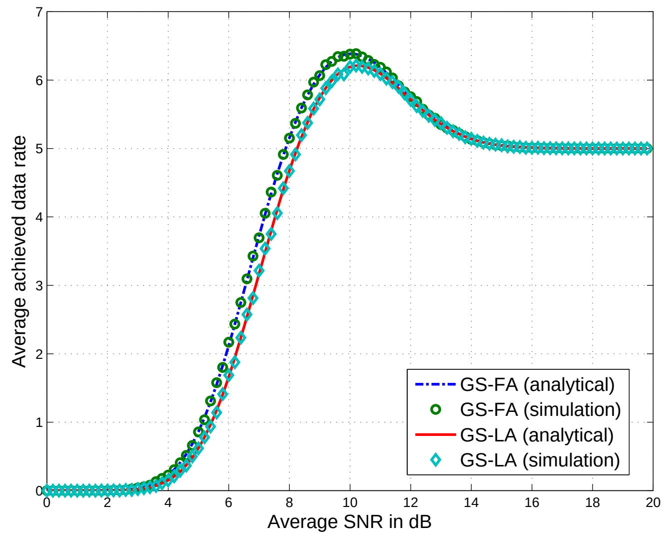


Figure 2.6: Average achieved rate with $R_T = 5$ bits/symbol for both options

Finally, in Figure 2.8 and Figure 2.9, we compare the average feedback load of the two options. Firstly, in Figure 2.8, both options lead to similar behavior as the average SNR increases. Specifically, it increases from 1 bit (indicating whether there will be a transmission or not) and saturates as 13 bits (one bit to indicate transmission, ten bits for subchannel selection result and 2 bits to indicate the modulation modes of selected subchannels). From Figure 2.9, we can see the obvious gap of feedback overhead between two options. In particular, GS-FA incurs 15 bits feedback (1 bit transmission indicator, ten bits for subchannel selection result and 4 bits to indicate the modulation modes of selected subchannels) whereas GS-LA still requires 13 bits. This is because the feedback load in GS-FA is related to the number of selected subchannels, since different modulation modes can be applied on these subchannels, whereas the feedback load in GS-LA has nothing to do with the number of selected subchannels, since all the selected subchannels have to use the same modulation mode. On the other hand, for both $R_T = 5$ bits/symbol and $R_T = 10$ bits/symbol cases, the feedback load of GS-LA option is a monotonically increasing function of the average SNR where that of GS-FA option demonstrates a peak over medium average SNR region. In addition, over all average SNR region, GS-LA option always leads to a smaller feedback load than GS-FA option.

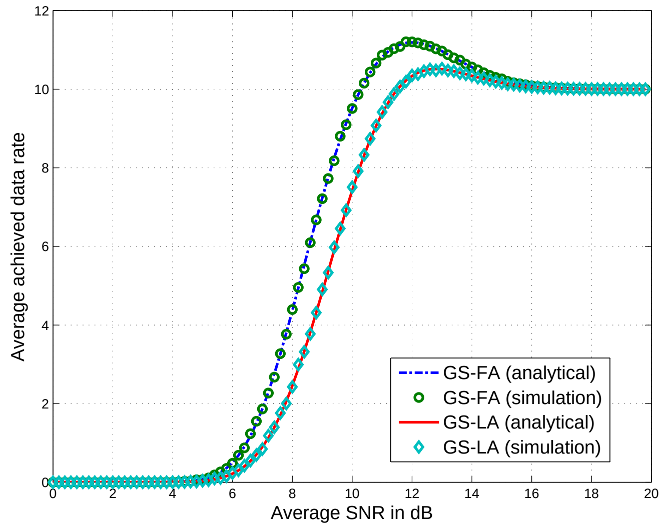


Figure 2.7: Average achieved rate with $R_T = 10$ bits/symbol for both options

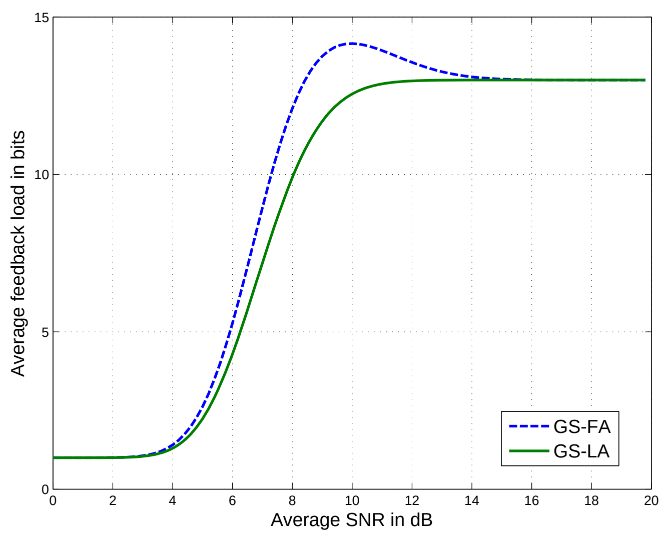


Figure 2.8: Average feedback load with $R_T = 5$ bits/symbol for both options

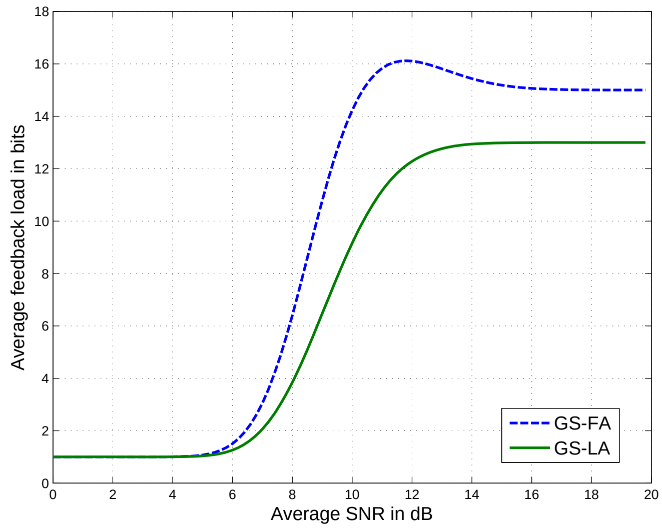


Figure 2.9: Average feedback load with $R_T = 10$ bits/symbol for both options

## 2.6 Conclusion

In this chapter, we propose a low-complexity greedy subchannel allocation algorithm for downlink adaptive OFDMA single-hop transmission systems. We analyze the performances of the proposed algorithm with two different options, depending on whether a common modulation mode is used on all selected subchannels or not. From the selected numerical examples based on the analysis, we demonstrate the tradeoff between performance versus feedback load between both options. We can see that the greedy selection with limited adaptivity (GS-LA) can achieve very similar performance (in terms of the average number of selected subchannels and the average achieved data rate) as that with full adaptivity case (GS-FA) but with much lower feedback load, especially when the target data rate is high (e.g., $R_T = 10$ bits/symbol case).

# Chapter 3

# Extensions to Dual-Hop Relay Transmission

## 3.1 Introduction

One key factor for the next generation wireless communications is to provide wide range HDR services with limited radio spectrum. For fixed bandwidth and power, a large amount of base stations have to be deployed to cover all the service range in cellular networks with traditional cellular architectures, which is impractical due to the prohibitive cost. Relaying systems combined with OFDM/OFDMA can provide HDR coverage with low cost, which is promising for the next generation wireless communications.

In general, relaying transmission systems can be classified into two main categories: Amplified-and-Forward (AF) and Decode-and-Forward (DF) relaying. In AF systems, received signals are only amplified and retransmitted by the relay without performing any decoding. Intuitively, we can see that AF scheme is of low-complexity. Meanwhile, AF systems are sometimes referred to the non-regenerative systems and the relays in AF systems play the role of analog repeater. In DF systems, relays first decode the received signal and verify the correctness and then re-encode and re-transmit the signal to the destination. DF scheme is more complicated than AF scheme, but it can effectively reduce noise propagation by performing decoding and re-encoding [19]. During recent years, many works have focused on performance analysis of resource allocation or scheduling for OFDMA systems with multiple source and relays or relay-aided cellular OFDMA systems [20] [21]. However, those schemes with optimal solutions (if solutions exist) usually lead to complex optimization problems with high computational complexity. They usually need complete channel state information of all channels at both base station and relay side, incurring a large amount of feedback overhead. Therefore, it is of great interest to find low-complexity resource allocation schemes for OFDMA relaying transmission systems.

In this chapter, we propose four subchannel allocation schemes for adaptive OFDMA dual-hop relay transmission systems. The ordered subchannel selection with adaptive modulation scheme is applied to both DF and AF scenarios. Based on this, the resulting four schemes are DF with full adaptivity (DF-FA), AF with full adaptivity (AF-FA), AF with limited adaptivity (AF-LA), and AF with limited adaptivity and subchannel mapping (AF-LA with SCM), respectively. The objective is to achieve fair scheduling while satisfying the rate requirement of scheduled users with a minimum system resource in terms of subchannels. Users are considered sequentially with known priority. For each user, the minimum number of the best subchannels are allocated, which can satisfy its rate requirement. Again to simplify the bit and power loading process and minimize the feedback load, constant-power variable-rate adaptive modulation scheme is applied on the selected subchannels [17].

In the following sections, performances of the proposed schemes are evaluated through mathematical analysis. To be specific, analytical expression for the average number of selected subchannels, average achieved data rate and outage probability are derived. Selected numerical examples are presented to illustrate the mathematical formulism. In the end, performance comparisons among different schemes are given.

## 3.2 System and Channel Models

We consider a single-cell downlink adaptive OFDMA system with dual-hop relaying, where one source $S$ (Base Station) communicates with multiple destination $D$ (Users) via one relay $R$. The system model is shown in Figure 3.1. Similar to single-hop transmission case, subchannel allocation and data transmission happens on a frame-by-frame basis. And each transmission frame is further divided into two sub-

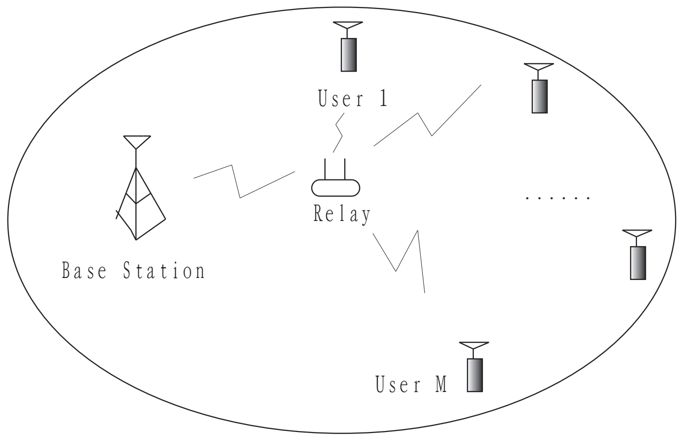


Figure 3.1: System Model

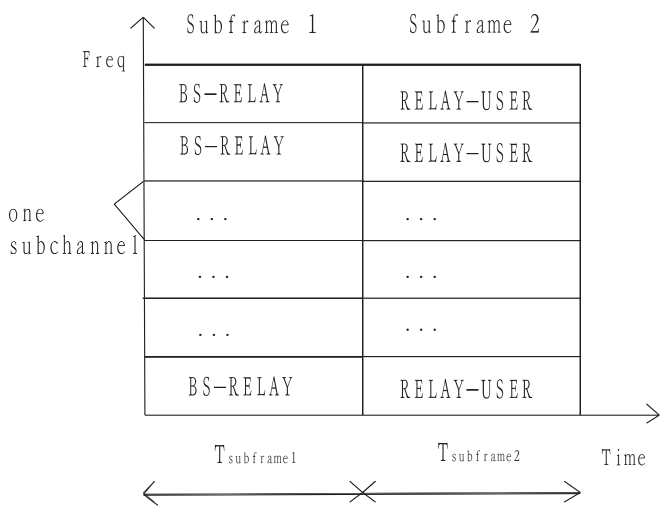


Figure 3.2: Frame Structure

frames with equal time division. In the first subframe, $S$ transmits an OFDM symbol to $R$; in the second subframe, $R$ helps to transmit that OFDM symbol from $S$ to $D$. During each transmission frame, the total bandwidth in the frequency domain is divided into $N$ subchannels for each hop to serve users with known priority. The frame structure is shown in Figure 3.2. In our work, we assume the number of subchannels on the source-relay hop and the relay-destination hop are the same (both are $N$ subchannels). In the real-world system, these two hops may have asymmetrical number of subchannels, e.g., the source-relay hop may have more subchannels than the relay-destination hop. But our analysis will still apply regardless of whether the number of subchannels are the same for both hops or not.

