PHENOMENOLOGICAL CALCULATION OF THE CROSS SECTION FOR THE REACTION

$\pi^{+} d \longrightarrow p p \gamma$

by

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ABSTRACT

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A phenomenological calculation of the differential cross sections for the reaction $\pi^+ d \longrightarrow pp\gamma$ is presented. The scattering matrix is derived from the first order Feynman diagrams. When the angles of the protons are chosen close to their elastic ($\pi^+ d \longrightarrow pp$) values, the energy of the photon is small. The scattering matrix can then be simplified since magnetic-moment coupling is insignificant for low photon energies. Expansion in powers of the energy of the photon, and the application of gauge invariance allows the scattering matrix to be written in terms of mass-shell strong amplitudes. The cross section can then be determined from the elastic data.

The pion energy range considered is 150-300 MeV. The geometry is chosen such that the two protons make equal angles with the incident direction, and results are given for various values of those angles.

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CHAPTER 1

INTRODUCTION

The study of quantum field theory has led to several different approaches to the problem of interactions of particles and fields. These approaches are ultimately equivalent in that when they are applied to specific problems, they lead to the same physical predictions. Schwinger (1958) has developed field theory using a strict variational approach. Canonical field theory, with its elaborate mathematical structure, has been thoroughly treated by Schweber (1961). There have also been attempts at a purely axiomatic field theory.

A less formal but more direct approach, which is applicable to quantum electrodynamics, is propagator theory. This theory was developed by Feynman, and it can be found in various texts of modern physics (see, for example, Bjorken and Drell 1964). The propagator for a certain particle is determined by the wave equation which describes that particular particle. In fact, the propagator is just the Green's function $G(x',x)$ of the appropriate wave equation, and represents the probability amplitude for a particle wave to originate at x and propagate to x′. Application of propagator theory to scattering processes involving electromagnetic interactions leads directly to a perturbation expansion of the scattering matrix in powers of the proton charge e. Each term in the expansion can be interpreted in terms of corresponding Feynman diagrams of the process. The rules used in the analysis of these diagrams are identical to those developed from a rigorous canonical field theory.

The number and complexity of the diagrams required to evaluate the terms in the scattering matrix increases rapidly with increasing powers of e. The success of the procedure lies in the fact that e is small. Thus, the contribution from the complex higher order terms is comparatively small. It is well known that calculations in which only the first order diagrams are considered have produced results in good agreement with experimental data.

Calculations of scattering amplitudes using Feynman diagrams is intuitively appealing and much simpler than a canonical field-theoretic calculation, since it allows one to write the scattering matrix directly from simple graphs which describe the process. One of the interesting features of these graphs is the appearance of virtual particles. Real particles, the particles which describe the initial and final states for example, always satisfy the Einstein condition that the square of the four-momentum of the particle equals the square of its rest mass. Virtual particles, however, are described by propagators in momentum space, and do not satisfy the Einstein condition. The exchange of virtual particles is considered to be responsible for the type of interaction involved. The electromagnetic interaction is described by the exchange of virtual photons. Analogously, one tries to describe the strong interaction in terms of the exchange of virtual pions. In Feynman diagrams an external (real) particle is drawn as a line which connects a vertex (interaction) to a region outside the interaction. An internal (virtual) particle is drawn as a line which connects two vertices and is always described by a propagator in the scattering matrix.

Virtual particles are also known as particles that are off the mass shell. Since a virtual process temporarily violates the usual relation between energy and momentum, the durations of such violations must of course be compatible with the uncertainty principle.

The dynamics of strong interaction processes depend on whether or not the particles involved are on their mass shells. Consider the case in which an incident particle emits a photon in the field of the target particle and then interacts strongly with that particle. After the incident particle emits the photon (which is classically forbidden by energy-momentum conservation) it becomes an off-shell particle and must be described by its appropriate propagator. This bremsstrahlung scattering matrix is thus described by off-shell strong amplitudes.

Bremsstrahlung reactions involving nucleons and mesons have received considerable experimental and theoretical attention in recent years. Refinement of bremsstrahlung experiments and calculations is necessary in order to explicitely determine the off-shell behavior of the strong interactions. The results of such investigations are useful in the testing of strong-interaction models and in the study of nuclear reactions.

Since nucleons have been the most readily available nuclear probes, nucleon-nucleon bremsstrahlung has been extensively studied and used as a test of different potential models. Pearce, Gale and Duck (1967) have calculated proton-proton bremsstrahlung cross sections using the Tabakin separable potential to generate off-shell scattering matrix elements for low partial waves and off-shell one-pion-exchange amplitudes for higher partial waves. Their results were in good agreement with experiments in which the nucleon energies were above 30 MeV. Drechsell and Maximon (1968) calculated the cross section for the same reaction using the Hamada-Johnston and Reid (soft-core) potentials. Agreement is obtained with experimental data for angular distributions and cross sections integrated over the photon directions.

The only phenomenological calculations for nucleon-nucleon bremsstrahlung were carried out by Nyman (1967) and Felsner (1967). Their results are obtained by using only the elastic (without bremsstrahlung) nucleon-nucleon scattering data. Even though the method used is applicable to the production of soft, or low energy, photons, they found good agreement with experiment even for photons with energies comparable to the incident nucleon energy.

With the coming availability of high-intensity pion beams, new information and understanding of the strong interaction can be obtained by using pions as nuclear probes. There are several advantages of utilizing pion interactions in nuclear structure studies (Jean 1964). The pion can be absorbed by the nucleus, and, since it is a boson, the kinematical analysis of the final state is simplified. This occurs also with photons as nuclear probes, but the pion's charge makes it easier to control experimentally than the photon. In addition, since charge exchange is possible with pions, there is a greater number of possible investigations. Study of the reactions $\pi N \rightarrow \pi N \gamma$ and $\pi d \rightarrow NN\gamma$ is attractive because their strongly interacting parts correspond to quasi-elastic nuclear scattering $(\pi,\pi N)$ and $(\pi,2N)$, which are scattering events in which the nucleus takes no dynamic part in the interaction.

Calculations for πN bremsstrahlung have been carried out recently by Picciotto (1969). In the present work similar phenomenological calculations are carried out for the reaction $\pi^+ d \rightarrow pp\gamma$ for pion energies of 150-300 MeV and various angles of the final particles. First-order electromagnetic effects are added to the reaction $\pi^+ d \rightarrow pp$, with the approximation that magnetic-moment coupling is insignificant. This approximation is valid for soft photons and provides good estimates for future experiments.

The constraints introduced by the gauge-invariance of the electromagnetic interaction relate the amplitudes due to photon emission from external particles to the amplitude due to emission from within the strong interaction. This result is a very important tool of quantum electrodynamics, and it is used here in the following way. An expression is gauge-invariant if its physical content is not affected by the substitution $A_\mu \longrightarrow A_\mu + \partial f/\partial x^\mu$, where A_μ is the four-potential of the interaction (see Appendix A for notation). It can be shown (Bjorken and Drell 1964) that gauge-invariance of the electromagnetic interaction is equivalent to the conservation of overall electromagnetic current. The continuity equation $\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$ is written covariantly as $\partial_\mu j^\mu = 0$. In quantum mechanics ∂_μ is proportional to k_μ, thus $k_\mu j^\mu = 0$. Here j^μ is the total four-current which produces the photons of four-momentum k_μ. It is also known from quantum electrodynamics that the total matrix element for emission of a free photon of polarization ϵ_μ is proportional to $\epsilon_\mu j^\mu$. It follows that if ϵ_μ in the total matrix element is replaced by k_μ, the resulting expression must vanish identically. In the present problem the condition of gauge-invariance allows the unknown part of the matrix element (photon emission from within the strong interaction) to be determined. This allows the first two terms in the expansion of the scattering matrix in powers of k/E, where k is the energy of the photon, and E is the total energy available to the photon, to be written in terms of strong amplitudes evaluated on the mass-shell, and thus the cross section can be determined from the data for the elastic reaction alone.

CHAPTER 2

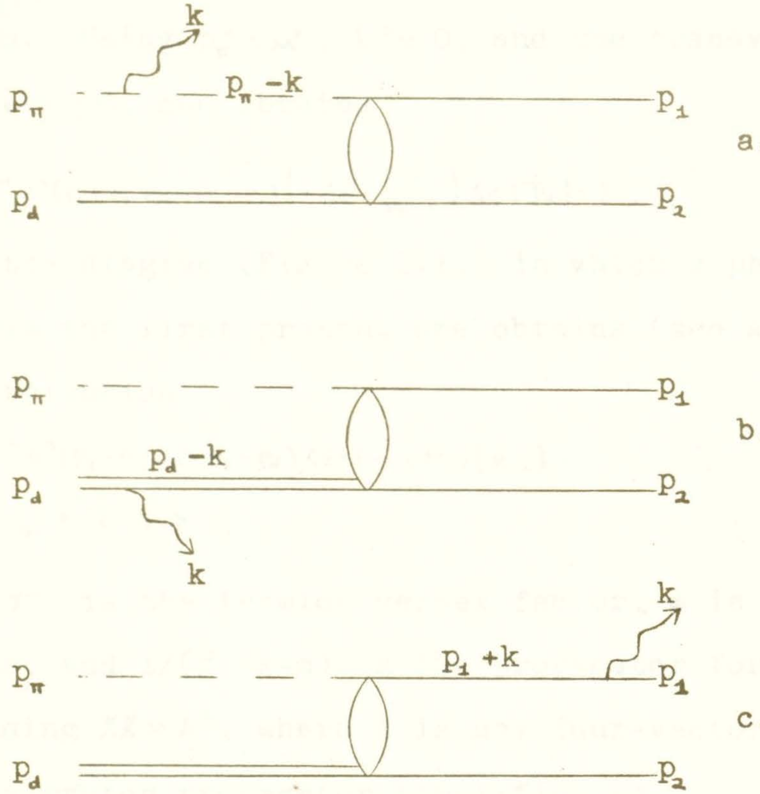
CALCULATION OF THE SCATTERING MATRIX

In calculating bremsstrahlung matrix elements to first order in e, all possible Feynman diagrams in which one photon is emitted must be considered. When a positively charged pion strikes a deuteron, and two protons emerge, a photon can be emitted from the pion, deuteron, either proton, or from within the structure of the strong interaction. Referring to the Feynman diagram (Figure 2.1a) and the rules for its analysis (Appendix A), the contribution to the scattering matrix due to photon emission from the pion is, in momentum space,

