AN ANALYSIS OF VISUALLY EQUAL-STEPPED GREY VALUES FOR AUTOMATED CARTOGRAPHY

by

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ABSTRACT

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Automated mapping is a new frontier in cartography. Commercialized maps are for the most part still in the experimental stage of production because many standard cartographic procedures have yet to be mechanized. This study deals with the automation of areal symbols for the presentation of choropleth and isarithmic maps.

Areal symbols on an achromatic map are formed by a gradation of grey values ranging from white to black. To maximize map-reading efficiency, these grey values must be chosen carefully to allow an optimal number of values with optimal spacing between consecutive values. Thus, the selection of grey values is done in accordance with both psychophysical and cartographic rules.

Existing literature from the field of psychophysics is examined thoroughly for concepts that might apply to cartographic areal symbols. Because many past approaches to areal symbols in mapping have been based on experimental results from disciplines which have used non-cartographic symbols as stimuli, inconsistencies and unfounded assumptions have degraded the resulting cartographic grey scales.

This study investigates the relationships between physical measurement and psychological sensation which are applicable to cartographic areal symbols. A set of rules based on a cartographic parameter of "per cent ink", rather than the psychophysical standard of "per cent reflectance", is used as a basis for construction of an optimal grey scale series.

Before a series of grey values can be applied to a distribution map, the data must be generalized into a set of intervals that best portrays the distribution. Graphical and mathematical methods that deal with this difficult generalization process are discussed for both theoretical and practical applications.

Because the series of grey values derived in this study were drawn by the ink-line plotter, they reflect the mechanical limitations of the plotter. Although the first grey series derived had correct grey values, the precision of the plotter produced a pronounced pattern to several of the tones. This necessitated development of several greys whose values deviated slightly from the ideals, but were of a regular pattern. The final grey scale, therefore, is not ideal, but closely approaches the required per cent ink parameters.

An experiment was conducted to test the automated grey scale for equal visual sensation steps. The inconsistencies in the experimental results are reflected in the limitations of the ink-line plotter.

This study produces a workable approach for automated areal symbols, and derives a set of conditions for the construction of an equal-stepped grey scale applicable to cartography.

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CHAPTER I

INTRODUCTION

Automation and Cartography

The expanded use of the computer over the last decade has directly or indirectly affected all academic disciplines. Although the computer is basically a numeric tool, it has been used recently for graphic output. Automation in graphics has changed the focus of cartography. The application of computer science to mechanized drafting and other cartographic processes has revolutionized the methodology of cartographic institutions. New concepts and procedures constantly are being sought to transform standard compilation and drafting processes into computerized formats. Certain limitations are realized, however, because automated graphics, in a cartographic sense, is still in the incubating stage, and the potential that automation possesses is far from being exploited fully. Many of the steps involved in producing a quality map have yet to be converted into mechanized processes.

One of the elements of a map that has not been satisfactorily converted to machine output is areal symbolization. The areal distribution of geographical factors is a mapping characteristic that must be portrayed by visually contrasting symbols. The shapes and sizes of the areal units are either determined by statistical manipulation of the data, or by already-existing physical or political boundaries. Isarithms often are used to depict statistically-manipulated surfaces, whereas choropleths are used when existing boundaries are used.

Isarithms and choropleths define the two primary types of statistical surfaces to which areal symbolization is most applicable. The only way to differentiate the symbols visually is to vary the degree of contrast. For an achromatic map, the most efficient method of areal symbolization is to employ a gradation of the grey spectrum, which ranges from white through grey to black.

Purpose and Need

The purpose of this study is twofold: first, to research the available literature in both psychophysics and cartography in order to find the correct relationships between physical stimuli and psychological measurement. This then can be used to develop a set of rules that yields visually equal-stepped grey values. Secondly, to construct a series of grey values on a line plotter according to these rules, and then test the grey values in a map environment for equal-step sensation.

This ability to construct a grey scale is vital to the transition from the traditional multi-step photomechanical manner of producing areal symbols on a distribution map, to a single-step automated construction of areal symbols.

Interval Consideration

For maximum efficiency in the use of shaded areal symbols, each grey value in the series should have an equal degree of visual separation from its adjacent grey values. By using a perceptually equal-stepped grey series, the number of grey steps that can be distinguished visually can be maximized. Correspondingly, the number of meaningful intervals used on a distribution map can be maximized also. However, the psychological limit to the number of grey tones that can be separated visually over an areal context by a map reader is approximately eight.[1] Thus, in order to distinguish various magnitudes of a geographical factor, while still giving clarity and unity to a map, areal representations rarely exceed eight categories.

The problem of sectioning the data into a given number of meaningful intervals involves careful consideration by the cartographer. This selection of interval breaks is prerequisite to the application of graduated grey values to the distribution. The problems of interval selection, a review of the literature, and the presentation of several workable models, both graphical and mathematical, are presented in Chapter II.

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[1] A. H. Robinson, Elements of Cartography, third ed., New York: John Wiley & Sons, 1969, p. 257.

Grey Scales

The psychophysical and cartographic complexities concerning the derivation and use of grey scales are numerous. Nearly all work concerning grey value theory and application has occurred in the field of psychophysics. The third chapter of this study outlines the procedures and conclusions recorded during many years of experimentation in this discipline. All concepts applicable to cartography and automated graphics are freely borrowed and applied.

Very little literature is available on grey scale construction using stimuli directly concerned with mapping. Because much of the grey scale theory used in cartography is of psychophysical origin, and because some of the concepts are not applicable to both cartography and psychophysics, grey scales often are used incorrectly in mapping. Chapter IV discusses the variables and considerations necessary for the construction and application of grey scales.

Equipment and Materials Used

It was decided to use commonly-available equipment and materials in this research. Thus, the line plotter used was the CalComp Model 563 belonging to the University of Victoria Computing Centre. This particular plotter has a minimal pen movement of .005 inches. The pen mounted in the plotter was a Mars 0.2 millimeter nib. This study does not deal with theoretical black and white as in psychophysics, but rather, practical white, as in any text paper, and a good quality black ink. Pelikan TT ink was used for drawing on standard CalComp white paper.

The state of grey scale application to cartography is still insufficient for complete automated purposes. This study does, however, attempt to gather all pertinent literature and theory together, and produces a workable procedure for automated grey scales. New guidelines for further research and application are developed and problem areas needing further experimentation are isolated.

CHAPTER II

THE USE OF GREY SCALES AS AN AREAL SYMBOL

Application of Areal Symbols

A distribution map using areal symbols can be either an isarithm or choropleth presentation. The latter of the two lends itself more readily to automation and at present is in increasingly wide use in distribution analysis. In a choropleth map, existing boundaries are used to define areas, each of which receives an areal symbol representing information depicting that area. Two examples are the use of political boundaries for a population density map, and a region of a homogeneous characteristic. The boundary line of the area has no specific value but the enumerated data assigned to the area within the perimeter may be thought of as being spread evenly over the surface of that area.

The dasymetric map, a variety of the choropleth map, differs from the simple choropleth in that the lines dividing areas of different data values are not limited to the existing boundaries. That is, a census tract, within which a geographic factor such as population density changes abruptly, would be divided by a dasymetric line at the point where the change occurs.[2] The simple choropleth, in contrast, would use a single density value symbol throughout the whole census tract. Because the simple choropleth requires much less compilation and mathematical manipulation to prepare, it is employed much more often.

The other form of distribution mapping that lends itself to grey scales is isarithms. It differs from the choropleth in the kind of statistical surface that is symbolized. Isarithms are line symbols that join points of constant magnitude. They represent a statistical surface that is a continuous gradient rather than the discrete stepped surfaces of a choropleth.

There are two distinct types of isarithms depending upon the nature of the data: isolines and isopleths. Isolines join data values which can occur at a real point, whereas isopleths form statistical surfaces from data which cannot exist at a real point, but rather are expressed as a ratio quantity for an area. Contour lines, isotherms, isogons, and isohyets are all examples of isolines. An isopleth joining all counties of the same population density would pass through areas of a given average value. Because the isopleth can be positioned anywhere within the area of each county it cannot therefore be said to join points of equal magnitude. Thus, an isopleth is open to more placement error than an isoline.

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[2] Ibid., p. 145.

The areas between isarithmic lines can be shaded to give symbolic representation of their magnitude. This shading is commonly done on isopleth maps. Isolines, however, usually are just numbered to show the direction of the surface gradient. A common example is contours.

The choice of whether to use a choropleth or an isopleth presentation of the statistical surface is already somewhat predetermined for the cartographer. By its very nature the simple choropleth must show a locational distribution since the unit area boundaries already exist. The statistical surface should be visualized as a stepped sequence of <u>z</u> value areas as shown in Figure 2. Because the isopleth is more concerned with the form of the statistical surface, that is, with the steepness and direction of the continuous gradient, a two dimensional isopleth surface should be visualized as three dimensional distribution (see Figure 1).

Class Intervals for Data Generalization

No matter which method is used to portray a distribution of relationships, the data will have to be generalized into class intervals by some logical procedure. J. R. MacKay emphatically states that the selection of class intervals is one of the most difficult problems facing the cartographer who wishes to map a distribution using isopleths or choropleths.[3]

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[3] J. R. MacKay, "An Analysis of Isopleth and Choropleth Class Intervals", <u>Economic Geography</u>, V. 31, (1955), p. 71.

Source: PhD Thesis: L. J. Evenden. The Settlement Hierarchy in South-East Scotland, University of Edinburgh, 1970.

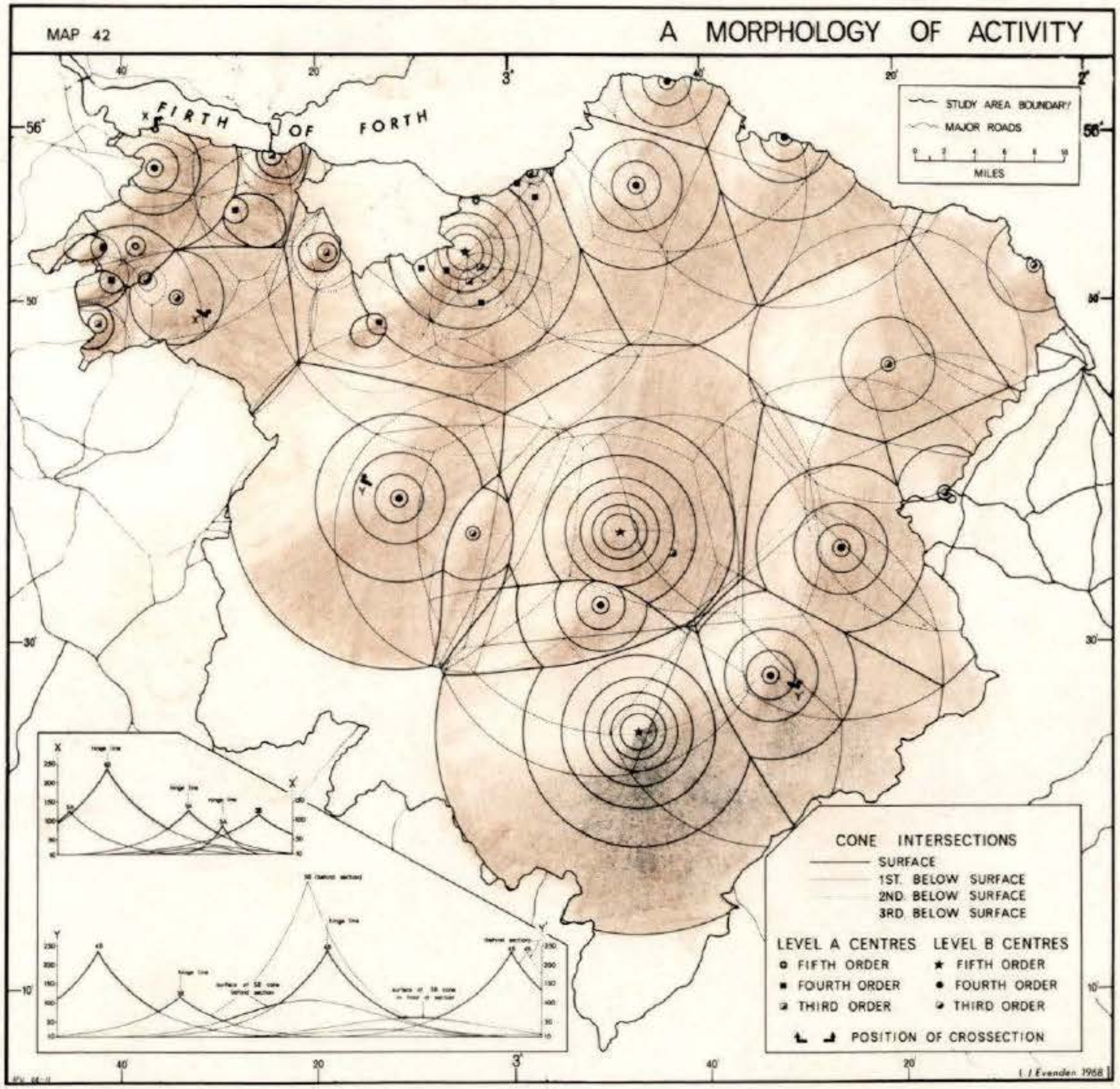
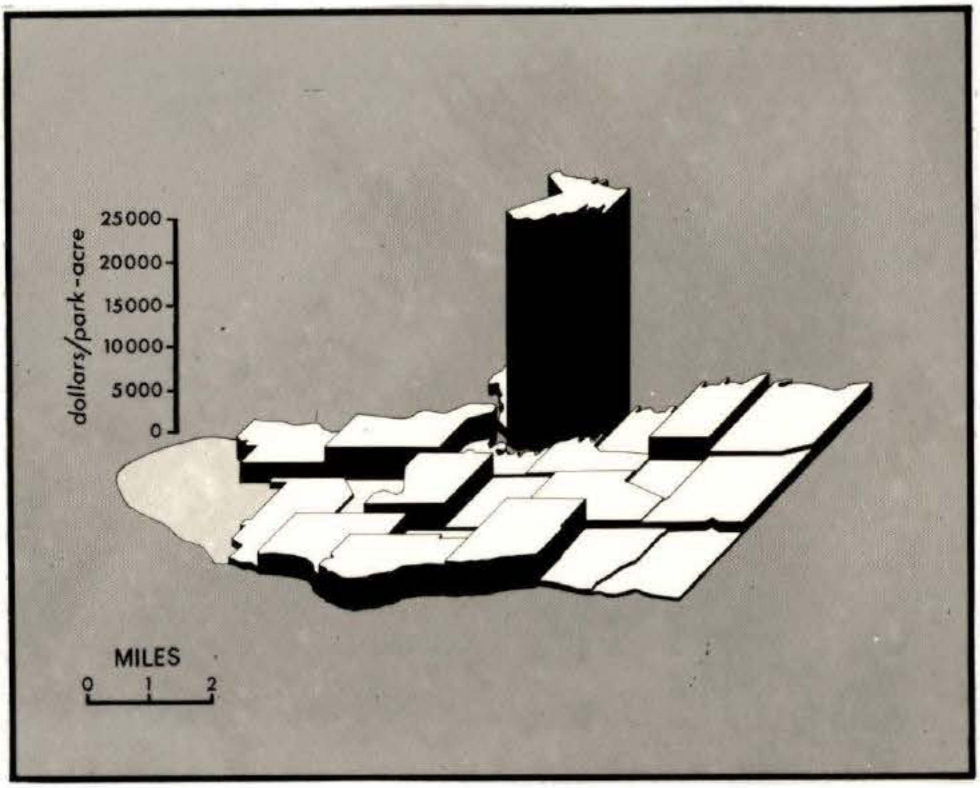


Figure 1 Continuous Isopleth Surface

Figure 2



Stepped Choropleth Surface

Source: PhD Thesis: E. Gibson. <u>Social Conditions Influencing Landscape Change: A Geographical Study of Vancouver, Canada</u>. Simon Fraser University (unpublished).

Indeed, much literature has been devoted to the problem.

Statistical data, as was discussed earlier, must be thought of by the cartographer as a three dimensional surface during the compilation stages of the map distribution. G. F. Jenks points out the immense difficulties in trying to form a mental image of a statistical surface when the data are arranged in tabular, or even spatial, position.[4] It is only logical, then, that methods must be employed to analyze the data before selection of class intervals. The important point here is that the cartographer can present to the map reader an infinite number of distributions by varying his choice of class intervals. The choropleth maps in Figure 3 were all constructed with six class intervals and the shading used was identical for all six maps. The different distributions of the same data arose strictly from different methods of generalizing the data into intervals. The responsibility, then, is on the cartographer to exercise rational judgment in selecting the most desirable set of intervals. A poorly chosen set can mask valuable aspects of the distribution. Discrete manipulation of data will produce a meaningful surface, and hence a meaningful distribution.

The degree of generalization must be considered. The greater the number of class intervals used, the less

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[4] G. F. Jenks, "Generalization in Statistical Mapping", Annals of the Association of American Geographers, V. 53, (1963), p. 15.

generalized the map will be. However, the extreme but impractical limit of one interval for each data value must be avoided. The general rule is that the degree of generalization varies with the number of categories used.[5] Unfortunately, there is an upper limit to the eye's ability to distinguish stepped grey scales over an areal distribution, and therefore the number of class intervals that can be used is limited. There is no consensus on a maximum number of intervals, but most cartography and psychology texts place the upper limit at from eight to ten intervals. This psychological limit permits less rigourous drafting techniques as well as less complicated manipulation of the data.[6] For example, the drafting of twenty to thirty intervals for a choropleth map would be extremely demanding when one takes account of the numerous overlays needed, plus the further complexity of shades and patterns for each class interval. The resulting map would resemble a checker-board of mismatched squares and would have no use whatsoever as a tool for geographic analysis. Another important consideration is that statistical data seldom lends themself to more than ten meaningful categories. In a comprehevsive study of geographic literature, MacKay rarely found isopleth or choropleth maps that employed more

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[5] Ibid., p. 20.