We assume that subchannels on hop1 and hop2 experience independent but not necessarily identically distributed Rayleigh fading while all subchannels within hop1 and hop2 experience i.i.d. Raleigh fading. As such, the PDF of the instantaneous SNR on different subchannels of hop1 and hop2, denoted as $p_1(\gamma)$ and $p_2(\gamma)$, are commonly given by

$$p_1(\gamma) = \frac{1}{\bar{\gamma_1}} \exp(-\frac{\gamma}{\bar{\gamma_1}}) \tag{3.1}$$

and

$$p_2(\gamma) = \frac{1}{\bar{\gamma_2}} \exp(-\frac{\gamma}{\bar{\gamma_2}}), \tag{3.2}$$

respectively, where $\bar{\gamma_1}$ and $\bar{\gamma_2}$ are the average SNR per subchannel for hop1 and hop2, respectively. And the CDF (Cumulative Distribution Function) of the instantaneous SNR on different subchannels of hop1 and hop2, denoted as $\mathrm{P}_1(\gamma)$ and $\mathrm{P}_2(\gamma)$, can be derived as

$$\mathrm{P}_1(\gamma) = 1 - \exp(-\frac{1}{\bar{\gamma_1}} \cdot \gamma) \tag{3.3}$$

and

$$\mathrm{P}_2(\gamma) = 1 - \exp(-\frac{1}{\bar{\gamma_2}} \cdot \gamma), \tag{3.4}$$

respectively.

## 3.3 Proposed Schemes

We propose four schemes based on the objective of serving scheduled users with minimum best system resource in terms of subchannels to satisfy their data rate requirements under a certain error rate performance restriction. During each transmission frame, a group of users are sequentially considered according to their known priority. The focus is still on the first scheduled user in a group with a data rate requirement of $R_T$ bits per OFDM symbol and note that the remaining users in the group will be served in a similar way with less available subchannels. Since either hop in relaying transmission system only works for half of transmission time, the actual achieved data rate with the selected subchannels is half of total data rate that can be supported by those selected subchannels if using the whole time of a transmission frame. Therefore, in order to satisfy the user's rate requirement $R_T$, the total data rate of the selected subchannels should be at least twice of user's rate requirement $2R_T$. Like single-hop transmission case, the condition of subchannels on both hops will be re-estimated and the subchannels on both hops will be re-ordered in the next transmission frame where a new group of users will be served in the same method as previous served group.

### 3.3.1 AF-FA scheme

In AF relaying systems, the relay only amplifies the received signal and does not decode it. In AF-FA scheme, subchannels for both hops are sequentially combined to form an end-to-end subchannel. Different modulation modes can be used on different end-to-end subchannels. Therefore, we can simplify the analysis by focusing on the end-to-end SNRs of subchannels. Then, after proper channel estimation for both hop1 and hop2 at the destination $D$, $D$ will rank the end-to-end subchannels in a descending order based on the end-to-end instantaneous SNR values. After acquiring the ordered end-to-end subchannels, the destination $D$ will select the minimum number of best end-to-end subchannels whenever the user's rate requirement is satisfied. The mode of operation can be expressed as selecting $Ns$ end-to-end subchannels where $Ns$ satisfies $\sum_{j=1}^{Ns-1} R_j < 2R_T$ but $\sum_{j=1}^{Ns} R_j \geq 2R_T$. And after the selection, the destination $D$ first informs the relay $R$ the selected end-to-end subchannels and the corresponding modulation modes to use through feedback. Then $R$ forwards such information to the source $S$. Finally $S$ starts to transmit. However, if the user's rate requirement still can not be satisfied by utilizing all end-to-end subchannels, there will be no transmission and the user will be announced an outage.

### 3.3.2 DF-FA scheme

In DF relaying systems, the relay needs to decode the received signal first and then re-encode it before re-transmission. In DF-FA scheme, different subchannels can be chosen for both hops and different kinds of modulation modes can be applied on the selected subchannels for both hops as well. After proper channel estimation at the relay $R$, $R$ ranks the SNR of all subchannels for hop1 in a descending order. After acquiring the ordered subchannels for hop1, $R$ selects the best $N_{D_1}(\leq N)$ subchannels for hop1 according to the instantaneous SNR of hop1 whenever the user's rate requirement is satisfied on hop1, i.e., $N_{D_1}$ satisfies $\sum_{j=1}^{N_{D_1}-1} R_j < 2R_T$ but $\sum_{j=1}^{N_{D_1}} R_j \geq 2R_T$. And after proper channel estimation at the destination $D$, $D$ ranks the the SNR of all subchannels for hop2 in a descending order. After acquiring the ordered subchannels for hop2, $D$ selects the best $N_{D_2}(\leq N)$ subchannels for hop2 according to the instantaneous SNR of hop2 whenever the user's rate requirement is satisfied on hop2, i.e., $N_{D_2}$ satisfies $\sum_{j=1}^{N_{D_2}-1} R_j < 2R_T$ but $\sum_{j=1}^{N_{D_2}} R_j \geq 2R_T$. After the selections at both $R$ and $D$, $D$ informs $R$ the selected subchannels and the corresponding modulation modes to use for hop2 through feedback. Then $R$ informs the source $S$ the selected subchannels and the corresponding modulation modes to use for hop1 through feedback. Finally, $S$ starts to transmit. But in the worst case that the user's rate requirement still can not be achieved by using all subchannels for either hop, no transmission will occur and the user will experience outage.

### 3.3.3 AF-LA scheme

AF-LA scheme is very similar to AF-FA scheme, except that in AF-LA scheme all selected end-to-end subchannels have to use the same modulation mode to satisfy the user's rate requirement. For AF-LA scheme, the process of generating and ranking end-to-end subchannels are the same as AF-FA scheme. After acquiring the ordered end-to-end subchannels, the destination $D$ will select the minimum number of best end-to-end subchannels whenever the user's rate requirement is satisfied, i.e., $n$ best end-to-end subchannels are selected when $(n-1)R_{n-1} < 2R_T$ but $nR_n \geq 2R_T$. The feedback procedure of AF-LA scheme is similar to AF-FA scheme, except that the feedback load for AF-LA scheme is lower than AF-FA scheme. It is because that only one index of modulation mode corresponding to all selected end-to-end subchannels needs to be fed back for AF-LA scheme, whereas indexes of modulation modes corresponding to each selected end-to-end subchannel need to be fed back for AF-FA scheme. After acquiring all the feedback information, the source $S$ will start to transmit. However, in the worst case that the user's rate requirement still can not be satisfied by utilizing all end-to-end subchannels, there will be no transmission and the user will be announced an outage.

### 3.3.4 AF-LA with SCM scheme

In AF-LA with SCM scheme, all selected subchannel pairs also have to use the same modulation mode as AF-LA scheme. But the processes of generating subchannel pairs and feedback for AF-LA with SCM scheme are different from AF-LA scheme. In AF-LA with SCM scheme, after proper channel estimation at the relay $R$, $R$ ranks the SNR of all subchannels of hop1 in a descending order. And after proper channel estimation at the destination $D$, $D$ ranks the the SNR of all subchannels of hop2 in a descending order and feeds back the indexes and SNR values of these subchannels to $R$. Then $R$ does subchannel mapping based on the instantaneous SNR of both hops, i.e., $R$ chooses the subchannel with the best SNR of hop1 and the subchannel with the best SNR of hop2 to form the best subchannel pair, chooses the the subchannel with the second best SNR of hop1 and the subchannel with the second best SNR of hop2 to form the second best subchannel pair and so forth. After acquiring all ordered subchannel pairs, $R$ will select the minimum number of best subchannel pairs whenever the user's rate requirement is satisfied, i.e., $n$ best subchannel pairs are selected when $(n-1)R_{n-1} < 2R_T$ but $nR_n \geq 2R_T$. Finally, the relay $R$ informs the source $S$ and the destination $D$ the selected subchannel pairs and the corresponding modulation mode to use through feedback. Intuitively, we can see that AF-LA with SCM scheme is more complicated than AF-LA scheme but is expected to provide better performance. In addition, AF-LA with SCM scheme incurs higher feedback overhead, since the destination $D$ has to feed back additional indexes and SNR values of the ordered subchannels of hop2 to the relay $R$ as a compensation of mapping process compared with AF-LA scheme. The worst case happens when the user's rate requirement still can not be achieved by using all subchannel pairs. In that case, there will be no transmission and the user will be announced an outage.

## 3.4 Performance Analysis

### 3.4.1 AF-FA scheme

The performance analysis of AF-FA scheme can be carried out in a similar fashion as the single-hop case while based on the end-to-end SNRs of end-to-end subchannels. Assuming the instantaneous SNR of the subchannels of hop1 and hop2 are $\gamma_1$ and $\gamma_2$, respectively, the end-to-end instantaneous SNR of the end-to-end subchannels are given by [22],

$$\gamma = \frac{\gamma_1 \cdot \gamma_2}{\gamma_1 + \gamma_2 + 1} \approx \min[\gamma_1, \gamma_2]. \tag{3.5}$$

Since PDF of the exact end-to-end instantaneous SNR is unavailable, we use the approximate end-to-end instantaneous SNR (minimum instantaneous SNR of two hops) through out the analysis for AF-FA and AF-LA schemes. By referring to eq.(3.1) and eq.(3.2) and utilizing order statistics [23], $\min[\gamma_1, \gamma_2]$ is also exponential distributed with PDF

$$p(\gamma) = \frac{\bar{\gamma}_1 + \bar{\gamma}_2}{\bar{\gamma}_1 \cdot \bar{\gamma}_2} \exp(-\frac{\bar{\gamma}_1 + \bar{\gamma}_2}{\bar{\gamma}_1 \cdot \bar{\gamma}_2} \cdot \gamma). \tag{3.6}$$

The performance analysis for AF-FA scheme is omitted here, since it can be conducted in a similar way as GS-FA option in single-hop case of chapter 2 except that the probability analysis is based on end-to-end SNR of end-to-end subchannels.