$$S_\pi = (2\pi)^4 \delta^4(p_1 + p_2 + k - p_\pi - p_d) \langle f | \hat{T}_\pi \times \frac{i}{(p_\pi - k)^2 - \mu^2} \times [-ie(2p_\pi - k)] \cdot \epsilon | i \rangle , \tag{2.1}$$

where p_π, p_d, k, p_1, and p_2 are the four-momenta of the pion, deuteron, photon and two protons respectively; μ is the mass of the charged pion. The four-dimensional Dirac delta function expresses overall energy-momentum conservation. The factor $i/[(p_\pi - k)^2 - \mu^2]$ is the pion propagator. The scattering of the pion from momentum p_π to $p_\pi - k$ contributes the vertex factor $-ie(2p_\pi - k)$, and the

Figure 2.1


k
p_π
$p_\pi - k$
p_1
a.
p_d
p_2
p_π
p_1
b.
$p_d - k$
p_d
p_2
k
k
$p_1 + k$
p_π
p_1
c.
p_d
p_2
p_π
p_1
d.
p_d
$p_2 + k$
p_2
k
p_π
p_1
k
e.
p_d
p_2

free photon vertex contributes ϵ, the photon polarization four-vector. The operator $\hat{T}_\pi$ characterizes the strong interaction. Using $p_\pi^2 = \mu^2$, $k^2 = 0$, and the transversality condition $\epsilon \cdot k = 0$, one obtains

$$S_\pi = (2\pi)^4 \delta^4(P_1 + P_2 + K - P_\pi - P_d)\left(-e\epsilon \cdot \frac{P_\pi}{P_\pi \cdot K}\right) \langle f | \hat{T}_\pi | i \rangle . \tag{2.2}$$

From the diagram (Figure 2.1c) in which a photon is emitted from the first proton, one obtains (see Appendix A) the contribution

$$S_1 = (2\pi)^4 \delta^4(P_1 + P_2 + K - P_\pi - P_d) \langle f | (-ie\gamma^\mu)(\epsilon_\mu) \times \frac{i}{\not{P}_1 + \not{K} - m} \times \hat{T}_1 | i \rangle , \tag{2.3}$$

where $(-ie\gamma^\mu)$ is the fermion vertex factor, m is the proton mass, and $i/(\not{p}_1 + \not{K} - m)$ is the propagator for the proton. Using $\not{A}\not{A} = A^2$, where A is any four-vector, and $p_1^2 = m^2$, the proton propagator simplifies to

$$\frac{i}{\not{P}_1 + \not{K} - m} = \frac{i(\not{P}_1 + \not{K} + m)}{(P_1 + K)^2 - m^2} = \frac{i}{2K \cdot P_1} \times (\not{P}_1 + \not{K} + m) \quad ; \tag{2.4}$$

therefore equation (2.3) becomes

$$S_1 = (2\pi)^4 \delta^4(P_1 + P_2 + K - P_\pi - P_d) \frac{e}{2P_1 \cdot K} \langle f | \not{\epsilon} (\not{P}_1 + \not{K} + m) \hat{T}_1 | i \rangle . \tag{2.5}$$

Using the identity $\not{A}\not{B} = -\not{B}\not{A} + 2A \cdot B$, one obtains

$$\langle f | \not{\epsilon} (\not{P}_1 + \not{K} + m) \hat{T}_1 | i \rangle = \langle f | (-\not{P}_1 + m) \not{\epsilon} \hat{T}_1 | i \rangle + \langle f | \not{\epsilon} \not{K} \hat{T}_1 | i \rangle + 2 \epsilon \cdot P_1 \langle f | \hat{T}_1 | i \rangle . \tag{2.6}$$

The first term in equation (2.6) vanishes since the final state $\langle f|$ includes the spinor $\bar{u}(p_1, s_1)$, and by the Dirac equation, $\bar{u}(p_1, s_1)(\not{p}_1 - m) = 0$. Now, using $\not{\epsilon}\not{k} = \epsilon \cdot k - i\sigma_{\mu\nu}\epsilon^\mu k^\nu$, where $\sigma_{\mu\nu}$ is defined in Appendix A, and substituting equation (2.6) into equation (2.5) gives

$$S_1 = (2\pi)^4 \delta^4(p_1 + p_2 + k - p_\pi - p_d) \frac{e}{2p_1 \cdot k} \left[2\epsilon \cdot p_1 \langle f|\hat{T}_1|i\rangle + \langle f|(-i\lambda\sigma_{\mu\nu}\epsilon^\mu k^\nu)\hat{T}_1|i\rangle \right] , \tag{2.7}$$

where λ is the total magnetic moment of the proton.

The contributions from the graphs in which the photon is emitted from the deuteron and second proton are calculated in the same manner. The scattering matrix is the sum of the contributions from all the graphs in Figure 2.1 and is given by

$$\begin{aligned} S_{fi} = {} & (2\pi)^4 \delta^4(p_1 + p_2 + k - p_\pi - p_d) \langle f| - e\epsilon \cdot \left(\frac{p_\pi}{p_\pi \cdot k}\hat{T}_\pi + \frac{p_d}{p_d \cdot k}\hat{T}_d + \hat{T}_{int} \right. \\ & \left. - \frac{p_1}{p_1 \cdot k}\hat{T}_1 - \frac{p_2}{p_2 \cdot k}\hat{T}_2 \right) + \frac{e}{2p_1 \cdot k}(-i\lambda\sigma_{\mu\nu}\epsilon^\mu k^\nu \hat{T}_1) + \\ & \frac{e}{2p_2 \cdot k}(-i\lambda\sigma_{\mu\nu}\epsilon^\mu k^\nu \hat{T}_2) + \frac{e}{2p_d \cdot k}(i\lambda_d \sigma_{\mu\nu}\epsilon^\mu k^\nu \hat{T}_d)|i\rangle , \end{aligned} \tag{2.8}$$

where λ_d is the total magnetic moment of the deuteron. The terms with $\hat{T}_{int}$, $\hat{T}_2$, and $\hat{T}_d$ correspond to the graphs in Figure 2.1e, d, and b respectively. The terms which contain $\sigma_{\mu\nu}$ are due to the magnetic-moment coupling of the photon.

In the soft-photon approximation all components of k^μ are small compared with the components of p_1^μ and p_2^μ. This approximation greatly simplifies an otherwise extremely difficult problem, since the magnetic-moment coupling is no longer significant, and the scattering matrix reduces to

$$S_{fi} = (2\pi)^4 \delta^4(P_1 + P_2 + K - P_\pi - P_d) \langle f | e\epsilon \cdot \Big(\frac{P_1}{P_1 \cdot K} \hat{T}_1 + \frac{P_2}{P_2 \cdot K} \hat{T}_2 + - \frac{P_\pi}{P_\pi \cdot K} \hat{T}_\pi - \frac{P_d}{P_d \cdot K} \hat{T}_d + \hat{T}_{int} \Big) | i \rangle \; . \tag{2.9}$$

The final state is represented by

$$\langle f | = \langle N_f \, u(P_1, P_2, s_1, s_2) \, \phi_f | \; , \tag{2.10}$$

where N_f is the product of the normalization factors for the photon and two protons; u includes the spinors (without normalization) for the protons, and depends on their momenta and spins (s_1 and s_2). ϕ_f is the unnormalized photon wave function. The initial state is written as $|N_i \phi_i\rangle$, where ϕ_i represents the wave function for the pion and deuteron, and N_i are their normalization factors. The scattering matrix can now be written

$$S_{fi} = (2\pi)^4 \delta^4(P_1 + P_2 + K - P_\pi - P_d) \, N_i N_f \, e\epsilon \cdot (M_{ext} + M_{int}) \; , \tag{2.11}$$

where

$$M_{ext} = \frac{P_1}{P_1 \cdot K} T_1 + \frac{P_2}{P_2 \cdot K} T_2 - \frac{P_\pi}{P_\pi \cdot K} T_\pi - \frac{P_d}{P_d \cdot K} T_d \; , \tag{2.12}$$

in which

$$T_1 = \langle u(P_1, P_2, s_1, s_2) \phi_f | \hat{T}_1 | \phi_i \rangle \; , \tag{2.13}$$

and similarly for T_2, T_π, and T_d; and

$$M_{int} = \langle u(P_1, P_2, s_1, s_2) \phi_f | \hat{T}_{int} | \phi_i \rangle \; . \tag{2.14}$$

The Lorentz-invariant amplitudes T_1, T_2, T_π, and T_d depend on the masses of the respective particles and on two independent kinematic variables. These variables describe the kinematics of the strong interaction. The amplitudes have spin dependence, but it does not have to be determined explicitely since it will automatically be included when the amplitudes are extracted from the elastic data.

