A Biophysical Model of the Industrial Revolution

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Abstract

Several biophysical characteristics underlay Britain's Industrial Revolution: improvements in agricultural productivity; large increases in use of coal energy supply; and physical construction of infrastructure for industrialization and urbanization. These characteristics are represented in a four sector model of Britain's economy (1760 to 1913) including: agriculture; mining; construction of capital; and the production of goods and services. The model has a novel mathematical representation of a dynamic general equilibrium between capital, labour and energy in an economy. Historical data is used to calibrate the model for growth of Britain's capital stock, coal use and employment during the Industrial Revolution (first and second periods). Model simulations explore the impacts of two biophysical constraints: stagnation in agricultural productivity; and reduced efficiency in coal mining in the absence of steam engines. Both scenarios exhibit substantial reductions in the growth of capital stock and significant changes to the distribution of labour.

Keywords: capital, energy, labour, endogenous growth, Great Britain, general equilibrium model, coal, construction, agriculture

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1. Introduction

To industrial ecologists, the most outstanding characteristic of the Industrial Revolution (starting in Great Britain in ~1760) was that it entailed a transformation from a society based on organic energy sources to one overwhelmingly supported by fossil fuels, and in particularly coal (Kander and colleagues 2013; Kennedy, 2020; Krausmann and colleagues, 2008; Krausmann and Fischer-Kowalski, 2013; Schandl and Schulz, 2002; Wrigley, 2010, 2013). The classical Industrial Revolution (~1760 to ~1840) was a period of remarkable invention in Great Britain, though punctuated by many years of war, and with economic growth now considered to be quite modest (Crafts 1985, 2005; Mokyr, 2018). This was followed by a second period – *the railway era* (~1840 to ~1913) – in which exploitation of earlier inventions in steam engines, steel making and other technologies produced more rapid economic growth. Over the period 1760 to 1913, while the population of England & Wales increased by a factor of 3.7, use of organic energy sources only rose by ~2.4; yet coal energy use increased by a factor of~35 (Kennedy, 2020, based on Warde, 2007).

The nature and consequences of the Industrial Revolution were so profound, that many interpretations of its causes exist (Allen, 2009; Deane and Cole, 1967; Mokyr, 2018; Wrigley, 2010). Some of the underlying factors are of a biophysical nature: increases in agricultural productivity in Britain (Allen, 1994; O'Brien, 1985; Turner et al., 2001); widespread exploitation of coal reserves (Kennedy, 2020; Wrigley, 2013); and urbanization– requiring massive capital investment in buildings and infrastructure (Wrigley, 2010). Other factors, beyond the biophysical, include: Britain's relatively high wages, sophisticated markets and patent laws; technological invention; skilled artisans, mechanics and engineers; well-specified property rights; population growth; existing vibrant cities, notably London;

foreign trade and political factors (Allen, 2009; Mokyr, 2018; Wrigley, 1967, 2010). Many of these factors are complex and interwoven; for example, Britain's relatively high wages at the outset of the Industrial Revolution could relate to the already significant use of coal. Some of the factors are quite contested – the role of foreign trade, for example – but the focus of this paper is on the biophysical factors.

Economists have formulated a variety of macroeconomic models to further understanding of the Industrial Revolution. Without attempting to be comprehensive, a few examples are summarized here. Stokey (2001) developed a model for Great Britain from 1780 to 1850, with inputs of capital, labour and land; and outputs in three sectors (agriculture, industry and other). The model highlighted the role of foreign trade in economic growth, and the importance of technological development in manufacturing and to a lesser extent energy supply. In a similar way, Voigtländer and Voth (2006) calibrated a two-sector model of the English economy for 1700 to 1880. Incorporating aspects of unified growth theory (Galor, 2005), as well as stochastic weather shocks impacting agricultural productivity, they examined the timing of industrialization in England. Allen (2009b) constructed an aggregate model of Britain capturing both the growth and distribution of income from 1760 to 1913. Others have applied more general, theoretical growth models to the Industrial Revolution (e.g., Jones, 1999), while another approach has been to develop general equilibrium models for specific years (e.g., Harley and Crafts, 2000). These models have helped to test theories and develop understanding of growth during the Industrial Revolution.

Attention to biophysical factors in the above economic models is mixed. Voigtländer and Voth (2006) give close consideration to agricultural productivity, while largely ignoring energy use. The model of

Stokey (2001) is perhaps the most observant to the role of energy in the Industrial Revolution. She considers mechanical power to industry as a variable in the model, but it is as an intermediate good, dependent upon primary inputs of land, labour and capital. This misses a key point that energy is not only required in the use of capital, but in the construction of capital too (Kennedy, 2020b). The urbanization that accompanied the Industrial Revolution is included in the models in so much as capital is a variable in all models. That much of this capital investment is for the construction of buildings and infrastructure – the physical parts of cities – is generally not explicitly recognized.

While the transition to a fossil-fuel based society was a central feature of the Industrial Revolution (Kander and colleagues 2013; Wrigley, 2010, 2013), understanding of the massive growth of coal use has been an enigma for economists (Kennedy, 2020). Reasons for this are apparent on the surface; while there were huge increases in the demand for coal, there was little change in the labour productivity of coal mining (Crafts, 2004). This has lead to conclusions ranging from: advances in coal mining were less significant than those in manufacturing (Stokey, 2001), to coal mining did not even have an indirect impact on incomes (Clark & Jacks, 2007). Thinking more broadly, however, I argue that mainstream economics has yet to develop suitable techniques for incorporating energy constraints into economic growth models - of any period. This is an issue that ecological economists and others have recognized for some time (Daly, 1997; Georgescu-Roegen, 1971; Hall and Klitgaard, 2011; Kummel and colleagues, 2002; Stern, 2011). Hence, in this paper I will demonstrate a radically different way of modelling Britain's growth during the Industrial Revolution – using biophysical terms. In particular, I shall demonstrate how capital, energy and labour intersect with a complex equilibrium in the macroeconomy.

The objective of this paper is to develop a biophysical model of Britain's growth during the Industrial Revolution (first and second periods). In some respects, the mathematical formulation borrows from macroeconomics. It is a dynamic general equilibrium model - and could even be considered a type of endogenous growth model (Romer, 1994) - at least with respect to capital. There again, the model is very much different to any conventional form of macroeconomic model. The general equilibrium is expressed between capital, labour and energy – not prices and quantities of goods. There is no profit maximization by firms, nor utility maximization by households – because neither are explicitly represented. The model does not even pay attention to economic output – although this perhaps could be added with relative ease, following the calculations of capital and labour.

In the next section, I describe the equilibrium between capital, energy and labour in the macro-economy, and provide the equations for the theoretical model. I then review data on capital, energy and labour in Great Britain from 1760 to 1913, for the four sectors of the model. This is followed by a brief methodological section on model solution and calibration. Finally, the model is used to explore two scenarios of deviations from historical record; these are: i) a stagnation in agricultural productivity (or equivalently a decline in food imports); and ii) a reduction in technological progress, specifically in coal mining.

2. Theory

The biophysical model of the economy captures the equilibrium between capital, energy and labour over four sectors: agriculture; coal mining; construction of capital; and the production of goods and services (Figure 1). The first three of these sectors are quite specific, while the fourth is essentially the rest of the economy, including all of the economic activities in manufacturing, transportation, distribution, retail and all other services. Each of the three broad variables – capital, energy and labour – are subject to balance equations. Before stating these equations, I will first describe the dynamic equilibrium between them that is represented by the model.