[6] MacKay, op. cit., p. 79.

than ten class intervals.[7] The minimum number of class intervals is two because at least two classes are needed to show a distribution.

There are several interval attributes that affect isarithms and not choropleths. Because the family of isarithms represents a continuous smooth surface, the relative change in relief is denoted by the spacing of the isarithms (i.e., the closer the spacing, the steeper the slope). But this holds true only if the interval is constant, for it is extremely difficult for the reader to visualize a surface where the interval used varies. For this reason isometric maps, such as contours and isotherms, very rarely have an irregular interval. However, a change in the interval does allow the cartographer to concentrate on one portion of the distribution. Sometimes this is desirable when a large data range produces overly-large intervals which obscure desired map detail. Again it is the task of the cartographer to weigh any gain in using irregular isarithmic intervals against the probable mental discomfort and frustration of the reader as he jumps from one vertical scale to another. MacKay has discovered that isopleth maps, unlike isoline maps, are more often than not used with irregular intervals.[8]

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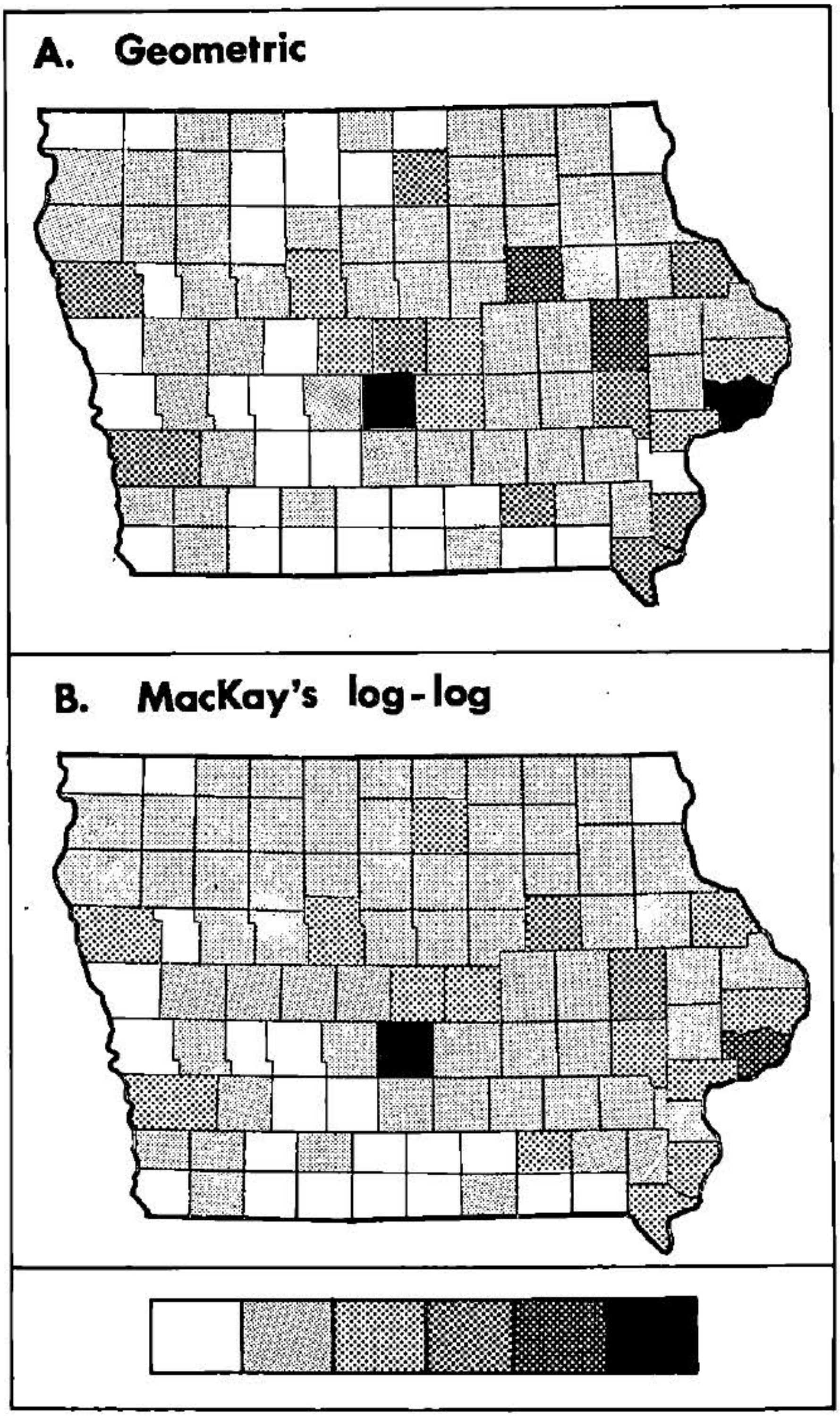
[7] Ibid., p. 78.

[8] J. R. MacKay, "Isopleth Class Intervals: A Consideration in their Selection", Canadian Geographer, V. 7, (1963), p. 42.

Figure 3a



A. Geometric

B. MacKay's log-log

Figure 3b

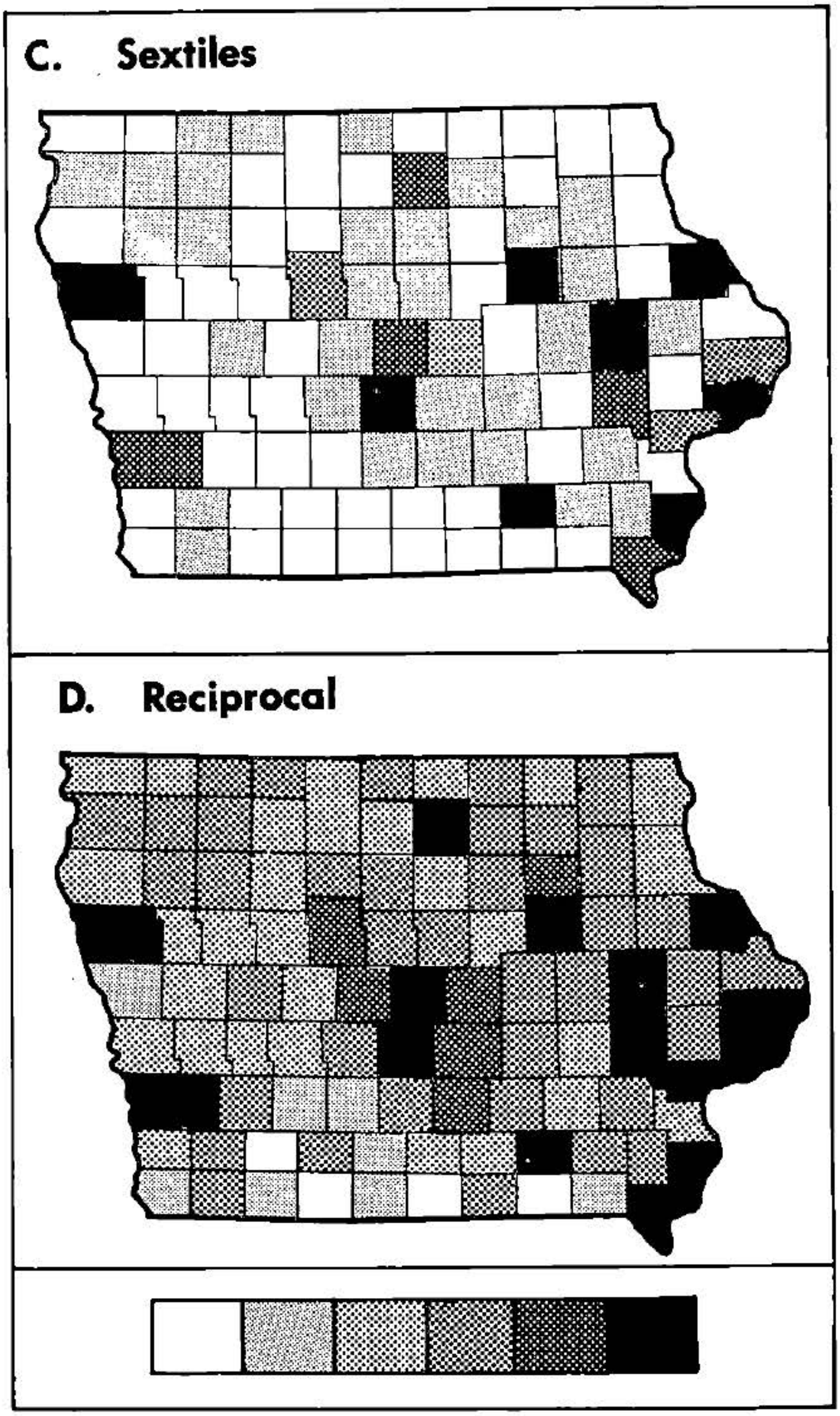
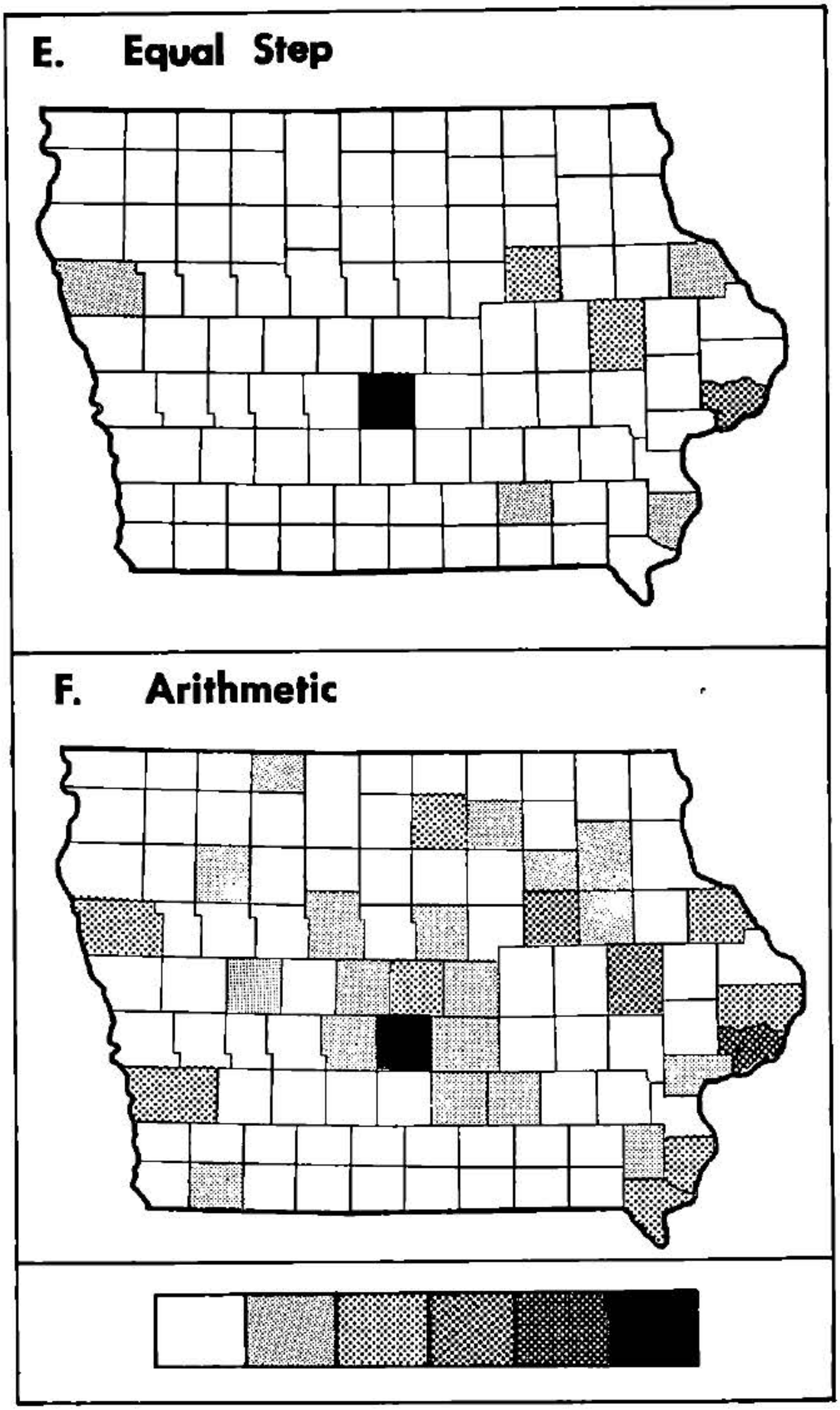


Figure 3c



Models for Interval Selection

This section discusses the more widely used methods of defining class intervals. Because of the differences in the nature of statistical surfaces described by isopleths and choropleths, these two mapping forms often are considered separately for the procedures of interval selection, although for the most part the methods are quite interchangeable. The specific methods available for interval selection are endless, but MacKay categorizes all the different series into two prominent types: equal step and unequal step.[9]

The first method, and most conventional way of interval selection, is the simple equal step, or constant series. Equal step intervals are used primarily in isarithmic mapping. The second method is the unequal step, where the interval can vary either systematically, as in an arithmetic or geometric progression, or irregularly.

Both MacKay and Robinson point out that when an interval varies systematically, invariably the smaller interval is placed at the lower end.[10] This can be explained by the preoccupation in geographic studies with relative differences, rather than absolute differences, in phenomena. For example, demographers attach more significance to the population

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[9] MacKay, "An Analysis of Isopleth and Choropleth Class Intervals", p. 77.

[10] Robinson, op. cit., p. 166.

density interval 0-5, than to the interval 100-105, although both intervals have equal absolute ranges. Also, many frequency distributions of geographic data are skewed towards the left tail (positively), and for this reason a larger assignment of interval width is desirable in this region of the data array.[11] Some of the commonly used mathematical models for systematic selection are discussed below.

Graphical Selection of Intervals

The other type of variable interval, the irregular variable interval, makes use of unique methods for decisions on class break points. These intervals can be derived from frequency graphs, cumulative graphs, and clinographs. Jenks notes that although graphical methods of interval setting are the least used of all, this method is the most appropriate when the cartographer wishes to isolate a certain characteristic of the data array.[12]

The psychologically determined limit of 8-10 applies only when one is trying to differentiate classes over a spatial context. Often the cartographer will want to form intervals for the preparation of histograms, ogives, and other descriptive graphic aids, which in turn can be used

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[11] L. J. King, Statistical Analysis in Geography, Englewood Cliffs: Prentice-Hall, Inc., 1969, p. 15.

[12] Jenks, op. cit., p. 21.

as a method for interval selection for maps. If there are many observations, more than ten intervals might be advantageous. Huntsberger offers a formula for estimating the appropriate number of class intervals for a data array of given size:[13]

$$K = 1 + 3.3 \log_{10} N$$

where: K = the number of class intervals, and

N = the total number of observations.

According to this formula, more than ten class intervals would be needed only if the number of observations exceeded 550.

Defining Class Intervals by Graphic Aids

The three most familiar graphic aids are those mentioned above: the frequency graph, the cumulative frequency graph, and the clinographic curve. The frequency graph, the most common of the three, is formed by identifying data values along the abscissa (in groups, according to Huntsberger, if the observations are numerous), and marking the frequency of their occurrence along the ordinate. If the frequency is shown by bars instead of lines, the diagram is termed a histogram. Gaps occurring along the X axis between groupings show discontinuities in the data and are optimal places for interval breaks. However, there is always the possibility

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[13] D. V. Huntsberger, Elements of Statistical Inference, Boston: Allyn & Bacon, Inc., 1961, p. 10.

that these breaks may be so oddly arrayed that the resulting class intervals would be confusing to the reader. Frequency graphs are more often used as a supplementary visual aid to determine the mean, median, and standard deviation that describe a data array, rather than a tool for class interval decisions.

A cumulative graph is a diagram where the frequencies of the data values are accumulated along the Y axis, while the variable is scaled arithmetically along the X axis. Therefore, each point represents the accumulated total of all preceding data scores of lower value. This type of a graph is often converted to an ogive, which is a line joining the points.

The last graphic interval aid to be examined is the clinograph. This method of determining class breaks is quite similar to the cumulative frequency graph in that sharp breaks in the curve profile indicate a change in the degree of relationship in the data. These slope changes are examined for significant causal relationships and therefore may be rational points for class interval breaks. As in the cumulative graph, the data are arrayed in an equal-spaced arithmetic sequence along the Y axis, and the variable is plotted on the X axis. The X axis is divided into square root intervals, so that the value 5 lies $\sqrt{5}$ from the origin, the value 25 at 5 units from the origin and so on. As in the cumulative frequency graph, cumulative totals may be read directly from the clinographic curve.

MacKay states that when using graphic aids to determine class intervals, all three methods discussed above should be scrutinized for class breaks and combined to yield the most meaningful intervals.[14] This becomes apparent when one realizes that all three graphic methods suggest different interval limits for the same data.