### 3.4.2 DF-FA scheme

The PDF of instantaneous SNR of subchannels for both hops are defined in eq.(3.1) and eq.(3.2), respectively. The modulation mode selection for both hops are based on these instantaneous SNR values. We assume that there are $L$ different modulation modes with corresponding threshold intervals, which is similar to single-hop case. By referring to eq.(3.1) and eq.(3.2), we derive the probability that the SNR is in the $l$th interval $\Pr[\gamma_{T_l} < \gamma < \gamma_{T_{l+1}}]$ and, as such, model $\mathcal{M}_l$ can be used as

$$P_{1l} = \int_{\gamma_{T_l}}^{\gamma_{T_{l+1}}} p_1(\gamma)\,\mathrm{d}\gamma = \exp(-\frac{\gamma_{T_l}}{\bar{\gamma}_1}) - \exp(-\frac{\gamma_{T_{l+1}}}{\bar{\gamma}_1}) \tag{3.7}$$

and

$$P_{2l} = \int_{\gamma_{T_l}}^{\gamma_{T_{l+1}}} p_2(\gamma)\,\mathrm{d}\gamma = \exp(-\frac{\gamma_{T_l}}{\bar{\gamma}_2}) - \exp(-\frac{\gamma_{T_{l+1}}}{\bar{\gamma}_2}), \tag{3.8}$$

for hop1 and hop2, respectively, where $l = 1, 2, \ldots, L$.
The probability that a subchannel can not support $\mathcal{M}_l$ can also be derived as

$$P_{1l}^{-} = \Pr[\gamma < \gamma_{T_l}] = 1 - \exp(-\frac{\gamma_{T_l}}{\bar{\gamma}_1}) \tag{3.9}$$

and

$$P_{2l}^{-} = \Pr[\gamma < \gamma_{T_l}] = 1 - \exp(-\frac{\gamma_{T_l}}{\bar{\gamma}_2}), \tag{3.10}$$

for hop1 and hop2, respectively.

Following the similar analytical approach for GS-FA in previous chapter, we can determine the probability that the $i$th mode $\mathcal{M}_i$ is the lowest modulation mode used on hop1 to satisfy the user's rate requirement given that $j_{1_L}$ subchannels can support mode $\mathcal{M}_L$, $j_{1_{L-1}}$ subchannels can support mode $\mathcal{M}_{L-1}$, ..., $j_{1_{i+1}}$ subchannels can support mode $\mathcal{M}_{i+1}$, denoted as $P_{1\mathcal{M}_i}(j_{1_L}, j_{1_{L-1}}, \ldots, j_{1_{i+1}})$, as:

$$P_{1\mathcal{M}_i}(j_{1_L}, j_{1_{L-1}}, \ldots, j_{1_{i+1}}) = \binom{N}{j_{1_L}} P_{1_L}^{j_{1_L}} P_{1_L}^{-\,(N-j_{1_L})}$$
$$\prod_{q=i+1}^{L-1} \binom{N-\sum_{k=q+1}^{L-1} j_{1_k}}{j_{1_q}} (\frac{P_{1_q}}{P^-_{1_{q+1}}})^{j_{1q}} (\frac{P^-_{1_q}}{P^-_{1_{q+1}}})^{(N-\sum_{q=i+1}^{L} \; j_{1_q})}$$
$$\sum_{j_{1_i}=\lceil \frac{2R_T-\sum_{l=i+1}^{L} \; R_l j_{1_l}}{R_i} \rceil}^{N-\sum_{l=i+1}^{L} \; j_{1_l}} \binom{N-\sum_{l=i+1}^{L} \; j_{1_l}}{j_{1_i}}$$
$$(\frac{P_{1_i}}{P^-_{1_{i+1}}})^{j_{1_i}} (\frac{P^-_{1_i}}{P^-_{1_{i+1}}})^{(N-\sum_{l=i}^{L} \; j_{1_l})} \;, \tag{3.11}$$

where $j_{1_l}$, $l = L, L-1, \cdots, i+1$, satisfy $0 < j_{1_l} < \lceil \frac{2R_T-\sum_{q=l+1}^{L} \; R_q j_{1_q}}{R_l} \rceil$.

The probability for hop2 can be similarly obtained. Consider all the possible realizations of hop1 and hop2, while noting that hop1 and hop2 are independent, we can obtain the outage probability over two hops as:

$$P_{\text{out}} = 1 - \sum_{i=1}^{L} \sum_{\substack{0 \le j_{1k} \le \lceil \frac{2R_T-\sum_{m=k+1}^{L} R_m j_{1_m}}{R_{m+1}} \rceil -1 \\ i+1 \le k \le L}} P_{1\mathcal{M}_i}(j_{1_L}, j_{1_{L-1}}, \ldots, j_{1_{i+1}}) \cdot$$
$$\sum_{i'=1}^{L} \sum_{\substack{0 \le j_{2h} \le \lceil \frac{2R_T-\sum_{n=h+1}^{L} R_n j_{2_n}}{R_{n+1}} \rceil -1 \\ i'+1 \le h \le L}} P_{2\mathcal{M}_{i'}}(j_{2_L}, j_{2_{L-1}}, \ldots, j_{2_{i'+1}}), \tag{3.12}$$

where $P_{2\mathcal{M}_{i'}}(j_{2_L}, j_{2_{L-1}}, \ldots, j_{2_{i'+1}})$ is defined in the same way as $P_{1\mathcal{M}_i}(j_{1_L}, \ldots, j_{1_{i+1}})$ for hop2. The average number of selected subchannels over two hops can be determined as the average of those selected on both hops. Again, considering all possible realizations of $j$ vectors for hop1 and hop2 (note that for a given j vector, the number of selected subchannels of hop1 is $(\sum_{s=i+1}^{L} \; j_{1_s} + \lceil \frac{2R_T-\sum_{s=i+1}^{L} \; R_s j_{1_s}}{R_i} \rceil))$, the average number of selected subchannels is obtained as

$$\bar{N_D} = \sum_{i=1}^{L} \sum_{\substack{0 \le j_{1k} \le \lceil \frac{2R_T - \sum_{m=k+1}^{L} R_m j_{1_m}}{R_{m+1}} \rceil - 1 \\ i+1 \le k \le L}} P_{1\mathcal{M}_i}(j_{1_L}, j_{1_{L-1}}, \ldots, j_{1_{i+1}}) \cdot$$
$$\sum_{i'=1}^{L} \sum_{\substack{0 \le j_{2h} \le \lceil \frac{2R_T - \sum_{n=h+1}^{L} R_n j_{2_n}}{R_{n+1}} \rceil - 1 \\ i'+1 \le h \le L}} P_{2\mathcal{M}_{i'}}(j_{2_L}, j_{2_{L-1}}, \ldots, j_{2_{i'+1}}) \cdot$$
$$\frac{1}{2}\Bigg( (\sum_{s=i+1}^{L} j_{1_s} + \lceil \frac{2R_T - \sum_{s=i+1}^{L} R_s j_{1_s}}{R_i} \rceil) +$$
$$(\sum_{q=i'+1}^{L} j_{2_q} + \lceil \frac{2R_T - \sum_{q=i'+1}^{L} R_q j_{2_q}}{R_{i'}} \rceil) \Bigg) \tag{3.13}$$

Finally, the average achieved rate over dual-hop with DF-FA can be determined, while noting that the instantaneous rate is the minimum of the rates supportable by both hops (note that the rate supported by hop1 is $(\sum_{s=i+1}^{L} R_s j_{1_s} + R_i \lceil \frac{2R_T - \sum_{s=i+1}^{L} R_s j_{1_s}}{R_i} \rceil)$ for a particular j vector), as

$$\bar{R_D} = \frac{1}{2} \sum_{i=1}^{L} \sum_{\substack{0 \le j_{1k} \le \lceil \frac{2R_T - \sum_{m=k+1}^{L} R_m j_{1_m}}{R_{m+1}} \rceil - 1 \\ i+1 \le k \le L}} P_{1\mathcal{M}_i}(j_{1_L}, j_{1_{L-1}}, \ldots, j_{1_{i+1}}) \cdot$$
$$\sum_{i'=1}^{L} \sum_{\substack{0 \le j_{2h} \le \lceil \frac{2R_T - \sum_{n=h+1}^{L} R_n j_{2_n}}{R_{n+1}} \rceil - 1 \\ i'+1 \le h \le L}} P_{2\mathcal{M}_{i'}}(j_{2_L}, j_{2_{L-1}}, \ldots, j_{2_{i'+1}}) \cdot$$
$$\min\Bigg( (\sum_{s=i+1}^{L} R_s j_{1_s} + R_i \lceil \frac{2R_T - \sum_{s=i+1}^{L} R_s j_{1_s}}{R_i} \rceil),$$
$$(\sum_{q=i'+1}^{L} R_q j_{2_q} + R_{i'} \lceil \frac{2R_T - \sum_{q=i'+1}^{L} R_q j_{2_q}}{R_{i'}} \rceil) \Bigg), \tag{3.14}$$

### 3.4.3 AF-LA scheme

While the method in chapter 2 can directly apply to this scenario, we use a new analytical method to AF-LA scheme to calculate the outage probability, average achieved data rate and the average number of selected end-to-end subchannels in this chapter. The idea is to consider $N$ mutually exclusive cases, where exact $n$ end-to-end subchannels can satisfy the rate requirement ($n = 1, 2, \ldots, N$). The core of the analysis is to figure out the probability that the $n$th end-to-end subchannel can support rate $\frac{2R_T}{n}$ while the $n-1$th end-to-end subchannel, $n-2$th end-to-end subchannel, ..., 1st end-to-end subchannel, can not support rate $\frac{2R_T}{n-1}$, $\frac{2R_T}{n-2}$, ..., and $\frac{2R_T}{1}$, respectively. Such probability is denoted as $\Pr_{(\mathrm{n})}$ throughout the analysis for AF-LA scheme.

Referring to eq.(3.6), CDF of the instantaneous end-to-end SNR of the end-to-end subchannels can be derived as

$$P(\gamma) = 1 - \exp(-\frac{\bar{\gamma}_1 + \bar{\gamma}_2}{\bar{\gamma}_1 \cdot \bar{\gamma}_2} \cdot \gamma). \tag{3.15}$$

By utilizing order statistics, the joint PDF of end-to-end SNR of the $n$th and $n-1$th, $n-2$th, $n-3$th, ..., 1st largest end-to-end subchannels, given by [24], is

$$\begin{aligned} &p_{\gamma_{(1)},\gamma_{(2)},\ldots,\gamma_{(n-1)},\gamma_{(n)}}(\alpha_1, \alpha_2, \ldots, \alpha_{n-1}, \alpha_n) = n!\binom{N}{n}[P(\alpha_n)]^{N-n} \cdot \prod_{i=1}^{n} p(\alpha_i), \\ &\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_{n-1} \geq \alpha_n, \end{aligned} \tag{3.16}$$

where $n = 1, 2, \ldots, N$, and $\gamma_{(1)}$, $\gamma_{(2)}$, ..., $\gamma_{(n-1)}$ and $\gamma_{(n)}$ are end-to-end SNRs of the 1st, 2nd, ..., $n-1$th and $n$th end-to-end subchannels, respectively, and $p(\alpha_i)$ and $P(\alpha_n)$ are defined in eq.(3.6) and eq.(3.15), respectively.