The agricultural sector is the source of all labour in the model through, of course, the production of food. Agricultural labour must produce sufficient food to provide for the needs of the population, with adjustments for net imports in agricultural products. Increase in the productivity of agriculture was one of the key factors underlying the Industrial Revolution, as identified above. With increased productivity, a lower fraction of the labour force was required in agriculture, hence freeing up labour for other sectors. The distribution of labour between mining, construction and the production of goods and services depends, however, on the provision of physical capital assets in these sectors.

Formation of capital is core to economic growth and drives demand for both energy and labour in the economy. Capital and labour are not treated as substitutes in the model, as in conventional growth theory, because this has been found not to hold during the Industrial Revolution (see Allen, 2009b, and studies cited within). Moreover, quantification of economic output (e.g., GDP), is not sought for this model. Rather, capital and labour are considered to be fully complementary, with growth in the capital stock providing increased opportunities for labour (endnote 1). The stocks of capital in the mining sector and in agriculture are treated separately from the rest of the capital stock, which is used for the production of goods and services. Investment in each of these three forms of capital requires labour in the construction sector, which can broadly be considered to include production of materials used in capital construction (such as bricks and steel). Capital accumulation provides the dynamics to the model and is represented in a conventional way from the balance of investment and capital depreciation.

Energy is required in the economy both to form the capital stock and produce goods and services from it. These two uses of energy and the energy used in coal mining itself are subject to conservation of energy, as shown in my previous work on *Energy and Capital* (Kennedy, 2020b). The model presented here (Figure 1) only applies an energy balance to coal energy. There was substantial use of non-fossil fuel energy sources during the early Industrial Revolution (Warde, 2007), but much of this was from humans and animals, which was generated and used within the agricultural sector. (I will return this later.). Coal can be primarily seen as the energy source behind the capital investment for building of machinery, buildings, and infrastructure in the Industrial Revolution. The growth and use of the capital stock is constrained by the coal energy balance equation.

The biophysical model as shown in Figure 1 is of a semi-open economy, with the dimension of trade only partially included. The model resolves balance equations for capital, energy and labour, but each of these balances can also be impacted by trade. During the Industrial Revolution, especially after the abolition of the *Corn Laws*, Britain's import of agricultural products increased the availability of excess labour beyond the agricultural sector. The reciprocal to agricultural imports was export of manufactured goods – requiring necessary capital investment and labour. Britain also exported substantial quantities of coal especially towards the end of the nineteenth century, which has an impact on the energy balance. The model does not include a strict balance of trade, although adjustments for traded quantities are made where necessary when applying the model.

Labour

Excess labour, $\hat{L}$, available to other sectors after provision of labour for agriculture, L_A, is given by:

$$\hat{L} = L - L_A = L_K + L_M + L_D \tag{1}$$

where L is the total labour force; L_K is the labour employed in capital production (i.e., construction and material production sectors); L_M is labour employed in mining; and L_D is labour employed in the production of goods and services. The three labour terms on the right-side of equation (1) are dependent upon capital stock and investment as follows:

$$L_K = \alpha_K I \quad (2)$$

$$L_M = \alpha_M K_M \quad (3)$$

$$L_D = \alpha_D K_D \quad (4)$$

where I is capital investment in all sectors; K_M is the capital stock in the mining sector and K_D is the capital stock for production of goods and services. The parameters α_K, α_M and α_D are labour intensities of capital formation, mining and production of goods and services respectively.

Capital

Total capital investment is given by:

$$I = I_A + I_M + I_D \quad (5)$$

where I_A, I_M and I_D are annual investments in agriculture, mining and production of goods and services respectively. Note that capital investment in the construction sector itself (e.g., buildings owned by construction companies, brick kilns, or metal refineries) could be added as a separate term in equation (5), but due to lack of data it is simply included within I_D for this model.

The accumulation of capital is given by conventional macroeconomic equations:

$$K_{A,t} = K_{A,t-1}(1 - \delta_A) + I_{A,t} \quad (6)$$

$$K_{M,t} = K_{M,t-1}(1 - \delta_M) + I_{M,t} \quad (7)$$

$$K_{D,t} = K_{D,t-1}(1 - \delta_D) + I_{D,t} \quad (8)$$

where the subscript t denotes time; and δ_A, δ_M and δ_D are annual rates of depreciation in respective sectors.

Energy

The energy balance equation and relationships between capital and energy are similar to those developed in a previous paper (Kennedy, 2020b). Net available energy, $\hat{E}$, after use by the mining sector itself, E_M, is given by:

$$\hat{E} = E - E_M = E_D + E_K \tag{9}$$

where:

E is the available useful energy generated from domestic resources (net of energy exports where applicable) or obtained through international trade;

E_K is the energy required for gross fixed capital formation, i.e., investment, within the economy;

E_D is the energy demanded in producing goods and services in the economy, for domestic consumption and export, from the existing capital stock.

Energy use in specific sectors is related to capital stocks via:

$$\hat{E} = \hat{e}_M K_M \tag{10}$$

$$E_K = e_K I \tag{11}$$

$$E_D = e_D K_D \tag{12}$$

where:

$\hat{e}_M$ is the net energy production per monetary unit of capital stock in the coal mining sector (K_M);

e_K is the energy required to produce capital per monetary unit of investment (I);

e_D is the energy required to produce goods and services per monetary unit of capital stock (K_D).

Note that the energy intensities $\hat{e}_M$ and e_D are defined slightly differently here to Kennedy (2020b).

The twelve-equation model has nine parameters; these are the capital depreciation rates (δ_A, δ_M and δ_D), labour intensities of capital (α_K , α_M and α_D) and energy intensities of capital ($\hat{e}_M$, e_K and e_D). Calculation of these parameters from underlying capital, energy and labour data is undertaken in the following sections.

3. Capital, Energy and Labour in Great Britain, 1760 to 1913

Labour

Estimates of the size and components of Britain's labour force from 1760 to 1913, are available from several sources including Broadberry et al. (2015), Dean & Cole (1967), Feinstein (1996) and Mitchell (1962). A Bank of England database provides two annual time series of the total labour force from 1700 to 1870 (endnote 2). These are a series from Broadberry et al. (2015) that is used in this study, and a series with slightly higher estimates, based on Feinstein (1996) (Figure 2a). From 1841, the census provides measures of the labour force every decade, although values in the first census of 1841 are limited (Deane & Cole, 1967). The census is the basis of Mitchell's (1962) values shown in Figure 2a, which are used here after 1871, with linear interpolation. A further set of decadal estimates from Dean & Cole (1967) are coarser estimates derived from census data after 1851, but extending back to 1801. Dean & Cole's (1967) series is in fairly close agreement with the values from Broadberry et al. (2015) and Mitchell (1962) used in this study.

The consolidated data series (see Supplementary Information 1) shows the total labour force growing from 3.3 million in 1760, to 20.3 million in 1913 (Figure 2a). The population of Great Britain increased from 7.65 million to 36 million over the same time period (also based on Broadberry et al., 2015 and Mitchell, 1962).

