The Fisher Equation and Short-Term Interest Rates

by

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ABSTRACT

Over the past twenty years real interest rates, measured *ex-post*, have varied considerably. Recent economic theory has isolated the role of expectations as a factor in decision and policy-making processes. The relationship between inflation expectations and real interest rates was formalized by Irving Fisher, who defined nominal interest rates in terms of two unobservable variables: real interest rates and inflation expectations. The question which remains unresolved is the relationship between the two unobserved variables. The Fisher hypothesis claims that since the real interest rate is constant and unaffected by inflation expectations, the nominal rate reflects completely changes in inflation expectations.

That the Fisher hypothesis is unresolved is mostly due to the difficulty in measuring the variables, as the measurement defines the relationship, and on the model used to test the relationship. Recent economic literature has produced a range of conflicting values.

In order to pursue this question in a Canadian setting, a reduced-form model of the nominal interest rates is developed, incorporating the models of loanable funds and liquidity preference, and substituted into the Fisher equation. The model differs from previous studies, to which it is compared, because it accounts for international capital flows, and corrects for autocorrelation. The model sets nominal after-tax interest rates dependent on inflation expectations, inflation uncertainty, government deficits, economic activity, unexpected money supply changes and international capital flows.

A number of conclusion were drawn from the results. First, the role of unexpected money supply changes and economic output were significant. Second, like government deficits, inflation uncertainty was rejected, a conclusion consistent with most previous studies. Third, international capital movements proved to be a strong influence. Lastly, rejection of the Fisher hypothesis indicates that inflation expectations inversely affect the real interest rate in the short term. This is primarily due to a real balance effect which causes less non-interest bearing assets to be held (the Mundell effect), and to price setting by the Bank of Canada (the "Inverted" Fisher effect).

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Table of Contents

List of Tables

Chapter 1: Literature Survey

While inflation rates have fluctuated widely since the early 1970's, nominal interest rates have followed suit, as lenders have had to protect the purchasing power of their money by allowing for inflation. Thus nominal interest rates are normally considered to be made up of a real interest rate and the expected inflation rate:

$$i = r + \pi^e \qquad (1 - 1)$$

where i is the nominal interest rate from time t to t + 1, r is the real interest rate and π^e is the expected inflation rate for the same time period[1]. The real rate r represents the true cost of capital or the opportunity cost of foregone consumption of capital, while i represents the nominal price of capital. Because interest rates are calculated within a framework of inflation or eroding value of currency, inflationary expectations are included to compensate the lender for the expected decrease in value of the nominal amount of principle.

This theory was formalised in 1930, when Irving Fisher more narrowly defined the relationship in *The Theory of Interest*. The 'Fisher Hypothesis' claims that the

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[1]The true equation should be:

$$i = r + \pi^2 + r\pi^2$$

since both the principle and interest should be corrected for loss in value due to inflation. However, for simplicity this last term is left out because its magnitude is negligible.

appropriate equation for the nominal interest rate is (1-1), and that the real rate r is constant; while the variation in the expected inflation term is reflected completely in the nominal rate. The implication of this notion is that the real rate of interest is unaffected by inflationary expectations.

Unfortunately, both the real rate of interest and inflation expectations are not observable, and since there are several different measurements of each variable, their true values are difficult to determine. Consider the measurement of the ex-post real rate over the last twenty years, detailed in Table 1-1: Annual Inflation and Interest Rates, which shows the Bank Rate and the 91 day Treasury Bill rate averaged annually, and the corresponding ex-post inflation rate. The real rates are the difference between the inflation rate and the nominal (observed) rate. In the late seventies, the annual real rate dipped into negative territory, while in the early eighties it reached levels beyond eight percent. Since the real rate represents not only the price of capital, but also the time preference of consumption, such variation over a long time period makes little sense. Because the measurement of the real interest rates depends on the accuracy of the measurement of expected inflation, changes in the real rate may reflect to a large extent problems in measuring inflationary expectations.

Table 1-1

Annual Inflation and Interest Rates

Year	Inflation Rate	Bank Rate Nominal	Bank Rate Real	90 Day TB Nominal	90 Day TB Real
1973	7.7	6.1	-1.6	5.4	-2.3
1974	10.9	8.5	-2.4	7.8	-2.3
1975	10.8	8.5	-2.3	7.4	-3.4
1976	7.5	9.2	1.8	8.9	1.4
1977	8.0	7.7	0.2	7.3	-0.6
1978	8.8	8.9	0.2	8.6	-0.2
1979	9.2	12.1	2.9	11.6	2.6
1980	10.2	12.8	2.7	12.7	2.5
1981	12.5	17.9	5.4	17.8	5.4
1982	10.8	13.9	3.2	13.8	3.0
1983	5.8	9.5	3.7	9.3	3.5
1984	4.4	11.3	7.0	11.1	6.8
1985	4.0	9.6	5.6	9.5	5.5
1986	4.1	9.2	5.1	9.0	4.9
1987	4.4	8.4	4.0	8.2	3.8
1988	4.05	9.7	5.6	9.4	5.3

Source: Bank of Canada

However, the extent of the variation in Table 1-1 is considerable enough to indicate that this example is more than a statistical anomaly. This suggests that inflation expectations were often wildly inaccurate, or that fluctuations in nominal rates (or Equation (1-1)) must be explained by more components than the two unobserved variables.

The two relationships defined by Fisher, that the real rate is constant and that it is unaffected by inflation expectation, remain unresolved for short term interest rates.

A great portion of past research has either rejected or failed to accept the hypothesis of constant real rates. Fama (1975), Nelson and Schwert (1977), Levi and Makin (1978, 1979, 1981), Hoffman and Schlagenhauf (1985), and Neumann (1977) are all notable members of this club. Some theories have attempted to isolate the apparent causes of real rate movements. Mundell (1963) and Tobin (1965) posited a relationship between anticipated inflation and real rates through real balances, while Tanzi (1980) examined the relationship between real rate movements and economic activity. Darby (1975) introduces taxes as a wedge between nominal interest rates and both anticipated inflation and real interest rates. More recently, Levi and Makin (1978) and Hoffman and Schlagenhauf (1985) link inflation uncertainty to real rate movements. Another body of literature examines the effects of budget deficits on interest rates and the possible crowding out of private investment. The most notable study is that of Evans (1985).

International studies of this nature are few, and limit themselves to the constancy of real interest rates across time and countries. Aliber (1972), Litzenberger and Rolfo (1984) and Hoffman and Schlagenhauf (1985) compare real rates across countries. Only one empirical study using purely Canadian data, Carr and Smith (1972), accepted Fisher's hypothesis, contradicting the results found by Hoffman and

Schlagenhauf. However, no Canadian study has used an open economy setting, although Neumann (1978) examined German interest rates under flexible exchange rates, and Makin (1983) proposes a long-term interest rate model under flexible exchange rates.

The wide range of results may be a reflection of the variety of models of interest rate determination. Early empirical examinations have focused on a simple regression of inflation expectations on nominal interest rates. More recent and sophisticated models have realised the importance of related macroeconomic variables and incorporated these into the analysis. Such models are based on two economic theories governing interest rate movements: loanable funds and liquidity preference. In both cases, a reduced form equation for real interest rates is developed, which, when substituted into a Fisher equation, yields a testable model of the nominal interest rate, allowing for the more complicated relationship between real and nominal interest rates in the economy.

Empirical evidence has supported the competing theories. Table 1-2: Survey of Results surveys the empirical findings of a variety of models. The table presents estimated coefficients and test statistics for inflation expectations (π^e), government deficits (D^e), economic output (Y), inflation uncertainty (σ^2), money supply growth (M), and the foreign interest rate (i^*).

Although loanable funds and the liquidity preference models lend an intuitive grasp of the underlying forces governing interest rate movements, their usefulness in providing clear empirical results has been hampered by difficulties in measuring inflation expectations and real interest rates.

Table 1-2

Survey of Results

STUDY and Dependent Variable	π^e	D^e	Y	σ^2	M	i*	R^2
Carr, Smith (1976)	0.95						0.71
91 day Treasury Bill	(13.6)						
Thomas (1988)	0.79	1.04	0.10		-0.02		0.72
Long-Term Interest rate	(5.2)	(7.4)	(1.9)		(-1.2)		
Feldstein (1970)	0.23	2.04	7.59		-8.48		0.99
Long-Term Interest rate	(3.83)	(2.29)	(10.5)		(10.5)		
Taylor (1981)	0.41		0.02	0.02			0.77
91 day Treasury Bill	(1.9)		(0.49)	(0.06)			
Levi and Makin (1979)	1.00		-0.13	-1.01			0.69
91 day Treasury Bill	(8.44)		(-3.41)	(-3.97)			
Levi and Makin (1981)	0.96		-0.14	-1.01	0.05		0.70
91 day Treasury Bill	(6.9)		(-3.4)	(-3.91)	(0.63)		
Neumann (1977)	0.36		1.10	-1.04		0.26	0.95
90 day Term Deposit	(2.03)		(1.54)	(-3.94)		(9.1)	

Note: test-statistics below estimates.

An inflation expectations coefficient of less than one indicates a reduction in real rates as inflation expectations rise, since nominal rates do not mirror the increase. The survey of estimates is inconclusive, as coefficients range from 0.23 to 1.00. The inclusion of macroeconomic variables does little to identify a clear relationship. The signs on the other macroeconomic variables for the most part display the expected signs, although some variation suggests mis-specification error.

These problems might arise for three reasons. First, the reduced interest rate equations may be mis-specified. Second, because inflation expectations is not observable, the generation of the expectations data may often not be appropriate. Third, a large variety of data exist for the nominal interest rate employed as the dependent variable. The data used would certainly affect the results. For instance, the bank rate would be more likely to suggest an "Inverted" Fisher[1] effect, since it is largely dependent on monetary policy.

This paper attempts to examine the movement of short-term real and nominal interest rates, and to test the validity of the Fisher hypothesis in an open economy context. Therefore, it does not follow exactly any model used in past studies. Other issues addressed include the magnitude and direction of the effect of unanticipated monetary shocks, the impact of uncertainty, the direct effect of anticipated inflation, the possible crowding-out by government deficits, and the sensitivity of the domestic interest rate to international interest rate differentials. Through an IS-LM framework, including an aggregate supply schedule and a balance of payments

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[1]See p. 1-12

function, a reduced-form equation for the interest rate will be derived.

In order to understand the relevance of any new models, an outline of the issues addressed above is warranted. As expected inflation is an essential component in each model, it is developed first.

Expected Inflation

The Fisher equation suggests that the real interest rate in equation (1) is unaffected by inflation expectations, or that:

$$\frac{\partial r}{\partial \pi^e} = 0 \qquad (1 - 2)$$

and

$$\frac{\partial i}{\partial \pi^e} = 1 \qquad (1 - 3)$$

These hypotheses have been tested successfully in the past with American Data: see for instance Yohe and Karnosky (1970) and Cargill and Meyer (1977). Criticism in the form of more recent articles have made these early results suspect, most notable Fama (1975), but also Carlson (1977), Joines (1977), Levi and Makin (1979) and Nelson and Schwert (1977), among others. Results different from that proposed by Fisher are most commonly attributed to two forces. Mundell (1963) showed a negative relationship between expected inflation and real interest rates, and Darby (1975) modified the Fisher Equation to introduce a tax wedge between expected inflation and nominal interest rates.