When considering the elastic collision ($\pi^+ d \rightarrow pp$), it is convenient to use the kinematic variables $t_1 = (p_2 - p_d)^2$ and $s_1 = (p_1 + p_2)^2$, or $t_2 = (p_1 - p_\pi)^2$ and $s_2 = (p_\pi + p_d)^2$, because they are Lorentz-invariant. Since total four-momentum is conserved, these sets are identical. However, when considering the inelastic collision ($\pi^+ d \rightarrow pp\gamma$) the sets are no longer the same, and it is convenient to use the variables determined by the average of s_1 and s_2, and the average of t_1 and t_2. This averaging process leads to the variables

$$\begin{aligned} \nu &= P_\pi \cdot P_d + P_1 \cdot P_2 \\ \Delta &= P_\pi \cdot P_1 + P_d \cdot P_2 \end{aligned} \qquad (2.15)$$

The strong amplitudes in equation (2.12) therefore have the functional dependence $T(\mu_s^2, M_s^2, m_{1s}^2, m_{2s}^2, \nu_s, \Delta_s)$, where μ, M, m_1, and m_2 are the masses of the pion, deuteron, first, and second protons respectively. The subscript on the variables indicates that they correspond to the off-shell strong interaction. Substituting the four-momenta for each amplitude, as shown in Figure 2.1, into the corresponding terms in equation (2.12) gives

$$\begin{aligned} M_{ext} = & \frac{P_1}{P_1\cdot K} T(\mu^2, M^2, m_1^2+2K\cdot P_1, m_2^2, \nu+K\cdot P_2, \Delta+K\cdot P_\pi) \\ & + \frac{P_2}{P_2\cdot K} T(\mu^2, M^2, m_1^2, m_2^2+2K\cdot P_2, \nu+K\cdot P_1, \Delta+K\cdot P_d) \\ & - \frac{P_\pi}{P_\pi\cdot K} T(\mu^2-2K\cdot P_\pi, M^2, m_1^2, m_2^2, \nu-K\cdot P_d, \Delta-K\cdot P_1) \\ & - \frac{P_d}{P_d\cdot K} T(\mu^2, M^2-2K\cdot P_d, m_1^2, m_2^2, \nu-K\cdot P_\pi, \Delta-K\cdot P_2) \, . \end{aligned} \tag{2.16}$$

Using an approximation procedure introduced by Low (Low 1958), the right side of equation (2.16) is expanded in powers of k. The first two terms in this expansion will give sufficient accuracy provided that k is small, and T is a smooth function of all six variables. The result of this expansion is

$$\begin{aligned} M_{ext} = & \left(\frac{P_1}{P_1\cdot K} + \frac{P_2}{P_2\cdot K} - \frac{P_\pi}{P_\pi\cdot K} - \frac{P_d}{P_d\cdot K}\right) T(\mu^2, M^2, m_1^2, m_2^2, \nu, \Delta) \\ & + \frac{\partial T}{\partial \nu}\left[\frac{P_2(P_1\cdot K)}{P_2\cdot K} + \frac{P_1(P_2\cdot K)}{P_1\cdot K} + \frac{P_d(P_\pi\cdot K)}{P_d\cdot K} + \frac{P_\pi(P_d\cdot K)}{P_\pi\cdot K}\right] \\ & + \frac{\partial T}{\partial \Delta}\left[\frac{P_2(P_d\cdot K)}{P_2\cdot K} + \frac{P_1(P_\pi\cdot K)}{P_1\cdot K} + \frac{P_d(P_2\cdot K)}{P_d\cdot K} + \frac{P_\pi(P_1\cdot K)}{P_\pi\cdot K}\right] \\ & + \frac{\partial T}{\partial m_2^2}(2P_2) + \frac{\partial T}{\partial m_1^2}(2P_1) + \frac{\partial T}{\partial M^2}(2P_d) + \frac{\partial T}{\partial \mu^2}(2P_\pi) \, . \end{aligned} \tag{2.17}$$

Now, by gauge invariance $k\cdot M_{total} = k\cdot(M_{ext} + M_{int})$ vanishes, thus

$$k \cdot M_{ext} = -k \cdot M_{int} . \tag{2.18}$$

Therefore

$$\begin{aligned} k \cdot M_{int} = -k \cdot \Big[&(P_1 + P_2 + P_\pi + P_d)\left(\frac{\partial T}{\partial \nu} + \frac{\partial T}{\partial \Delta}\right) + \\ &2\left(\frac{\partial T}{\partial \mu^2} P_\pi + \frac{\partial T}{\partial M^2} P_d + \frac{\partial T}{\partial m_1^2} P_1 + \frac{\partial T}{\partial m_2^2} P_2\right)\Big] , \end{aligned} \tag{2.19}$$

and, to this order

$$\begin{aligned} M_{int} = &-(P_1 + P_2 + P_\pi + P_d)\left(\frac{\partial T}{\partial \nu} + \frac{\partial T}{\partial \Delta}\right) \\ &- 2\left(\frac{\partial T}{\partial \mu^2} P_\pi + \frac{\partial T}{\partial M^2} P_d + \frac{\partial T}{\partial m_1^2} P_1 + \frac{\partial T}{\partial m_2^2} P_2\right) . \end{aligned} \tag{2.20}$$

Adding equation (2.17) to equation (2.20) gives

$$\begin{aligned} M_{ext} + M_{int} = &\left(\frac{P_1}{P_1 \cdot K} + \frac{P_2}{P_2 \cdot K} - \frac{P_\ell}{P_\ell \cdot K} - \frac{P_\pi}{P_\pi \cdot K}\right) T(\mu^2, M^2, m_1^2, m_2^2, \nu, \Delta) \\ &+ \frac{\partial T}{\partial \nu}\left[\frac{P_2(P_1 \cdot K)}{P_2 \cdot K} + \frac{P_1(P_2 \cdot K)}{P_1 \cdot K} + \frac{P_d(P_\pi \cdot K)}{P_d \cdot K} + \frac{P_\pi(P_\ell \cdot K)}{P_\pi \cdot K} - P_1 \right. \\ &\left. - P_2 - P_\pi - P_d\right] + \frac{\partial T}{\partial \Delta}\left[\frac{P_2(P_\ell \cdot K)}{P_2 \cdot K} + \frac{P_1(P_\pi \cdot K)}{P_1 \cdot K} + \frac{P_d(P_2 \cdot K)}{P_\ell \cdot K} \right. \\ &\left. + \frac{P_\pi(P_1 \cdot K)}{P_\pi \cdot K} - P_1 - P_2 - P_\pi - P_d\right] . \end{aligned} \tag{2.21}$$

It is seen that, since no derivatives with respect to the masses appear in equation (2.21), the matrix elements to order k^0 are functions of the mass-shell scattering amplitudes and their derivatives with respect to ν and Δ. By comparing equation (2.21) with equation (2.12), it is observed that the k^{-1} term in the matrix element consists only of the graphs with a photon emitted by external lines, while the contribution of the internal emission diagram is to modify the external graphs in such a way that the strong-interaction amplitudes are taken on the mass-shell and at the average values of the kinematic variables. At this time there is not enough experimental data to determine $\partial T/\partial \nu$ and $\partial T/\partial \Delta$, so only the leading term will be calculated here. This gives a good approximation as long as k is small, that is, if the proton angles are chosen close to their elastic values. Substituting the k^{-1} term of equation (2.21) into equation (2.9), and writing out the normalization factors gives the soft-photon scattering matrix

$$S_{fi} = (2\pi)^4 \delta^4(P_1 + P_2 + K - P_\pi - P_d)\left(\frac{m}{E_1 V}\,\frac{m}{E_2 V}\,\frac{1}{2E_\pi V}\,\times\right.$$
$$\left.\frac{1}{2kV}\right)^{1/2} e\,\epsilon\cdot\left(\frac{P_1}{P_1\cdot K} + \frac{P_2}{P_2\cdot K} - \frac{P_\pi}{P_\pi\cdot K} - \frac{P_d}{P_d\cdot K}\right) T, \tag{2.22}$$

where E_1, E_2, E_π, E_d, and k are the energies of the protons, pion, deuteron, and photon. V is the normalization volume.

The scattering matrix in equation (2.22) has resulted from the first two terms in the expansion of M_{ext} in powers of k. When higher order terms are included, the resulting scattering matrix will have the form $S_{fi} = k^{-1}A_{-1} + A_0 + kA_1 + k^2A_2 + \ldots$, where the coefficients A_1, A_2, etc. will depend on derivatives of T with respect to the masses of the particles. Very accurate experiments are necessary in order to determine the significance of these coefficients. The geometry used for such experiments would have to be chosen carefully in order to obtain large values of k. The data from these experiments would be useful in testing strong interaction models which predict different values for these coefficients.