Estimates of employment by specific sectors are more challenging, especially prior to the 1841 census. For employment in agriculture (including fishing & forestry), which is a key input to the model, there are estimates from Broadberry et al (2015), Mitchell (1962), Deane & Cole (1967) and Shaw-Taylor et al. (2019). Broadberry et al (2015) and Shaw-Taylor et al. (2019) conveniently provide similar estimates for agricultural employment in 1759 and 1761, respectively, at the beginning of the study period (Figure 2b). (Shaw-Taylor et al (2019) provide estimates for England and Wales, which I have proportionally scaled for Britain's workforce). Broadberry et al (2015) also give an estimate for agricultural labour for 1800, although the difference with Dean & Cole's (1967) estimate for that year is notable (1.42 million vs. 1.70 million); this gives a sense of the uncertainty in the overall data. To be pragmatic, I adopt a mid-point value between Broadberry et al (2015) and Deane & Cole (1967) for 1800, and then follow Deane & Cole's (1967) decadal estimates, before picking up Mitchell's (1962) series in 1851, based on the census. Linear interpolation is used to complete the data set.

The overall trend in agricultural employment is a gradual increase from 1.2 million in 1759, to a peak of 2.1 million in 1851 (Figure 2b). The peak may relate to the abolition of the *Corn Laws* in 1849, which increased the competitiveness of food imports. Agricultural labour then declined over the second half of the nineteenth century, before rising again in 1910. The contrast with labour in the other specific sectors in the model – mining and capital construction – is notable. Employment in construction (including material production) was roughly half of agricultural employment at the middle of the nineteenth century, rising to be double by the end of the century (Figure 2b). Employment in the mining sector also rose rapidly, approaching a level similar to agriculture by 1910.

Data on employment in capital construction comes from Shaw-Taylor et al. (2019) before 1841, and from Mitchell (1962) thereafter. This sector, as I define it here, is broader than the conventional construction sector; it also includes the metal manufacturing sector, plus bricks, cement, pottery and glass (i.e., it reflects all of the employment involved with the creation of physical capital assets, beyond the mining sector). To estimate labour employed in capital construction from Shaw-Taylor et al. (2019), I have summed their occupational categories of: iron and steel manufacture; machine and tool making; and building and construction. These categories are slightly narrower than those used from Mitchell (1962) of: metal manufacture, machines, implements, vehicles, precious metals; bricks, cement, pottery and glass; and building and construction. The difference in composition, and scaling applied to the Shaw-Taylor et al. (2019) data, can explain the discrepancy between the two datasets. My estimates for employment in capital construction for 1761 and 1817 (based Shaw-Taylor et al., 2019), of 227,000 and 470,000, respectively are likely low.

The mining sector data includes employment in quarries and other types of mines beyond collieries. Mitchell (1962) provides employment data specifically for coal mining from 1864 onward, which is about 80% of the sector (Figure 2b). Since the datasets for capital investment and coal use (discussed below) are for the broader mining sector, including other minerals other than coal, then I have pragmatically used the full mining and quarrying sector throughout this paper. Mining sector employment for 1761 and 1817, of 37,000 and 116,000 respectively are estimated from Shaw-Taylor et al. (2019). These are reasonably consistent with speculative calculations from Flinn (1984) that coal mines might have employed about 83,000 people in 1801, and 174,000 in 1831.

Capital

Estimates of capital formation in Great Britain from 1760, are well established by Feinstein & Pollard (1988), both for asset types and industrial sectors. Decadal values for gross domestic fixed capital formation and net stock of reproducible fixed assets for Great Britain are provided up to 1860. From 1850 onwards, Feinstein & Pollard (1988) provide annual time series for the United Kingdom (i.e., including Ireland). Data from the period of overlap (1850 to 1860), indicates that Great Britain's capital stock was 94.4% of the UK's capital stock. In the absence of other data, the factor 0.944 was used in estimating Great Britain's level of capital stock and investments from 1851 to 1913. A second factor of 0.893 was used to used to convert constant prices in 1900 pounds sterling to prices based on 1851.

Great Britain's capital stock grew substantially from £248 million in 1760, to £3,737 million in 1913 (in 1851 pounds sterling; Figure 3). This was an increase by a factor 15.1, or a factor of 3.2 in per capita terms. The agriculture and mining sectors both have relatively small fractions of the total capita stock, but changes to them are profound. In 1760, agriculture accounted for 31% of the capital stock, at a value of £77 million. By 1913, agricultural capital had increased to £209 million, but by then was only 5.6% of the total capital stock. The stock of capital assets in the mining sector was almost negligible in 1760, of the order £1 million (or 0.4% of total), but it grew to £95 million (or 2.5% of total) by 1913, making it close to half the value of agricultural capital.

Coal

The model in the paper only considers energy provided through coal combustion, ignoring other forms. Coal was the dominant source of energy in England & Wales over the nineteenth century, for example accounting for 77% of end-use energy in 1800, rising to 91% by 1850, and 95% by 1900 (Warde, 2007; Kennedy, 2020). The share of coal energy use was also significant in the eighteenth century. In 1760, coal made up 64% of energy use in England & Wales, with draught livestock (13%), human labour (12%), firewood (9%), wind (1.2%) and water (0.5%) accounting for the rest (Warde, 2007). Much of the human and draught livestock energy would be used in the agricultural sector; but energy use in this sector is explicitly not included in the model (Figure 1). Some draught animal energy was used in transportation (horses) as was wind energy for sailing ships. Firewood was also used by industry and for domestic heating purpose. Peat was used as an energy source in Scotland, though the quantities are unknown (Humphrey & Stanislaw, 1979). Although these forms of energy use are important in relative terms in the eighteenth century; their overall significance over the period to 1913 was small.   Coal accounted for over 90% of total energy use from 1760 to 1913.

Data on coal production, consumption and export for Great Britain and the UK are provided by Flinn (1984) and Church (1984), but some assumptions are required to achieve consistent time series. Flinn (1984) gives estimates for coal production in Great Britain in 25 to 30 year intervals between 1750 and 1830. There is a steady upward trend, so annual values can be established by linear interpolation with reasonable confidence. Tonnage of coal exports, including those to Ireland, can similarly be determined, and then subtracted to establish total amounts of domestic coal use.

From 1830 onwards, Church (1984) provides annual data on coal production for the UK. This time series of UK data can essentially also be used for coal production for Great Britain, since Ireland’s coal mining was insignificant. Irish coal production was of the order 100,000 tons in 1907 (see Table 8.16 in

Bielenberg, 1994), while British production was 288 million tons. The export of coal from Great Britain to Ireland after 1830, however, poses a data challenge. The UK's total overseas coal exports after 1830 are given by Mitchell (1962), but data on the export from Great Britain to Ireland has not been found – so has to estimated. In 1830, Great Britain exported 750,000 tons of coal to Ireland, which was 2.5% of total British production (Table 7.13 in Flinn, 1984). Aside from the shipyards and linen making in Belfast, Ireland's industry went into significant decline after 1841 (O'Malley, 1981). In 1907, Ireland accounted for 4.2% of the UK's industrial employment, but only 2.5% of employment in the large energy intensive sector of iron/steel/engineering and ships (Table 1 in Bielenberg, 2008). After considering these factors a continuing export of 2.5% of Great Britain's coal production is assumed to go to Ireland from 1830 onwards.

Great Britain's net coal energy supply, after subtracting exports, is divided into the quantity used for capital formation, and that used in the production of goods and services (i.e., all other uses; Figure 4). This is achieved using my previously established annual time series estimates of energy embodied in the UK's capital assets (Kennedy, 2020). The coal used for capital formation in Great Britain is given by multiplying the UK value by 94.4%; this is the same factor that was used in estimating Britain's capital assets, as discussed above.

