THE FORECASTING POWER OF FORWARD INTEREST RATES AND THE SLOPE OF THE YIELD CURVE IN THE CANADIAN TREASURY BILL MARKET

by

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ABSTRACT

This thesis analyzes the power of the forward interest rate and the slope of the yield curve in predicting future interest rates, by comparing their predictive ability against that of the martingale model. It also examines whether an adjustment for a risk premium improves the predictive power of the forward and the slope. The premium is calculated on several different bases, the most effective of which is based on an ARIMA model. The data in this thesis are 3- and 6-month Canadian Treasury Bills over the period January 1960 to March 1988.

The finding of this thesis is that the forward rate and the slope of the yield curve are little better than the martingale model at forecasting future interest rates. While an adjustment for a risk premium improves their predictive power, the improvement is only marginal.

This finding has important implications to monetary authorities, who may seek to influence interest rate movements, and to private (or public) sector borrowers or lenders who might attempt to extract information about future interest rates from the forward or slope.

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1.0 Introduction and Literature Review

Interest rates, with their far-reaching effects on individuals and global economies, have been the subject of considerable study for many years. Levels of interest rates and changes in those levels have a profound effect on the world around us; they affect a wide spectrum of our lives, from broad world trade patterns to national employment prospects to an individual's ability to purchase a house. An understanding of interest rate behaviour is also significant from a policy perspective, because it will affect the manner in which monetary authorities choose to conduct and implement monetary policy actions, and it will help to determine how monetary action aimed at short-term interest rates will affect long-term interest rates. In addition, it will provide insight both about and for borrowers and lenders in the financial markets who must regularly make financing decisions; the borrowers providing the demand for funds and seeking to minimize interest cost, while the lenders provide the supply of funds and seek to maximize interest earnings. It is no surprise, then, that many minds have turned their attention to the study of interest rate phenomenon.

1.1 Description of the Interest Rate Family

A reference to interest rates in a general sense will typically refer to a subset of the family of interest rates. The interest rate family can most simply be segregated by risk and term. For any particular term, the higher the risk associated with a transaction, the higher will be the applicable interest rate. (This is to encourage an investor to enter into a transaction where he runs some risk of not receiving his interest and principal as contracted.) Similarly, for any particular risk level, the interest rate will vary, depending on the term of the transaction. Typically, a longer-term transaction will be at a higher interest rate than a shorter-term transaction.[1] The higher rate is normally viewed as a compensation to the investor for risk associated with a longer-term transaction.[2]

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[1]This will not always be the case and, in fact, it is the purpose of the thesis to determine how the relationship between long- and short-term interest rates develops.

[2]This also is the subject of some debate and is one of the key concepts considered in this thesis.