Graphic methods can apply realistically only when the areas considered are relatively uniform in size. MacKay points out, however, that if the Y axis on a frequency diagram were scaled off in units of area, rather than the usual integer numbers, extensive variation in the size of the unit areas would not produce a misleading class division.[15]

Class interval selection by graphic methods generally finds limited use in isopleth and choropleth maps. The main drawback of these methods is that the graph axes are a continuum of values, so that the segment of a graph representing a map area bears no relation to the area's original spatial location on the map. For example, although a specific number of areas have the same X value on a frequency graph, these same areas could be juxtaposed or widely dispersed on the map. For this reason, graphic aids

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[14] MacKay, "An Analysis of Isopleth and Choropleth Class Intervals", p. 75.

[15] Ibid., p. 72.

are more valuable for defining class breaks when the nature of the statistical surface is more important than the geographical distribution of the data values.

Defining Class Intervals by Mathematical Aids

The following models are by no means a complete list, nor is any one model the "best" one to use. They are, rather, a listing to introduce techniques to lessen the difficulties of class selection by sorting the data array or distribution into a visual statistical surface.

Equal Step, or Constant Series

The most elementary of the equal step series is the type (0.0 - 4.9, 5.0 - 9.9, 10.0 - 14.9, ...). Another frequently employed method is the selection of quantities, in which the data are arrayed in tabular form in ascending order, and then sectioned off depending upon how many intervals are desired. For example, if five intervals (quintiles) are required, then one merely counts off one-fifth of the total observations starting at the lowest end, which becomes the first interval, and so on. This technique is popular in choropleth mapping but is restricted to data areas of uniform spatial extent.

If the data are normally distributed, a third equal-step method can be used to show each area's variation from the mean. First, the data is normalized by the standard formula:

$$Z_i = \frac{X_i - \overline{x}}{S_x}$$

where Z_i = normalized i^{th} value,

X_i = i^{th} value,

$\overline{x}$ = mean, and

S_x = standard deviation of the set of observations.

Then the intervals are marked off on the idealized normal curve, where $\overline{x} = 0$, and $S_x = 1$. The data would be grouped as to whether a particular z score was within one S_x of the mean, two S_x's of the mean, etc.

Robinson mentions one other method, ... "equal area steps, or in a sense, geographical quantiles".[16] The geographic region portrayed is divided into equal subregions, the number subjectively chosen by the cartographer. A frequency graph then can be used to derive the class limits.

Many distribution maps, especially of the choropleth variety, have statistical surfaces whose lowest value is greater than zero. In this case one simply finds the range of data values, divides this range by the number of classes desired, and then adds this class width repeatedly to the smallest data score. Map C of Figure 3 was calculated in this manner (see Appendix, p. 77).

Unequal step Series

This series is divided into two main categories,

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[16] Robinson, op. cit., p. 168.

arithmetic progressions and geometric progressions, although it is also possible to have a progression which varies in a random fashion. MacKay points out that it is not uncommon to find a series of intervals that starts off arithmetically or geometrically and then progresses into a random interval.[17] This occurs because a large range of data cannot be covered adequately by an arithmetic progression in which one hopes to isolate the small relative change at the lower end of the scale. The random class limits often are assigned to the least important parts of the data.

Arithmetic Series

In a pure arithmetic series, each class interval limit differs from the preceding one by a numerical quantity that is not a constant, but rather a multiple of a constant. The arithmetic series equations used by Robinson and Jenks differ slightly.

Jenks uses an equation of the type:[18]

$$H = L + D\sum_{i=1}^{n} i$$

where H = the highest value,

L = the lowest value,

n = the number of classes used, and

D = the chosen difference between successive class limits.

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[17] MacKay, "An Analysis of Isopleth and Choropleth Class Intervals", p. 79.

[18] Jenks, <u>op. cit.</u>, p. 22.

This equation produces the series

$$L,\ L + D,\ L + 3D,\ L + 6D + \ldots L + \Sigma iD = H.$$

Map F of Figure 3 uses this method (see Appendix, p. 78).

Robinson's formula is somewhat more flexible, as it allows more variation in the interval sizes:[19]

$$H = L + \sum_{i=1}^{n} D_i$$

where H = the highest value,

L = the lowest value,

D_i = the calculated difference between the class limits of the ith interval, and

n = the number of classes used.

D_i is calculated by the formula

$$D_i = cB_i$$

where c is a scaling factor defined by $c = \frac{H - L}{B_i}$, and

B_i is the value of the ith term in the arithmetic series defined by the equation

$$B_i = a + (i-1)d$$

where a = the value of the first term of the series,

i = the ordinal number of the term being determined, and

d = the stated difference between two successive terms of the series, as chosen by the cartographer.

Robinson's method is more flexible than Jenks' because by varying _d_, the intervals can be made to decrease toward the top or increase toward the top. It is apparent that some care must be exercised when choosing _d_. In addition, the

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[19] Robinson, _op. cit_., pp. 168-169.

use of the scaling factor c enables a developed progression to be applied simply to any data distribution.

Geometric Series

Successive geometric intervals are related by a ratio rather than by a difference. The geometric progression formula used by Robinson is identical to his arithmetic formula. However, for a geometric progression, the ith term is evaluated by setting

$$B_i = gr^{i-1}$$

where g = the value of the first non-zero term,

r = the stated ratio of two successive class limits, as chosen by the cartographer, and

i = the ordinal number of the term being determined.[20]

Again a wide range of flexibility in calculating class breaks is possible by letting r assume any positive, negative, or variable quantity. If, for example, r is made a fractional quantity, concentration of the interval series will be at the upper end, rather than at the more common lower end.

Jenks uses a logarithmic difference series to obtain geometric class limits:[21]

$$\log H = \log L + nD$$

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[20] Ibid., p. 168.

[21] Jenks, op. cit., p. 22.

where H = the highest value,

L = the lowest value,

n = the number of classes used, and

D = the logarithmic difference between successive class limits, calculated by the formula $D = \frac{\log H - \log L}{n}$.

Map A of Figure 3a uses this method (see Appendix, p. 75). Although this method is simpler, it does not have the flexibility of Robinson's equation.

Choropleth Summary

It must be remembered that these methods are only suggested guidelines to help the cartographer visualize possible generalizations of the statistical surface. Even though the cartographer might know what aspects he wants to make prominent on the map, he may have a difficult time trying to visualize mentally the surface of the distribution while he allocates less important geographical factors into less salient intervals. The methods given in this section are only suggestions that the cartographer can use.

For a map of a given area, it is obvious that each array of data lends itself to a different generalization. Similarly, a set of intervals is needed to show the correlation of two different geographic phenomena over the same area. The cartographer must make a series of judgments to express this correlation by a meaningful statistical surface, or distribution. These judgments, according to Jenks, can be conceived as three interrelated processes:

"(1) The concept of the statistical surface,

(2) generalizations resulting from the number of classes, and

(3) generalizations dependent upon the mathematics or graphics of classing data."[22]

These three processes are not independent and must be judged simultaneously. The decisions determine the form of the generalized statistical surface which is then symbolized to represent the abstract data.

Special Considerations for Isarithmic Maps

Isarithmic maps are slightly more complicated than choropleths, although much of the discussion so far is applicable to both map presentations. Although even spacing of isarithm interval breaks is the exception in cartographic practice, it is sometimes nearly mandatory. For example, on a topographic map, the morphology of the surface can be visualized easily by the reader because of the use of equal step intervals. The map reader knows that the compression and rarefaction of the isarithms are functions of slope, and after a mental understanding of the topography has been formed, he can physically measure the slope angle to quantify the qualitative visualization. This is not so with other isarithms. All relationships of compression and rarefaction

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[22] Ibid.

of isarithms to 'steepness' and 'flatness' are lost if class intervals fluctuate.

Because of the primary interest in the relative change of geographic factors, the first interval limit for a series of data is easiest to choose. Similarly, with knowledge of the range of data, the last class limit also is easily determined. The real problem is the assignment of the middle breaks. In a paper presented to the Canadian Geographer, J. R. MacKay describes a useful method of defining class breaks for isopleths.[23] MacKay states that if the cartographer can define three basic "givens" -- (1) upper and lower limits of the first interval, (2) the lower limit of the last interval, and (3) the number of intervals to be used in the distribution -- a formula can be used to derive the distribution of the remaining intervals, because the middle intervals are controlled by the three "givens":[24]

$$L_i = L_o + (L_1 - L_o)i^x$$

where L_i = the upper limit of the i^{th} class interval,

L_o = the lower limit of the first class interval (0 is permissible),

L_1 = the upper limit of the first class interval,

i = the ordinal number of the class interval, and

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[23] MacKay, "Isopleth Class Intervals: A Consideration in their Selection", p. 43.

[24] Ibid.

$$x = \frac{\log (L_{n-1} - L_o) - \log (L_1 - L_o)}{\log (n-1)},$$

where L_{n-1} = the lower limit of the last (n^{th}) class interval, and

n = the number of class intervals.

This equation was compiled from a study of class intervals that appeared in a number of geographic publications over a period of years. It was defined both to fit the average, and to allow flexibility by manipulation of the parameters. If Q is defined by

$$Q = \frac{L_{n-1} - L_o}{L_1 - L_o}$$

the intervals will be constant if n equals Q. The intervals will increase if n is less than Q, and will decrease if n is greater than Q.

Again, as before, this method is only a guideline and ... "is of as much theoretical as practical interest, because it summarizes much of current cartographic procedure relating to the spacing of class intervals".[25]

Summary

An important cartographic consideration when selecting intervals is that the intervals at the lower end should cover a greater areal extent. This is because a map is more pleasing to the eye and more easily understood if dark areas are kept to a minimum, and by convention light shades are

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[25] Ibid.

employed for lower values. Much of the time, geographical data such as rainfall or population count is distributed so that the cartographer need not be concerned about the lower intervals covering a greater areal extent because this is already determined by the data characteristics.

Isarithmic surfaces are better suited to a larger range of areal symbols than choropleths. This is because the surface is continuous and the eye can follow the gradation sequence since the intervals are always adjacent to the next interval in the series. Choropleths, however, are discrete areas, and the eye must jump from one area to another without the mental benefit of a side by side comparison.

The argument may be raised concerning all procedures, mathematical and graphical alike, that the cartographer is not free to select intervals of his choice. All class interval selection methods are merely suggestions to help break the data down into meaningful portions which can be used to construct mentally, or physically, the statistical surface. The cartographer is free to accept or reject the results of the interval selection methods, but at least by following prescribed suggestions, he is made aware of the variety of possible outcomes when both class number and class limits are free to vary. Methods exist for the statistically minded to test variation of a class interval

CHAPTER III

SELECTION OF GREY VALUES

Psychological and Physical Considerations

The final consideration for a single colour choropleth or isopleth map, after the class intervals have been chosen, is the designation of a grey value for each interval to give areal symbolization to the map. In many cases it is possible to have the map "visually exact", where the density of the grey area is directly correlated to the quantitative magnitude of the interval. For example, if the intervals of an isohyet map range from 0 to 80 inches in eight even intervals of ten inches, then the eighth interval's grey should have twice the visual intensity of the fourth interval's. Unfortunately this is not so simple to achieve as it first appears. A series of grey tones that are in constant series (10 per cent, 20 per cent, 30 per cent, etc.) will not appear to be in equal steps to the eye. The problem of defining and constructing a series of grey values that are at equal sensation steps to the eye has been under study for some time, predominantly in the field of psychophysics.

Definition of Terms

Definitions of terms to be used are included because the terminology used throughout the years in the study of grey scales varies. The following definitions, terminology, and examples are derived from Evans and Jenks.[27]

Reflectance

Reflectance is the fraction of light reflected under given conditions by a surface. It is a calculated or measured physical magnitude and not a psychological sensation. In essence, it is the ratio of incident light to reflected light, evaluated with the eye considered as a standardized receptor. It does not take into account the effect of the surface's surroundings or the adaptive state of the eye at the time. It is the property of a surface alone. Often the prefix "luminous" is attached, but for the purpose of this paper the terms reflectance and luminous reflectance are synonymous.

Brightness

Brightness is the mental interpretation of light of a given luminance (intensity) as perceived under given

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[27] R. M. Evans, An Introduction to Color, New York: John Wiley & Sons, 1965, pp. 157-160, and

G. F. Jenks and D. S. Knos, "The Use of Shading Patterns in Graded Series", Annals of the Association of American Geographers, V. 51 (1961), p. 321.

series and to arrive at an error term for between-class variation against total variation.[26]

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[26] See G. F. Jenks and M. R. Coulson, "Class Intervals for Statistical Maps", International Yearbook of Cartography, V. 3 (1963), pp. 119-134, and

G. F. Jenks, F. C. Caspall, and D. L. Williams, "The Error Factor in Statistical Mapping", Abstracts of Papers Presented at 64th Annual Meeting of the Association of American Geographers, Washington, D. C., 1968, p. 45.

conditions. It is the mental reaction to the light, and is properly described as the apparent luminance of the light. The word "apparent" is applicable because the same luminance does not necessarily produce the same brightness if the conditions are varied. Brightness is poorly defined, and equality of two brightnesses generally means only that the two lights have equal psychological intensities and may or may not be equal physically.

Lightness

Just as brightness refers to the mental perception of luminance, lightness refers to the mental perception of reflectance. This is crucial to the psychophysics of grey scales because two surfaces of equal lightness (i.e., apparently having the same reflectance), do not necessarily have the same actual reflectance. Conversely, two surfaces that are physically calculated equal do not necessarily have the same lightness. This is clear if one remembers that brightness and lightness are psychological, whereas luminance and reflectance are physical. These relationships will be looked at in more detail below. The term "value" is used to scale lightness. In the Munsell colour notation system, as well as others, value runs from 0 through 10. Zero per cent reflectance (full black) is designated 0 on the scale, and 100 per cent reflectance (full white) is designated 10. The intermediate values, defined by the integers 1 through 9, have been calculated to appear equally spaced to the eye. That is, the value 3 is twice as dark psychologically as the value 6. The luminous reflectances of these two value steps, however, are not in the ratio 2:1.

Colour Constancy

Another important phenomenon is what psychologists term "colour constancy". This process is based on the fact that light perceived as apparent reflectance appears more intense to the eye than light perceived as apparent luminance. This is more easily understood through the following example. If one places two identical light grey patches of paper in the centres of two large squares of white cardboard, no matter what the intensity of the illumination (within standard reading limits), the cardboard borders will always appear white. This is true even if the two samples are illuminated so that the light from the grey patch on one cardboard is many times the intensity of the white on the second cardboard. What this colour constancy concept means in essence is that grey will always occur when there is a brighter area in the field of view. The latter stages of this chapter discuss the importance of this concept when classifying grey scales according to the darkness or lightness (per cent reflectance) of their surrounds. Equally important is the concept that grey should be thought of as a sensation of lesser relative brightness.

History of Grey Scale Experimentation

The JND Method

Experimentation in the construction of equal-stepped grey scales began more than two hundred years ago. The first method used to obtain an equal sensation scale was what is now termed the "just noticeable difference", or JND method. Basically this method consists of projecting two illuminated targets on a test field. One target is increased in brightness until the observer can perceive a just-noticeable change in lightness sensation. Then this target is held constant while the other target is increased in brightness. By alternating the two brightness fields and noting the number of JNDs from darkness to the absolute brightness threshold where no further increase produces a different sensation, an equal-step grey scale can be constructed. The assumption used by early scientists in classifying the step of a grey scale was that one JND step represented a "unit sensation". It was this underlying assumption which held the discipline of psychophysics in the throes of controversy over sensation measurement for more than one hundred years.

The experiments by P. Bouguer back in 1760 concluded that the JND in sensation was a constant fraction.[28] This

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[28] A. E. O. Munsell, L. L. Sloan, and I. H. Godlove, "Neutral Value Scales. I. Munsell Neutral Value Scales", Journal of the Optical Society of America, V. 23 (1933), p. 395.

value was set by Bouguer for his own eye at 1.5 per cent. This simply means that a brightness, (B2), that could be perceived just brighter than B1 was 1.5 per cent larger than B1. The fraction constant later became known as the "Fechner fraction" and was brought forth into the literature as the "Weber Law". This law and Fechner's assumption that the JND unit sensation was an interval scale unit subject to the laws of mathematical manipulation, allowed Fechner to compute the famous Weber-Fechner law of sensation equivalence. The formula reads:[29]

$$V = c \log R + k,$$

where V = value (sensation),

R = reflectance of grey surface (stimulus), and

c and k are constants.

Further experiments in later years showed that if the intensities of illumination were extended past those of everyday viewing conditions, the fraction broke down. Because cartographers rarely work at limit conditions this should not detract from the important consideration that the Fechner fraction is independent of the intensity of illumination under average viewing conditions.

The Value Step Method

A second method of obtaining an equal step grey scale

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[29] *Ibid.*, p. 396.

was formulated at the end of the eighteenth century. The procedure, the "value step method", is initiated by an observer choosing a grey value which he feels is midway between a lighter and a darker surface. After the first middle grey is selected, the range between either end and the middle grey is then halved, and the process carried on until a sufficient number of values are obtained. In 1873, Plateau used a variation of this method, having his observers choose a middle grey value whose sensation produced a constant ratio when compared to the lighter and darker stimuli (e.g. the ratio middle:light was equal to the ratio dark:middle).[30] The Plateau formula derived to express the new relationships was a power function:

$$V = kR^c$$

$$\log V = \log k + c \log R.$$

The difference between this formula and that of Weber-Fechner is in the logarithmic relationship of V.

Plateau later dropped his formula for the logarithmic type, a decision much bemoaned by modern day psychophysicists because Plateau's original concepts on the measurement of sensation display the basics of present consensus on the subject. However, the important point in reference to this paper is that a second workable method was derived for the construction of an equal-sensation grey scale.

