Infrared Regularization in Relativistic Chiral Perturbation Theory

by

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Abstract

Chiral perturbation theory is a useful tool in the study of low energy reactions involving light particles. However the inclusion of heavy particles in chiral perturbation theory results in large contributions from loop diagrams which violate the standard power counting scheme. We review two methods, referred to as heavy baryon chiral perturbation theory and infrared regularization, which remove the high energy effects of the heavy particles and which therefore do not violate the power counting scheme. We then use these two methods to calculate the amplitude for pion photoproduction to fourth order and prove that the two amplitudes are equivalent.

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Chapter 1

Introduction

In the Standard Model mesons and baryons are composed of quarks which are bound together by strong interactions. Quantum chromodynamics is used to model these interactions, but at low energies the strong interactions of quarks and gluons become quite large and therefore the perturbation series fails to converge. As a result it is difficult to model low energy reactions using the Standard Model.

One alternative method of studying low energy reactions is to use an effective field theory, referred to as chiral perturbation theory, in which the mesons and baryons are used as the fundamental fields. The observed symmetries of QCD are used to build a set of effective vertices, with the relative magnitude of the couplings determined experimentally. Although the number of possible effective vertices is infinite, each vertex can be assigned a chiral dimension which corresponds to the number of small parameters it contains. Each Feynman diagram is then assigned a chiral dimension, and the calculations are performed at low energies where higher chiral dimensions correspond to small corrections to the results [1]. Reviews of chiral perturbation theory are given in [2, 3, 4]

Chiral perturbation theory has proven to be a useful tool in studying low energy reactions which involve light particles such as pions and kaons. However when applied to reactions involving heavier particles such as nucleons, the power counting methods begin to fail [5]. In diagrams which include a loop that contains a heavy particle, the loop momentum can become much larger than the typical low energy scale. The result is that a diagram with an arbitrary number of loops will be of the same magnitude as a tree diagram, though the chiral dimension of the loop diagram is greater than the chiral dimension of the tree diagram. As such the usual perturbation series will fail to converge.

The first method which was proposed for studying reactions involving heavy particles using ChPT was heavy baryon chiral perturbation theory [6], which was based on effective field theories used for heavy quarks. The mass of the nucleon is assumed to be significantly larger than the typical energies in the reaction, and as such the effects of antinucleon fields can be removed. The transformation which removes these effects results in a $1/M_N$ power series expansion of each effective vertex, and the resulting HBChPT vertices are quite complicated. As a result the amplitude for a reaction can usually only be calculated to third order, with higher order terms being too complicated to include in general.

A second method has been proposed [7, 8, 9] in which the effects of high energy loops are removed by separating the poles in the loop integrals. The infrared poles correspond to low energy loops which do not lead to problems with the power counting methods, and so they can be included in the calculations. The remaining poles in the loop integral correspond to the high energy loops that violate the power counting scheme. However these terms can be absorbed into the low energy constants and as such can be removed from the calculation. This method is referred to as infrared regularization when applied to a single heavy particle, and effective field theory dimensional regularization when applied to multiple heavy particles.

The two methods produce the same results, with the amplitudes calculated using HBChPT corresponding to a low energy expansion of the amplitudes calculated using infrared regularization. Since the method of infrared regularization does not require transforming the effective vertices, the resulting amplitudes tend to be simpler and can be calculated to higher orders. Infrared regularization also maintains Lorentz invariance, and so the results are not restricted to the nucleon rest frame. Furthermore, the method of infrared regularization can be applied to reactions which involve multiple heavy particles [10, 11] such as nucleon-nucleon scattering.

In order to study both methods and compare the results, the amplitude for pion photoproduction and radiative pion capture will be calculated using infrared regularization. The amplitude for this reaction and the related radiative pion capture amplitude have previously been calculated to third order using HBChPT [12]. It will be shown in Chapter 6 that the HBChPT amplitude can be produced by expanding the infrared regularization amplitude at low energies.

A review of chiral perturbation theory will be presented in Chapter 2, with the πN Lagrangian derived to second order. The heavy baryon formalism and the infrared regularization methods will be presented in Chapter 3 and Chapter 4 respectively. In Chapter 5 the method of infrared regularization will be used to calculate the nucleon wavefunction renormalization constant, the electromagnetic and pion nucleon interactions, and the amplitude corresponding to the irreducible diagrams involved in pion photoproduction. In Chapter 6 these calculations will be used to calculate the amplitude for pion photoproduction and radiative pion capture, with the resulting amplitude compared to the results of the heavy baryon calculation.

Chapter 2

Chiral Perturbation Theory

2.1 Introduction

The basis of chiral perturbation theory is that at low energies an effective Lagrangian can be used to describe strong interactions in place of the standard Lagrangian of quantum chromodynamics. In the effective Lagrangrian, the quark and gluon fields of QCD are replaced with meson and baryon fields. The result is that the calculations required for a specific reaction are simplified by replacing all of the quark interactions by a series of effective vertices. Since the effective vertices must reproduce the results of QCD, they must satisfy the same symmetry properties. It is the Chiral, Lorentz, P, C, and T symmetries which give the form of the effective vertices. Each effective vertex also includes one or more low energy constants which may in principle be calculated using QCD, though in practice these constants are usually determined experimentally. At higher energies, the effective Lagrangian and the low energy constants cease to approximate the physical properties of the particles, and chiral perturbation theory ceases to provide an accurate model.

In general the meson Lagrangian can be used to describe the complete pseudoscalar and vector meson octets, though we will only derive the part of the Lagrangian which describes the three lowest energy mesons, known as pions. While the complete SU(3) Lagrangian is not difficult to calculate, the additional vertices that it contains are not required in pion nucleon reactions. Similiarly the meson-baryon Lagrangian which contains interactions of the baryon octet with the pseudoscalar meson octet can be reduced to contain only pion-nucleon interactions.

In this chapter, chiral symmetry will be defined and both the pion Lagrangian and the pion-nucleon Lagrangian will be calculated.

2.2 Chiral Symmetry

2.2.1 Chiral Symmetry in QCD

The Lagrangian of QCD is

$$L_{QCD} = -\frac{1}{2} < G_{\mu\nu} G^{\mu\nu} > + \overline{q}(i\not{D} - M)q \tag{2.1}$$

where we have used the notation $\not{D} = \gamma^\mu D_\mu$. The matrix $G_{\mu\nu}$ denotes the gluon field strength tensor, the vectors $\overline{q}$ and q have the N quark fields as components, D_μ is the gauge covariant derivative, and M is the quark mass matrix.

The quark fields can be written as a sum of the left and right handed helicity components,

$$q_L = \tfrac{1}{2}(1-\gamma^5)q \qquad q_R = \tfrac{1}{2}(1+\gamma^5)q \tag{2.2}$$

The QCD Lagrangian is then written as

$$\begin{aligned} L_{QCD} = &-\frac{1}{2} < G_{\mu\nu}G^{\mu\nu} > +\overline{q_R}(i\not{D} - M)q_R + \overline{q_L}(i\not{D} - M)q_L \\ &- \overline{q_R}Mq_L - \overline{q_L}Mq_R \end{aligned} \tag{2.3}$$

which leads to the observation that if M=0, the two helicity components do not interact. In that case q_R and q_L are each invariant under an SU(N) transformation, and so there is a new symmetry group $SU(N)_R \times SU(N)_L$ which is referred to as the chiral symmetry group.

The state containing no quarks, known as the vacuum state, is invariant under SU(N) transformations, but does not obey chiral symmetry. As a result of this spontaneous symmetry breaking, Goldstone's theorem predicts the existence of N^2-1 massless bosons with $J^P = 0^-$. If the physical masses of the quarks do not vanish but are negligible, then the chiral symmetry is also broken explicitly by the quark mass. It is the explicit symmetry breaking which gives the N^2-1 bosons their observed masses.

The three lightest quarks, known as up, down, and strange, have masses that are small compared with the typical reaction energies while the other three quarks are much heavier than the typical energies. This implies that the chiral symmetry is valid for the light quarks and so we will take

$$q = \begin{bmatrix} u \\ d \\ s \end{bmatrix} \tag{2.4}$$

which is invariant under the chiral symmetry group $SU(3)_R \times SU(3)_L$. The spontaneous breaking of this chiral symmetry predicts eight massless bosons, which form the pseudoscalar octet.

At very low energies, the strange quark mass may be considered to be large as well. In this case only the up and down quarks are included

$$q = \begin{bmatrix} u \\ d \end{bmatrix} \tag{2.5}$$

and the chiral symmetry group is $SU(2)_R \times SU(2)_L$. When spontaneously broken, Goldstone's theorem predicts the existence of three massless bosons, which are the three pions.

2.2.2 Chiral Symmetry of the Pions

The pion fields also obey an SU(2) symmetry which is based on the isospin symmetry originally developed to describe the approximate symmetry of the proton and the neutron. The nucleon masses are nearly identical, which led to the idea that in addition to spin there is another quantum number known as isospin. Then the nucleons can be treated as a single particle in either an isospin up or a isospin down state, with each state representing either the proton or the neutron. A general nucleon wavefunction can be constructed as

$$\Psi = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \psi_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \psi_n \tag{2.6}$$

where ψ_p and ψ_n are the proton and neutron wavefunctions.

A similiar isospin symmetry is used for the three pion states, with the isospin quantum number taking values of $I_3 = 0, \pm 1$ instead of $I_3 = \pm 1/2$, although this is not the representation that is used in chiral perturbation theory. It will be simpler to use the SU(2) symmetry with a three dimensional basis comprised of the Pauli matrices. Then a general pion wavefunction can be written as

$$\phi = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \pi^3 + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \pi^1 + \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \pi^2 \tag{2.7}$$

The pion wavefunctions π^1 and π^2 cannot be observed independently, so we will use the physical pion fields defined by

$$\pi^+ = \frac{\pi^1 - i\pi^2}{\sqrt{2}}$$

$$\pi^- = \frac{\pi^1 + i\pi^2}{\sqrt{2}}$$

$$\pi^0 = \pi^3 \tag{2.8}$$

which gives the general pion wavefunction as

$$\phi = \begin{bmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{bmatrix} \tag{2.9}$$

The same method can also be used to describe the full pseudoscalar octet by using the Gell-Mann matrices as an eight dimensional basis for SU(3).

In chiral perturbation theory, a nonlinear function of the meson field ϕ is used in the Lagrangian [1] . The most common choices are

$$U = \sqrt{1 - \frac{\phi^2}{F^2}} + \frac{i\phi}{F} \tag{2.10}$$

and

$$U = e^{\frac{i\phi}{F}} \tag{2.11}$$

where F is a constant with the appropriate dimensions. This constant is usually taken to be the pion decay constant, though other choices are possible. We will use the exponential form of U in the deriving the chiral Lagrangian.

Each term in the chiral Lagrangian must be invariant under chiral symmetry. A chiral transformation on a matrix is defined as the local transformation

$$A(x) \rightarrow R(x)A(x)L^{\dagger}(x) \qquad R(x), L(x) \in SU(2) \tag{2.12}$$

and the matrices which transform in this manner are referred to as chiral matrices.

2.3 Pion Chiral Perturbation Theory

2.3.1 Definitions

The basis of chiral perturbation theory is the construction of a general Lagrangian which obeys the same symmetries as QCD. The Lagrangian contains every combination of fields and derivatives of fields which satisfy chiral, Lorentz, P,C and T symmetries. Since there are an infinite number of such terms, the Lagrangian is organized into a power series with the smallest powers of energy and momentum giving the largest contribution to the Lagrangian. In this section only the terms in the Lagrangian which describe pion interactions will be constructed.

The pion fields are introduced into the Lagrangian using the representation from equation 2.11. The matrix U(x) is a chiral matrix, as is its adjoint $U^{\dagger}(x)$. The matrix U(x) is also unitary so that

$$U^{\dagger}U = 1$$

and has unit determinant. The amplitude of the pion fields does not depend explicitly on the energy of the pions, and so it is said to be of zeroth order.

The chiral Lagrangian will also include derivatives of U(x) which are first order terms. However the derivative of U(x) is not itself a chiral matrix, since the derivative of the transformed field is

$$\begin{aligned}\partial_\mu U(x) \rightarrow\ & \partial_\mu (R(x)U(x)L^{\dagger}(x)) \\ & = (\partial_\mu R(x))U(x)L^{\dagger}(x) + R(x)(\partial_\mu U(x))L^{\dagger}(x) + R(x)U(x)(\partial_\mu L^{\dagger}(x))\end{aligned} \tag{2.13}$$

The first and third term must vanish if $\partial_\mu U(x)$ is a chiral matrix. In order to restore chiral symmetry, a new derivative must be defined which transforms in the correct way. This new derivative, denoted D_μ, is called the covariant derivative of a chiral matrix and is defined as

$$D_\mu A(x) = \partial_\mu A(x) - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu) \tag{2.14}$$

The two new fields v_μ and a_μ are gauge fields which will be used later to introduce gauge field couplings. The combinations of the two gauge fields are often denoted by

$$r_\mu \equiv v_\mu + a_\mu$$
$$l_\mu \equiv v_\mu - a_\mu$$

and must transform as

$$r_\mu \to R(x)(r_\mu + i\partial_\mu)R^\dagger(x) \tag{2.15}$$
$$l_\mu \to L(x)(l_\mu + i\partial_\mu)L^\dagger(x)$$

Using these transformation laws, a chiral transformation applied to $D_\mu A(x)$ gives

$$\begin{aligned} D_\mu A \to\ & \partial_\mu(RUL^\dagger) - iRr_\mu R^\dagger RUL^\dagger + iRUL^\dagger Ll_\mu L^\dagger + R\partial_\mu R^\dagger RUL^\dagger - RU\partial_\mu L^\dagger \\ & = RD_\mu UL^\dagger + \partial_\mu RUL^\dagger - R\partial_\mu R^\dagger RUL^\dagger \\ & = RD_\mu UL^\dagger \end{aligned} \tag{2.16}$$

where in the last line we use the identity

$$R\partial_\mu R^\dagger = -\partial_\mu RR^\dagger \tag{2.17}$$

which is true for any unitary matrix. Since the covariant derivative has now been shown to be a chiral matrix, and since the proof only assumes that U is a chiral matrix, it follows that multiple covariant derivatives will also form a chiral matrix.

The chiral Lagrangian must also contain terms describing the gauge fields, which leads to the defininition of field strength tensors,

$$\begin{aligned} F_{\mu\nu} &\equiv i[D_\mu, D_\nu]U \\ &= F^R_{\mu\nu}U - UF^L_{\mu\nu} \end{aligned} \tag{2.18}$$

where

$$\begin{aligned} F^R_{\mu\nu} &= \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu] \\ F^L_{\mu\nu} &= \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu] \end{aligned} \tag{2.19}$$

We will also need the adjoint of the field strength tensors,

$$(F_{\mu\nu}U)^\dagger = U^\dagger F^R_{\mu\nu} - F^L_{\mu\nu}U^\dagger \tag{2.20}$$

Under a chiral transformation, these two tensors transform as

$$\begin{aligned} F^R_{\mu\nu} &\to RF^R_{\mu\nu}R^\dagger \\ F^L_{\mu\nu} &\to LF^L_{\mu\nu}L^\dagger \end{aligned} \tag{2.21}$$

and so it is the combinations $F^R_{\mu\nu}U$ and $UF^L_{\mu\nu}$ which are chiral matrices. The field strength tensor contributes two powers of energy and momentum, and so they are second order terms.

The final component in the chiral Lagrangian is a term to describe the explicit symmetry breaking created by the non-zero quark mass. This is introduced as a complex matrix

$$\chi = 2B_0(s + ip) \tag{2.22}$$

where s is a scalar density and p is a pseudoscalar density. By definition χ transforms as

$$\chi \rightarrow R\chi L^{\dagger} \tag{2.23}$$

It is then assumed that for real reactions the scalar density is the quark mass matrix

$$M = \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{bmatrix} \tag{2.24}$$

while the pseudoscalar density vanishes. Although χ is linear in terms of the quark masses, it is considered to be a second order term in constructing the Lagrangian due to the nature of the constant B_0.

2.3.2 Chiral Invariants

The terms which appear in the Lagrangian must be invariant under a chiral transformation. This requires that each chiral matrix in the Lagrangian appear with the adjoint of a chiral matrix, since the adjoint of a chiral matrix transforms as

$$A^{\dagger} \rightarrow L(x)A^{\dagger}(x)R^{\dagger}(x) \tag{2.25}$$

and that the trace of such matrices is taken. The terms are also restricted by the relations

$$U^{\dagger}U = 1 \tag{2.26}$$

$$D_{\mu}U^{\dagger}U = -U^{\dagger}D_{\mu}U \tag{2.27}$$

which together imply that each term should contain the minimum number of factors of U. If extra factors of U are included in one of the terms, then there must be an equal number of factors of $U^{\dagger}$ to maintain chiral invariance. Then using the anticommutation rule from Eq. 2.27 and the unitary relation Eq. 2.26 the extra factors can be removed. It should also be noted that the Lagrangian is a Lorentz scalar, so that Lorentz indices may only appear as contracted pairs.

The lowest order contribution to the Lagrangian can contain only zeroth order terms. The only possible terms are $< U^{\dagger}U >, < U^{\dagger}U^{\dagger}UU >, ...$ which are constants. Since a constant term in the Lagrangian will not affect the results, the zeroth order part of the Lagrangian can be dropped. Next we must consider terms of order O(p), which can be built from factors of U, $U^{\dagger}$ and a single factor of $D_{\mu}U$ or its adjoint. However the single derivative contains a Lorentz index which cannot be contracted with any other index, and so there are no first order terms in the Lagrangian. The same argument can be applied to all odd orders, since in each case there must be a derivative which contains an uncontracted index.

The first non-trivial terms in the chiral Lagrangian appear at second order. These terms must contain either two derivatives with their indices contracted, or a single factor of χ or $\chi^{\dagger}$. The field strength tensors are also

$O(p^2)$ terms, but as the only possible contraction of the two Lorentz indices is

$$F_{\mu}^{R\mu} = F_{\mu}^{L\mu} = 0 \tag{2.28}$$

these tensors do not contribute at second order. There are then four terms which obey Lorentz and chiral symmetry,

$$< D^{\mu}U^{\dagger}D_{\mu}U > \tag{2.29}$$

$$< D^{\mu}D_{\mu}U^{\dagger}U > \tag{2.30}$$

$$< \chi^{\dagger}U > \tag{2.31}$$

$$< U^{\dagger}\chi > \tag{2.32}$$

This method is also used to derive the fourth order chiral invariant terms [13]. Each term is composed of either four derivatives, two derivatives and a factor of χ, two derivatives and a field strength tensor, two factors of χ, or two field strength tensors. Due to the large number of chiral invariant terms, they will not be written explicity.

2.3.3 Parity

The chiral Lagrangian can be further simplified if it is assumed that each term is invariant under parity transformations. The original QCD Lagrangian possesses this symmetry, and so the effective Lagrangian of chiral perturbation theory must also be invariant under parity transformations. The parity transformation acts on the fields as follows [13, 14, 15].

$$
\begin{aligned}
&U(t,\mathbf{x}) \rightarrow U^{\dagger}(t,-\mathbf{x}) \\
&\partial_{\mu}U(t,\mathbf{x}) \rightarrow \partial^{\mu}U^{\dagger}(t,-\mathbf{x}) \\
&v_{\mu}(t,\mathbf{x}) \rightarrow v^{\mu}(t,-\mathbf{x}) \\
&a_{\mu}(t,) \rightarrow -a^{\mu}(t,-\mathbf{x}) \\
&s(t,\mathbf{x}) \rightarrow s(t,-\mathbf{x}) \\
&p(t,\mathbf{x}) \rightarrow -p(t,-\mathbf{x})
\end{aligned}
\tag{2.33}
$$

which then leads to the transformation rules for the remaining objects

$$\chi \rightarrow \chi^{\dagger}$$

$$D_\mu U \rightarrow D^\mu U^\dagger \tag{2.34}$$

$$F^R_{\mu\nu} \rightarrow F^{L\mu\nu}$$

with all coordinates on the right reversed in sign by the transformation.

Using these transformation properties, we can take linear combinations of the four chiral invariant terms to produce a set of terms which are both chiral invariant and parity invariant. The result is

$$< D^\mu U^\dagger D_\mu U > \tag{2.35}$$

$$< D^\mu D_\mu U^\dagger U + D^\mu D_\mu U U^\dagger > \tag{2.36}$$

$$< \chi^\dagger U + \chi U^\dagger > \tag{2.37}$$

The $O(p^4)$ parity invariant terms are calculated using this same procedure [13, 16, 17], but again are too numerous to list.

2.3.4 Charge Conjugation

The final symmetry of the QCD Lagrangian which will be included in the chiral Lagrangian is charge conjugation. This symmetry requires the terms in the chiral Lagrangian to be invariant under the transformations [13, 14]

$$U \rightarrow U^T$$

$$\partial_\mu U \rightarrow \partial_\mu U^T$$

$$v_\mu \rightarrow -v_\mu^T$$

$$a_\mu \rightarrow a_\mu^T \tag{2.38}$$

$$s \rightarrow s^T$$

$$p \rightarrow p^T$$

which leads to the transformation rules

$$\chi \rightarrow \chi^\dagger$$

$$D_\mu U \rightarrow D_\mu U^T \tag{2.39}$$

$$F_{\mu\nu}^R \rightarrow -F_{\mu\nu}^{LT}$$

The parity invariant terms in the lowest order Lagrangian are also invariant under charge conjugation without modification. The trace of a matrix has the property that $< A^T > = < A >$, and for a pair of matrices,

$$< A^T B^T > = < (BA)^T > = < BA > = < AB >$$

As a result, only the higher order terms which contain the field strength tensors or which contain more than two matrices will need to be modified to satisfy charge conjugation invariance.

2.3.5 The Lagrangian

The chiral Lagrangian can be further simplified by removing terms which are related. The only relation which can be used at lowest order is Eq 2.27, which allows us to write

$$\begin{aligned} D^{\mu}D_{\mu}U^{\dagger}U + D^{\mu}D_{\mu}UU^{\dagger} &= -D^{\mu}U^{\dagger}D_{\mu}U - D^{\mu}UD_{\mu}U^{\dagger} \\ &= -2D^{\mu}U^{\dagger}D_{\mu}U \end{aligned} \tag{2.40}$$

The result is that two of the invariant monomials are related, and as such there is no loss of generality if only one of these terms is included in the Lagrangian.

At second order, there are only two independent terms and so there should be two independent constants in the Lagrangian. The constants are taken to be the pion decay constant F and B_0 from the definition of χ. The resulting Lagrangian is

$$L_2 = \frac{F^2}{4} < D^{\mu}U^{\dagger}D_{\mu}U > + \frac{F^2}{4} < \chi^{\dagger}U + \chi U^{\dagger} > \tag{2.41}$$

At fourth order, there are twelve terms which satisfy the same symmetries. The low energy constants in this part of the Lagrangian cannot be written in a simple form using known physical constants as was done in L_2, and so we will define a new set of constants L_i and H_i. The fourth order Lagrangian [16, 17] is then

$$
\begin{aligned}
L_4 &= L_1 < D^\mu U^\dagger D_\mu U >^2 + L_2 < D^\mu U^\dagger D^\nu U >< D_\mu U^\dagger D_\nu U > \\
&+ L_3 < D^\mu U^\dagger D_\mu U D^\nu U^\dagger D_\nu U^\dagger > + L_4 < D^\mu U^\dagger D_\mu U >< \chi^\dagger U + \chi U^\dagger > \\
&+ L_5 < D^\mu U^\dagger D_\mu U (\chi^\dagger U + \chi U^\dagger) > + L_6 < \chi^\dagger U + \chi U^\dagger >^2 \\
&+ L_7 < \chi^\dagger U - \chi U^\dagger >^2 + L_8 < \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger > \\
&- i L_9 < F^R_{\mu\nu} D^\mu U D^\nu U^\dagger + F^L_{\mu\nu} D^\mu U^\dagger D^\nu U > + L_{10} < F^R_{\mu\nu} U F^L_{\mu\nu} U^\dagger > \\
&+ H_1 < F^{R\mu\nu} F^R_{\mu\nu} + F^{L\mu\nu} F^L_{\mu\nu} > + H_2 < \chi^\dagger \chi >
\end{aligned}
\tag{2.42}
$$

There are also two additional fourth order terms which are included when the pions are off-shell [13],

$$
L_{4off} = P_1 < D^\mu D_\mu U^\dagger D^\nu D_\nu U > + P_2 < D^\mu D_\mu U^\dagger \chi + D^\mu D_\mu U \chi^\dagger > \tag{2.43}
$$

though these terms may be removed by a transformation of the Lagrangian. The sixth order Lagrangian has been calculated [18, 19], but as the pion-nucleon Lagrangian is only known to $O(p^4)$ these terms and higher order terms can be omitted without a loss of precision.

2.4 Pion-Nucleon Lagrangian

2.4.1 Definitions

The derivation of the pion-nucleon Lagrangian is more difficult, since the presence of fermions requires that the Dirac matrices be included. It is also complicated due to the magnitude of the typical energy of a nucleon. In the derivation of the pion Lagrangian it was assumed that the energy and momentum of the fields were much less than the energy scale of 1 GeV, and so each derivative was counted as one order of magnitude. However the nucleon rest mass is close to the 1 GeV limit, which suggests that an expansion in terms of the nucleon energy will fail to converge. In this section the pion-nucleon Lagrangian will be derived up to second order, while the convergence will be studied in Section 2.5.

As in the derivation of the pion Lagrangian, the first stage in deriving the pion nucleon Lagrangian will be to determine all of the factors which are to be included. The nucleon fields will be included as the doublet

$$\Psi = \begin{bmatrix} \psi_p \\ \psi_n \end{bmatrix} \tag{2.44}$$

where ψ_p and ψ_n are the proton and neutron wavefunctions. The nucleon doublet has an additional symmetry which must be included, known as axial symmetry. When the quarks are taken to be massless in QCD, the quark fields are invariant under a U(1) transformation of the form [20]

$$q \rightarrow e^{-i\theta\gamma^5} q \tag{2.45}$$

The axial symmetry should also be satisfied in the chiral Lagrangian. In the pion Lagrangian derived previously the terms are unaffected by the axial transformations, and so it was ignored. However the terms in the pion-nucleon Lagrangian are transformed, and the axial symmetry must now be included.

The symmetry group used in the pion-nucleon Lagrangian is $SU(2)_R \times SU(2)_L \times U(1)_A$ and the nucleon fields transform as [2, 21, 22]

$$\Psi \rightarrow h(g,\phi)\Psi \qquad \overline{\Psi} \rightarrow \overline{\Psi} h^{-1}(g,\phi) \tag{2.46}$$

where the compensator $h(g, \Phi)$ is a nonlinear function of the pion fields ϕ and an element of the chiral symmetry group $g \in SU(2)_R \times SU(2)_L$. The explicit form of the compensator is complicated, but if the factors from the pion Lagrangian are redefined by

$$u_\mu = i(u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger) = iu^\dagger D_\mu U u^\dagger$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u \tag{2.47}$$

$$F^\pm_{\mu\nu} = u^\dagger F^R_{\mu\nu} u \pm u F^L_{\mu\nu} u^\dagger$$

then each factor transforms as

$$A \to h(g,\phi) A h^{-1}(g,\phi) \tag{2.48}$$

and the terms in the Lagrangian transform as

$$\overline{\Psi} A \Psi \to \overline{\Psi} h^{-1} h A h^{-1} h \Psi = \overline{\Psi} A \Psi \tag{2.49}$$

It should also be noted that the pion fields enter the pion-nucleon Lagrangian through the terms u and $u^\dagger$ which are defined by

$$U = u^2 \qquad U^\dagger = u^{\dagger 2} \tag{2.50}$$

which in the exponential parameterization gives

$$u = exp(\frac{i\phi}{2F}) \tag{2.51}$$

The pion Lagrangian can be written using this form of the pion fields and the terms given in Eq 2.47, but to be consistent with other papers we have used the more traditional form of the pion Lagrangian.

The next factor in the pion-nucleon Lagrangian is the nucleon covariant derivative. As with the pion Lagrangian, a derivative is required which transforms as

$$D_\mu \Psi \rightarrow h(\phi, g) D_\mu \Psi \tag{2.52}$$

If an ordinary derivative were used, the result of a tranformation would be

$$\partial_\mu \Psi \rightarrow \partial_\mu h(\phi, g)\Psi + h(\phi, g)\partial_\mu \Psi \tag{2.53}$$

The first term must be removed by adding an additional term Γ_μ to the derivative. The covariant derivative is given by

$$D_\mu = \partial_\mu + \Gamma_\mu \qquad \Gamma_\mu = \tfrac{1}{2}(u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger) \tag{2.54}$$

The result is that terms of the form $[D_\mu, A]$ transform as

$$[D_\mu, A] \rightarrow h(\phi, g)[D_\mu, A]h^{-1}(\phi, g) \tag{2.55}$$

while terms of the form $D_\mu \Psi$ and $D_\mu D_\nu \Psi$ transform as required in Eq 2.52.

	u_μ	$\chi+$	$\chi-$	$F^+_{\mu\nu}$	$F^-_{\mu\nu}$	D_μ
chiral dimension	1	2	2	2	2	1
parity	-	+	-	+	-	+
charge conjugation	+	+	+	-	+	+
hermitian conjugation	+	+	-	+	+	+

Table 2.1: Pion and Gauge Field Transformation Properties

	γ_5	γ_μ	$\gamma_\mu\gamma_5$	$\sigma_{\mu\nu}$	$g_{\mu\nu}$	$\epsilon_{\lambda\mu\nu\rho}$	$D_\mu\Psi$
chiral dimension	1	0	0	0	0	0	0
parity	-	+	-	+	+	-	+
charge conjugation	+	-	+	-	+	+	-
hermitian conjugation	-	+	+	+	+	+	-

Table 2.2: Transformation Properties of Nucleon Fields and Dirac Matrices

The pion-nucleon Lagrangian also includes the Dirac matrices which were not present in the pion Lagrangian. The transformation properties of these matrices are given in Table 2.2, with charge conjugation and the parity transformation also transposing the matrices. There is also a complication with the chiral dimension, which is not well defined for the Dirac matrices. It is expected that since the matrices have no dependence on energy or momentum, they should all be zeroth order terms. However the matrix γ^5 is taken to be a first order term, though the products $\gamma_\mu\gamma^5$ and $\gamma^5\gamma^5$ are both $O(p^0)$ [22].

Another complication in the derivation of the pion-nucleon Lagrangian is that the energy and mass of a nucleon is too large to be considered as small parameters. As a result, $D_\mu\Psi$ and M_N are both taken to be $O(p^0)$ in the expansion. However the difference of the two quantities is assumed to be small, so that the combination $(i\not{D} - M_N)$ has a chiral dimension of 1.

As in the derivation of the pion Lagrangian, the terms of the pion-nucleon

Lagrangian can now be constructed by forming monomials from these factors which are invariant under all of the symmetry transformations used before. The only differences are that we can now construct terms with odd order since we have $O(p^0)$ objects that contain a Lorentz index, and that the matrices can now be included outside of a trace since the product of the matrices with the nucleon doublets result in scalar terms.

2.4.2 Chiral Invariants

In this section we will combine the quantities listed in Tables 2.1 and 2.2 to form a complete list of O(p) and $O(p^2)$ monomials which are invariant under chiral and Lorentz transformations. The higher order terms in the Lagrangian have been calculated up to fourth order [22, 23, 21, 24, 25]. The $O(p^3)$ terms will be listed in Section 2.4.5, while the $O(p^4)$ terms which are required for the calculations will be introduced in the Chapter 5.

We expect that the leading term in the pion-nucleon Lagrangian will be the Lagrangian for a free Dirac field

$$L_{free} = \overline{\Psi}(i\not{D} - M_N)\Psi \tag{2.56}$$

which is O(p). As a result there cannot exist any terms in the Lagrangian at $O(p^0)$, although this is not apparent based on the possible combinations of factors which have a chiral dimension of zero.

Since the pion fields and guage field terms in Table 2.1 have non-zero chiral dimension, they cannot be used to construct $O(p^0)$ terms. One set of possible $O(p^0)$ terms are the combinations of the Dirac matrices

$$\gamma_\mu\gamma^\mu, \gamma^\mu\gamma^5\gamma_\mu, \sigma_{\mu\nu}\gamma^\mu\gamma^\nu, \sigma_{\mu\nu}\gamma^\mu\gamma^\nu, \epsilon_{\mu\nu\lambda\rho}\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\lambda, \ldots$$

However each of these terms can be reduced to a constant term, or to a constant multiple of γ^5. The constant terms do not alter the physical results and thus can be ignored, while constant multiples of γ^5 are O(p) terms. This is an example of one of the complications in the pion-nucleon Lagrangian, where the product of the three zeroth order terms $g^{\mu\nu}$, γ_μ, and $\gamma_\nu\gamma^5$ forms a first order term $4\gamma^5$. The other possible $O(p^0)$ terms include the covariant derivative of a nucleon field or the nucleon mass. However for each term of the form

$$\overline{\Psi}A^\mu D_\mu\Psi$$

there is a corresponding term of the form

$$\overline{\Psi}A^\mu\gamma_\mu M_N\Psi$$

The two terms can be rewritten in terms of $(i\not{D} - M_N)$ and M_N, the first part being a higher order contribution while the second part is a constant which can be dropped.