4. Model Solution and Calibration

The model described by equations (1) to (12) has fourteen variables, so two of them have to be specified in order to provide a solution. As improvement in agricultural productivity - freeing up labour - is seen as one of the driving factors of the Industrial Revolution, capital investment in agriculture, K_A, and net available labour, $\hat{L}$, are taken as exogenous variables in model simulations. Specifying net available labour also removes the need to model population growth and labour participation rates; and quantification of agricultural trade also becomes unnecessary. Beyond exogenous specification of the agricultural sector and the labour supply, the remaining equations form an endogenous growth model. Their solution determines the growth of the capital stock, the distribution of labour between sectors and the production and distribution coal energy supplies.

Whilst the twelve equations could be solved using matrix algebra, I provide an analytical solution in Appendix 1. The procedure for applying this solution involves specifying the initial capital assets for mining, K_M, and the production of goods and services, K_D, for a base year – and values for the nine parameters for all years. The analytical solution is then stepped through, resolving the depreciation of capital, the growth of new capital and the distributions of labour and energy each year. The model parameters are specified by drawing upon the estimates for labour, capital and energy in the previous section. The parameters are uniquely specified for each modelled simulation year, so that the model is initially calibrated to simulate the historically observed values of labour, capital and energy (Figure 5).

Determination of the parameters provide some additional insights into the biophysical characteristics of the Industrial Revolution. A summary of parameter values is given in Table 1. The energy produced per unit of capital stock in the mining sector shows a clear downward trend from 1760 to 1913. This is consistent with a decreasing labour intensity of mining and greater reliance on capital as mines became deeper. The energy intensity of 8.2 tons coal per £(1851), for 1775, based on an estimate of coal production from Flinn (1978) and capital stock from Feinstein and Pollard (1988) aligns with the trend, but it is a sole data point. The energy intensity of capital formation is higher in the early nineteenth century, than in the eras before or after. This is consistent with an increasing use of steel in capital formation, to make assets such as plant, machinery, equipment, rolling stock, ships and vehicles, over the nineteenth century – plus improvements in the energy efficiency of steel production in the second half of the century (Kennedy, 2020). The observation that the energy intensity of capital formation for 1800-1830, is about four times greater than that for capital use in producing good and services was previously seen from my input-output analysis for 1841 (Kennedy, 2020).

All three labour intensity parameters show a trend of declining over time (Table 1). Such decreases in the labour required per unit of capital (investment or stock) might be attributed to technological development, increasing labour productivity, economies of scale, or some combination thereof. The declining intensity of labour use is more noted for mining and production of goods & services. The construction and materials sector does not exhibit much change in labour per unit of capital formation after 1800. Moreover, the higher value for 1760 (31 persons per £) is only based on one data point.

Rates of capital depreciation are calculated from Feinstein & Pollard's (1998) time series estimates for capital investment and capital stock. The depreciation rates for agriculture and production of goods and services rise gradually over the second half of the nineteenth century, while depreciation rates for mining capital are highest in the eighteenth century (Table 1). As the investment and stock data is decadal before 1850, the estimated depreciation rates sometimes exhibit abnormal corrections at the end of each decade, particular with the transition to annual capital data in 1851. Nonetheless, the calibrated model is able to reproduce the historical capital stock very closely, as shown in Figure 5a.

5. Exploring Biophysical Constraints

The significance of Britain's high level of agricultural productivity in contributing to its growth during the Industrial Revolution can be demonstrated using a simulation of the biophysical model. During the eighteenth and nineteenth centuries, Britain had higher labour productivity in agriculture than most of her European neighbours – having larger farms and higher yields than France, for example (O'Brien, 1985; O'Brien, et al. 1977). In 1760, Britain was fed with 37% of the labour force working in agriculture (aided by some food imports), and by 1913, agricultural employment had fallen to 8.7%. To develop a scenario with lower labour productivity, I examined model cases with much higher percentages of workers in agriculture – and all else unchanged. With the capital stock fixed at the given value for 1760, however, the agricultural labour force could not be simulated above 43% without immediately incurring destruction in capital (and negative employment in capital construction). So the scenario chosen has agricultural employment fixed at 43% of labour force – and held fixed at this percentage for the entire period through to 1913.

In this scenario, with all other model parameters unchanged, by 1913, the capital stock for production of goods and services would be about two-thirds of that historically observed (£2.18 billion in the scenario vs. £3.43 billion in the historical data; Figure 6a). Employment in production of goods and services in the scenario tracks closely to the employment in agriculture (Figure 6b); this is dramatically different to the historical record (Figure 5c). There is also lower employment in mining and capital construction – which is consistent with the lower levels of capital investment. In this scenario, with lower agricultural productivity and a higher proportion of the labour force remaining in agriculture, there would clearly be less urbanization too.

In a second scenario, I examine the impacts of technological progress – specifically in the mining sector. The key invention for British coal-mining was the steam engine, which was required for dewatering of coal seams. Coal mining did take place without the steam engine, but it required coal deposits to be above the water table, be close to a source of water-power, or, more commonly, rely on the use of teams of up tethered horses turning gins (Flinn, 1984; p.112-114). The Newcomen steam engine was first used at a mining site in 1712 (Flinn, 1984; Allen, 2009); Watt improved the energy efficiency of steam engines by a factor of 5 to 10 (Jevons, 1866); and further efficiencies were achieved in the nineteenth century (Allen, 2009). To examine the impacts of steam engines – just in the mining sector – I create a scenario in which the energy output per unit of capital is reduced by a factor of ten (i.e. values of $\hat{e}_M$ are set to 10% of those in the calibrated model).

Economic growth still occurs under this scenario, as access to coal is still, somewhat optimistically, considered possible. The size of the capital stock for production of goods and services, by 1913, however, is 59% of that historically observed (Figure 7a). To achieve this form of growth a substantially different distribution of labour is required. Employment in mining increases by a factor of 5 to 10, depending on the year – and is similar to that for goods and services by 1913 (Figure 7b). This is offset by lower employment in capital construction and production of goods and services. Agricultural employment remains fixed in this scenario, but theoretically it may have increased if Britain's approach to de-watering mines required many large teams of tethered horses. Increased agricultural production would be required to feed the horses. The Industrial Revolution would have been remarkably different with horses in place of steam engines!

6. Conclusions

The Industrial Revolution entailed increases in agricultural productivity, with excess labour exiting the countryside to work in new industries in Britain's growing towns and cities (Allen, 2009; Deane and Cole, 1967; Mokyr, 2018; Wrigley, 2010). To understand this phenomenon in biophysical terms, the model presented here focuses on the construction sector (including material production and manufacture of machinery), mining and agriculture. The construction sector physically created the machinery, buildings and infrastructure that underlay industrialization and urbanization, while the mining sector provided the necessary energy supply. Progress in agriculture determined the size of the excess labour force. The dynamic between these three key sectors provides the basis for understanding growth in Britain's capital stock for production of goods and services.

In methodological terms, a significant contribution of this paper is the mathematical representation of a general equilibrium between capital, labour and energy in an economy. Each of these primary factor inputs are subject to balance equations. They intersect through specification of energy and labour intensities of capital (use and formation), which are representative of technology and the wider business environment. An analytical solution was developed for the relatively simple model developed here. More advanced models capturing the general equilibrium between capital labour and energy, could be developed for other contexts beyond Britain during the Industrial Revolution.

The biophysical model presented here is not a conventional economic model in the sense that it does not include prices or quantities of good and services (other than tons of coal). Nonetheless, as capital is expressed in monetary terms, and labour is specified, there may be potential to add further monetary dimensions to the model. Other ways in which the current model could be enhanced include: fuller representation of trade (e.g., potentially including a balance of trade); and inclusion of other energy sources (e.g., organic energy supply).