CHAPTER 3

CALCULATION OF THE CROSS SECTION

In order to obtain the transition probability per unit time w_{fi}, one must sum over the final polarization states of the square of the scattering matrix and divide by the arbitrarily large normalization four-volume VT, thus

$$W_{fi} = (VT)^{-1}\sum |S_{fi}|^2 = (2\pi)^8\left[\delta^4(P_1+P_2+K-P_\pi-P_d)\right]^2 (VT)^{-1} \times \frac{m^2 e^2}{8E_1E_2E_\pi E_d K V^5} \times \sum_f |M_{fi}|^2 \quad , \tag{3.1}$$

where

$$\sum_f |M_{fi}|^2 = \sum_f \left\{\left[\epsilon\cdot\left(\frac{P_1}{P_1\cdot K}+\frac{P_2}{P_2\cdot K}-\frac{P_\pi}{P_\pi\cdot K}-\frac{P_d}{P_d\cdot K}\right)\right]^2 |T|^2\right\} \tag{3.2}$$

Since the term $\left[\epsilon\cdot\left(\frac{P_1}{P_1\cdot K}+\frac{P_2}{P_2\cdot K}-\frac{P_\pi}{P_\pi\cdot K}-\frac{P_d}{P_d\cdot K}\right)\right]^2$ has spin dependence through the photon only, the sum in equation (3.2) can be written

$$\sum_f |M_{fi}|^2 = \sum_{phot}\left[\epsilon\cdot\left(\frac{P_1}{P_1\cdot K}+\frac{P_2}{P_2\cdot K}-\frac{P_\pi}{P_\pi\cdot K}-\frac{P_d}{P_d\cdot K}\right)\right]^2 \times \sum_f |T|^2 \tag{3.3}$$

where the first sum is over the photon polarizations. Since $\frac{P_1}{P_1\cdot K}+\frac{P_2}{P_2\cdot K}-\frac{P_\pi}{P_\pi\cdot K}-\frac{P_d}{P_d\cdot K}$ is a conserved current, one can apply the equation

$$\sum_{phot}(\epsilon\cdot a)^2 = -a^2 \tag{3.4}$$

to obtain

$$\sum_f |M_{fi}|^2 = \left[\frac{2P_2\cdot P_d}{(P_2\cdot K)(P_d\cdot K)} + \frac{2P_2\cdot P_\pi}{(P_2\cdot K)(P_\pi\cdot K)} + \frac{2P_1\cdot P_d}{(P_1\cdot K)(P_d\cdot K)} + \frac{2P_1\cdot P_\pi}{(P_1\cdot K)(P_\pi\cdot K)} - \frac{2P_d\cdot P_\pi}{(P_d\cdot K)(P_\pi\cdot K)} - \frac{2P_1\cdot P_2}{(P_1\cdot K)(P_2\cdot K)} - \frac{m^2}{(P_2\cdot K)^2} - \frac{m^2}{(P_1\cdot K)^2} - \frac{M^2}{(P_d\cdot K)^2} - \frac{\mu^2}{(P_\pi\cdot K)^2}\right] \sum_f |T|^2 . \qquad (3.5)$$

Returning to equation (3.1) and using

$$\left[\delta^4(P_1+P_2+K-P_\pi-P_d)\right]^2 = (2\pi)^{-4}(V\Upsilon)\,\delta^4(P_1+P_2+K-P_\pi-P_d) , \qquad (3.6)$$

one obtains the transition probability per unit time

$$W_{fi} = (2\pi)^4\delta^4(P_1+P_2+K-P_\pi-P_d)\frac{m^2e^2}{8E_\pi E_d E_1 E_2 K V^5} \sum_f |M_{fi}|^2 . \qquad (3.7)$$

The cross section $d\sigma$ can now be found by dividing the transition probability per unit time by the incident flux and then multiplying by the density of final states. The incident current is

$$J_i = \rho_\pi \rho_d |\bar{v}_\pi - \bar{v}_d| = V^{-2}|\bar{v}_\pi - \bar{v}_d| , \qquad (3.8)$$

where ρ_π and ρ_d are the particle densities of pions and deuterons. $\bar{v}_\pi$ and $\bar{v}_d$ are the velocity vectors of the pion and deuteron. The density of final states is

$$dN_f = \left[\frac{V}{(2\pi)^3}\right]^3 d^3K\, d^3P_1\, d^3P_2 , \qquad (3.9)$$

where k, p_1 and p_2 are now used as the magnitudes of the three-vectors $\bar{k}$, $\bar{p}_1$ and $\bar{p}_2$ respectively. The cross section is therefore

$$d\sigma = (2\pi)^{-5}\,\delta^4(P_1+P_2+K-P_\pi-P_d)\times\frac{m^2e^2\sum|M_{fi}|^2}{E_\pi E_d|\bar{V}_\pi-\bar{V}_d|}\times\frac{d^3K}{2K}\,\frac{d^3P_1}{2E_1}\,\frac{d^3P_2}{2E_2}\,. \quad (3.10)$$

In the laboratory frame $\bar{v}_d = 0$, and the cross section reduces to

$$d\sigma = \frac{m^2e^2}{(2\pi)^5\cdot 8P_\pi M}\times\delta^4(P_1+P_2+K-P_\pi-P_d)\sum_f|M_{fi}|^2(E_1E_2K)^{-1}d^3K\,d^3P_1\,d^3P_2\,, \quad (3.11)$$

where

$$\begin{aligned}\sum_f|M_{fi}|^2 = \Big[&\frac{2E_2}{K(P_2\cdot K)}+\frac{2P_2\cdot P_\pi}{(P_2\cdot K)(P_\pi\cdot K)}+\frac{2E_1}{K(P_1\cdot K)}\\&+\frac{2P_1\cdot P_\pi}{(P_1\cdot K)(P_\pi\cdot K)}-\frac{m^2}{(P_2\cdot K)^2}-\frac{m^2}{(P_1\cdot K)^2}-\frac{1}{K^2}\\&-\frac{\mu^2}{(P_\pi\cdot K)^2}-\frac{2P_1\cdot P_2}{(P_1\cdot K)(P_2\cdot K)}-\frac{2E_\pi}{K(P_\pi\cdot K)}\Big]\sum_f|T|^2\,.\end{aligned} \quad (3.12)$$

Using

$$\begin{aligned}&(E_1E_2K)^{-1}d^3K\,d^3P_1\,d^3P_2 =\\&(P_1P_2K)\sin\theta_K\,d\phi_K\,dK\,dE_1\,dE_2\,(d\Omega_1\,d\Omega_2\,d\theta_K)\,,\end{aligned} \quad (3.13)$$

one obtains the differential cross section

$$\begin{aligned}\frac{d\sigma}{d\Omega_1 d\Omega_2 d\theta_K} = \frac{m^2e^2}{(2\pi)^5\,8P_\pi M}\int&(P_1P_2K)\,\delta^4(P_1+P_2+K-P_\pi-P_d)\times\\&\sin\theta_K\,d\phi_K\,dK\,dE_2\,dE_1\sum_f|M_{fi}|^2.\end{aligned} \quad (3.14)$$

For the coplanar geometry of Figure 3.1, the four-dimensional delta function can be written

$$\begin{aligned}
\delta^4(P_1+P_2+K-P_\pi-P_d) &= \delta(E_1+E_2+K-E_i)\times \\
\delta^3(\bar{P}_1+\bar{P}_2+\bar{K}-\bar{P}_i) &= \delta(E_1+E_2+K-E_i)\times \\
&\delta_x(P_1\sin\theta_1-P_2\sin\theta_2+K\sin\theta_K\cos\phi_K)\times\delta_y(K\sin\theta_K\sin\phi_K)\times \\
&\delta_z(P_1\cos\theta_1+P_2\cos\theta_2+K\cos\theta_K-P_i),
\end{aligned} \tag{3.15}$$

where $E_i=E_\pi+E_d$ and $\bar{p}_i=\bar{p}_\pi+\bar{p}_d$. When integrating over $d\phi_K$, δ_y can be expressed as

$$\delta_y(K\sin\theta_K\sin\phi_K) = \frac{\delta(\phi_K)+\delta(\phi_K-\pi)}{\left.\frac{\partial(K\sin\theta_K\sin\phi_K)}{\partial\phi_K}\right|_{0,\pi}} = \frac{\delta(\phi_K)+\delta(\phi_K-\pi)}{K\sin\theta_K}. \tag{3.16}$$