The first terms in the pion-nucleon Lagrangian which cannot be removed are at O(p). The terms which are chiral invariant and Lorentz invariant are

$$\overline{\Psi}(i\not{D} - M_N)\Psi \tag{2.57}$$

$$\overline{\Psi}\gamma^{\mu}u_{\mu}\Psi \tag{2.58}$$

$$\overline{\Psi}\gamma^{\mu}\gamma^{5}u_{\mu}\Psi \tag{2.59}$$

$$\overline{\Psi}\gamma^{5}\Psi \tag{2.60}$$

$$\overline{\Psi}u_{\mu}D^{\mu}\Psi + h.c. \tag{2.61}$$

At second order, the chiral invariant terms are

$$\overline{\Psi}u_{\mu}u^{\mu}\Psi \tag{2.62}$$

$$\overline{\Psi}\sigma^{\mu\nu}u_{\mu}u_{\nu}\Psi \tag{2.63}$$

$$\overline{\Psi}D^{\mu}u_{\mu}\Psi \tag{2.64}$$

$$\overline{\Psi}\gamma^{5}u_{\mu}D^{\mu}\Psi + h.c. \tag{2.65}$$

$$\overline{\Psi}\sigma^{\mu\nu}D_{\mu}u_{\nu}\Psi \tag{2.66}$$

$$\overline{\Psi}u_{\mu}u_{\nu}D^{\mu}D^{\nu}\Psi + h.c. \tag{2.67}$$

$$\overline{\Psi} < u_\mu u_\nu > D^\mu D^\nu \Psi + h.c. \tag{2.68}$$

$$\overline{\Psi} u_\mu u_\nu D^\nu D^\mu \Psi + h.c. \tag{2.69}$$

$$\overline{\Psi} < u_\mu u_\nu > D^\nu D^\mu \Psi + h.c. \tag{2.70}$$

$$\overline{\Psi} \epsilon^{\mu\nu\rho\lambda} u_\nu u_\mu D_\rho D_\lambda \Psi + h.c. \tag{2.71}$$

$$\overline{\Psi} \chi^+ \Psi \tag{2.72}$$

$$\overline{\Psi} < \chi^+ > \Psi \tag{2.73}$$

$$\overline{\Psi} \chi^- \Psi \tag{2.74}$$

$$\overline{\Psi} < \chi^- > \Psi \tag{2.75}$$

$$\overline{\Psi} \sigma^{\mu\nu} F^+_{\mu\nu} \Psi \tag{2.76}$$

$$\overline{\Psi} \sigma^{\mu\nu} < F^+_{\mu\nu} > \Psi \tag{2.77}$$

$$\overline{\Psi} F^+_{\mu\nu} D^\mu D^\nu \Psi + h.c. \tag{2.78}$$

$$\overline{\Psi} F^{+}_{\mu\nu} D^{\nu} D^{\mu} \Psi + h.c. \tag{2.79}$$

$$\overline{\Psi} \epsilon^{\mu\nu\rho\lambda} F^{+}_{\mu\nu} D_{\rho} D_{\lambda} \Psi + h.c. \tag{2.80}$$

$$\overline{\Psi} < F^{+}_{\mu\nu} > D^{\mu} D^{\nu} \Psi + h.c. \tag{2.81}$$

$$\overline{\Psi} < F^{+}_{\mu\nu} > D^{\nu} D^{\mu} \Psi + h.c. \tag{2.82}$$

$$\overline{\Psi} \epsilon^{\mu\nu\rho\lambda} < F^{+}_{\mu\nu} > D_{\rho} D_{\lambda} \Psi + h.c. \tag{2.83}$$

$$\overline{\Psi} \sigma^{\mu\nu} F^{-}_{\mu\nu} \Psi \tag{2.84}$$

$$\overline{\Psi} \sigma^{\mu\nu} < F^{-}_{\mu\nu} > \Psi \tag{2.85}$$

$$\overline{\Psi} F^{-}_{\mu\nu} D^{\mu} D^{\nu} \Psi + h.c. \tag{2.86}$$

$$\overline{\Psi} F^{-}_{\mu\nu} D^{\nu} D^{\mu} \Psi + h.c. \tag{2.87}$$

$$\overline{\Psi} \epsilon^{\mu\nu\rho\lambda} F^{-}_{\mu\nu} D_{\rho} D_{\lambda} \Psi + h.c. \tag{2.88}$$

$$\overline{\Psi} < F^{-}_{\mu\nu} > D^{\mu} D^{\nu} \Psi + h.c. \tag{2.89}$$

$$\overline{\Psi} < F^{-}_{\mu\nu} > D^{\nu} D^{\mu} \Psi + h.c. \tag{2.90}$$

$$\overline{\Psi} \epsilon^{\mu\nu\rho\lambda} < F^{-}_{\mu\nu} > D_{\rho} D_{\lambda} \Psi + h.c. \tag{2.91}$$

The terms which involve derivatives of the nucleon field Ψ are not hermitian, and so these terms represent the sum of the term and its hermitian conjugate.

Some of the terms can be removed using the properties of the factors involved and other relations between the terms. However these relations will not be considered until after parity and charge conjugation have been included.

2.4.3 Parity

The list of chiral invariants can be further reduced by allowing only combinations of terms which are invariant under a parity transformation. The parity transformation of each factor is given in Tables 2.1 and 2.2. The O(p) parity invariant terms are

$$\overline{\Psi}(i\not{D} - M_N)\Psi \tag{2.92}$$

$$\overline{\Psi}\gamma^{\mu}\gamma^{5} u_{\mu}\Psi \tag{2.93}$$

The $O(p^2)$ parity invariant terms are

$$\overline{\Psi} u^{\mu} u_{\mu} \Psi \tag{2.94}$$

$$\overline{\Psi} < u^{\mu}u_{\mu} > \Psi \tag{2.95}$$

$$\overline{\Psi}\sigma^{\mu\nu}u_{\mu}u_{\nu}\Psi \tag{2.96}$$

$$\overline{\Psi}\sigma^{\mu\nu} < u_{\mu}u_{\nu} > \Psi \tag{2.97}$$

$$\overline{\Psi}\gamma^{5}u_{\mu}D^{\mu}\Psi + h.c. \tag{2.98}$$

$$\overline{\Psi}\gamma^{5} < u_{\mu} > D^{\mu}\Psi + h.c. \tag{2.99}$$

$$\overline{\Psi}u_{\mu}u_{\nu}D^{\mu}D^{\nu}\Psi + h.c. \tag{2.100}$$

$$\overline{\Psi} < u_{\mu}u_{\nu} > D^{\mu}D^{\nu}\Psi + h.c. \tag{2.101}$$

$$\overline{\Psi}u_{\mu}u_{\nu}D^{\nu}D^{\mu}\Psi + h.c. \tag{2.102}$$

$$\overline{\Psi} < u_{\mu}u_{\nu} > D^{\nu}D^{\mu}\Psi + h.c. \tag{2.103}$$

$$\overline{\Psi}\chi^{+}\Psi \tag{2.104}$$

$$\overline{\Psi} < \chi^{+} > \Psi \tag{2.105}$$

$$\overline{\Psi}\sigma^{\mu\nu}F^{+}_{\mu\nu}\Psi \tag{2.106}$$

$$\overline{\Psi}\sigma^{\mu\nu} < F^{+}_{\mu\nu} > \Psi \tag{2.107}$$

$$\overline{\Psi}F^{+}_{\mu\nu}D^{\mu}D^{\nu}\Psi + h.c. \tag{2.108}$$

$$\overline{\Psi}F^{+}_{\mu\nu}D^{\nu}D^{\mu}\Psi + h.c. \tag{2.109}$$

$$\overline{\Psi} < F^{+}_{\mu\nu} > D^{\mu}D^{\nu}\Psi + h.c. \tag{2.110}$$

$$\overline{\Psi} < F^{+}_{\mu\nu} > D^{\nu}D^{\mu}\Psi + h.c. \tag{2.111}$$

$$\overline{\Psi}\epsilon^{\mu\nu\rho\lambda}F^{-}_{\mu\nu}D_{\rho}D_{\lambda}\Psi + h.c. \tag{2.112}$$

$$\overline{\Psi}\epsilon^{\mu\nu\rho\lambda} < F^{-}_{\mu\nu} > D_{\rho}D_{\lambda}\Psi + h.c. \tag{2.113}$$

2.4.4 Charge Conjugation

The parity invariant O(p) terms which are also invariant under charge conjugation are

$$\overline{\Psi}(i\not{D} - M_N)\Psi \tag{2.114}$$

$$\overline{\Psi}\gamma^{\mu}\gamma^{5}u_{\mu}\Psi \tag{2.115}$$

and the $O(p^2)$ invariant terms are

$$\overline{\Psi} < u^\mu u_\mu > \Psi \tag{2.116}$$

$$\overline{\Psi}\sigma^{\mu\nu}[u_\mu, u_\nu]\Psi \tag{2.117}$$

$$\overline{\Psi}(< u_\mu u_\nu > D^\mu D^\nu \Psi + h.c. \tag{2.118}$$

$$\overline{\Psi}\chi^+\Psi \tag{2.119}$$

$$\overline{\Psi} < \chi^+ > \Psi \tag{2.120}$$

$$\overline{\Psi}\sigma^{\mu\nu} F^+_{\mu\nu}\Psi \tag{2.121}$$

$$\overline{\Psi}\sigma^{\mu\nu} < F^+_{\mu\nu} > \Psi \tag{2.122}$$

$$\overline{\Psi}\epsilon^{\mu\nu\rho\lambda} F^-_{\mu\nu} D_\rho D_\lambda \Psi + h.c. \tag{2.123}$$

$$\overline{\Psi}\epsilon^{\mu\nu\rho\lambda} < F^-_{\mu\nu} > D_\rho D_\lambda \Psi + h.c. \tag{2.124}$$

The chiral Lagrangian must also be hermitian and invariant under time reversal, though neither of these symmetries will reduce the number of terms or require the terms to be combined.

2.4.5 The Lagrangian

The construction of the second order pion-nucleon Lagrangian is almost complete, with two O(p) terms and nine $O(p^2)$ terms which satisfy the required symmetries. The list of terms can be further reduced using the curvature relation [22]

$$[D_\mu, D_\nu] = \frac{1}{4}[u_\mu, u_\nu] - \frac{i}{2}F^+_{\mu\nu} \tag{2.125}$$

The terms on the right hand side are both second order quantities, and so the term 2.123 can be rewritten as

$$\overline{\Psi}\epsilon^{\mu\nu\rho\lambda}F^-_{\mu\nu}(D_\lambda D_\rho + O(p^2))\Psi \approx \overline{\Psi}\epsilon^{\mu\nu\rho\lambda}F^-_{\mu\nu}D_\lambda D_\rho\Psi \tag{2.126}$$

The Lorentz indices λ and ρ can be interchanged, which gives

$$\overline{\Psi}\epsilon^{\mu\nu\lambda\rho}F^-_{\mu\nu}D_\rho D_\lambda\Psi = -\overline{\Psi}\epsilon^{\mu\nu\rho\lambda}F^-_{\mu\nu}D_\rho D_\lambda\Psi \tag{2.127}$$

The result is that the term 2.123 is not actually a second order term, since it has now been shown that it is of the form

$$\overline{\Psi}\epsilon^{\mu\nu\rho\lambda}F^-_{\mu\nu}D_\rho D_\lambda\Psi = \frac{1}{2}\overline{\Psi}\epsilon^{\mu\nu\rho\lambda}F^-_{\mu\nu}A_{\rho\lambda}\Psi \tag{2.128}$$

where $A_{\lambda\rho}$ has chiral dimension 2. The same proof can be applied to eliminate 2.124.

The first order pion-nucleon Lagrangian contains two independent terms, and so it will also contain two low energy constants. We expect the first term to be the free nucleon Lagrangian, and so we can set its constant equal to one, while the simplest choice for the second term is the axial coupling constant g_A.

$$L^{(1)}_{\pi N} = \overline{\Psi}(i\not{D} - M_N + \frac{g_A}{2}\gamma^\mu\gamma^5 u_\mu)\Psi \tag{2.129}$$

The second order pion-nucleon Lagrangian can be formed with the seven terms listed above, with each given a low energy constant that can be determined experimentally. However the convention is to rewrite the pion mass term as

$$\chi^+ \rightarrow \tilde{\chi}^+ = \chi^+ - \frac{1}{2} < \chi^+ > \tag{2.130}$$

and to define the low energy constants c_i as follows,

$$\begin{aligned} L^{(2)}_{\pi N} =& \overline{\Psi}(c_1 < \chi^+ > - \frac{c_2}{8M_N^2}(< u_\mu u_\nu > D^\mu D^\nu + h.c.) \\ &+ \frac{c_3}{2} < u^\mu u_\mu > + \frac{ic_4}{4}\sigma^{\mu\nu}[u_\mu, u_\nu] + c_5\tilde{\chi}^+ + \frac{c_6}{8M_N}\sigma_{\mu\nu}F^+_{\mu\nu} \\ &+ \frac{c_7}{8M_N}\sigma^{\mu\nu} < F^+_{\mu\nu} >)\Psi \end{aligned} \tag{2.131}$$

The third order pion-nucleon Lagrangian is given by 23 terms [25, 21], with low energy constants d_i

$$
\begin{aligned}
L_{\pi N}^{(3)} =& \overline{\Psi}(-\frac{d_1}{2M_N}[u_\mu,[D_\nu,u^\mu]D^\nu - \frac{d_2}{2M_N}[u_\mu,[D^\mu,u_\nu]D^\nu \\
&+ \frac{d_3}{12M_N^3}[u_\mu,[D_\nu,u_\rho]]D^\mu D^\nu D^\rho - \frac{d_4}{2M_N}\epsilon^{\mu\nu\rho\lambda} < u_\mu u_\nu u_\rho > D_\lambda \\
&+ \frac{id_5}{2M_N}[\chi^-,u_\mu]D^\mu + \frac{id_6}{2M_N}[D^\mu,F_{\mu\nu}^+]D^\nu + \frac{id_7}{2M_N}[D^\mu,< F_{\mu\nu}^+ >]D^\nu \\
&+ \frac{id_8}{2M_N}\epsilon^{\mu\nu\rho\lambda} < F_{\mu\nu}^+ u_\rho > D_\lambda + \frac{id_9}{2M_N}\epsilon^{\mu\nu\rho\lambda} < F_{\mu\nu}^+ > u_\rho D_\lambda \\
&+ \frac{d_{10}}{2}\gamma^\mu\gamma^5 < u^2 > u_\mu + \frac{d_{11}}{2}\gamma^\mu\gamma^5 < u_\mu u_\nu > u^\nu \\
&- \frac{d_{12}}{8M_N^2}\gamma^\mu\gamma^5 < u_\rho u_\nu > u_\mu D^\rho D^\nu - \frac{d_{13}}{8M_N^2}\gamma^\mu\gamma^5 < u_\mu u_\nu > u_\lambda D^\lambda D^\nu \\
&+ \frac{id_{14}}{4M_N}\sigma^{\mu\nu} < [D_\lambda,u_\mu]u_\nu > D^\lambda + \frac{id_{15}}{4M_N}\sigma^{\mu\nu} < u_\mu[D_\nu,u_\lambda] > D^\lambda \\
&+ \frac{d_{16}}{2}\gamma^\mu\gamma^5 < \chi^+ > u_\mu + \frac{d_{17}}{2}\gamma^\mu\gamma^5 < \chi^+ u_\mu > + \frac{id_{18}}{2}\gamma^\mu\gamma^5[D_\mu,\chi^-] \\
&+ \frac{id_{19}}{2}\gamma^\mu\gamma^5[D_\mu,<\chi^->] - \frac{id_{20}}{8M_N^2}\gamma^\mu\gamma^5[\tilde{F}_{\mu\nu}^+,u_\lambda]D^\lambda D^\nu \\
&+ \frac{id_{21}}{2}\gamma^\mu\gamma^5[\tilde{F}_{\mu\nu}^+,u^\nu] + \frac{d_{22}}{2}\gamma^\mu\gamma^5[D^\nu,F_{\mu\nu}^-] \\
&+ \frac{d_{23}}{2}\gamma_\mu\gamma^5\epsilon^{\mu\nu\rho\lambda} < u_\nu F_{\rho\lambda}^- >)\Psi
\end{aligned}
\tag{2.132}
$$

It should be noted that each term in $L_{\pi N}^{(3)}$ also includes all possible permutations of the Lorentz indices, and for each term which involves a derivative of the nucleon field Ψ there is an analagous term that includes a derivative of the nucleon field $\overline{\Psi}$. These additional terms have not been listed, but they will be used in the form factor calculations in Chapter 5 .

The $O(p^4)$ Lagrangian has also been calculated [22] and contains 118 terms with low energy constants labelled as e_i. Since only a few of the fourth order terms will contribute to our calculations using infrared regularization, and since the previous calculations using heavy baryon chiral perturbation theory have not included fourth order contributions, the Lagrangian $L_{\pi N}^{(4)}$ will not be reproduced. Those terms which are required for the calculation will be listed in Chapter 5.

2.5 Power Counting

In this section the order of a Feynman diagram will be defined analagous to the order of each term in the Lagrangian. It will then be shown that any series of Feynman diagrams which involves heavy particles will either converge slowly or will not converge, with the two most common solutions of this problem given in Chapters 3 and 4.

In traditional quantum field theory, all of the tree level diagrams must be calculated first, with single loop diagrams representing a small correction. However in chiral perturbation theory there are an infinite number of vertices and thus an infinite number of tree level diagrams. The single loop diagrams cannot be considered to be small correction terms, as the contribution to the amplitude from loop diagrams is larger than the contribution from most of the tree level diagrams. The power counting scheme which is used in chiral perturbation theory assigns each Feynman diagram a number, which is referred to as the chiral dimension or the order of the diagram. Then the contributions from first order Feynman diagrams are calculated, with the higher order diagrams representing corrections.

The chiral dimension of a diagram is calculated by summing the chiral dimension of each vertex, each propagator, and each loop integral. The chiral dimension of a vertex is the same as the order of the corresponding term in the chiral Lagrangian. For example, each vertex derived from an $O(p^n)$ term in the chiral Lagrangian contributes n to the order to the diagram. Each propagator of the form

$$\frac{i}{k^2 - m^2 + i\epsilon}$$

contributes -2 to the order of the diagram, while each propagator of the form

$$\frac{i(\not{k} + m)}{k^2 - m^2 + i\epsilon}$$

reduces the order of the diagram by one. If the diagram contains a loop, then the momentum integral increases the order of the diagram by four.

In reactions which do not involve heavy particles, the power counting scheme provides an ordering of the diagrams which simplifies the calculations. The higher order diagrams can be omitted as they provide a very small correction to the amplitude. Consider the reaction $\gamma\pi^a \rightarrow \gamma\pi^b$ and the corresponding Feynman diagrams in Figure 2.1. This is not the complete set of diagrams for pion Compton scattering, but the power counting scheme can be applied to the remaining diagrams with no complications.

In the first diagram there are two leading order vertices, which are $O(p^2)$ terms in the pion Lagrangian, and a pion propagator which is $O(p^{-2})$. Therefore the lowest order diagram has chiral dimension 2. The second diagram is identical to the first, except that one of the $O(p^2)$ vertices is replaced with an $O(p^4)$ vertex and so it is a fourth order diagram. In the remaining tree level diagram, both vertices are $O(p^4)$, and so it is a sixth order diagram. The sixth order diagrams are usually omitted, since they include diagrams

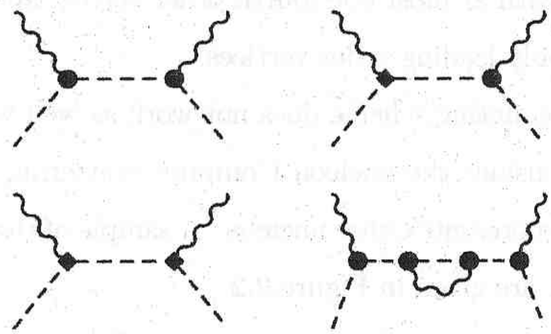

Figure 2.1: Pion Compton Scattering Diagrams

which contain either a single sixth order vertex or two loops. Although the sixth order Lagrangian has been calculated [18, 19] and two loop diagrams can be included, both types of diagram lead to complicated amplitudes.

The fourth diagram contains only leading order vertices, and represents the lowest order contribution from a loop. In this case the loop momentum also contributes to the order of the diagram. The lowest order loop diagrams are all fourth order. Using the topological equation

$$V - E + L = 1 \tag{2.133}$$

with V,E, and L representing the number of vertices, propagators, and loops respectively, it can be shown that any diagram with n_i vertices of order d_i , and L loops will have order [21]

$$D = 2 + 2L + \sum n_i(d_i - 2) \tag{2.134}$$

If the sixth order diagrams are omitted, then we only need to consider tree level diagrams with at most one fourth order vertex, and one loop diagrams which contain only leading order vertices.

The power counting scheme does not work as well when heavy particles are present. Consider the nucleon Compton scattering reaction, $N + \gamma \rightarrow N + \gamma$, where N represents either nucleon. A sample of the Feynman diagrams for this reaction are given in Figure 2.2.

The leading order diagram contains two O(p) vertices and a nucleon propagator which has chiral dimension -1, and so diagram has chiral dimension D=1. In the second diagram, one of the vertices is replaced by a second order vertex, and so the diagram has chiral dimension D=2. In the third diagram both vertices are second order, and so the diagram has chiral dimension D=3. The chiral dimension of the loop diagram is calculated using the same method, with the loop momentum integral assumed to have chiral dimension 4, and is also D = 3. Then for a general diagram which contains n_i vertices of order d_i, E_N external nucleon lines, and L loops, the chiral dimension is [21, 3]

$$D = 2 + 2L - \frac{E_B}{2} + \sum_{\pi N} n_i(d_i - 1) + \sum_{\pi} n_i(d_i - 2) \tag{2.135}$$

where the sums are over pion vertices and pion-nucleon vertices.

The power counting scheme for nucleon reactions produces the correct ordering of the terms, but in practice the contribution from the loop diagrams is larger than expected. In pion Compton scattering, the loop integral was taken to be an $O(p^4)$ term because the only poles in the integrand were in the pion propagator and were of the same order as the pion mass. When

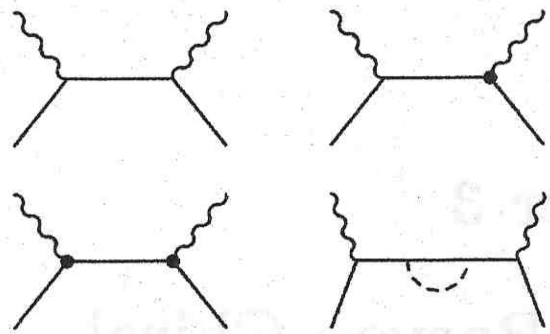

Figure 2.2: Nucleon Compton Scattering Diagrams

the loop integral includes a nucleon propagator, there exists a pole in the integrand at the nucleon mass. As a result the loop integral is separated into a series of terms corresponding to low energy poles and a series of terms which correspond to the high energy poles. The first series is an $O(m_\pi^4) \approx O(p^4)$ term as required by the power counting scheme, but the second series is an $\mathrm{O}(M_N^4)$ term.

The result is that the second part of each loop integral will effectively have a smaller chiral dimension, and therefore the loop diagram in Figure 2.2 may be larger than the O(p) leading order tree diagram. Higher order diagrams which contain multiple loops may also contribute at O(p). As a result, the calculations of any reaction involving heavy particles at any order must include an infinite number of loop diagrams. In Chapters 3 and 4 two methods of resolving this problem will be given.

Chapter 3

Heavy Baryon Chiral Perturbation Theory

3.1 Introduction

In this chapter we will introduce a modified form of chiral perturbation theory referred to as heavy baryon chiral perturbation theory [6]. This theory will allow heavy baryon fields, such as the nucleon fields, to be included in the Lagrangian without introducing the high energy poles that are normally present in the corresponding propagators. As the propagators have only low energy poles, the power counting scheme introduced in Section 2.5 will be valid for Feynman diagrams derived from the heavy baryon Lagrangian.

However heavy baryon chiral perturbation theory requires a complicated Lagrangian which is not necessarily Lorentz invariant. Each vertex in the Feynman diagrams will be a complicated series of terms, with each vertex from the chiral Lagrangian being transformed into an expansion in powers of the nucleon mass. As a result the calculations which have been performed have been restricted to $O(p^3)$ diagrams.

The derivation of the heavy baryon Lagrangian also requires the nucleon kinematics to be nonrelativistic, which restricts the use of HBChPT to reactions involving a single nucleon. This method is further complicated by the heavy baryon Lagrangian not being manifestly Lorentz invariant. Once the amplitude for a reaction is calculated, it must be rewritten in a Lorentz invariant form.

3.2 Heavy Baryon Expansion

3.2.1 Free Dirac Lagrangian

Heavy baryon chiral perturbation theory is based on the assumption that the baryon mass is signficantly larger than any of the other masses or energies involved in the reaction. In this case the baryon can be treated at first order as an infinite mass particle which remains at rest during the reaction. The recoil effects are introduced at higher orders and are proportional to powers of the inverse baryon mass.

The derivation of the heavy baryon Lagrangian from the chiral Lagrangian is complicated, since each term in the pion-nucleon part of the chiral Lagrangian must be expanded in powers of $1/M_N$. To introduce the methods of deriving the heavy baryon Lagrangian with less complication, in this section we will derive only the part of the heavy baryon Lagrangian which corresponds to a free nucleon in the chiral Lagrangian,

$$L_{free} = \overline{\Psi}(i\partial\!\!\!/ - M_N)\Psi \tag{3.1}$$

Since the nucleon is considered to be very heavy, its four-momentum can be written as

$$p_\mu = M_N v_\mu + l_\mu \tag{3.2}$$

where v_μ is the nucleon four-velocity which has the property $v^2 = 1$, and l_μ is a small off-shell momentum which satisfies $v \cdot l \ll M_N$. Then the nucleon field can be separated into two components using the velocity projection operators [26]

$$P_v^\pm = \frac{1 \pm v}{2} \tag{3.3}$$

This results in two fields defined by

$$\begin{aligned} N_v &= e^{iM_N v\cdot x} P_v^+ \Psi \\ H_v &= e^{iM_N v\cdot x} P_v^- \Psi \end{aligned} \tag{3.4}$$

and the nucleon field is

$$\Psi = e^{-iM_N v\cdot x}(N_v + H_v) \tag{3.5}$$

In the nucleon rest frame, $v_\mu = (1, 0, 0, 0)$, this transformation reduces the nucleon spinor to its upper and lower components.

The form of the nucleon field given in Eq 3.5 is then substituted into the Lagrangian,

$$L_{free} = e^{iM_N v\cdot x}(\overline{N_v} + \overline{H_v})(i\partial\!\!\!/ - M_N)e^{-iM_N v\cdot x}(N_v + H_v)$$

$$= (\overline{N_v} + \overline{H_v})(M_N v\!\!\!/ + i\partial\!\!\!/ - M_N)(N_v + H_v)$$

$$= \overline{N_v}(i\partial\!\!\!/)N_v + \overline{H_v}(i\partial\!\!\!/ - 2M_N)H_v + \overline{N_v}(i\partial\!\!\!/ - 2M_N)H_v + \overline{H_v}(i\partial\!\!\!/)N_v \tag{3.6}$$

where we have used the properties

$$v\!\!\!/ N_v = N_v$$

$$\tag{3.7}$$

$$v\!\!\!/ H_v = -H_v$$

These two equations can be used to prove several other identities [3], such as

$$\overline{N_v}\gamma^\mu N_v = \overline{N_v}\gamma^\mu v\!\!\!/ N_v = 2\overline{N_v}v^\mu N_v - \overline{N_v}\gamma^\mu N_v = \overline{N_v}v^\mu N_v \tag{3.8}$$

$$\overline{H_v}\gamma^\mu H_v = -\overline{H_v}\gamma^\mu v\!\!\!/ H_v = -2\overline{H_v}v^\mu H_v - \overline{H_v}\gamma^\mu H_v = -\overline{H_v}v^\mu H_v \tag{3.9}$$

$$\overline{N_v}H_v = \overline{N_v}v\!\!\!/ v\!\!\!/ H_v = -\overline{N_v}H_v = 0 \tag{3.10}$$

These three identities allow the Lagrangian to be rewritten in terms of v_μ,

$$L_{free} = \overline{N_v}(iv\cdot\partial)N_v - \overline{H_v}(iv\cdot\partial + 2M_N)H_v + \overline{N_v}(i\partial\!\!\!/)H_v + \overline{H_v}(i\partial\!\!\!/)N_v \tag{3.11}$$

The equation of motion of the field N_v is the Dirac equation for a massless particle at lowest order

$$v \cdot \partial N_v = 0 \tag{3.12}$$

and the propagator of N_v is then

$$S(v \cdot k) = \frac{i}{v \cdot k + i\epsilon} \tag{3.13}$$

where k is the four-momentum of the field N_v. If the nucleon is at rest, then the spacetime representation of the propagator is [3]

$$\tilde{S}(t, \bar{r}) = \Theta(t)\delta^{(3)}(\bar{r}) \tag{3.14}$$

which is consistent with an infinite mass static source.

3.2.2 General Expansion

In HBChPT the same method is used with the free Lagrangian replaced by the πN Lagrangian. The nucleon spinor is separated into two component fields, and then substituted into the πN Lagrangian. In this section the heavy baryon expansion will be applied to a general Lagrangian which is expanded in a power series

$$L = L^{(1)} + L^{(2)} + L^{(3)} + \ldots$$

while the method will be applied to the chiral πN Lagrangian in Section 3.3. Although this method is valid for any heavy baryon field, we will restrict the discussion to nucleon fields. The generalization to the baryon octet is not difficult, and requires only that the nucleon mass is replaced by an average baryon mass [24].

The explicit form of the expanded Lagrangian is often complicated, but it will take the form [21, 2]

$$L = \overline{N_v} A N_v + \overline{H_v} B N_v + \overline{N_v} \gamma^0 B^\dagger \gamma^0 H_v - \overline{H_v} C H_v \tag{3.15}$$

where A,B, and C are operators which are expressed as chiral expansions

$$A = A_{(1)} + A_{(2)} + A_{(3)} + ... \tag{3.16}$$

$$B = B_{(1)} + B_{(2)} + B_{(3)} + ... \tag{3.17}$$

$$C = C_{(1)} + C_{(2)} + C_{(3)} + ... \tag{3.18}$$

and the terms $A_{(i)}, B_{(i)}, C_{(i)}$ are calculated using the terms from $L^{(i)}$. The explicit form of these terms for the πN Lagrangian will be given in Section 3.3.

The terms in the Lagrangian in Eq 3.15 lead to four possible forms for the vertices. The $\overline{N_v} A N_v$ term produces a series of vertices in which light fields couple to the light nucleon field N_v. The $\overline{H_v} C H_v$ term represents the vertices in which light fields can interact with the heavy nucleon field H_v. The remaining two terms produce vertices in which a light field can be transformed into a heavy field, which allows diagrams with only light external lines to contain heavy propagators.

In low energy reactions it is not possible to produce a heavy field, and as such the terms which involve H_v can be removed from the Lagrangian. It is possible that heavy virtual particles will be produced in a reaction, and so these effects must be included in the Lagrangian. The decoupling theorem states that if the low energy theory is renormalizable, then all of the effects of the heavy fields are included as either a renormalization of the coupling constants or are suppressed by powers of the heavy mass [20]. In HBChPT the effects are included as correction terms which include powers of the nucleon mass.

In order to decouple the two fields, the Lagrangian must be rewritten as

$$\begin{aligned} L &= \overline{N_v}AN_v - (\overline{H_v} - \overline{N_v}\gamma^0 B^\dagger \gamma^0 C^{-1})C(H_v - C^{-1}BN_v) \\ &\quad + \overline{N_v}\gamma^0 B^\dagger \gamma^0 C^{-1} BN_v \\ &= \overline{N_v}(A + \gamma^0 B^\dagger \gamma^0 C^{-1}B)N_v + \overline{H_v'}CH_v' \end{aligned} \tag{3.19}$$

where the H_v' is a new heavy field defined by

$$H_v' = H_v - C^{-1}BN_v \tag{3.20}$$

The heavy and light fields have now been decoupled. As a result the heavy field can be dropped from the Lagrangian at low energies, since the only diagrams which contain the heavy field are those that have a heavy external line.