The Mundell effect[1] essentially states that an increase in inflation expectations will result in a permanent decrease in real interest rates, and thus the nominal interest rate will rise by less than the increase in inflation expectations. The mechanism at work here is a real balance effect, in which an increase in inflation expectations causes real balances (money demanded divided by the price level) to shrink, since a rise in inflation expectations, with the actual price level unchanged, will cause people to hold less non-interest bearing cash to prevent a deterioration of the asset's value. If consumption is assumed to react positively to changes in real balances, then the decrease in real balances will lead to an increase in savings. In an IS-LM framework, the LM curve shifts in due to lower real balances, while the IS curve, in reaction, shifts in due to the increase in savings. If the decrease in real balances is a stronger influence than the increase in savings, a net decrease in real interest rates will result. Thus a rise in inflation expectations may not only be mirrored by a rise in nominal interest rates (in reaction to increased inflation expectations), but also in a reduction in real interest rates. The permanence of such a change is essential to contradict Fisher in the long run, but not in the short run. However, any pressure on real interest rates is unlikely to hold in the long term, since the real rate is the time preference of holding money. As human nature changes little this can be assumed to be relatively stable in the long run. Thus the

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[1]This is also commonly called the Mundell-Tobin effect, acknowledging Tobin (1965).

first implication of the Fisher Hypothesis, that the real rate is constant, is in the long run necessarily true. The Mundell effect is a short to medium term phenomenon. In this case, short run pressures on real interest rates can be considered 'permanent' if they last into the medium term.

Makin (1978) provides some theoretical evidence to support Mundell. In a model of a closed economy, this real interest effect may occur, but in a more realistic open setting, the increase in savings from an increase in inflation expectations may be matched by a fall in imports, since consumption has decreased. The changes in savings and imports may be equal. These conclusions, though, depend on the assumption that "interest rate parity is continually maintained by capital movements and accompanying foreign exchange transactions whereas full purchasing power parity, accomplished through commodity arbitrage, holds only with a lag."[1] Unfortunately, even if interest rate parity is instantaneously achieved (i.e. perfect capital mobility), it does not mean that the domestic interest rate is immune to the adjustment process. Moreover, the sum of changes in imports and savings will not occur until the purchasing power adjustment is complete, a process reaching beyond the short term.

The inclusion of taxes may contradict the Fisher Hypothesis. Darby argues that taxes may pose a problem for Fisherians because their analysis ignores the transfer of income tax liability on the part of the interest payment representing a return of

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[1]Makin (1978), p. 288.

real capital[1], so that the true relationship is more in the form:

$$i = \frac{r + \pi^e}{(1 - \tau)} \qquad (1 - 4)$$

where τ is the effective marginal tax rate each time period. Therefore, assuming Mundell to be wildly mistaken and $\partial r/\partial \pi^e = 0$, the relationship between nominal interest rates and inflation expectations is

$$\frac{\partial i}{\partial \pi^e} = \frac{1}{(1 - \tau)} \qquad (1 - 5)$$

so that $1/(1 - \tau)$ is the tax wedge. The effect of such a wedge should push the elasticity coefficient on the expectations variable above unity in a model of nominal interest rates. Following an increase in inflation expectations the nominal rate of interest must rise by enough to cover both the loss of principle due to inflation and the taxes on the return of principle. The Darby effect should not reduce the real interest rate, because the marginal tax rate τ is relatively stable. Where variation in τ causes some unanticipated tax outlays, it may affect the real rate through a real balance effect. This is likely to be negligible, so that the Darby Hypotheses remains a contradiction to Fisher in models of nominal interest rates.

Both the Mundell and the Darby effect may cause the interest elasticity of inflation expectations to stray from its proposed unitary value. In the case that

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[1]Carr, Smith and Pesando (1976), p. 259.

Mundell's real balance effect is negated by Makin's open economy effect in the short run, *ceteris paribus*, the coefficient on expectations should be $1/(1 - \tau)$. In a model without taxes, the coefficient should be one. It is important to note that some studies, most notable Hoffman and Schlagenhauf (1984), have found that coefficients on the pre-tax model variables do not differ significantly (statistically speaking) from coefficients on after-tax models variables.

Another theory suggesting a less than unitary coefficient on inflation expectations is the "Inverted Fisher" hypothesis. Introduced by Carmichael and Stebbing (1983), the hypothesis claims that interest rates over the past few decades have been controlled by monetary authorities and have not been free to wander in reaction to demand and supply for capital. As was explained on a theoretical level, the fixed real rate of interest underlying the Fisher hypothesis was justified by associating it with the opportunity cost of, or the real rate of return to, capital, which is determined in part by the technology and the rate of time preference. On the empirical level, however, the interest rates used in tests of the Fisher equation have usually been a measure of the value of money rather than capital. Therefore, empirical investigations should find that nominal rates will be relatively fixed by financial institutions, and real rates of return absorbing price expectation fluctuations. In this case, the amount of variation in interest rates depends on the central bank's own price expectations. In this case one would expect a coefficient of zero on the inflation expectations variable. Carmichael and Stebbing and Groenewold (1989) find evidence supporting this theory using both the simple Fisher equation and increasingly complete macroeconomic models more recently in vogue. Therefore, it does present another reason to expect the coefficient on the expectations variable to fall below one.

Loanable Funds and Liquidity Preference

The two theories governing interest rate determination, loanable funds and liquidity preference, identify sources of variation in real interest rates. Some of these variables are the same, but the mechanism through which their effects are explained differ.

The theory of loanable funds considers interest rates as the market clearing price between consumers and users of loanable funds. Supply and demand is determined by the four sectors of the economy: households, government, business and foreign investors. The equilibrium interest rate prevails when savings (supply) is equal to investment (demand). Savings (supplied primarily by households and foreign investors, not the government in recent times) are dependent on and vary directly with the real interest rate, income and real balances. Investment, on the other hand, varies with the real interest rate and inflation uncertainty. Business and the government tend to be net borrowers.

Liquidity preference theory sets money supply equal to money demand; the nominal interest rate is the market-clearing price. Money supply is dependent on monetary policy. Money demand varies with the price level and interest rates. Since nominal rates often spark large international flows in interest rate-sensitive capital, the money demand is dependent on foreign interest rates as well.

The two theories of interest rate determination identify five other variables which should enter into the real interest rate model: money supply growth or monetary policy, government fiscal balances, cash demand by business, inflation uncertainty and international capital flows.

Monetary Policy

Monetary policy is often considered to have a large effect on interest rates since basic theory on money links rising interest rates to a tight money supply. This has in fact been a common explanation for the high real rates in the early and late 1980's. The difficulty in measuring the effects, however, is separating the nominal from the real movements, because a change in the money supply can also affect inflation expectations. Part of this process has been referred to as the Wicksell Effect[1], which explains the pressure on interest rates and inflation resulting from a one time increase in the money supply. The total real supply of loanable funds increases when the money supply increases and prices have not yet adjusted. The increased supply of funds forces real rates downward. The change in money supply will eventually cause the price level to increase, reducing the level of real balances and reducing the real supply of loanable funds. One time changes in money supply are therefore temporary, with no effect on expectations and an equilibrium level of

[1]Carr and Smith (1972).

real interest rates not far from its starting point.

In a world where the money supply is continually growing, however, inflation expectations will form, increasing the difference between the real and nominal rate. Where the growth rate is constant and money growth expectations are the same as reality, then the real rate will once again be unaffected, and inflation expectations will affect only nominal rates through the Fisher effect.

Since monetary policy must be characterised independently of both real and nominal rates, a variable measuring unexpected changes in monetary growth should be used, because such deviations will not immediately be reflected in the price level. Once the unexpected part has been reflected real rates should return to near its initial equilibrium level, so that in the medium to long term, monetary policy should have little or no effect on the real interest rate.

This theory was formalized by Barro (1977), whose model has commonly been used as a framework for interest rate studies[1]. Although positive unexpected money growth is not reflected in expected future prices, it is reflected in actual future prices. The future price response will be larger than the immediate price response because realisation of the unexpected growth will negate the real balance effect, recalling output back to its original level. In the end prices alone adjust to the unanticipated money growth as inflation rises and the real rate of interest declines. The nominal rate of interest may also fall in the short run in order to clear the money market.

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[1]For instance Barth and Bradley (1989).

In the medium term, inflation should cause expectations to rise, pushing up nominal rates. This, combined with the return of *ex-post* real rates to normal, should push nominal rates past their original level. As inflation expectations decline (barring further monetary surprises) nominal rates would also settle to their pre-surprise level.

Government Deficits

Government deficits are widely perceived as a large source of pressure on both real and nominal interest rates. All introductory and intermediate macroeconomic textbooks devote ample space to government deficits and the 'crowding-out' theory, a celebrated consequence in loanable funds models. An increase in the government deficit must be financed either at home or abroad. At home, if it is monetized, it represents an unanticipated money supply increase. These monetized deficits not neutralized by open market operations will affect the interest rate solely through the money surprise variable. Neutralized deficits (through open market operations) would leave the money supply unchanged, and in effect would differ little from debt-financing in its effect on interest rates[1]. A deficit variable should therefore include only borrowing which directly puts pressure on the credit market. Furthermore, it should include only domestically financed debt, since deficits financed externally would not put pressure on domestic credit markets. Also, these deficits are too small to influence foreign interest rates and indirectly influence international capital flows.

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[1]Only negligible amounts of the government debt in the past has been monetized.

There are two ways in which budget deficits could contribute to high interest rates. The first is the well-known crowding out effect, where increased spending and higher demand for funds raises the nominal interest rates. In the short term the nominal interest rate will also increase relative to inflation, pushing up real interest rates. Higher financing costs will eventually have their price effects, pushing the real rates back down and keeping the nominal rate temporarily higher until the price expectations component dissipates. The second is the result of persistent budget deficits. These imply rising government indebtedness and debt servicing burdens, which may cause financial markets to expect increasing monetary accommodation[1]. Price expectations would as a result remain high, contributing to greater and more prolonged nominal interest rate hikes. This second effect would have less of an effect on real interest rates because the monetary accommodation is anticipated.

All of the previous studies incorporating budget deficits in their models have used American data, and the resulting coefficients have mostly been insignificant. However, similar results should not be expected for Canadian data for two reasons. First, the government deficit is relatively higher in Canada than in the United States. Since the deficit as a percentage of GDP is higher in Canada it will have a greater effect on Canadian credit markets. Second, while Canadian provincial governments are also running deficits, American state governments are generally running a substantial surplus[2]. Such surpluses account for the insignificant results in American

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[1]Atkinson, p. 13.

[2]*Ibid*, p. 13.

studies. In Canada the pressure on credit markets is heightened with the inclusion of provincial deficits. Thus one would expect to see more significant results.

Specification of the budget deficit variable should account for all of these factors. Unfortunately it is impossible to allow for the method of financing the deficit. Therefore, confidence in the results is limited by the inclusion of accommodated monetized debt and externally financed debt. The variable will allow for business cycle conditions and the relative size of the economy by standardizing by the level of GNP. Thus cyclical deficit adjustment will be accounted for. Finally, provincial and municipal deficits will be added to the federal amounts in order to provide a more accurate measure of public sector pressure on credit markets.

Business Cycle Conditions

Financing requirements and consumer spending habits also produce pressure on real and nominal interest rates. Since these indicate the level of economic activity, business cycle conditions play a significant role in interest rate movements. The relationship has two components, which indicate that an increase in GDP should raise real and nominal rates temporarily. First, an increase in economic activity or an upturn in the business cycle would put pressure on the credit market as financing capital becomes scarcer. Thus real and nominal interest rates are pushed up directly as rates rise to equate demand and supply.