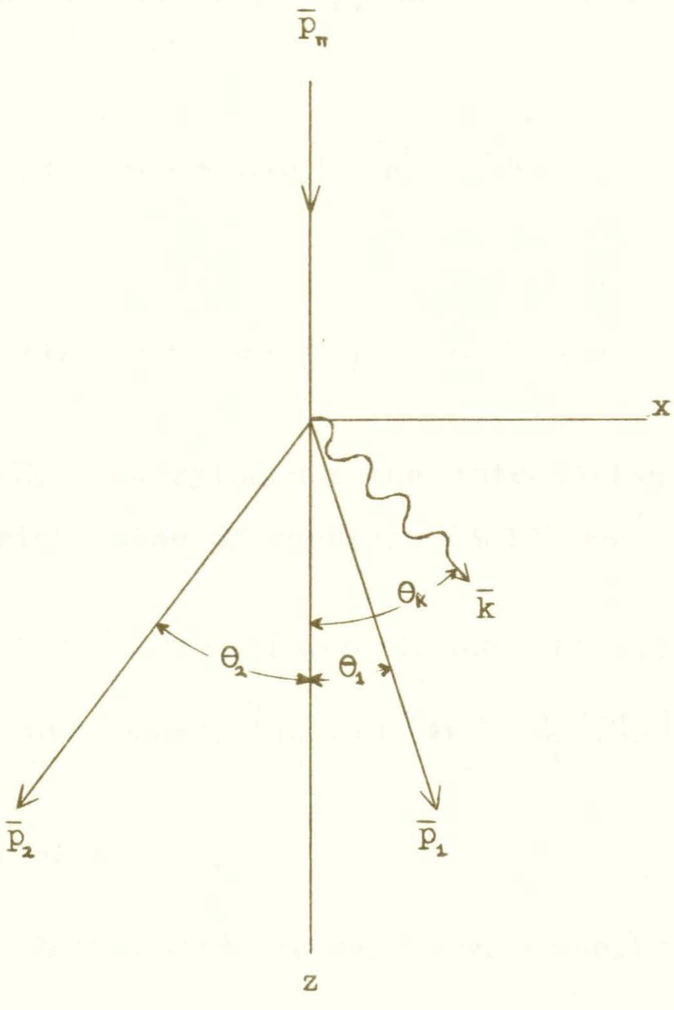
Carrying out the integration over ϕ_K gives

$$\begin{aligned}
&\int(P_1P_2K)\delta^4(P_1+P_2+K-P_\pi-P_d)\sin\theta_K\,d\phi_K\,dK\,dE_2\,dE_1\sum_f|M_{fi}|^2 = \\
&\int P_1P_2\,\delta(E_1+E_2+K-E_i)\,\delta_z(P_1\cos\theta_1+P_2\cos\theta_2+K\cos\theta_K-P_i)\times \\
&[\delta_{x1}(P_1\sin\theta_1-P_2\sin\theta_2+K\sin\theta_K)+\delta_{x2}(P_1\sin\theta_1-P_2\sin\theta_2-K\sin\theta_K)] \\
&\times dK\,dE_2\,dE_1\sum_f|M_{fi}|^2,
\end{aligned} \tag{3.17}$$

with the conditions that δ_{x1} is used when $\phi_K=0$ and δ_{x2} is used when $\phi_K=\pi$. When integrating over E_2, δ_z becomes

$$\delta_z(P_1\cos\theta_1+P_2\cos\theta_2+K\cos\theta_K-P_i) = \frac{\delta(E_2-E_{2o})}{\left|\frac{\partial(P_1\cos\theta_1+P_2\cos\theta_2+K\cos\theta_K)}{\partial E_2}\right|}, \tag{3.18}$$

Figure 3.1



where E_{2o} is the value of E_2 which makes $p_1\cos\theta_1+p_2\cos\theta_2+k\cos\theta_k-p_i$ vanish. Since p_1, p_2 and k are still independent,

$$\frac{\partial}{\partial E_2}\left(p_1\cos\theta_1+p_2\cos\theta_2+k\cos\theta_k\right)=\frac{E_2}{p_2}\cos\theta_2 \quad . \tag{3.19}$$

Therefore

$$\delta_z(p_1\cos\theta_1+p_2\cos\theta_2+k\cos\theta_k-p_i)=|\cos\theta_2|^{-1}\beta_2\,\delta(E_2-E_{2o}), \tag{3.20}$$

where $\beta_2=p_2/E_2$. Carrying out the integration over E_2 reduces the right side of equation (3.17) to

$$\begin{aligned}\int p_1p_2\beta_2\,\delta(E_1+E_2+k-E_i)\big[&\delta_{x1}(p_1\sin\theta_1-p_2\sin\theta_2+k\sin\theta_k)\\ &+\delta_{x2}(p_1\sin\theta_1-p_2\sin\theta_2-k\sin\theta_k)\big]\,|\cos\theta_2|^{-1}\,dk\,dE_1\sum_f|M_{fi}|^2 .\end{aligned} \tag{3.21}$$

When integrating over k,

$$\begin{aligned}&\delta_{x1}(p_1\sin\theta_1-p_2\sin\theta_2+k\sin\theta_k)+\delta_{x2}(p_1\sin\theta_1-p_2\sin\theta_2-k\sin\theta_k)=\\ &\frac{\delta_1(k-k_{o1})}{\left|\frac{\partial}{\partial k}(p_1\sin\theta_1-p_2\sin\theta_2+k\sin\theta_k)\right|}+\frac{\delta_2(k-k_{o2})}{\left|\frac{\partial}{\partial k}(p_1\sin\theta_1-p_2\sin\theta_2-k\sin\theta_k)\right|},\end{aligned} \tag{3.22}$$

where k_{o1} is the value of k which makes $p_1\sin\theta_1-p_2\sin\theta_2+k\sin\theta_k$ vanish, and k_{o2} is the value of k which makes $p_1\sin\theta_1-p_2\sin\theta_2-k\sin\theta_k$ vanish. Since p_1 is still an independent variable, $\partial p_1/\partial k = 0$, but by using the condition that $p_1\cos\theta_1 + p_2\cos\theta_2 + k\cos\theta_K - p_i$ vanishes,

$$\frac{\partial p_2}{\partial k} = \frac{-\cos\theta_K}{\cos\theta_2} . \tag{3.23}$$

Therefore the right side of equation (3.22) can be written as

$$|\cos\theta_2|\left[\frac{\delta_1(k-k_{01})}{|\sin(\theta_K+\theta_2)|} + \frac{\delta_2(k-k_{02})}{|\sin(\theta_K-\theta_2)|}\right] . \tag{3.24}$$

It is now convenient to introduce the variable θ_K' defined by

$$\theta_K = \theta_K' \quad \text{when} \quad \phi_K = 0$$

$$\theta_K = 2\pi - \theta_K' \quad \text{when} \quad \phi_K = \pi \quad . \tag{3.25}$$

The right side of equation (3.22) can then be written, for either ϕ_K, as

$$|\cos\theta_2|\frac{\delta(k-k_0)}{|\sin(\theta_K'+\theta_2)|} , \tag{3.26}$$

where k_0 is the value of k which makes $p_1\sin\theta_1 - p_2\sin\theta_2 + k\sin\theta_K'$ vanish. Carrying out the integration over k reduces the right side of equation (3.21) to

$$\int p_1 p_2 B_2 \delta(E_1+E_2+k-E_i)|\sin(\theta_K'+\theta_2)|^{-1} dE_1 \sum_f |M_{fi}|^2 . \tag{3.27}$$

When integrating over E_1,

$$\delta(E_1+E_2+k-E_i)=\frac{\delta(E_1-E_{10})}{\left|1+\frac{\partial E_2}{\partial E_1}+\frac{\partial k}{\partial E_1}\right|}, \tag{3.28}$$

where E_{1c} is the value of E_1 which makes $E_1+E_2+k-E_i$ vanish. Differentiating with respect to E_1 the condition which resulted from the integral over $\delta(k-k_o)$ results in

$$\frac{\partial p_1}{\partial E_1}\sin\theta_1-\frac{\partial p_2}{\partial E_2}\frac{\partial E_2}{\partial E_1}\sin\theta_2+\frac{\partial k}{\partial E_1}\sin\theta_K'=0\,. \tag{3.29}$$

Differentiation of the condition which resulted from the integral over $\delta(E_2-E_{20})$ results in

$$\frac{\partial p_1}{\partial E_1}\cos\theta_1+\frac{\partial p_2}{\partial E_2}\frac{\partial E_2}{\partial E_1}\cos\theta_2+\frac{\partial k}{\partial E_1}\cos\theta_K'=0\,. \tag{3.30}$$

Using $\partial p_1/\partial E_1=E_1/p_1=\beta_1^{-1}$ and $\partial p_2/\partial E_2=\beta_2^{-1}$, equations (3.29) and (3.30) are written as

$$\begin{aligned}&\beta_2\sin\theta_1-\beta_1\frac{\partial E_2}{\partial E_1}\sin\theta_2+\beta_1\beta_2\frac{\partial K}{\partial E_1}\sin\theta_K'=0\\&\beta_2\cos\theta_1+\beta_1\frac{\partial E_2}{\partial E_1}\cos\theta_2+\beta_1\beta_2\frac{\partial K}{\partial E_1}\cos\theta_K'=0\,.\end{aligned} \tag{3.31}$$

These are solved simultaneously for $\partial E_2/\partial E_1$ and $\partial k/\partial E_1$, thus giving

$$\delta(E_1+E_2+K-E_i) = \frac{\delta(E_1-E_{10})}{\left|1 - \frac{\beta_2 \sin(\theta_K'-\theta_1)}{\beta_1 \sin(\theta_K'+\theta_2)} - \frac{\sin(\theta_1+\theta_2)}{\beta_1 \sin(\theta_K'+\theta_2)}\right|} \cdot \quad (3.32)$$

Carrying out the integration over E_1, and replacing θ_K' by θ_K gives

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 d\theta_K} = \frac{m^2 e^2}{(2\pi)^5 \cdot 8 P_\pi M} \times \frac{P_1 P_2 \beta_2 \beta_1 \sum_i |M_{fi}|^2}{\beta_2 \sin(\theta_K-\theta_1) - \beta_1 \sin(\theta_K+\theta_2) + \sin(\theta_1+\theta_2)} \cdot \quad (3.33)$$

According to the defining equations (3.25), the final integration over θ_K (actually θ_K') in equation (3.33) covers the range $(0,2\pi)$. This will finally yield $\frac{d^2\sigma}{d\Omega_1 d\Omega_2}$.