The decoupled form of the Lagrangian also contains a factor of C^{-1}, which cannot be calculated explicitly. In the heavy baryon Lagrangian, the operator C^{-1} is included as a power series expansion with powers of $1/M_N$. Each power of $1/M_N$ is added to the chiral dimension of the resulting vertex, so that an $O(p^n)$ term in the chiral Lagrangian will appear in the heavy baryon Lagrangian as separate terms at every order m $\geq$ n.

The final form of the heavy baryon Lagrangian is

$$L = \overline{N_v}(A + \gamma^0 B^\dagger \gamma^0 C^{-1} B) N_v \tag{3.21}$$

where C^{-1} is a power series expansion.

3.3 HBChPT Lagrangian

The HBChPT Lagrangian can now be derived using the pion-nucleon Lagrangian from Chapter 2 and the general form of the heavy baryon Lagrangian from Eq 3.21. The first step is to calculate the explicit form of the operators A,B and C from the πN Lagrangian with the nucleon wavefunction written in terms of H_v and N_v as in Eq 3.5.

The lowest order contribution to the HBChPT Lagrangian is $A_{(1)}$, which is given by [21]

$$A_{(1)} = iv \cdot D + g_A S \cdot u \tag{3.22}$$

where

$$S_\mu = \frac{i}{2}\gamma^5 \sigma_{\mu\nu} v^\nu \tag{3.23}$$

is the spin-vector or spin operator. The first order part of the Lagrangian is

$$L_{HB}^{(1)} = \overline{N_v}(iv \cdot D + g_A S \cdot u) N_v \tag{3.24}$$

The second order terms can be either from $A_{(2)}$ in which case they contain the low energy constants c_i, or from the expansion of $C_{(1)}$ in which they contain a factor of $1/M_N$. The terms from $A_{(2)}$ are [23, 21]

$$\begin{aligned} A_{(2)} =& c_1 < \chi^+ > + \frac{c_2}{2} < (v \cdot u)^2 > + \frac{c_3}{2} < u \cdot u > + \frac{c_4}{2}[S^\mu, S^\nu][u_\mu, u_\nu] \\ & + c_5 \tilde{\chi}^+ - \frac{ic_6}{4M_N}[S^\mu, S^\nu] F^+_{\mu\nu} - \frac{ic_7}{4M_N}[S^\mu, S^\nu] < F^+_{\mu\nu} > \end{aligned} \tag{3.25}$$

The remaining second order terms are given by the first term in the chiral expansion of B,

$$B_{(1)} = i(\not{D} - \not{v} v \cdot D) - \frac{g_A}{2}(v \cdot u)\gamma^5 \tag{3.26}$$

and the power series expansion of C^{-1},

$$C^{-1} = \frac{1}{iv \cdot D + 2M_N + g_A u \cdot S} = \frac{1}{2M_N} - \frac{iv \cdot D + g_A(u \cdot S)}{4M_N^2} + O(p^2) \tag{3.27}$$

The terms in the Lagrangian are given by

$$\overline{N_v}\gamma^0 B^\dagger_{(1)}\gamma^0 C^{-1} B_{(1)} N_v = \frac{1}{2M_N}\overline{N_v}((v \cdot D)^2 - D^2 - ig_A\{S \cdot D, v \cdot u\})N_v \tag{3.28}$$

The term $(v \cdot D)^2$ is proportional to the equation of motion, $v \cdot D$, and as such it can be removed by a transformation of the nucleon field [21]. The result of this transformation is to renormalize the low energy constants. Although we will not transform away the terms proportional to the equation of motion, the procedure is given in Section 3.3.1 for comparison with previous calculations.

The third order terms in the heavy baryon Lagrangian are calculated using the same method [21, 22]. The first set of terms are $\overline{N_v} A_{(3)} N_v$ where

$$\begin{aligned}
A_{(3)} =& id_1[u_\mu, [v \cdot D, u^\mu]] + id_2[u_\mu, [D^\mu, v \cdot u]] + id_3[v \cdot u, [v \cdot D, v \cdot u]] \\
&+ id_4 \epsilon^{\mu\nu\rho\lambda} v_\lambda < u_\mu u_\nu u_\rho > + d_5[\chi^-, v \cdot u] + d_6 v^\nu [D^\mu, \tilde{F}^+_{\mu\nu}] \\
&+ d_7 v^\nu [D^\mu, < F^+_{\mu\nu} >] + d_8 \epsilon^{\mu\nu\rho\lambda} v_\lambda < \tilde{F}^+_{\mu\nu} u_\rho > + d_9 \epsilon^{\mu\nu\rho\lambda} v_\lambda < F^+_{\mu\nu} > u_\rho \\
&+ d_{10} S \cdot u < u^2 > + d_{11} S^\mu u^\nu < u_\mu u_\nu > + d_{12} S \cdot u < (v \cdot u)^2 > \\
&+ d_{13} < S \cdot u v \cdot u > v \cdot u + id_{14}[S^\mu, S^\nu] < [v \cdot D, u_\mu] u_\nu > \\
&+ id_{15}[S^\mu, S^\nu] < u_\mu [D_\nu, v \cdot u] > + d_{16} S \cdot u < \chi^+ > + d_{17} < S \cdot u \chi^+ > \\
&+ id_{18}[S \cdot D, \chi^-] + d_{19}[S \cdot D, < \chi^- >] + id_{20} S^\mu v^\nu [\tilde{F}^+_{\mu\nu}, v \cdot u] \\
&+ id_{21} S^\mu [\tilde{F}^+_{\mu\nu}, u^\nu] + d_{22} S^\mu [D^\nu, F^-_{\mu\nu}] + d_{23} \epsilon^{\mu\nu\rho\lambda} S_\mu < u_\nu F^-_{\rho\lambda} >
\end{aligned} \tag{3.29}$$

The remainder of the term are derived from the expansion of C^{-1}

$$\frac{1}{2M_N}(\gamma^0 B^\dagger_{(2)}\gamma^0 B_{(1)} + \gamma^0 B^\dagger_{(1)}\gamma^0 B_{(2)}) - \frac{1}{4M_N^2}\gamma^0 B^\dagger_{(1)}\gamma^0(iv\cdot D + g_A S\cdot u)B_{(1)}$$

$$= \frac{g_A}{8M_N^2}[D^\mu,[D_\mu, S\cdot u]] - \frac{g_A^2}{32M_N^2}\epsilon^{\mu\nu\rho\lambda}v_\rho S_\lambda < F^-_{\mu\nu} v\cdot u > - \frac{i}{4M_N^2}(v\cdot D)^3$$

$$- \frac{g_A}{4M_N^2} v\cdot \overline{D} S\cdot uv\cdot D + \frac{1}{8M_N^2}(iD^2 v\cdot D) - \frac{g_A}{4M_N^2}(\{S\cdot D, v\cdot u\}v\cdot D)$$

$$+ \frac{3g_A^2}{64M_N^2}(i < (v\cdot u)^2 v\cdot D > + \frac{1}{32M_N^2}(\epsilon^{\mu\nu\rho\lambda}v_\rho S_\lambda[u_\mu, u_\nu]v\cdot D)$$

$$- \frac{1}{16M_N^2}(i\epsilon^{\mu\nu\rho\lambda}v_\rho S_\lambda \tilde{F}^+_{\mu\nu} v\cdot D) - \frac{g_A}{8M_N^2}(S\cdot uD^2) - \frac{g_A}{4M_N^2}(S\cdot \overline{D}u\cdot D)$$

$$- \frac{1+2c_6}{8M_N^2}(i\epsilon^{\mu\nu\rho\lambda}v_\rho S_\lambda \tilde{F}^+_{\mu\sigma} v^\sigma D_\nu) - \frac{c_7}{4M_N^2}(i\epsilon^{\mu\nu\rho\lambda}v_\rho S_\lambda < F^+_{\mu\sigma} > v^\sigma D_\nu)$$

$$+ \frac{1+g_A^2+8M_N c_4}{16M_N^2}(\epsilon^{\mu\nu\rho\lambda}v_\rho S_\lambda[u_\mu, v\cdot u]D_\nu) + \frac{g_A}{32M_N^2}(i\epsilon^{\mu\nu\rho\lambda}v_\rho F^-_{\mu\nu}D_\lambda)$$

$$- \frac{g_A^2}{16M_N^2}(iv\cdot uu\cdot D) + i\frac{1+8M_N c_4}{32M_N^2}[v\cdot u, [D^\mu, u_\mu]]$$

$$+ \frac{c_2}{2M_N}(i < v\cdot uu_\mu > D^\mu) \tag{3.30}$$

where $\overline{D}_\mu$ is the covariant derivative acting on $\overline{N}_v$. Several of the terms listed can be included as a renormalization of the low energy constants d_i, and the heavy baryon Lagrangian can be further reduced if the terms proportional to the equation of motion $v \cdot D$ are eliminated by a transformation of the nucleon field. These renormalized constants are listed in Section 3.3.1.

3.3.1 Reduction of the Lagrangian

The form of the HBChPT Lagrangian derived in Section 3.3 can be simplified by removing terms proportional to $v \cdot D$, and by defining a new set of low energy constants [21]. The redefined low energy constants are measured experimentally and will not complicate the Lagrangian. However it has been argued that the equation of motion terms should not be eliminated, and several amplitudes have been calculated without removing these terms. The method of removing the EOM terms will be presented in this section, although both forms of the HBChPT Lagrangian will be used.

The equation of motion terms in the Lagrangian are of the form

$$L_{EOM} = \overline{N_v}(X(iv \cdot D)^3 - v \cdot \overleftarrow{D} Y v \cdot D + Z iv \cdot D - iv \cdot \overleftarrow{D} Z^\dagger) N_v \tag{3.31}$$

where X,Y, and Z are combinations of pion fields and operators. In order to eliminate these terms, a new field N_v' is defined by

$$\begin{aligned} N_v = \quad & (1 - \tfrac{X}{2}(iv \cdot D)^2 + \tfrac{1}{2}(Y + g_A X S \cdot U) iv \cdot D + \tfrac{g_A}{2} X [iv \cdot D, S \cdot u] \\ & - \tfrac{g_A^2}{2} X (S \cdot u)^2 - \tfrac{g_A}{2} Y S \cdot u - Z^\dagger) N_v' \end{aligned} \tag{3.32}$$

When this transformation of the field is applied to the HBChPT Lagrangian the equation of motion terms are removed, but additional terms of the form

$$L_{new} = \overline{N}_v'(-g_A^3 X(S\cdot u)^3 + \tfrac{g_A^2}{2} X[S\cdot u,[iv\cdot D, S\cdot u]] \\ -g_A^2 S\cdot uYS\cdot u - g_A(ZS\cdot u + S\cdot uZ^\dagger))N_v' \tag{3.33}$$

are produced [21]. These new terms can be expressed as a linear combination of the terms in A, and as such the low energy constants can be redefined to include the field transformation.

The Lagrangian can be further simplified if the redefined low energy constants also include the contribution from the expansion of $\gamma^0 B^\dagger \gamma^0 C^{-1} B$. Then the second order HBChPT Lagrangian can be written as

$$L_{HB}^{(2)} = \overline{N}_v(-\tfrac{1}{2M_N}(D^2 + ig_A\{S\cdot D, v\cdot u\}) \\ +\tfrac{1}{M_N}(a_1 < u\cdot u > +a_2 < (v\cdot u)^2 > +a_3 < \chi^+ > +a_4\tilde{\chi}^+ \\ +\epsilon^{\mu\nu\rho\lambda} v_\rho S_\sigma(ia_5 u_\mu u_\nu + a_6 F_{\mu\nu}^+ + a_7 < F_{\mu\nu}^+ >)))N_v \tag{3.34}$$

where N_v is the transformed field and the new low energy constants a_i are defined by

$$a_1 = M_N\frac{c_3}{2} + \frac{g_A^2}{16} \quad a_2 = M_N\frac{c_2}{2} - \frac{g_A^2}{8} \quad a_3 = M_N c_3 \quad a_4 = M_N c_5$$

$$a_5 = M_N c_4 - \frac{1-g_A^2}{4} \quad a_6 = M_N c_6 + \frac{1}{4} \quad a_7 = M_N c_7 + \frac{1}{2} \quad .$$

The new form of the third order HBChPT Lagrangian is

$$
\begin{aligned}
L_{HB}^{(3)} =& \overline{N}_v(\frac{g_A}{8M_N^2}[D_\mu,[D^\mu,S\cdot u]] + \frac{1}{2M_N^2}((i(a_5 - \frac{1-3g_A^2}{8})u_\nu u_\mu \\
&+ (a_6 - \frac{1}{8})F_{\mu\nu}^+ + (a_7 - \frac{1}{4}) < F_{\mu\nu}^+ >)\epsilon^{\mu\nu\rho\sigma}S_\sigma iD_\rho \\
&+ \frac{g_A}{2}S\cdot Du\cdot D - \frac{g_A^2}{8}\{v\cdot u, u_\mu\}\epsilon^{\mu\nu\rho\sigma}v_\rho S_\sigma D_\nu \\
&- \frac{ig_A}{16}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^- v_\rho D_\sigma + \frac{1}{(4\pi F)^2}(ib_1[u_\mu,[v\cdot D,u^\mu]] \\
&+ ib_2[u_\mu,[D^\mu,v\cdot u]] + ib_3[v\cdot u,[v\cdot D,v\cdot u]] \\
&+ ib_4 < u_\mu v\cdot u > D^\mu + ib_5 v_\lambda\epsilon^{\lambda\mu\nu\rho} < u_\mu u_\nu u_\rho + b_6[\chi^-,v\cdot u] \\
&+ b_7[D^\mu,F_{\mu\nu}^+]v^\nu + b_8\partial^\mu < F_{\mu\nu}^+ > v^\nu + b_9\epsilon^{\mu\nu\rho\lambda} + b_{10} < F_{\mu\nu}^+ > u_\rho v_\sigma \\
&+ b_{11}S\cdot u < u\cdot u > + b_{12}u_\mu S_\nu < u^\mu u^\nu > + b_{13}S\cdot u < (v\cdot u)^2 > \\
&+ b_{14}v\cdot uS_\mu < u^\mu v\cdot u > + b_{15}\epsilon^{\mu\nu\rho\lambda}v_\rho S_\lambda < [v\cdot D,u_\mu]u_\nu > \\
&+ b_{16}\epsilon^{\mu\nu\rho\sigma}v_\rho S_\sigma < u_\mu[D_\nu,v\cdot u] > + b_{17}S\cdot u < \chi^+ > \\
&+ b_{18}S^\mu < u_\mu\chi^+ > + ib_{19}S^\mu[D_\mu,\chi^-] + ib_{20}S^\mu < \partial_\mu\chi^- > \\
&+ ib_{21}S^\mu v^\nu[F_{\mu\nu}^+,v\cdot u] + ib_{22}S^\mu[F_{\mu\nu}^+,u^\nu] \\
&+ b_{23}S^\mu[D^\mu,F_{\mu\nu}^-] + b_{24}\epsilon^{\mu\nu\rho\sigma}S_\mu < u_\nu F_{\rho\sigma}^- >))N_v
\end{aligned}
\tag{3.35}
$$

The low energy constants which will be required in the pion photoproduction calculation are related to the relativistic low energy constants,

$$\begin{aligned} b_{11} &= (4\pi F)^2 (d_{10} - \frac{g_A^3}{8M_N^2}) \\ b_i &= (4\pi F)^2 d_{i-1} \qquad i = 9, 10, 17, 18, 19, 20, 21, 22, 23 \end{aligned} \tag{3.36}$$

The fourth order HBChPT Lagrangian has been calculated [22], but as yet no calculations have been performed with the reduced form of the Lagrangian. As the redefined fourth order low energy constants will not be needed, the reduced form of the Lagrangian will not be listed.

Chapter 4

Infrared Regularization

4.1 Introduction

The method of infrared regularization [7, 8, 9] has been proposed as an alternative to heavy baryon chiral perturbation theory. Infrared regularization is based on separating each loop integral into two components. One component contains only poles from light particle propagators and is retained, while the remainder of the integral is absorbed into the low energy constants of the Lagrangian. The advantages of infrared regularization are that it does not require a new form of the Lagrangian, and that manifest Lorentz invariance is maintained. It also can be applied to reactions which contain multiple heavy particles. In this chapter, two forms of infrared regularization will be presented.

The first infrared regularization method [7], which will be presented in Section 4.2, is based on an expansion of all loop integral integrands. The loop momentum is assumed to be significantly smaller than the nucleon mass, so that the integrand can be expanded in powers of k/M_N. Once the expansion is performed, the lowest order terms in the series are integrated while the terms which are of higher order can be omitted. The remainder of the loop integral is removed by redefining the low energy constants. This method evaluates the part of the loop integral which corresponds to low loop momenta while removing the high energy component which violates the power counting scheme.

The second form of infrared regularization [9], which will be presented in Section 4.3, also separates the loop integrals into a low energy component and a high energy component. However the separation of the integral is based on classifying each singularity in Feynman parameter space as either an infrared singularity or a regular singularity. The integral over the regions of parameter space which contain infrared singularities is the infrared component of the loop integral. The remaining part of the integral is absorbed into the low energy constants. This method also has an advantage in that it can be applied to reactions with multiple heavy particles [10, 11].

4.2 Chiral Expansion Method

In this section the original method of infrared regularization [7, 8] proposed by H.B. Tang will be presented. Although alternate methods have since been developed which simplify the calculation of loop integrals and which provide more accurate results, the chiral expansion used in this method will be necessary to prove that infrared regularization and heavy baryon chiral perturbation theory provide consistent results.

The method of infrared regularization uses the πN Lagrangian derived in Section 2.4 and the corresponding Feynman graphs for a given reaction. The tree level diagrams will not violate the power counting scheme, and so the regularization scheme will not be applied to them. The loop diagrams will be modified, with the regularization scheme applied to each loop integral.

Let G represent a one loop Feynman diagram, with loop momentum q and let Q represent a low energy of the same order as the pion mass. Then the diagram can be separated into two components,

$$\begin{aligned} G &= \tilde{R}\tilde{S}G + (G - \tilde{R}\tilde{S}G) \\ &= I + R \end{aligned} \tag{4.1}$$

where $\tilde{S}$ is an operator that eliminates the high energy part of the loop and $\tilde{R}$ is an operator which renormalizes the diagram. The two components are often referred to as the soft part or infrared part I, and the regular part R. The unrenormalized soft part of the diagram is calculated by assuming that the loop momentum is of order Q, and then expanding the integrand in powers of Q/M_N. The individual terms of the resulting series are then integrated to produce $\tilde{S}G$. The infrared component of the diagram is derived from $\tilde{S}G$ by standard method of renormalization.

As an example, consider the self energy diagram in Figure 4.1. The loop integral in this diagram is

$$G = -\frac{3g_A^2}{4F^2}\int \frac{d^4q}{(2\pi)^4}\frac{\not{q}\gamma^5(\not{P}+\not{q}+M_N)\not{q}\gamma^5}{(q^2-m_\pi^2+i\epsilon)((P+q)^2-M_N^2+i\epsilon)} \tag{4.2}$$

where P^μ is the nucleon four-momentum and q^μ is the loop momentum. It is assumed that the loop momentum is of the same order as the pion mass, and the integrand is expanded in powers of q/M_N,

$$\frac{\not q\gamma^5(\not P+\not q+M_N)\not q\gamma^5}{(q^2-m_\pi^2+i\epsilon)((P+q)^2-M_N^2+i\epsilon)} \tag{4.3}$$
$$=\frac{1}{q^2-m_\pi^2+i\epsilon}\left(\frac{\not q\gamma^5(\not P+\not q+M_N)\not q\gamma^5}{2P\cdot q+P^2-M_N^2+i\epsilon}-\frac{q^2\not q\gamma^5(\not P+\not q+M_N)\not q\gamma^5}{(2P\cdot q+P^2-M_N^2+i\epsilon)^2}+...\right)$$

The series is then integrated term by term, with each term containing only low energy poles. The high energy poles which resulted from the nucleon propagator are removed when the order of integration and summation is reversed.

$$\tilde{S}G=\frac{3g_A^2}{4F^2}\int\frac{d^4q}{(2\pi)^4}\frac{2P\cdot q\not q-(\not P+M_N)q^2}{(q^2-m_\pi^2+i\epsilon)(2P\cdot q+P^2-M_N^2+i\epsilon)}+... \tag{4.4}$$

For this example, the pole at $q^2=m_\pi^2$ allows each factor of q^2 in the series to be replaced by m_π^2. This allows the self energy loop integral to be written as

$$\tilde{S}G=\frac{3g_A^2}{4F^2}\int\frac{d^4q}{(2\pi)^4}\frac{(\not P+M_N)m_\pi^2-(2P\cdot q+m_\pi^2)\not q}{(q^2-m_\pi^2+i\epsilon)(2P\cdot q+P^2-M_N^2+m_\pi^2+i\epsilon)} \tag{4.5}$$

which can then be evaluated using dimensional regularization.

In order to prove that the operation $\tilde{S}$ extracts the soft part of each diagram, consider the integral over the time component of the loop momentum q. The contour is closed with a semicircle at infinity, and the contributions to the integral are due to the integral over the semicircle, the low energy or soft poles of order Q, and the high energy or hard poles of order M_N. It must be assumed that the poles can be separated into soft poles and hard

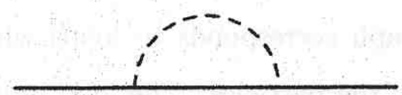

Figure 4.1: Nucleon Self-Energy Loop

poles, which is valid for theories containing only Goldstone bosons and massive baryons. For other fields this separation may not be valid, in which case infrared regularization cannot be used.

The standard renormalization methods of quantum field theory can be used to remove any divergent terms produced by the semicircle, and the remainder of the terms produced by the semicircle can be included in the infrared component or absorbed into the low energy constants. The soft poles in the loop integral are not affected by the operation $\tilde{S}$ since they cannot be expressed as a Q/M_N expansion. The hard poles can be represented as a Q/M_N expansion since we have assumed that the loop momentum is of order Q and that the hard poles are of order M_N.

The resulting series expansion of the integrand will contain only soft poles, since hard poles do not appear in the terms in the expansion. When each term is integrated, only the soft poles will contribute, and so the sum of the integrated terms will contain only contributions from the soft poles. Since the original soft poles were not affected by the operator $\tilde{S}$, and since the hard poles are removed by exchanging the order of summation and integration, $\tilde{S}G$ is the soft part of the graph G.

Since $\tilde{S}G$ contains all of the soft poles, it follows that the regular part of the graph, $(G - \tilde{R}\tilde{S}G)$ must contain only hard poles. The hard poles correspond to large momenta and small interaction distances, and so the regular part of the graph corresponds to loops which are small compared with the wavelength of the particles. The loop integral must also obey the same symmetry properties as the chiral Lagrangian. Therefore the regular part of the graph can be included as a term in a modified form of the chiral Lagrangian,

$$L'_{\pi N} = L_{\pi N} + \delta L_{\pi N}$$

where $L_{\pi N}$ is πN Lagrangian derived in Section 2.4, and $\delta L_{\pi N}$ is the contribution from the regular part of the graph. Since the chiral Lagrangian contains all terms which are invariant under the required symmetry transformations, it follows that $\delta L_{\pi N}$ can be written as a combination of terms in the Lagrangian

$$\delta L_{\pi N} = \sum \delta c_i L_i$$

where L_i are the terms in the chiral Lagrangian, and that the regular part of the graph can be absorbed into the low energy constants through a transformation

$$c_i \rightarrow c_i - \delta c_i$$

Although the low energy constants have been transformed to absorb the regular part of the loop integral, the modification cannot be detected experimentally. A reaction which includes a diagram containing a specific vertex from the πN Lagrangian must also include all possible diagrams that contain a one loop corrections to the vertex. As a result the original low energy constants cannot be measured independent of the contribution from the high energy loop integrals.

The proof that infrared regularization produces the same result as HBChPT has been completed for several one loop and two loop diagrams [8, 27, 28], but a general proof has not been attempted as it would require complicated diagrams with an arbitrary number of loops. As an example we will consider the nucleon self-energy diagram shown in Figure 4.1. The loop integral for this diagram was given in Eq 4.2, with the soft part of the integral given in Eq 4.4.

The first step in comparing the two methods is to express the soft part of the loop integral in terms of the non-relativistic momentum of the nucleon and to replace the nucleon field with the light components of the nucleon field. In HBChPT the momentum of the nucleon is written as

$$P_\mu = M_N v_\mu + l_\mu$$

while the light components of the nucleon field are extracted using the projection operator $\frac{1}{2}(1 + \not{v})$. The light components of the nucleon field are extracted by multiplying $\tilde{S}G$ by the projection operators

$$\tilde{S}G \to \frac{1+\slashed{v}}{2}\tilde{S}G\frac{1+\slashed{v}}{2}$$

Then using the relation

$$\left(\frac{1+\slashed{v}}{2}\right)\slashed{q}\gamma^5\left(\frac{1+\slashed{v}}{2}\right) = 2q\cdot S\frac{1+\slashed{v}}{2} \tag{4.6}$$

the soft part of the integral can be written as

$$\frac{1+\slashed{v}}{2}\tilde{S}G\frac{1+\slashed{v}}{2} = \frac{1+\slashed{v}}{2}\frac{3g_A^2}{F^2}\int\frac{d^4q}{(2\pi)^4}\frac{(S\cdot q)^2}{(q^2-m_\pi^2+i\epsilon)(v\cdot l - v\cdot q+i\epsilon)}+\ldots \tag{4.7}$$

which is identical to the loop integral obtained from HBChPT.

4.3 Infrared Regularization

In this section an alternate form of infrared regularization [9] will be presented. The loop integrals will again be separated into an infrared component and a regular component, with the regular component absorbed into the low energy constants. However the separation is performed using Feynman parameters and does not require a chiral expansion. As a result infrared regularization can be applied to diagrams in which the chiral expansion does not converge and to diagrams which are too complicated to calculate using the heavy baryon methods.

4.3.1 Nucleon Self-Energy

To introduce the method of infrared regularization, we will first apply it to the nucleon self energy diagram in Figure 4.1. The loop integral in d dimensions is given by

$$H = -i \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m_\pi^2 + i\epsilon)((P-k)^2 - M_N^2 + i\epsilon)} \tag{4.8}$$

and contains a soft pole at $k^2 = m_\pi^2$ and a hard pole at $(P-k)^2 = M_N^2$. As in Section 4.2 the loop integral is divided into an infrared component which contains the soft pole and a regular component which contains the hard pole.

$$H = I + R \tag{4.9}$$

However in this method the separation of the two components is based on isolating the infrared singularities using the Feynman parameterization.

In order to evaluate a loop integral which contains multiple propagators, the denominators of the propagators are combined into a single polynomial using the Feynman parameterization. For two propagators a single Feynman parameter is used,

$$\frac{1}{ab} = \int_0^1 \frac{dz}{((1-z)a + zb)^2} \tag{4.10}$$

which leads to two terms

$$\frac{1}{ab} = \frac{1}{a(b-a)} - \frac{1}{b(b-a)} \tag{4.11}$$

If 1/(b-a) does not contain a pole, then this form of the integrand separates the two poles in the integrand. It will be shown that if 1/a is the pion propagator and 1/b is the nucleon propagator then the first term is the infrared component and the second term is the regular component of the loop integral.

The nucleon self energy loop integral can be written as

$$H = -i \int_0^1 dz \int \frac{d^d k}{(2\pi)^d} \frac{1}{((1-z)(k^2 - m_\pi^2) + z((P-k)^2 - M_N^2) + i\epsilon)^2} \tag{4.12}$$

and the integral over the loop momentum can be evaluated using dimensional regularization,

$$H = \kappa \int_0^1 dz C^{\frac{d}{2}-2} \qquad \kappa = (4\pi)^{-\frac{d}{2}} M_N^{d-4} \Gamma(2 - \frac{d}{2}) \tag{4.13}$$

$$C = z - (\frac{P}{M_N})^2 z(1-z) + (\frac{m_\pi}{M_N})^2 (1-z) - i\epsilon$$

The infrared singularity is located at z = 0, where C = $(m_\pi/M_N)^2$ vanishes in the chiral limit. Since the integrand vanishes as $z \to 0$ in the chiral limit of $m_\pi = 0$, the infrared singular part of the integral can be extracted by extending the upper limit to ∞

$$I = \kappa \int_0^\infty dz C^{\frac{d}{2}-2} \tag{4.14}$$

and the regular part is

$$R = H - I = -\kappa \int_1^{\infty} dz C^{\frac{d}{2}-2} \tag{4.15}$$

As in Section 4.2 the regular part of the integral can be absorbed into the low energy constants.

When the dimension d is not an integer, this separation of the loop integral can also be expressed as a separation of the terms in the chiral expansion of the integrand as in Section 4.2. In d-dimensional spherical coordinates, with $d \approx 4$, the factor of d^dk can be written as

$$d^dk = k^{d-1}\sin^{d-2}(\theta_1)\sin^{d-3}(\theta_2)dkd\theta_1 d\theta_2 d\phi \tag{4.16}$$

and as such the location of the poles in the integrand will appear in the integral with a non-integer power. In the infrared component the poles are at small values of the loop momentum and the expansion of $C^{\frac{d}{2}-2}$ produces terms with fractional powers,

$$I = O(p^{d-3}) + O(p^{d-2}) + O(p^{d-1}) + ... \tag{4.17}$$

while the expansion of the regular part of the integral, which contains large values of the loop momentum, is a Taylor series

$$R = O(p^0) + O(p^1) + O(p^2) + ... \tag{4.18}$$

The fractional powers in the expansion also indicate that only soft poles are contained in the infrared component of the integral, while the absence of fractional powers in the regular part indicate that it contains no soft poles.

4.3.2 General Infrared Regularization

The method of infrared regularization can now be expanded to the most general form of the loop integral,

$$H_{mn}^{\mu_1 \ldots \mu_r} = -i \int \frac{d^d k}{(2\pi)^d} \frac{k^{\mu_1} \ldots k^{\mu_r}}{a_1 \ldots a_m b_1 \ldots b_n} \tag{4.19}$$

where a_i and b_i represent the pion and nucleon propagators

$$a_i = (k - q_i)^2 - m_\pi^2 + i\epsilon \tag{4.20}$$

$$b_i = (P_i - k_i)^2 - M_N^2 + i\epsilon \tag{4.21}$$

and where q_i and P_i represent the pion and nucleon momenta. The numerator is generated by the derivative couplings in the vertices and by the numerators of the nucleon propagators. However Lorentz invariance allows the integrals to be written as tensor polynomials in which the factors of the loop momentum in the numerator are replaced with factors of the external momenta. As a result each loop integral can be written as a combination of scalar integrals H_{mn}.

If the loop contains only pion propagators, then the loop integral contains only soft poles and the separation is trivial

$$I_{m0} = H_{m0} \qquad R_{m0} = 0 \tag{4.22}$$

Similiarly if the loop contains only nucleon propagators, the loop integral does not contain soft poles.

$$I_{0n} = 0 \qquad R_{0n} = H_{0n} \tag{4.23}$$

If the loop contains both pion and nucleon propagators, then the loop integral will contain both an infrared part and a regular part.

The general form of the loop integral is separated into two components by first combining the pion propagators into a single polynomial A, and combining all of the nucleon propagators into a second polynomial B. The pion propagators are combined using the formula

$$\frac{1}{a_1 ... a_m} = (\frac{\partial}{\partial m_\pi^2})^{(m-1)} \int_0^1 dx_1 ... \int_0^1 dx_{m-1} \frac{X}{A} \tag{4.24}$$

where the numerator is given by

$$X = x_2 (x_3)^2 ... (x_{m-1})^{m-2} \tag{4.25}$$

and the denominator is defined by the recursive relation

$$\begin{gathered} A_{p+1} = x_p A_p + (1 - x_p) A_{p-1} \\ A = A_m \qquad A_1 = a_1 \end{gathered} \tag{4.26}$$

The nucleon propagators are combined using a similiar formula

$$\frac{1}{b_1 ... b_n} = (\frac{\partial}{\partial M_N^2})^{n-1} \int_0^1 dy_1 ... \int_0^1 y_{n-1} \frac{Y}{B} \tag{4.27}$$

where

$$Y = y_2(y_3)^2...(y_{n-1})^{n-2} \tag{4.28}$$

and

$$\begin{gathered} B_{p+1} = y_p B_p + (1 - y_p) B_{p-1} \\ B = B_n \qquad B_1 = b_1 \end{gathered} \tag{4.29}$$

The loop integral can then be written as

$$H_{mn} = (\frac{\partial}{\partial m_\pi^2})^{m-1} (\frac{\partial}{\partial M_N^2})^{n-1} \int_0^1 dx dy XY (-i \int \frac{d^d k}{(2\pi)^d} \frac{1}{AB}) \tag{4.30}$$

where the integral over the m+n-2 Feynman parameters is represented by $\int_0^1 dxdy$.