Second, the presence of the short run Phillips curve suggests that increases in the growth rate of GDP will give rise to inflation and nominal interest rates indirectly through wages and other inputs. Ex post real interest rates may also be affected as anticipated inflation takes time to catch up to actual inflation. A discussion of this transmission is outlined elsewhere in this paper. In an open economy increased growth in GDP will increase import spending and pressure the Canadian dollar downward, counteracting the appreciation resulting from the increase in nominal interest rates. Thus the open economy would, under a floating exchange rate, serve to dampen the influence of the business cycle. This effect, however, is likely to be small[1].

Inflation Uncertainty

Uncertainty about the future rate of inflation may also affect interest rates, as outlined by Milton Friedman in his Nobel Lecture[2]. According to Friedman, increased inflation uncertainty leads to a decrease in the marginal productivity of capital and a subsequent softening of interest rates. Higher levels of inflation are associated with greater volatility in the actual and anticipated inflation rate. Institutional rigidities in the form of medium and long term contracts and government price fixing, along with high volatility of inflation means a "reduction in the capacity of the price system to guide economic activity. . . . These developments

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[1]This may also be the case for fixed exchange rates, where the targets are short-term. A three month fixed target may, over the course of the year, float considerably.

[2]In M. Friedman, "Nobel Lecture: Inflation and Unemployment." *Journal of Political Economy* (June, 1977).

clearly lower economic efficiency."[1] Thus high inflation levels may cause investment decisions to be delayed or cancelled, causing aggregate demand and real interest rates to decline. Therefore, increased inflation uncertainty is generally associated with reduced growth rates in GDP.

Inclusion of the uncertainty variable therefore implies more specific behaviour for the growth variable (economic output or activity), and therefore a joint hypothesis. One model, Makin (1978), sought to allow for this phenomenon without including uncertainty in the model, by using a dummy variable on GNP growth when growth and inflation were high. However, since both GNP and inflation uncertainty can have their individual effects, inclusion of both variables should have more explanatory power.

The effect of a change in inflation uncertainty may be ambiguous, as noted in Hoffmann and Schlagenhauf (1985), because increased inflation variance may reduce savings. This would offset the pressure from changes in investment by increasing aggregate demand and raising real rates. The contradiction of these two effects prevents any *a priori* determination of the sign of the uncertainty variable.

International Capital Movements

The determination of interest rates in an open economy is influenced not only by domestic factors, but also by the international financial environment. This is

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[1] *Ibid*, p. 466.

especially true for Canada, which has historically relied on an influx of foreign capital in its development. Unfortunately, very few interest rate models have incorporated an open economy setting. Neumann (1977) developed a general macroeconomic model of the Federal Republic of Germany to include several markets. His results, calculated in a period of fixed exchange rates, found that the foreign interest rate appeared to have an independent positive effect on the domestic interest rate. Although such a model would be inappropriate for Canada, which moved to managed (more flexible) exchange rates in 1972, Neumann's findings suggest that a model without the foreign sector would be mis-specified. Such a mis-specification was the basis for Carr, Pesando and Smith (1976), who provide the only empirical Canadian study of interest rates. Although they estimate a number of models, none of them allows for international capital flows, so their results cannot be viewed with much confidence. Makin (1978) introduced a model of anticipated inflation and interest rates in an open economy intended specifically for Canadian data, but sadly provided no empirical support for his conclusions. It is clear from his analysis, though, that international capital movements must be included in order to calculate realistic coefficients (recall the earlier discussion of falling imports compensating the real balance effect on saving from a change in inflation expectations).

In his model of fixed exchange rates, Neumann (1978) again included foreign interest rates. Under a flexible exchange rate regime, a more comprehensive approach would incorporate forward exchange premiums or discounts (i.e. capital gains). Interest parity requires that

$$i = i^f + \frac{F + S}{S} \qquad (1 - 6)$$

where F is the forward rate or expected future spot rate and S is the current spot rate. The foreign interest rate is represented by i^f. Thus (F + S) / S represents the forward premium or discount. When equation (1-6) does not hold, all four of the terms in (1-6) adjust until interest parity once again holds. Therefore, both the foreign interest rate and the premium or discount can affect the domestic interest rate. In a world with perfect capital mobility (Makin's (1978) model), equation (1-6) will always hold. In Canada's case capital is mobile, but there is no reason to assume that parity conditions will hold in the short run. The amount the domestic interest rate will react depends on the sensitivity of the domestic interest rate to the premium and discount, the extent to which the parity conditions are out of equilibrium, and the way in which monetary authorities react to the conditions. For instance, the Bank of Canada's concern to keep the price of the Canadian dollar high may cause the domestic interest rate to increase more than usual when the spot rate is low by historical standards.

Makin (1978) also introduces purchasing power parity into his model by assuming that prices equalize only after a lag. Unfortunately, because it is not an empirical paper, the study does not suggest lag lengths. However, assuming that purchasing power parity holds only in the medium to long term is an unnecessary specification for the short term model.

Conclusion

Support for the short-term Fisher hypothesis should not rely on isolating the theory from surrounding economic factors, as early studies have done. These inquiries simply estimated the Fisher equation to measure the variance in the expectations coefficient. The hypothesis was often not rejected.

Instead, and increasingly more common, the models derive reduced-form models for interest rates and substitute these into the equation. This has two advantages. First, it reduces the unobservable variables included in the estimation to the expectations variable (recall that real interest rates are also unobservable). This increases measurement accuracy. Second, it allows other structural parameters to be modelled and included in the analysis, providing a more sophisticated approach to the inquiry. The next Chapter develops such a model.

The conclusion to be drawn, should Fisher withstand the study, is not that the correlation between nominal interest rates and expected inflation is perfect, but that the underlying forces causing loanable funds and other forms of money to be supplied and consumed cause nominal rates to reflect expectations, in the short term.

Chapter 2: A Model of Real Interest Rates

A variety of models have been developed to model real interest rates, largely because both anticipated inflation and ex-ante real interest rates cannot be observed. Resulting coefficients can vary, and without an absolute means of comparison the reliability of different models is difficult to determine. Some models in the past have assumed that ex-post real rates differ from ex-ante real rates only by a white noise disturbance term, for example Mishkin (1982). Others substitute the reduced form equation for the real interest rate into the Fisher equation in order to limit the number of proxies for unobservable variables included in estimation. Both of these methods will be used to compare the sensitivity of results to the procedure. The framework from which the reduced form equation is drawn varies considerably, including loanable funds models[1], general equilibrium frameworks[2], demand for money equations[3], Keynesian IS-LM frameworks[4], and the Fisher equation with

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[1]This is the same approach as used by Thomas and Abderrezak (1988) and Sargent (1969).

[2]See Levi and Makin (1978,1979), Makin (1978), Neumann (1978).

[3]For instance Feldstein and Eckstein (1970).

[4]Carr and Smith (1972), Feldstein and Eckstein (1970).

various modifications[1]. Despite the wide range of approaches, all of the reduced form equations for the nominal or real interest rate have been similar (or similarly ad hoc). The differences between the models may be less than imagined because, for instance, the general equilibrium approach just substitutes its reduced-form equation for the real interest rate into the Fisher equation. Because the modelling approach of Hoffman and Schlagenhauf, like Levi and Makin, includes inflation uncertainty, it is used as a basis for the model employed in this study. It is chosen over the latter modelling technique because, except for Neumann (1978) and Makin (1978), none of the models uses an open economy setting. Since Canada maintains a large international posture with a voluminous current account and balance of payments, foreign influences are important to include. Furthermore, the econometric implications of Hoffman and Schlagenhauf's IS-LM model are clearly defined in their model. Therefore, the model employed here is an IS-LM framework modified for an open economy. Much of the theory, however, is common to all of the models, and among the IS-LM designs there is little variation.

The framework is made up of four behavioral equations and two identities:

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[1]Carr, Pesando, Smith (1976), Carr and Smith (1972), Yohe and Karnovsky (1969).

$$y_t = a_0 - a_1 r_t^* + a_2 \sigma_t^2 + a_3(m_t^s - p_t) + a_4 D_t + a_5(X_t - Z_t) + u_{1t} \quad (2-1)$$

$$(m_t^s - p_t) = b_0 + b_1 y_t - b_2 i_t^* + u_{2t} \quad (2-2)$$

$$(X_t - Z_t) = -c_0(i_t^* - i_t^f - f_t^e) + u_{3t} \quad (2-3)$$

$$y_t = d_0 + d_1(m_t - m_t^e) + u_{4t} \quad (2-4)$$

$$i_t^* = i_t(1 - \tau) \quad (2-5)$$

$$i_t^* = r_t^* + \pi_t^e \quad (2-6)$$

where:

y_t	logarithm of real output
r_t^*	real interest rate
σ_t^2	inflation uncertainty
m_t^s	logarithm of money supply
p_t	logarithm of the price level
i_t^*	nominal, after-tax interest rate
i_t	pre-tax interest rate
D_t	deficit in government spending
$X_t - Z_t$	net exports
i_t^f	foreign interest rate
f_t^e	exchange rate forward premium or discount
m_t	rate of growth in the money supply
m_t^e	expected money supply growth rate
τ	tax rate
π_t^e	expected inflation rate for time t, at end of time t-1
u_{it}	error term for non-identity models

Equation (2-1) is an IS schedule which can be derived from the aggregate expenditure equation[1], and in this reduced form postulates the dependence of

[1] Aggregate Expenditure = Consumption + Investment Spending + Government Spending + (Exports - Imports)

aggregate demand on the real interest rate, inflation uncertainty, real balances, deficits in government spending and net exports. The effects of a change in r*, m-p, D and X-Z are clear. Investment and savings decisions are made based on the real interest rate. A higher rate of interest dampens investment spending and lowers economic activity. Higher real balances increase consumption leading to higher aggregate demand. An increase in the deficit means higher net government expenditure, a boost to economic activity. Similarly, net exports are positively linked to aggregate demand. The coefficient on the uncertainty variable is more difficult to sign since there are two opposing forces. One is Friedman's (1977) notion that higher levels of inflation are associated with greater volatility in the actual and anticipated rate of inflation, as explained in the last chapter. High inflation levels may cause investment decisions to be delayed or cancelled, causing aggregate demand and real interest rates to decline. However, high inflation uncertainty may also dampen savings, increasing consumption and aggregate demand. Therefore the sign of this coefficient is *a priori* ambiguous.

Equation (2-2) is an LM schedule. The demand for real balances hinges on aggregate demand and the after-tax nominal interest rate. If holding cash balances, unlike bonds, is considered to have zero return, then an increase in the nominal interest rate will reduce cash holdings. Higher aggregate demand will also clearly increase the demand for real balances.

Equation (2-3) is an external balance of payments equation assuming freely flexible exchange rates. The left hand side is the net export function which measure the current account. This is matched in value by the capital account, or net capital inflows, on the right hand side of the equation. In the short term these capital flows can be modelled with the covered interest rate differential, which measures the difference in yields between domestic and foreign interest rates plus expected capital gains through exchange rate depreciation from holding money in the foreign currency. The term f^e is the forward exchange rate discount and is the percentage difference between the forward rate (F) and the spot rate (S):

$$f^e = \frac{F - S}{S} \qquad (2 - 7)$$

Equation (2-4) is a Lucas supply function, also known as a new classical surprise function or an aggregate supply schedule, where output depends on unanticipated money growth. Since unexpected shifts in aggregate demand are misinterpreted by economic agents with limited information as relative shifts, attempts to take advantage of what are believed to be temporary relative price increases lead to increases in economic activity.