CHAPTER 4

EXTRACTION OF STRONG AMPLITUDES AND NUMERICAL INTEGRATION

The integration over θ_k must be performed numerically. For each value of θ_k the energy and momentum conservation equations must be solved, again numerically (see Appendix B), for p_1, p_2, and k. Everything on the right side of equation (3.33) except $\sum_f |T|^2$ is determined directly from p_1, p_2, and k. The quantity $\sum_f |T|^2$ is extracted from the elastic data through the variables ν and Δ. In the laboratory frame ν and Δ are determined by

$$\begin{aligned} \nu &= E_\pi M + E_1 E_2 - P_1 P_2 \cos(\theta_1 + \theta_2) \\ \Delta &= E_1 E_\pi - P_1 P_\pi \cos\theta_1 + E_2 M \end{aligned} \tag{4.1}$$

Since the elastic data is given in terms of center-of-momentum variables (see Appendix C), the theoretical differential cross section for this reaction is calculated in the CM frame. The only Feynman diagram for the elastic reaction is the one with the pion and deuteron entering and the two protons leaving the strong interaction without any emission of radiation. The analysis of this diagram gives the scattering matrix

$$S_{fi} = (2\pi)^4 \delta^4(P_1 + P_2 - P_\pi - P_d)\left(\frac{1}{2E_dV}\,\frac{1}{2E_\pi V}\,\frac{m}{E_1V}\,\frac{m}{E_2V}\right)^{1/2} T\,, \tag{4.2}$$

where $T = T(\mu^2, M^2, m_1^2, m_2^2, \nu, \Delta)$ as before. The transition probability per unit time is then found to be

$$W_{fi} = (2\pi)^4 \delta^4(P_1 + P_2 - P_\pi - P_d) \times \frac{m^2}{4E_dE_\pi E_1E_2V^4} \times \sum_f |T|^2\,. \tag{4.3}$$

The current and density of final states are

$$J_i = V^{-2}|\bar{v}_\pi - \bar{v}_d| \tag{4.4}$$

$$dN_f = \left[\frac{V}{(2\pi)^3}\right]^2 d^3P_1\, d^3P_2 \quad .$$

Therefore the elastic cross section is

$$d\sigma_{el} = (2\pi)^{-2} \delta^4(P_1 + P_2 - P_\pi - P_d)\, \frac{m^2 \sum |T|^2}{4E_dE_\pi E_1E_2|\bar{v}_\pi - \bar{v}_d|} \times d^3P_1\, d^3P_2\,. \tag{4.5}$$

In the CM frame $\bar{p}_\pi = -\bar{p}_d$, thus giving

$$E_dE_\pi|\bar{v}_\pi - \bar{v}_d| = P_\pi(E_d + E_\pi) \quad . \tag{4.6}$$

The elastic cross section in the CM frame is then

$$d\sigma_{el} = \frac{m^2 \sum |T|^2}{(2\pi)^2 \cdot 4P_\pi(E_d + E_\pi)} \times \delta^4(P_1 + P_2 - P_\pi - P_d)\, \frac{d^3P_1}{E_1}\, \frac{d^3P_2}{E_2} \quad . \tag{4.7}$$

The differential cross section is given by

$$\frac{d\sigma_{el}}{d\Omega} = \frac{m^2}{(2\pi)^2 \cdot 4P_\pi(E_\pi + E_d)} \times \int \delta^4(P_1 + P_2 - P_\pi - P_d)\, \frac{P_1^2}{E_1E_2} \times d^3P_2\, dP_1 \sum_f |T|^2\,. \tag{4.8}$$

Using the identity

$$\frac{d^3P_2}{E_2} = 2\int \delta(E_2^2 - \bar{P}_2^2 - m^2)\, d^4P_2\,, \tag{4.9}$$

$$\int \delta^4(p_1+p_2-p_\pi-p_d)(E_1E_2)^{-1}p_1^2\,dp_1\,d^3p_2 =$$

$$2\int E_1^{-1}p_1^2\,\delta(E_2^2-\bar{p}_2^2-m^2)\,dp_1\,, \tag{4.10}$$

with the conditions

$$\bar{p}_2 = \bar{p}_\pi + \bar{p}_d - \bar{p}_1$$

$$E_2 = E_\pi + E_d - E_1\,. \tag{4.11}$$

When integrating over p_1,

$$\delta(E_2^2-\bar{p}_2^2-m^2) = \frac{\delta(p_1-p_{10})}{2\left|E_2\frac{\partial E_2}{\partial p_1} - \bar{p}_2\cdot\frac{\partial \bar{p}_2}{\partial p_1}\right|}\,, \tag{4.12}$$

where p_{10} is the value of p_1 which makes $E_2^2 - \bar{p}_2^2 - m$ vanish. But the conditions (4.11) give

$$\frac{\partial E_2}{\partial p_1} = -\frac{\partial E_1}{\partial p_1} = -\frac{p_1}{E_1}$$

$$\frac{\partial \bar{p}_2}{\partial p_1} = -\frac{\partial \bar{p}_1}{\partial p_1} = -\bar{u}_1\,, \tag{4.13}$$

where $\bar{u}_1 = \bar{p}_1/p_1$. It follows that

$$\delta(E_2^2-\bar{p}_2^2-m^2) = \frac{\delta(p_1-p_{10})}{2\left|p_2\cos(\bar{p}_1,\bar{p}_2) - E_2\frac{p_1}{E_1}\right|}\,. \tag{4.14}$$

But in the CM frame $\cos(\bar{p}_1,\bar{p}_2) = -1$, $E_1 = E_2 = E$, and $p_1 = p_2 = p$, therefore

$$\delta(E_2^2-\bar{p}_2^2-m^2) = (4p)^{-1}\delta(p_1-p_{10})\,. \tag{4.15}$$

Carrying out the integration over p_1 gives the differential cross section

$$\frac{d\sigma_{el}}{d\Omega} = \frac{m^2}{(2\pi)^2 \; 8 P_\pi (E_\pi + E_d)} \times \frac{P}{E} \times \sum_f |T|^2 . \tag{4.16}$$

This can be written as

$$\frac{d\sigma_{el}}{d\Omega} = \frac{m^2}{32\pi^2 P_\pi E_i} \times \left[1 - \left(\frac{2m}{E_i}\right)^2\right]^{1/2} \sum_f |T|^2 , \tag{4.17}$$

where

$$E_i = (P_\pi^2 + \mu^2)^{1/2} + (P_\pi^2 + M^2)^{1/2} . \tag{4.18}$$

In the CM frame

$$\nu = E_\pi E_d + P_\pi^2 + 2E^2 - m^2$$

$$\Delta = 2(E^2 - P P_\pi \cos\theta) \quad . \tag{4.19}$$

These can be solved to give

$$P_\pi = \left[\frac{(\nu - \frac{1}{2}\mu^2 - \frac{1}{2}M^2 + m^2)^2 - 4\mu^2 M^2}{2(2\nu + \mu^2 + M^2 + 2m^2)}\right]^{1/2}$$

$$\cos\theta = \frac{\nu - \Delta - E_\pi E_d - P_\pi^2 + m^2}{P_\pi P} \quad . \tag{4.20}$$

Thus $\sum_f |T|^2$ is extracted from the elastic data, and the numerical integration is carried out.

CHAPTER 5

RESULTS OF NUMERICAL INTEGRATION

The differential cross section $\frac{d^2\sigma}{d\Omega_1 d\Omega_2}$ was calculated for pion laboratory energies 150 MeV, 200 MeV, 250 MeV, and 300 MeV. The proton angles were chosen to be equal ($\theta_1 = \theta_2 = \theta$). The values of θ were chosen different from the proton angles in the elastic collision. Thus the complications of the infrared divergence which appear in radiative correction calculations are not present. The conservation equations, with equal proton angles, can be solved to give the elastic angles in terms of the pion energy. The result is

$$\cos\theta_{el} = \left[\frac{E_\pi^2 - \mu^2}{(E_\pi + M)^2 - 4m^2}\right]^{1/2} \tag{5.1}$$

Results of the numerical integration are given in Figures 5.1 and 5.2. Figure 5.1 gives the differential cross section as a function of pion energy for several proton angles, while Figure 5.2 gives it as a function of proton angles for different pion energies. Figure 5.3 shows $\frac{d^3\sigma}{d\Omega_1 d\Omega_2 d\theta_k}$ as a function of θ_k for the typical cases $T_\pi = 250$ MeV, $\theta = 64°$; and $T_\pi = 150$ MeV, $\theta = 67°$.