The form of the loop integral in Eq 4.30 is the same as the nucleon self energy loop given in Eq 4.8 with A replacing the pion propagator and B replacing the nucleon propagator. Following the method used to derive the infrared component of the self energy integral, the infrared part of the general loop integral is derived by using a Feynman parameter to combine the two factors in the denominator. The upper limit of integration is extended to ∞ to isolate the infrared singularities, and the infrared component is given by

$$I_{mn} = (\frac{\partial}{\partial m_\pi^2})^{m-1} (\frac{\partial}{\partial M_N^2})^{n-1} \int_0^1 dx dy XYI \tag{4.31}$$

where

$$I = -i\int_0^\infty dz \int \frac{d^dk}{(2\pi)^d}\frac{1}{((1-z)A+zB)^2} = -i\int \frac{d^dk}{(2\pi)^d}\frac{1}{A(B-A)} \qquad (4.32)$$

The regular part of the loop integral is given by

$$R_{mn} = (\frac{\partial}{\partial m_\pi^2})^{m-1}(\frac{\partial}{\partial M_N^2})^{n-1}\int_0^1 dxdyXYR \qquad (4.33)$$

where

$$R = i\int_1^\infty dz \int \frac{d^dk}{(2\pi)^d}\frac{1}{((1-z)A+zB)^2} = -i\int \frac{d^dk}{(2\pi)^d}\frac{1}{B(B-A)} \qquad (4.34)$$

However as in Section 4.2 the regular part of the loop integral can be absorbed into the low energy constants and is not required in the amplitude calculations.

The combined propagators defined in Eq 4.26 and Eq 4.29 can be written as quadratic polynomials in k,

$$A = (k-\overline{q})^2 - \overline{A} + i\epsilon \qquad (4.35)$$

and

$$B = (\overline{P}-k)^2 - \overline{B} + i\epsilon \qquad (4.36)$$

where $\overline{q}$ and $\overline{P}$ are linear combinations of the momenta q_i and P_i respectively, and $\overline{A}$ and $\overline{B}$ are constant terms. Then the integral in Eq 4.32 gives the same result as the infrared part of the nucleon self energy loop integral in Eq 4.14 if P,M_N and m_π are transformed as

$$P \to \overline{P} - \overline{q} \qquad m_\pi^2 \to \overline{A} \qquad M_N^2 \to \overline{B} \tag{4.37}$$

Therefore the general form of the infrared integral is

$$I_{mn} = \frac{\Gamma(m+n-d/2)}{(4\pi)^{d/2}} \int_0^1 dxdy \int_0^\infty dz(1-z)^{m-1} z^{n-1} C^{d/2-m-n} \tag{4.38}$$

$$C = (1-z)\overline{A} + z\overline{B} - z(1-z)(\overline{P} - \overline{q})^2 - i\epsilon$$

The explicit form of the infrared components for the common loop integrals are listed in Appendix A.

Chapter 5

Form Factors

To simplify the pion photoproduction amplitude calculation in Chapter 6, we will first use the method of infrared regularization to calculate the form factors for each of the three nucleon vertices as well as the overall renormalization constant. The pion electromagnetic form factor and renormalization constant, which were calculated in [13, 29], will also be required but as they do not include nucleons the method of infrared regularization leaves them unchanged.

By first calculating the form factors, the amplitude calculation in Chapter 6 will require only four tree diagrams, with each vertex replaced with the corresponding form factors. The remainder of the diagrams which include higher order vertices and loops are included in the calculation of the form factors. The renormalization constant is required to compensate for the effects of loops in the nucleon propagator and external nucleon lines.

5.1 Nucleon Self Energy

5.1.1 The Nucleon Propagator

The nucleon propagator which has been used in previous chapters represents the amplitude for a free nucleon to propagate from one point in spacetime to another. However it does not include the effects from the self interactions of the nucleon, such as the possibility that the nucleon emits a pion and then re-absorbs it. Such particle loops will result in a shift in the physical mass of the nucleon, and will require the external nucleon fields to be renormalized.

Let G(P) represent the free nucleon propagator,

$$G(P) = \frac{i}{\not{P} - M_0 + i\epsilon} \tag{5.1}$$

where M_0 is the bare nucleon mass, and let - i $\Sigma(P)$ represent the sum of all one particle irreducible diagrams. Then the full propagator can be written as

$$\begin{aligned}
\Gamma(P) &= G(P) + G(P)(-i\Sigma(P))G(P) + G(P)(-i\Sigma(P))G(P)(-i\Sigma(P))G(P) + ... \\
&= G(P) + G(P)(-i\Sigma(P))(G(P) + G(P)(-i\Sigma(P))G(P) + ...) \\
&= G(P) + G(P)(-i\Sigma(P))\Gamma(P)
\end{aligned} \tag{5.2}$$

The full nucleon propagator can then be written in terms of the free nucleon propagator,

$$\Gamma(P) = \frac{G(P)}{1 - G(P)(-i\Sigma(P))} = \frac{i}{\not{P} - M_0 - \Sigma(P) + i\epsilon} \tag{5.3}$$

The propagator still has a simple pole, but the position of the pole is shifted away from $P^2 = M_0^2$.

The physical mass of the nucleon is given by the pole in the full propagator. The mass of the nucleon is given by $\not{P}_0 = M_N$ where P_0 satisfies

$$\not{P}_0 - M_0 - \Sigma(P_0) = 0 \tag{5.4}$$

When the nucleon momentum is close to the pole, the nucleon propagator is of the form

$$\Gamma(P) = \frac{iZ(\not{P} + M_N)}{P^2 - M_N^2 + i\epsilon} \tag{5.5}$$

where the renormalization constant is

$$Z^{-1} = 1 - \frac{\partial \Sigma}{\partial \not{P}}(P_0) \tag{5.6}$$

The external nucleon fields must also be renormalized, which results in a factor of $\sqrt{Z}$ in the amplitude for each external nucleon field.

5.1.2 O(p^4) Calculation

In this section the nucleon self energy $\Sigma(P)$ will be calculated to fourth order using the method of infrared regularization. This same calculation was performed in [9] in which $\Sigma(P)$ was shown to be equivalent to the results obtained with HBChPT, and the nucleon renormalization constant Z_N was calculated in [30] to renormalize the nucleons in the nucleon electromagnetic form factor.

The five Feynman diagrams which must be included are given in Figure 5.1, with the standard notation of a solid line for a nucleon propagator, a dashed line for a pion propagator, and the solid dot, square and diamond denoting vertices of order p^2, p^3 and p^4. The number of diagrams can be reduced by absorbing the second order insertion in diagrams 5.1(a) and 5.1(c) into the bare nucleon mass [30], however we will include these terms in the calculation.

The leading order term in $\Sigma(P)$ is given by the second order diagram 5.1(a) and involves a single second order vertex. This vertex arises from the $\overline{\Psi} c_1 < \chi^+ > \Psi$ term in $L^{(2)}_{\pi N}$. The leading term in the expansion of χ^+ is

$$\begin{aligned} < \chi^+ > &= < \chi + \chi^\dagger > \\ &= 4B_0(m_u + m_d) \\ &= 4m_\pi^2 \end{aligned} \tag{5.7}$$

and so the contribution to $(-i\Sigma(P))$ is $4\,\mathrm{i}\,c_1 m_\pi^2$, and the leading term in the self energy is

$$\Sigma_a = -4c_1 m_\pi^2 \tag{5.8}$$

The loop diagram 5.1(b) is the only third order contribution to $\Sigma(P)$, as the first order vertex with two pions vanishes by symmetry. The vertices are

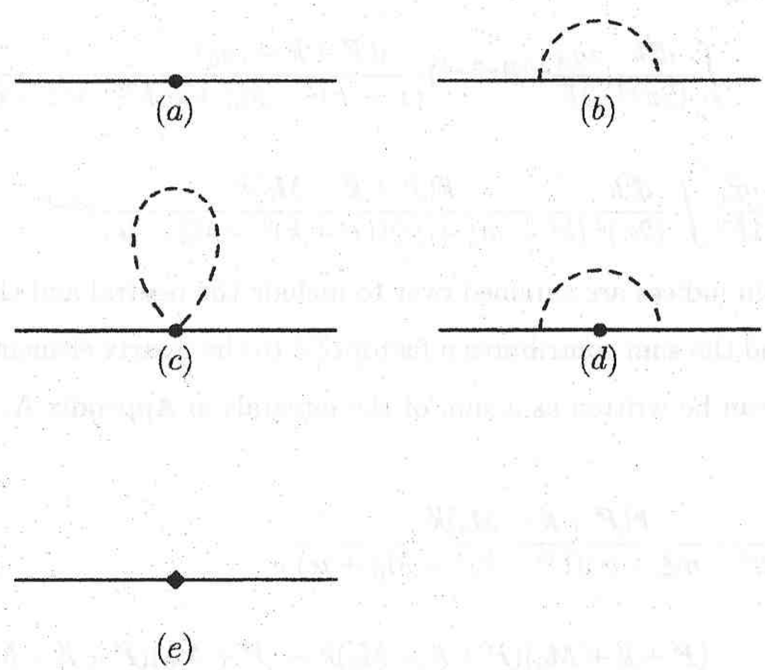

Figure 5.1: Nucleon Self-Energy Diagrams

given by the first order term

$$\bar{\Psi}\frac{g_A}{2}\gamma^\mu\gamma^5 u_\mu \Psi = \bar{\Psi}\frac{-g_A}{2F}\gamma^\mu\gamma^5\partial_\mu\pi^a\tau^a\Psi \tag{5.9}$$

where a is the pion isospin index. Then using the commutation relations for γ^5, the contribution to the matrix element from each pion-nucleon vertex is $\frac{ig_A}{2F}\gamma^5\not{\partial}\pi^a\tau^a$. The total matrix element is

$$-i\Sigma_b = -i\int\frac{d^dk}{(2\pi)^d}\left(\frac{ig_A}{2F}\gamma^5\not{\partial}\pi^a\tau^a\right)\frac{i(\not{P}+\not{k}+M_0)}{(P+k)^2-M_0^2+i\epsilon}\frac{i}{k^2-m_\pi^2+i\epsilon}\left(\frac{ig_A}{2F}\gamma^5\not{\partial}\pi^b\tau^b\right)$$

$$= \frac{ig_A^2}{4F^2}\int\frac{d^dk}{(2\pi)^d}\frac{\not{k}(\not{P}+\not{k}-M_0)\not{k}}{(k^2-m_\pi^2+i\epsilon)((P+k)^2-M_0^2+i\epsilon)}\tau^a\tau^b$$

The isopin indices are summed over to include the neutral and charged pion loops, and the sum contributes a factor of 3 to the matrix element. The loop integral can be written as a sum of the integrals in Appendix A,

$$\frac{\not{k}(\not{P}+\not{k}-M_0)\not{k}}{(k^2-m_\pi^2+i\epsilon)((P+k)^2-M_0^2+i\epsilon)}$$

$$= \frac{(\not{P}+\not{k}+M_0)(\not{P}+\not{k}-M_0)\not{k}-(\not{P}+M_0)(\not{P}+\not{k}-M_0)\not{k}}{(k^2-m_\pi^2+i\epsilon)((P+k)^2-M_0^2+i\epsilon)}$$

$$= \gamma_\mu\Delta_\pi^\mu - (\not{P}+M_0)m_\pi^2 I(P) + (P^2-M_0^2)\not{P}I^{(1)}(P)$$

Then the third order contribution to $\Sigma(P)$ is

$$\Sigma_b = \frac{3g_A^2}{4F^2}((\not{P}+M_0)m_\pi^2 I(P) + (M_0^2-P^2)\not{P}I^{(1)}(P)) \tag{5.10}$$

The remaining three diagrams are fourth order contributions to the self energy. The contribution from diagram (c) is given by the three terms in the Lagrangian which contain two pion fields, $c_1\overline{\Psi} < \chi^+ > \Psi$, $-\frac{c_2}{4M_N^2} < u_\mu u_\nu > \overline{\Psi} D^\mu D^\nu \Psi$, and $\frac{c_3}{2} < u_\mu u^\mu > \overline{\Psi}\Psi$. The second nonvanishing term in the expansion of $< \chi^+ >$ is

$$< \chi^+ >=< \chi U^\dagger + \chi^\dagger U >= -\frac{2m_\pi^2}{F^2}\tau^a\pi^a\tau^b\pi^b \tag{5.11}$$

which leads to

$$-i\Sigma_{c_1} = -\frac{6ic_1m_\pi^2}{F^2}\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2 - m_\pi^2 + i\epsilon} \tag{5.12}$$

and

$$\Sigma_{c_1} = \frac{6c_1m_\pi^2}{F^2}\Delta_\pi$$

The other two terms can be calculated in the same manner, with the final result for Σ_c

$$\Sigma_c = \frac{3m_\pi^2}{F^2}\Delta_\pi\{2c_1 - \frac{P^2}{M_0^2 d}c_2 - c_3\} \tag{5.13}$$

The contribution from diagram 5.1(d) is

$$-i\Sigma_d = -\frac{3ic_1g_A^2m_\pi^2}{F^2}\int \frac{d^dk}{(2\pi)^d}\frac{\not{k}(\not{P} + \not{k} - M_0)^2\not{k}}{(k^2 - m_\pi^2 + i\epsilon)((P+k)^2 - M_0^2 + i\epsilon)^2} \tag{5.14}$$

As the loop integral in Σ_d can be written as the derivative of the loop integral in Σ_b with respect to M_0, it follows that Σ_d can be written as

$$\Sigma_d = -4c_1 m_\pi^2 \frac{\partial \Sigma_b}{\partial M_0} \tag{5.15}$$

The fourth order insertion in diagram 5.1(e) is calculated using the term $-\frac{e_1}{16} < \chi^+ >^2 \overline{\Psi}\Psi$ from $L^{(4)}_{\pi N}$. The expansion of $< \chi^+ >$ is given in Eq 5.7, which leads to

$$\Sigma_e = e_1 m_\pi^4 \tag{5.16}$$

The total self energy $\Sigma(P)$ at $O(p^4)$ is the sum of the terms calculated

$$\Sigma(P) = -4c_1 m_\pi^2 + \Sigma_b + \Sigma_c + \Sigma_d + e_1 m_\pi^2 \tag{5.17}$$

5.1.3 Nucleon Mass Shift

The self energy of the nucleon can be used to calculate the physical mass of the nucleon. Letting P_0 be a solution of

$$P_0 - M_0 - \Sigma(P_0) = 0 \tag{5.18}$$

and defining the physical mass by

$$M_N = M_0 + \Sigma(P_0) \tag{5.19}$$

it follows that

$$\begin{aligned} M_N - M_0 =& \Sigma(P_0) = -4c_1 m_\pi^2 + \frac{3g_A^2 M_0 m_\pi^2}{2F^2} I \\ &+ \frac{3m_\pi^2}{F^2}\Delta_\pi(2c_1 - \frac{c_2}{4} - c_3) - \frac{6c_1 g_A^2 m_\pi^4}{F^2}(I + M_0\frac{\partial I}{\partial M_0}) \end{aligned} \tag{5.20}$$

The infrared integrals can be expanded in powers of the bare mass [9], which gives the nucleon mass shift as

$$M_N - M_0 = \Sigma(P_0) = -4c_1 m_\pi^2 + \frac{3g_A^2 m_\pi^3}{32\pi F^2} + k_1 m_\pi^2 \ln\frac{m_\pi}{M_0} + k_2 m_\pi^4 \tag{5.21}$$

where

$$k_1 = -\frac{3}{32\pi^2 F^2 M_0}(g_A^2 - 8c_1 M_0 + c_2 M_0 + 4c_3 M_0) \tag{5.22}$$

$$k_2 = \overline{e_1} - \frac{3}{128\pi^2 F^2 M_0}(2g_A^2 - c_2 M_0) \tag{5.23}$$

and where $\overline{e_1}$ is the renormalized coupling constant e_1 given by

$$\overline{e_1} = e_1 - \frac{3M_N^{d-5}}{32\pi^2 F^2}(\frac{1}{d-4} - \frac{1}{2}(\log 4\pi + \Gamma'(1) + 1))(g_A^2 - 8c_1 M_N + c_2 M_N + 4c_3 M_N) \tag{5.24}$$

5.1.4 Renormalization

The renormalization constant, defined by

$$Z_N^{-1} = 1 - \frac{\partial \Sigma}{\partial \not{P}}\Big|_{\not{P}=M_N} \tag{5.25}$$

is used to renormalize the nucleon propagator and the external nucleon fields. Using the $O(p^4)$ nucleon self energy given in Eq 5.17, the renomalization constant is [9]

$$\begin{aligned} Z_N =& 1 + \frac{3g_A^2}{4F^2}\{m_\pi^2 I(M_N^2) - 2M_N^2 I^{(1)}(M_N^2) \\ &+ 4M_N^2 m_\pi^2 (I_{12}(0) - 2I_{12}^{(1)}(0))\} - c_2 \frac{3m_\pi^2}{2M_N F^2}(\Delta_\pi - \frac{m_\pi^2}{32\pi^2}) \end{aligned} \tag{5.26}$$

5.2 Nucleon Electromagnetic Form Factors

5.2.1 The Nucleon Electromagnetic Vertex Function

The electromagnetic interactions of the nucleons can be written in terms of two form factors for each of the nucleons[31, 30]. In this section the calculation of each of the fourth order form factors using the method of infrared regularization, originally derived in [30], will be presented.

According to quantum electrodynamics, the form of the nucleon electromagnetic vertex must be

$$V_{\gamma NN} = ie\overline{\Psi}\Gamma^\mu \Psi A_\mu \tag{5.27}$$

where e is the electric charge of the proton, A_μ represents the photon field, and Γ^μ is a combination of Dirac matrices. At leading order the vertex is derived from the covariant derivative in the Lagrangian

$$L = \overline{\Psi}(i\not{D} - M_N)\Psi = \overline{\Psi}(i\not{\partial} - M_N)\Psi + e\overline{\Psi}\gamma^\mu \Psi A_\mu \tag{5.28}$$

and so the leading order term in Γ^μ is γ^μ. In general Γ^μ will also depend on the momenta of the nucleon fields $\overline{\Psi}$ and Ψ, denoted by p^μ and p'^μ, as well as the nucleon mass, the electric charge, and the low energy constants. Since the electromagnetic interaction is assumed to be parity conserving, Γ^μ will not contain factors of γ^5 or $\epsilon^{\mu\nu\rho\lambda}$ [31].

Lorentz invariance requires Γ^μ to transform as a vector, and so it must be a linear combination of vectors. The nucleon momenta are combined for convenience, and the general form of Γ^μ is

$$\Gamma^\mu = \gamma^\mu A + (p^\mu + p'^\mu)B + (p'^\mu - p^\mu)C \tag{5.29}$$

The coefficients A,B and C could involve factors of $\not{p}$ or $\not{p}'$, however since $\not{p}\Psi = M_N\Psi$ and $\overline{\Psi}\not{p}' = \overline{\Psi}M_N$ these factors can be removed. Γ^μ can be further simplified using the Ward identity,

$$q_\mu\Gamma^\mu = (p_\mu - p'_\mu)\Gamma^\mu = 0 \tag{5.30}$$

The first two terms in Γ^μ vanish, but the third does not and as a result C=0.

By convention the coefficients A and B are not used as the form factors. Instead the Gordon identity

$$\overline{\Psi}\gamma^\mu\Psi = \overline{\Psi}\{\frac{p^\mu + p'^\mu}{2M_N} + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\}\Psi \tag{5.31}$$

is used to rewrite Γ^μ in the form

$$\Gamma^\mu = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}F_2(q^2) \tag{5.32}$$

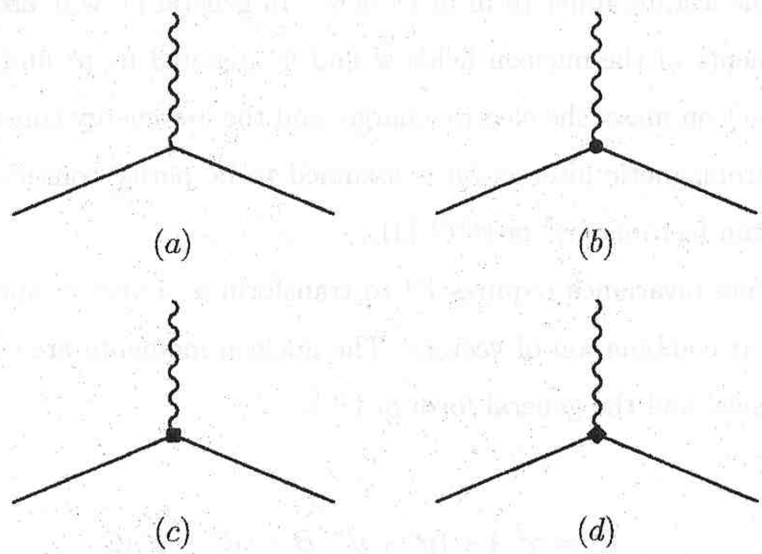

Figure 5.2: Nucleon Electromagnetic Form Factor Tree Diagrams

where $F_1(q^2)$ and $F_2(q^2)$ are the nucleon electromagnetic form factors.

5.2.2 Tree Level Calculations

The first step in calculating $F_1(q^2)$ and $F_2(q^2)$ is to calculate the contribution of the electromagnetic interactions which are included as terms in the chiral Lagrangian. The corresponding tree diagrams are given in Figure 5.2. The leading order diagram corresponds to the vertex given in Eq 5.28. It was observed that the contribution from this vertex is $\Gamma^\mu = \gamma^\mu$, and at lowest order $F_1(0) = 1$ and $F_2(0) = 0$.

There are two $O(p^2)$ vertices which contribute to diagram 5.2(b). The second order pion-nucleon Lagrangian contains the terms

$$\frac{c_6}{8M_N}\sigma^{\mu\nu}F^+_{\mu\nu} + \frac{c_7}{8M_N}\sigma^{\mu\nu} < F^+_\mu >$$

which contribute to $F_2(q^2)$. The field strength tensor $F^+_{\mu\nu}$ can be written as

$$F^+_{\mu\nu} = 2F_{\mu\nu} = -2ieQ(q_\mu A_\nu - q_\nu A_\mu) \tag{5.33}$$

where Q is the charge matrix for the nucleons

$$Q = \frac{1+\tau_3}{2} \tag{5.34}$$

As a result, the two second order vertices are

$$V^{(2)}_{\gamma NN} = -\frac{ec_6}{2M_N}\sigma^{\mu\nu}q_\nu\frac{1+\tau_3}{2} - \frac{ec_7}{2M_N}\sigma^{\mu\nu}q_\nu \tag{5.35}$$

and the contribution to $F_2(q^2)$ is

$$F_2^{(5.2b)}(q^2) = c_6\frac{1+\tau_3}{2} + c_7 \tag{5.36}$$

The third order tree diagram 5.2(c) contributes to both form factors. The electromagnetic terms in the third order pion-nucleon Lagrangian are

$$(\frac{id_6}{2M_N}[D^\mu, \tilde{F}^+_{\mu\nu}]D^\nu + h.c.) + (\frac{id_7}{2M_N}[D^\mu, < F^+_{\mu\nu} >]D^\nu + h.c.)$$

and $F^+_{\mu\nu}$ can be written in terms of the photon momentum as in Eq 5.33. The resulting vertex is

$$V^{(3)}_{\gamma NN} = \frac{ie}{M_N}(d_6\frac{\tau_3}{2} + d_7)(q^2 - q_\nu q^\mu)(p^\nu + p'^\nu) \tag{5.37}$$

The term $q_\nu q^\mu$ can be removed using the transversality condition $\mathbf{q} \cdot \epsilon = \mathbf{0}$ and the Coulomb gauge with $\epsilon^0 = 0$. The vertex can be put into the required form using the Gordon identity,

$$V^{(3)}_{\gamma NN} = ie(d_6\tau_3 + 2d_7)q^2(\frac{i\sigma^{\mu\nu}q_\nu}{2M_N} - \gamma^\mu) \tag{5.38}$$

and the contribution to the form factors is

$$F_1^{5.2c}(q^2) = -(d_6\tau_3 + 2d_7)q^2 \tag{5.39}$$

$$F_2^{5.2c}(q^2) = (d_6\tau_3 + 2d_7)q^2 \tag{5.40}$$

The fourth order Lagrangian contains four electromagnetic terms which contribute to $F_2(q^2)$,

$$-\frac{e_{54}}{2}\sigma^{\mu\nu}[D^\rho, [D_\rho, < F^+_{\mu\nu} >]] - \frac{e_{74}}{2}\sigma^{\mu\nu}[D^\rho[D_\rho, \tilde{F}^+_{\mu\nu}]] \tag{5.41}$$

$$-\frac{e_{105}}{2}\sigma^{\mu\nu} < F^+_{\mu\nu} >< \chi^+ > - \frac{e_{106}}{2}\sigma^{\mu\nu}\tilde{F}^+_{\mu\nu} < \chi^+ >$$

The contribution of the first two terms is calculated in the same manner as the contributions third order electromagnetic interactions, while the second two terms are calculated in the same manner as the contribution from the second order diagrams.

$$F_2^{(5.2d)}(q^2) = 2M_N(2e_{54} + \tau_3 e_{74})q^2 - 8M_N m_\pi^2(2e_{105} + \tau_3 e_{106}) \qquad (5.42)$$

5.2.3 Loop Calculations

The next step in calculation $F_1(q^2)$ and $F_2(q^2)$ is to include the contributions from the loop diagrams given in Figure 5.3 and Figure 5.4. In each diagram the pion propagator represents the three possible pion states, although in most of the diagrams one of the pion states vanishes. The calculation of the loop diagrams will also require the infrared regularized loop integrals which are listed in Appendix A.

The amplitude corresponding to diagram 5.3(a) is

$$M = \frac{ieg_A^2}{8F^2}(3 - \tau_3)H \qquad (5.43)$$

where H is the loop integral

$$H = i\int \frac{d^dk}{(2\pi)^d} \frac{\not{k}(\not{P}' + \not{k} - M_N)\gamma^\mu(\not{P} + \not{k} - M_N)\not{k}}{((P' + k)^2 - M_N^2 + i\epsilon)(k^2 - m_\pi^2 + i\epsilon)((P + k)^2 - M_N^2 + i\epsilon)} \qquad (5.44)$$

To evaluate the loop integral, the first factor of $\not{k}$ is replaced with $(\not{P}' + \not{k} + M_N) - (\not{P}' + M_N)$ and the second factor of $\not{k}$ is replaced with $(\not{P} + \not{k} + M_N) - (\not{P} + M_N)$. The first term in the integral is

$$i\int \frac{d^dk}{(2\pi)^d} \frac{\gamma^\mu}{k^2 - m_\pi^2 + i\epsilon} = \gamma^\mu \Delta_\pi \qquad (5.45)$$

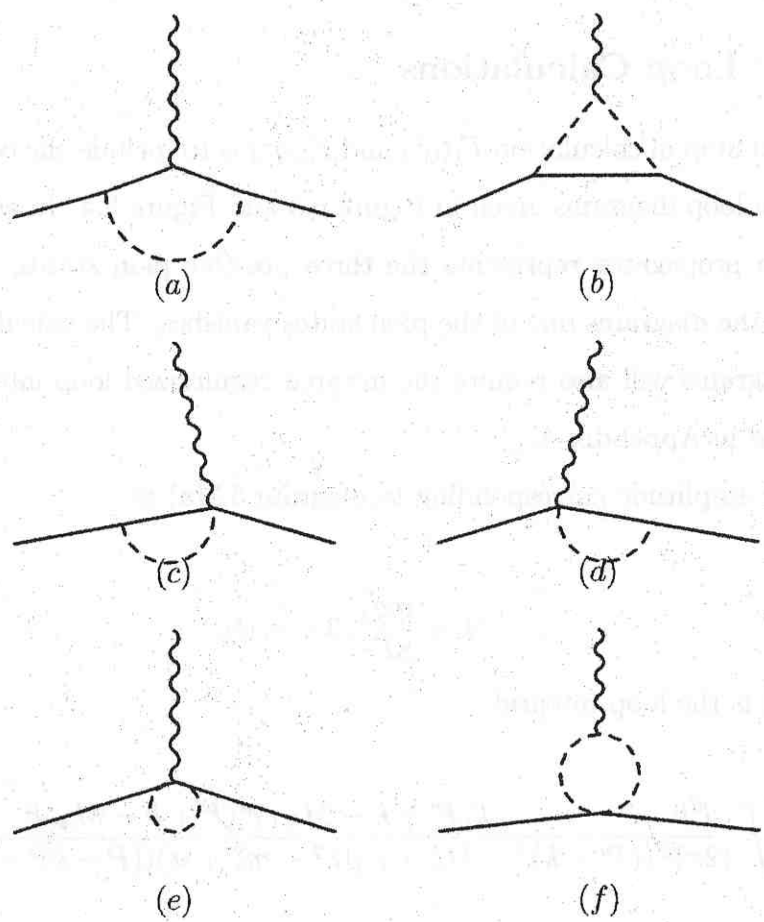

Figure 5.3: $O(p^3)$ Nucleon Electromagnetic Form Factor Loop Diagrams

and the remaining terms are

$$-i\int \frac{d^dk}{(2\pi)^d}\frac{(\not{P}'+M_N)(\not{P}'+\not{k}-M_N)\gamma^\mu}{((P'+k)^2-M_N^2+i\epsilon)(k^2-m_\pi^2+i\epsilon)} = 2M_N^2\gamma^\mu I^{(1)}(M_N^2) \tag{5.46}$$

$$-i\int \frac{d^dk}{(2\pi)^d}\frac{\gamma^\mu(\not{P}+\not{k}-M_N)(\not{P}+M_N)\gamma^\mu}{((P+k)^2-M_N^2+i\epsilon)(k^2-m_\pi^2+i\epsilon)} = 2M_N^2\gamma^\mu I^{(1)}(M_N^2) \tag{5.47}$$

$$i\int \frac{d^dk}{(2\pi)^d}\frac{(\not{P}'+M_N)(\not{P}'+\not{k}-M_N)\gamma^\mu(\not{P}+\not{k}-M_N)(\not{P}+M_N)}{((P'+k)^2-M_N^2+i\epsilon)(k^2-m_\pi^2+i\epsilon)((P+k)^2-M_N^2)} \tag{5.48}$$

$$= -4M_N^2m_\pi^2I_A(q^2)+8M_N^2\gamma^\mu I_A^{(2)}(q^2)+16M_N^3(p^\mu+p'^\mu)I_A^{(3)}$$

The Gordon identity can be applied to the last term to eliminate the factor of $(p^\mu + p'^\mu)$. The contribution to the form factors is

$$F_1^{5.3a}(q^2) = \frac{g_A^2}{8F^2}(3-\tau_3)\{\Delta_\pi - 4M_N^2I^{(1)}(M_N^2) - 4M_N^2m_\pi^2I_A(q^2) + 8M_N^2I_A^{(2)}(q^2) + 32M_N^4I_A^{(3)}(q^2)\} \tag{5.49}$$

$$F_2^{5.3a}(q^2) = -\frac{4g_A^2M_N^4}{F^2}(3-\tau_3)I_A^{(3)}(q^2) \tag{5.50}$$

For diagram 5.3(b) the amplitude is

$$M = -\frac{ieg_A^2}{4F^2}\tau_3H \tag{5.51}$$

where H is the loop integral

$$H = i\int \frac{d^dk}{(2\pi)^d} \frac{\not{k}(\not{P}+\not{k}-M_N)(2q^\mu - k^\mu)\not{k}}{(k^2 - m_\pi^2 + i\epsilon)((P+k)^2 - M_N^2 + i\epsilon)((q-k)^2 - m_\pi^2 + i\epsilon)} \tag{5.52}$$

As before the integral is evaluated by replacing the second factor of $\not{k}$ with $(\not{P}+\not{k}+M_N) - (\not{P}+M_N)$ and using the Gordon identity to replace factors of $(p^\mu + p'^\mu)$. The resulting contribution to the form factors is

$$F_1^{5.3b}(q^2) = -\frac{g_A^2}{F^2}\tau_3\{q^2 J^{(1)}(q^2) + 4M_N^2 I_{21}^{00}(q^2) + 16M_N^4 I_{21}^{QQ}(q^2)\} \tag{5.53}$$

$$F_2^{5.3b}(q^2) = -\frac{16g_A^2}{F^2}\tau_3 M_N^4 I_{21}^{QQ}(q^2) \tag{5.54}$$

The amplitudes for diagram 5.3c and 5.3d are related, and as such the sum of the two amplitudes will be calculated. Using the pion-photon vertex given in Eq 5.107, the amplitude is found to be

$$M = \frac{ieg_A^2}{2F^2}\tau_3 H \tag{5.55}$$

where

$$H = -i\int \frac{d^dk}{(2\pi)^d}\{\frac{\gamma^\mu(\not{P}+\not{k}-M_N)\not{k}}{(k^2 - m_\pi^2 + i\epsilon)((P+k)^2 - M_N^2 + i\epsilon)} \tag{5.56}$$

$$+\frac{\not{k}(\not{P}'+\not{k}-M_N)\gamma^\mu)}{(k^2 - m_\pi^2 + i\epsilon)((P'+k)^2 - M_N^2 + i\epsilon)}\}$$