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[30] Ibid.

The Adams-Cobb Parameter

Experiments continued in the field of psychophysics and many modifications were made to Fechner's basic logarithmic formula. All results followed the same general trend, but values fluctuated enough that no one scale could be deemed ideal. It was not until 1920 that E. Q. Adams and P. W. Cobb formulated a procedure to control the variation in grey scale experiments.[31] They showed that the brightness of the area surrounding the grey patch under consideration was a major determinant in the perception of the grey sensation because it controlled the adaptive state of the observer's eye. Adams and Cobb constructed the following formula to include this important variable:[32]

$$V = \frac{cR}{R+Ra},$$

where V = value from a reference point V_o,

R = reflectance of the grey patch,

Ra = background brightness, and

c is a constant.

In 1940 this formula was modified by D. B. Judd to:[33]

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[31] Ibid., p. 397.

[32] Ibid.

[33] G. Wyszecki and W. S. Stiles, Color Science: Concepts and Methods, Quantitative Data and Formulas, New York: John Wiley and Sons, 1967, p. 453.

$$V = \frac{.1R\ (Rb+100)}{Rb + Y} \quad ,$$

where V = grey value (0 to 10),

Y = reflectance

R = reflectance of the grey patch, and

Rb = background reflectance.

The constants in Judd's formula were made to conform to the ten point value scale which is now the standard number for nearly all value scales. Thus, black is designated as 0, and absolute white is designated as 10 (theoretically 100% reflectance). Figure 6 clearly illustrates the dependence of grey sensation (lightness) on the adaptive state of the eyes, which in turn is controlled by the background or surround reflectance. Figure 5 illustrates the same principle using brightness and luminance as the basis for psychological and physical measurement.

Rationale of Sensation Measurement

Mention should be made of psychophysicists' efforts to rationalize the idea of attaching an interval scale quantity to such an indirect concept as sensation (perception of a JND). The rationalization is culminated in the work of the eminent psychologist S. S. Stevens, who refutes the measuring assumptions of the Weber-Fechner law of equal sensation steps.

The various literary attacks on Fechner throughout the past century have centred around two basic lines of thought: Fechner's constancy fraction relating subjective magnitude to stimulus magnitude, and his definition of a JND as a measurable unit sensation. Rigourous study of the subject, begun in the 1930's, were climaxed in the mid-1950's by S. S. Stevens. Stevens first attacked Fechner's principle that ... "the unit of measurement is given by resolving power",[34] that is, that the JND unit is an indirect form of measurement resulting from the perception of a difference in sensation by the observer. Stevens suggests that ... "equal stimulus ratios produce equal subjective ratios",[35] and that these ... "ratio scales are related to the stimulus by a power function of one degree or another".[36] By contrast, the Weber-Fechner law of equal sensation units is defined by a logarithmic formula.

Stevens' main concentration of criticism is on the allocation of a unit value to the JND. He shows that the summation of JNDs is not a straight-line function as the Fechner constancy fraction would show, because the JNDs vary in subjective size for different stimuli. Thus, Fechner's law, showing that a grey value of 3 (on a 10 point scale) is half as many JNDs away from 0 as a grey value 6, is invalid. Stevens goes on to show that the main reason for an exponential non-linear formula in equal-sensation

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[34] S. S. Stevens, "On the Psychophysical Law", The Psychological Review, V. 64 (1957), p. 154.

[35] Ibid., p. 153.

[36] Ibid.

grey scales is ... "variation in the subject's sensitivities to differences".[37]

The grey scale methods of Plateau (1813) after he changed from his power function, Delboeuf (1873), Priest (1920), Munsell et al. (1933), and Adams and Cobb (1922), were all based on the classical assumption of constant unit stimuli. Every time these lightness scales are duplicated by others using the same constant conditions, and variations occur in results, Stevens' theory is verified further. This is because the unit value JND is not actually a constant but is subject to variation in an observer's perception of a sensation magnitude. For example, in the determination of a grey scale by Munsell et al. in 1933, the observers averaged 351 JNDs for the range from 0 to 100 per cent reflectance. The curve of the JNDs versus the reflectance was then plotted and equal value steps counted off, so that value 1 lay 35.1 JNDs from the origin, value 2 lay 70.2 JNDs, etc. However, if different observers had been used in this experiment, there certainly would have been a different set of variations in perception of the JNDs, therefore a different average to plot, and consequently a slightly different grey scale.

The Munsell group computed the variations they found in their grey scale compared to others using the same testing

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[37] *Ibid.*, p. 155.

conditions, and, although these deviations are slight, they are substantial enough to support Stevens' theory.[38] It is interesting to note that because Munsell *et al*. recognized that the variations could be due to differences in the magnitude of the JNDs, the Munsell renotation system switched to a power function formula in 1943. Most of the later grey scales -- Munsell renotation (1943), Ladd and Pinney (1953), Glasser (1958), and Wyszechi (1963) -- all use exponential formulas rather than Fechner's logarithmic law for the ... "simple reason that resolving power (JND) is not constant in psychological units but is roughly proportional to psychological magnitude".[39]

The formula for Stevens' lightness scale is:

$$V = kR^n,$$

where V = value,

R = the luminous reflectance of stimuli,

n = the exponent for lightness scales (determined to be 1.2 by Stevens), and

k is a constant.

Regardless of all the work accomplished in psychophysics over the last 35 years concerning the measurement of sensation, lightness scales based on the Fechner concept are still being

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[38] See I. H. Godlove, "Neutral Value Scales II. A Comparison of Results and Equations Describing Value Scales", *Journal of the Optical Society of America*, V. 23 (December 1933), p. 424.

[39] S. S. Stevens, *op. cit*., p. 179.

used. This is because most people who find use for grey scales in business, industry, and academic disciplines are not concerned with "scientific exactness", so that an error of ± 2 per cent reflectance is considered unimportant. Most lightness scales are constructed for what is termed "average viewing conditions", and evaluated for a "standard observer". Cartography is no exception. Because the Fechner concept does hold up fiarly well under these condition and gives a "good" representative grey scale, the concept is still useful for cartographic purposes.

Grey Scales Applied to Cartography

There exist several problems in the application of lightness scales to area symbols on maps. Until recently, many map makers have shown little concern for exact visual differences in grey magnitude, and were satisfied if the grey quantitative areas simply looked distinguishably different. Therefore most cartographic laboratories were equipped with only nominal per cent screens (i.e., screens at intervals of 10 per cent). Further, because tone screens are based on per cent transmittance and not per cent reflectance, the cartographer seldom had means to measure the reflectance of a resulting tone from a particular screen.

Manufacturers of cartographic aids brought out many varieties of pre-drawn adhesive tones and patterns with stated percentage values, but these had two major drawbacks. First, results of a sampling by Jenks of various brands of stickdown area symbols showed that actual reflectance and per cent ink claims by manufacturers were highly variable and deviated by as much as ± 15 per cent.[40] Secondly, and by far the most important, cartographers had no way of knowing if patterns behave perceptually the same way as flat tone. It was obvious that experiments on equal appearing grey scales were needed using cartographic methods and materials.

Grey Scales Using Cartographic Stimuli

One of the first to experiment with grey scales with particular reference to area symbols on maps was R. L. Williams. He realized that the conditions used in psychophysics might not be applicable for use on monochromatic maps. Williams set out to determine ... "what values printed screens should have to give even-appearing visual steps from white to full color".[41] The measuring scale was not reflectance but rather a more easily computed parameter, per cent ink area. The test samples used in the experiment were dot screens and line screens with known percentages of ink occupying equal size areas. The dot patterns ranged from 2 per cent to 100 per cent. The most closely grouped of these intervals were from 2 per cent to 22 per cent. The line screens ranged

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[40] G. F. Jenks and D. S. Knos, op. cit., p. 326.

[41] R. L. Williams, "Map Symbols: Equal-Appearing Intervals for Printed Screens", Annals of the Association of American Geographers, V. 48, (1958), p. 132.

from 5 per cent to 100 per cent in 5 per cent intervals.

The comparability of the two screen types used is open to criticism. Dot screens give the visual impression of an even grey tone when viewed from a distance because the pattern effect merges into a flat tone. Williams' line screens, however, consist of a constant number of lines per unit area. Thus, for darker tones, the width of the lines was increased rather than the number of lines, producing a strong line pattern even from a considerable viewing distance. The ever-present effect of induction (see Figure 7) may have modified the sensation aroused in an observer by a strong line pattern when compared with a less obvious dot pattern of the same per cent ink area. Although Williams' results of the value sensations for line screens plotted very close to those for the dot screens, the two types of screens were not compared simultaneously and there is still doubt whether a map using both dot and line patterns in juxtaposition are compatible for an equal step scale.

After rigourous testing, Williams summarized his results in his "curve of the grey spectrum" (Figure 4). Because the resulting curve is not a linear function of the log of the per cent inked area, Williams concluded that the Fechner law is ... "not applicable to equal appearing intervals of a grey scale".[42] This conclusion is questionable

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[42] Ibid., p. 135.

because there is doubt whether a flat tone stimulus evokes the same sensation as a patterned stimulus. Because Williams' experiment was set up with average lighting conditions, similar to those used by Munsell et al. where the Fechner constant did apply, it is highly likely that flat tone and pattern tone do evoke different discriminatory sensations.

Further weight to this concept is supplied by the Jenks-Knos experiment.[43] Jenks and Knos set up a test to determine the most practical grey scale using cartographic symbols. They made the dangerous assumption that ... "a printed screen will cause the same visual reaction as the materials used in developing the above psychophysical scales".[44] They further assumed that ... "reflectance from a grey surface, whether a flat tone, a lighted target, or a section of shading media will cause the same reaction".[45] Jenks and Knos then proceeded to transform several psychophysically determined grey scales to a common per cent inked base, because ... "per cent of area inked was considered to be analogous to stimulus and reflectance".[46] According

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[43] G. F. Jenks and D. S. Knos, op. cit., pp. 316-334.

[44] Ibid., p. 321.

[45] Ibid.

[46] Idem.

to their conversion, a 7.5 value on the Priest scale (Weber-Fechner), corresponding to a luminous reflectance of 28.2 per cent, would require 96 per cent of the area to be inked. Not only is this conversion intuitively questionable, but Williams states that tests show linear conversion from reflectance to area of ink is not workable.[47] Indeed, any conversion from a flat tone to per cent ink area, especially when the conversion results in a pattern tone, is based on unfounded assumption.

The work done by L. A. Jones[48] reinforces the theory that pattern tones and flat tones evoke different responses.

Jones states:

> ... if we were dealing with areas of uniform reflectance -- that is, not line structure but greys printed solidly without structure -- a scale printed according to the data in the Munsell standard value scale of the table would meet all the requirements of equal sensory steps between adjacent members. When, however, we are dealing with a scale in which the end members are of uniform reflectance and the four medial members all have structure that is the same in the four cases, there is little doubt in my mind that the magnitude of the sensory or perceptual step between the white area and the adjacent ruled area having an average reflectance of 48.5 per cent is considerably greater than that between the same white area and an adjacent uniformly grey (non-structured) area having a reflectance of 48.5 per cent. In other words, the very presence of the

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[47] R. L. Williams, "Map Symbols: The Curve of the Grey Spectrum -- An Answer", Annals of the Association of American Geographers, V. 48 (1958), p. 489.

[48] J. K. Wright, L. A. Jones, L. Stone, and T. W. Birch, "Notes on Statistical Mapping: With Special Reference to the Mapping of Population Phenomena", American Geographical Society, 1938, pp. 21-23.

> structure is of itself a factor that modifies profoundly the magnitude of our estimate of the perceptual or sensory differences between two areas ...[49]

Therefore it is not at all surprising that the end result of the Jenks-Knos experiment was that the Williams grey scale was the one best suited to stickdown patterns. It would have been very surprising indeed if the results were different, because all of the scales studied, except Williams', were constructed with flat tones.

Summary

There exist in the literature, then, three basic grey scale formulas, those of Plateau, Weber-Fechner, and Judd. The differences in the formulas are manifestations of different experimental conditions alone. Because the various grey scales used in science and industry were derived with different surround reflectances, variation in the value scales occurs. Thus, if one is using a grey scale for tone variation, he must select a grey scale that was derived with a background reflectance close to the one he is using in order to insure correctness of equal-appearing intervals. Several widely used lightness scales are included in Appendix B.

From a perusal of the literature of the psychophysics

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[49] R. L. Williams, "Map Symbols: The Curve of the Grey Spectrum -- An Answer", p. 488.

of grey scales, it is possible to summarize some of the results that are important in the use of grey scales in cartography.[50]

(1) The colour of the illumination (fluorescent white, natural, or tungsten-yellow) has no appreciable effect on the uniform grey scale evaluation (within practical limits).

(2) The size of the stimulus field used has no appreciable effect on the uniform grey scale evaluation.

(3) The intensity of illumination (within the ordinary indoor illumination range) on the stimulus surface and surround has no appreciable effect on the evaluation of a uniform grey scale.

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[50] W. R. J. Brown, "The Influence of Luminance Level on Visual Sensitivity to Color Differences", Journal of the Optical Society of America, V. 41 (1951), pp. 684-688, and

R. W. Burnham, R. M. Evans, and S. M. Newhall, "Prediction of Color Appearance with Different Adaptation Illuminations", Journal of the Optical Society of America, V. 47 (1957), pp. 35-42, and

R. W. Burnham, "Predictions of Shifts in Color Appearance with a Change from Daylight to Tungsten Adaptation", Journal of the Optical Society of America, V. 49 (1958), pp. 254-263, and

J. S. Keates, "The Perception of Colour in Cartography", Reprinted from the Proceedings of the Cartographic Symposium, Edinburgh, 1962, and

I. H. Godlove, op. cit., pp. 419-425, and

A. E. O. Munsell et al, op. cit., pp. 394-411, and

D. B. Judd, Color in Business, Science and Industry, New York: John Wiley and Sons, Inc., 1957.

(4) With similar experimental conditions, grey scales constructed from the JND method and the value step method agree closely.

(5) Grey scales are a function of both the reflecting properties of the stimulus surface and the surround to which the observations are adapted.

(6) The Fechner fraction of constancy applies only over a limited range of illumination (from 4 to 18 millilamberts).

(7) The Stevens principle, that equal stimulus ratios tend to produce equal ratio results, proves that a scale of equal lightness steps is not correctly constructed by limiting sensation steps (JND) to a constant unit.

Cartographers cannot freely apply all the principles of grey scales constructed by psychophysical experiments. Those of psychophysics deal with flat tones and reflectance measurement, whereas cartography deals with flat tone and pattern tone, both measured in transmittance percentage. Care must be exercised when using a grey scale that was constructed without cartographic stimuli. The feasible transformations from psychophysical grey scales are discussed in the next chapter. It is not the purpose of this study to judge the various lightness scales on the basis of correct or incorrect procedures, but rather to set up a background of evidence on which to base theories and methods that might pertain to the automatic construction of a grey scale for areal symbology.