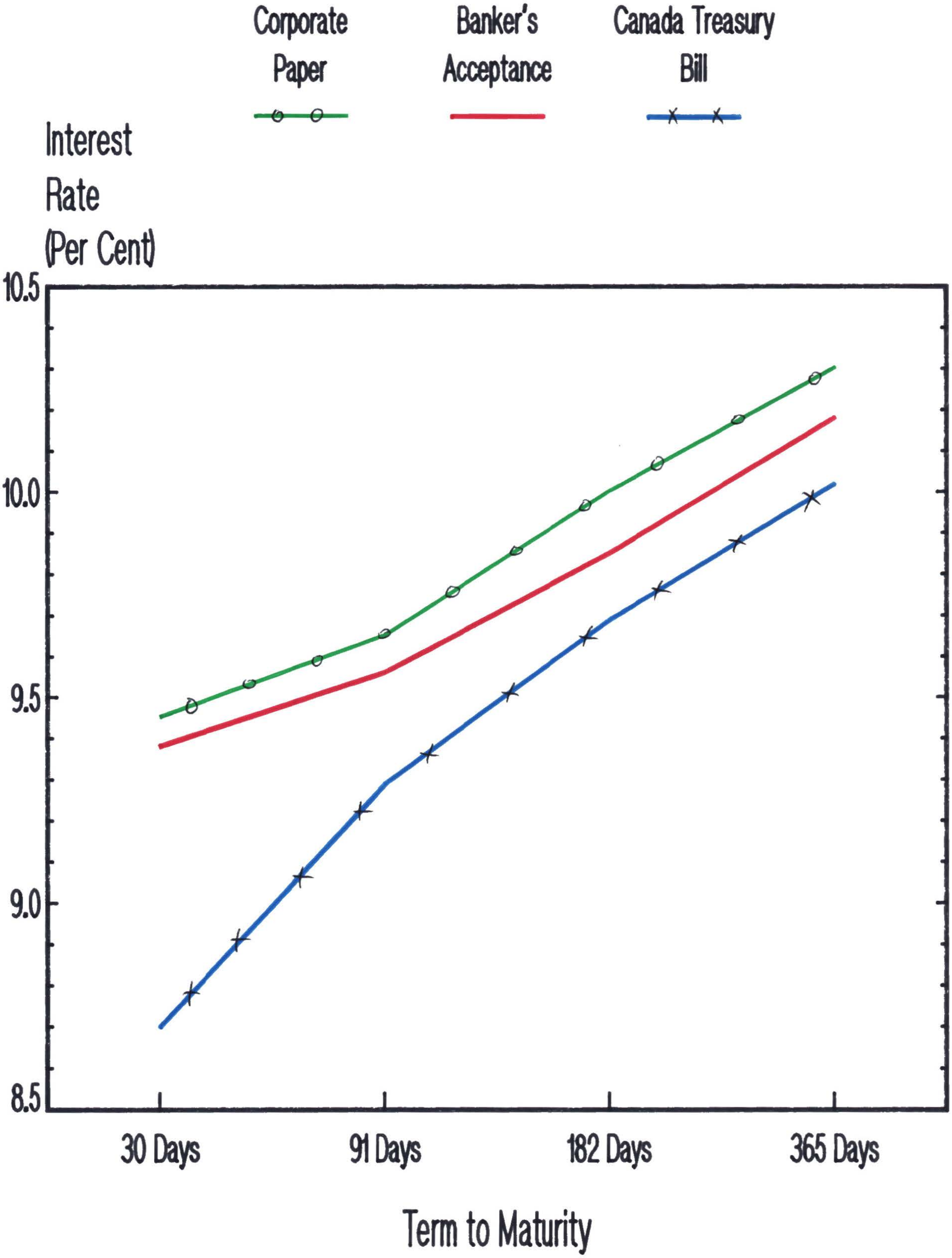
The relationship between interest rate levels and various risk levels and term structures can be most simply described with reference to the yield curve which, in turn, can be best described graphically, as shown in Figure 1.

The vertical axis marks the yield (interest rate) associated with a particular security, while the term to maturity of the security is represented on the horizontal axis. An individual yield curve therefore represents the spectrum of interest rates associated with securities of a particular risk, where the different rates are a function of the term to maturity.

Figure 1 also illustrates that securities of different risks will have different yield curves. As mentioned earlier, the larger the risk associated with a security, the higher the interest rate and hence the higher the yield curve. In this example, the yield curve for Canadian corporate commercial paper is 30 - 80 basis points higher than for

Figure 1

YIELD CURVES
As at August 4, 1988



Government of Canada Treasury Bills, depending on term, based on August 4, 1988 data.[3]

1.2 Theoretical Models

A number of attempts have been made to determine what factors determine interest rates and how these can be modelled to provide forecast estimates of future interest rates. The theories fall into four basic categories: (1) pure expectations, (2) liquidity preference, (3) market segmentation; and (4) preferred habitat.

The pure expectations hypothesis suggests that long-term interest rates are simply a function of current and expected future short-term rates. The rationale for this hypothesis is that investors will be indifferent between holding a long-term asset with a particular yield to maturity and a shorter-term asset with a different yield to maturity, provided that the differential accurately reflects their views on future interest rates. In other words, if

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[3]One basis point is equivalent to 1/100 of one percentage point.

long-term rates depend on current and expected short-term rates, then the yield curve will reflect the market view of future rates. If rates are not so related in the market, investors will change their portfolio mix (thereby affecting the price, and consequently the yield, of the assets in the market) until the postulated relation holds. Thus, long-term rates will depend on current and future interest rates. (This process will be described more fully in Section 1.3.)

The liquidity preference model, which was first clearly articulated by Hicks (1939), accepts that long-term rates are largely a function of expected short-term rates, but suggests that a premium is also built into the return on longer-term securities -- and the longer the term of the security, the larger the premium. The theory hinges on the fact that a short-term security will have a known, risk-free return, while a longer-term security will not. As an illustration, consider the following two securities: a one-period (short-term) security and a two-period (long-term) security. The return on the one-period security is risk-free because the principal is repaid at the end of the first period and hence is known.