FIGURE 5.1

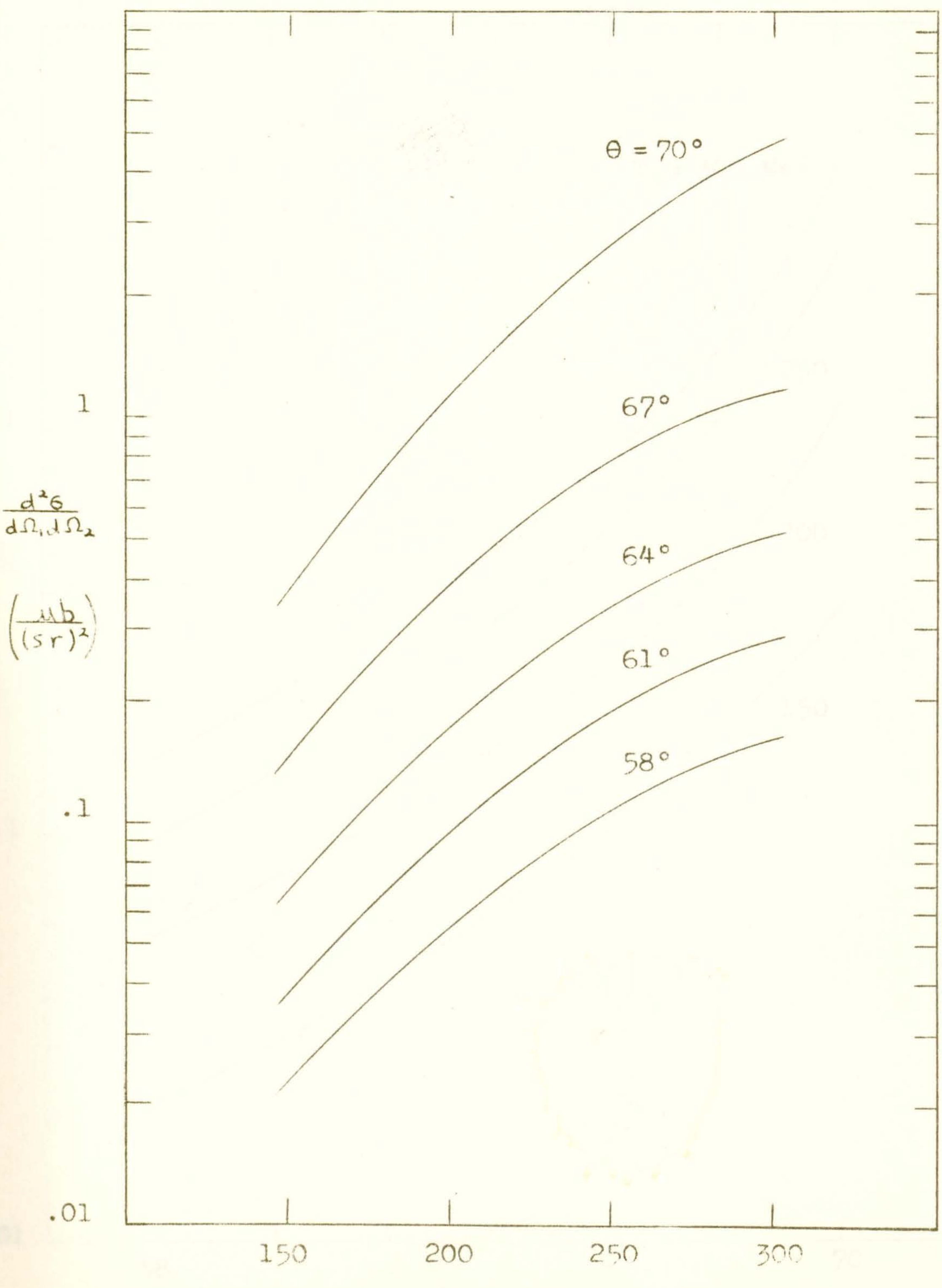
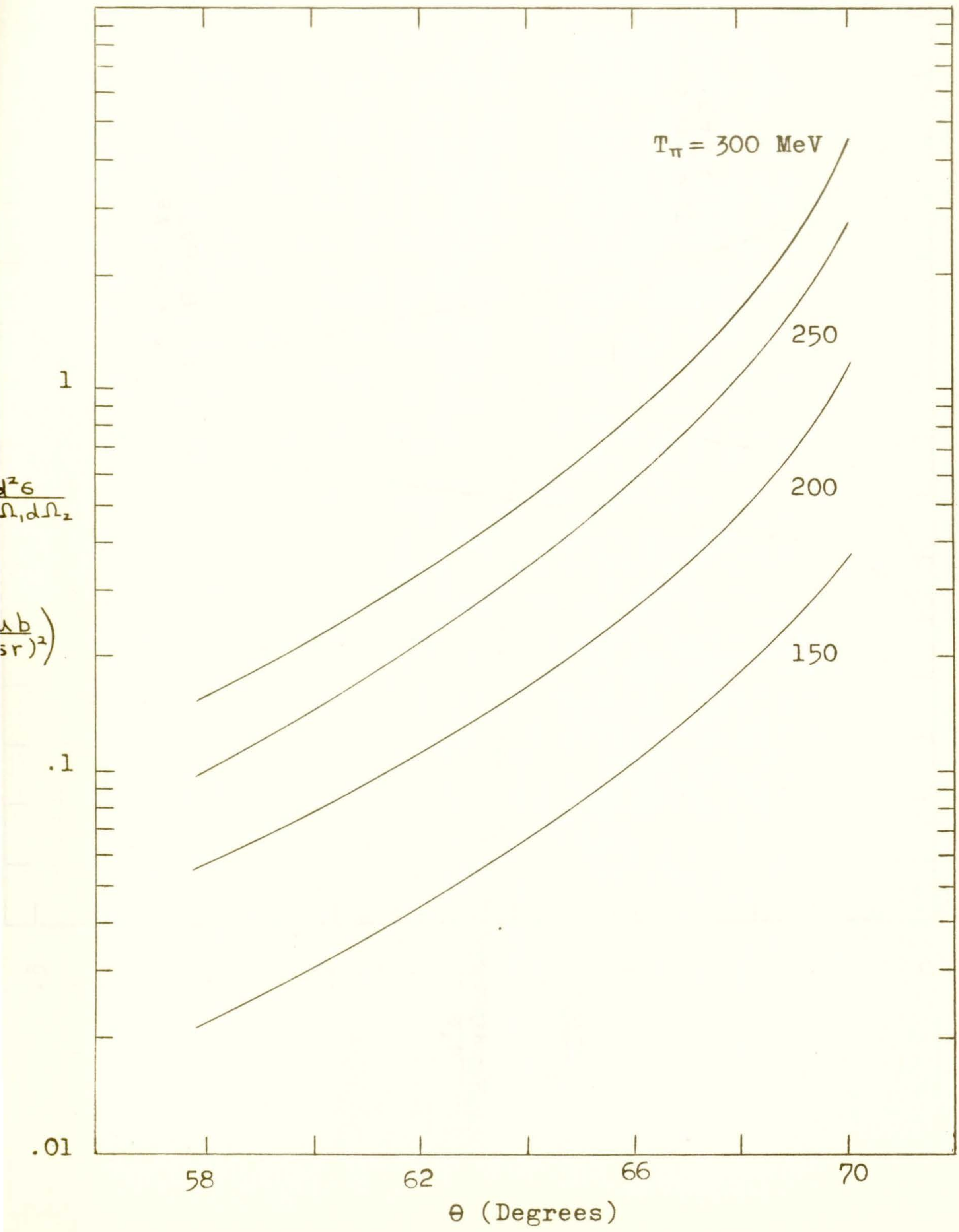
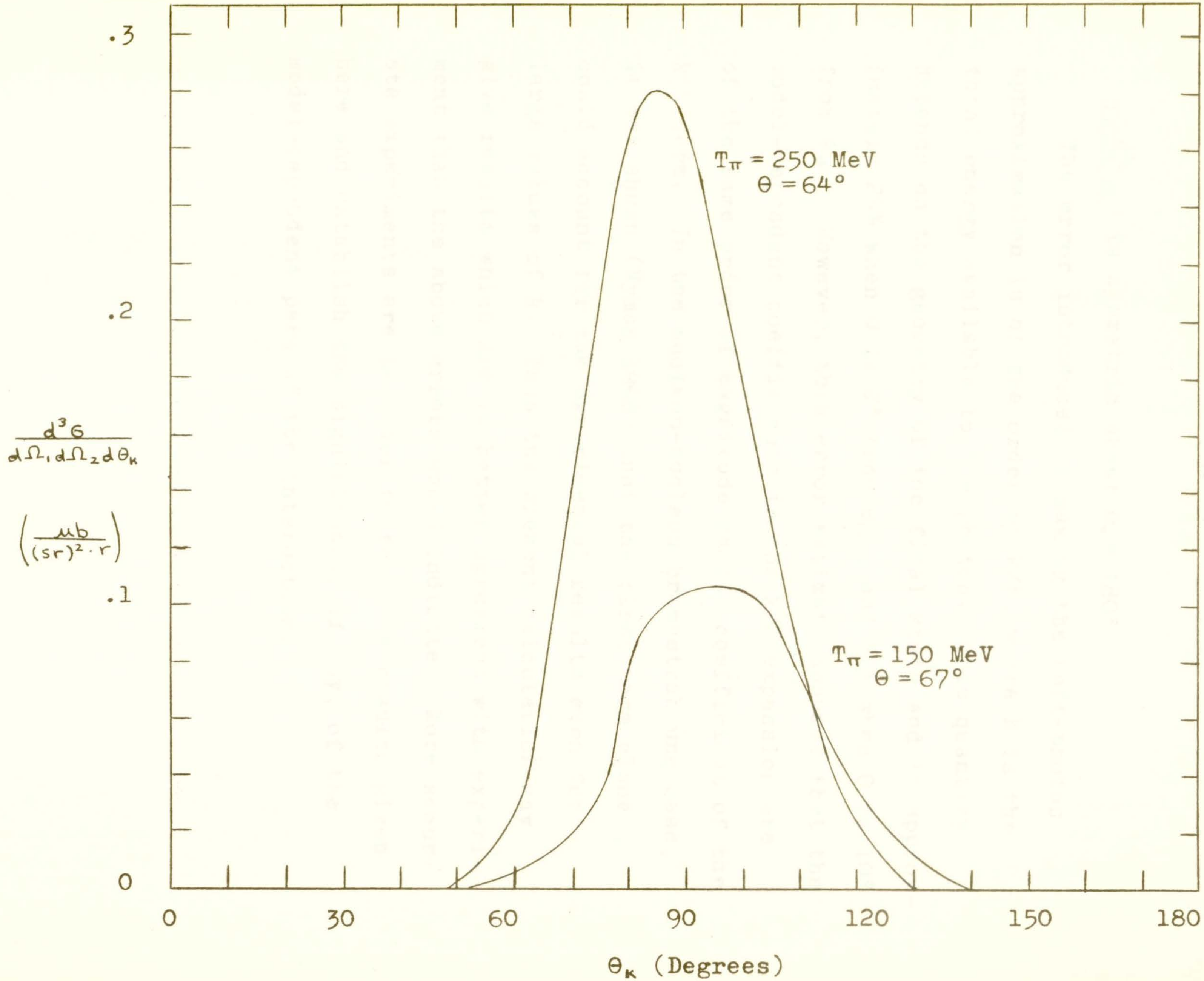