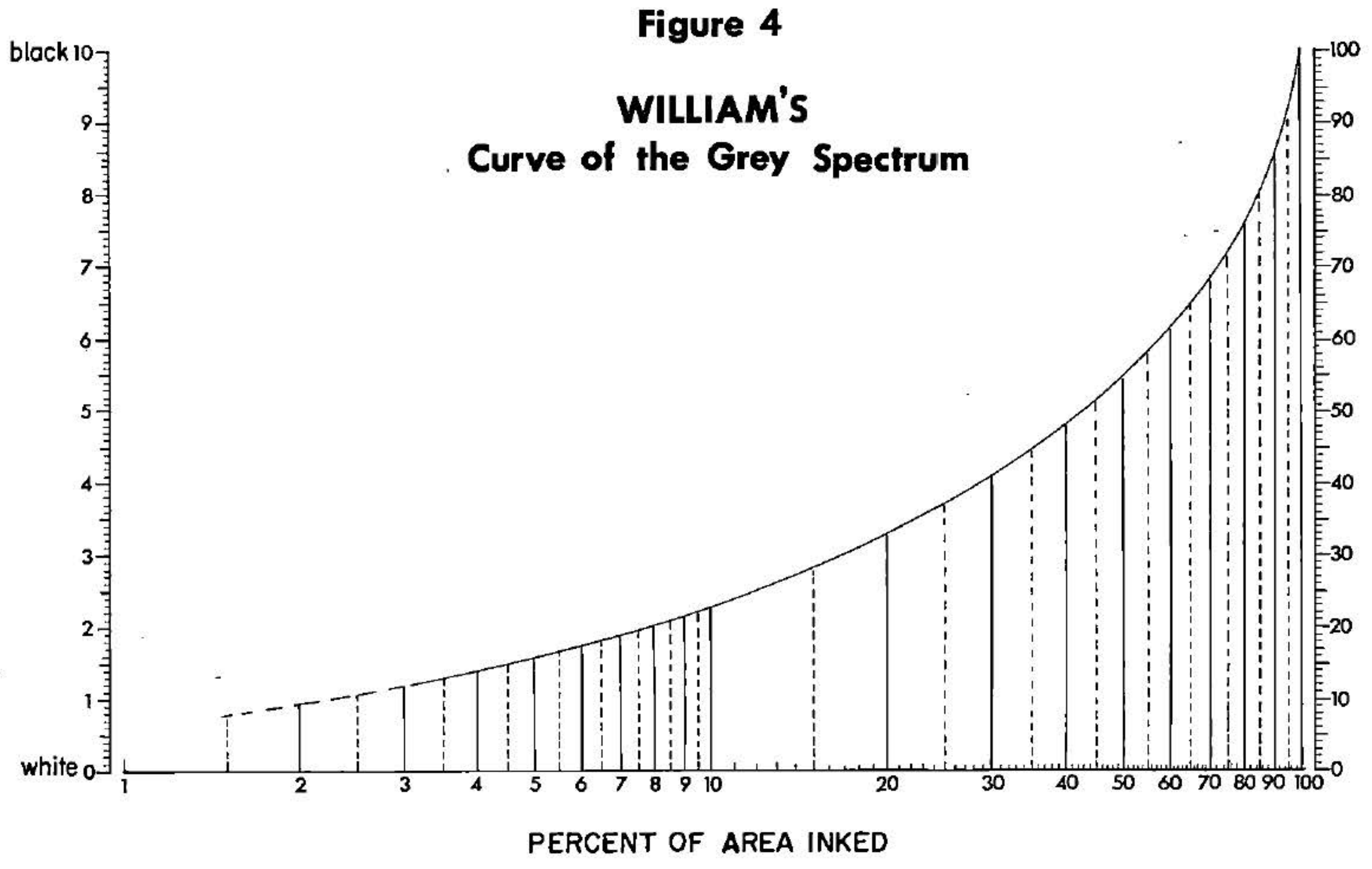
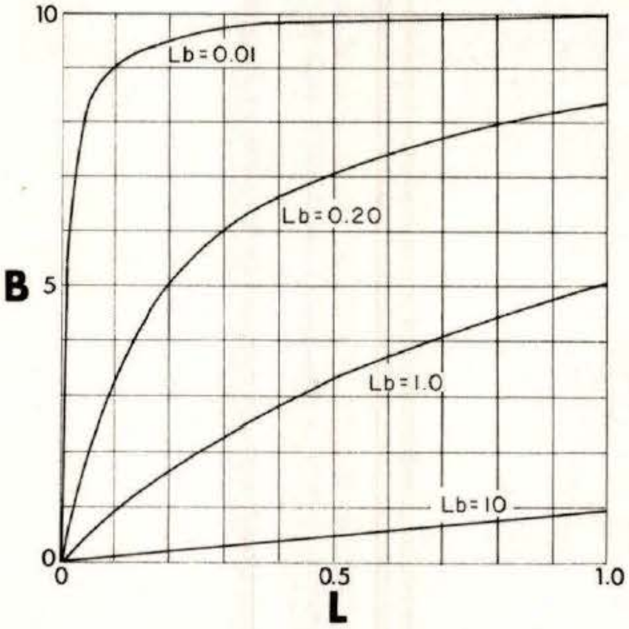


Figure 4

WILLIAM'S
Curve of the Grey Spectrum

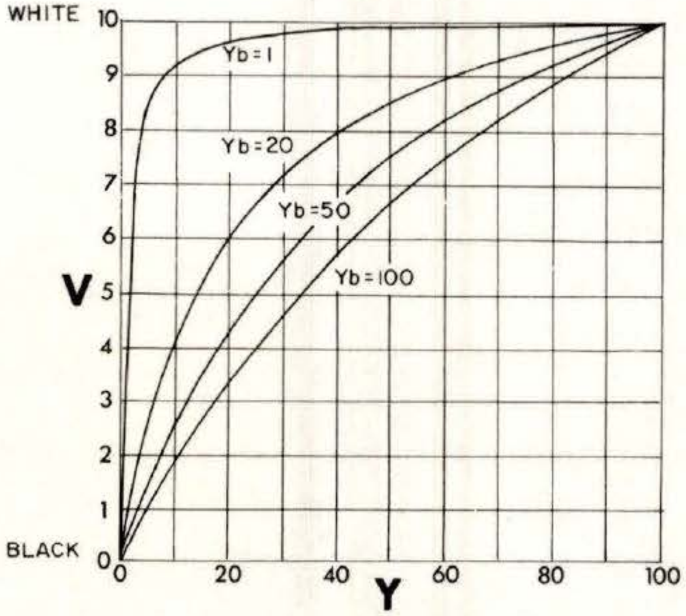
Source: R. L. Williams. Map Symbols: *Equal-Appearing Intervals for printed screens*. *Annals of the A.A.G.* *50*, 1960

Figure 5



Dependence of brightness (B) on luminance (L), observed for varying background luminance (Lb)

Figure 6



Dependence of grey scale (V) on reflectance (Y), when background reflectance (Yb) is varied.

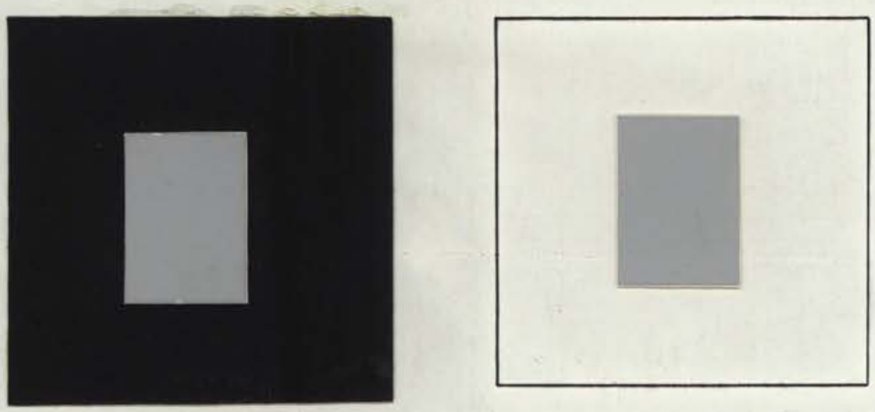
Source: G. Wyszecki, and W. S. Stiles. Color Science: Concepts and Methods, Quantitative Data and Formulas. New York: J. Wiley & Sons, 1967.

Figure 7

Induction Phenomenon of Adjacent Grey Values

Figure 8

The Effect of Background on the Perception of Lightness



CHAPTER IV

AUTOMATED GREY SCALE CONSTRUCTION

Grey Scales from Cartographic Stimuli

Only one of the grey scales discussed in the previous chapter is applicable to automated drafting. Because inkline plotters produce a visible line pattern, not a flat surface tone, computerized areal symbols must be constructed without reference to reflectance. The only empirically derived grey scale which uses such cartographic map symbols is the one constructed by Williams. All others are conversions from existing psychophysical flat tone scale to per cent ink magnitudes. Even though it is possible to measure the per cent reflectance of a per cent ink scale, the resulting series of reflectance numbers that would equate per cent ink with reflectance would be strictly conditional. The feasibility of such an equation between per cent ink area and per cent reflectance is discussed in Chapter V.

In 1959 the U. S. Aeronautical Chart and Information Center developed a grey scale in order to standardize the gradation of areal symbols used on aeronautical charts. They used dot screens of fine enough quality so the stimulus area appeared as a flat tone to the eye. The A.C.I.C. scale was directly concerned with mapping, and because their grey scale was defined by screen percentages instead of reflectance magnitudes, the value series was widely accepted in cartography. The per cent screen values for equal step grey tone according to the A.C.I.C. series are 0, 5, 10, 17, 28, 35, 42, 58, 76, and 100. Much of the time these screen values can only be approximated because few cartographic establishments, academic or otherwise, have tone screens of the exact values stated. The A.C.I.C. scale does not state any deviation limits around the screen values, and therefore it is not actually known whether, for example, a 60 per cent screen can be used in place of the 58 per cent screen without upsetting the equal step sequence. Cartographic institutions in past years have shown little concern with precision in screen values, and many lightness scales are approximations of the A.C.I.C. scale to the nearest 5 per cent. More recently, however, the concern for precision in cartography has become more apparent.

Quantification in Cartography

Throughout the history of cartography as a recognized discipline, it has been housed and nurtured within the confines of geography. The situation is rapidly changing. No longer is cartography a skill in drafting, design, and other basic techniques that can be "picked up" during one's

undergraduate years in a geography department. As A. H. Robinson states, ... "no longer can one, even in jest, describe a cartographer as a 'geographer who can also make maps'".[51] Cartography is becoming a separate discipline that must be self-administered in accordance with its own methodology, theory, and practical applications. Cartography, then, cannot be a subject that is wholly administered by a geography department. This is not saying that cartography and geography will, or should, undergo a complete split, for geography depends on cartography as an analytical tool which gives a visual display of quantitative ideas. However, cartography is becoming more concerned with engineering and mathematics.

In the last decade, one of the major factors pulling cartography in this direction has been the improvement of the computer and the consequent development of automated graphics. It can be said that automation has deprived cartography of much of its artistic heritage. Whether this is deemed fortunate or unfortunate is probably a personal matter. Even if one compares a present day atlas map drawn by standard procedures to one drawn during the seventeenth century by one of the great map-publishing establishments such as Mercator, Blaeau, or Hondius, a striking loss

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[51] A. H. Robinson, "Cartography -- Which Way", The Journal of Geography, V. 16 (January, 1967), p. 5.

of artistic flourish is evident. Today's maps are simply a matter of efficiency, and automation is carrying efficiency and precision to new thresholds. Computerized graphics is the most exciting and challenging aspect of cartography today.

Automation in Cartographic Drafting

As data gathering and processing increase in precision, it is increasingly important that cartography be concerned with precise measurement and calculations. Because it is no more trouble to digitize 34.3681 per cent than 34.0 per cent, and the computer handles both quantities with the same ease and speed, why not be exact. Precision, speed, and flexibility are only a few of the benefits automation offers to graphics.

There are two common types of automated devices which produce "hard copies": computer printers, and ink-line plotters. A printer produces a rather crude image similar to a typewriter picture, whereas a plotter draws a more refined image composed of individual lines.

SYMAP is a widely used program, developed by the Computer Graphics Laboratory at Harvard University, which constructs maps by computer printout. The SYMAP program, however, has many limitations, the most crucial of which is that all symbols, or combinations of symbols, are confined to those found on a typewriter. SYMAP produces interval shading by printing characters, singly or superimposed, within a given area. Different combinations of characters produce different percentages of black, yielding visually-different areas. Because little, if any, control can be exercised in per cent ink area calculation, and no control whatsoever over per cent reflectance, equal sensation steps for areal intervals become a matter of educated guess. The cartographer must choose sets of typewriter characters which when overprinted will give the ink weight necessary to differentiate areas. One other important limitation is that the precision of this type of map is quite low. Because the type characters are discrete symbols of fixed dimensions, the precision is limited to the characteristics of the computer printer, normally 6 or 8 intervals per inch in the Y direction, and 10 intervals per inch in the X direction. This means that straight lines can be represented successfully only if they are parallel to the axes. Thus, all choropleth and isopleth distribution maps utilizing printer output have low quality line work and areal patterns. SYMAP does offer some attractive advantages. The printout map is completed at great speed and at a relatively low cost. If one needs a quick visual representation where outline generalization is not crucial, then SYMAP application of automated graphics is a very useful analytical tool.

A line plotter is a graphical output device of relatively fine precision. All graphic output is by a pen which is actived by computer generated instructions. The line weight drawn can be varied by attaching any desired diameter pen nib. This system is much more flexible than the computer printer because the choice of characters is unlimited, and the character size is variable. A common minimal increment movement is .005 inches, so that curved lines and straight lines at various angles are easily constructed.

Pattern Rationale

As discussed in Chapter II, a cartographer has a choice of three different means of creating inked grey scales. The two most commonly used are dot and line patterns. The third style, irregularly shaped figures, such as those used with stickdown to depict vegetation coverage, is rarely used as a quantitative symbol.

The dot pattern is most preferred by the map reader for aesthetic appeal.[52] However, construction of a dot pattern on an ink-line plotter is impractical because of the lack of rigid control over per cent ink area and the time required for drawing of the dots. This suggests a line symbol instead, even though the line symbol is at a distinct disadvantage in the lower per cent ink ranges of the value scale. For example, a value of 2, which appears to the eye to be a

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[52] G. F. Jenks and D. S. Knos, <u>op. cit.</u>, p. 319.

20 per cent grey, requires only 7.5 per cent of the area to be inked. This means that very few lines are required, and the resulting pattern is quite harsh because of the large gaps of white space. A dot pattern using the same amount of ink is better suited to the lower values because the dots give a better visual distribution to the inked area. Robinson and Raisz both agree that fine textured patterns are preferred by the map reader over more coarse textures.[53]

Although there is very little control over texture at lower values when using a line pattern, if the pen width is held constant as more line work is added, the texture becomes quite fine and "tone like" to the eye. In Figure 10 the difference in fineness of tone between values 3 and 9 is quite pronounced. Loss of visual continuity for a graduated sequence of line work is unfortunate but unavoidable. However, had the pen width been increased as in Williams' experiment, the coarseness of texture would have been much more displeasing.

Grey Scale Construction

The grey values chosen to be constructed ranged from 2 to 10 at integer increments. Using Williams' scale from Figure 4, the corresponding per cent ink quantities are: 7.5, 17, 28, 43, 57, 73, 85, 94, and 100 per cent. It was

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[53] Ibid., p. 318.

Figure 9

Automated
Grey Scale Sample

plotter increment not controlled

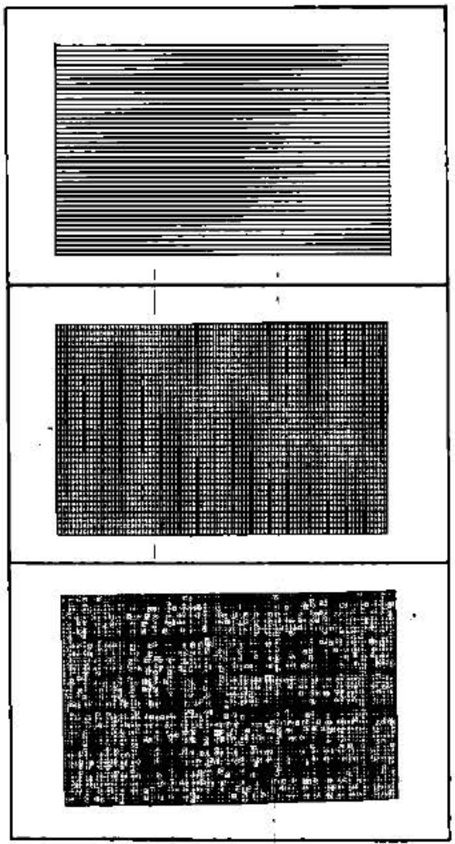
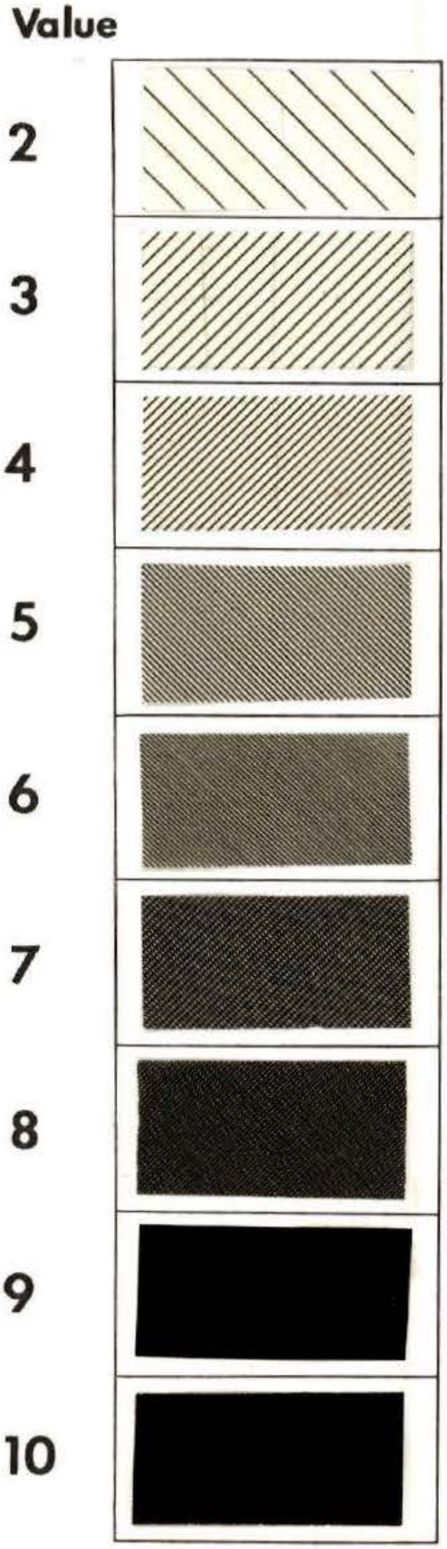


Figure 10



Automated
Equal Step Grey
Scale
plotter increment = .005"

decided to leave the line width constant throughout the value series and increase the number of lines to produce the respective per cent ink requirements.

A pen size of 0.2 millimeters was chosen, which theoretically converts to a .008 inch line weight. A sampling of linework by this pen size showed the average line width after spread on the CalComp plotter paper to be .0108 inches. This value of line width was used for all calculations. The construction of values 2 through 6 proved relatively simple because no crossing of line work was necessary to meet Williams' requirements. In order to produce the four highest grey values, it was necessary to add lines crossing at right angles. Value 6 was used as a base for values 7, 8, and 9. The line crossing procedure added complications because the areas of overlap had to be considered in the calculation of the per cent inked area of each test patch.