In the first integral, $\not{k}$ is replaced with $(\not{P}+\not{k}+M_N) - (\not{P}+M_N)$ to give

$$H_1 = \gamma^\mu \Delta_\pi - \gamma^\mu 2M_N^2 I^{(1)}(M_N^2) \tag{5.57}$$

In the second integral $\not{k}$ is replaced with $(\not{P}' + \not{k} + M_N) - (\not{P}' + M_N)$ to give

$$H_2 = \gamma^\mu \Delta_\pi - \gamma^\mu 2M_N^2 I^{(1)}(M_N^2) \tag{5.58}$$

The contribution to $F_1(q^2)$ from these two diagrams is then

$$F_1^{5.3c+d}(q^2) = \frac{g_A^2}{F^2}\tau_3\{\Delta_\pi - 2M_N^2 I^{(1)}(M_N^2)\} \tag{5.59}$$

The O(p) vertex in diagram 5.3(e) is

$$V^{(1)}_{NN\pi\pi\gamma} = -\frac{ie}{4F^2}\gamma^\mu(2\pi^+\pi^-\tau_3) \tag{5.60}$$

This vertex also contains two terms in which the pion fields cannot be contracted into a single loop, and as such these terms have been omitted. The amplitude for the diagram is

$$M = -\frac{ie}{2F^2}\gamma^\mu\tau_3(i\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2 - m_\pi^2 + i\epsilon}) \tag{5.61}$$

The loop integral is the definition of Δ_π, and so

$$F_1^{5.3e}(q^2) = -\frac{\tau_3}{2F^2}\Delta_\pi \tag{5.62}$$

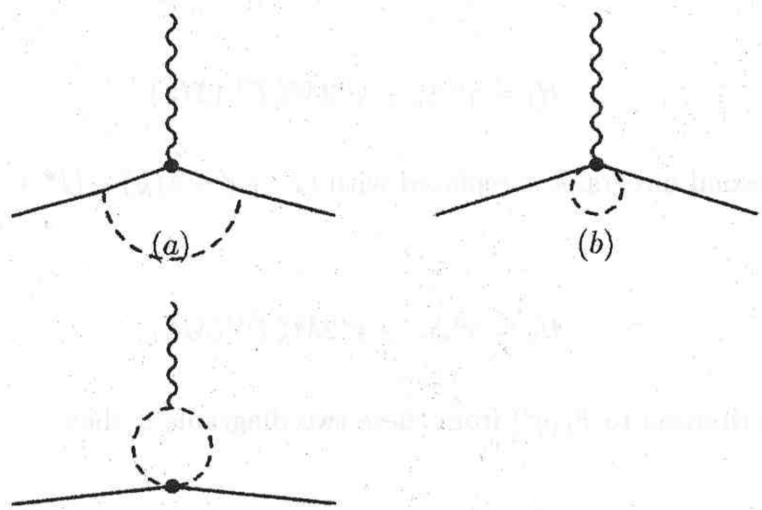

Figure 5.4: $O(p^4)$ Nucleon Electromagnetic Form Factor Loop Diagrams

Diagram 5.3(f) is calculated in the same manner using the two pion vertex,

$$V^{(1)}_{NN\pi\pi} = \frac{1}{4F^2}\gamma^\mu \tau_3(\pi^-\partial_\mu\pi^+ - \pi^+\partial_\mu\pi^-)) \tag{5.63}$$

This vertex also contains additional terms which do not contribute to the pion loop and which have been omitted. The contribution to the form factors is

$$F_1^{5.3f}(q^2) = \frac{q^2}{F^2}\tau_3 J^{(1)}(q^2) \tag{5.64}$$

The remaining three diagrams, listed in Figure 5.4, are fourth order corrections to the form factors. The calculation of each diagram is analagous to the third order calculations, and so only the results will be listed.

$$F_1^{5.4a}(q^2) = \frac{g_A^2 M_N^2}{F^2}((3-\tau_3)c_6 + 6c_7)q^2 I_A^{(3)}(q^2) \tag{5.65}$$

$$F_2^{5.4a}(q^2) = -\frac{M_N^4 g_A^2}{8F^2}((3-\tau_3)c_6 + 6c_7)\{\Delta_\pi - 4M_N^2 I^{(1)}(M_N^2)$$
$$+ 4M_N^2 m_\pi^2 I_A(q^2) - 16M_N^2 I_A^{(2)}(q^2) + 8M_N^2 q^2 (I_A^{(3)}(q^2) - I_A^{(4)}(q^2))\} \quad (5.66)$$

$$F_1^{5.4b}(q^2) = \frac{3c_2 m_\pi^2}{4M_N F^2}(1+\tau_3)\{\Delta_{pi} - \frac{m_\pi^2}{32\pi^2}\} \quad (5.67)$$

$$F_2^{5.4b}(q^2) = -\frac{3c_2 m_\pi^2}{4M_N F^2}(1+\tau_3)\{\Delta_\pi - \frac{m_\pi^2}{32\pi^2}\} - \frac{c_6\tau_3}{2F^2}\Delta_\pi \quad (5.68)$$

$$F_2^{5.4c}(q^2) = \frac{4c_4 M_N}{F^2}\tau_3 q^2 J^{(1)}(q^2) \quad (5.69)$$

5.3 Pion Nucleon Form Factors

5.3.1 The Nucleon Pion Vertex Function

The pion nucleon interactions can also be written in terms of four form factors. In this section the four pion form factors will be calculated to fourth order using the method of infrared regularization.

The first order pion-nucleon Lagrangian contains a single interaction term,

$$L_{\pi N}^{(1)} = \overline{\Psi}(i\not{D} - M_N)\Psi + \overline{\Psi}\frac{g_A}{2}\gamma^\mu\gamma^5 u_\mu \Psi$$

The single pion term in the expansion of u_μ is $-\frac{\tau_a \partial_\mu \pi^a}{F}$, which gives the first order pion-nucleon vertex as

$$V_{\pi NN} = -\frac{i g_A}{2F}\gamma^\mu\gamma^5\tau_a\partial_\mu\pi^a \quad (5.70)$$

As in Section 5.2, the higher order contributions to the pion nucleon vertex can be included by replacing γ^μ with Γ^μ where Γ^μ is a combination of Dirac matrices and momenta. The general form of Γ^μ is [31]

$$\Gamma^\mu = \gamma^\mu A_5 + (p^\mu + p'^\mu)B_5 + (p'^\mu - p^\mu)C_5 \tag{5.71}$$

where A_5,B_5, and C_5 represent the axial form factors. However since the vertex contains a factor of $\partial_\mu \pi^a = i({p'}_\mu - p_\mu)\pi^a$ the second term vanishes, and as such B_5 can be set to zero without loss of generality.

The result is that the complete pion nucleon vertex is of the form

$$V_{\pi NN} = \frac{1}{2F}(\not{q}G_1^a(q^2) + \frac{q^2}{2M_N}G_2^a(q^2))\gamma^5\pi^a \tag{5.72}$$

where $G_1(q^2)$ and $G_2(q^2)$ are the two form factors for a single nucleon and a represents the pion isospin index. Using Eq 5.70, it is expected that at lowest order $G_1^a(m_\pi^2) = g_A\tau^a$ and $G_2^a(m_\pi^2) = 0$.

If the pion is on-shell and the incoming and outgoing nucleons satisfy Dirac's equation, then the interaction can be written as

$$V_{\pi NN} = \frac{1}{2F}(2M_N G_1^a(m_\pi^2) + \frac{m_\pi^2}{2M_N}G_2^a(m_\pi^2))\gamma^5\pi^a \tag{5.73}$$

The pion-nucleon interaction can also be written as [31, 20]

$$V_{\pi NN} = G_{\pi NN}\gamma^5\tau^a \tag{5.74}$$

which gives the relation

$$G_{\pi NN}\tau_a = \frac{M_N G_1^a(m_\pi^2)}{F} + \frac{m_\pi^2 G_2^a(m_\pi^2)}{4M_N F} \tag{5.75}$$

The pion coupling constant $G_{\pi NN}$ can also be written in terms of physical axial coupling constant, G_A, using the Goldberger-Treiman relation [1] [31, 20]

$$G_{\pi NN} = \frac{M_N G_A}{F} \tag{5.76}$$

which gives

$$G_A\tau_a = G_1^a(m_\pi^2) + \frac{m_\pi^2}{4M_N^2} G_2^a(m_\pi^2) \tag{5.77}$$

5.3.2 Tree Level Calculations

The tree level diagrams which contribute to the form factors are given in Figure 5.5. The first diagram corresponds to the first order vertex in Eq 5.70 and gives the leading order axial term[2] $G_1(q^2) = g_A\tau^a$. The only higher order tree contribution to the form factors occurs at $O(p^3)$ since there are no $O(p^2)$ or $O(p^4)$ vertices which contain a single pion field.

At third order there are four terms in the Lagrangian which can contribute,

[1] The form of the Goldberger-Treiman relation which is used is only valid in the chiral limit with the axial current conserved. In general there is an additional term which is proportional to average of the two quark masses [31]. However as this correction term is only required at higher orders, is has not been included.

[2] The pion isospin index has been dropped from the pion form factors in this section to avoid confusion. Hence G_1 and G_2 will denote the form factors, while G_1^x and G_2^x will represent the contribution to the form factors from diagram x.

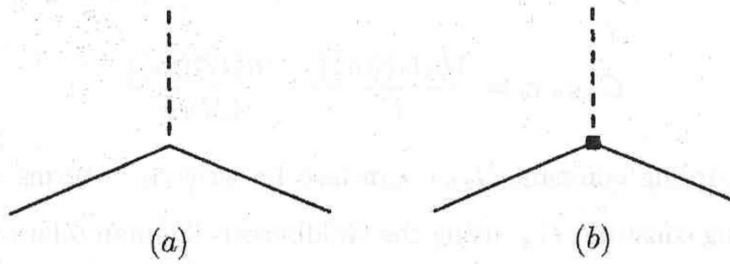

Figure 5.5: Pion Nucleon Form Factor Tree Diagrams

$$\frac{d_{16}}{2}\gamma^{\mu}\gamma^{5} < \chi_{+} > u_{\mu} + \frac{d_{17}}{2}\gamma^{\mu}\gamma^{5} < \chi_{+} u_{\mu} >$$

$$\frac{id_{18}}{2}\gamma^{\mu}\gamma^{5}[D_{\mu}, \chi_{-}] + \frac{id_{19}}{2}\gamma^{\mu}\gamma^{5}[D_{\mu}, < \chi_{-} >]$$

In each vertex the single pion field term in the expansion of u_{μ} is used,

$$u_{\mu} \rightarrow -\tau_{a}\frac{\partial_{\mu}\pi^{a}}{F}$$

When the first term in the expansion of χ_{+} is used, the first vertex becomes

$$-\frac{4id_{16}m_{\pi}^{2}}{2F}\gamma^{\mu}\gamma^{5}\tau_{a}\partial_{\mu}\pi^{a} \tag{5.78}$$

and the contribution to $G_{A}(q^{2})$ is 4 $d_{16}m_{\pi}^{2}\tau_{a}$. The next vertex contains a factor of $< \chi_{+}\tau_{a} >$. At lowest order $\chi_{+} = 4BM_{q}$ where M_{q} is the quark mass matrix, and so

$$< \chi_{+}\tau_{a} > = (m_{u} - m_{d})\delta_{a3} \tag{5.79}$$

In the isospin limit $m_u = m_d$ and so this vertex vanishes. The third term gives a contribution of $-2d_{18}m_\pi^2\tau_a$, and the fourth term in the Lagrangian vanishes in the isospin limit. Therefore the total contribution to the form factors is

$$G_1^{5.5b}(q^2) = 2m_\pi^2(2d_{16} - d_{18})\tau_a \tag{5.80}$$

5.3.3 Loop Calculations

The loop diagrams which will be required to calculate the axial form factors are given in Figure 5.6 and Figure 5.7, with each internal pion line representing the three pion states. The loop integrals will be written in terms of the infrared regularized integral listed in Appendix A.

The amplitude corresponding to the first loop diagram is

$$M = -\frac{g_A^3}{8F^3}((3 + 2\tau_3)\delta_{c3} - \tau_c)\gamma^5 H \tag{5.81}$$

where

$$H = i\int \frac{d^dk}{(2\pi)^d} \frac{\not{k}(\not{P}' + \not{k} + M_N)\not{q}(\not{P} + \not{k} - M_N)\not{k}}{((P' + k)^2 - M_N^2 + i\epsilon)((P + k)^2 - M_N^2)(k^2 - m_\pi^2 + i\epsilon)} \tag{5.82}$$

The first factor of $\not{k}$ can be replaced with $(\not{P}' + \not{k} - M_N) - (\not{P}' - M_N)$ and the second factor is replaced with $(\not{P} + \not{k} + M_N) - (\not{P} + M_N)$. The result is a four simple integrals,

$$H_1 = i\int \frac{d^dk}{(2\pi)^d} \frac{\not{q}}{k^2 - m_\pi^2 + i\epsilon} = \not{q}\Delta_\pi \tag{5.83}$$

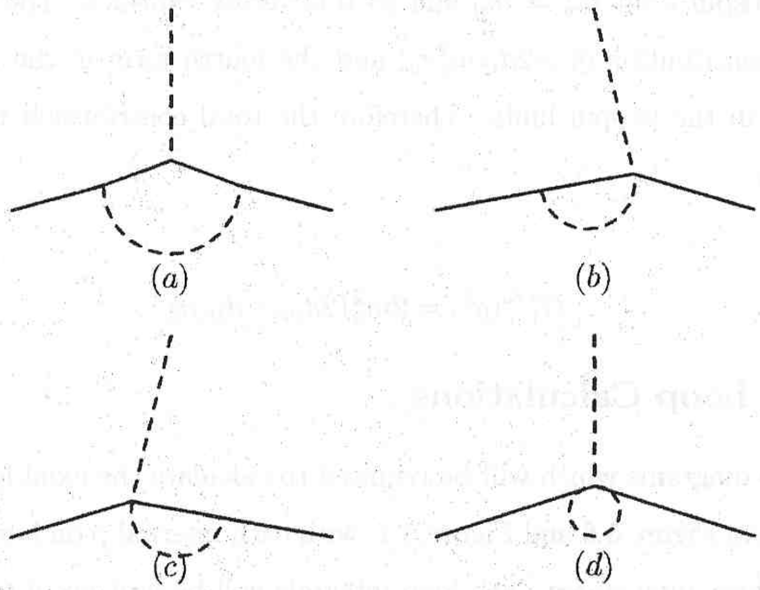

Figure 5.6: $O(p^3)$ Pion Nucleon Form Factor Loop Diagrams

$$H_2 = -i\int \frac{d^dk}{(2\pi)^d}\frac{(\not{P}' - M_N)\not{k}\not{q}}{((P'+k)^2 - M_N^2 + i\epsilon)(k^2 - m_\pi^2 + i\epsilon)} = -2\not{q}M_N^2 I^{(1)}(M_N^2) \tag{5.84}$$

$$H_3 = -i\int \frac{d^dk}{(2\pi)^d}\frac{\not{q}\not{k}(\not{P} + M_N)}{((P+k)^2 - M_N^2 + i\epsilon)(k^2 - m_\pi^2 + i\epsilon)} = -2\not{q}M_N^2 I^{(1)}(M_N^2) \tag{5.85}$$

$$H_4 = i\int \frac{d^dk}{(2\pi)^d}\frac{(\not{P}' - M_N)\not{k}\not{q}\not{k}(\not{P} + M_N)}{((P+k)^2 - M_N^2 + i\epsilon)((P'+k)^2 - M_N^2 + i\epsilon)(k^2 - m_\pi^2 + i\epsilon)} \tag{5.86}$$

$$= 4M_N^2 m_\pi^2 \not{q} I_A(q^2) - 8M_N^2 \not{q} I_A^{(2)}(q^2) - 8M_N^2 q^2 \not{q} I_A^{(4)}(q^2)$$

The total contribution to $G_1(q^2)$ is

$$G_1^{5.6a}(q^2) = \frac{g_A^3}{4F^2}((3+2\tau_3)\delta_{a3} - \tau_a)\{\Delta_\pi - 4M_N^2 I^{(1)}(M_N^2) + 4M_N^2 m_\pi^2 I_A(q^2) - 8M_N^2 I_A^{(2)}(q^2) - 8M_N^2 q^2 I_A^{(4)}(q^2)\} \tag{5.87}$$

The two diagrams 5.6(b) and 5.6(c) are related, and so the sum of the two diagrams is calculated. These diagrams require the first order two pion vertex, which is derived from the covariant derivative in the first order Lagrangian

$$\overline{\Psi}\frac{i}{2}\gamma^\mu(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger)\Psi \tag{5.88}$$

and gives the vertex

$$V_{\pi\pi NN} = \frac{1}{4F^2}\gamma^\mu(\tau_+(\pi^+\partial_\mu\pi^0 - \pi^0\partial_\mu\pi^+) + \tau_-(\pi^0\partial_\mu\pi^- - \pi^-\partial_\mu\pi^0) + \tau_3(\pi^-\partial_\mu\pi^+ - \pi^+\partial_\mu\pi^-)) \tag{5.89}$$

Then the amplitude for the two diagrams is

$$M = -\frac{g_A}{4F^3}\gamma^5(H_b + H_c)\tau_a \tag{5.90}$$

where the loop integrals are

$$H_b = i\int \frac{d^dk}{(2\pi)^d}\frac{(\not{q}+\not{k})(\not{P}+\not{k}-M_N)\not{k}}{((P+k)^2 - M_N^2 + i\epsilon)(k^2 - m_\pi^2 + i\epsilon)} \tag{5.91}$$

and

$$H_c = i\int \frac{d^d k}{(2\pi)^d} \frac{\not{k}(\not{P}' + \not{k} + M_N)(\not{q} - \not{k})}{((P'+k)^2 - M_N^2 + i\epsilon)(k^2 - m_\pi^2 + i\epsilon)} \tag{5.92}$$

The integral H_b can be evaluated by rewriting the factor of $\not{k}$ in the numerator as $(\not{P} + \not{k} + M_N) - (\not{P} + M_N)$. Then the integral can be written as

$$H_b = \not{q}\Delta_\pi - 2M_N^2 \not{q} I^{(1)}(M_N^2) - 2M_N m_\pi^2 I(M_N^2) \tag{5.93}$$

Using the same procedure with $\not{k} \rightarrow (\not{P}' + \not{k} - M_N) - (\not{P}' - M_N)$, H_c can be written as

$$H_c = \not{q}\Delta_\pi - 2M_N^2 \not{q} I^{(1)}(M_N^2) + 2M_N m_\pi^2 I(M_N^2) \tag{5.94}$$

Then the amplitude is

$$M = -\frac{g_A}{2F^3}\gamma^5(\Delta_\pi - 2M_N^2 I^{(1)}(M_N^2))\tau_a \not{q} \tag{5.95}$$

and the contribution to $G_1(q^2)$ is

$$G_1^{5.6(b+c)}(q^2) = \frac{g_A}{F^2}\tau_a(\Delta_\pi - 2M_N^2 I^{(1)}(M_N^2)) \tag{5.96}$$

The amplitude for diagram 5.6(d) is calculated by summing each of the possible contractions of two of the pion fields in the three pion vertex,

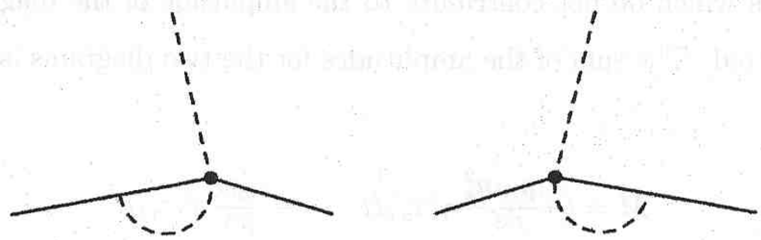

Figure 5.7: O(p^4) Pion Nucleon Form Factor Loop Diagrams

$$
\begin{aligned}
V_{\pi\pi\pi NN} = & \frac{i g_A}{12F^3}\gamma^\mu\gamma^5((\pi^0\pi^0 + 2\pi^+\pi^-)\tau_a\partial_\mu\pi^a \\
& - \tau_a\pi^a(\pi^0\partial_\mu\pi^0 + \pi^+\partial_\mu\pi^- + \pi^-\partial_\mu\pi^+))
\end{aligned}
\tag{5.97}
$$

However the pion fields which appear as a derivative do not need to be included, as the symmetry of the loop cancels odd powers of the loop momentum in the numerator. The amplitude for this diagram is

$$
M = -\frac{g_A}{6F^3}\not{q}\gamma^5\Delta_\pi\tau_a \tag{5.98}
$$

and

$$
G_1^{5.6d}(q^2) = -\frac{g_A}{3F^2}\Delta_\pi\tau_a \tag{5.99}
$$

The two $O(p^4)$ diagrams are given in Figure 5.7, and contain part of the second order two pion vertex

$$
V^{(2)}_{\pi\pi NN} = \frac{-2ic_1 m_\pi^2}{F^2}(\pi^0\pi^0 + 2\pi^+\pi^-) + \frac{ic_3}{F^2}(\partial^\mu\pi^0\partial_\mu\pi^0 + 2\partial^\mu\pi^+\partial_\mu\pi^-) \tag{5.100}
$$

The terms which do not contribute to the amplitude of the diagrams have been omitted. The sum of the amplitudes for the two diagrams is

$$M = (\frac{c_1 g_A m_\pi^2}{F^3}\gamma^5\tau_a)H_1 + (-\frac{c_3 g_A}{F^3}\gamma^5\tau_a)H_2 \tag{5.101}$$

where

$$\begin{aligned} H_1 = i\int \frac{d^d k}{(2\pi)^d}\{&\frac{\not{k}(\not{P}' + \not{k} + M_N)}{(k^2 - m_\pi^2 + i\epsilon)((P' + k)^2 - M_N^2 + i\epsilon)} \\ &+ \frac{(\not{P} + \not{k} - M_N)\not{k}}{(k^2 - m_\pi^2 + i\epsilon)((P + k)^2 - M_N^2 + i\epsilon)}\} \\ = 2\Delta_\pi - 4M_N^2 I^{(1)}(M_N^2) \end{aligned} \tag{5.102}$$

$$\begin{aligned} H_2 = -i\int \frac{d^d k}{(2\pi)^d}\{&\frac{\not{k}(\not{P}' + \not{k} + M_N)q^\mu k_\mu}{(k^2 - m_\pi^2 + i\epsilon)((P' + k)^2 - M_N^2 + i\epsilon)} \\ &- \frac{q^\mu k_\mu(\not{P} + \not{k} - M_N)\not{k}}{(k^2 - m_\pi^2 + i\epsilon)((P + k)^2 - M_N^2 + i\epsilon)}\} \\ = -2M_N(2\not{q}I^{(2)}(M_N^2) + M_N q^2 I^{(3)}(M_N^2)) \end{aligned} \tag{5.103}$$

The contribution to the form factors is

$$G_1^{5.7(a+b)}(q^2) = -\frac{8c_3 M_N^2 g_A}{F^2}\tau_a I^{(2)}(M_N^2) \tag{5.104}$$

$$\begin{aligned} G_2^{5.7(a+b)}(q^2) =& \frac{8M_N c_1 g_A m_\pi^2}{q^2 F^2}\tau_a(\Delta_\pi - 2M_N^2 I^{(1)}(M_N^2)) \\ &- \frac{4M_N^2 c_3 g_A}{F^2}\tau_a I^{(3)}(M_N^2) \end{aligned} \tag{5.105}$$

5.4 Nucleon Pion-Photon Vertex

The final vertex which must be included in the pion photoproduction calculation is the contact term in which the pion and photon couple to the nucleon at the same point. The first order vertex is derived from the axial term in the Lagrangian,

$$L^{(1)}_{\pi N} = \overline{\Psi}(i\not{D} - M_N)\Psi + \overline{\Psi}\frac{g_A}{2}\gamma^\mu\gamma^5 u_\mu \Psi$$

The photon is included in the covariant derivative in u_μ. For a single pion and photon u_μ is given by

$$\begin{aligned} u_\mu &= iu^\dagger(eQA_\mu U)u^\dagger \\ &= (\frac{ie}{2F}A_\mu\{\phi, Q\}) + (\frac{ie}{F}A_\mu Q\phi) \\ &= \frac{ie}{2F}A_\mu[\phi, Q] \end{aligned} \tag{5.106}$$

where Q is the charge matrix. Then the first order pion-photon vertex is

$$\begin{aligned} V_{\pi^a\gamma NN} &= -\frac{eg_A}{4F}\gamma^\mu\gamma^5[\tau_a, Q]\pi^a \\ &= -\frac{eg_A}{2F}\gamma^\mu\gamma^5(\tau_+\pi^+ - \tau_-\pi^-) \end{aligned} \tag{5.107}$$

5.4.1 Tree Diagrams

The three tree diagrams which contribute to the pion-photon vertex are given in Figure 5.8. The first diagram is the first order contribution to the vertex function given in Eq 5.107. The second diagram is a third order contribution which results from several terms in the Lagrangian $L^{(3)}_{\pi N}$,

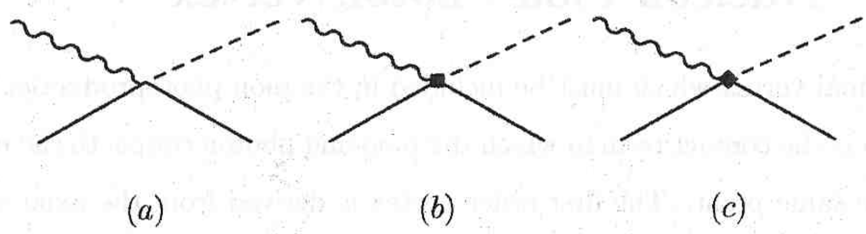

Figure 5.8: Pion-Photon Vertex Tree Diagrams

$$
\begin{aligned}
&(\frac{id_8}{2M_N}\epsilon^{\mu\nu\alpha\beta} < \tilde{F}^{+}_{\mu\nu}u_\alpha > D_\beta + h.c.) + (\frac{id_9}{2M_N}\epsilon^{\mu\nu\alpha\beta} < \tilde{F}^{+}_{\mu\nu} > u_\alpha D_\beta + h.c.) \\
&+ \frac{d_{16}}{2}\gamma^\mu\gamma^5 < \chi^+ > u_\mu - (\frac{id_{20}}{8M_N^2}\gamma^\mu\gamma^5[\tilde{F}^{+}_{\mu\nu}, u_\lambda]D^\lambda D^\nu + h.c.) \\
&+ \frac{id_{21}}{2}\gamma^\mu\gamma^5[\tilde{F}^{+}_{\mu\nu}, u^\nu] + \frac{d_{22}}{2}\gamma^\mu\gamma^5[D^\nu, F^{-}_{\mu\nu}] +
\end{aligned}
$$

The resulting third order vertex is

$$
\begin{aligned}
V^{(3)}_{\pi\gamma NN} =& -\frac{2e}{F}\epsilon^{\mu\nu\alpha\beta}(k_\nu q_\alpha(P'_\beta + P_\beta)(d_8\pi^0 + d_9\tau_a\pi^a) \\
& - \frac{em_\pi^2(2d_{16} - d_{18})}{F}\gamma^\mu\gamma^5(\tau_+\pi^+ - \tau_-\pi^-) \\
& - \frac{e}{F}\gamma^\mu\gamma^5 F_{\mu\nu}(d_{20}\frac{P'^\lambda P'^\nu + P^\lambda P^\nu}{4M_N^2}q_\lambda - d_{21}q^\nu)(\tau_+\pi^+ - \tau_-\pi^-) \\
& - \frac{ed_{22}(k^2 + q^\mu k_\mu)}{2F}\gamma^\mu\gamma^5(\tau_+\pi^+ - \tau_-\pi^-)
\end{aligned}
\tag{5.108}
$$

At fourth order the contribution to the vertex is

$$
\begin{aligned}
V^{(4)}_{\pi\gamma NN} = &-\frac{ieF_{\lambda\mu}}{M_N F}\gamma^5\{(e_{48}\tau_a + e_{67}\delta_{a3})q^\lambda q_\nu \gamma^\mu (P'^\nu + P^\nu) \\
&+ (e_{49}\tau_a + e_{68}\delta_{a3})q^\lambda q_\nu \gamma^\nu (P'^\mu + P^\mu) \\
&- \frac{1}{6}(e_{50}\tau_a + e_{69}\delta_{a3})q_\nu q_\rho \gamma^\lambda (P'^\mu P'^\nu P'^\rho + P^\rho P^\nu P^\mu) \\
&+ \frac{1}{2}(e_{51}\tau_a + e_{70}\delta_{a3})q \cdot k \gamma^\lambda (P'^\mu + P^\mu) \\
&+ \frac{1}{2}(e_{52}\tau_a + e_{71}\delta_{a3})q_\mu k^\lambda \gamma^5 \gamma^\mu (P'^\nu + P^\nu) \\
&+ \frac{1}{2}(e_{53}\tau_a + e_{72}\delta_{a3})q_\nu k^\lambda \gamma^5 \gamma^\mu (P'^\nu + P^\nu)\}
\end{aligned}
\tag{5.109}
$$

The terms which depend on e_{52}, e_{53}, e_{71} and e_{72} vanish when the photon is on-shell. The terms in the Lagrangian which contribute only to isospin violation have been omitted.

5.4.2 O(p^4) Loop Diagrams

The O(p^3) loop diagrams which contribute to the pion-photon vertex are given in Figures 5.9 and 5.10, and the O(p^4) loop diagrams are given in Figure 5.11. As the method of calculating the infrared regularized loop integrals has already been presented in detail in previous calculations, only the results of these calculations will be listed.