The last two equations, (2-5) and (2-6), are identities. The first states that the nominal after-tax interest rate is equal to the observable pre-tax nominal rate discounted by the marginal or effective tax rate. The second is the Fisher equation, in which the after tax nominal interest rate is composed of the after tax real interest rate and inflation expectations for the period of the interest rate.

Equations (2-1) to (2-4) and equation (2-6) can be used to solve for a reduced-form equation (see Appendix A) for the real interest rate r*:

$$r_t^* = \beta_0 + \beta_1\pi_t^e + \beta_2\sigma_t^2 + \beta_3 y_t + \beta_4 D_t + \beta_5(m - m^e)_t + \beta_6(i^f + f^e)_t + v_t \qquad (2 - 8)$$

where the ßs are the reduced from coefficients described in Appendix A. Since r* is not observable the Fisher equation can again be used to solve for the nominal rate of interest:

$$i_t^* = \alpha_0 + \alpha_1\pi_t^e + \alpha_2\sigma_t^2 + \alpha_3 y_t + \alpha_4 D_t + \alpha_5(m - m^e)_t + \alpha_6(i^f + f^e)_t + v_t \qquad (2 - 9)$$

The αs in equation (2-8) are the same as the ßs in equation (2-9) except for α_1, which is equal to $\beta_1 + 1$. However, i* is also not observable. Using equation (2-5), equation (2-9) can be expressed using the pre-tax nominal interest rate:

$$(1 - \tau)i_t = \alpha_0 + \alpha_1\pi_t^e + \alpha_2\sigma_t^2 + \alpha_3 y_t + \alpha_4 D_t + \alpha_5(m - m^e)_t + \alpha_6(i^f + f^e)_t + u_t \qquad (2 - 10)$$

Equation (2-10) is now in a form where the dependent variable is observable and only expected inflation and the money surprise variables are not. Therefore, this model will serve as the basis for estimation. The coefficients in Equation (2-10), after dividing both sides by $(1-\tau)$, are related to the underlying structural parameters outlined in Appendix A, and are presented below in Table 2-1: Structural Parameters.

Table 2-1

Structural Parameters

Coefficient	Parameters
α_0	$\dfrac{a_0 - d_0 - a_3 b_0}{a_1 + a_5 c_0 - a_3 b_2}$
α_1	$\dfrac{a_1}{a_1 + a_5 c_0 - a_3 b_2}$
α_2	$\dfrac{a_2}{a_1 + a_5 c_0 - a_3 b_2}$
α_3	$\dfrac{- a_3 b_1}{a_1 + a_5 c_0 - a_3 b_2}$
α_4	$\dfrac{a_4}{a_1 + a_5 c_0 - a_3 b_2}$
α_5	$\dfrac{- d_1}{a_1 + a_5 c_0 - a_3 b_2}$
α_6	$\dfrac{a_5 c_0}{a_1 + a_5 c_0 - a_3 b_2}$

Assuming that the denominator in each coefficient is positive, the expected signs are those displayed in the numerator. The sign of the intercept is uncertain and depends on the magnitudes of the second and third terms in the numerator. The magnitude of α_1 determines the validity of the Fisher hypothesis. If there is a one-to-one relationship between anticipated inflation and the nominal interest rate, i.e. $\partial i/\partial\pi^e = 1/(1-\tau)$, and $\partial r/\partial\pi^e = 0$, then $a_5c_0-a_3b_2=0$. In this case Fisher was correct and the real interest rate is immune to effects of anticipated inflation. If $a_5c_0-a_3b_2<0$ then α_1 in equation (2-10) is less than one and anticipated inflation has a less than equal effect on the nominal after-tax interest rate. This is the Mundell-Tobin hypothesis, and the conclusion here is that the less than unity increase in nominal rates is accompanied by a decrease in real interest rates.

The remaining coefficients are relatively straight forward, and the *a priori* sign expectations of these variables are those indicated by their reduced-form expressions. A positive coefficient is expected on the deficit variable if government borrowing to finance a deficit causes some crowding out of private investment. Unanticipated money supply increases would lower rates through a real balance effect due to a temporary increase in loanable funds, until the price effects (as explained by Barro) kick in. Thus a negative coefficient is expected. A positive sign on the output variable should arise as, *ceteris paribus*, an increase in economic activity would decrease the supply of funds available for financing investment. The foreign interest rate differential should also affect domestic interest rates positively since an increase in foreign rates causes and outflow of short-term capital. The decreased supply of funds causes borrowers to increase their domestic rates to clear the market and re-attract lost capital.

Definition of Variables

The definition of the expected inflation variable has shown most variety in past studies, primarily because a consensus does not exist in its definition. However, although anticipated inflation is not observable, many of the studies suggesting a number of possible strategies in the calculations found that the coefficients obtained were fairly insensitive to the measurement technique used. In this study a similar path will be followed, and two different measurements will be tested: a univariate ARIMA process, or the rational expectations approach; and a simple, "conventional" measure. The "conventional" measure is simply the previous quarter's actual inflation rate, and is included as a simple contrast to rational expectations. There is however, a rationale for this measurement. As inflation is difficult to predict, investors may assume that the future inflation rate will not vary considerably from the most recent. Therefore, they expect that the inflation rate will be what it now is. In this case, changes in the inflation rate are considered a "random walk."

Another conventional measure is survey data[1]. Although it may be a very accurate measure of expectations, this approach is not incorporated into the analysis because of resulting problems in biased estimators, and the fact that this data does not exist for Canada.

A number of other measures following the Rational Expectations approach are

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[1]Most of the studies, notably Levi and Makin, use Livingston data. A description of the data's limitations can be found in any study not using this data, for instance Fama and Gibbons (1984) and Mishkin (1982).

commonly used, mostly as distributed lags, adaptive expectations and extrapolative models (Lahiri (1981) uses all three). However, there is no theoretical reason to choose any of these models, and an ARIMA process is the best approximation under the criterion of the most efficient estimator of π at time t given all of the costless information at time t-1. Since the best estimator of π_t is π_{t-1}, the ARIMA process fulfils this criterion, and is the "Economically Rational Expectations"[1] variable.[2] An in-depth discussion of the various measures of inflation expectations can be found in Pepper (1985) and Reichenstein and Elliott (1987).

The majority of studies use this form of Muthian Rational Expectations Hypothesis (REH) in which:

$$_{t-1}\pi_t^e = E(\pi_t \mid I_{t-1}) + \text{random error}$$

where:

$_{t-1}\pi_t^e$:rational expectation of π for time t, generated at t-1

$E(\pi_t \mid I_{t-1})$: Conditional mathematical expectation of π for time t based on information set I at time t-1.

Since I_{t-1} is the complete information set that determines the economic variable, the Muthian notion of expectation is that it is rational if is equal to the prediction of relevant theory. Thus, unlike adaptive expectations and similar models, forecast errors are random and not persistently positive or negative.[3]

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[1]Feige and Pearce (1976).

[2]Pepper (1985), p. 31.

[3]Pepper (1985), p. 23.

In order to fulfil this notion of rationality, it is necessary to specify relevant theory and extract the appropriate predictions. However, finding REH solutions from complete macroeconomic models is difficult because there is no consensus on the proper empirical structure of these models.[1] For simplicity's sake this limitation is not pursued and the process outlined in Lahiri (1981) is used. Assuming I_{t-1} includes all of the macroeconomic variables available at time t-1, then a REH estimation of ${}_{t-1}\pi_t^e$ is the forecast from the regression of inflation on macroeconomic variables known at t-1, such as lagged inflation rates, output and the money supply. This process is repeated until the errors from this regression are white noise, and thus the forecasted expectations differ from the true expectations by only a random error term, which is not correlated with the error term in the reduced interest rate model. The Box-Jenkins Arima process thus fulfils this form of rational expectations because this year's price level contains all the price information for this year. Since a Box-Jenkins specification reduces the errors to white noise, the conditions have been met.

Inflation uncertainty is proxied by the variance in the inflation expectations because actual uncertainty is unobservable. Past studies used the variance in their survey data, but since no such data exists for Canada this approach is not an option. Two other methods using generated data are common. One is the variance in the actual inflation rate around its mean for a certain period of time (for instance a

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[1]Hoffman and Schlagenhauf (1984), p. 288.

year). The other is the current squared actual forecast error, which can be derived from the calculation of the inflation expectations variable. This last approach is favoured, although it is a noisy proxy of the true variance, because it is related to the true variance[1] and because is forward looking, representing expectations in the spirit of the REH variable itself.

The economic activity variable Y is the growth rate of real GDP, not seasonally adjusted. In contrast, the money surprise variable is the difference between the log of the money supply (M2+) growth and the log of the expected money supply growth in nominal terms. Expected money supply growth is determined using an ARIMA process.

The deficit variable will include federal, provincial and local government budget surpluses or deficits. Since the smaller governments also run deficits, crowding out is more likely to occur in Canada than in the United States, where state surpluses are common and counteract somewhat the effects of the federal deficit. To remove the structural component of the deficit, the variable is divided by GDP[2]. Furthermore, the deficit will put pressure on interest rates only if it crowds out private investment. The availability of capital is a pivotal measure because it determines the price of capital. Since domestic savings are the largest source of such capital, the deficit would put pressure on capital markets when savings are low. Therefore, a more appropriate measure of government deficits in the model is the government deficit

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[1]Hoffman and Schlagenhauf (1984) p. 289.

[2]See Groenewold (1989).

net of domestic savings.

Ninety-one day Treasury Bill rates will be used for the international capital flow portion of the model. The foreign interest rate is represented by the 91-day Treasury Bill rate in the United States. The forward premium or discount is measured using the spot Canada-U.S. dollar exchange rate and the three month forward rate for the same currencies.

The tax rate presents more difficulties, since there is no standard measure. Either the effective tax rate, i.e. the total tax revenues collected divided by GDP, or some weighted average of marginal rates can be used. The effective tax rate will be used as a better measure of changes in tax conditions, rather than a series of different arbitrarily assigned weights as tax brackets are changed.

Econometric Considerations

Although a study of this nature poses no particularly difficult econometric problems, the presence of the uncertainty variable along with the expected inflation does cause some concern. Since inflation uncertainty is derived from the errors of the model to calculate inflation expectations, both heteroscedasticity and correlation with the error term in the final model can arise.

The fact that the variance in inflation expectations is included as a variable assumes that the inflation expectations variable exhibits heteroscedasticity. Although resulting coefficients are unbiased and consistent, test statistics would be exaggerated and misleading, justifying the use of a weighted least squares process. This suggests a two-step process. First, the estimation of inflation expectations produces the errors which are used to calculate the uncertainty variable. Second, the generated expectations variable serves as weights in the final model.

Another complicating feature is the implication of using the uncertainty variable. Since this variable is derived from the errors in the expectations variable, correlation between uncertainty and the errors in the final model can be expected. In order to overcome this problem, Hoffman and Schlagenhauf (1985) try a number of different insturmental variables, including commodity prices, as proxies. As the proxies failed to improve the model's performance, they used the other variables in the equation as proxies (instrumental variables). This same process will be followed in this study.

This process is simplified when the uncertainty variable is eliminated. This would be the case in the preliminary estimation, where the null hypothesis that inflation uncertainty is significant, is rejected. In this case, the model can be estimated using ordinary least squares.

Data

Because this study focuses on short-term relationships, the term of the interest rate is three months. Thus quarterly measurements of the 91-day Treasury Bill rate for both Canada and the United States can be used. This is appropriate because "quarterly data on three month rates have the advantage that the data are nonoverlapping and timing problems that would arise with monthly data are avoided."[1] Finally, forward exchange rate values are available for three month terms. The Bank of Canada, in its monthly report, also reports the premium and discount on three month forward rates between the United States and Canada. The interest parity variable will use the premium or discount from the first of each month of each quarter.