Two types of line patterns are possible: even spacing and uneven spacing. In the first set of grey scale patches produced, the computer was instructed to draw the lines at intervals specified to six decimal places. Because of the plotter's precision limitation of .005 inches, this method produced an uneven line pattern (Figure 9). The non-uniform pattern resulted because the plotter could only approximate the correct spacing to the closest multiple of .005 inches. Uneven patterns, such as those illustrated in Figure 9, are displeasing to the eye and distract from equal step sensation.

A second set of grey scales was produced using an even multiple of .005 inches for the spacing between lines. Some rigour is lost by the second method in meeting Williams' requirements, but it is minimal, especially when it eliminates the highly-undesirable uneven spacing. Table I shows the loss in precision when the lines are evenly spaced.

Table I

The Per Cent Error Due to Machine Limitations

Value	Per Cent Ink Unevenly Spaced Lines (Williams' Ideal)	Evenly Spaced Lines	Discrepancy
2	7.5	7.5	0.
3	17.0	17.3	+ .3
4	28.0	28.1	+ .1
5	43.0	44.9	+1.9
6	57.0	56.1	- .9
7	73.0	70.6	-2.4
8	85.0	87.1	+2.1
9	94.0	92.2	-1.8
10	100.0	100.0	0.

Figure 10 contains evenly-spaced samples which are the final result of an automated grey scale mechanically drafted to meet Williams' curve of the grey spectrum.

Experiment to Determine Equal Visually-Stepped Scale

At this point it is not known whether the series of grey patches generated by the computer (Figure 10) is an equal stepped scale or not because Williams' per cent ink presentation was not strictly followed. As stated earlier, it was decided that, unlike Williams' original equal step scale, the automated scale presented in this paper would hold line weight constant while varying the number of lines per value. This latter method eliminates the gross pattern effect found in Williams' grey scale, and makes the individual steps more "tone-like" to the eye.

Another important consideration that may have affected the equal step properties of the automated scale is that the limits of the line plotter did not allow exact duplication of Williams' per cent ink quantities. Therefore, it was decided that the grey scale produced by the line plotter would be presented in an actual map environment and tested for equal step sensation.

An important advantage in an equal step grey scale is that it allows for the maximum number of greys that can be identified in a spatial context by an observer. For this reason it was important to test equal step sensations for grey values in both adjacent and non-adjacent positions.

Method

Subjects

The subjects (Ss) used in the experiment consisted of students from the University of Victoria. Male and female students of both the graduate and undergraduate levels were approached at random and asked to be subjects in the experiment.

Stimuli

The base map of the state of Iowa used in Figure 3 was chosen as the test stimulus. Values 4 through 9 of the automated grey scale (Figure 10) were randomly assigned to each of the 99 counties that comprise the state of Iowa. The random selection of grey values to the 99 data-units allowed adjacent, as well as non-adjacent, comparisons of grey value sequences, a necessary requisite for the equal step experiment. The resulting map stimulus used in testing is displayed in Figure 11.[54]

Eight transparent overlays were prepared and registered to the base map (test stimulus). Each overlay had three coloured dots on it (yellow, red and blue). The dots were placed over the value patches that were to be tested for equal step. The six grey patches that made up the test map (values 4 through 0) allowed four unique triad combinations (values: 4-5-6, 5-6-7, 6-7-8, and 7-8-9). The first four

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[54] note: Some of the grey tone value was lost in the photographic reproduction, especially in the higher value (8-9) ranges. The original test map used the exact value patches as in Figure 10.

overlays tested the four triad combinations with the grey patches adjacent to each other. Overlays 5 through 8 tested the same triad combinations except that the grey patches of each triad were non-adjacent in proximity. In each overlay the yellow dot was always over the lightest of the three values, the red dot over the middle value, and the blue dot over the darkest of the triad.

The response sheet for each S consisted of four horizontal lines, each 100 mm. in length. Each line was marked "yellow" on the left edge, and "blue" at the right edge. (See Appendix, p.89.)

Procedure

Each S made four responses, one for each of the triads. The four triads (4-5-6, 5-6-7, 6-7-8, 7-8-9) were randomly ordered for each S. The four overlays presented to each S were of one set, either adjacent or non-adjacent. The task of each S was to make a mark on the 100 mm. line where he or she felt the grey patch under the red dot best fit. For example, if the S felt that the patch under the red dot was closer in lightness to the blue dot patch than the yellow dot patch, he would mark the line toward the blue end, aware that his mark on the 100 mm. line was an indication of "how much" closer. If the S perceived the red dot patch was one-third closer to the blue dot patch than the yellow dot patch then he would mark the line, as best he could, two-thirds, or 67 mm. from the yellow end.

Forty Ss were tested, 20 of them making judgements for the 6 values in an adjacent context, and 20 making judgements for the same 6 values in a non-adjacent context.

The hypothesis tested states that there is no variation in the mean difference between value steps. That is, distance (psychological sensation) between values 4 and 5, is the same as between values 5 and 6, is the same as between values 6 and 7, etc.

The mean difference between value steps was computed by the scaling procedure which is outlined in Appendix C. The value steps are values 4 through 9 of Figure 10.

After the data were scaled, the means of the differences between value steps, for both adjacent and non-adjacent groups; were transformed by a square root function and subjected to an analysis of variance (ANOVA).

Results

(1) A non-significant result for the adjacent and non-adjacent conditions (F = .055), indicates that Ss judged lightness distances in the same manner for both adjacent and non-adjacent conditions. There was no appreciable difference in evaluating grey scales when the grey stimuli are juxtaposed or spatially separated. This is a valuable consideration in using grey values as a quantitative symbol on a map.

(2) The highly significant finding for the value steps

portion ($F = 29.53$) suggests that the series from value 4 through 9 is not a visually equal-stepped sequence.

(3) The significant interaction between value steps measured over adjacent and non-adjacent stimuli indicates that the mean differences of the value steps for the adjacent series is significantly different from the mean differences of the values for the non-adjacent series. Therefore, the value steps are dependent upon whether the grey values being compared for equal step are adjacent or non-adjacent in location.

The results of the 2 (adjacent vs. non-adjacent) x 5 (value steps) ANOVA for a factorial design are presented in Table II.

Table II

Source of Variation	SS	df	MS	F	p
Between *Ss*	1.382	39	.035		
adjacent-nonadjacent	.002	1	.002	.055	
error (a)	1.380	38	.037		
Within *Ss*	21.881	160	.137		
value steps	9.173	4	2.293	29.53	<.001
steps x adjacent-nonadjacent	.904	4	.226	2.91	<.025
error (b)	11.805	152	.078		
TOTAL	23.263	199			

The conclusions concerning the interaction indicate only that at least two of the mean differences of the values differ. Because the *a priori* hypothesis was not accepted, and because it was of importance to the analysis to discover which of the automated value steps held an equal sensation relationship, a t-test for the differences among means was applied. All possible combinations of adjacent and non-adjacent values were tested and reported in Table III.

Discussion

The automated grey scale in this paper is not a scale with equal visual steps. Apart from the ANOVA results, this is quite evident from Figure 14 [55] which shows a very uneven trend of the mean of the difference between values. Ideally the graph should show a horizontal line across the value ranges. A trend analysis showed significant results on linear, quadratic, and even cubic shape for the adjacent and non-adjacent curves. Therefore no one shape can be said to best fit the data points of the two groups.

Much of the failure to achieve an equal sensation scale is reflected in Table I. The discrepancy in per cent ink lost or exceeded in meeting the plotter's limitations mirrors the discrepancies in mean differences between values (see raw data, Appendix C). This is especially prominent over value steps 7, 8 and 9, where value 7 is drawn toward value 6, while value 8 further compounds the effect by having an excess of ink, thereby pushing it towards value 9 in lightness sensation. Value 9 is drawn towards value 8 because it has a negative amount of required ink. The end result is that

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[55] see Appendix D. p. 92.

Table III

t-test for the Differences among Means for Value Step Differences

$\alpha = .05$

ADJACENT DIFFERENCES

NON-ADJACENT DIFFERENCES

	4-5	5-6	6-7	7-8	8-9	
4-5		+	+	+	−	4-5
5-6	+	▨	+	+	−	5-6
6-7	−	−	▨	+	−	6-7
7-8	−	+	+	▨	−	7-8
8-9	−	−	−	−		8-9
	4-5	5-6	6-7	7-8	8-9	

ADJACENT DIFFERENCES

NON-ADJACENT DIFFERENCES

+ non-significant (equal step)

− significant (not equal step)

NON-ADJACENT DIFFERENCES

4-5	−	−	−	−	−
5-6	+	+	−	+	−
6-7	+	+	+	−	−
7-8	+	+	+	+	−
8-9	−	−	−	−	−
	4-5	5-6	6-7	7-8	8-9

ADJACENT DIFFERENCES

+ non-significant (equal step)

− significant (not equal step)

the visual difference between values 6 and 7 is less than intended, while value 8 appears very close in lightness to value 9. The average of the mean differences in the data is directly correlated to the plotter's limitations.

An obvious fault of the construction of an equal step automated scale comes to light with the failure to achieve such an equal sensation scale. This study's deviation from the parameters of the original stimului, to avoid patterned greys, has produced per cent ink quantities which deviate in a corresponding manner from the established grey scale. A more precise approach would have been to produce a series of grey patches from the plotter varying each by 2 per cent ink area. Then by using either the JND or the value step methods outlined in Chapter 3, an equal sensation grey scale could be constructed free from any unfounded assumptions incited by exchanging pattern effect for tonal effect.

CHAPTER 5

CONCLUSIONS

Analysis of Results

When Williams set us a line pattern to be tested for equal sensation steps, he held the humber of lines per unit area constant and increased their widths to produce the required per cent ink. The result was a pronounced pattern easily discernible throughout the entire value range. All his observers were cued to this pattern effect and his resulting curve of the grey spectrum was constructed with this parameter as a constant.

The grey scale created in this study differs from Williams' study in that the line width is held constant and the number of lines vary. The results (Figure 10) show that the line pattern acquires a flat tone appearance from value 6 on. Therefore, because the four value steps, 6 through 9, deviated from Williams' test parameters, it was questionable whether the desired equal visual step results had been achieved.

An experiment to test for equal distance sensation among values 4 through 9 was performed on 40 observers. One portion of the experimental design was to test the performances of the observers over two conditions of adjacent grey patches and non-adjacent grey patches. The results, as outlined in Chapter 4, were disappointing because the most important aspect of the analysis, the visual equality of step values, proved to be unequal.

Much of the explanation for the failure can be attributed to loss of precision because of the line plotter's physical limitations. But this does not explain all of the deficiencies of the automated scale. The scale created in this study was different in structure from Williams', and therefore the experiment to determine equal step should not have relied so heavily on his per cent ink parameters. A more sound approach, perhaps, would have been to create a separate grey scale, independent of Williams', using a series of automated patches and the value step method. This is only an after-the-fact evaluation of the experiment, and the question still remains whether an equal step scale would have resulted if the plotter could have produced the exact per cent ink requirements formulated by Williams.

Precision is important in any form of quality mapping, but one must work within the limitations of the tools used. The plotter used for the grey scale construction was adequate, but grey values deviated as much as 2.4 per cent from the ideal values, which upset the equal step sequences. This deviation can be reduced by the use of plotters that operate at smaller increments and pen widths which produce finer line weights. The added precision of these refined pieces of equipment would result in more controlled ink percentages. But regardless of the increased precision, a pattern tone will always be visible at the lower end of the value scale.

Reflectance Measurement and Per Cent Ink Measurement

There exists no feasible method, short of making some dangerous assumptions, of converting a per cent ink value to a reflectance value or vice versa. Williams set up an empirical scale where he assigned a number system to his stimulus of known per cent ink. In no way was he concerned with reflectance measurements of the ink or the paper. He states in defence of his grey curve:

> I deliberately avoided reflectance as a measure of stimulus. The solid black and the white paper are two end points on a visual scale and can be assigned only arbitrary numbers...It is immaterial, within very wide limits, what the reflectance of the ink or paper is. The eye is very adaptable, and a surface that reflects as little as 50 per cent of the light will appear white because of the color constancy phenomenon as it is...the brightest surface in the field of view.[56]

Therefore any linear conversion of reflectance to per cent ink, or vice versa, is meaningless. Williams had very precise reflectance measurements done on his sample grey values to show conclusively that per cent ink and reflectance cannot be equated to form an interchangeable value scale. The reason this conversion does not work is because reflectance measurements are a function of the reflectance properties of both the ink and the paper. To obtain reflectance measurements for the grey scales of this thesis would add nothing to the content because the reflectance qualities of the paper and ink used are unique to these

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[56] R.L. Williams, op. cit.,p. 489. "Map Symbols -- An Answer".

particular materials. The properties of different inks, and the quality of paper surfaces are far too variable to have one workable transformation formula based on per cent ink. It was for this reason that Williams excluded reflectance measurement and based his grey spectrum entirely on per cent ink area. The per cent ink parameter yields results that are constant regardless of ink or paper reflectance qualities.

Standard Conditions

Chapter 3 outlined the importance that the background assumes in the perception of a grey sensation. Realistically, very few achromatic maps, charts, or any other graphic displays are printed on anything but a white background, or viewed under anything but average lighting conditions, and from average viewing distances. The grey scale sample constructed in this study follows these conditions.

The spacing of pattern tone must also follow a standard format. The ideal grey scale, which is approximated by the uneven tones of Figure 10, resulted from lack of precision by the plotter. Thus the deviation from the exact percentages required by Williams' rules was unavoidable. Exact spacing for per cent ink line work is a minor problem, and as more sophisticated machinery becomes readily available for cartographic use, automated drafting will assume the quality of hand drafting.

Future Aspects of Automated Graphics

In this study, all graphics produced on the line plotter were achromatic. However, multi-coloured work can be produced in two ways: by overprinting using different coloured inks, or by plotting separate images which can be used to prepare printing plates for multicolour printing. The ultimate for graphics constructed by the line plotter is to have printing plates made directly from the finished graphics.

The grey tones generated in this study probably are too coarse for the multiple printing of maps. For single-purpose thematic maps, where no plate is required and only a few copies are needed for demonstration or visual aid purposes, the precision and pattern work of the CalComp 563 is sufficient.

Experimentation is required to determine the feasibility of producing grey tones on the plotter by dot pattern rather than line pattern. A dot pattern would eliminate much of the harshness on the lower values and make the gradation scale more pleasing to the eye.

Updating and error correction often is long and tedious when maps are constructed from negatives, overlays, and screens produced by non-automated methods, and frequently the entire set of graphics must be redrawn and reprocessed to correct a single mistake, or update a single geographic factor. This procedure is avoided by utilizing the flexibility of computerized drafting. For example, a single modification of the plotting program achieved the improvement from uneven pattern to even pattern for this study. Automated storage and retrieval of map data also hold a potential advantage over conventional methods.

To have only one guide to follow for the application of areal symbols to distribution maps is sadly inadequate. The best way progress can be made in this direction is to have cartographers, rather than psychophysicists or engineers, search out new methods and theoretical frameworks for automated mapping. The need is apparent when one reads articles written for cartographic purposes that are theoretically incorrect because the author had to borrow concepts from another discipline because of a lack of cartographic source material. Just as S. S. Stevens amended the lightness laws of psychophysics because of variation in the subjects' sensitivity to differences, so must Williams' scales be standardized for variations of line weights, and for "flat tone" patterns. Automation in cartography is here to stay. No longer can a competent cartographer ignore the advantages that mechanized mapping processes have to offer. This does not mean that hand drafting is a passing requisite of cartography. It means that the computer-plotter is a new tool that must be added to a cartographer's equipment list, for cartography is becoming increasingly more scientific.

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APPENDIX A

Methods of Deriving Step Intervals for Sample Data

The intervals produced by these six methods are presented graphically in Figure 12. The maps drawn according to the six sets of steps are shown in Figure 3.

A. Geometric Method (used in Figure 3a)

Formula used: $\log H = \log L + nD$

Data values: $H = 449$

$L = 15$

$n = 6$ and

$D = .2460$

Calculations:

$\log_{10}449 = 2.6523$, $\log_{10}15 = 1.1761$

log difference $= 2.6523 - 1.1761 = 1.4762$

step log difference $= \frac{1.4761}{6} = .2460$

Interval logs: $2.6523 - .2460 = 2.4063$

$2.4063 - .2460 = 2.1603$, etc.

Interval antilogs: $\text{antilog}_{10}\ 2.4063 = 252$

$\text{antilog}_{10}\ 2.1603 = 145$, etc.

Intervals produced: 15, 26, 47, 82, 145, 252, 449

B. MacKay's Log-Log method (used in Figure 3a)

Formula used: $L_i = L_o + (L_1 - L_o)i^x$

where $x = \frac{\log(L_{n-1}-L)-\log(L_1-L_o)}{\log(n-1)}$

Data values:

L_i is undefined

$L_o = 15$

$L_1 = 25$

i = ordinal number of the class interval ($2 \leq i \leq n$)

$L_{n-1} = 400$ and

$n = 6$

Sample calculations:

$$x = \frac{\log_{10}385 - \log_{10}10}{\log_{10}5} = 2.36$$

Second interval $= 15 + 10x2^{2.236} = 48.17$

Third interval $= 15 + 10x3^{2.36} = 120.85$

Interval limits: $15 + 48.17 = 63.17$

$15 + 120.85 = 135.85$, etc.

Intervals produced: 15, 25, 63, 136, 246, 400

C. Sextiles (used in Figure 3b)

Data description: 99 data units which are to be broken into 6 intervals.

Method: The data units are arrayed in ascending order of magnitude. The intervals are obtained by counting off equal groups of data units.