Appendix: Solution to Biophysical Model

The following is the solution to equations (1) to (12) for given values of excess labour, $\hat{L}$, and agricultural capital stock K_A.

Substituting equations (2), (3) and (4) into (1) gives:

$$\hat{L} = \alpha_K I + \alpha_M K_M + \alpha_D K_D \tag{A1}$$

Substituting equations (10), (11) and (12) into (9) gives:

$$\hat{e}_M K_M = e_K I + e_D K_D \tag{A2}$$

Substituting equations (6), (7) and (8) into (5) gives:

$$I = K_A + K_M + K_D - K^* \tag{A3}$$

where

$$K^* = K_{A,t-1}(1-\delta_A) + K_{M,t-1}(1-\delta_M) + K_{D,t-1}(1-\delta_D) \tag{A4}$$

Substituting (A3) into (A1) and rearranging, we get:

$$\hat{L} + \alpha_K(K^* - K_A) = K_M(\alpha_K + \alpha_M) + K_D(\alpha_K + \alpha_D) \tag{A5}$$

Substituting (A3) into (A2) and rearranging, we get:

$$K_M = K_D\left(\frac{e_I + e_D}{\hat{e}_M - e_I}\right) + (K_A - K^*)\left(\frac{e_I}{\hat{e}_M - e_I}\right) \tag{A6}$$

Substituting (A6) into (A5) gives:

$$\hat{L} + \alpha_K(K^* - K_A) = (\alpha_K + \alpha_M)\left\{K_D\left(\frac{e_I + e_D}{\hat{e}_M - e_I}\right) + (K_A - K^*)\left(\frac{e_I}{\hat{e}_M - e_I}\right)\right\} + K_D(\alpha_K + \alpha_D) \tag{A7}$$

Rearranging (A7) gives:

$$K_D = \frac{\hat{L} + (K^* - K_A)\left\{\alpha_K + \frac{e_I}{\hat{e}_M - e_I}(\alpha_K + \alpha_M)\right\}}{(\alpha_K + \alpha_M)\left(\frac{e_I + e_D}{\hat{e}_M - e_I}\right) + (\alpha_K + \alpha_D)} \tag{A8}$$

Starting with initial values for all capital terms in a base year (t - 1), and annual time series for $\hat{L}$ and K_A, the solution procedure for the model is as follows:

- K^* is calculated from (A4)
- K_D is calculated from (A8)
- K_M is calculated from (A6)
- I is calculated from (A3)
- L_M, L_K and L_D are calculated from equations (2), (3) and (4)

- Ê, E_M and E_D are calculated from equations (10), (11) and (12)

End Note

1. Allen (2009b, p.425) observed that the elasticity of substitution between capital and labour was "close to zero." A small substitution effect is still captured in my model through changes in the labour intensities of capital, α, over time.
2. Data on Britain's total labour force is from Version 3.1 of The Bank of England's database of historical macroeconomic and financial statistics (at https://www.bankofengland.co.uk/statistics/research-datasets).

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Table 1. Energy intensity, labour intensity and capital depreciation parameters for a biophysical model of the British economy by era.

Parameters	**1760-75**	**1800-1830**	**1850-1913**
Energy intensity (ton coal per £(1851))			
$\hat{e}_M$: net energy production per mining capital stock	8.2	7.0 – 4.6	4.2 – 2.0
e_K : energy intensity of capital investment	0.09	0.19 (av.)	0.12 (av.)
e_D : energy intensity of capital use for production of goods and services	0.04	0.050 (av.)	0.056 (av.)
Labour intensity (persons per £(1851))			
α_M : mining	37	32	26 -14
α_K : capital formation	31	24	22 (av.)
α_D : production of goods and services	11	10	6.3 – 3.6
Annual capital depreciation rate (£ per £)			
δ_A : agriculture	0.029 (av.)	0.027 (av.)	0.038 (av.)
δ_M : mining	0.06	0.034	0.036
δ_D : production of goods and services	0.024	0.025	0.020 – 0.032

List of Figures

Figure 1. Four sector model of equilibrium between capital, energy and labour for Great Britain during the Industrial Revolution. (See text for definition of variables.)

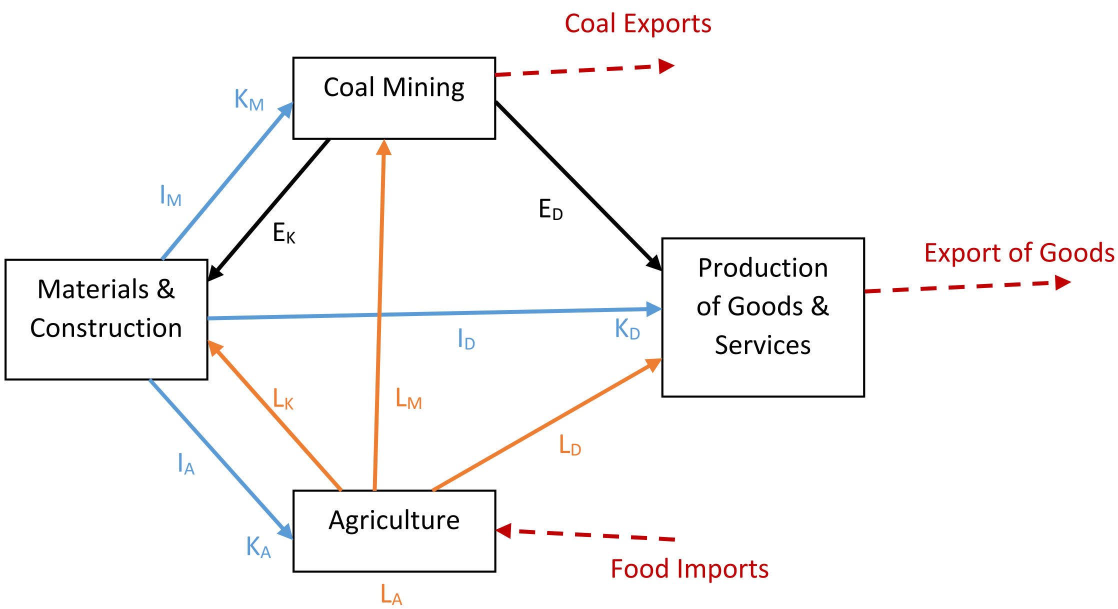


Figure 2. Employment in Great Britain, 1760 to 1913: a) Total labour force; b) Agriculture, capital construction and mining sectors. (B = Broadberry et al, 2015; D&C = Deane & Cole, 1967; F = Flinn, 1984; M = Mitchell, 1962; S-T = Shaw-Taylor et al., 2019). Data is provided in Supplementary Information 1.

a)