The data for the deficit variable comes from Statistics Canada's quarterly National Income and Expenditure Accounts, no. 13-001. The rest of the data comes from the Bank of Canada's Quarterly Review. The data are limited to the post-1972 era of flexible exchange rates in Canada, in order to avoid the need to account for structural changes at that time. Therefore, the data will consist of 68 observations, from 1973 I to 1989 IV.

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[1]Mishkin (1984) p. 290.

Chapter 3: The Levi-Makin Model

It is useful at this stage to use the Canadian data collected for the model developed in the last chapter to test against one of the other models outlined earlier in the paper. The developed model was assembled in response to the claim that earlier models of short term interest rates were misspecified, either because they simply tested the Fisher hypothesis without taking account of influential macroeconomic variables, or because their account failed to be complete. It would, however, be far beyond the limits of intellectual modesty to suggest that the model conceived in this investigation is an ultimate specification. Therefore, applying the Canadian data to an earlier model would make a useful reference point.

Selection of the appropriate model for this 'benchmark' estimation is not obvious, because the model defined in this paper draws on elements common to many of the past studies, and little variation exists among the IS-LM frameworks. For this class of model, almost all of the studies used or tested some combination of prices expectation, money supply growth (expected or unexpected), government debt, short-term international capital flows, and Canadian data. Amongst the possibilities are Makin (1978), Neumann (1978), Groenewold (1989), Levi and Makin (1982), and Hoffmann and Schlagenhauf (1984).

Much of the theory and econometric approach follows Hoffman and Schlagenhauf, yet their model tests a distributed lag structure of the 91-day Treasury

Bill (TB) rate on past TB rates, unexpected money, inflation expectations and inflation uncertainty. Neither an autoregressive nor a distributed lag structure was suggested in the model presented in the last Chapter. The Groenewold model includes the necessary variables, but uses commercial paper rates as the dependent variables. Comparisons are then difficult to make because the price setting nature of treasury bills is a fundamental requirement for reasons outlined earlier. The Levi and Makin (1982) model may also not be appropriate because it uses survey data as a measure of inflation expectations. However, the model does employ 90 day TB rates as the dependent variable, and the reduced form interest rate equation is most similar to the model developed in this paper. As a result, this model deviates least from the chosen path. For expository purposes, both the Levi-Makin and the Hoffman-Schlagenhauf models are presented in Appendix B.

The Levi and Makin model is comprised of three equations representing equilibrium conditions in the market for commodities, money supply and labour:

$$I\ (r, \sigma) - S\ \{y\ (N, \sigma), M/P\} = 0$$

$$L\ \{y\ (N, \sigma), i, P\} - M = 0$$

$$P\ y'\ (N, \sigma) - W\ (P, N\sigma) = 0$$

The first equation sets real investment, I, equal to real savings, S. I is expressed as a function of the real interest rate, r, and inflation uncertainty, σ (in this form to follow the authors' notation; in this paper it is represented as σ^2). Savings is expressed as a function of real income, itself determined by the quantity of labour employed (N), and inflation uncertainty. The second equation sets nominal money demand, dependent on real income, the nominal interest rate and the price level, equal to the exogenous nominal money supply. The last equation sets money wage offered (the value of the marginal product of labour) equal to the money wage demanded. Wage demands are dependent on the price level, P, and labour supply.

Supplementing this set of equations with the Fisher equation, $i^* = r^* + \pi^e$, Levi and Makin suggest a model to test the Fisher hypothesis, taking into account the various effects:

$$i_t = B_0 + B_1 \pi_t^e + B_2 y_t + B_3 \sigma_t + B_4 M_t + u_t \qquad (3 - 1)$$

where i is the after-tax 90 day TB rate, π^e is inflation expectation, y is the growth rate of real GDP, σ_t is inflation uncertainty, and M_t is the growth rate of the money supply. The difference between this model and that developed for estimation is the omission of the government deficit and international capital flows in this model, and in the definition of some of the variables. The Levi-Makin inflation expectations and uncertainty variables are derived from biased survey data. The money supply variable is simply the growth rate in the Levi-Makin case, whereas in the developed model it is represented by unexpected money supply growth.

Estimation by Levi and Makin of this model over the period 1947 to 1975 gave the following results:

	Constant	πe_t	y_t	σ_t	m_t	R^2	DW
Coefficient	3.29	0.54	-0.16	-0.38	0.12	0.71	1.13
t-statistic	(9.54)	(6.57)	(-4.10)	-(3.03)	(1.53)		

The results show an insignificant money supply coefficient at the 95% level. Re-estimating the equation excluding this variable produced:

	Constant	πe_t	y_t	σ_t	R^2	DW
Coefficient	3.53	0.62	-0.14	-0.42	0.70	1.02
t-statistic	(11.29)	(9.71)	(-3.76)	(-3.30)		

Levi and Makin base their conclusions, in which they fail to accept the Fisher hypotheses, on these results. Athough the coefficients display reasonable values, the Durbin-Watson statistic, as 1.02, falls far short of the lower limit. Because this indicated autocorrelation in the model, the test statistics are biased and the standard errors high.

Estimating the model with Canadian data will allow for the correction of autocorrelation. First, the appropriate variables must be developed; then they may be applied to the model.

Variable Creation

The unobserved variables were created primarily through Box-Jenkins (ARIMA) modelling of inflation expectations (from which was derived inflation uncertainty) and money supply growth. The resulting data was then used in both the benchmark (Levi-Makin) and the developed model.

Box-Jenkins methodology entails a multi-step process. First, the data must be stationary, a process which involves differencing or taking logarithms of the data. Second, autocorrelations and partial autocorrelations are calculated in order to indicate the autoregressive and moving average components of the series. Third, the model or models, estimated through Maximum Likelihood[1], that these autocorrelations suggest are tested both by the standard t-tests of the estimates, and by the autocorrelations of the error terms. As a general rule, autocorrelations and partial autocorrelations are considered significant if they exceed $\frac{2}{\sqrt{n}}$, where n represents the number of observations in the sample. Error terms which are white noise (i.e. which are not significantly different from zero) indicate that the chosen model explains adequately the underlying series. This test is formalized with the use of the Box-Pierce Q-statistic[2], which follows the Chi-square distribution, and tests the joint hypothesis that all of the autocorrelations are zero. The forecasts generated from the time series model become the expectations variable. The difference between forecasts and actuals form the surprise variables.

Table 3-1: Time Series Estimates outlines the time series modelling of the price series and of the money supply.

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[1]Maximum Likelihood is a non-linear iterative procedure which seeks to minimize the mean square error of the estimates. For reference, see Judge, Hill, Lee, et. al. (1988).

[2]Judge, G.G., et al. (1988).

Table 3-1

Time Series Estimates

	Constant	AR(1)	AR(2)	AR(3)	D.F.	Chi^2
Price level						
AR(1)	2.06	0.72			11	17.7
	(8.24)	(7.6)				
AR(3)	1.09	0.50	0.04	0.31	9	9.3
	(4.12)	(0.35)	(2.52)	(1.22)		

	Constant	AR(1)	MA(1)	D.F.	Chi^2
M1					
ARMA(1,1)	0.67	0.87	0.45	10	7.4
	(96.42)	(9.25)	(2.72)		

	Constant	MA(1)	MA(2)	MA(3)	MA(4)	D.F.	Chi^2
M2							
MA(4)	0.03	-0.34	0.22	-0.24	-0.50	8	11.5
	(8.85)	(-3.03)	(1.92)	(-2.06)	(-4.37)		

A single differencing of the price data created a stationary series whose autocorrelations and partial autocorrelations suggested two models. The first and third partial autocorrelations (in the series of 0.712, 0.201, and 0.293) are greater than the cut-off value of 0.246[1]. The two tested models are therefore an AR(1) and

[1]following the general rule, $\frac{2}{\sqrt{66}}=0.246$.

an AR(3). Both models produce significant estimates at the 95% level (aside from the AR(2) parameter). The null hypothesis that the errors are white noise can be rejected if the Box-Pierce Q is greater than the critical Chi-square[1]. The critical Chi-square for the AR(1) with 11 degrees of freedom is 17.28 at the 90% level, and 19.68 at the 95% level. The test statistic, at 17.7 does not pass both tests (that the residuals are not white noise). On the other hand, the critical Chi-Square for AR(3) with 9 degrees of freedom is 14.68 at the 90% level and 16.92 at the 95% level. Since the test statistic of 9.3 is well below both critical values, one can consider the AR(3) model to be a better model specification. Thus the AR(3) forecasts are used for the π^e series, while the squared errors are used for the uncertainty variable.

This same process was followed for the creation of the money surprise variable, using both M1 and M2a. In these cases, however, the autocorrelations and partial autocorrelations suggested only one model. For M1 the critical Chi-square value at the 90 % level (the stronger test) is 15.99; for M2a it is 13.36. Both test statistics are well below their respective critical values. The forecast errors are used as the surprise variable, while the forecasts are used as the growth expectations in the Levi-Makin model.

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[1]The Box-Pierce statistic can be described as follows:

$$Q_{(K-p-q)} = n\sum_{k=1}^{m} r_k^2$$

where K is the number of lags, p and q are the autoregressive and moving average parameters respectively, and r is the value of the autocorrelation.

Levi-Makin Model Estimation

The creation of the price and money supply variables now allow the Levi-Makin model to be estimated. A series of models is tested to allow for specification and econometric considerations. The Darby effect is tested by estimating the model with after-tax nominal rates. Using Inflation expectations as weights for the other variables accounts for heteroscedasticity, while regressing uncertainty on itself and incorporating the forecasts accounts for correlation with the error terms.

The Presence of Autocorrelation

The equations in Table 3-2: Levi-Makin Reestimation estimate the model and correct for the presence of auto- (or serial) correlation.

Table 3-2

Levi-Makin Re-estimation

Equation #		Constant	π^e_t	y_t	m_t	σ_t	R^2	DW
	Pre-Tax - OLS							
(3-1)	Coefficient	-193.88	0.96	15.42	-53.50	0.07	0.70	1.03
		(-1.52)	(9.52)	(17.80)	(-5.88)	(0.17)		
	OLS							
(3-2)	Coefficient	-193.57	0.96	15.39	-53.19		0.70	1.02
		(-7.60)	(9.71)	(7.89)	(-6.01)			
	AUTO							
(3-3)	Coefficient	-135.27	0.35	11.18	-24.52	-0.10	0.85	1.97
		(-1.99)	(3.06)	(2.11)	(-4.75)	(-0.43)		
	AUTO							
(3-4)	Coefficient	-135.77	0.34	11.22	-24.52		0.85	1.97
		(-6.64)	(7.86)	(6.88)	(-4.78)			
	After Tax - OLS							
(3-5)	Coefficient	-101.47	0.49	8.08	-27.72	0.04	0.70	.99
		(-7.58)	(9.33)	(7.87)	(-5.86)	(0.20)		
	OLS							
(3-6)	Coefficient	-101.22	0.49	8.06	-27.52		0.70	.98
		(-7.65)	(9.51)	(7.95)	(-5.99)			
	AUTO							
(3-7)	Coefficient	-72.22	0.18	5.97	-13.01	-0.01	0.85	1.98
		(-2.08)	(3.07)	(2.21)	(-4.89)	(-0.09)		
	AUTO							
(3-8)	Coefficient	-72.23	0.18	5.97	-13.01		0.85	1.97
		(-2.10)	(3.14)	(2.22)	(-4.93)			

Equations (3-1) and (3-2) are the simple reproduction of the Levi-Makin models. Despite an opposite sign on the output variable, the results are very similar to those obtained by Levi and Makin. First, the coefficient on the expectations term is very close to one. In fact, at the 95% level, the null hypothesis that it equals one cannot be rejected. Furthermore, the uncertainty variable is insignificant in the first model; in the second its exclusion does not alter the results. Third, the Durbin-Watson statistic is low, in the second equation below its critical lower value of 1.50.