Data Values:

Number of data units = 17
Number of intervals desired = 6

Sample Calculations:

Data units per interval = 99/6 = 17

Interval limits: 15, 24, 27, 32, 39, 87, 449

D. Reciprocals (used in Figure 3b)

Method: The difference of the reciprocals of the two extreme values is calculated. This difference then is divided by the number of class intervals, giving the step multiplier for the interval limits.

Data values:

$L = 15$
$H = 449$

Sample calculations:

Reciprocals: $\frac{1}{449} = .0022, \frac{1}{15} = .0666$
Difference $= .0666 - .0022 = .0644$
Step multiplier $= \frac{.0644}{6} = .0107$
Steps: $.0666 - .0107 = .0559$
$.0559 - .0107 = .0452$
etc.

Reciprocal conversion:

$\frac{1}{.0559} = 18$ $\frac{1}{.0452} = 22$

Intervals produced: 15, 18, 22, 29, 42, 76, 449.

E. Equal Step Method (used in Figure 3c)

Method: The data range is calculated as the difference of the extreme values. This data range then is divided by the number of intervals used, giving the step increment for the interval limits.

Sample calculations:

Data range $= 449 - 15 = 434$

Step increment $= \frac{434}{6} = 72.33$

Interval limits: $15 + 72.33 = 87.33$
$87.33 + 72.33 = 159.66$
$159.66 + 72.33 = 231.99$ etc.

Intervals produced: 15, 87, 160, 232, 304, 377, 449

F. Arithmetic Method (used in Figure 3c)

Formula used:

$$D = \frac{H - L}{\sum_{i=1}^{n} i}$$

Data values:

H = 449
L = 15
n = 6

Sample calculations:

$$D = \frac{H - L}{\sum_{i=1}^{6} i} = \frac{449 - 15}{21} = 20.67$$

Interval limits:

$$15 + 20.67 = 35.67$$
$$35.67 + 2(20.67) = 77.01$$
$$77.01 + 3(20.67) = 139.03 \text{ etc.}$$

Intervals produced: 15, 36, 77, 139, 221, 324, 449

APPENDIX B

Description of Grey Scale Formulas

The following five formulas define luminous reflectance as a function of grey value. The formulas are graphed in Figure 13, and sample values are tabulated in Table III. Where necessary, the formulas have been modified so that all give per cent luminous reflectance (Y) as a function of grey value (V).

1. Priest, Gibson and MacNicholas square root formula (1920):[57]

   $Y = 0.1V^2$

   This formula is similar in form and results to that of the original Munsell system (Munsell *et al*, 1933). It is best suited to a white background.

2. Newhall, Nickerson and Judd (Munsell renotation formula) (1943):[58]

   $Y = 1.2219V - .23111V^2 + .23951V^3 - .021009V^4 + .008404V^5$

   This formula applies best when the observer is adapted to a middle-grey portion of the scale. Y in this formula is on a slightly different scale, being relative to the reflectance of magnesium oxide (97.5 per cent reflectance). This accounts for $Y = 102.568$ when $V = 10$ in Table III.

3. Ladd and Pinney cube root formula (1955):[59]

---

[57] G. Wyszecki and W. S. Stiles, *op. cit.*, pp. 451-453. See pp. 32, 38. Also D.B. Judd, *op. cit.*, p. 225.

[58] *Ibid.*

[59] G. Wyszecki and W. S. Stiles, *op. cit.*, p. 451. D.B. Judd, *op. cit.*, p. 226.

$$Y = \left(\frac{V + 1.636}{2.468}\right)^3$$

This formula is an approximation of the lengthy Munsell renotation formula (formula 2 above). It should be used with the same reflectance conditions as formula number 2.

4. Foss logarithmic formula (1944):[59]

$$Y = 10^{((V-10.25)/5)}$$

This Weber-Fechner based formula is best suited to a background that is close in value to the grey chips being compared for value difference. The Color Harmony Manual uses this value step method, in which black (V-0) is defined as 9 per cent reflectance, and white (V-10) as 89.1 per cent reflectance.

5. Richter logarithmic DIN-System (1953) formula:[60]

$$Y = 10^{((V/6.1732)-1/40.7)}$$

This formula is a modification of the original Delboeuff formula. The grey scale used in the DIN color chart is defined by this formula.[61] This formula is best suited for a background whose reflectance is approximately 50 per cent.

---

[59] Ibid., pp. 451-453. See p. 33.

[60] Ibid., p. 451-453.

[61] Ibid., p. 452. See p. 38.

Table III

Reflectance Y (per cent)

Value (V)	formula (1)	formula (2)	formula (3)	formula (4)	formula (5)
0.0	0.0	0.0	.291	.889	0.0
0.5	.250	.582	.648	1.122	.504
1.0	1.000	1.210	1.218	1.412	1.111
1.5	2.250	2.021	2.051	1.778	1.842
2.0	4.000	3.126	3.198	2.238	2.724
2.5	6.250	4.614	4.707	2.818	3.786
3.0	9.000	6.555	6.628	3.548	5.066
3.5	12.250	9.003	9.012	4.467	6.608
4.0	16.000	12.001	11.909	5.623	8.467
4.5	20.250	15.580	15.368	7.079	10.706
5.0	25.000	19.766	19.439	8.912	13.405
5.5	30.250	24.583	24.173	11.220	16.657
6.0	36.000	30.053	29.618	14.125	20.576
6.5	42.250	36.202	35.826	17.783	25.298
7.0	49.000	43.063	42.845	22.387	30.989
7.5	56.250	50.677	50.726	28.184	37.846
8.0	64.000	59.099	59.519	35.481	46.109
8.5	72.250	68.398	69.273	44.668	56.066
9.0	81.000	78.665	80.039	56.234	68.064
9.5	90.250	90.009	91.865	70.794	82.523
10.0	100.000	102.568	104.804	89.125	99.946

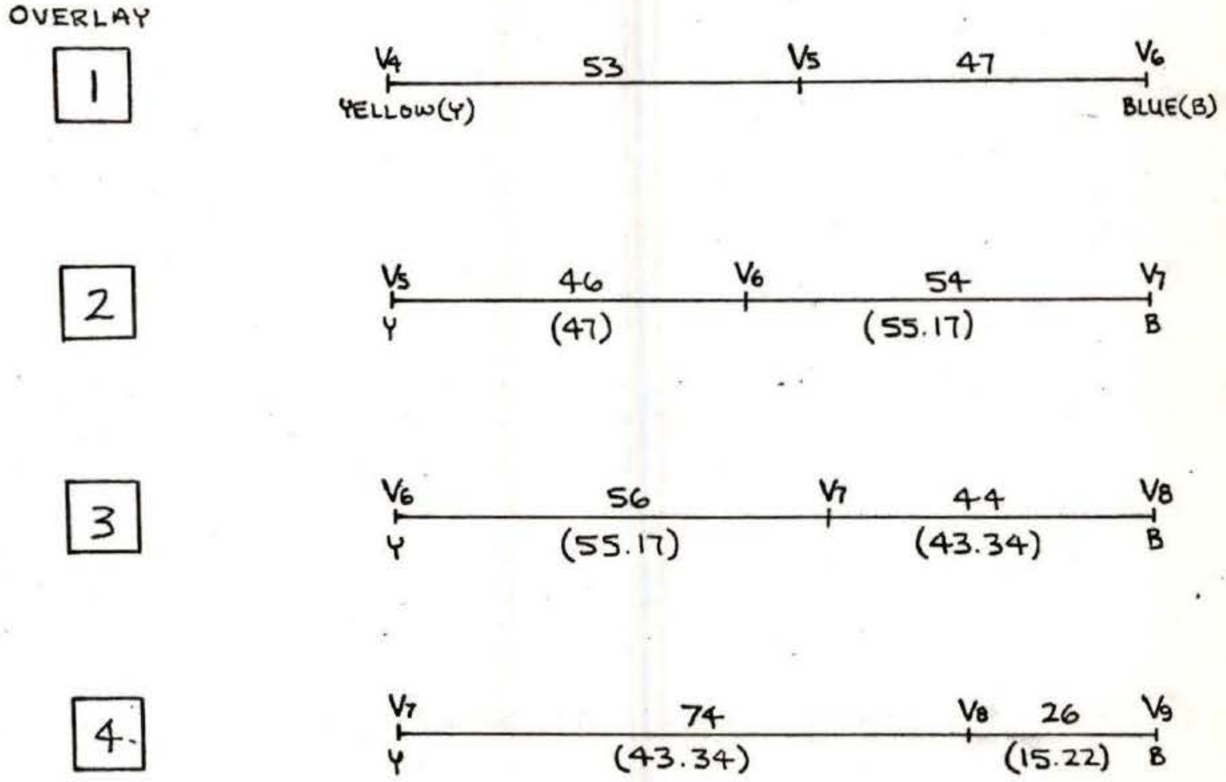
note: the reflectance refers to CIE source C and CIE (1931) standard observer

APPENDIX C

Scaling of the Experimental Data

When a S was presented the overlays for his task of the experiment, three of the five distances occurred more than once. For example, the distance 5-6 occurs in overlay 1 (triad 4-5-6), as well as overlay 2 (triad 5-6-7). Therefore the initial response of the S to overlay 1 for distance 5-6 must be taken into account when he responds to the same distance on overlay 2. For example, when a S was presented overlay 1 (triad values 4-5-6), the distance relationship between values was fixed by the S marking the line. But, when presented overlay 2 (triad values 5-6-7), the distance 5-6 had to be scaled to comply with distance 5-6 of overlay 1. This is because the distance from 6 to 7 of overlay 2 is a function of sensation distance 5-6 of overlay 2, which in turn is dependent upon distance 5-6 of overlay 1. Similarly, the remaining two triad presentations were adjusted to meet the distance relationship of the preceding overlay.

The following example is an actual test result for S20 on the four non-adjacent overlays. The four 100 mm. lines are marked off to the same distances as the test sheet for S20. Then:



Where: V4 = value 4

V5 = value 5

V6 = value 6

V7 = value 7

V8 = value 8, and

V9 = value 9

The integers (without brackets) above each line represent the distance in mm. from the middle value to the end values.

In order to begin with a relative scale, V4 of overlay 1 was set equal to 0, and V6 of overlay 1 was set equal to 100. Therefore, V5 of overlay 1 has the value 53 because S20 marked the line 53 mm. from V4. The S was then presented overlay 2, and he marked V6 a distance of 46 mm. from overlay V5. But, the distance between 5 and 6 was already fixed at 47 mm. from overlay 1. Therefore the distances on overlay 2 had to be scaled to comply with overlay 1.

Hence: distance 5-6 on overlay 2 = 47, and

distance 6-7 on overlay 2 = $\frac{47}{46} \times 54 = 55.17$.

Similarly, for the remaining overlays:

distance 6-7 on overlay 3 = 55.17, and

distance 7-8 on overlay 3 = $\frac{55.17}{56} \times 44 = 43.34$,

distance 7-8 on overlay 4 = 43.34, and

distance 8-9 on overlay 4 = $\frac{43.34}{74} \times 26 = 15.22$.

The resulting distances that have been scaled are displayed in brackets beneath each 100 mm. line.

Adding the successive scaled distances to the two preset values (V4 and V6) results in the following series:

V4 = 0
V5 = 53
V6 = 100
V7 = 155.17
V8 = 198.51
V9 = 213.73, then

using the formula: Vscale = (5/Vmax(Vi) + 4) where; Vscale = original scale values 4 through 9

Vmax = the maximum value in the scale data (V9), and

Vi goes from V4 through V9 of the scaled values.

Sample calculations:

$$V4 = \frac{5}{213.73} \times 0 + 4 = 4$$

$$V5 = \left(\frac{5}{213.73} \times 53\right) + 4 = 5.24$$

$$V6 = \left(\frac{5}{213.73} \times 100\right) + 4 = 6.34, \text{ etc.}$$

The scaled values for S20 non-adjacent compute to:

4.0, 5.24, 6.34, 7.63, 8.64, 9.0

The scores of all 40 Ss were scaled using the above procedure. The differences between successive pairs (i.e. 5.24-4.0 = 1.24, 6.34-5.24 = 1.10 , etc.) were then transformed by a square root function and the resulting treated data were used in the ANOVA.

MAP STIMULI FOR EQUAL VALUE STEP TEST

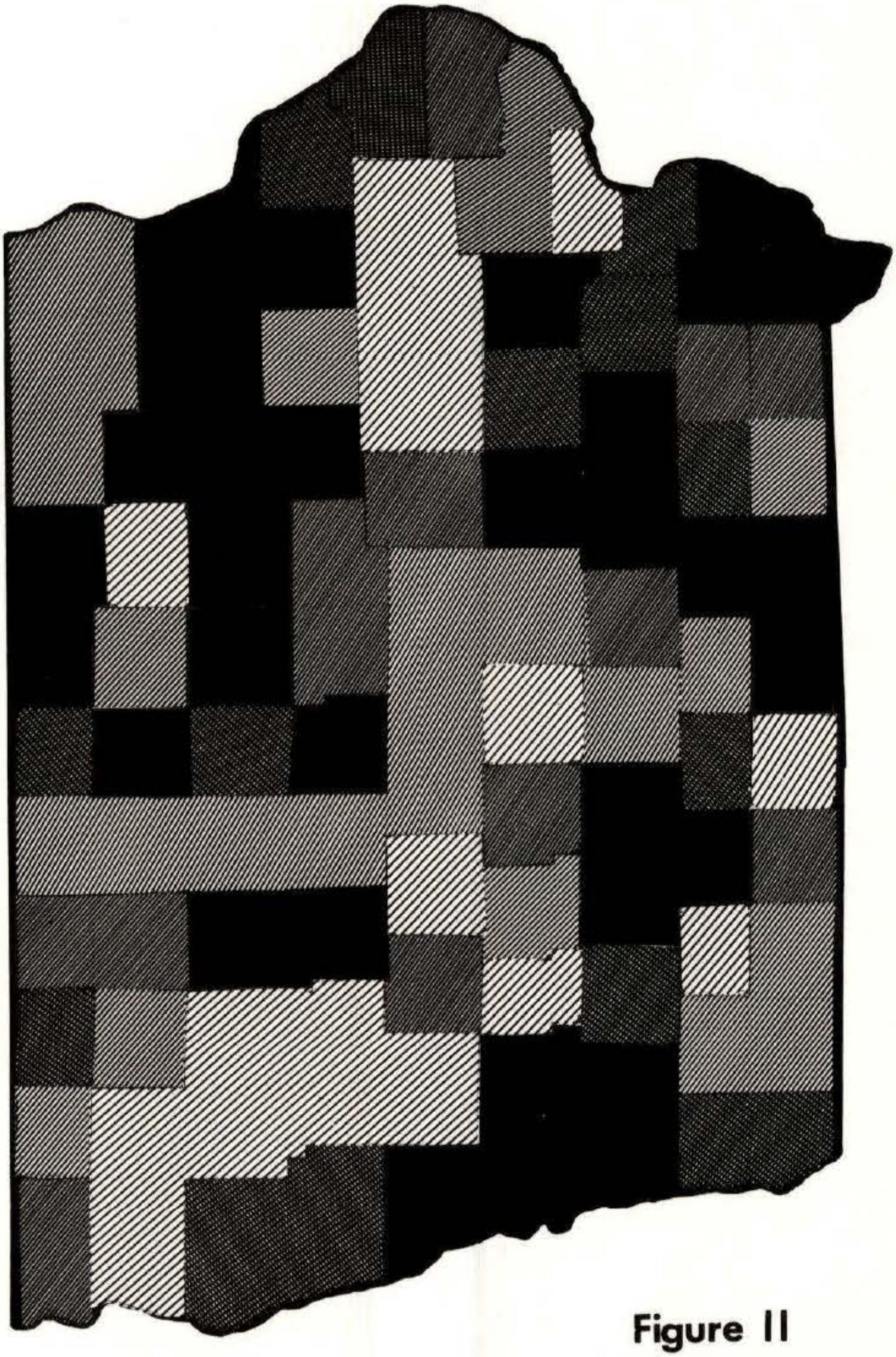


Figure 11

Square Root Transformation

NON-ADJACENT STIMULI GROUP

VALUE STEP DIFFERENCES

	4-5	5-6	6-7	7-8	8-9	Σx
S1	1.88	.79	.41	.81	0.0	3.89
2	1.10	1.37	.83	1.06	.24	4.60
3	1.49	1.02	.75	.96	.49	4.71
4	1.29	.93	.68	1.28	.61	4.79
5	1.34	1.45	.79	.62	.28	4.48
6	1.20	1.20	.94	1.10	.14	4.58
7	1.33	1.28	.89	.81	.37	4.68
8	1.30	1.25	.78	.88	.59	4.80
9	1.16	1.19	1.11	.91	.41	4.78
10	1.38	1.01	.82	1.04	.55	4.80
11	1.26	1.25	.77	1.21	.55	5.04
12	.87	.73	.89	1.37	.00	3.86
13	1.33	1.00	.88	1.31	.43	4.95
14	1.45	1.11	.95	.96	.24	4.71
15	1.38	.80	.90	.85	.57	4.50
16	.73	1.19	.98	1.49	.79	5.18
17	.77	1.46	1.16	1.19	.48	5.06
18	1.34	1.09	.50	.75	.48	4.16
19	1.08	1.04	.92	.93	.96	4.93
20	1.11	1.01	1.13	1.00	.60	4.85
Σx	24.79	22.17	17.08	20.53	8.78	93.35
Σx^2	32.01	25.39	15.24	22.04	4.96	99.64
$\bar{x}$	1.2395	1.1085	.8540	1.0265	.4390	
s	.2535	.2019	.1813	.2195	.2349	
s^2	.0642	.0407	.0328	.0481	.0551	