To calculate $\Pr_{(\mathrm{n})}$, we note that the end-to-end SNR of the $n$th end-to-end subchannel should be equal to or greater than the threshold $\gamma_{T_i}$, where $i$ satisfies $i = \min_k\{k \mid R_k \geq \frac{2R_T}{n}\}$ and $R_k$ corresponds to the rate which modulation mode $\mathcal{M}_k$ can support. Similarly, the end-to-end SNR of the $l$th end-to-end subchannel should be smaller than the threshold $\gamma_{T_{j_l}}$, where $j_l$ satisfies $j_l = \min_t\{t \mid R_t \geq \frac{2R_T}{l}\}$ and $R_t$ corresponds to the rate which modulation mode $\mathcal{M}_t$ can support, where $i = 1, 2, \ldots n$ and $l = 1, 2, \ldots, n-1$.

Therefore, referring to eq.(3.16), the probability $\mathrm{Pr}_{(\mathrm{n})}$ can be derived as

$$\begin{aligned}
&\mathrm{Pr}_{(\mathrm{n})} = \mathrm{Pr}[\gamma_{(n)} > \gamma_{T_i}, \gamma_{(n-1)} < \gamma_{T_{j_{n-1}}}, \gamma_{(n-2)} < \gamma_{T_{j_{n-2}}}, \ldots, \gamma_{(1)} < \gamma_{T_{j_1}}] \\
&= \int_{\gamma_{T_i}}^{\gamma_{T_{j_{n-1}}}} \int_{\gamma_{(n)}}^{\gamma_{T_{j_{n-1}}}} \int_{\gamma_{(n-1)}}^{\gamma_{T_{j_{n-2}}}} \cdots \int_{\gamma_{(2)}}^{\gamma_{T_{j_1}}} \\
&p_{\gamma_{(1)},\gamma_{(2)},\ldots,\gamma_{(n-1)},\gamma_{(n)}}(\alpha_1, \alpha_2, \ldots, \alpha_{n-1}, \alpha_n)\mathrm{d}\alpha_1\mathrm{d}\alpha_2, \ldots \mathrm{d}\alpha_{n-1}, \mathrm{d}\alpha_n, \\
&n \geq 2
\end{aligned} \tag{3.17}$$

and

$$\begin{aligned}
&\mathrm{Pr}_{(1)} = \mathrm{Pr}[\gamma_{(1)} > \gamma_{T_i}] \\
&= \int_{\gamma_{T_i}}^{\infty} p_{\gamma_{(1)}}(\alpha_1)\mathrm{d}\alpha_1 \\
&= N \cdot \sum_{k=0}^{N-1} \frac{(-1)^{k+1} \cdot \binom{N-1}{k}}{k+1} \exp\big(-(k+1)\Gamma\gamma_{T_i}\big), \\
&n = 1,
\end{aligned} \tag{3.18}$$

where $\Gamma = \frac{\bar{\gamma}_1 + \bar{\gamma}_2}{\bar{\gamma}_1 \cdot \bar{\gamma}_2}$.

It follows that the average number of selected end-to-end subchannels and outage probability, denoted as $\bar{N}$ and $P_{out}$, can be calculated as

$$\bar{N} = \sum_{n=1}^{N} \mathrm{Pr}_{(\mathrm{n})} \cdot (n) \tag{3.19}$$

and

$$P_{out} = 1 - \sum_{n=1}^{N} \mathrm{Pr}_{(\mathrm{n})}, \tag{3.20}$$

respectively. For the data rate analysis, since $n$ end-to-end subchannels have to use the same modulation mode, the total data rate with $n$ selected end-to-end subchahnels should be $n$ times of data rate $R_i$, which satisfies $i = \min_k\{k \mid R_k \geq \dfrac{2R_T}{n}\}$. Note that the minimum value of $R_k$ is $\lceil \frac{2R_T}{n} \rceil$. It is because that the rate requirement $R_T$ may not be an integer, but the modulation mode used to achieve it is commonly with integer supportable data rate. Therefore, the total data rate with $n$ selected end-to-end subchahnels is equal to $n\lceil\frac{2R_T}{n}\rceil$. Based on this, the average achieved data rate, denoted as $\bar{R}$, can be calculated as

$$\bar{R} = \frac{1}{2}\sum_{n=1}^{N} \mathrm{Pr}_{(\mathrm{n})} \cdot \ (n) \cdot \ \lceil\frac{2R_T}{n}\rceil. \tag{3.21}$$

It is very challenging, if not impossible, to find the general closed-form expression of $\mathrm{Pr}_{(\mathrm{n})}$ when $n > 1$, since it is difficult to find the normal pattern of the multiple integral in eq.(3.17). As an illustration, we present the closed-form expression of $\mathrm{Pr}_{(\mathrm{n})}$ for $n = 2$ and $n = 3$ in what follows.

When $n = 2$,

$$\begin{aligned}\mathrm{Pr}_{(2)} &= \mathrm{Pr}[\gamma_{(2)} > \gamma_{T_i}, \gamma_{(1)} < \gamma_{T_{j_1}}] \\ &= \int_{\gamma_{T_i}}^{\gamma_{T_{j_1}}} \int_{\gamma_{(2)}}^{\gamma_{T_{j_1}}} p_{\gamma_{(1)},\gamma_{(2)}}(\alpha_1, \alpha_2)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\ &= 2! \cdot \binom{N}{2} \cdot \Gamma \cdot \big( -\exp(-\Gamma\gamma_{T_{j_1}})A + B\big),\end{aligned} \tag{3.22}$$

where $p_{\gamma_{(1)},\gamma_{(2)}}(\alpha_1, \alpha_2)$ can be obtained from eq.(3.16) and

$$A = \sum_{i=0}^{N-2} \frac{\binom{N-2}{i}}{(i+1)\Gamma}(-1)^{i+1}\Big(\exp\big(-(i+1)\Gamma\gamma_{T_{j_1}}\big) - \exp\big(-(i+1)\Gamma\gamma_{T_i}\big)\Big) \tag{3.23}$$

and

$$B = \sum_{i=0}^{N-2} \frac{\binom{N-2}{i}}{(i+2)\Gamma}(-1)^{i+1}\Big(\exp\big(-(i+2)\Gamma\gamma_{T_{j_1}}\big) - \exp\big(-(i+2)\Gamma\gamma_{T_i}\big)\Big), \tag{3.24}$$

respectively.

$\mathrm{Pr}_{(3)}$ can be derived in a similar way. Note that the values of all thresholds in $n = 3$ case are different from $n = 2$ case, although the notations are the same. We have

$$\begin{aligned}
\Pr_{(3)} &= \Pr[\gamma_{(3)} > \gamma_{T_i}, \gamma_{(2)} < \gamma_{T_{j_2}}, \gamma_{(1)} < \gamma_{T_{j_1}}] \\
&= \int_{\gamma_{T_i}}^{\gamma_{T_{j_2}}} \int_{\gamma_{(3)}}^{\gamma_{T_{j_2}}} \int_{\gamma_{(2)}}^{\gamma_{T_{j_1}}} p_{\gamma_{(1)},\gamma_{(2)},\gamma_{(3)}}(\alpha_1, \alpha_2, \alpha_3) \mathrm{d}\alpha_1 \mathrm{d}\alpha_2 \mathrm{d}\alpha_3 \\
&= 3! \cdot \binom{N}{3} \cdot \Gamma \cdot (A_1 \cdot A + B_1 \cdot B + C_1 \cdot A + D_1 \cdot D),
\end{aligned} \tag{3.25}$$

where $p_{\gamma_{(1)},\gamma_{(2)},\gamma_{(3)}}(\alpha_1, \alpha_2, \alpha_3)$ can be obtained from eq.(3.16) and

$$A_1 = \exp(-\Gamma\gamma_{T_{j_1}})\exp(-\Gamma\gamma_{T_{j_2}}), \tag{3.26}$$

$$A = \sum_{i=0}^{N-3} \frac{\binom{N-3}{i}}{(i+1)\Gamma} (-1)^{i+1} \Big( \exp\big(-(i+1)\Gamma\gamma_{T_{j_1}}\big) - \exp\big(-(i+1)\Gamma\gamma_{T_i}\big) \Big), \tag{3.27}$$

$$B_1 = -\exp(-\Gamma\gamma_{T_{j_1}}), \tag{3.28}$$

$$B = \sum_{i=0}^{N-3} \frac{\binom{N-3}{i}}{(i+2)\Gamma} (-1)^{i+1} \Big( \exp\big(-(i+2)\Gamma\gamma_{T_{j_1}}\big) - \exp\big(-(i+2)\Gamma\gamma_{T_i}\big) \Big), \tag{3.29}$$

$$C_1 = -\frac{1}{2}\exp(-2\Gamma\gamma_{T_{j_2}}), \tag{3.30}$$

$$D_1 = \frac{1}{2}, \tag{3.31}$$

and

$$D = \sum_{i=0}^{N-3} \frac{\binom{N-3}{i}}{(i+3)\Gamma} (-1)^{i+1} \Big( \exp\big(-(i+3)\Gamma\gamma_{T_{j_1}}\big) - \exp\big(-(i+3)\Gamma\gamma_{T_i}\big) \Big), \tag{3.32}$$

respectively.

### 3.4.4 AF-LA with SCM scheme

AF-LA with SCM scheme also only allows different subchannel pairs to use the same modulation mode. However, it includes one more step of subchannel mapping compared with AF-LA scheme. The performance analysis of AF-LA with SCM scheme can be carried out in a similar way as AF-LA scheme. We can still focus on the analysis of the probability that the user's rate requirement can be satisfied by using $n$ subchannel pairs. But unlike AF-LA scheme, in which the joint PDF of SNR of the $n$th and $n-1$th, $n-2$th, $n-3$th, $\ldots$, 1st largest end-to-end subchannel is available, the mapping process changes the distribution of subchannel pairs of AF-LA with SCM scheme, i.e., the PDF of SNR of all subchannel pairs are no longer exponential distributed and are unknown after the mapping process.