This last result is somewhat disturbing, since it indicates the presence of autocorrelation, and was not accounted for in the Levi-Makin study. Autocorrelation reduces the efficiency of estimators (no longer BLUE), which causes standard errors to be smaller than the true values, the R^2 to be inflated and hypothesis test unreliable. As a result, conclusions about the precision of estimates may be erroneous. Levi and Makin rationalize their inaction: "In light of the low Durbin-Watson statistic, care should be taken in interpreting these 't' statistics. We confine our discussion to point estimates."[1]

Equations (3-3) and (3-4) re-estimate the models while accounting for the autocorrelation. The results, which now produce a Durbin-Watson statistic well above the upper limit of 1.70, show the greatest change in the coefficient on the inflation expectations variable. The final coefficient of 0.34, while still significant, no longer lies within the confidence band around one. This considerable reduction

[1]Levi and Makin (1979), p. 49 fn.

indicates a misspecification error on the part of Levi and Makin[1]. Not surprisingly, the magnitude of the other coefficients are reduced as a result of the correction.

Changes to the size of the coefficient on the expectations variable now allow the null hypothesis to be rejected. The range of coefficients, with corresponding one tail (upper limit) tests at the 95% confidence level, is set out below.

Equation	Estimate	Upper Limit (One Tail)
(3-1)	0.96059	1.35181
(3-2)	0.96275	1.34865
(3-3)	0.35140	0.79769
(3-4)	0.33975	0.77287
(3-5)	0.48863	0.69183
(3-6)	0.49001	0.69000
(3-7)	0.18153	0.41225
(3-8)	0.18014	0.40415

Equations (3-1) to (3-4) show this difference, as the upper limits of the confidence interval are well below unity. In both cases, (3-3) and (3-4), the hypothesis that the expectations coefficient is different from unity can be rejected with a one tail test.

In the next set of equations (3-5) to (3-8), using after-tax data, a similar change in the inflation expectations coefficient appears in response to the correction for serial correlation. Equations (3-5) and (3-6) both have coefficients lying around 0.49, while in the more appropriately specified models (3-7) and (3-8), the coefficients are less than half their previous values. Once again, accounting for the insignificant

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[1]This same series of models was estimated using M2a, a broader measure of the money supply. This new data, however, does not provide significant results, and reduces the R^2 of each equation. Furthermore, the correction for autocorrelation maintains the Durbin-Watson statistic at the upper limit. As a result, M1 was used for the remaining models estimated in this part of the study.

uncertainty variable and the low Durbin Watson statistic, the final coefficient becomes 0.18[1].

Equations (3-5) through (3-8) show the effect of taxes on the model. Recall from Appendix C that the Fisher hypothesis can only be properly tested using after tax rates of interest. In each equation the coefficient is significantly reduced; by examining the difference between pre- and post-tax interest rates, the sensitivity of the model to the inclusion of taxes can be seen. This sensitivity can be calculated as the difference in the coefficients on the expectations variable, between the pre-tax model and the post-tax model, the latter adjusted for taxes:

$$\textit{Difference} = \beta_{pre\text{-}tax} - \frac{\beta_{post\text{-}tax}}{(1 - \tau)}$$

This difference can be calculated in the case of each equation[2]:

Equations	$\beta_{pre\text{-}tax}$	$\beta_{after\text{-}tax}$	Difference
(3-1,5)	0.96059	0.48863	0.038
(3-2,6)	0.96275	0.49001	0.038
(3-3,7)	0.35140	0.18153	0.009
(3-4,8)	0.33975	0.18014	0.000

This range in the last column shows that the estimates are sensitive to the model

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[1]The final equations used do not include the uncertainty variable, since it is not significant in regression estimates so far. Correcting for suspected heteroscedasticity using weighted least squares does not improve matters, since estimates become insignificant, even after correction for autocorrelation. Since these corrections do not improve the explanatory power of the uncertainty variable, its inclusion is considered a misspecification. This allows the remaining models to be estimated using ordinary least squares without having to correct for heteroscedasticity.

[2]The average percentage tax revenue of GDP for the sample is 0.4700.

specification. The wedge is higher for regressions exhibiting autocorrelation, but little or no wedge at all for the corrected version. The model without uncertainty shows almost no wedge at all. As the specification of the model accounts for some of the problems, such as serial correlation, the impact of taxes on the expectations coefficient diminishes.

The final coefficients now lie in the range of 0.18 in equations (3-7) and (3-8). In these last two models, the coefficients are not sensitive to taxes, a conclusion consistent with Hoffman and Schlagenhauf. Unlike the Levi and Makin results, which cannot reject Fisher's hypothesis with a coefficient of 1.00 and 0.96, the hypothesis can be rejected in favour of the Mundell effect, both before and after accounting for the Darby effect.

Other Variables

Another deviation from the Levi-Makin results is the coefficient on the output variable. Their results showed a small negative relationship between nominal interest rates and output changes; the re-estimation here showed a stronger positive result. This sign is more consistent with the theory outlined earlier.

The money supply variable both in the Levi-Makin case and in the re-estimation show a small negative relationship consistent with the real balance effect.

Conclusion

The initial estimation of the Levi-Makin model supported the conclusions arrived at by the pair of researchers, which rejected the Mundell/Tobin hypothesis. In each case the role of inflation uncertainty was rejected. However, like Levi and Makin, the model suffered from autocorrelation. Upon correction, the Mundell hypothesis failed to be rejected. Had Levi and Makin also revised the specification of their model based on the low Durbin-Watson, they may also have reached different conclusions. The low coefficients on the inflation expectations variable, dipping to as low as 0.18, suggests that real rates buffer much of the change in expectations in the short term.

Levi and Makin gave little consideration to Darby, which, in an examination of real interest rates, can be a major factor. In both the uncorrected and corrected models, the coefficients are approximately $1/(1-\tau)$ times the coefficient after taxes. The evidence from their model, again corrected for autocorrelation, indicates a negligible difference between pre-tax and after-tax rates.

Enthusiasm over the results of these last models, however, must be tempered. This primarily due to the presumed misspecification of the Levi-Makin model. While it includes many of the important variables, it ignores the possibility of foreign capital flows, which are particularly conducive to a Canadian setting, and it makes no allowance for the possibility that government deficits may put pressure on capital markets, thus becoming an important variable in the explanation of interest rate movements. In order to assess the importance of these, a more comprehensive model must be estimated.

Chapter 4: A More Fully Specified Model

In Chapter 2 a model based on underlying structural equations was developed in order to provide such a comprehensive explanation of interest rate movements. Estimation of the Levi-Makin model in the last Chapter suggested that model specification is a crucial element in the analytical procedure, as the results and conclusions vary dramatically as the specification changed. In those estimates, results varied depending on the data used (M1 or M2), the variables included, and the inclusion of taxes.

Thus one can expect to find the same complications in the next model. As a result, specification detail will be focused on the breadth of the money supply data, the inclusion of taxes, and the variables included (primarily inflation uncertainty, government deficit and foreign interest arbitrage). As in the last model, attention will also be given to econometric details, such as correlation between the error term and the uncertainty variable.

Recall that the full specification of the new model is:

$$(1 - \tau)i_t = \alpha_0 + \alpha_1\pi_t^e + \alpha_2\sigma_t^2 + \alpha_3 y_t + \alpha_4 D_t + \alpha_5(m - m^e)_t + \alpha_6(i^f + f^e)_t + u_t \qquad (2\text{-}10)$$

The inclusion of inflation uncertainty causes one to suspect correlation with the error term. In order to remove this problem, following Hoffman and Schlagenhauf, the uncertainty term was regressed on lagged values of itself and the forecasts were used as an instrumental variable for uncertainty[1]. Equations (4-1) through (4-4) in Table 4-1 express these results using M1 and M2+ for both after tax and pretax models. These models, generated using ordinary least squares, perform better than the Levi-Makin models. A high R^2, around .95 in each case, shows a continued good overall fit. Unlike the Levi-Makin model, however, the inclusion of the new variables eliminated extensively the autocorrelation. The upper and lower bounds for the Durbin Watson statistic for 65 observations and five explanatory variables are 1.44 and 1.77 respectively. Equations (4-1) and (4-3), including taxes, both have a DW statistic above 1.5, which lies in the uncertain region between the critical upper and lower values. Equations (4-3) and (4-4) have test statistics above 1.8, which is well above the critical value. Thus we can reject the hypothesis that autocorrelation plagues our model to an extent where corrective action is necessary, an impossible conclusion in the Levi-Makin case.

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[1]Since the estimation of this model in 1990, a wide body of literature has developed to show that ordinary least squares breaks down with the inclusion of generated regressors such as inflation expectations and unexpected money supply changes. In other words, test statistics may be biased and variances inefficient. Furthermore, diagnostic checking, such as for the stability of the expectation equation and more general testing for model specification and trending variables were not performed on the independent variables. The two-step estimation procedure followed in this paper should be viewed with caution. See Oxley and McAleer (1993).

Table 4-1

Full Model Specification

	Const.	p_t*	Y_t	m_t	σ_t	D_t	I_t*	R^2	DW
M1-tax									
(4-1)	-31.131	0.13437	2.4365	-5.1264	-0.02287	2.9114	0.47919	.9496	1.56
	(-4.29)	(3.75)	(4.31)	(-2.03)	(-0.24)	(0.564)	(14.38)		
M1									
(4-2)	-54.277	0.24771	4.2251	-8.4660	-0.04671	0.09212	0.09178	0.9581	1.92
	(-4.23)	(3.91)	(4.23)	(-1.90)	(-0.28)	(0.01)	(15.59)		
M2-tax									
(4-3)	-26.06	.098019	2.0356	4.0167	-0.02800	-1.3173	0.49348	0.9488	1.51
	(-3.75)	(3.04)	(3.77)	(1.79)	(-0.30)	(-0.26)	(15.31)		
M2									
(4-4)	-45.791	.18673	3.5544	7.8191	-0.04928	-7.2144	0.93978	0.9583	1.84
	(-3.77)	(3.32)	(3.77)	(2.00)	(-0.30)	(-0.83)	(16.68)		

Table 4-1

Full Model Specification

	Const.	p_t*	y_t	m_t	σ_t	D_t	I_t*	R^2	DW
M1-tax									
(4-5)	-31.229	.13319	2.4443	-5.2091		2.7144	0.47824	0.9495	1.56
	(-4.34)	(3.78)	(4.36)	(-2.1)		(0.53)	(14.57)		
M1									
(4-6)	-54.478	.24530	4.2410	-8.6348		-0.30993	0.91590	0.9580	1.92
	(-4.29)	(3.94)	(4.29)	(-1.97)		(-0.034)	(15.79)		
M2-tax									
(4-7)	-26.084	.095852	2.0373	4.0966		-1.6431	0.49259	0.9487	1.49
	(-3.78)	(3.08)	(3.81)	(1.85)		(-0.34)	(15.46)		
M2									
(4-8)	-45.827	.18292	3.5573	7.9897		-7.7877	0.93821	0.9583	1.84
	(-3.80)	(3.36)	(3.8)	(2.06)		(-0.92)	(16.86)		

Since the uncertainty variable was rejected in the Levi-Makin model, Equations (4-5) through (4-8) re-estimate the model without the uncertainty variable[1]. The results are not an improvement over the first four the results obtained in Equations (4-1) to (4-4). None of the other coefficient estimates changes as a result of these new equations, so the comments above apply similarly in this case.