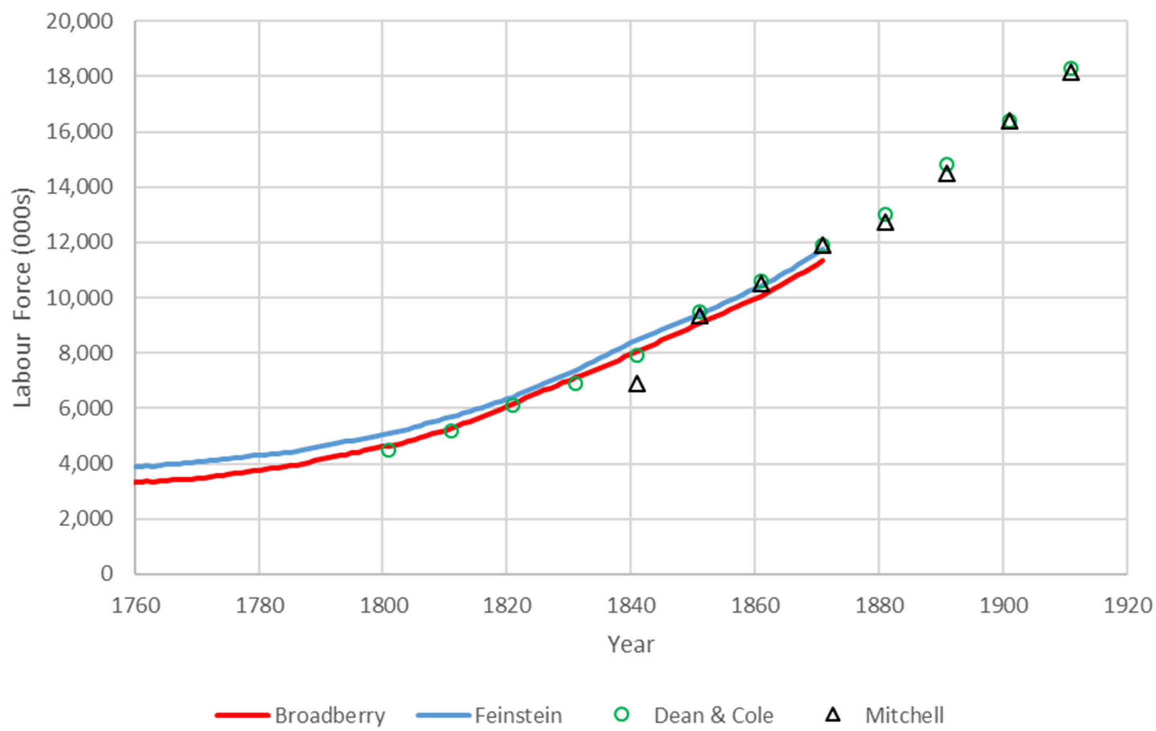


Figure 2 b)

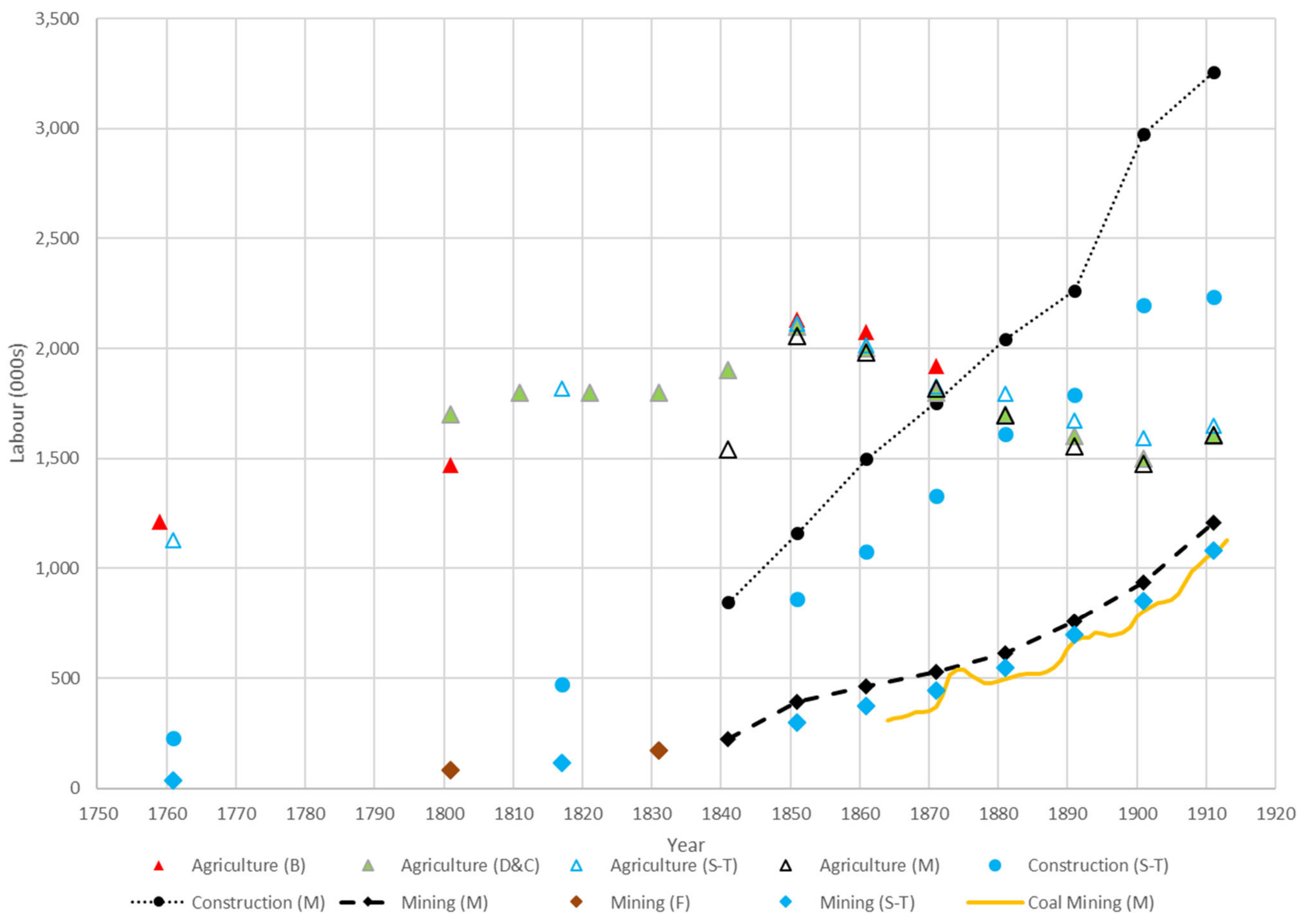

3,500
3,000
2,500
2,000
1,500
1,000
500
0
Labour (000s)
1750
1760
1770
1780
1790
1800
1810
1820
1830
1840
1850
1860
1870
1880
1890
1900
1910
1920
Year
Agriculture (B)
Agriculture (D&C)
Agriculture (S-T)
Agriculture (M)
Construction (S-T)
Construction (M)
Mining (M)
Mining (F)
Mining (S-T)
Coal Mining (M)

Figure 3. Britain's Capital Stock, 1760 to 1913 (£ million, 1851; Source: Feinstein & Pollard, 1988). Data is provided in Supplementary Information 1.