The amplitudes for the diagrams in Figure 5.9 written in terms of the variable $t = (q-k)^2$ are

$$
\begin{aligned}
M_a = &\frac{eg_A^3}{8F^3}(\tau_+ - \tau_-)\{\Delta_\pi - 4M_N^2 I^{(1)}(M_N^2) + 4M_N^2 m_\pi^2 I_A(t) \\
&- 8M_N^2 I_A^{(2)}(t) - 8M_N^2 t I_A^{(4)}(t)\}\gamma^\mu\gamma^5
\end{aligned}
\tag{5.110}
$$

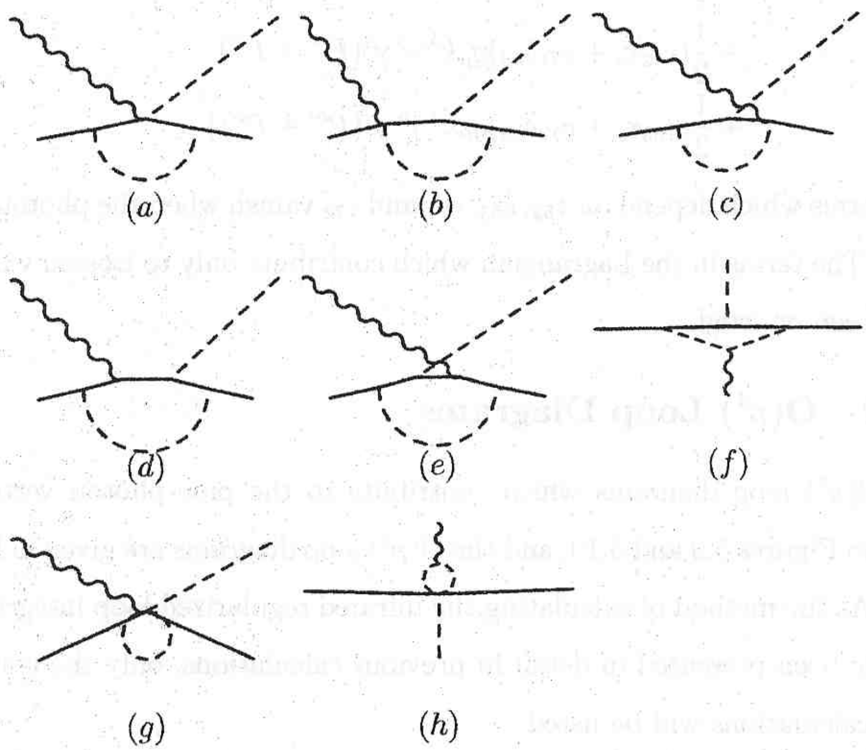

Figure 5.9: Pion-Photon Vertex Loop Diagrams

$$\begin{aligned}M_{b+c} =& -\frac{eg_A^3}{4F^3}\delta_{a3}\gamma^\mu\{(\not{q}-\not{k})I(M_N^2)+tI^{(1)}(M_N^2)\\&+2M_N(\not{P}'-M_N)\not{q}I_A(t)\}\gamma^5\end{aligned} \tag{5.111}$$

$$M_{d+e} =\frac{eg_A^3}{8F^3}(2\delta_{a3}-\tau_+ +\tau_-)\gamma^5\{M_N q^\mu I^{(1)}(M_N^2)-4M_N m_\pi^2\gamma^\mu I_A^{(1)}(t)\} \tag{5.112}$$

$$M_f =-\frac{2ieg_A^3}{F^3}\delta_{a3}\sigma^{\mu\nu}q_\nu\gamma^5 I_{21}^{00}(k^2) \tag{5.113}$$

$$M_g =\frac{eg_A}{4F^2}\Delta_\pi(\tau_+ -\tau_-)\gamma^\mu\gamma^5 \tag{5.114}$$

$$M_h =-\frac{eg_A}{4F^2}(\tau_+ -\tau_-)J(k^2)q^\mu\not{q}\gamma^5 \tag{5.115}$$

The amplitudes for diagrams d, e and f also contain several higher order terms which do not contribute to the fourth order calculation and have been omitted. The amplitudes for the diagrams in Figure 5.10 are

$$M_{a+e} =-\frac{eg_A}{2F^3}\delta_{a3}\gamma^\mu\gamma^5(\Delta_\pi-2M_N^2 I^{(1)}(M_N^2)) \tag{5.116}$$

$$\begin{aligned}M_{b+g} =&\frac{eg_A}{8F^3}(\tau_a+\delta_{a3})\{(4M_N\not{q}+4m_\pi^2)\gamma^\mu I^{(1)}(M_N^2)\\&-2\Delta_\pi\gamma^\mu-2M_N q^\mu I^{(1)}(M_N^2)\}\end{aligned} \tag{5.117}$$

$$\begin{aligned}M_{c+f} =&\frac{eg_A}{8F^3}(\tau_a-\delta_{a3})\{2\Delta_\pi\gamma^\mu+4M_N^2\gamma^\mu I^{(1)}(M_N^2)\\&+2M_N q^\mu I^{(1)}(M_N^2)\}\gamma^5\end{aligned} \tag{5.118}$$

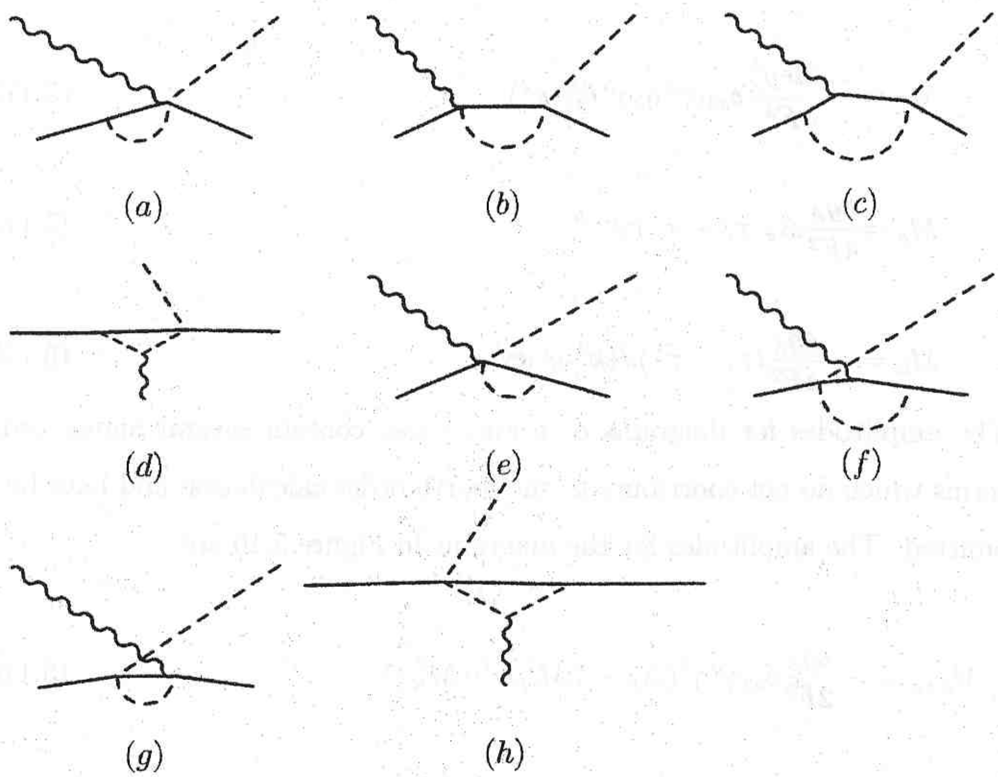

Figure 5.10: Pion-Photon Vertex Loop Diagrams

$$M_{d+h} = -\frac{eg_A}{F^3}\delta_{a3}\{(k^2 J^{(1)}(k^2) + 4M_N^2 I_{21}^{00}(k^2) - 16M_N^3 I_{21}^Q(k^2))\gamma^\mu\gamma^5 \quad (5.119)$$
$$+ 8iM_N^2 I_{21}^Q(k^2)\sigma^{\mu\nu}k_\nu\gamma^5\}$$

The amplitudes M_{c+f} and M_{d+h} also contain higher order terms which do not contribute to the O(p^4) pion photoproduction calculation.

The $O(p^4)$ diagrams are given in Figure 5.11, and the corresponding amplitudes are

$$M_{a+b} = -\frac{eg_A^3}{16M_N F^3}(2c_6 - (c_6 + 2c_7)(\tau_+ - \tau_-))\gamma^5\{M_N(\not{k}q^\mu - q\cdot k\gamma^\mu)I^{(1)}(M_N^2) \quad (5.120)$$
$$+ 4iM_N m_\pi^2 \sigma^{\mu\nu}k_\nu I_A^{(1)}(t)\}$$

$$M_{c+i} = 0 \quad (5.121)$$

$$M_{d+j} = \frac{2c_1 eg_A m_\pi^2}{F^3}(\tau_+ - \tau_-)(2M_N\gamma^\mu - i\sigma^{\mu\nu}q_\nu)I(M_N^2) \quad (5.122)$$

$$M_{e+k} = \frac{4ec_1 g_A M_N^2 m_\pi^2}{F^3}(\tau_a + \delta_{a3})\gamma^\mu\gamma^5(\Delta_\pi - 2M_N^2 I^{(1)}(M_N^2)) \quad (5.123)$$
$$+ \frac{2c_3 eg_A M_N^2}{F^3}(\tau_a + \delta_{a3})\gamma^\mu\gamma^5(\not{q}I^{(2)}(M_N^2) + q^2 I^{(3)}(M_N^2))$$

$$M_{f+k} = \frac{ieg_A}{16M_N F^3}((c_6 + 2c_7)\tau_a - c_6\delta_{a3})\{2\Delta_\pi\sigma^{\mu\nu}k_\nu + 4M_N^2\sigma^{\mu\nu}k_\nu I^{(1)}(M_N^2) \quad (5.124)$$
$$+ 2iM_N(2\not{k}q^\mu - k\cdot q\gamma^\mu)I^{(1)}(M_N^2)\}\gamma^5$$

$$M_{g+h} = 0 \quad (5.125)$$

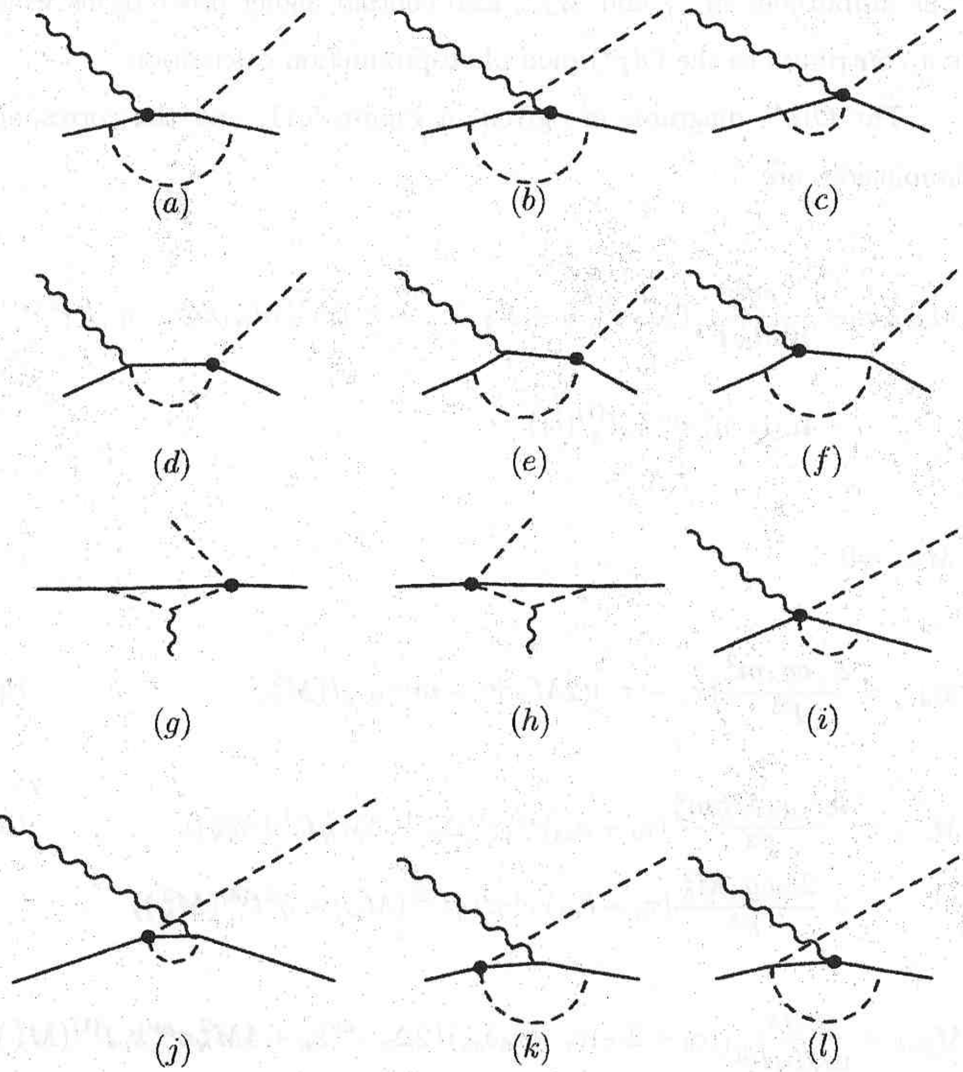

Figure 5.11: Pion-Photon Vertex Loop Diagrams

where the higher order terms have been omitted. The amplitudes M_{c+i} and M_{g+h} contain only terms of fifth order or higher and can be set to zero for this calculation.

Chapter 6

Pion Photoproduction

In this chapter the form factors derived in Chapter 5 will be used to calculate the amplitudes for the pion photoproduction reaction,

$$\gamma(k) + N(P) \to \pi^a(q) + N(P')$$

and the radiative pion capture reaction,

$$\pi^a(q) + N(P) \to \gamma(k) + N(P')$$

at fourth order. The amplitude can then be written as a low energy expansion and compared to the results obtained using the heavy baryon methods outlined in Chapter 3.

6.1 Kinematics

In the pion photoproduction amplitude calculation the external fields correspond to physical particles, and as such the momentum of each field is taken to be on shell,

$$P^2 = P'^2 = M_N^2 \qquad k^2 = 0 \qquad q^2 = m_\pi^2$$

The results of the calculation will be given in the center of mass frame, and in that frame the particle four-momenta can be written as

$$\begin{aligned}
P^\mu &= (E_N, 0, 0, -\omega) \\
P'^\mu &= (E'_N, 0, -|\mathbf{q}|\sin(\theta), -|\mathbf{q}|\cos(\theta)) \\
k^\mu &= (\omega, 0, 0, \omega) \\
q^\mu &= (E_\pi, 0, |\mathbf{q}|\sin(\theta), |\mathbf{q}|\cos(\theta))
\end{aligned} \tag{6.1}$$

where θ is the angle between the photon and pion three-momenta.

In the calculation, the amplitudes will be written in terms of the Mandelstam variables

$$\begin{aligned}
s &= (P+k)^2 = M_N^2 + 2P\cdot k \\
&= (P'+q)^2 = M_N^2 + m_\pi^2 + 2P'\cdot q
\end{aligned} \tag{6.2}$$

$$\begin{aligned}
t &= (P'-P)^2 = 2M_N^2 - 2P'\cdot P \\
&= (q-k)^2 = m_\pi^2 - 2q\cdot k
\end{aligned} \tag{6.3}$$

$$\begin{aligned}
u &= (P'-k)^2 = 2M_N^2 - 2P'\cdot k \\
&= (q-P)^2 = m_\pi^2 + M_N^2 - 2P\cdot q
\end{aligned} \tag{6.4}$$

so that the amplitude does not depend on the reference frame used. In the center of mass frame, the definition of the four-momenta given in Eq 6.1 can be used to write the Mandelstam variables as

$$\begin{aligned} s &= M_N^2 + 2\omega(E_N + \omega) = (E_N + \omega)^2 \\ &= M_N^2 + 2(E_N E_\pi + |q|^2) = (E'_N + E_\pi)^2 \end{aligned} \tag{6.5}$$

$$\begin{aligned} t &= 2(M_N^2 - E_N E'_N - \omega q \cos(\theta)) \\ &= m_\pi^2 - 2(E_\pi \omega + \omega |q| \cos(\theta)) \end{aligned} \tag{6.6}$$

$$\begin{aligned} u &= 2(M_N^2 - E'_N E_\pi - \omega q \cos(\theta)) \\ &= m_\pi^2 + M_N^2 - 2E_N E_\pi + 2\omega |q| \cos(\theta)) \end{aligned} \tag{6.7}$$

These relations will be required when the results of the amplitude calculation are compared to the results obtained using HBChPT and when the results are compared to experimental data.

In the center of mass frame, the differential cross section for pion photoproduction with the initial particles unpolarized is given by [31]

$$\begin{aligned} \left(\frac{\partial \sigma}{\partial \Omega}\right)_{CM} &= \frac{1}{4\omega E_N(1+\frac{\omega}{E_N})} \frac{|\mathrm{q}|}{16\pi^2\sqrt{s}} \frac{1}{4} \sum_{pol} |M|^2 \\ &= \frac{|q|}{64\pi^2 \omega s} \frac{1}{4} \sum_{pol} |M|^2 \end{aligned} \tag{6.8}$$

where M is the amplitude for the reaction and $|q|$ is the magnitude of the pion momentum. The squared amplitude for radiative pion capture is equivalent to the squared amplitude for pion photoproduction, and so the differential cross section is

$$\begin{aligned}\left(\frac{\partial\sigma}{\partial\Omega}\right)_{CM} &= \frac{1}{4E_\pi E'_N\left(\frac{|\mathbf{q}|}{E'_N}+\frac{|\mathbf{q}|}{E_\pi}\right)}\frac{\omega}{16\pi^2\sqrt{s}}\frac{1}{4}\sum_{pol}|M|^2 \\ &= \frac{\omega}{64\pi^2|\mathbf{q}|s}\frac{1}{4}\sum_{pol}|M|^2\end{aligned} \tag{6.9}$$

It should be noted that in [12] the nucleon fields are normalized in a different manner, resulting in a different form of the amplitude and a different relation between the differential cross section and square of the amplitude. Although this will not affect the results, it will be important when the amplitude calculated using infrared regularization is compared with the results of the HBChPT calculation given in [12].

6.2 Pion Photoproduction Amplitude

The pion photoproduction amplitude will be calculated in two parts. In the first section, the first and second order tree diagrams in Figure 6.1 will be used to calculate the amplitude to second order. The amplitudes for these diagrams do not contain loops, and as such can be calculated without using infrared regularization or heavy baryon methods. In the second section the $O(p^4)$ amplitude will be calculated using the form factors given in Chapter 5. The amplitude will then be expanded and compared with the results obtained from HBChPT [12].

6.2.1 O(p^2) Amplitude

The diagrams used in the O(p^2) amplitude calculation include the vertices

$$V^{(1)}_{\gamma\pi^+\pi^-} = -eA^\mu(\pi^-\partial_\mu\pi^+ - \pi^+\partial_\mu\pi^-)$$

$$V^{(1)}_{\gamma NN} = ie\gamma^\mu A_\mu$$

$$V^{(2)}_{\gamma NN} = -\frac{e\sigma^{\mu\nu}k_\nu}{2M_N}(c_6\frac{1+\tau_3}{2} + c_7)$$

$$V^{(1)}_{\pi^a NN} = \frac{g_A}{2F}\not{q}\gamma^5\tau_a\pi^a$$

$$V^{(1)}_{\pi^a\gamma NN} = \frac{eg_A}{2F}\gamma^\mu\gamma^5(\tau_+\pi^+ - \tau_-\pi^-)$$

which were derived in Chapter 5. Using these vertices and the Mandelstam variables defined in the previous section gives the amplitude corresponding to each diagram in Figure 6.1 as

$$M_a = -\overline{\Psi}\epsilon_\mu\frac{eg_A}{2F}\frac{1}{s-M_N^2}(\tau_a\frac{1+\tau_3}{2})\not{q}\gamma^5(\not{P}+\not{K}+M_N)\gamma^\mu\pi^a\Psi \tag{6.10}$$

$$M_b = -\overline{\Psi}\epsilon_\mu\frac{eg_A}{2F}\frac{1}{u-M_N^2}(\frac{1+\tau_3}{2}\tau_a)\gamma^\mu(\not{P}-\not{q}+M_N)\not{q}\gamma^5\pi^a\Psi \tag{6.11}$$

$$M_c = \overline{\Psi}\epsilon_\mu\frac{eg_A}{2F}\gamma^\mu\gamma^5(\tau_+\pi^+ - \tau_-\pi^-)\Psi \tag{6.12}$$

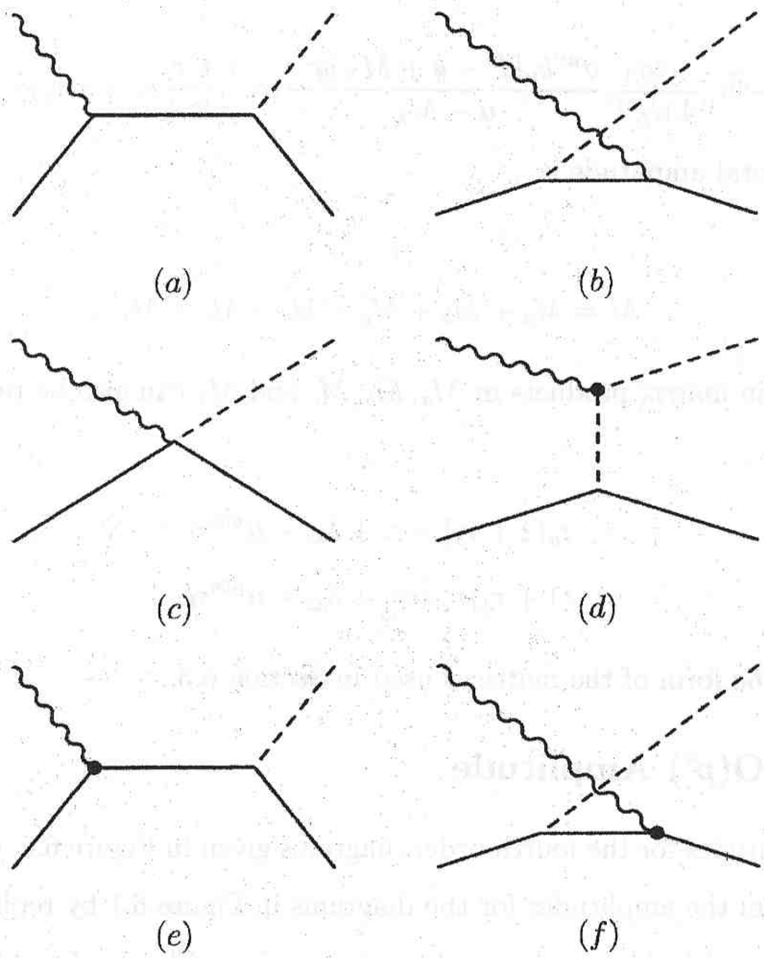

Figure 6.1: $O(p^2)$ Charged Pion Photoproduction Diagrams

$$M_d = \overline{\Psi}\epsilon_\mu \frac{eg_A}{F}\frac{q^\mu}{t-m_\pi^2}\not{q}\gamma^5(\tau_+\pi^+ - \tau_-\pi^-)\Psi \tag{6.13}$$

$$M_e = -\overline{\Psi}\epsilon_\mu \frac{ieg_A}{4M_N F}\frac{\not{q}\gamma^5(\not{P}+\not{K}+M_N)\sigma^{\mu\nu}k_\nu}{s-M_N^2}(c_6\tau_a\frac{1+\tau_3}{2} + c_7\tau_a)\pi^a\Psi \tag{6.14}$$

$$M_f = -\overline{\Psi}\epsilon_\mu \frac{ieg_A}{4M_N F}\frac{\sigma^{\mu\nu}k_\nu(\not{P}-\not{q}+M_N)\not{q}\gamma^5}{u-M_N^2}(c_6\frac{1+\tau_3}{2}\tau_a + c_7\tau_a)\pi^a\Psi \tag{6.15}$$

and the total amplitude is

$$M = M_a + M_b + M_c + M_d + M_e + M_f \tag{6.16}$$

The isospin matrix products in M_a, M_b, M_e and M_f can also be rewritten as

$$\begin{aligned}\tau_a(1+\tau_3) &= \tau_a + \delta_{a3} - i\epsilon^{a3b}\tau_b\\ (1+\tau_3)\tau_a &= \tau_a + \delta_{a3} + i\epsilon^{a3b}\tau_b\end{aligned}$$

which is the form of the matrices used in Section 6.3.

6.2.2 O(p^4) Amplitude

The amplitudes for the fourth order diagrams given in Figure 6.2, can be obtained from the amplitudes for the diagrams in Figure 6.1 by replacing each bare vertex with the appropriate form factors from Chapter 5, and multiplying each propagator and external field by the appropriate renormalization constants. The resulting amplitudes are

$$M_a = -\overline{\Psi}\epsilon_\mu \frac{e}{2F}\frac{Z_N^2\sqrt{Z_\pi}}{s-M_N^2}G_A F_1(0)\not{q}\gamma^5(\not{P}+\not{K}+M_N)\gamma^\mu\pi^a\Psi \tag{6.17}$$

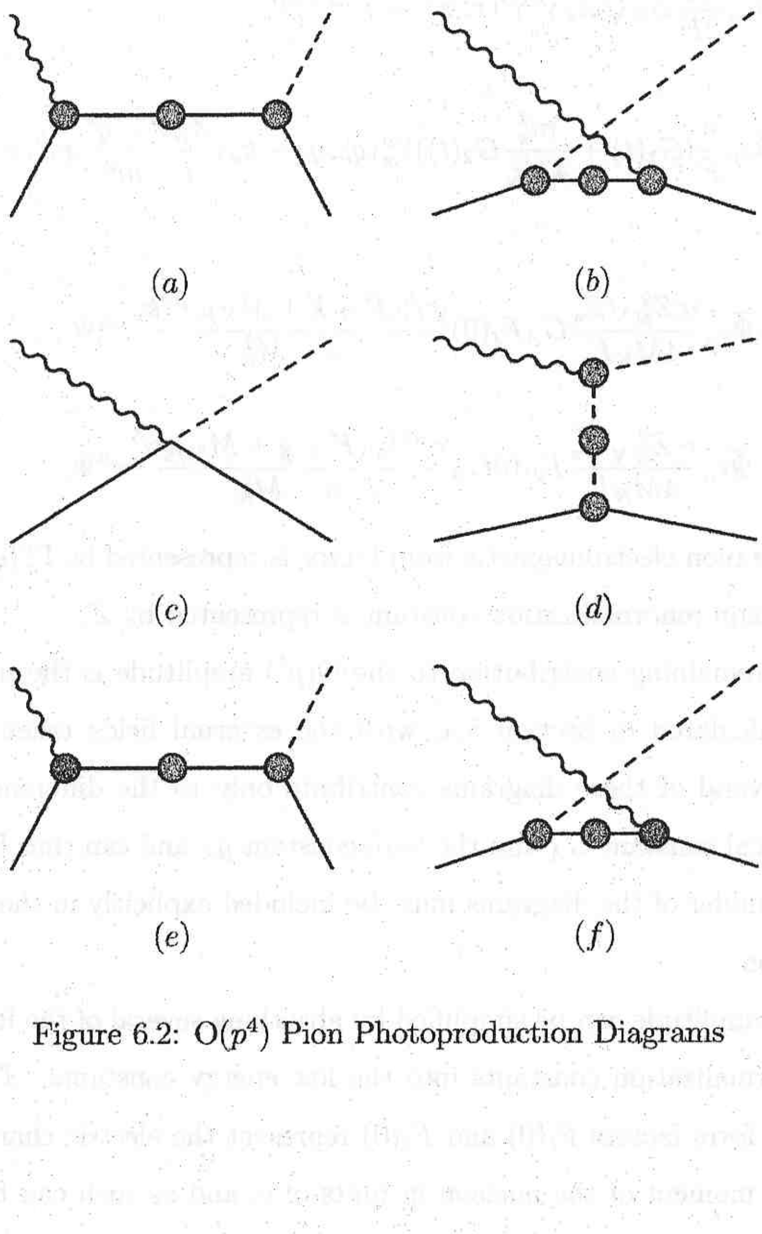

Figure 6.2: $O(p^4)$ Pion Photoproduction Diagrams

$$M_b = -\overline{\Psi}\epsilon_\mu \frac{e}{2F}\frac{Z_N^2\sqrt{Z_\pi}}{u - M_N^2} F_1(0) G_A \gamma^\mu (\not{P} - \not{q} + M_N)\not{q}\gamma^5 \pi^a \Psi \tag{6.18}$$

$$M_c = \overline{\Psi}\epsilon_\mu \frac{eg_A}{2F} Z_N \sqrt{Z_\pi}\gamma^\mu\gamma^5(\tau_+\pi^+ - \tau_-\pi^-)\Psi \tag{6.19}$$

$$M_d = \overline{\Psi}\epsilon_\mu \frac{e}{F}(G_1(t) + \frac{m_\pi^2}{4M_N^2}G_2(t))\Gamma_\pi^\mu(q_\nu, q_\nu - k_\nu)\frac{Z_N Z_\pi^{3/2} q^\mu}{t - m_\pi^2}\not{q}\gamma^5(\pi^+ - \pi^-)\Psi \tag{6.20}$$

$$M_e = -\overline{\Psi}\epsilon_\mu \frac{ieZ_N^2\sqrt{Z_\pi}}{4M_N F} G_A F_2(0)\frac{\not{q}\gamma^5(\not{P} + \not{K} + M_N)\sigma^{\mu\nu}k_\nu}{s - M_N^2}\pi^a \Psi \tag{6.21}$$

$$M_f = -\overline{\Psi}\epsilon_\mu \frac{ieZ_N^2\sqrt{Z_\pi}}{4M_N F} F_2(0) G_A \frac{\sigma^{\mu\nu}k_\nu(\not{P} - \not{q} + M_N)\not{q}\gamma^5}{u - M_N^2}\pi^a \Psi \tag{6.22}$$

where the pion electromagnetic form factor is represented by $\Gamma_\pi^\mu(q_\nu, q_\nu - k_\nu)$, and the pion renormalization constant is represented by Z_π.

The remaining contribution to the $O(p^4)$ amplitude is the pion-photon vertex calculated in Section 5.4, with the external fields taken to be on-shell. Several of these diagrams contribute only to the difference between the physical constant G_A and the bare constant g_A and can thus be omitted. The remainder of the diagrams must be included explicitly in the amplitude calculation.

The amplitude can be simplified by absorbing several of the form factors and renormalization constants into the low energy constants. The electromagnetic form factors $F_1(0)$ and $F_2(0)$ represent the electric charge and the magnetic moment of the nucleon in units of e, and as such can be replaced with the measured values. Using similiar arguments, the renormalization constants Z_N and Z_π can be absorbed in the nucleon and pion electromagnetic charge and the constant G_A.

The amplitude terms can then be written as

$$M_a = -\overline{\Psi}\epsilon_\mu \frac{eG_A}{4F}\frac{1}{s-M_N^2}\not{q}\gamma^5(\not{P}+\not{K}+M_N)\gamma^\mu(\tau_a+\delta_{a3}-i\epsilon^{a3b}\tau_b)\pi^a\Psi \quad (6.23)$$

$$M_b = -\overline{\Psi}\epsilon_\mu \frac{eG_A}{4F}\frac{1}{u-M_N^2}\gamma^\mu(\not{P}-\not{q}+M_N)\not{q}\gamma^5(\tau_a+\delta_{a3}+i\epsilon^{a3b}\tau_b)\pi^a\Psi \quad (6.24)$$

$$M_d = \overline{\Psi}\epsilon_\mu \frac{e(G_1(t)+m_\pi^2/4M_N^2G_2(t)}{F}\Gamma_\pi^\mu(q_\nu, q_\nu-k_\nu)\frac{q^\mu}{t-m_\pi^2}\not{q}\gamma^5(i\epsilon^{a3b}\tau_b)\pi^a\Psi \quad (6.25)$$

$$\begin{aligned} M_e = &-\overline{\Psi}\epsilon_\mu \frac{ieG_A}{4M_NF}\frac{\not{q}\gamma^5(\not{P}+\not{K}+M_N)\sigma^{\mu\nu}k_\nu}{s-M_N^2} \\ &\times((\mu_p+\mu_n)\tau_a+(\mu_p-\mu_n)\delta_{a3}-i(\mu_p-\mu_n)\epsilon^{a3b}\tau_b)\pi^a\Psi \end{aligned} \quad (6.26)$$

$$\begin{aligned} M_f = &-\overline{\Psi}\epsilon_\mu \frac{ieG_A}{4M_NF}\frac{\sigma^{\mu\nu}k_\nu(\not{P}-\not{q}+M_N)\not{q}\gamma^5}{u-M_N^2} \\ &\times((\mu_p+\mu_n)\tau_a+(\mu_p-\mu_n)\delta_{a3}-i(\mu_p-\mu_n)\epsilon^{a3b}\tau_b)\pi^a\Psi \end{aligned} \quad (6.27)$$

with the contact term M_c calculated separately. In M_d the pion form factors are used as the pion which is emitted by the nucleon is not on-shell. However these form factors can also be written as

$$G_1(t)+\frac{m_\pi^2}{4M_N^2}G_2(t) = G_A+(G_1(t)-G_1(m_\pi^2) \quad (6.28)$$

since the correction to G_A due to $G_2(t)$ is of higher order.

6.3 Low Energy Expansion

The pion photoproduction amplitude can now be written as a low energy expansion and compared with the results obtained from HBChPT. In this section the nucleon momentum will be taken to be nonrelativistic and will be written in terms of the nucleon four-velocity

$$P_\mu = M_N v_\mu \qquad {P'}_\mu = M_N {v'}_\mu \tag{6.29}$$

where

$$v_\mu = (1,0,0,-\frac{\omega}{E_N}) \quad {v'}_\mu = (1,0,\frac{|\mathbf{q}|\sin\theta}{{E'}_N},\frac{|\mathbf{q}|\cos\theta}{{E'}_N}) \tag{6.30}$$

At low energies the nucleon can be approximated as being at rest, and the nucleon three-momenta can be treated as a higher order correction. It should also be noted that in this form the external nucleons are taken to be slightly off-shell.