From order statistics results, the joint PDF of SNR of the $n$th and $n-1$th, $n-2$th, $\ldots$, 1st largest subchannel of hop1 and hop2, is given by [24]

$$p_{\gamma_{1(1)},\gamma_{1(2)},\ldots,\gamma_{1(n-1)},\gamma_{1(n)}}(\alpha_1,\alpha_2,\ldots,\alpha_{n-1},\alpha_n) = n!\binom{N}{n}[P_1(\alpha_n)]^{N-n}\cdot\prod_{i=1}^{n}p_1(\alpha_i),$$
$$\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_{n-1} \geq \alpha_n, \tag{3.33}$$

$$p_{\gamma_{2(1)},\gamma_{2(2)},\ldots,\gamma_{2(n-1)},\gamma_{2(n)}}(\alpha_1,\alpha_2,\ldots,\alpha_{n-1},\alpha_n) = n!\binom{N}{n}[P_2(\alpha_n)]^{N-n}\cdot\prod_{i=1}^{n}p_2(\alpha_i),$$
$$\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_{n-1} \geq \alpha_n, \tag{3.34}$$

respectively, where $n = 1, 2, \ldots, N$, and $\gamma_{1(n)}$, $\gamma_{1(n-1)}$, $\ldots$, $\gamma_{1(1)}$, $\gamma_{2(n)}$, $\gamma_{2(n-1)}$ $\ldots$, $\gamma_{2(1)}$ are SNRs of the $n$th, $n-1$th, $\ldots$, 1st largest subchannel of hop1 and hop2, respectively and $p_1(\cdot)$, $p_2(\cdot)$, $\mathrm{P}_1(\cdot)$ and $\mathrm{P}_2(\cdot)$ are defined in eq.(3.1)-eq.(3.4), respectively. We now derive the joint CDF of the end-to-end SNR of the $n$th and $n-1$th, $\ldots$, 1st subchannel pair after subchannel mapping, which are denoted by $\gamma_{(n)}$, $\gamma_{(n-1)}$, $\ldots$, $\gamma_{(1)}$ and related to the $n$th, $n-1$th, $n-2$th, $\ldots$, 1st largest single-hop SNRs, denoted as

$P_{\gamma_{(1)},\gamma_{(2)},\ldots,\gamma_{(n-1)},\gamma_{(n)}}$, as

$$
\begin{aligned}
&P_{\gamma_{(n)},\gamma_{(n-1)},\gamma_{(n-2)},\ldots,\gamma_{(1)}}(\alpha_n, \alpha_{n-1}, \alpha_{n-2}, \ldots, \alpha_1)\\
&= \Pr[\gamma_{(n)} < \alpha_n, \gamma_{(n-1)} < \alpha_{n-1}, \gamma_{(n-2)} < \alpha_{n-2}, \ldots, \gamma_{(1)} < \alpha_1]\\
&= 1 - \sum_{i=1}^{n} \Pr[\gamma_{(i)} > \alpha_i]\\
&+ \sum_{k_1=1}^{n} \sum_{k_2=1}^{n} \ldots \sum_{k_{n-2}=1}^{n} \sum_{k_{n-1}=1}^{n} \Pr[\gamma_{(k_1)} > \alpha_{k_1}, \gamma_{(k_2)} > \alpha_{k_2}, \ldots,\\
&\gamma_{(k_{n-2})} > \alpha_{k_{n-2}}, \gamma_{(k_{n-1})} > \alpha_{k_{n-1}}]\\
&- \sum_{k_1=1}^{n} \sum_{k_2=1}^{n} \ldots \sum_{k_{n-2}=1}^{n} \Pr[\gamma_{(k_1)} > \alpha_{k_1}, \gamma_{(k_2)} > \alpha_{k_2}, \ldots, \gamma_{(k_{n-2})} > \alpha_{k_{n-2}}]\\
&+ \sum_{k_1=1}^{n} \sum_{k_2=1}^{n} \ldots \sum_{k_{n-3}=1}^{n} \Pr[\gamma_{(k_1)} > \alpha_{k_1}, \gamma_{(k_2)} > \alpha_{k_2}, \ldots, \gamma_{(k_{n-3})} > \alpha_{k_{n-3}}]\\
&- \ldots + (-1)^{(n-1)} \sum_{k_1=1}^{n} \sum_{k_2=1}^{n} \Pr[\gamma_{(k_1)} > \alpha_{k_1}, \gamma_{(k_2)} > \alpha_{k_2}]\\
&+(-1)^n \Pr[\gamma_{(k_1)} > \alpha_{k_1}, \gamma_{(k_2)} > \alpha_{k_2}, \ldots,\\
&\gamma_{(k_{n-2})} > \alpha_{k_{n-2}}, \gamma_{(k_{n-1})} > \alpha_{k_{n-1}}, \gamma_{(k_n)} > \alpha_{k_n}],\\
&k_1 < k_2 < k_3 < \ldots < k_{n-1} \text{ and } \alpha_1 > \alpha_2 > \alpha_3 > \ldots \alpha_n.
\end{aligned}
\tag{3.35}
$$

Therefore, the joint PDF of end-to-end SNR of the $n$th and $n-1$th, $n-2$th, $\ldots$, 1st subchannel pair, denoted as $p_{\gamma_{(n)},\gamma_{(n-1)},\gamma_{(n-2)},\ldots,\gamma_{(1)}}(\alpha_n, \alpha_{n-1}, \alpha_{n-2}, \ldots, \alpha_1)$, can be derived as

$$
\begin{aligned}
&p_{\gamma_{(n)},\gamma_{(n-1)},\gamma_{(n-2)},\ldots,\gamma_{(1)}}(\alpha_n, \alpha_{n-1}, \alpha_{n-2}, \ldots, \alpha_1)\\
&= \frac{\partial^n P\gamma_{(n)}, \gamma_{(n-1)}, \gamma_{(n-2)}, \ldots, \gamma_{(1)}(\alpha_n, \alpha_{n-1}, \alpha_{n-2}, \ldots, \alpha_1)}{\partial\alpha_n \partial\alpha_{n-1} \partial\alpha_{n-2} \ldots \partial\alpha_1}
\end{aligned}
\tag{3.36}
$$

Note that the only thing which is related to the derivative in eq.(3.36) is the last item $(-1)^n \Pr[\gamma_{(k_1)} > \alpha_{k_1}, \gamma_{(k_2)} > \alpha_{k_2}, \ldots, \gamma_{(k_{n-2})} > \alpha_{k_{n-2}}, \gamma_{(k_{n-1})} > \alpha_{k_{n-1}}, \gamma_{(k_n)} > \alpha_{k_n}]$ in eq.(3.35) and the sign $(-1)^n$ will be cancelled after derivative. And this item can be re-written as

$$
\begin{aligned}
&(-1)^n \mathrm{Pr}[\gamma_{(k_1)} > \alpha_{k_1}, \gamma_{(k_2)} > \alpha_{k_2}, \dots, \gamma_{(k_{n-2})} > \alpha_{k_{n-2}}, \gamma_{(k_{n-1})} > \alpha_{k_{n-1}}, \gamma_{(k_n)} > \alpha_{k_n}] \\
&= (-1)^n \mathrm{Pr}[\gamma_{1(k_1)} > \alpha_{k_1}, \gamma_{1(k_2)} > \alpha_{k_2}, \dots, \\
&\gamma_{1(k_{n-2})} > \alpha_{1k_{n-2}}, \gamma_{1(k_{n-1})} > \alpha_{k_{n-1}}, \gamma_{1(k_n)} > \alpha_{k_n}] \\
&\cdot \mathrm{Pr}[\gamma_{2(k_1)} > \alpha_{k_1}, \gamma_{2(k_2)} > \alpha_{k_2}, \dots, \\
&\gamma_{2(k_{n-2})} > \alpha_{k_{n-2}}, \gamma_{2(k_{n-1})} < \alpha_{k_{n-1}}, \gamma_{2(k_n)} > \alpha_{k_n}].
\end{aligned}
\tag{3.37}
$$

Based on eq.(3.37), the eq.(3.36) can be calculated as

$$
\begin{aligned}
&p_{\gamma_{(n)},\gamma_{(n-1)},\gamma_{(n-2)},\dots,\gamma_{(1)}}(\alpha_n, \alpha_{n-1}, \alpha_{n-2}, \dots, \alpha_1) \\
&= p_{\gamma_{1(1)},\gamma_{1(2)},\dots,\gamma_{1(n-1)},\gamma_{1(n)}}(\alpha_1, \alpha_2, \dots, \alpha_{n-1}, \alpha_n) \cdot \\
&\int_{\alpha_1}^{\infty}\int_{\alpha_2}^{\infty}\cdots\int_{\alpha_{n-1}}^{\infty}\int_{\alpha_n}^{\infty} p_{\gamma_{2(1)},\gamma_{2(2)},\dots,\gamma_{2(n-1)},\gamma_{2(n)}}(x_1, x_2, \dots, x_{n-1}, x_n) \\
&\mathrm{d}x_1 \mathrm{d}x_2 \dots \mathrm{d}x_{n-1} \mathrm{d}x_n \\
&+ p_{\gamma_{2(1)},\gamma_{2(2)},\dots,\gamma_{2(n-1)},\gamma_{2(n)}}(\alpha_1, \alpha_2, \dots, \alpha_{n-1}, \alpha_n) \cdot \\
&\int_{\alpha_1}^{\infty}\int_{\alpha_2}^{\infty}\cdots\int_{\alpha_{n-1}}^{\infty}\int_{\alpha_n}^{\infty} p_{\gamma_{1(1)},\gamma_{1(2)},\dots,\gamma_{1(n-1)},\gamma_{1(n)}}(x_1, x_2, \dots, x_{n-1}, x_n) \\
&\mathrm{d}x_1 \mathrm{d}x_2 \dots \mathrm{d}x_{n-1} \mathrm{d}x_n.
\end{aligned}
\tag{3.38}
$$

Based on this, the probability that the user's rate requirement can be satisfied by using exact $n$ subchannel pairs and the performance measures based on such probability can be calculated in a similar way as AF-LA scheme. It turns out that obtaining the general closed-form expression of such probability is very challenging. We develop an approximate approach, where the probability that exact $n$ subchannel pairs are needed to support the rate requirement is approximated by the probability that $n$ subchannel pairs can support the rate requirement whereas $n-1$ subchannel pairs can not. We've conducted this approximate analysis but it is shown in the appendix that it is not working well, especially for high average SNR region. Note that though the accurate analysis results can not be obtained, the accurate simulation results are available and will be given in the numerical results part.

## 3.5 Numerical Results

In this section, the analytical results are illustrated with some selected numerical examples. Specifically, the performance comparisons are divided into three groups: DF-FA and AF-AF schemes, AF-FA and AF-LA schemes, and AF-LA and AF-LA with SCM schemes. To be specific, performances are measured in terms of the outage probability, average number of selected subchannels/end-to-end subchannels/subchannel pairs (DF/AF-FA, AF-LA/AF-LA with SCM scheme), and average achieved data rate. The simulation environment and setup are the same as numerical results part of single-hop case in chapter 2. Without loss of generality, the total number of subchannels is $N = 10$ for each hop and the user's data rate requirement is $R_T = 5$ bits/symbol. All the original figures are 3-Dimensional (X and Y axises are served as the average SNR per subchannel for hop1 and hop2, respectively and Z axis is served as the performance curve), but in order to show the performance more clearly, 2-Dimensional figures are shown (i.e., the case that average SNR per subchannel for both hops are the same) One original 3-Dimensional figure is shown for illustrative purpose.