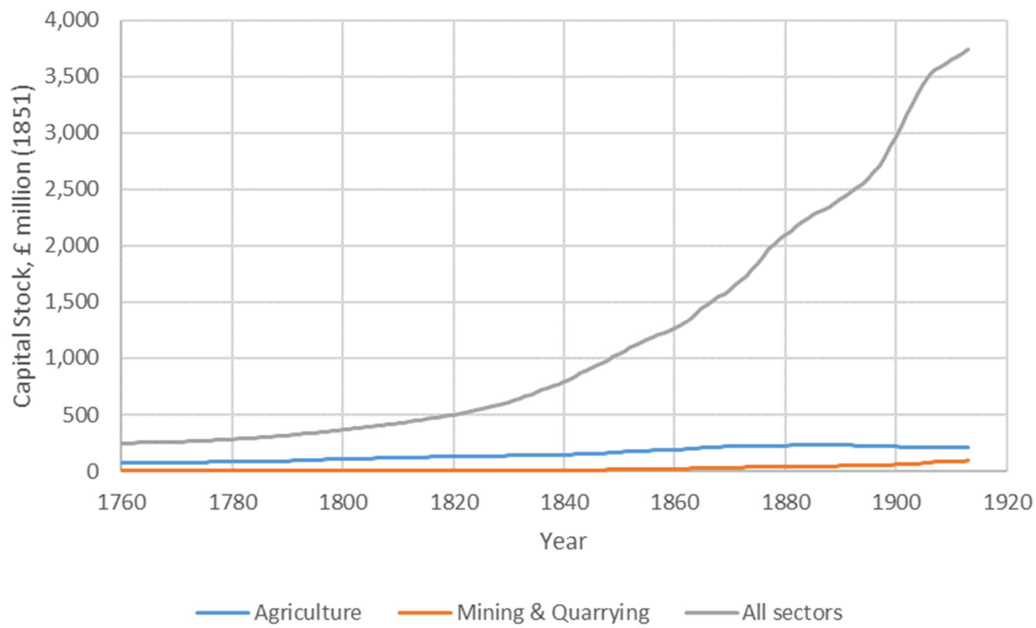


Figure 4. Uses of British coal, 1760 to 1913 (Source: Kennedy, 2020). Data is provided in Supplementary Information 1.

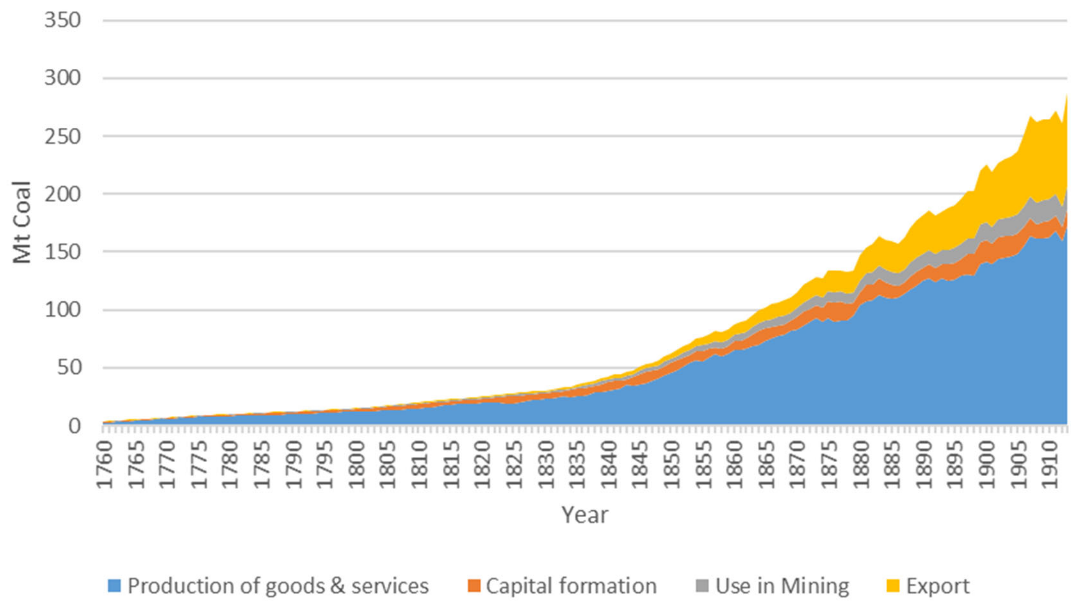
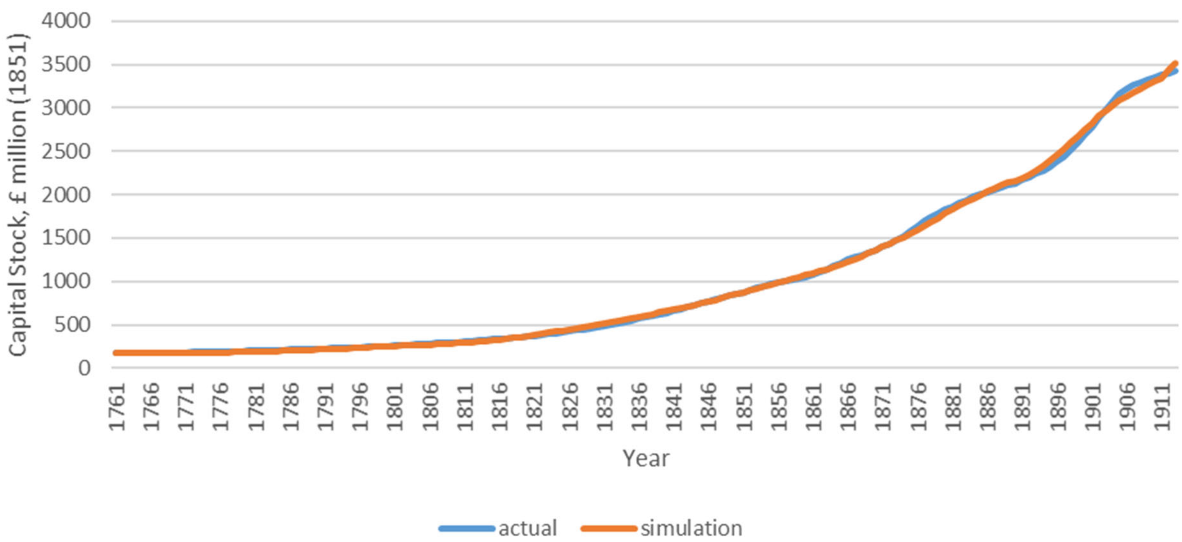
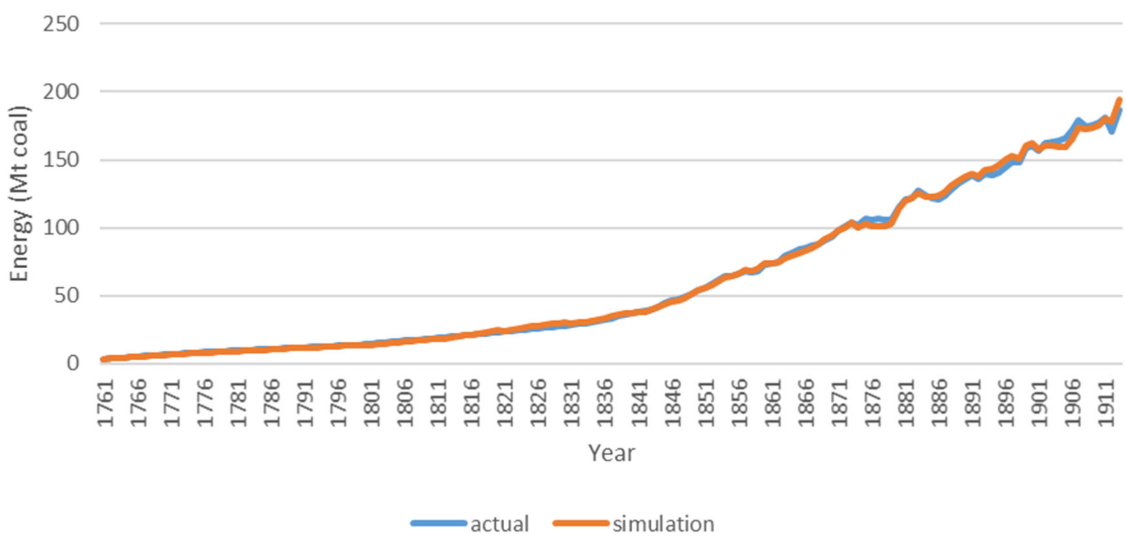


Figure 5. Calibrated results for the biophysical model of the Industrial Revolution: a) Capital for production of goods and services (£ million, 1851); b) Net available coal energy, Ê (Mt coal); c) Distribution of labour (000s). Data is provided in Supplementary Information 2.