The final equations[2] estimate the model without both the inflation uncertainty and the government deficit variable; the results are contained in Table 4-2 as equations (4-9) through (4-12). The exclusion of the deficit variable changed the results more than the exclusion of the uncertainty variable. Although the R^2, and the coefficient on international money flows changed little from Equations (4-5) through (4-8), the Durbin-Watson statistics and output coefficients declined slightly, while the coefficients on money supply and inflation expectations increased slightly. However, none of these changes altered the results enough to change the conclusions reached from the results of the other equations.

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[1]Further testing of the models without using instrumental variables also found the inflation uncertainty variable to insignificant.

[2]Structural breaks for the changes in monetary policy at the beginning of the 1980's were accounted for in the model through the use of dummy variables. However, the results did not constitute an improvement in the model.

Table 4-2

Full Model Specification

	Const.	p_t^*	y_t	m_t	I_t^*	R^2	DW
M1-tax							
(4-9)	-30.213	0.12377	2.3677	-4.7754	0.46879	0.9493	1.51
	(-4.38)	(4.18)	(4.40)	(-2.05)	(17.01)		
M1							
(4-10)	-54.594	0.24638	4.2498	-8.6843	0.91698	0.9580	1.92
	(-4.49)	(4.60)	(4.48)	(-2.11)	(18.89)		
M2-tax							
(4-11)	-26.51	0.10021	2.0682	4.0226	0.50011	0.9486	1.52
	(-3.94)	(3.56)	(3.95)	(1.84)	(22.02)		
M2							
(4-12)	-47.846	0.20357	3.7041	7.6087	0.97385	0.9577	1.90
	(-4.05)	(4.11)	(4.03)	(1.98)	(24.39)		

Unexpected Money Supply Changes

The use of M1 and M2+ affected the model results. Although the overall level of significance changed little (the R^2 and t-ratios were the same) the inflation expectations and output variable were lower in each case. The foreign interest rate almost doubled with M2+. The most disturbing result, however, was the coefficient on the money surprise variable itself. The coefficient here and in the Levi-Makin model (M1) produced the expected negative coefficient, althought the size of the coefficient is smaller in the present model. The broader money suprise variable (M2+) shows a positive sign in each case, which contradicts the theory outlined earlier. This incorrect sign suggests model misspecification or misdirected theory.

Government Deficit

The deficit variable varied considerably in magnitude and in sign, depending on the specification of the model. In each case, however, the t-ratios were well below the critical level, allowing it to be rejected as a significant factor. As a result, there is little evidence of crowding out in a short-run.

International Capital Flows

International interest arbitrage, as expected, is a consistently large factor in the Canadian equation. In the after-tax models, the coefficients ranged from 0.4688 to 0.5001; in the pre-tax models they ranged from 0.9159 to 0.9577. Neumann (1977) reported a coefficient of 0.26 in his model using Canadian data during fixed exchange rates. The present study confirms this conclusion in a period of freer exchange rate movement, suggesting even that this dependency has strengthened. One implication of this result is that perfect capital mobility does not hold in the short run (as suggested by Makin (1978)).

The strength of this relationship also explains the low coefficient on the inflation expectations variable. Recall that in the reduced form equation of nominal interest rates, the denominator of this coefficient incorporates the sensitivity of international capital flows to the interest rate differential. The stronger the relationship, the greater the denominator, and the lower the coefficient. The results here confirm this relationship.

Inflation Expectations

The coefficients on the inflation expectations variable are generally lower but displayed a similar range of values, depending on the inclusion of taxes, to that generated in the Levi-Makin re-estimation.

Equation	Estimate	Upper Limit (One Tail)
(4-1)	0.13437	0.21152
(4-2)	0.24770	0.37382
(4-3)	0.09802	0.16141
(4-4)	0.18673	0.29707
(4-5)	0.13319	0.20286
(4-6)	0.24530	0.36865
(4-7)	0.09585	0.15693
(4-8)	0.18292	0.28968
(4-9)	0.12377	0.18469
(4-10)	0.24638	0.35207
(4-11)	0.10021	0.15606
(4-12)	0.20357	0.30171

All of the estimates are significant, while the range of values are all below 0.25. In fact, all of the upper limits of the stronger one-tailed test (at the 95% level) lie below 0.4. These low values allow the Fisher hypothesis to be rejected in each case, and are considerable evidence for the Mundell hypothesis, in which the nominal interest rate, in the short run, does not match the increase in inflation expectations, while the real interest rate makes up the difference.

The coefficients found in this model are not inconsistent with other interest rate studies. Feldstein and Eckstein (1970) estimated a coefficient of 0.23; Taylor's (1981) Treasury Bill study estimated 0.41, and Neumann's (1977) term deposit study 0.36.

All of these are close to the range of estimates of 0.096 to 0.247.

The sensitivity of the analysis to taxes can also be calculated in this model[1]:

Equations	$\beta_{\text{pre-tax}}$	$\beta_{\text{after-tax}}$	Difference
(4-1,2)	0.247737	0.13437	-0.00581
(4-3,4)	0.18673	0.09802	0.001786
(4-5,6)	0.24530	0.13319	-.00600
(4-7,8)	0.18292	0.095852	0.002067
(4-9,10)	0.24638	0.12377	0.012851
(4-11,12)	0.20357	0.10021	0.014494

The first estimates of the difference are very close to zero. Only the last two differences are greater, reaching 0.012851 for M1 and 0.014494 for M2+. The difference in magnitudes is most likely a consequence of model specification, as the estimated value changes after eliminating insignificant variables. In the event that the difference is in fact positive, the real balance effect will be negligible (the impact of a wedge of 0.015 is 0.015 points of nominal interest rates). The conclusion here, similar to that reached by Hoffman and Schlagenhauf, is that the inclusion of taxes does not alter the implications of the model results.

Measurement Error

Although measurement error was detailed earlier, a caveat must accompany these conclusions, because the unobservable nature of the relationship of the variables exposes the model to much potential error. Inflation expectations, unexpected money supply growth and real interest rates are not observable. This has a number of

[1]The average tax revenue as a percentage of GDP for the sample is 0.4700.

implications. First, the inflation expectations data is based on an *ad hoc* model. The Rational Expectations Hypothesis was used to generate the expectations data, in contrast to many other studies which incorporated survey data, or assumed a specific lag structure. The expectations therefore, do not necessarily reflect those of the 'street'. This is similarily applicable to expectations in money supply growth.

As inflation uncertainty was an outgrowth the expectations variable, the accuracy of this data is also dependent on the expectations model.

Second, taxes can be measured in a number of different ways. The effective tax rate measures total tax revenue as a percentage of GDP. Other measurements, also used in other models, could be the effect tax rate on investment income, or the weighted average of personal and corporate tax rates.

Third, the nominal interest rate used for estimation has a profound impact on the conclusions drawn. The conclusions reached above have attributed the real interest rate movements to Mundell and Tobin's real balance effect. However, this could as easily be explained by the Inverted Fisher Hypothesis (that the nominal interest is set by financial institutions, so that general, mostly non-bank, inflation expectations will affect only the real rate). Therefore, to restrict the relationship to Mundell and Tobin would be misleading.

Thus there are a number of ways in which this issue could be developed. As the model results demonstrated, the coefficients, especially on the inflation expectations variable, were sensitive to the specification of the model. In order to account for this, a range of results were reported. However, other methods of generating data may present further variation in results. Thus further development of the theory would include sensitivity analysis incorporating the model of expectations, the measurement of taxes, and the nominal interest rate.

In order to pursue further this relationship between inflation expectations and real rates, a more sophisticated model would incorporate real balances in an effort to separate Mundell and Tobin from the Inverted Fisher Hypothesis.

Finally, the model of international flows of capital was somewhat simplistic, by incorporating only the interest rate differential. A more complete model would incorporate the purchasing power parity as a separate influence on real balances.

Conclusion

The models tested in this chapter show that, at least in an open economy setting such as Canada's, the Levi-Makin model is under-specified by not accounting for some structural economic variables. In the Levi-Makin re-estimation, the model was hampered by serial correlation when tax effects are modelled. The inclusion of the deficit and foreign interest rate reduced the serial correlation to the extent that it no longer needed attention.

The primary factor was the flow of international capital, which proved to be strongly significant in all of the equations. This variable improved the overall fit of the specification and accounted for the serial correlation by raising the Durbin Watson statistic. It also displayed a more prominent role than that seen in previous studies.

The rejection of government deficits as an explanation of interest rate movements has two implications. First, government spending has little effect on the level of nominal interest rates, contradicting the 'crowding out' theory. This may be primarily due to the second implication, which is a confirmation of the role of international capital movements. Thus a reduction in the supply of loanable funds due to government borrowing is offset by the supply of funds from external sources.

The more fully specified model confirms the conclusion regarding inflation expectations reached in the last chapter: that the Mundell hypothesis cannot be rejected. The Mundell effect, according to pre-tax interest rate models, accounts for approximately 75 to 82 percent of the effect on real interest rates of a change in inflation expectations. Consequently, in the short run an increase in inflation expectations will decrease real interest rates. After accounting for the Darby effect, the relationship is even weaker, reducing the nominal interest rate movements to as little as 0.1 of the change in inflation expectations.

Confidence in the two step estimation procedure followed here, though, must be limited. The primary reason is that the inclusion of generated regressors raises the potential of biased and inefficient regressors. Problems also exist with stationarity[1]. Therefore, a more comprehensive approach to the estimation (i.e. diagnostic tests for stationarity and model specification) would increase the strength of the inferences drawn from the model.

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[1]Oxley and McAleer (1993).

Chapter 5: Conclusion

The behaviour of real interest rates is gaining considerable importance in recent times, especially as inflation has set root in the economic process. This behaviour is important because it affects numerous activities, ranging from investment decision-making to the allocation of resources. Understanding this behaviour will allow more effective control over its movement, facilitating the process of stabilizing real interest rates and productive monetary and fiscal policy. Over the past twenty years real rates, measured *ex-post*, have varied considerably.

Recent economic theory has isolated the role of expectations as a factor in decision and policy-making processes. Sixty years ago Irving Fisher formalized the relationship between expected inflation rates and interest rates. The Fisher identity defines nominal interest rates in terms of two unobservable variables: real interest rates and inflation expectations. The question which remains unresolved is the relationship between the two unobserved variables. Fisher postulated that the real interest rate is constant, because real rates are unaffected by inflation expectations. However, competing theory gives numerous alternative points of view, none of which has been fully accepted. This may be mostly due to the difficulty in measuring the variables, as the measurement defines the relationship. Recent economic literature has produced a range of values, often contradictory in conclusion.

These recent studies have shown, though, that the specification of the model is fundamental to the strength of the conclusion. The evolution of models has progressed from simple Fisher equations to general equilibrium models. This process has underlined the importance of other macroeconomic variables in the relationship. The most significant variables are money supply growth, government deficits, economic activity, inflation uncertainty and international financial flows.

In order to pursue this question in a Canadian setting, a reduced form model of the nominal interest rates incorporating these variables was developed, the results of which can be extrapolated, through the Fisher equation, to the real interest rate theory.