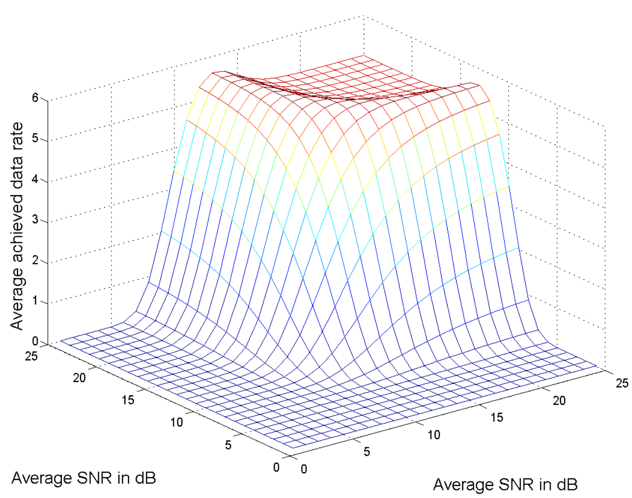


Figure 3.3: Average Achieved Data Rate with $R_T = 5$ bits/symbol for AF-FA

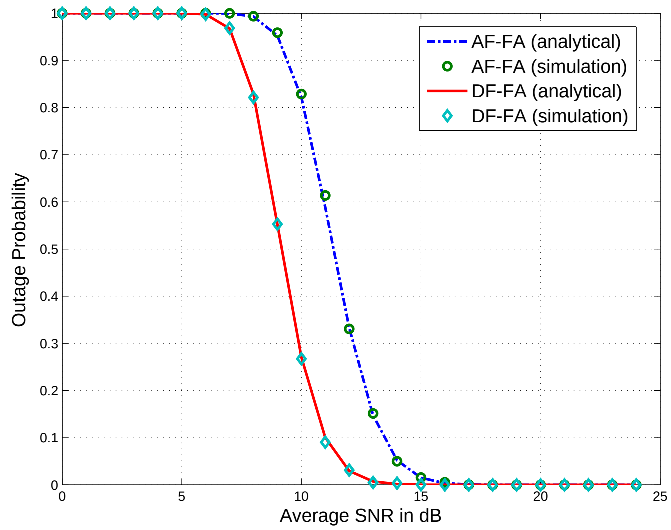


Figure 3.4: Outage Performance with $R_T = 5$ bits/symbol for AF-FA and DF-FA

In Figure 3.4, Figure 3.5 and Figure 3.6, we compare the outage performance of each group. From Figure 3.4, we can observe that AF-FA has higher outage probability than DF-FA over average SNR region of 6dB-16dB. The reason is that DF-FA enjoys more flexibility than AF-FA. This performance gap is obvious over medium average SNR region and disappears with increasing average SNR. However, a 2 dB performance gain can be achieved by applying DF-FA . From Figure 3.5, it can be observed that AF-LA always has a higher outage probability than AF-FA. It is because AF-FA can apply different modulation modes among selected end-to-end subchannels whereas AF-LA has to use the same modulation mode on all selected end-to-end subchannels. Again, this performance gap is obvious over medium average SNR region and disappears with increasing average SNR. In Figure 3.6, we can see that AF-LA with SCM always incurs a lower outage probability than AF-LA. This is because that although both options have to use the same modulation mode on all selected end-to-end subchannels/subchannel pairs, AF-LA with SCM conducts one more step of subchannel mapping compared with AF-LA. More channel gain can be obtained via this mapping process, since the most balanced subchannel pairs are chosen to improve the performance.

In Figure 3.7, Figure 3.8 and Figure 3.9, we compare the average number of selected subchannels/end-to-end subchannels/subchannel pairs for each group. From Figure

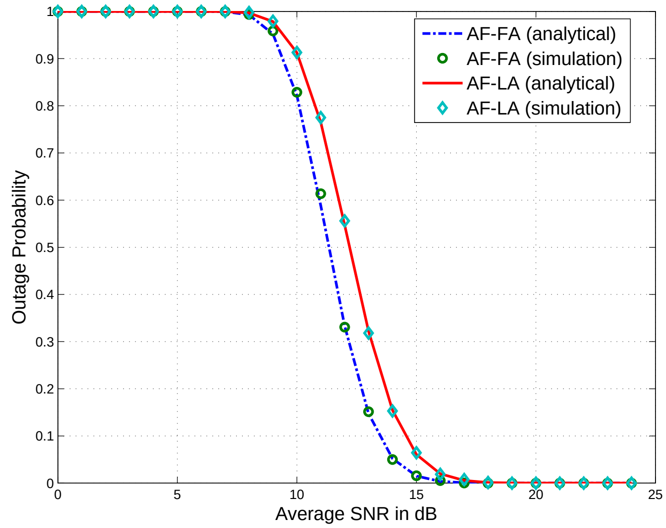


Figure 3.5: Outage Performance with $R_T = 5$ bits/symbol for AF-FA and AF-LA

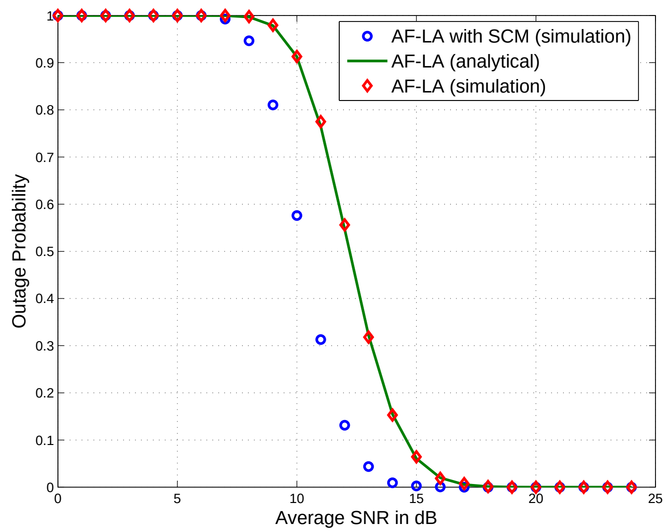


Figure 3.6: Outage Performance with $R_T = 5$ bits/symbol for AF-LA and AF-LA with SCM

3.7, we can first observe that over high average SNR region, the number of selected subchannels/end-to-end subchannels of both options eventually converge to 2 as expected. It is because that two subchannels on each hop which can support 32QAM can satisfy twice of the the target rate of $R_T = 5$ bits/symbol for DF-FA and using 2 end-to-end subchannels which can support 32QAM can satisfy the same requirement for AF-FA. Secondly, it can be observed that DF-FA always uses less resources and converges faster than AF-FA due to its flexibility of allowing different modulation

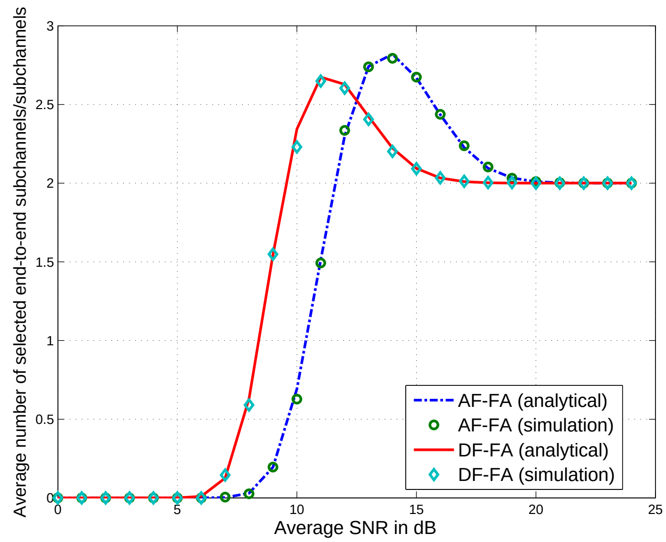


Figure 3.7: Average Number of Selected end-to-end subchannels/subchannels with $R_T = 5$ bits/symbol for AF-FA and DF-FA

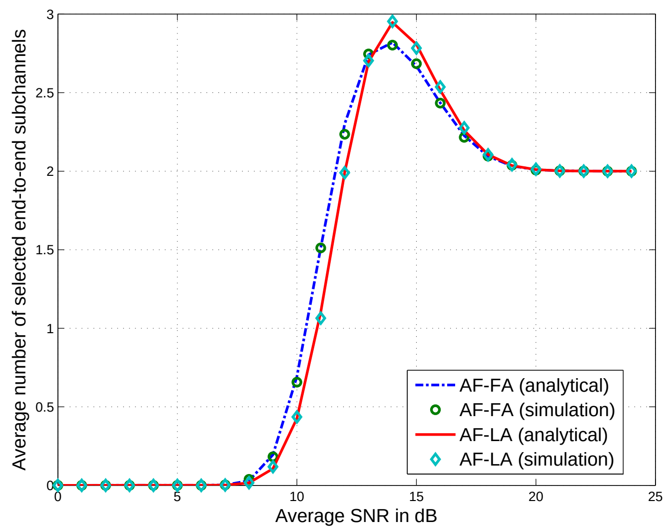


Figure 3.8: Average Number of Selected end-to-end subchannels with $R_T = 5$ bits/symbol for AF-FA and AF-LA

modes being used on different subchannels on both hops. Thirdly, we can see that a 2dB performance gain can be obtained by applying DF-FA. From Figure 3.8, it can be observed that the number of selected end-to-end subchannels with both options eventually converge to 2 in the end as expected. Secondly, we can observe that when the average SNR is in the medium range of 13dB-18dB, AF-FA uses less end-to-end

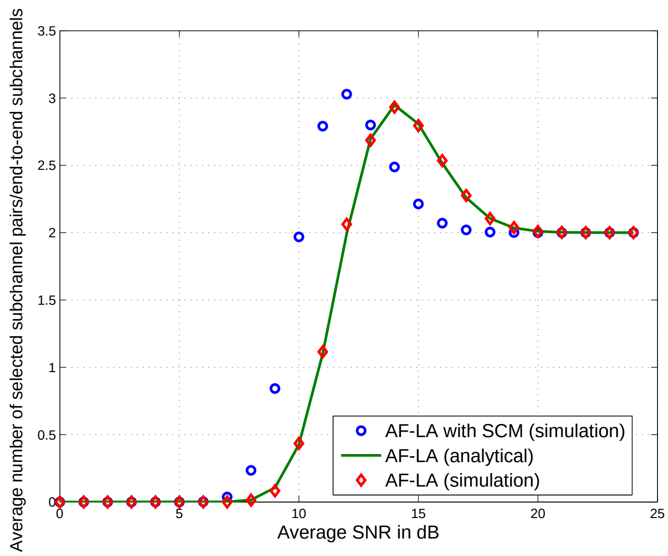


Figure 3.9: Average Number of Selected end-to-end subchannels/subchannel pairs with $R_T = 5$ bits/symbol for AF-LA and AF-LA with SCM

subchannels than AF-LA due to its flexibility of allowing different modulation modes being used among different end-to-end subchannels. Thirdly, over medium average SNR region AF-FA uses a little more end-to-end subchannels than AF-LA, since AF-LA has a higher outage probability than AF-FA over such average SNR region. From Figure 3.9, again it can be observed that the number of selected end-to-end subchannels/subchannel pairs with both options eventually converge to 2 over the high average SNR region as expected. Secondly, we can see that when the average SNR is in the medium range of 13dB-19dB, AF-LA with SCM uses less subchannel pairs than AF-LA due to its advantage of subchannel mapping process. Thirdly, over medium average SNR region, AF-LA with SCM uses a little more subchannel pairs than AF-LA. This is because that AF-LA option leads to more frequent outage events over that average SNR region, where transmission does not occur.