θ = 70°
67°
64°
61°
58°
1
.1
.01
$\frac{d^2\sigma}{d\Omega_1 d\Omega_2}$
$\left(\frac{\mu b}{(sr)^2}\right)$
150
200
250
300
T_π (MeV)

FIGURE 5.2





.3
.2
.1
0
$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 d\theta_K}$
$\left(\frac{\mu b}{(sr)^2 \cdot r}\right)$
$T_\pi = 250$ MeV
$\theta = 64°$
$T_\pi = 150$ MeV
$\theta = 67°$
0
30
60
90
120
150
180
θ_K (Degrees)

$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 d\theta_k}$ is symmetric about $\theta_k = 180°$.

The error introduced by making the soft-photon approximation is of the order of k/E, where E is the total energy available to the photon. This quantity depends on the geometry of the final state and is approximately 25% when θ is 2° from θ_{el}, and 55% when θ is 10° from θ_{el}. However, this error estimate assumes that the model-dependent coefficients in the k/E expansion are of the same order of magnitude as the coefficient of the k^{-1} term. In the nucleon-nucleon bremsstrahlung case, it was shown (Nyman 1968) that the first term alone could account for the experimental results even for large values of k. Thus the present calculation may give results which are in better agreement with experiment than the above errors would indicate. More accurate experiments are in order to test the results given here and establish the significance, if any, of the model-dependent part of the interaction.

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APPENDIX A

NOTATION AND RULES FOR FEYNMAN DIAGRAMS

The propagator approach to relativistic quantum field theory yields the intuitively appealing formulation of quantum electrodynamics in terms of Feynman diagrams (Bjorken and Drell 1964). Here a four-vector A has contravariant components $A^\mu = (A^0, \bar{A})$. The metric tensor is given by

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{A.1}$$

The covariant components of A are defined by

$$A_\mu = g_{\mu\nu} A^\nu = (A^0, -\bar{A}) \tag{A.2}$$

The scalar product of vectors A and B is

$$A \cdot B = A^\mu B_\mu = g_{\mu\nu} A^\mu B^\nu = A^0 B^0 - \bar{A} \cdot \bar{B} \tag{A.3}$$

The Feynman slash of a vector is a scalar defined by

$$\not{A} = \gamma^\mu A_\mu = \gamma^0 A^0 - \bar{\gamma} \cdot \bar{A} \tag{A.4}$$

where the gamma matrices are

$$\gamma^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad \gamma^1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

$$\gamma^2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix} \qquad \gamma^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{A.5}$$

Another frequently appearing combination is

$$\sigma^{\mu\nu} = \frac{i}{2}\left(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu\right) \tag{A.6}$$

In the representation of (A.5) the components of $\sigma^{\mu\nu}$ are

$$\sigma^{12} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad \sigma^{23} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\sigma^{31} = \begin{bmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{bmatrix} \qquad \sigma^{01} = \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix} \tag{A.6}$$

$$\sigma^{02} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \qquad \sigma^{03} = \begin{bmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix}$$

The analysis of the Feynman diagrams in the present problem is based on the rules given below.

A fermion vertex contributes the factor $-ie\gamma^{\alpha}$.

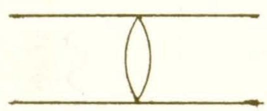
An internal fermion line of momentum p contributes $i/(\not{p}-m)$, where m is the mass of the fermion.

A vertex scattering a meson from p_{α} to p'_{α} contributes $-ie(p+p')_{\alpha}$.

An internal meson line of momentum q has propagator $i/(q^2-\mu^2)$, where μ is the mass of the meson.

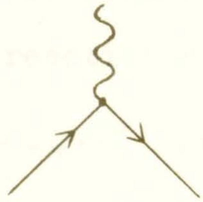
A vertex at which a free photon of polarization ϵ is emitted contributes a factor ϵ^{α}.

The strong interaction is represented by



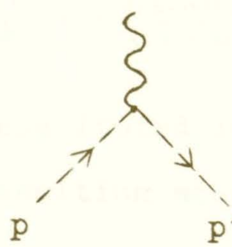
Photon emission from the deuteron is represented by

FIGURE A

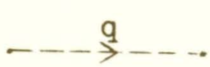


Fermion
Vertex

Internal
Fermion
Line



Meson
Vertex



Internal
Meson
Line

Free
Photon
Vertex

APPENDIX B

SOLUTION OF CONSERVATION EQUATIONS

The equations of conservation of energy and momentum for the inelastic reaction are, with $\theta_1 = \theta_2 = \theta$,

$$\begin{aligned} E_\pi + E_d &= k + (p_1^2 + m^2)^{1/2} + (p_2^2 + m^2)^{1/2} \\ p_\pi &= (p_1 + p_2)\cos\theta + k\cos\theta_k \\ k\sin\theta_k &= (p_2 - p_1)\sin\theta \quad . \end{aligned} \tag{B.1}$$

The last two equations are solved for p_1 and p_2 as functions of k. The result is

$$\begin{aligned} p_1 &= \frac{1}{2}\left[\frac{p_\pi}{\cos\theta} - k\left(\frac{\sin\theta_k}{\sin\theta} + \frac{\cos\theta_k}{\cos\theta}\right)\right] \\ p_2 &= \frac{1}{2}\left[\frac{p_\pi}{\cos\theta} + k\left(\frac{\sin\theta_k}{\sin\theta} - \frac{\cos\theta_k}{\cos\theta}\right)\right] . \end{aligned} \tag{B.2}$$

These are then substituted into the energy conservation equation. The resulting equation for k is

$$Ak^4 + Bk^3 + Ck^2 + Dk + E = 0 , \tag{B.3}$$

where

$$A = \frac{\sin^2\theta_k}{\sin^2\theta} + \frac{\cos^2\theta_k}{\cos^2\theta} - \left(\frac{\sin\theta_k\cos\theta_k}{\sin\theta\cos\theta}\right)^2 - 1$$

$$B = \frac{2 P_\pi \cos\theta_K}{\cos^2\theta}\left(\frac{\sin^2\theta_K}{\sin^2\theta} - 1\right) + (E_\pi + E_d)\times 2\left(2 - \frac{\sin^2\theta_K}{\sin^2\theta} - \frac{\cos^2\theta_K}{\cos^2\theta}\right)$$

$$C = (E_\pi + E_d)^2\left(\frac{\sin^2\theta_K}{\sin^2\theta} + \frac{\cos^2\theta_K}{\cos^2\theta} - 6\right) + \frac{P_\pi^2}{\cos^2\theta}\left(1 - \frac{\sin^2\theta_K}{\sin^2\theta}\right)$$
$$+ \frac{4 P_\pi \cos\theta_K}{\cos^2\theta}(E_\pi + E_d) + 4m^2$$

$$D = 2(E_\pi + E_d)\left\{(E_\pi + E_d)\left[2(E_\pi + E_d) - \frac{P_\pi \cos\theta_K}{\cos^2\theta}\right] - 4m^2 - \frac{P_\pi^2}{\cos^2\theta}\right\}$$

$$E = (E_\pi + E_d)^2\left[4m^2 + \frac{P_\pi^2}{\cos^2\theta} - (E_\pi + E_d)^2\right].$$

APPENDIX C

ELASTIC DATA

The differential cross section for the elastic reaction in the CM frame is (Richard-Cerre 1968)

$$\frac{d\sigma}{d\Omega} = .007\,T_\pi - .11 + (.023\,T_\pi - .1)\cos^2\theta \quad \text{for } 130\,\text{MeV} > T_\pi$$

$$= (7.7 - .037\,T_\pi)(.0037\,T_\pi - .21 + \cos^2\theta) \quad \text{for } 180\,\text{MeV} > T_\pi \geq 130\,\text{MeV}$$

$$= (3.36 - .013\,T_\pi)(.0037\,T_\pi - .21 + \cos^2\theta) \quad \text{for } T_\pi \geq 180\,\text{MeV} .$$

The units are mb/sr.

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