When the nucleon is at rest, the heavy baryon fields H_v and N_v correspond to the upper and lower components of the relativistic nucleon spinor which will be written as χ and η. Then at low energies, the nucleon spinors can be approximated as

$$\Psi = \begin{bmatrix} \chi \\ \eta \end{bmatrix} \to \begin{bmatrix} \chi \\ 0 \end{bmatrix} \tag{6.31}$$

The higher order effects of the spinor η are included using the separated form of Dirac's equation,

$$\sigma \cdot \mathbf{P}\eta = (P^0 - M_N)\chi \tag{6.32}$$

$$\sigma \cdot \mathbf{P}\chi = (P^0 + M_N)\eta \tag{6.33}$$

For $P^0 = M_N$, this relation becomes

$$\eta = \frac{\sigma \cdot \mathbf{P}}{2M_N}\chi \tag{6.34}$$

Since the nucleon spinor is divided into upper and lower components, it is convenient to write the Dirac matrices in terms of the Pauli matrices and the 2×2 identity matrix,

$$\gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{6.35}$$

$$\gamma^\alpha = \begin{bmatrix} 0 & \sigma^\alpha \\ -\sigma^\alpha & 0 \end{bmatrix} \quad \alpha = 1, 2, 3 \tag{6.36}$$

$$\gamma^5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{6.37}$$

Each of the remaining quantities in the amplitude can be expanded as a low energy power series, with $O(p^5)$ terms omitted. Using the non-relativistic off-shell momentum $P_\mu = M_N v_\mu + q_\mu$, the nucleon propagator can be written as

$$\frac{\not{P} + M_N}{P^2 - M_N^2} = \frac{M_N(1+\not{v}) + \not{q}}{2M_N v \cdot q} - \frac{q^2(M_N(1+\not{v}) + \not{q})}{(2M_N v \cdot q)^2} + \frac{q^4(M_N(1+\not{v}) + \not{q})}{(2M_N v \cdot q)^2} + O(p^5) \tag{6.38}$$

Using the kinematics given in Section 6.1 and the approximation $P^0 = P'^0 = M_N$, the factors of $1/v' \cdot q$ and $1/v \cdot q$ can be expanded as

$$\begin{aligned}\frac{1}{2M_N v' \cdot q} &= \frac{1}{2M_N(E_\pi + \frac{|\mathbf{q}|^2}{M_N})} \\ &= \frac{1}{2M_N E_\pi} - \frac{|\mathbf{q}|^2}{2M_N^2 E_\pi^2} + O(p^4)\end{aligned} \tag{6.39}$$

$$\begin{aligned}\frac{1}{2M_N v \cdot q} &= \frac{1}{2M_N(E_\pi + \frac{x\omega|\mathbf{q}|}{M_N})} \\ &= \frac{1}{2M_N E_\pi} - \frac{x\omega|\mathbf{q}|}{2M_N^2 E_\pi^2} + O(p^4)\end{aligned} \tag{6.40}$$

The propagators can also be expressed in terms of the photon four-momentum k_μ, with a similiar expansion. The low energy expansions of the infrared regularized loop integrals are given in Appendix A.4.

In [12] the amplitude and the renormalization of the nucleon spinors are not defined the same as in Section 6.2.1. The relation between the amplitude defined in [12] and the amplitudes derived in Section 6.2.1 and Section 6.2.2 is

$$M^{\gamma N \to \pi N} = \frac{1}{8\pi\sqrt{s}} M \tag{6.41}$$

and the nucleon spinor referred to as χ in [12] is equivalent to $\frac{\chi}{\sqrt{2M_N}}$ using the definition in Eq 6.31.

The amplitude can be written in terms of four structure amplitudes,

$$\begin{aligned} M^{\gamma N\to\pi N} &= \frac{M_N}{4\pi\sqrt{s}} T\cdot\epsilon \\ &= F_1(E_\pi,x) i\chi^\dagger\sigma\cdot\epsilon\chi + F_2(E_\pi,x)\chi^\dagger\sigma\cdot\hat{q}\sigma\cdot(\hat{k}\times\epsilon)\chi \\ &+ F_3(E_\pi,x) i\chi^\dagger\sigma\cdot\hat{k}\epsilon\cdot\hat{q}\chi + F_4(E_\pi,x) i\chi^\dagger\sigma\cdot\hat{q}\epsilon\cdot\hat{q}\chi \end{aligned} \tag{6.42}$$

where $x = \cos\theta$ measures the angle between the pion and photon momentum. Each structure amplitude is further divided into the three isospin channels,

$$F_i^a(E_\pi,x) = F_i^{(-)}(E_\pi,x) i\epsilon^{a3b}\tau_b + F_i^{(0)}(E_\pi,x)\tau_a + F_i^{(+)}(E_\pi,x)\delta_{a3} \tag{6.43}$$

The structure amplitudes written in terms of the physical pion states are

$$F_i^{\gamma n\to\pi^- p} = \sqrt{2}\left[F_i^{(0)} - F_i^{(-)}\right] \tag{6.44}$$

$$F_i^{\gamma p\to\pi^+ n} = \sqrt{2}\left[F_i^{(0)} + F_i^{(-)}\right] \tag{6.45}$$

$$F_i^{\gamma n\to\pi^0 n} = F_i^{(+)} + F_i^{(0)} \tag{6.46}$$

$$F_i^{\gamma p\to\pi^0 n} = F_i^{(+)} - F_i^{(0)} \tag{6.47}$$

The first and second order contributions to the structure amplitudes are the leading order terms in the low energy expansion of the amplitude given in Section 6.2.1. The amplitude M_a can be written as

$$
\begin{aligned}
M_a = & -\overline{\Psi}\epsilon_\mu \frac{eG_A}{4F}\left(\frac{1}{2M_N v'\cdot q} - \frac{m_\pi^2}{4M_N^2 v'\cdot q}\right)\not{q}\gamma^5(M_N\not{v}+\not{q}+M_N)\gamma^\mu \\
& \times(\tau_a+\delta_{a3}-i\epsilon^{a3b}\tau_b)\pi^a\Psi
\end{aligned} \tag{6.48}
$$

When the Dirac matrices are written in terms of the Pauli matrices, the product of the matrices can be written as

$$
\begin{aligned}
& \Psi\not{q}\gamma^5(M_N(1+\not{v})+\not{q})\not{\epsilon} \\
& = \chi^\dagger \left[\begin{array}{cc} 1 & -\frac{\sigma\cdot\mathbf{q}}{2M_N}\end{array}\right]\left[\begin{array}{cc} \sigma\cdot\mathbf{q} & E_\pi \\ -E_\pi & -\sigma\cdot\mathbf{q}\end{array}\right]\left[\begin{array}{cc} 2M_N+E_\pi & 0 \\ 0 & -E_\pi\end{array}\right] \\
& \times\left[\begin{array}{cc} 0 & \sigma\cdot\epsilon \\ -\sigma\cdot\epsilon & 0\end{array}\right]\left[\begin{array}{c} 1 \\ \frac{\sigma\cdot\mathbf{k}}{2M_N}\end{array}\right]\chi \\
& = \chi^\dagger(E_\pi^2\sigma\cdot\epsilon - \frac{E_\pi|\mathbf{q}|^2}{2M_N}\sigma\cdot\epsilon + \omega|\mathbf{q}|\sigma\cdot\hat{q}\sigma\cdot\epsilon\sigma\cdot\hat{k} \\
& -(\frac{\omega E_\pi^2|\mathbf{q}|}{4M_N^2})\sigma\cdot\hat{q}\sigma\cdot\epsilon\sigma\cdot\hat{k})\chi
\end{aligned} \tag{6.49}
$$

The fourth term is of higher order and can be dropped. The third term can be removed using a specific gauge which corresponds to the gauge $v\cdot\epsilon=0$ that was used in [12]. Then the amplitude is given by

$$
\begin{aligned}
M_a = & -\frac{eG_A}{4F}\left(\frac{E_\pi(E_\pi-|\mathbf{q}|^2/2M_N)}{2M_N v'\cdot q} - \frac{m_\pi^2 E_\pi^2}{4M_N^2(v'\cdot q)^2}\right)\chi^\dagger\sigma\cdot\epsilon\chi \\
& \times(\tau_a+\delta_{a3}-i\epsilon^{a3b}\tau_b)\pi^a
\end{aligned} \tag{6.50}
$$

$$
= -\frac{eG_A}{4F}\left(\frac{E_\pi}{2M_N} - \frac{3|\mathbf{q}|^2+m_\pi^2}{4M_N^2}\right)\chi^\dagger\sigma\cdot\epsilon\chi(\tau_a+\delta_{a3}-i\epsilon^{a3b}\tau_b)\pi^a
$$

and the contribution to the $O(p^4)$ structure amplitudes is

$$F_1^{(0)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}(-\frac{E_\pi}{4M_N} + \frac{3|\mathbf{q}|^2 + m_\pi^2}{8M_N^2}) \tag{6.51}$$

$$F_1^{(+)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}(-\frac{E_\pi}{4M_N} + \frac{3|\mathbf{q}|^2 + m_\pi^2}{8M_N^2}) \tag{6.52}$$

$$F_1^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}(-\frac{E_\pi}{4M_N} + \frac{3|\mathbf{q}|^2 + m_\pi^2}{8M_N^2}) \tag{6.53}$$

The same procedure can be used to derive the expansion of the amplitude M_b

$$\begin{aligned} M_b = & -\frac{eG_A}{4F}\frac{1}{2M_N v\cdot q} + \frac{m^2}{(2M_N v\cdot q)^2}\chi^\dagger((E_\pi^2 - \frac{E_\pi|\mathbf{q}|^2}{M_N})\sigma\cdot\epsilon \\ & - 2|\mathbf{q}|^2(1-\frac{E_\pi}{M_N})\hat{q}\cdot\epsilon\sigma\cdot\hat{q})\chi(\tau_a + \delta_{a3} + i\epsilon^{a3b}\tau_b)\pi^a \end{aligned} \tag{6.54}$$

$$\begin{aligned} = & -\frac{eG_A}{4F}(\frac{1}{2M_N E_\pi} - \frac{2x\omega|\mathbf{q}| - m_\pi^2}{4M_N^2 E_\pi^2})\chi^\dagger((E_\pi^2 - \frac{E_\pi|\mathbf{q}|^2}{M_N})\sigma\cdot\epsilon \\ & - 2|\mathbf{q}|^2(1-\frac{E_\pi}{M_N})\hat{q}\cdot\epsilon\sigma\cdot\hat{q})\chi(\tau_a + \delta_{a3} + i\epsilon^{a3b}\tau_b)\pi^a \end{aligned}$$

and the contribution to the structure amplitudes from M_b,

$$F_1^{(0)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}(-\frac{E_\pi}{4M_N} + \frac{|\mathbf{q}|^2}{2M_N^2} + \frac{2x\omega|\mathbf{q}| - m_\pi^2}{8M_N^2 E_\pi}) \tag{6.55}$$

$$F_1^{(+)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}(-\frac{E_\pi}{4M_N} + \frac{|\mathbf{q}|^2}{2M_N^2} + \frac{2x\omega|\mathbf{q}| - m_\pi^2}{8M_N^2 E_\pi}) \tag{6.56}$$

$$F_1^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}(\frac{E_\pi}{4M_N} - \frac{|\mathbf{q}|^2}{2M_N^2} - \frac{2x\omega|\mathbf{q}| - m_\pi^2}{8M_N^2 E_\pi}) \tag{6.57}$$

$$F_4^{(0)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\left(\frac{|\mathbf{q}|^2(1-E_\pi/M_N)}{2M_N E_\pi} - \frac{2x\omega|\mathbf{q}|^3 - |\mathbf{q}|^2 m_\pi^2}{4M_N^2 E_\pi^2}\right) \tag{6.58}$$

$$F_4^{(+)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\left(\frac{|\mathbf{q}|^2(1-E_\pi/M_N)}{2M_N E_\pi} - \frac{2x\omega|\mathbf{q}|^3 - |\mathbf{q}|^2 m_\pi^2}{4M_N^2 E_\pi^2}\right) \tag{6.59}$$

$$F_4^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\left(-\frac{|\mathbf{q}|^2(1-E_\pi/M_N)}{2M_N E_\pi} + \frac{2x\omega|\mathbf{q}|^3 - |\mathbf{q}|^2 m_\pi^2}{4M_N^2 E_\pi^2}\right) \tag{6.60}$$

The low energy expansion of M_c is

$$M_c = \frac{eg_A}{2F}\chi^\dagger(\sigma\cdot\epsilon + \frac{\omega|\mathbf{q}|}{4M_N^2}\sigma\cdot\hat{q}\sigma\cdot\epsilon\sigma\cdot\hat{k})\chi(i\epsilon^{a3b}\tau_b) \tag{6.61}$$

Then by using the identity

$$\sigma\cdot\hat{k}\sigma\cdot\epsilon = \hat{k}\cdot\epsilon + i\sigma\cdot(\hat{k}\times\epsilon)$$

and $\hat{k}\cdot\epsilon = 0$, the contribution to $F_1^{(-)}$ and $F_2^{(-)}$ is

$$F_1^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F} \tag{6.62}$$

$$F_2^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\left(-\frac{\omega|\mathbf{q}|}{4M_N^2}\right) \tag{6.63}$$

The low energy expansion of M_d is

$$M_d = \frac{eg_A}{F}\frac{1}{2q\cdot k}\chi^\dagger(\sigma\cdot(\mathbf{q}-\mathbf{k})\epsilon\cdot\mathbf{q})\chi(i\epsilon^{a3b}\tau_b) \tag{6.64}$$

which gives the contribution to $F_3^{(-)}$ and $F_4^{(-)}$,

$$F_3^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eg_A}{2F}\left(\frac{|\mathbf{q}|}{(E_\pi - |\mathbf{q}|x)} + \frac{m_\pi^2|\mathbf{q}|}{4M_N^2(E_\pi - x|\mathbf{q}|)}\right) \tag{6.65}$$

$$F_4^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eg_A}{2F}\left(-\frac{|\mathbf{q}|^2}{\omega(E_\pi - |\mathbf{q}|x)} - \frac{m_\pi^2|\mathbf{q}|^2}{4M_N^2\omega(E_\pi - x|\mathbf{q}|)}\right) \tag{6.66}$$

or using the expansion

$$\frac{1}{\omega} = \frac{1}{E_\pi} + \frac{m_\pi^2}{2M_N E_\pi^2} - \frac{m_\pi^2}{2M_N^2 E_\pi} + \frac{3m_\pi^2}{2M_N^3} - \frac{5m_\pi^2 E_\pi}{3M_N^4} + \frac{m_\pi^4}{4M_N^2 E_\pi^3} \tag{6.67}$$

the contribution to $F_4^{(-)}$ is

$$\begin{aligned} F_4^{(-)} =& \frac{M_N}{4\pi\sqrt{s}}\frac{eg_A}{2F}\left(-\frac{|\mathbf{q}|^2}{(E_\pi - |\mathbf{q}|x)}\right)\left(\frac{1}{E_\pi} + \frac{m_\pi^2}{2M_N E_\pi^2} - \frac{m_\pi^2}{2M_N^2 E_\pi} + \frac{3m_\pi^2}{2M_N^3}\right. \\ &\left. + \frac{m_\pi^4}{4M_N^2 E_\pi^3} - \frac{5m_\pi^2 E_\pi}{3M_N^4} - \frac{m_\pi^4}{2E_\pi^2 M_N^3}\right) \end{aligned} \tag{6.68}$$

The remaining second order contributions to the structure amplitudes are the leading order terms in the expansions of M_e and M_f. At lowest order the low energy constants c_6 and c_7 can be written in terms of the nucleon magnetic moments,

$$c_6 = \mu_p - \mu_n \qquad c_7 = \mu_n \tag{6.69}$$

The contribution to the structure amplitudes from the sum $M_e + M_f$ is

$$F_1^{(0)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\left\{\frac{\omega|\mathbf{q}|(1+E_\pi/2M_N)}{2M_N E_\pi} - \frac{x\omega^2|\mathbf{q}|^2}{2M_N^2 E_\pi^2} + \frac{m_\pi^2\omega|\mathbf{q}|}{4M_N^2 E_\pi^2}\right\}(\mu_p + \mu_n) \tag{6.70}$$

$$F_1^{(+)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{\omega|\mathbf{q}|(1+E_\pi/2M_N)}{2M_N E_\pi} - \frac{x\omega^2|\mathbf{q}|^2}{2M_N^2 E_\pi^2} + \frac{m_\pi^2\omega|\mathbf{q}|}{4M_N^2 E_\pi^2}\}(\mu_p - \mu_n) \tag{6.71}$$

$$F_1^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{\omega|\mathbf{q}|(1+E_\pi/2M_N)}{2M_N E_\pi} + \frac{x\omega^2|\mathbf{q}|^2}{2M_N^2 E_\pi^2} - \frac{m_\pi^2\omega|\mathbf{q}|}{4M_N^2 E_\pi^2}\}(\mu_p - \mu_n) \tag{6.72}$$

$$F_2^{(0)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{x|\mathbf{q}|^2}{4M_N^2}\}(\mu_p + \mu_n) \tag{6.73}$$

$$F_2^{(+)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{x|\mathbf{q}|^2}{4M_N^2}\}(\mu_p - \mu_n) \tag{6.74}$$

$$F_2^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{\omega|\mathbf{q}|}{2M_N E_\pi} + \frac{(E_\pi - x|\mathbf{q}|)|\mathbf{q}|}{4M_N^2} - \frac{\omega E_\pi|\mathbf{q}|}{8M_N^3}\}(\mu_p - \mu_n) \tag{6.75}$$

$$F_3^{(0)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{\omega|\mathbf{q}|(1-E_\pi/2M_N)}{2M_N E_\pi} + \frac{x\omega^2|\mathbf{q}|^2}{2M_N^2 E_\pi^2} - \frac{m_\pi^2\omega|\mathbf{q}|}{4M_N^2 E_\pi^2} - \frac{\omega(1+\omega/2M_N)|\mathbf{q}|}{4M_N^2}\}(\mu_p + \mu_n) \tag{6.76}$$

$$F_3^{(+)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{\omega|\mathbf{q}|(1-E_\pi/2M_N)}{2M_N E_\pi} + \frac{x\omega^2|\mathbf{q}|^2}{2M_N^2 E_\pi^2} - \frac{m_\pi^2\omega|\mathbf{q}|}{4M_N^2 E_\pi^2} - \frac{\omega(1+\omega/2M_N)|\mathbf{q}|}{4M_N^2}\}(\mu_p - \mu_n) \tag{6.77}$$

$$F_3^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{\omega|\mathbf{q}|(1-E_\pi/2M_N)}{2M_N E_\pi} - \frac{x\omega^2|\mathbf{q}|^2}{2M_N^2 E_\pi^2} + \frac{m_\pi^2\omega|\mathbf{q}|}{4M_N^2 E_\pi^2} + \frac{\omega(1+\omega/2M_N)|\mathbf{q}|}{4M_N^2}\}(\mu_p - \mu_n) \tag{6.78}$$

$$F_4^{(0)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{|\mathbf{q}|^2(1-E_\pi/2M_N)}{4M_N^2}\}(\mu_p+\mu_n) \tag{6.79}$$

$$F_4^{(+)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{|\mathbf{q}|^2(1-E_\pi/2M_N)}{4M_N^2}\}(\mu_p-\mu_n) \tag{6.80}$$

$$F_4^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{|\mathbf{q}|^2(1-E_\pi/2M_N)}{4M_N^2}\}(\mu_p-\mu_n) \tag{6.81}$$

When the higher order irreducible diagrams are included, the $O(p^4)$ structure amplitudes are given by

$$\begin{aligned}
F_1^{(0)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{&-\frac{E_\pi}{2M_N}+\frac{2x\omega|\mathbf{q}|-m_\pi^2}{8M_N^2E_\pi} \\
&+\frac{\omega|\mathbf{q}|(1+E_\pi/M_N)}{2M_NE_\pi}(\mu_p+\mu_n)+\frac{2xE_\pi|\mathbf{q}|b_{10}}{G_A(4\pi F)^2}+\frac{7|\mathbf{q}|^2}{8M_N^2} \\
&-\frac{x\omega^2|\mathbf{q}|^2}{2M_N^2E_\pi^2}(\mu_p+\mu_n)+\frac{m_\pi^2\omega|\mathbf{q}|}{4M_N^2E_\pi^2}(\mu_p+\mu_n) \\
&-\frac{G_A^2}{2F^2}(\frac{m_\pi^2}{2M_N}I(M_N^2)+m_\pi^2I^{(1)}(M_N^2)-4E_\pi I_{21}^{00}(I_{21}^{00}(0)) \\
&+\frac{8b_1M_N^2m_\pi^2}{(4\pi F^2)F^2}(\Delta_\pi-2M_N^2I^{(1)}(M_N^2))+\frac{4b_3E_\pi M_N^2}{(4\pi F)^2F^2}I^{(2)}(M_N^2) \\
&+\frac{\omega(\mu_p+\mu_n)}{8M_NF^2}(\Delta_\pi+(4M_N^2+E_\pi-x|\mathbf{q}|)I^{(1)}(M_N^2)) \\
&-\frac{4e_{48}\omega E_\pi(E_\pi-x|\mathbf{q}|)}{G_A}-\frac{8e_{50}\omega E_\pi^2}{3G_A}+\frac{2e_{51}\omega^2(E_\pi-x|\mathbf{q}|)}{G_A}\}
\end{aligned} \tag{6.82}$$

$$\begin{aligned}
F_1^{(+)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{&-\frac{E_\pi}{2M_N}+\frac{2x\omega|\mathbf{q}|-m_\pi^2}{8M_N^2E_\pi} \\
&+\frac{\omega|\mathbf{q}|(1+E_\pi/M_N)}{2M_NE_\pi}(\mu_p-\mu_n)+\frac{2xE_\pi|\mathbf{q}|b_9}{G_A(4\pi F)^2}+\frac{7|\mathbf{q}|^2}{8M_N^2} \\
&-\frac{x\omega^2|\mathbf{q}|^2}{2M_N^2E_\pi^2}(\mu_p-\mu_n)+\frac{m_\pi^2\omega|\mathbf{q}|}{4M_N^2E_\pi^2}(\mu_p-\mu_n)
\end{aligned} \tag{6.83}$$

$$
\begin{aligned}
&+\frac{G_A^2}{2F^2}(\frac{m_\pi^2}{2M_N}I(M_N^2)+(m_\pi^2-2\omega(E_\pi-x|\mathbf{q}|))I^{(1)}(M_N^2)\\
&+2M_N|\mathbf{q}|^2I_A(t)-4E_\pi I_{21}^{00}(0))-\frac{1}{2F^2}(\Delta_\pi+2M_N^2I^{(1)}(M_N^2)\\
&+2M_N^2I_{21}^{00}(0)-8M_N^3I_{21}^{0}(0)+4M_N^2\omega I_{21}^{Q}(0))\\
&+\frac{G_A^2}{4F^2}(x\omega|\mathbf{q}|I^{(1)}(M_N^2)+2m_\pi^2\omega(1-\frac{|\mathbf{q}|}{2M_N})I_A^{(1)}(t))(\mu_p-\mu_n)\\
&+\frac{8b_1M_N^2m_\pi^2}{(4\pi F^2)F^2}(\Delta_\pi-2M_N^2I^{(1)}(M_N^2))+\frac{4b_3E_\pi M_N^2}{(4\pi F)^2F^2}I^{(2)}(M_N^2)\\
&-\frac{\omega(\mu_p-\mu_n)}{8M_NF^2}(\Delta_\pi+(4M_N^2+E_\pi-x|\mathbf{q}|)I^{(1)}(M_N^2))\\
&-\frac{4e_{67}\omega E_\pi(E_\pi-x|\mathbf{q}|)}{G_A}-\frac{8e_{69}\omega E_\pi^2}{3G_A}+\frac{2e_{70}\omega^2(E_\pi-x|\mathbf{q}|)}{G_A}\}
\end{aligned}
$$

$$
\begin{aligned}
F_1^{(-)}=&\frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{1-\frac{2x\omega|\mathbf{q}|-m_\pi^2}{8M_N^2E_\pi}-\frac{\omega|\mathbf{q}|(1+E_\pi/M_N)}{2M_NE_\pi}(\mu_p-\mu_n)\\
&-\frac{2m_\pi^2b_{19}}{G_A(4\pi F)^2}+\frac{2E_\pi^2}{G_A(4\pi F)^2}(b_{21}^r(\mu)-\frac{G_A}{2}(1+G_A)\log\frac{m_\pi^2}{\mu})\\
&+\frac{E_\pi(E_\pi-x|\mathbf{q}|)}{G_A(4\pi F)^2}(2b_{22}^r+b_{23}+G_A^3\log\frac{m_\pi^2}{\mu})\\
&-\frac{|\mathbf{q}|^2}{8M_N^2}+\frac{x\omega^2|\mathbf{q}|^2}{2M_N^2E_\pi^2}(\mu_p-\mu_n)-\frac{m_\pi^2\omega|\mathbf{q}|}{4M_N^2E_\pi^2}(\mu_p-\mu_n)\\
&-\frac{G_A^2}{8F^2}(x\omega|\mathbf{q}|I^{(1)}(M_N^2)+2m_\pi^2\omega(1-\frac{|\mathbf{q}|}{2M_N})I_A^{(1)}(t))(\mu_p+\mu_n)\\
&+\frac{G_A^2}{2F^2}(-\frac{m_\pi^2}{2M_N}I(M_N^2)+m_\pi^2I^{(1)}(M_N^2)-4E_\pi I_{21}^{00}(0))\}
\end{aligned}
\tag{6.84}
$$

$$
\begin{aligned}
F_2^{(0)}=&\frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{E_\pi|\mathbf{q}|}{8M_N^2}-\frac{x|\mathbf{q}|^2}{4M_N^2}(\mu_p+\mu_n)+\frac{2E_\pi|\mathbf{q}|b_{10}}{G_A(4\pi F)^2}\\
&+\frac{2G_A^2\omega|\mathbf{q}|}{M_NF^2}I_{21}^{00}(0)+\frac{G_A^2m_\pi^2\omega|\mathbf{q}|}{4M_NF^2}I_A^{(1)}(t)\}
\end{aligned}
\tag{6.85}
$$

$$
F_2^{(+)}=\frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{E_\pi|\mathbf{q}|}{8M_N^2}-\frac{x|\mathbf{q}|^2}{4M_N^2}\}(\mu_p-\mu_n)+\frac{2E_\pi|\mathbf{q}|b_9}{G_A(4\pi F)^2}
\tag{6.86}
$$

$$+\frac{4M_N\omega|\mathbf{q}|^2}{F^2}+\frac{2G_A^2\omega|\mathbf{q}|}{M_NF^2}I_{21}^{00}(0)\}$$

$$\begin{aligned}F_2^{(-)} = {} & \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{\omega|\mathbf{q}|}{2M_NE_\pi}(\mu_p-\mu_n)+\frac{(E_\pi-x|\mathbf{q}|)|\mathbf{q}|}{4M_N^2}(\mu_p-\mu_n) \\ & -\frac{\omega E_\pi|\mathbf{q}|}{8M_N^3}(\mu_p-\mu_n)-\frac{\omega|\mathbf{q}|}{4M_N^2}-\frac{2G_A^2\omega|\mathbf{q}|}{M_NF^2}I_{21}^{00}(0)-\frac{G_A^2m_\pi^2\omega|\mathbf{q}|}{4M_NF^2}I_A^{(1)}(t)\}\end{aligned} \tag{6.87}$$

$$\begin{aligned}F_3^{(0)} = {} & \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{\omega|\mathbf{q}|(1-E_\pi/2M_N)}{2M_NE_\pi}(\mu_p+\mu_n)-\frac{2E_\pi|\mathbf{q}|b_{10}}{G_A(4\pi F)^2} \\ & +\frac{x\omega^2|\mathbf{q}|^2}{2M_N^2E_\pi^2}(\mu_p+\mu_n)-\frac{m_\pi^2\omega|\mathbf{q}|}{4M_N^2E_\pi^2}(\mu_p+\mu_n)-\frac{\omega(1+\omega/2M_N)|\mathbf{q}|}{4M_N^2}(\mu_p+\mu_n) \\ & -\frac{\omega|\mathbf{q}|(\mu_p+\mu_n)}{2F^2}I^{(1)}(M_N^2) \\ & +\frac{4e_{48}\omega E_\pi|\mathbf{q}|}{G_A}\}\end{aligned} \tag{6.88}$$

$$\begin{aligned}F_3^{(+)} = {} & \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{\omega|\mathbf{q}|(1-E_\pi/2M_N)}{2M_NE_\pi}(\mu_p-\mu_n)-\frac{2E_\pi|\mathbf{q}|b_9}{G_A(4\pi F)^2} \\ & -\frac{x\omega^2|\mathbf{q}|^2}{2M_N^2E_\pi^2}(\mu_p-\mu_n)+\frac{m_\pi^2\omega|\mathbf{q}|}{4M_N^2E_\pi^2}(\mu_p-\mu_n)+\frac{\omega(1+\omega/2M_N)|\mathbf{q}|}{4M_N^2}(\mu_p-\mu_n) \\ & +\frac{G_A^2}{2F^2}\frac{\omega|\mathbf{q}|}{2}I^{(1)}(M_N^2)+\frac{\omega|\mathbf{q}|(\mu_p-\mu_n)}{2F^2}I^{(1)}(M_N^2) \\ & +\frac{4e_{67}\omega E_\pi|\mathbf{q}|}{G_A}\}\end{aligned} \tag{6.89}$$

$$\begin{aligned}F_3^{(-)} = {} & \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{|\mathbf{q}|}{(E_\pi-|\mathbf{q}|x)}-\frac{\omega|\mathbf{q}|(1-E_\pi/2M_N)}{2M_NE_\pi}(\mu_p-\mu_n) \\ & -\frac{G_A^2E_\pi|\mathbf{q}|}{(4\pi F)^2}-\frac{2m_\pi^2|\mathbf{q}|b_{19}}{G_A(4\pi F)^2(E_\pi-x|\mathbf{q}|)} \\ & +\frac{E_\pi|\mathbf{q}|}{G_A(4\pi F)^2}(2b_{22}^r(\mu)+b_{23}+G_A^3\log\frac{m_\pi^2}{\mu}) \\ & +\frac{m_\pi^2|\mathbf{q}|}{4M_N^2(E_\pi-x|\mathbf{q}|)}+\frac{x\omega^2|\mathbf{q}|^2}{2M_N^2E_\pi^2}(\mu_p-\mu_n)-\frac{G_A^2}{4F^2}\frac{\omega|\mathbf{q}|}{2}I^{(1)}(M_N^2)\end{aligned} \tag{6.90}$$

$$- \frac{m_\pi^2 \omega |\mathbf{q}|}{4M_N^2 E_\pi^2}(\mu_p - \mu_n) - \frac{\omega(1+\omega/2M_N)|\mathbf{q}|}{4M_N^2}(\mu_p - \mu_n)\}$$

$$F_4^{(0)} = \frac{M_N}{4\pi\sqrt{s}} \frac{eG_A}{2F} \{ \frac{|\mathbf{q}|^2(1-E_\pi/M_N)}{2M_N E_\pi} - \frac{2x\omega|\mathbf{q}|^3 - |\mathbf{q}|^2 m_\pi^2}{4M_N^2 E_\pi^2} - \frac{|\mathbf{q}|^2(1-E_\pi/2M_N)}{4M_N^2}(\mu_p + \mu_n) + \frac{4e_{49}\omega|\mathbf{q}|^2}{G_A}\} \tag{6.91}$$

$$F_4^{(+)} = \frac{M_N}{4\pi\sqrt{s}} \frac{eG_A}{2F} \{ \frac{|\mathbf{q}|^2(1-E_\pi/M_N)}{2M_N E_\pi} - \frac{2x\omega|\mathbf{q}|^3 - |\mathbf{q}|^2 m_\pi^2}{4M_N^2 E_\pi^2} - \frac{|\mathbf{q}|^2(1-E_\pi/2M_N)}{4M_N^2}(\mu_p - \mu_n) + \frac{G_A^2}{2F^2}\frac{|\mathbf{q}|^2}{2} I^{(1)}(M_N^2) + \frac{4e_{68}\omega|\mathbf{q}|^2}{G_A}\} \tag{6.92}$$

$$\begin{aligned} F_4^{(-)} = & \frac{M_N}{4\pi\sqrt{s}} \frac{eG_A}{2F} \{ - \frac{|\mathbf{q}|^2(1-E_\pi/M_N)}{2M_N E_\pi} \\ & - \frac{|\mathbf{q}|^2}{(E_\pi - |\mathbf{q}|x)}(\frac{1}{E_\pi} + \frac{m_\pi^2}{2M_N E_\pi^2} \\ & - \frac{m_\pi^2}{2M_N^2 E_\pi} + \frac{3m_\pi^2}{2M_N^3} + \frac{m_\pi^4}{4M_N^2 E_\pi^3} - \frac{5m_\pi^2 E_\pi}{3M_N^4} - \frac{m_\pi^4}{2E_\pi^2 M_N^3}) + \frac{|\mathbf{q}|^2(1-E_\pi/2M_N)}{4M_N^2}(\mu_p - \mu_n) \\ & + \frac{2m_\pi^2|\mathbf{q}|^2 b_{19}}{G_A(4\pi F)^2 E_\pi(E_\pi - x|\mathbf{q}|)} + \frac{2x\omega|\mathbf{q}|^3 - |\mathbf{q}|^2 m_\pi^2}{4M_N^2 E_\pi^2} - \frac{|\mathbf{q}|^2}{2F} J(0) \\ & - \frac{G_A^2}{4F^2}\frac{|\mathbf{q}|^2}{2} I^{(1)}(M_N^2)\} \end{aligned} \tag{6.93}$$

The two low energy constants, b_{21} and b_{22}, are renormalized to remove higher order divergences and as such are dependent upon an energy scale μ.