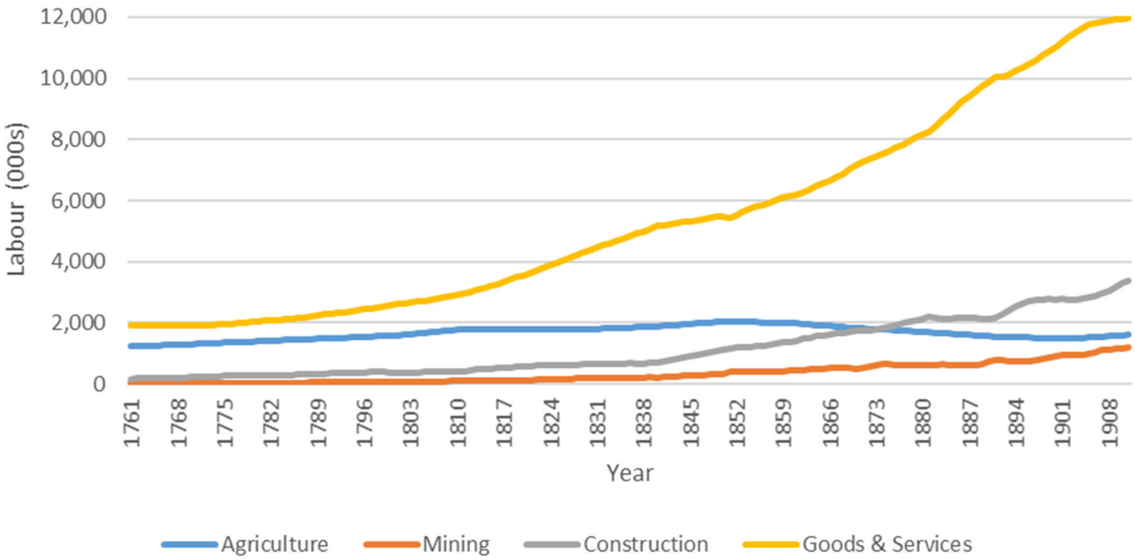
a)



b)



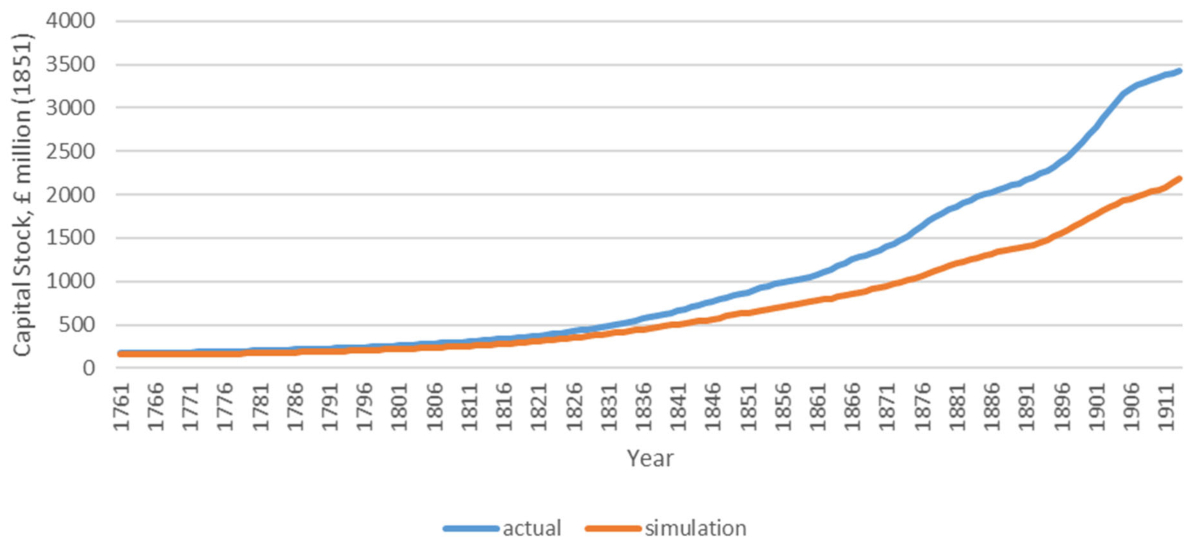
c)



12,000
10,000
8,000
6,000
4,000
2,000
0
Labour (000s)
1761
1768
1775
1782
1789
1796
1803
1810
1817
1824
1831
1838
1845
1852
1859
1866
1873
1880
1887
1894
1901
1908
Year
Agriculture
Mining
Construction
Goods & Services

Figure 6. Scenario results for lower agricultural productivity, with agricultural labour fixed at 43% of labour force: a) Capital for production of goods and services (£ million, 1851); b) Distribution of labour (000s). Data is provided in Supplementary Information 2.

a)



b)

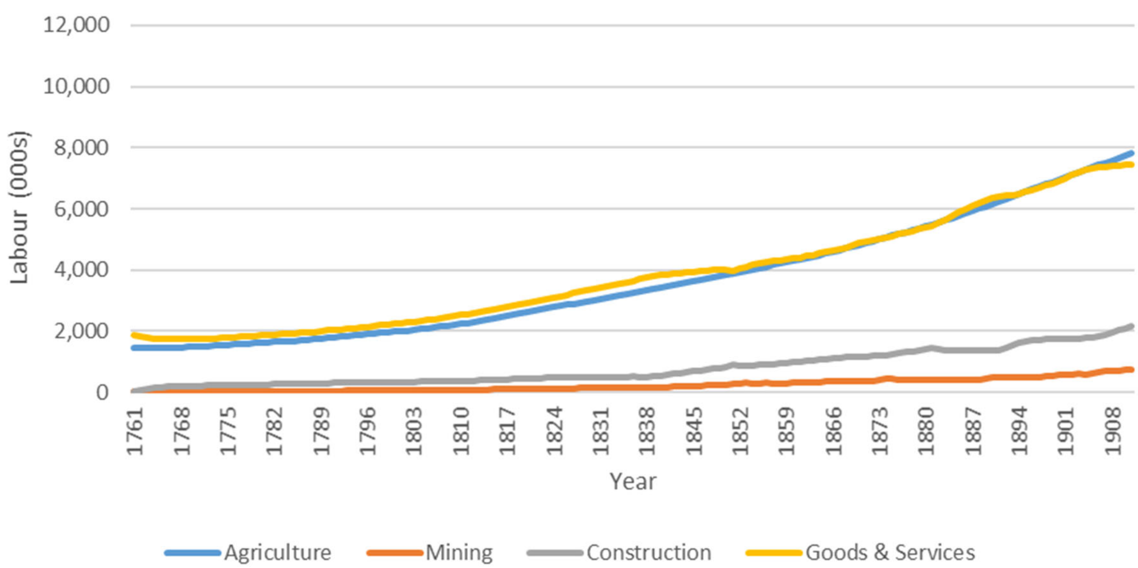


Figure 7. Scenario results for constrained mining technology, with energy intensity of mining sector capital, $\hat{e}_M$, at 10% of historical values: a) Capital for production of goods and services (£ million, 1851); b) Distribution of labour (000s). Data is provided in Supplementary Information 2.