The performance advantage of DF-FA over AF-FA, AF-FA over AF-LA and AF-LA with SCM over AF-LA can also be observed in Figure 3.10, Figure 3.11 and Figure 3.12, where the rate performance for each group is presented. From Figure 3.10, we can observe that though DF-FA and AF-FA can provide almost the same maximum achieved data rate at some average SNR value, DF-FA can provide a 2dB performance gain. Secondly, it worths noting that for medium SNR range, both options may lead

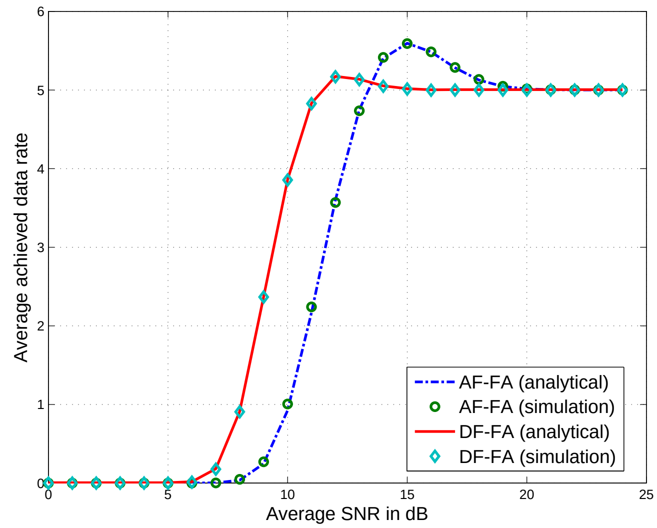


Figure 3.10: Average Achieved Data Rate with $R_T = 5$ bits/symbol for AF-FA and DF-FA

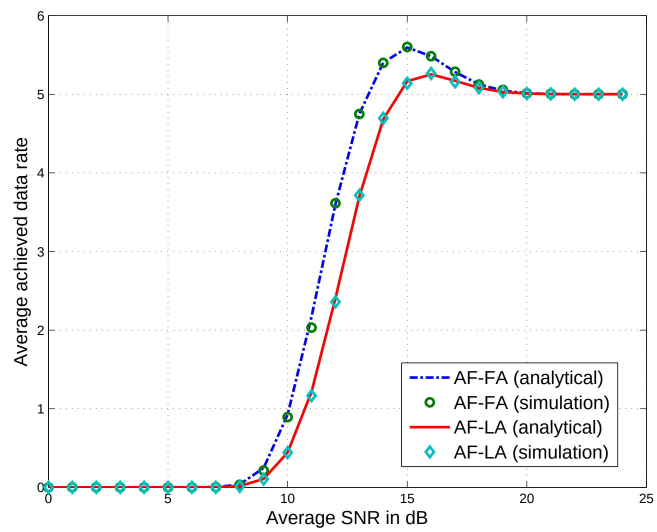


Figure 3.11: Average Achieved Data Rate with $R_T = 5$ bits/symbol for AF-FA and AF-LA

to a higher rate than the target. From Figure 3.11, we can see that AF-FA option always achieves higher rate than AF-LA option. Though the performance gap is not so significant as DF-FA over AF-FA, it still demonstrates the benefits of applying full adaptive modulation among different selected end-to-end subchannels of AF-FA over AF-LA. Again, the achieved data rate exceeds the target rate over some average

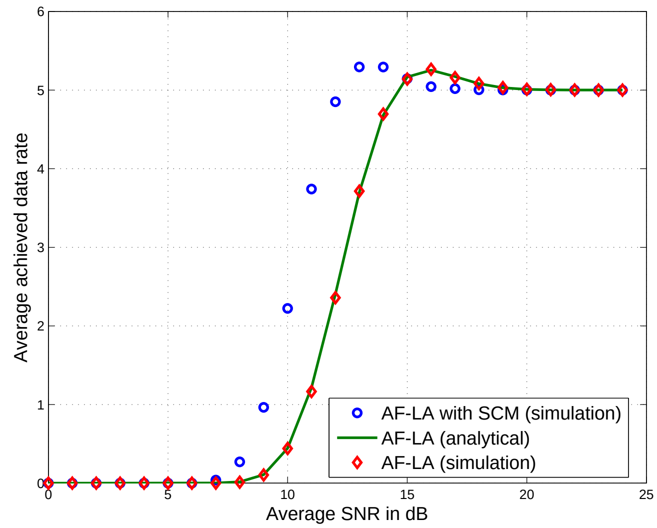


Figure 3.12: Average Achieved Data Rate with $R_T = 5$ bits/symbol for AF-LA and AF-LA with SCM

SNR region for both options. From Figure 3.12, it can be firstly observed that AF-LA with SCM can always provide higher data rate over average SNR region of 7dB-16dB. Secondly, AF-LA with SCM converges to the target rate faster than AF-LA with a performance gain of almost 2dB. And these benefits of AF-LA with SCM are obtained by the subchannel mapping process.

## 3.6 Conclusion

In this chapter, we propose four subchannel allocation schemes for downlink adaptive OFDMA dual-hop relaying systems. Performances are analyzed among different relaying schemes and ordered subchannel selection with adaptive modulation scheme, i.e., depending on whether AF or DF scheme is applied and whether a common modulation scheme is used on all selected subchannels or not. From the selected numerical results, we can see the benefits in terms of using less subchannels, achieving higher data rate and leading to lower outage probability of DF-FA scheme over AF-LA scheme, AF-FA scheme over AF-LA scheme and AF-LA with SCM scheme over AF-LA scheme.

# Chapter 4

# Summary and Future Work

## 4.1 Summary

The performance analysis of subchannel allocation schemes for downlink adaptive OFDMA systems is investigated. The analysis includes two main parts: single-hop transmission and dual-hop relaying transmission. For both scenarios, the overall objective is to satisfy the scheduled users' data rate requirement with minimum system resource, in terms of subchannels, under a certain error rate performance restriction. Users are served with known priority and the focus is on the first scheduled user. In order to simplify the bit and power loading process and minimize the feedback load, constant-power variable-rate adaptive modulation scheme is applied on the selected subchannels [17].

In chapter 2, we propose a low-complexity greedy subchannel allocation algorithm for single-hop scenario. The ordered subchannel selection schemes with adaptive modulation is applied. The algorithm includes two options depending on whether a common modulation scheme is applied on all selected subchannels or not. The option which allows different subchannels to use different modulation schemes enjoys more flexibility and is thus termed as greedy selection with full adaptivity (GS-FA). The other option which restricts all selected subchannels to use the same modulation scheme is termed as greedy selection with limited adaptivity (GS-LA). The performance is evaluated in terms of the outage probability, average number of selected subchannels and average achieved data rate. In addition, the trade-off of performance versus cost is quantized by feedback load. In the end, selected numerical examples are presented to illustrate the mathematical formulism. From the selected examples, it can be observed that GS-LA scheme can get similar performance as GS-FA scheme but incurs a much lower feedback load than GS-FA.

In chapter 3, we consider four subchannel allocation schemes for dual-hop relaying scenario. Two relaying schemes are investigated: Amplified-and-Forward (AF) and Decode-and-Forward (DF). Based on this, the ordered subchannel selection with adaptive modulation scheme is combined with both DF and AF schemes. To be specific, full adaptivity is applied to both DF and AF scheme (termed as DF-FA and AF-FA). And limited adaptivity is applied to AF scheme (termed as AF-LA). Furthermore, inspired by the concept of subcarrier mapping in [13], a revised subchannel mapping scheme is combined with AF-LA (termed as AF-LA with SCM). The performances of the four schemes are quantized in terms of the outage probability, average number of selected subchannels and average achieved data rate. Specifically, analysis of AF-FA and DF-FA schemes are conducted in a similar analytical method as single-hop scenario whereas analysis of AF-LA and AF-LA with SCM schemes are conducted through a new analytical method. The new analytical method is easier from the concept point of view, since it only needs to consider the probability of one event. However, it is difficult to get the closed-form expression of such probability, which hinders the acquisition of complete performance analysis. Therefore, we propose an approximate approach to analyze the performance of AF-LA with SCM scheme in the appendix, and the result shows that the approximation does not work well, especially for high average SNR region. For the purpose of comparison, we divide the numerical results into three groups: DF-FA and AF-FA schemes, AF-FA and AF-LA schemes, and AF-LA and AF-LA with SCM schemes. From selected examples, we can observe that DF-FA scheme has advantages of using less subchannels and incurring lower outage probability compared with AF-FA scheme. In addition, AF-FA scheme is over AF-LA scheme by the benefits of providing higher achieved data rate, using less end-to-end subchannels and incurring lower outage probability at the same time. Finally, though the analysis result for AF-LA with SCM scheme is not given, the simulation result can still demonstrate that after the process of subchannel mapping, AF-LA with SCM scheme can improve the system performance compared with pure AF-LA scheme.

In summary, the focus of this thesis in on the performance analysis of subchannel allocation schemes for donwlink adaptive OFDMA systems.

## 4.2 Future Work

### 4.2.1 Continuation of Analyzing AF-LA with SCM scheme

In this thesis, the closed-form expression of the probability that the user's rate requirement can be satisfied by using exact $n$ subchannel pairs of AF-LA with SCM scheme is not given due to its complexity. Though we've proposed an approximate analysis, the result shows that the approximation does not work well. Therefore, in order to evaluate accurate analytical performance of AF-LA with SCM scheme, further research should be conducted to figure out the accurate closed-form expression of the probability.

### 4.2.2 Performance analysis of subchannel allocation for multiple users

In this thesis, the subchannel allocation focuses on the scheduled users with known priority. Our schemes address the subchannel allocation for the first scheduled user and consider other users in a similar fashion. An interesting problem will be how to allocate subchannels to multiple users at the same time based on their limited feedback while considering the rate requirement. To be specific, if two users feed back their selected subchannels at the same time, some new strategy needs to be developed to resolve the possible conflicts between their requirements. Further research will be conducted to address this problem with similar low-complexity solutions.

# Appendix A

# Approximate Performance Analysis of AF-LA with SCM Scheme

We elaborate the approximate approach for AF-LA with SCM scheme we mentioned in chapter 3 here. Specifically, the probability that exact $n$ subchannel pairs are needed to support the rate requirement is approximated by the probability that $n$ subchannel pairs can support the rate requirement whereas $n-1$ subchannel pairs can not. This probability can be further transformed into the probability that the $n$th subchannel pair can support rate $\lceil \frac{2R_T}{n} \rceil$ while the $n-1$th subchannel pair can not support rate $\lceil \frac{2R_T}{n-1} \rceil$.

From order statistics results, the joint PDF of SNR of the $n-1$th and $n$th largest subchannel of hop1 and hop2, denoted as $p_{\gamma_{1(n-1)},\gamma_{1(n)}}$ and $p_{\gamma_{2(n-1)},\gamma_{2(n)}}$, can be derived as [25]

$$p_{\gamma_{1(n-1)},\gamma_{1(n)}}(x,y) = \frac{N!}{(N-n)!(n-2)!\bar{\gamma_1}^2} \cdot \left( \big(1-\mathrm{P}_1(x)\big)^{(n-2)} \cdot \ p_1(x) \cdot \ p_1(y) \cdot \big(\mathrm{P}_1(y)\big)^{(N-n)} \right) \tag{A.1}$$

and

$$p_{\gamma_{2(n-1)},\gamma_{2(n)}}(x,y) = \frac{N!}{(N-n)!(n-2)!\bar{\gamma_2}^2} \cdot \left( \big(1-\mathrm{P}_2(x)\big)^{(n-2)} \cdot \ p_2(x) \cdot \ p_2(y) \cdot \big(\mathrm{P}_2(y)\big)^{(N-n)} \right), \tag{A.2}$$

respectively, where $\gamma_{1(n-1)}$, $\gamma_{1(n)}$, $\gamma_{2(n-1)}$ and $\gamma_{2(n)}$ are SNRs of the $n-1$th and $n$th largest subchannel of hop1 and hop2, respectively and $p_1(\cdot)$, $p_2(\cdot)$, $\mathrm{P}_1(\cdot)$ and $\mathrm{P}_2(\cdot)$ are defined in eq.(3.1)-eq.(3.4), respectively. We now derive the joint CDF of the end-to-end SNR of the $n$th and $n-1$th subchannel pair after subchannel mapping, which are denoted by $\gamma_{(n)}$ and $\gamma_{(n-1)}$ and related to the $n$th and $n-1$th largest single-hop SNRs, denoted as $\mathrm{P}_{\gamma_{(n-1)},\gamma_{(n)}}$, as

$$\begin{aligned}
&\mathrm{P}_{\gamma_{(n-1)},\gamma_{(n)}}(\alpha_{n-1},\alpha_n) = \Pr[\gamma_{(n-1)} < \alpha_{n-1}, \gamma_{(n)} < \alpha_n] \\
&= 1 - \Pr[\gamma_{(n-1)} > \alpha_{n-1}] - \Pr[\gamma_{(n)} > \alpha_n] + \Pr[\gamma_{(n-1)} > \alpha_{n-1}, \gamma_{(n)} > \alpha_n] \\
&= 1 - \Pr[\gamma_{1(n-1)} > \alpha_{n-1}, \gamma_{2(n-1)} > \alpha_{n-1}] - \Pr[\gamma_{1(n)} > \alpha_n, \gamma_{2(n)} > \alpha_n] \\
&+ \Pr[\gamma_{1(n-1)} > \alpha_{n-1}, \gamma_{1(n)} > \alpha_n] \cdot \Pr[\gamma_{2(n-1)} > \alpha_{n-1}, \gamma_{2(n)} > \alpha_n], \\
&\alpha_{n-1} \geq \alpha_n. \qquad \text{(A.3)}
\end{aligned}$$