Square Root Transformation

ADJACENT STIMULI GROUP

VALUE STEP DIFFERENCES

	4-5	5-6	6-7	7-8	8-9	Σx
S1	1.13	1.23	1.15	.74	.57	4.83
2	1.02	1.01	1.00	1.14	.80	4.98
3	.97	1.04	.71	1.45	.58	4.77
4	1.23	1.22	.79	1.41	.61	5.27
5	1.24	1.32	.84	.81	.62	4.84
6	1.29	1.32	.88	.73	.53	4.76
7	.66	.69	.77	1.00	1.58	4.72
8	.57	.65	.75	1.45	1.26	4.69
9	.87	.92	.86	1.21	1.09	4.96
10	1.16	.74	1.11	.77	.45	4.24
11	1.10	1.08	1.46	1.69	.24	5.58
12	.71	.55	.71	.91	.92	3.81
13	1.23	1.25	.89	.61	.56	4.55
14	1.38	1.38	.86	1.37	.28	5.28
15	1.16	.81	.81	1.16	.69	4.64
16	.49	.74	.84	1.30	.40	3.78
17	.82	1.20	1.01	.57	.41	4.02
18	1.48	1.39	.72	.44	.22	4.26
19	1.55	.94	1.20	1.10	.26	5.06
20	.97	1.01	1.05	1.28	.85	5.17
Σx	21.03	20.49	18.41	21.14	12.92	94.21
Σx^2	24.02	22.30	17.67	24.62	10.75	99.38
$\bar{X}$	1.051	1.0245	.9205	1.0570	.6460	
S	.2683	.2551	.1897	.3374	.3468	
S^2	.0719	.0650	.0359	.1138	.1202	

GREY SCALE
ANALYSIS

☐ ADJACENT

☐ NON-ADJACENT

NUMBER

☐ YELLOW ↑——↑ BLUE

☐ YELLOW ↑——↑ BLUE

☐ YELLOW ↑——↑ BLUE

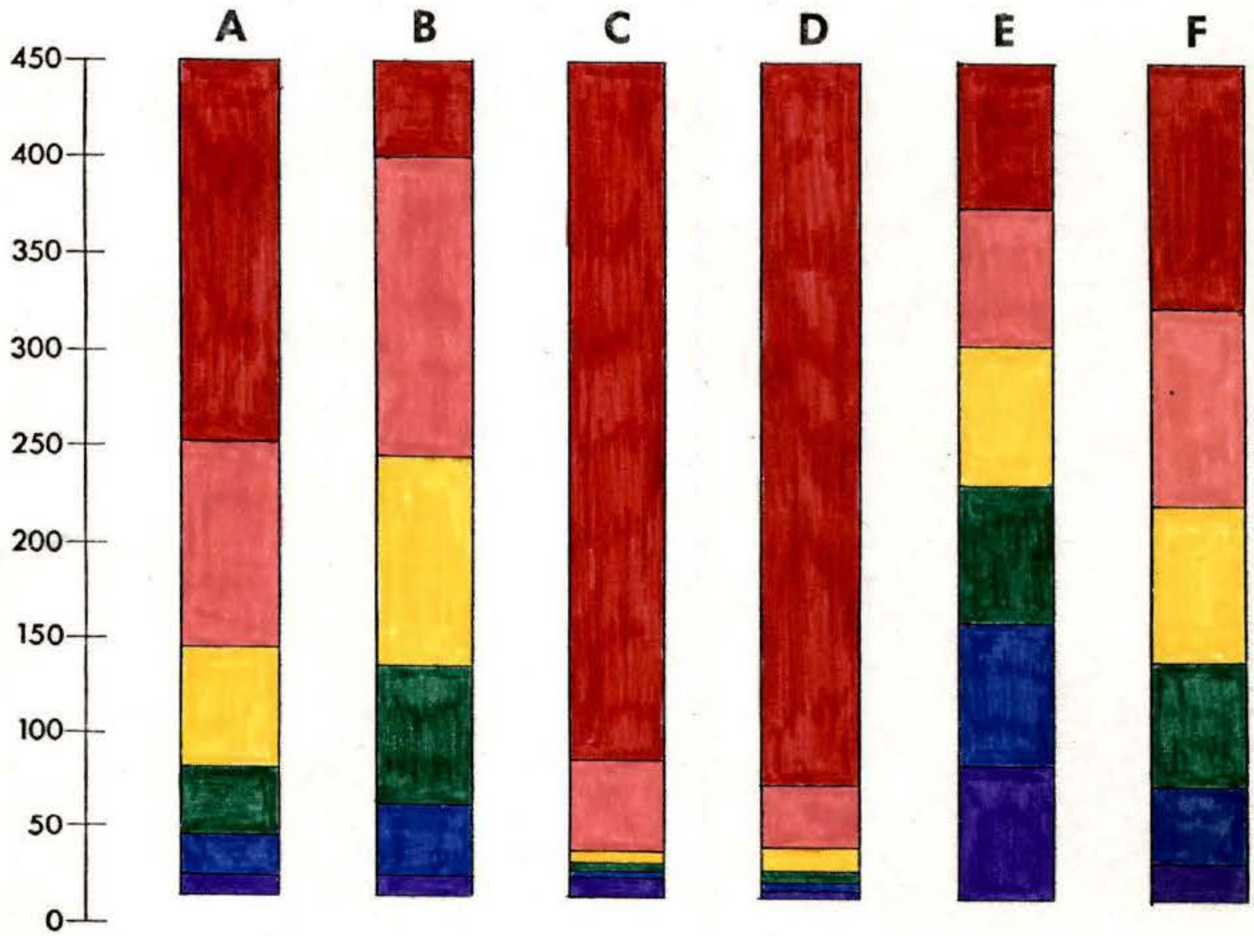
☐ YELLOW ↑——↑ BLUE

View 14 D

Figure 12

A GRAPHIC COMPARISON OF THE METHODS USED FOR INTERVAL SELECTION

A -- Geometric
B -- MacKay's Log-Log
C -- Sextiles
D -- Reciprocals
E -- Equal-Step
F -- Arithmetic



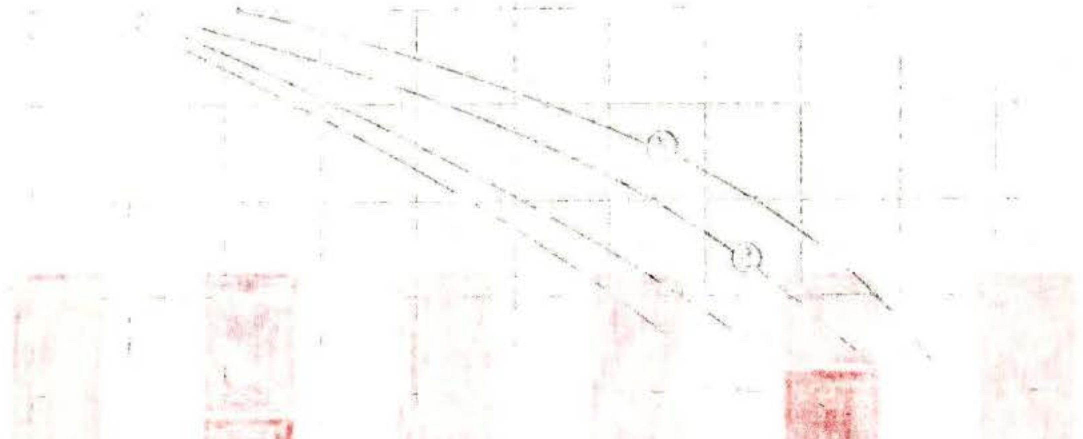
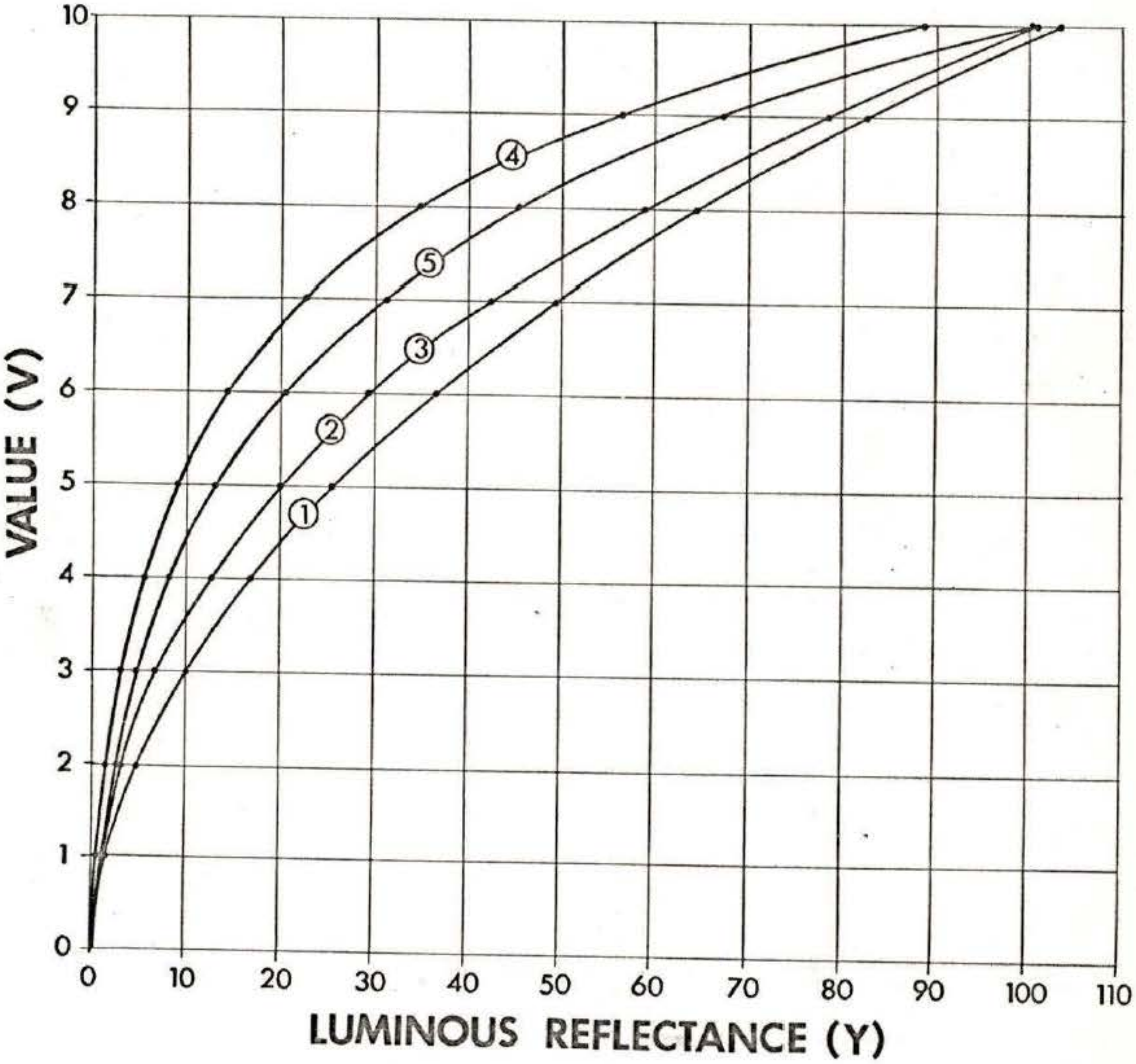


Figure 13

RELATIONSHIP BETWEEN VALUE (V) AND LUMINOUS REFLECTANCE (Y) BASED ON DIFFERENT EXPERIMENTS

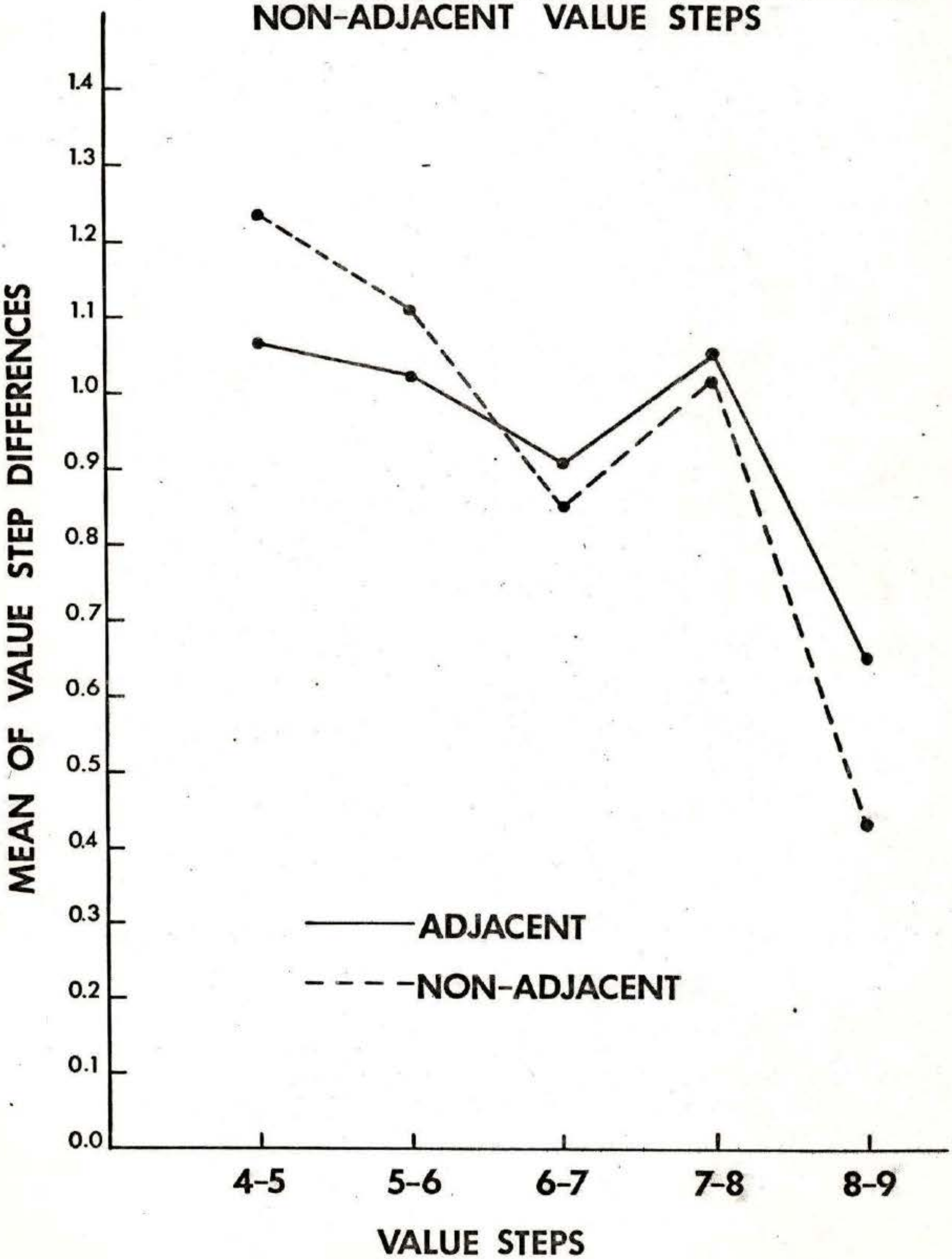
see Table III



Source: G. Wyszecki, and W. S. Stiles. Color Science: Concepts and Methods, Quantitative Data and Formulas. New York: J. Wiley & Sons, 1967.

Figure 14

INTERACTION OF THE MEAN DIFFERENCES BETWEEN ADJACENT AND NON-ADJACENT VALUE STEPS



GREY SCALE PROGRAM LISTING

```
C----THIS PROGRAM DRAWS GREY SCALES ACCORDING TO WILLIAMS'
     PER CENT INK RULES.

C----A. W. MEINHARDT.  MAY, 1970.
     DIMENSION WORK (10000)
     CALL PLOTS (WORK, 10000)

C----POSITION PLOTTER
     CALL PLOT (0., -10., 23)
     CALL PLOT (0., 1., -3)

C----THIS IS A PROGRAM FOR PLOTTER INCREMENT = .005 INCHES

C----VALUE 2
     CALL GRID (0., 0., 23, .15, 1, 3.5)
     CALL PLOT (5.5, 0., -3)

C----VALUE 3
     CALL GRID (0., 0., 54, .065, 1, 3.5)
     CALL PLOT (5.5, 0., -3)

C----VALUE 4
     CALL GRID (0., 0., 91, .04, 1, 3.5)
     CALL PLOT (5.5, 0., -3)

C----VALUE 5
     CALL GRID (0., 0., 138, .025, 1, 3.5)
     CALL PLOT (5.5, 0., -3)

C----VALUE 6
     CALL GRID (0., 0., 184, .02, 1, 3.5)
     CALL PLOT (5.5, 0., -3)

C----VALUE 7
     CALL GRID (0., 0., 184, .02, 121, .03)
     CALL PLOT (5.5, 0., -3)

C----VALUE 8
     CALL GRID (0., 0., 184, .02, 211, .015)
     CALL PLOT (5.5, 0., -3)

C----VALUE 9
     CALL GRID (0., 0., 211, .015, 211, .015)
     CALL PLOT (5.5, 0., -3)

C----VALUE 10
     CALL GRID (0., 0., 350, .01, 350, .01)

C----CLEAR PRINTER
     CALL PLOT (15., 1., 999)
     CALL EXIT
     END
```

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