Using the low energy expansion of the loop integrals and the expansion of ω in terms of E_π and m_π, the contribution to the O(p^3) structure amplitudes from the third order diagrams as

$$F_1^{(0)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{-E_\pi}{2M_N} + \frac{x|\mathbf{q}|}{2M_N}(\mu_p + \mu_n) + \frac{2xE_\pi|\mathbf{q}|b_{10}}{G_A(4\pi F)^2} \tag{6.94}$$
$$+ \frac{1}{4M_N^2}\left[-|\mathbf{q}|^2 - \frac{1}{2}xE_\pi|\mathbf{q}| + (2E_\pi^2 - m_\pi^2 + xE_\pi|\mathbf{q}| - 2x^2|\mathbf{q}|^2)(\mu_p + \mu_n)\right]\}$$

$$F_1^{(+)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{-E_\pi}{2M_N} + \frac{x|\mathbf{q}|}{2M_N}(\mu_p - \mu_n) + \frac{2xE_\pi|\mathbf{q}|b_9}{G_A(4\pi F)^2} \tag{6.95}$$
$$+ \frac{1}{4M_N^2}\left[-|\mathbf{q}|^2 - \frac{1}{2}xE_\pi|\mathbf{q}|(2E_\pi^2 - m_\pi^2 + xE_\pi|\mathbf{q}| - 2x^2|\mathbf{q}|^2)(\mu_p - \mu_n)\right]\}$$

$$F_1^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{1 - \frac{x|\mathbf{q}|}{2M_N}(\mu_p - \mu_n) \tag{6.96}$$
$$+ \frac{1}{4M_N^2}\left[E_\pi^2 - \frac{1}{2}m_\pi^2 + (2E_\pi^2 - m_\pi^2 + xE_\pi|\mathbf{q}| - 2x^2|\mathbf{q}|^2)(\mu_p - \mu_n)\right]$$
$$- \frac{2m_\pi^2 b_{19}}{G_A(4\pi F)^2} + \frac{2E_\pi^2}{G_A(4\pi F)^2}(b_{21}^r - \frac{G_A}{2}(1 + G_A^2)\ln\frac{m_\pi^2}{M_N^2})$$
$$+ \frac{E_\pi(E_\pi - x|\mathbf{q}|)}{G_A(4\pi F)^2}(2b_{22}^r + b_{23} + G_A^3\ln\frac{m_\pi^2}{M_N^2}) + \frac{1}{4(4\pi F)^2}(\pi^2 m_\pi^2$$
$$- 8E_\pi|\mathbf{q}|\log(\frac{E_\pi + |\mathbf{q}|}{m_\pi}) + 4i\pi m_\pi^2\log(\frac{E_\pi + |\mathbf{q}|}{m_\pi}) - 4m_\pi^2\ln(\frac{E_\pi + |\mathbf{q}|}{m_\pi})^2$$
$$+ 4i\pi E_\pi|\mathbf{q}|) + \frac{xG_A^2E_\pi|\mathbf{q}|}{(4\pi F)^2}(2 - \frac{2|\mathbf{q}|}{E_\pi}\ln(\frac{E_\pi + |\mathbf{q}|}{m_\pi}) + \frac{\pi^2 m_\pi^2}{4E_\pi^2} - \frac{2\pi m_\pi}{E_\pi}$$
$$+ \frac{m_\pi^2}{E_\pi^2}(\ln(\frac{E_\pi + |\mathbf{q}|}{m_\pi}))^2)\}$$

$$F_2^{(0)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{E_\pi|\mathbf{q}|}{8M_N^2} - \frac{x|\mathbf{q}|}{4M_N^2}(\mu_p + \mu_n) + \frac{2E_\pi|\mathbf{q}|b_{10}}{G_A(4\pi F)^2}\} \tag{6.97}$$

$$F_2^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{|\mathbf{q}|}{2M_N}(\mu_p-\mu_n)+\frac{|\mathbf{q}|}{4M_N^2}\left[E_\pi-(E_\pi-x|\mathbf{q}|)(\mu_p-\mu_n)\right]$$

$$+\frac{G_A^2E_\pi|\mathbf{q}|}{2(4\pi F)^2}\left[\frac{\pi^2m_\pi^2}{E_\pi^2}-\frac{4\pi m_\pi}{E_\pi}-2\pi i\frac{|\mathbf{q}|}{E_\pi}+2\pi i\frac{m_\pi^2}{E_\pi^2}\ln(\frac{E_\pi+|\mathbf{q}|}{m_\pi})\right]\} \tag{6.98}$$

$$F_2^{(+)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{E_\pi|\mathbf{q}|}{8M_N^2}-\frac{x|\mathbf{q}|}{4M_N^2}(\mu_p-\mu_n)+\frac{2E_\pi|\mathbf{q}|b_9}{G_A(4\pi F)^2}\} \tag{6.99}$$

$$F_3^{(0)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{|\mathbf{q}|}{2M_N}(\mu_p+\mu_n)+\frac{E_\pi|\mathbf{q}|}{8M_N^2}\left[3-2(\mu_p+\mu_n)\right] \tag{6.100}$$

$$+\frac{x|\mathbf{q}|^2}{2M_N^2}(\mu_p+\mu_n)-\frac{2E_\pi|\mathbf{q}|b_{10}}{G_A(4\pi F)^2}\}$$

$$F_3^{(-)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{|\mathbf{q}|}{(E_\pi-|\mathbf{q}|x)}+\frac{|\mathbf{q}|}{2M_N}(\mu_p-\mu_n)-\frac{m_\pi^2|\mathbf{q}|}{4M_N^2(E_\pi-x|\mathbf{q}|)} \tag{6.101}$$

$$+\frac{|\mathbf{q}|}{M_N^2}\left[-\frac{E_\pi}{4}+\frac{m_\pi^2}{8(E_\pi-x|\mathbf{q}|)}+\frac{1}{4}(E_\pi-2x|\mathbf{q}|)(\mu_p-\mu_n)\right]-\frac{2G_A^2E_\pi|\mathbf{q}|}{(4\pi F)^2}$$

$$-\frac{2m_\pi^2|\mathbf{q}|b_{19}}{G_A(4\pi F)^2(E_\pi-x|\mathbf{q}|)}+\frac{E_\pi|\mathbf{q}|}{g_A(4\pi F)^2}(2b_{22}^r+b_{23}+G_A^3\ln\frac{m_\pi^2}{E_\pi^2})$$

$$+\frac{G_A^2E_\pi|\mathbf{q}|}{(4\pi F)^2}\left[\frac{2|\mathbf{q}|}{E_\pi}\ln(\frac{E_\pi+|\mathbf{q}|}{m_\pi})-\frac{\pi^2m_\pi^2}{4E_\pi^2}-\frac{m_\pi^2}{E_\pi^2}(\ln(\frac{E_\pi+|\mathbf{q}|}{m_\pi}))^2+\frac{2\pi m_\pi}{E_\pi}\right]\}$$

$$F_3^{(+)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{|\mathbf{q}|}{2M_N}(\mu_p-\mu_n)+\frac{E_\pi|\mathbf{q}|}{8M_N^2}\left[3-2(\mu_p-\mu_n)\right] \tag{6.102}$$

$$+\frac{x|\mathbf{q}|^2}{2M_N^2}(\mu_p+\mu_n)-\frac{2E_\pi|\mathbf{q}|b_9}{G_A(4\pi F)^2}\}$$

$$F_4^{(0)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{|\mathbf{q}|^2}{2M_NE_\pi}+\frac{1}{M_N^2}\left[\frac{E_\pi^2}{4}-\frac{m_\pi^2}{4E_\pi^2}-\frac{x|\mathbf{q}|^3}{2E_\pi}-\frac{|\mathbf{q}|^2}{4}(\mu_p+\mu_n)\right]\} \tag{6.103}$$

$$F_4^{(+)} = \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{\frac{|\mathbf{q}|^2}{2M_N E_\pi} + \frac{1}{M_N^2}\left[\frac{E_\pi^2}{4} - \frac{m_\pi^2}{4E_\pi^2} - \frac{x|\mathbf{q}|^3}{2E_\pi} - \frac{|\mathbf{q}|^2}{4}(\mu_p - \mu_n)\right]\} \tag{6.104}$$

$$\begin{aligned} F_4^{(-)} = & \frac{M_N}{4\pi\sqrt{s}}\frac{eG_A}{2F}\{-\frac{|\mathbf{q}|^2}{E_\pi(E_\pi - |\mathbf{q}|x)} - \frac{|\mathbf{q}|^2}{2M_N E_\pi}\left[1 + \frac{m_\pi^2}{E_\pi(E_\pi - x|\mathbf{q}|)}\right] \\ & - \frac{|\mathbf{q}|^2}{4M_N^2}(1 + \frac{m_\pi^2}{E_\pi^2} - \frac{2x|\mathbf{q}|}{E_\pi} - (\mu_p - \mu_n) - \frac{3m_\pi^2}{2E_\pi(E_\pi - x|\mathbf{q}|)} \\ & + \frac{m_\pi^4}{E_\pi^3(E_\pi - x|\mathbf{q}|)}) + \frac{2m_\pi^2|\mathbf{q}|^2 b_{19}}{G_A(4\pi F)^2 E_\pi(E_\pi - x|\mathbf{q}|)}\} \end{aligned} \tag{6.105}$$

which are equal to the structure amplitudes obtained using HBChPT [12].

In [27, 28] it was indicated that HBChPT would not reproduce certain terms in the low energy expansion of the loop integrals which could not be written as an integer power of the nucleon mass. However in the case of pion photoproduction and radiative pion capture these terms do not appear until the $O(p^4)$, and as such it is not possible to determine if this claim is true.

Chapter 7

Summary & Conclusions

Using the chiral πN Lagrangian to model the possible interactions of a nucleon, the method of infrared regularization was used to derive the physical nucleon mass, the nucleon wavefunction renormalization constant, and the nucleon electromagnetic and axial form factors. The nucleon mass and renormalization constant were then compared with the results obtained using HBChPT and found to be equivalent.

The contribution to the four-point Green's function $G_{\gamma\pi NN}(k_\mu, q_\mu, P_\mu, P'_\mu)$ by irreducible diagrams, denoted by $\Gamma_{\gamma\pi NN}$, was also calculated using the method of infrared regularization. However due to the number of form factors which are required to describe the four-point interaction, these terms were left in their original form and not written in terms of form factors. It was determined that $\Gamma_{\gamma\pi NN}$ depends on the previously calculated $O(p^3)$ low energy constants $b_{10}, b_{19}, b_{21}, b_{22}$ and b_{23}, as well as b_9 for the special case of a neutral pion. At $O(p^4)$ the Green's function also depends on the six low energy constants $e_{48}, e_{49}, e_{50}, e_{51}, e_{52}$, and e_{53}, with the neutral pion case also depending on $e_{67}, e_{68}, e_{69}, e_{70}, e_{71}$, and e_{72}. However these constants have not yet been calculated from experimental data. The additional terms in the πN Lagrangian which explicitly break chiral symmetry and which vanish when $m_u = m_d$ were not included.

In Chapter 6 the nucleon form factors and the Green's function $\Gamma_{\gamma\pi NN}$ were used to calculate the $O(p^4)$ infrared regularized amplitude for pion photoproduction and radiative pion capture. The resulting amplitude was then separated into the twelve structure amplitudes which are required for analysis of the amplitude in terms of multipoles. It was then shown that when the infrared regularized amplitude is expanded at low energies, the resulting contribution to the structure amplitudes is equivalent to the $O(p^3)$ structure amplitudes which were previously calculated using HBChPT. However it was noted that the when the amplitude corresponding to a specific diagram was calculated using the two methods, the results were not equivalent and as such the comparison cannot be performed for individual diagrams.

The $O(p^3)$ structure amplitudes calculated from HBChPT are known to be consistent with the experimental data, and as such the $O(p^3)$ infrared regularized structure amplitudes are consistent with the experimental data. The $O(p^4)$ structure amplitudes contain eight fourth order low energy constants which have not been measured, and as a result numerical values for the fourth order multipoles and cross sections cannot be calculated. It is expected that the $O(p^4)$ differential cross section can be fitted to accurate data to measure the values of these LECs, though there does not appear to be sufficient data at present.

In calculating the pion photoproduction amplitude, it was observed that the method of infrared regularization reproduced the results of HBChPT.

However this method does not require a transformation of the πN Lagrangian and as a result the third order calculations were less complicated and involved fewer diagrams. The fourth order terms could not be calculated with the heavy baryon methods due to the complexity of the required vertices, however it was possible to calculate these terms using infrared regularization. Although the two methods were compared using the pion photoproduction amplitude, it is expected that in general the two methods will produce equivalent results for any reaction and that the method of infrared regularization will require less calculation than the heavy baryon method.

Appendix A

Infrared Regularized Loop Integrals

A.1 Notation

$$\bar{\lambda} = \frac{M_N^{d-4}}{(4\pi)^2}\{\frac{1}{d-4} - \frac{1}{2}(\log 4\pi + \Gamma'(1) + 1)\}$$

$$\alpha = \frac{m_\pi}{M_N} \qquad \Omega = \frac{s - M_N^2 - m_\pi^2}{2M_N m_\pi}$$

A.2 Definition of the Loop Integrals

$$\Delta_\pi = i\int_I \frac{d^dk}{(2\pi)^d}\frac{1}{k^2 - m_\pi^2} \tag{A.1}$$

$$\Delta_\pi^\mu = i\int_I \frac{d^dk}{(2\pi)^d}\frac{k^\mu}{k^2 - m_\pi^2} \tag{A.2}$$

$$\Delta_N = i\int_I \frac{d^dk}{(2\pi)^d}\frac{1}{k^2 - M_N^2} \tag{A.3}$$

$$\Delta_N^\mu = i\int_I \frac{d^dk}{(2\pi)^d}\frac{k^\mu}{k^2 - M_N^2} \tag{A.4}$$

$$\{J, J^\mu, J^{\mu\nu}\} = -i\int_I \frac{d^dk}{(2\pi)^d}\frac{\{1, k^\mu, k^\mu k^\nu\}}{(k^2 - m_\pi^2)((q-k)^2 - m_\pi^2)} \tag{A.5}$$

$$\{I, I^\mu, I^{\mu\nu}\} = -i\int_I \frac{d^dk}{(2\pi)^d}\frac{\{1, k^\mu, k^\mu k^\nu\}}{(k^2 - m_\pi^2)((P-k)^2 - M_N^2)} \tag{A.6}$$

$$\{I_{21}, I_{21}^\mu, I_{21}^{\mu\nu}\} = i\int_I \frac{d^dk}{(2\pi)^d}\frac{\{1, k^\mu, k^\mu k^\nu\}}{(k^2 - m_\pi^2)((k-q)^2 - m_\pi^2)((P-k)^2 - M_N^2)} \tag{A.7}$$

$$\{I_A, I_A^\mu, I_A^{\mu\nu}\} = i\int_I \frac{d^dk}{(2\pi)^d}\frac{\{1, k^\mu, k^\mu k^\nu\}}{(k^2 - m_\pi^2)((P'-k)^2 - M_N^2)((P-k)^2 - M_N^2)} \tag{A.8}$$

$$I_{13} = -i\int_I \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2 - m_\pi^2)((P_1-k)^2 - M_N^2)((P_2-k)^2 - M_N^2)((P_3-k)^2 - M_N^2)} \tag{A.9}$$

In general each of these integrals also includes a factor of μ^{4-d}, where μ is a constant with dimensions of energy, so that the integral will have the correct units.

A.3 Infrared Integrals

1 Meson

$$\Delta_\pi = 2m_\pi^2(\overline{\lambda} + \frac{1}{16\pi^2}\ln\alpha) \tag{A.10}$$

$$\Delta_\pi^\mu = 0 \tag{A.11}$$

1 Nucleon

$$\Delta_N = 0 \tag{A.12}$$

$$\Delta_N^\mu = 0 \tag{A.13}$$

2 Mesons

$$J(q^2) = \frac{1}{8\pi^2}\left(1 - \sqrt{\frac{4m_\pi^2 - q^2}{q^2}} \arcsin\frac{q}{2m_\pi}\right) - 2\bar{\lambda} - \frac{1}{16\pi^2}(2\ln\alpha + 1) \tag{A.14}$$

$$J^\mu(q^2) = \frac{1}{2} q^\mu J(q^2) \tag{A.15}$$

$$J^{\mu\nu}(q^2) = (q^\mu q^\nu - g^{\mu\nu} q^2) J^{(1)}(q^2) + q^\mu q^\nu J^{(2)}(q^2) \tag{A.16}$$

$$J^{(1)}(q^2) = \frac{1}{4q^2(d-1)}\{(q^2 - 4m_\pi^2) J(q^2) + 2\Delta_\pi\} \tag{A.17}$$

$$J^{(2)}(q^2) = \frac{1}{4} J(q^2) - \frac{1}{2q^2}\Delta_\pi \tag{A.18}$$

1 Meson, 1 Nucleon

$$I(P^2) = -\frac{1}{8\pi^2}\frac{\alpha\sqrt{1-\Omega^2}}{1+2\alpha\Omega+\alpha^2}\arccos(-\frac{\alpha+\Omega}{\sqrt{1+2\alpha\Omega+\alpha^2}}) \tag{A.19}$$

$$-\frac{1}{16\pi^2}\frac{\alpha(\alpha+\Omega)}{1+2\alpha\Omega+\alpha^2}(2\ln\alpha-1)-\frac{P^2-m_\pi^2-M_N^2}{P^2}\bar{\lambda} \tag{A.20}$$

$$I^\mu(P^2) = P^\mu I^{(1)}(P^2) \tag{A.21}$$

$$I^{\mu\nu}(P^2) = (g^{\mu\nu}I^{(2)}(P^2) + P^\mu P^\nu I^{(3)}(P^2)) \tag{A.22}$$

$$I^{(1)}(P^2) = \frac{1}{2P^2}\{(P^2 - M_N^2 + m_\pi^2)I(P^2) + \Delta_\pi\} \tag{A.23}$$

The integrals $I^{(2)}(P^2)$ and $I^{(3)}(P^2)$ are related to $I^{(1)}(P^2)$ and $I(P^2)$, however the relation is complicated and as such only the low energy expansion of these integrals will be given.

2 Mesons, 1 Nucleon

$$I_{21}(q^2) = \frac{1}{32M_N m_\pi}\{\sqrt{\frac{m_\pi^2}{q^2}}\ln\frac{2+\sqrt{q^2/m_\pi^2}}{2-\sqrt{q^2/m_\pi^2}} - \ln\left(1+\frac{\alpha}{\sqrt{4-q^2/m_\pi^2}}\right)\} \tag{A.24}$$

$$+\frac{1}{32\pi^2 M_N^2}\{\frac{2(2-q^2/m_\pi^2)}{\sqrt{q^2/m_\pi^2}}(4-q^2/m_\pi^2)\arcsin\sqrt{\frac{q^2}{4m_\pi^2}}$$

$$+\frac{\pi}{\sqrt{4-q^2/m_\pi^2}} + 2\ln\alpha + 1\} + \frac{4\bar{\lambda}}{\sqrt{q^2/m_\pi^2(4M_N^2-q^2)}}\arcsin\frac{q}{2M_N}$$

$$I_{21}^\mu(q^2) = (P^\mu + P'^\mu)I_{21}^Q(q^2) + \frac{1}{2}q^\mu I_{21}(q^2) \tag{A.25}$$

$$I_{21}^{\mu\nu}(q^2) = g_{\mu\nu} I_{21}^{00}(q^2) + (P^\mu + P'^\mu) I_{21}^{QQ}(q^2) + q^\mu q^\nu I_{21}(q^2) \tag{A.26}$$

$$I_{21}^{Q}(q^2) = \frac{1}{2(4M_N^2 - q^2)}\{(2m_\pi^2 - q^2) I_{21}(q^2) - 2I(M_N^2) + 2J(q^2)\} \tag{A.27}$$

$$I_{21}^{00}(q^2) = \frac{1}{4(2-d)}\{2I(M_N^2) - (4m_\pi^2 - q^2) I_{21}(q^2) + 2(2m_\pi^2 - q^2) I_{21}^{Q}(q^2)\} \tag{A.28}$$

$$I_{21}^{QQ}(q^2) = \frac{1}{4(d-2)(4M_N^2 - q^2)}\{2I(M_N^2) - 2(d-2) I^{(1)}(M_N^2) - (4m_\pi^2 - q^2) I_{21}(q^2) + 2(d-1)(2m_\pi^2 - q^2) I_{21}^{Q}(q^2)\} \tag{A.29}$$

$$I_{21}^{qq}(q^2) = \frac{1}{4(d-2)q^2}\{-2(d-3) I(M_N^2) + 2(d-2) I^{(1)}(M_N^2) - (4m_\pi^2 - (d-1)q^2 I_{21}(q^2) + 2(2m_\pi^2 - q^2) I_{21}^{Q}(q^2)\} \tag{A.30}$$

In [9, 32] a different notation is used. For comparison

$$I_{21}^{Q}(q^2) = I_{21}^{(1)}(q^2) \qquad I_{21}^{00}(q^2) = I_{21}^{(2)}(q^2) \tag{A.31}$$
$$I_{21}^{QQ}(q^2) = I_{21}^{(3)}(q^2) \qquad I_{21}^{qq}(q^2) = I_{21}^{(4)}(q^2)$$

1 Meson, 2 Nucleons

$$I_A(q^2) = -\{\bar{\lambda} + \frac{1}{16\pi^2}(\ln\alpha + \frac{1}{2})\}\frac{2}{\sqrt{q^2(q^2-4)}} \ln \frac{\sqrt{4M_N^2 - q^2} + \sqrt{-q^2}}{\sqrt{4M_N^2 - q^2} - \sqrt{-q^2}} \tag{A.32}$$

$$+\frac{\alpha}{32\pi^2 M_N^2}\int_0^1 dx \frac{M_N^2 \arccos(\frac{-m_\pi}{2\sqrt{M_N^2 - x(1-x)q^2}})}{(M_N^2 - x(1-x)q^2)\sqrt{1-\alpha^2/4 - x(1-x)q^2/M_N^2}}$$

$$I_A^\mu(q^2) = (P'^\mu + P^\mu) I_A^{(1)}(q^2) \tag{A.33}$$

$$I_A^{\mu\nu}(q^2) = g^{\mu\nu} I_A^{(2)}(q^2) + (P'^\mu + P^\mu)(P'^\nu + P^\nu) I_A^{(3)}(q^2) + q^\mu q^\nu I_A^{(4)}(q^2) \tag{A.34}$$

$$I_A^{(1)}(q^2) = \frac{1}{4M_N^2 - q^2}\{I(M_N^2) + m_\pi^2 I_A(q^2)\} \tag{A.35}$$

$$I_A^{(2)}(q^2) = \frac{m_\pi^2}{2}\{I_A(q^2) - I_A^{(1)}(q^2)\} \tag{A.36}$$

$$I_A^{(3)}(q^2) = \frac{1}{8M_N^2 - 2q^2}\{3m_\pi^2 I_A^{(1)}(q^2) - m_\pi^2 I_A(q^2) + I^{(1)}(M_N^2)\} \tag{A.37}$$

$$I_A^{(4)}(q^2) = \frac{1}{2q^2}\{m_\pi^2 I_A^{(1)}(q^2) - m_\pi^2 I_A(q^2) - I^{(1)}(M_N^2)\} \tag{A.38}$$

For comparison, the integrals in [9, 32] are related by

$$\begin{aligned} I_A(q^2) &= I_{12}(M_N^2, q^2) & I_A^{(1)}(q^2) &= I_{12}^{(1)}(M_N^2, q^2) \\ I_A^{(2)}(q^2) &= I_{12}^{(3)}(M_N^2, q^2) & I_A^{(3)}(q^2) &= I_{12}^{(4)}(M_N^2, q^2) \\ I_A^{(4)}(q^2) &= I_{12}^{(5)}(M_N^2, q^2) \end{aligned} \tag{A.39}$$

The integrals $I_{12}^{(2)}(s,t)$ and $I_{12}^{(6)}(s,t)$ vanish when $s = M_N^2$.

A.4 Low Energy Expansion

The low energy expansions of the integrals can be written in terms of the constant

$$\lambda_\pi = \overline{\lambda} + \frac{1}{16\pi^2}\ln\alpha \tag{A.40}$$

and the three functions

$$f(\Omega) = \frac{1}{8\pi^2}\sqrt{1-\Omega^2}\arccos(-\Omega) \tag{A.41}$$

$$\overline{J}(\tau) = J(\tau) - J(0) = \frac{1}{8\pi^2}\{1 - \sqrt{\frac{4-\tau}{\tau}}\arcsin\frac{\sqrt{\tau}}{2}\} \tag{A.42}$$

$$g(\tau) = \frac{1}{32\pi\sqrt{\tau}}\ln\frac{2+\sqrt{\tau}}{2-\sqrt{\tau}} - \frac{1}{32\pi}\ln\{1+\frac{\alpha}{\sqrt{4-\tau}}\} \tag{A.43}$$

$$+\frac{\alpha}{32\pi^2}\{1+\frac{\pi}{\sqrt{4-\tau}}+\frac{2(2-\tau)}{\sqrt{\tau(4-\tau)}}\arcsin\frac{\sqrt{\tau}}{2}\} \tag{A.44}$$

2 Mesons

$$J(t) = \overline{J}(\frac{t}{m_\pi^2}) - \frac{1}{16\pi^2} - 2\lambda_\pi \tag{A.45}$$

$$J^{(1)}(t) = \frac{t-4m_\pi^2}{12t}\overline{J}(\frac{t}{m_\pi^2}) - \frac{1}{576\pi^2} + (\frac{m_\pi^2}{t} - \frac{1}{6})\lambda_\pi \tag{A.46}$$

$$J^{(2)}(t) = \frac{1}{4}\overline{J}(\frac{t}{m_\pi^2}) - \frac{1}{64\pi^2} - \frac{2m_\pi^2+t}{2t}\lambda_\pi \tag{A.47}$$

1 Meson, 1 Nucleon

$$I(P^2) = -\alpha(1-2\alpha\Omega)f(\Omega) + \frac{\alpha(\Omega-\alpha)}{16\pi^2} - 2\alpha(\Omega+\alpha-2\alpha\Omega^2)\lambda_\pi + O(\alpha^3) \tag{A.48}$$

$$I^{(1)}(P^2) = -\alpha^2(\Omega+\alpha-4\alpha\Omega^2)f(\Omega) + \frac{\alpha^2\Omega^2(1-2\alpha\Omega)}{16\pi^2} + \alpha^2(1-2\Omega^2-6\alpha\Omega+8\alpha\Omega^3)\lambda_\pi + O(\alpha^4) \tag{A.49}$$

$$I^{(2)}(P^2) = \frac{M_N^2\alpha^3}{3}(1-4\alpha\Omega)(\Omega^2-1)f(\Omega) + \frac{M_N^2\alpha^3\Omega}{144\pi^2}(6-5\Omega^2-15\alpha\Omega+14\alpha\Omega^3) - \frac{M_N^2\alpha^3}{3}(3\Omega-2\Omega^3+3\alpha-12\alpha\Omega^2+8\alpha\Omega^4)\lambda_\pi + O(\alpha^5) \tag{A.50}$$

$$I^{(3)}(P^2) = \frac{\alpha^3}{3}(1-4\Omega^2-12\alpha\Omega+24\alpha\Omega^3)f(\Omega) + \frac{\alpha^3\Omega}{72\pi^2}(7\Omega^2-3+18\alpha\Omega-30\alpha\Omega^3) + \frac{2\alpha^3}{3}(3\Omega-4\Omega^3+3\alpha-24\alpha\Omega^2+24\alpha\Omega^4)\lambda_\pi + O(\alpha^5) \tag{A.51}$$

2 Mesons, 1 Nucleon

$$I_{21}(t) = \frac{1}{M_N^2\alpha}(g(\frac{t}{m_\pi^2}) + \alpha\lambda_\pi) + O(\alpha) \tag{A.52}$$

$$I_{21}^{(1)}(t) = \frac{1}{8M_N^2}\{\alpha(2-\frac{t}{m_\pi^2})g(\frac{t}{m_\pi^2}) + 2\bar{J}(\frac{t}{m_\pi^2}) - \frac{1-\alpha\pi}{8\pi^2} - 4\lambda_\pi\} + O(\alpha^2) \tag{A.53}$$

$$I_{21}^{(2)}(t) = \frac{\alpha}{16}\{2(4-\frac{t}{m_\pi^2})g(\frac{t}{m_\pi^2}) + \alpha(\frac{t}{m_\pi^2}-2)\bar{J}(\frac{t}{m_\pi^2}) + \frac{4\pi+\alpha(\frac{t}{m_\pi^2}-4)}{16\pi^2} + 4\alpha(4-\frac{t}{m_\pi^2})\lambda_\pi\} + O(\alpha^3) \tag{A.54}$$

$$I_{21}^{(3)}(t) = \frac{1}{64M_N^2}\{2\alpha(\frac{t}{m_\pi^2}-4)g(\frac{t}{m_\pi^2}) + 3\alpha^2(2-\frac{t}{m_\pi^2})\bar{J}(t/m_\pi^2) - \frac{\alpha(4\pi-\alpha\frac{t}{m_\pi^2})}{16\pi^2} + 8\alpha^2(\frac{t}{m_\pi^2}-4)\lambda_\pi\} + O(\alpha^3) \tag{A.55}$$

$$I_{21}^{(4)}(t) = \frac{m_\pi^2}{16M_N^2\alpha t}\{2(3\frac{t}{m_\pi^2}-4)g(t/m_\pi^2) + (2-\frac{t}{m_\pi^2}\alpha J(t/m_\pi^2) + \frac{4\pi+\alpha(8-\frac{t}{m_\pi^2})}{16\pi^2} + \frac{8\alpha t}{m_\pi^2}\lambda_\pi\} + O(\alpha) \tag{A.56}$$

1 Meson, 2 Nucleons

$$I_{12}(s,t) = -\frac{1}{4M_N^2\Omega^2}(2\Omega-\alpha-2\alpha\Omega^2)f(\Omega) - \frac{1}{M_N^2}(1-\alpha\Omega)\lambda_\pi + \frac{1}{64M_N^2\pi^2\Omega^2}(2\pi\Omega+2\Omega^2-\pi\alpha-2\alpha\Omega+2\alpha\Omega^3) + O(\alpha^3) \tag{A.57}$$

$$I_{12}^{(1)}(s,t) = -\frac{\alpha}{24M_N^2\Omega}(6\Omega+\alpha-16\alpha\Omega^2)f(\Omega) + \frac{\alpha}{1152M_N^2\pi^2\Omega}(18\Omega^2+3\pi\alpha-21\alpha\Omega-8\alpha\Omega^3) - \frac{1}{12M_N^2}\alpha(6\Omega+9\alpha-16\alpha\Omega^2)\lambda_\pi + O(\alpha^3) \tag{A.58}$$

$$I_{12}^{(2)}(s,t) = -\frac{\alpha}{12M_N^2\Omega^2}(2+\Omega^2)f(\Omega) + \frac{\alpha}{576\pi^2M_N^2\Omega^2}(6\pi+12\Omega-\Omega^3) - \frac{\alpha\Omega}{6M_N^2}\lambda_\pi + O(\alpha^2) \tag{A.59}$$

$$I_{12}^{(3)}(s,t) = \frac{\alpha^2}{12\Omega^2}(\Omega^2-1)(2\Omega-\alpha-6\alpha\Omega^2)f(\Omega) \tag{A.60}$$
$$+\frac{\alpha^2}{576\pi^2\Omega^2}\{2\Omega(3\pi+6\Omega-5\Omega^3)-\alpha(3\pi+6\Omega+13\Omega^3-18\Omega^5)\}$$
$$-\frac{\alpha^2}{6}(3-2\Omega^2-8\alpha\Omega+6\alpha\Omega^3)\lambda_\pi+O(\alpha^4)$$

$$I_{12}^{(4)}(s,t) = \frac{\alpha^2}{48M_N^2\Omega^2}\{2\Omega-8\Omega^3-\alpha(1+14\Omega^2-36\Omega^4)\}f(\omega) \tag{A.61}$$
$$-\frac{\alpha^2}{2304M_N^2\pi^2\Omega^2}\{2\Omega(3\pi+6\Omega-14\Omega^3)-\alpha(3\pi+6\Omega+22\Omega^3-72\Omega^5)\}$$
$$+\frac{\alpha^2}{12M_N^2}(3-4\Omega^2-16\alpha\Omega+18\alpha\Omega^3)\lambda_\pi+O(\alpha^4)$$

$$I_{12}^{(5)}(s,t) = O(\alpha^2) \tag{A.62}$$

$$I_{12}^{(6)}(s,t) = -\frac{\alpha^2}{24M_N^2\Omega}(1+2\Omega^2)f(\Omega)+\frac{\alpha^2}{1152M_N^2\pi^2\Omega}(3\pi+6\Omega+4\Omega^3) \tag{A.63}$$
$$-\frac{\alpha^2\Omega^2}{6M_N^2}\lambda_\pi+O(\alpha^3)$$

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