A number of conclusion can be made from the results. First, the role of unexpected money supply changes and economic output were validated. Unexpected money supply changes affect real rates through a real balance effect in the short term. Thus unanticipated increases in the money supply increase the real supply of loanable funds until prices increase sufficiently to return real balances to normal. Economic activity increased real rates in the short term as an increased demand for loanable funds raised the market-clearing nominal rate, before inflation expectations have time to adjust to inflation arising through the Phillips curve.

Second, inflation uncertainty was not validated, a conclusion consistent with most previous studies. This result may not be a useful indication of its role however, as inflation uncertainty has offsetting effects. Recall Friedman's theory that higher levels of inflation (uncertainty) delays investment decisions, reducing economic activity and the demand for funds, reducing the real rate. At the same time, suggested Hoffman and Schlagenhauf, savings may decrease sufficiently to reduce the supply of funds and reduce the change in real rates. The ability to reject the role of inflation uncertainty in the model can mean that either both theories have negligible effects, or both are equally significant.

Third, international capital movements proved to be a strong influence. The absence of instantaneous purchasing power parity, in the short term, has developed a role for these cross-border capital movements in the determination of nominal interest rates. However, the supply of funds is sufficiently elastic to prevent any crowding out of domestic investment as a consequence of government borrowing. In this respect the elasticity has a stabilizing effect on real interest rates, since unexpected government deficits do not put downward pressure on real interest rates.

Lastly, the rejection of the Fisher hypothesis indicates that inflation expectations do indeed affect the real interest rate in the short term, through the real balance effect described by Mundell. This is primarily due to a real balance effect which causes less non-interest bearing assets to be held. A compensating increase in savings does not offset the real balance over the short term perhaps due to the fall in imports, as suggested by Makin. Another strong factor is the "Inverted" Fisher effect, which refers to price setting by the Bank of Canada. A low coefficient on the inflation expectations variable can also signify the rigidity of nominal interest rates causing the *ex-post* real rates to buffer the changes in expectations. This implies not only that rates are volatile in the short run, but also that the formation of expectations has less of an influence than Fisher claimed.

This last conclusion has significant implications for fiscal and monetary policy, since the weaker role of expectations increases the relative significance of more manageable (and observable) macroeconomic variables, such as economic activity, money supply growth and international capital movements. The result obtained here become an insight into the dynamics of the current 'sluggish' recovery, in which low inflation expectations have been accompanied by slow economic growth. This can be explained in part by the decrease in inflation expectations, which cause the real interest rate to rise. Because investment decisions are based on real interest rates, this causes a dampening in economic activity.

Care should be taken in the case of monetary policy, however, as much of the weakened relationship between real interest rates and inflation expectations may be made up in the longer term, as prices have time to adjust to changes in money balances and expectations. Therefore a question that remains unanswered is the longer term dynamics of real interest rates, and the speed at which the real interest rate returns to its normal level.

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Appendix A: A More Fully Specified Model

The model is a standard IS-LM model supplemented by an aggregate supply curve and a balance of payments restriction.

The IS equation holds only investment and savings as endogenous, and includes inflation uncertainty as outlined in Levi and Makin (1979).

$$I = f(r^*, \sigma^2)$$

$$S = f(r^*, y, m\text{-}p)$$

Substituting these into the aggregate expenditure equation (I-S) = (T-G) - (X-Z) and solving for y leaves the IS function:

(1) $y_t = a_0 - a_1 r_t^* + a_2 \sigma_t^2 + a_3(ms_t - p_t) + a_4 D_t + a_5(X_t - Z_t)$

The LM equation is standard, where in equilibrium money supply is equal to money demand. Money demand is endogenous:

$$\frac{M^d}{P} = L(Y, i^*)$$

The LM function is:

(2) $(ms_t - p_t) = b_0 + b_1 y_t - b_2 i_t^*$

The Balance of Payments equation is a short-run model in which purchasing power parity need not hold. Short-run international capital movements can be characterised by the covered interest differential, i^*-i^f-e^e. The value of these financial assets is the capital account, whose deficit, assuming freely flexible exchange rates, is equal to a current account surplus. When the capital account plus the current account is zero the Balance of Payments is in equilibrium. In this model, the current account is represented by net exports, or (X-Z):

$$(X\text{-}Z) + \text{Net Capital Inflows} = 0$$

$$\text{Net Capital Inflows} = f(i^* - i^f - e^e)$$

(3) $(X_t - Z_t) = -c_0(i_t^* - i_t^f - e_e^t)$

The aggregate supply schedule is that used in Hoffman and Schlagenhauf (1984). In this equation output is a function of unanticipated changes in the money supply. Barro (1980) formalized this idea as a short-run phenomenon by considering that positive unexpected money growth is not reflected in expected future prices but in actual future prices. Thus the expected price response is larger than the actual price response, negating the real balance effect which stimulated output originally. This money surprise-output relationship is, as a consequence, a short-run phenomenon, since in the long run prices alone adjust to the monetary phenomenon and real output retreats to its formal level. The aggregate supply function is:

(4) $y_t = d_0 + d_1(m_t - m_t^e)$

Two identities are required to round out the model. The first identity simply defines the after-tax interest rate as:

(5) $i_t^* = i_t(1-\tau)_t$

The second identity is the Darby (1975) formulation of the Fisher equation, in which the after-tax nominal interest rate is the sum of the after tax real interest rate and the anticipated rate of inflation.

(6) $i_t^* = r_t^* + \pi_t^e$

Since taxes can introduce a wedge between the nominal and real interest rate, pushing nominal rates up and real rates down, taxes may mask other forces influencing real rates in the other direction, specifically the Mundell hypothesis. By accounting for taxes in the structure of the model, the estimated coefficients are more accurate indications of the Fisher effect. While Darby found that

$$\frac{\partial i}{\partial \pi^e} = \frac{1}{(1-\tau)}$$

and the Mundell hypothesis indicates that

$$\frac{\partial i}{\partial \pi^e} < 1$$

which obviously act in opposite directions, the two views can be reconciled using after-tax interest rates, according to Equation (5). Thus for Fisher to be correct and real interest rates to be insulated form inflation expectations,

$$\frac{\partial i^*}{\partial \pi^e} = 1$$

This term will be less than one for the opposing hypothesis.

The dependent variable in the reduced-form estimated question must therefore be observable and be expressed in after-tax terms. By equating equation (1) and (4) and substituting in equations (2) and (3) one can solve for the after-tax real interest rate r*. Using equations (5) and (6) the real rate can be expressed in terms of the pre-tax nominal rate, which is observable. Solving for i:

$$i = \frac{a_0 - d_0 - a_3 b_0}{B} + \frac{a_1}{B}\pi^{e_t} + \frac{a_2}{B}\sigma^{2_t} - \frac{a_3 b_1}{B} y_t + \frac{a_4}{B} D_t - \frac{d_1}{B}(m - m^e)_t + \frac{a_5 c_0}{B}(i^f + e^e)_t$$

Where:

$$B = a_1 + a_5 c_0 - a_3 b_2$$

Appendix B: Other Models

Levi and Makin

The model in Levi and Makin (1979) and later models are essentially the same. That introduced in 1979, and followed in Chapter 4, is presented here[1]. From the general equilibrium model comprised of four structural equations (IS, LM, the bond market and the labour market), the bond market is set to zero:

$$I\ (r, \sigma) - S\{y\ (N, \sigma), M/P\} = 0$$

$$L\ \{y\ (N, \sigma)), i, P\} - M = 0$$

$$P\ y'\ (N, \sigma) - W\ (P, N\sigma) = 0$$

Putting each of the equations into elasticity form, where Σab signifies the cross-elasticity between a and b, and assuming $\Sigma \sigma^2 y = 1$, constant returns to labour, and that the marginal product of labour is unaffected by inflation uncertainty (rates of change are expressed with bars overhead):

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[1]This model is a short representation, for expository purposes, of that outlined in the appendix of Levi and Makin (1979), which details the model further.

$$\sum Ir\bar{r}+(\sum i\sigma^2-\sum y\sigma^2)\bar{\sigma}^2-\sum yN\bar{N}-\sum \sigma^2 m\bar{M}+\sum \sigma^2 m\bar{p}=0$$

$$\sum Ly\sum yN\bar{N}+\sum Ly\sum y\bar{\sigma}^2+\sum Li\frac{\partial i}{i}+\sum Lp\bar{p}-\bar{M}=0$$

$$(1-\sum wp)\bar{p}-\sum wN\bar{N}=0$$

$$\frac{\partial r}{r}=\frac{\partial i}{i}-\frac{\partial \pi^e}{r}$$

Assuming $\Sigma yN=\Sigma Ly=\Sigma Lp=\Sigma wN=1$, and taking M=0, they are able to solve for $\frac{\partial i}{\partial \pi^e}$. However, their derivation of the equation to be estimated is not a reduced form equation derived from these four structural equations, but is an ad hoc decision based on the individual theories (for instance the Mundell and Friedman effects). This would account for their eagerness to add and drop variables in the regression equation. The final reduced form equation includes the same variables as that estimated in Chapter 4.

Hoffman and Schlagenhauf

The Hoffman and Schlagenhauf model allows for changing real interest rates:

$$y_t=a_0-a_1r_t^*+\sum_{j=0}^{\infty} a_{2j}\sigma_{t-j}^2+\sum_{j=0}^{\infty} a_{3j}(m^s-p)_{t-j}+a_4D_t+a_5(X_t-M_t)+e_{1t}$$

$$(m_t^s-p_t)=b_0+b_1y_t-b_2i_t^*+e_{2t}$$

$$(X_t-M_t)=-c_0(i_t^*-i_t^f-e_t^e)+e_{3t}$$

$$y_t=d_0+d_1(m_t-m_t^e)+e_{4t}$$

$$i_t^*=i_t(1-\tau_t)$$

$$i_t^*=r_t^*+\pi_t^e$$

where:

y_t	Logarithm of real output
r_t*	real interest rate
σ_t^2	inflation uncertainty
ms_t	logarithm of the money supply
p_t	logarithm of the price level
i_t*	nominal, after-tax interest rate
i_t	pretax interest rate
m_t	rate of growth in the money supply
τ_t	tax rate
π_t^e	expected inflation rate for time t, mate at end of time t-1

One easily notices that the model framework is similar to the one presented in Appendix B. The major difference lies here in the missing equation describing short-term international capital flows (Equation (4) in Appendix B), and the missing deficit variable in the Hoffman Schlagenhauf model.

Reducing the above system into one single equation for the real interest rate:

$$r_t* = \mu + \sum_{j=0}^{\infty} a_j \pi^e_{t-j+1} + \sum_{j=0}^{\infty} b_j (m - m^e)_{t-j} + \sum_{j=0}^{\infty} c_j \sigma^2_{t-j} + \sum_{j=0}^{\infty} d_{jr_{t-j}} * + \sum_{j=0}^{\infty} f_{je_{t-j}}$$

where it is a linear combination of the disturbance terms in the first three equations.

Appendix C: List of Variables

For the model developed in Chapter 2, the variables and their definitions are listed below for easy reference:

y_t	logarithm of real output
r_t^*	real interest rate
σ_t^2	inflation uncertainty
m_t^s	logarithm of money supply
p_t	logarithm of the price level
i_t^*	nominal, after-tax interest rate
i_t	pre-tax interest rate
D_t	deficit in government spending
$X_t - Z_t$	net exports
i_t^f	foreign interest rate
f_t^e	exchange rate forward premium or discount
m_t	rate of growth in the money supply
m_t^e	expected money supply growth rate
τ	tax rate
π_t^e	expected inflation rate for time t, at end of time t-1
u_{it}	error term for non-identity models

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