Therefore, the joint PDF of end-to-end SNR of the $n$th and $n-1$th subchannel pair, denoted as $p_{\gamma_{(n-1)},\gamma_{(n)}}(\alpha_{n-1},\alpha_n)$, can be derived as

$$\begin{aligned}
&p_{\gamma_{(n-1)},\gamma_{(n)}}(\alpha_{n-1},\alpha_n) = \frac{\partial^2 \mathrm{P}\gamma_{(n-1)}, \gamma_{(n)}(x,y)}{\partial x \partial y} \\
&= p_{\gamma_{1(n-1)},\gamma_{1(n)}}(\alpha_{n-1},\alpha_n) \cdot \Big( \int_{\alpha_{n-1}}^{\infty} \int_{\alpha_n}^{\infty} p_{\gamma_{2(n-1)},\gamma_{2(n)}}(x,y) \mathrm{d}x \mathrm{d}y \Big) \\
&+ p_{\gamma_{2(n-1)},\gamma_{2(n)}}(\alpha_{n-1},\alpha_n) \cdot \Big( \int_{\alpha_{n-1}}^{\infty} \int_{\alpha_n}^{\infty} p_{\gamma_{1(n-1)},\gamma_{1(n)}}(x,y) \mathrm{d}x \mathrm{d}y \Big) \\
&= \; p_{\gamma_{1(n-1)},\gamma_{1(n)}}(\alpha_{n-1},\alpha_n) \cdot F_{\gamma_2} + \; p_{\gamma_{2(n-1)},\gamma_{2(n)}}(\alpha_{n-1},\alpha_n) \cdot F_{\gamma_1}, \qquad \text{(A.4)}
\end{aligned}$$

where

$$\begin{aligned}
&F_{\gamma_1} = \frac{N!}{(N-n)!(n-1)!} \cdot \Big( \exp\big( -(n-1)\Gamma_1 \alpha_{n-1} \big) \Big) \cdot \\
&\sum_{k=1}^{N-n+1} \frac{(-1)^{k-1} \binom{N-n}{k-1} \exp\big( -k\Gamma_1 \alpha_n \big)}{k} \qquad \text{(A.5)}
\end{aligned}$$

and

$$F_{\gamma_2} = \frac{N!}{(N-n)!(n-1)!} \cdot \Bigg( \exp\big( -(n-1)\Gamma_2 \alpha_{n-1} \big) \Bigg) \cdot \sum_{k=1}^{N-n+1} \frac{(-1)^{k-1}\binom{N-n}{k-1} \exp\big( -k\Gamma_2 \alpha_n \big)}{k}, \tag{A.6}$$

respectively, and $\Gamma_1 = \frac{1}{\bar{\gamma_1}}$ and $\Gamma_2 = \frac{1}{\bar{\gamma_2}}$, respectively.

Then, we can calculate the $\mathrm{Pr}_{(\mathrm{n})}$ as
When $n \geq 2$

$$\begin{aligned}
&\mathrm{Pr}_{(\mathrm{n})} = \mathrm{Pr}[\gamma_{(n)} > \gamma_{T_i}, \gamma_{(n-1)} < \gamma_{T_{j_{n-1}}}] \\
&= \int_{\gamma_{T_i}}^{\gamma_{T_{j_{n-1}}}} \int_{\alpha_n}^{\gamma_{T_{j_{n-1}}}} p_{\gamma_{(n-1)},\gamma_{(n)}}(\alpha_{n-1}, \alpha_n) \mathrm{d}\alpha_{n-1} \mathrm{d}\alpha_n \\
&= \int_{\gamma_{T_i}}^{\gamma_{T_{j_{n-1}}}} \int_{\alpha_n}^{\gamma_{T_{j_{n-1}}}} F_{\gamma_1} \cdot \ p_{\gamma_{2(n-1)},\gamma_{2(n)}}(\alpha_{n-1}, \alpha_n) \mathrm{d}\alpha_{n-1} \mathrm{d}\alpha_n \\
&+ \int_{\gamma_{T_i}}^{\gamma_{T_{j_{n-1}}}} \int_{\alpha_n}^{\gamma_{T_{j_{n-1}}}} F_{\gamma_2} \cdot \ p_{\gamma_{1(n-1)},\gamma_{1(n)}}(\alpha_{n-1}, \alpha_n) \mathrm{d}\alpha_{n-1} \mathrm{d}\alpha_n \\
&= Q_1 + Q_2,
\end{aligned} \tag{A.7}$$

where

$$Q_1 = \big( \frac{N! \cdot \Gamma_2}{(N-n)!(n-1)!} \big)^2 \cdot \frac{1}{\Gamma_1 + \Gamma_2} \cdot (A \cdot B_1 + D_1) \tag{A.8}$$

and

$$Q_2 = \big( \frac{N! \cdot \Gamma_1}{(N-n)!(n-1)!} \big)^2 \cdot \frac{1}{\Gamma_1 + \Gamma_2} \cdot (A \cdot B_2 + D_2), \tag{A.9}$$

respectively.

In eq. (A.8) and (A.9),

$$A = -\exp\big( -(n-1)(\Gamma_1 + \Gamma_2)\gamma_{T_{j_{n-1}}} \big), \tag{A.10}$$

$$B_1 = \sum_{k=1}^{N-n+1} \frac{(-1)^{k-1}\binom{N-n}{k-1}}{k} \sum_{q=1}^{N-n+1} \frac{(-1)^{q-1}\binom{N-n}{q-1}}{-k\Gamma_1 - q\Gamma_2}$$
$$\Bigg\{\exp\bigg(\Big(-k\Gamma_1 - q\Gamma_2\Big)\gamma_{T_{j_{n-1}}}\bigg) -$$
$$\exp\bigg(\Big(-k\Gamma_1 - q\Gamma_2\Big)\gamma_{T_i}\bigg)\Bigg\}, \tag{A.11}$$

$$B_2 = \sum_{k=1}^{N-n+1} \frac{(-1)^{k-1}\binom{N-n}{k-1}}{k} \sum_{q=1}^{N-n+1} \frac{(-1)^{q-1}\binom{N-n}{q-1}}{-k\Gamma_2 - q\Gamma_1}$$
$$\Bigg\{\exp\bigg(\Big(-k\Gamma_2 - q\Gamma_1\Big)\gamma_{T_{j_{n-1}}}\bigg) -$$
$$\exp\bigg(\Big(-k\Gamma_2 - q\Gamma_1\Big)\gamma_{T_i}\bigg)\Bigg\}, \tag{A.12}$$

$$D_1 = \sum_{k=1}^{N-n+1} \frac{(-1)^{k-1}\binom{N-n}{k-1}}{k} \sum_{l=1}^{N-n+1} \frac{(-1)^{l-1}\binom{N-n}{l-1}}{(1-k-n)\Gamma_1 - (n+l-1)\Gamma_2}$$
$$\Bigg\{\exp\bigg(\Big((1-k-n)\Gamma_1 - (n+l-1)\Gamma_2\Big)\gamma_{T_{j_{n-1}}}\bigg) -$$
$$\exp\bigg(\Big((1-k-n)\Gamma_1 - (n+l-1)\Gamma_2\Big)\gamma_{T_i}\bigg)\Bigg\} \tag{A.13}$$

and

$$D_2 = \sum_{k=1}^{N-n+1} \frac{(-1)^{k-1}\binom{N-n}{k-1}}{k} \sum_{l=1}^{N-n+1} \frac{(-1)^{l-1}\binom{N-n}{l-1}}{(1-k-n)\Gamma_2 - (n+l-1)\Gamma_1}$$
$$\Bigg\{\exp\bigg(\Big((1-k-n)\Gamma_2 - (n+l-1)\Gamma_1\Big)\gamma_{T_{j_{n-1}}}\bigg) -$$
$$\exp\bigg(\Big((1-k-n)\Gamma_2 - (n+l-1)\Gamma_1\Big)\gamma_{T_i}\bigg)\Bigg\}, \tag{A.14}$$

respectively.

When $n = 1$,

$$\begin{aligned}
&\Pr_{(1)} = \Pr[\gamma_{(1)} > \gamma_{T_i}] \\
&= \Pr[\gamma_{1(1)} > \gamma_{T_i}, \gamma_{2(1)} > \gamma_{T_i}] \\
&= \Pr[\gamma_{1(1)} > \gamma_{T_i}] \cdot \Pr[\gamma_{2(1)} > \gamma_{T_i}] \\
&= \int_{\gamma_{T_i}}^{\infty} p_{\gamma_{1(1)}}(\alpha_1)\mathrm{d}\alpha_1 \cdot \int_{\gamma_{T_i}}^{\infty} p_{\gamma_{2(1)}}(\alpha_1)\mathrm{d}\alpha_1 \\
&= N^2 \sum_{k=1}^{N} \frac{(-1)^k \cdot \binom{N}{k-1}}{k} \exp\Big( - k\Gamma_1\gamma_{T_i}\Big) \cdot \\
&\sum_{q=1}^{N} \frac{(-1)^q \cdot \binom{N}{q-1}}{q} \exp\Big( - q\Gamma_2\gamma_{T_i}\Big).
\end{aligned} \tag{A.15}$$

With the above results, the outage probability, average number of selected subchannel pairs and average achieved data rate with AF-LA with SCM scheme can be calculated in a similar fashion as for AF-LA scheme.

However, after realizing this approximate analysis by matlab programming, we found that this approximation does not work well, especially for high average SNR region. Part of the data of outage probability of approximate analysis v.s. simulation result is in the following table.

The reason of the different data for analysis and simulation in the tables should be that the simplification of only considering the relevence of the $n$th and $n-1$th subchannel pair is not enough, since other orderd subchannel pairs may also have significant relevence with the $n$th subchannel pair, which will affect the analysis result.

| Average SNR (dB) | Analysis | Simulation |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 1 | 1 |
| 2 | 1 | 1 |
| 5 | 1 | 1 |
| 7 | 0.9701 | 0.9990 |
| 9 | -0.1797 | 0.8020 |

Table A.1: Comparison of Outage Probability for Approximated Analysis of AF-LA with SCM Scheme

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