The two-period security, on the other hand, carries some risk because the principal is not repaid at the end of the first period. If it is liquidated before maturity (e.g. at the end of period one), it has the potential for a capital gain or loss. Because of this potential loss, we would expect an investor to prefer to hold a short-term asset, unless the longer-term asset carries a premium sufficient to compensate for risk.

The market segmentation hypothesis, stressed by a number of authors, including Culbertson (1957), suggests that both borrowers and lenders have strong preferences for particular maturities. While this may be attributable to institutional or regulatory conditions, as well as to individual idiosyncrasies, the result is that these definite preferences will lead to particular investment and borrowing decisions being constrained to certain maturities. This will lead to different markets being developed for different maturities, each with their own supply and demand schedules.

The preferred habitat theory, developed by Modigliani and Sutch (1966), is a synthesis of all three of the above hypotheses. It accepts that the yield structure is based principally on the equality of expected returns, modified by risk premiums, but suggests that the risk premium will depend on an individual investor's preferred habitat. While the Hicksian model assumes all investors wish to invest in short-term assets unless they are adequately compensated for risk for investing longer term, Modigliani and Sutch point out that some investors, who have longer-term interest return objectives, may prefer to purchase long-term assets where their long-term return on investment is known, over short-term assets where they run reinvestment risk. These investors would therefore require a premium for shortening term rather than for lengthening term. According to Modigliani and Sutch, then, the relationship between short- and long-term rates will depend on the expected change in long-term rates, the supply of short- and long-term assets, risk aversion profiles and transactions costs.

1.3 Expectations Models and the Yield Curve

Historically, the evidence in the literature has been most supportive of the expectations-based hypothesis.

It is this hypothesis which underlies the investigations in this thesis.

The expectations hypothesis implies that the yield curve contains predictive information about the market's expectations of the future course of interest rates. The objective of this thesis is to consider how expectations, in tandem with information contained in the yield curve, can be used to predict future interest rates.

The thesis will investigate the predictive ability of two types of information contained in the yield curve (the slope of the yield curve and the forward rate). While the details will be described more fully in Section 2, the basic concept is that an investor will be indifferent between a one-period security and a two-period security, provided that the differential in yield represents his view on future interest rates. Thus, market equilibrium will ensure that the differential in market interest rates represents market expectations of future rates. This can be denoted as

$$[1] \quad (1 + {}_tr_2)^2 = (1 + {}_tr_1)\,(1 + E_t[{}_{t+1}r_1])$$

where

$_tr_2$ is the current yield on a two-period security

$_tr_1$ is the current yield on a one-period security

$E_t[_{t+1}r_1]$ is the current expectation of the next-period yield on a one-period security.

This thesis then postulates that i) the forward rate and ii) the slope of the yield curve are good proxies for expected future interest rates.

In the first case, where the forward rate is assumed to represent current expectations,

$$[2] \quad _{t+1}F_{1,t} = E_t[_{t+1}r_1]$$

where

$_{t+1}F_{1,t}$ is the current implied one-period forward rate applicable to the next period

E is the expectations operator.

Substitution of Eq. [2] into Eq. [1] yields

$$[3]\quad (1 + {}_tr_2)^2 = (1 + {}_tr_1)\,(1 + {}_{t+1}F_{1,t})$$

In the second case, where the slope of the yield curve is assumed to represent interest rate expectations, the hypothesis suggests that the slope (the existing differential between one- and two-period rates) represents the change in interest rates expected by the markets, which can be denoted as

$$[4]\quad {}_tr_2 - {}_tr_1 = E_t[{}_{t+1}r_1] - {}_tr_1$$

This, in turn, can be rewritten as

$$[5]\quad {}_tr_2 = E_t[{}_{t+1}r_1].$$

1.3.1 Predictive Power of the Forward Rate

The hypothesis that forward rates of interest implicit in the yield curve should provide an accurate reflection of the market's expectation of future rates has led to a series of tests about the predictive ability of forward rates vis-a-vis future interest rates.

Many of the results have been disappointing. Hamburger and Platt (1975), for example, using U.S. Treasury Bill data from 1961 to 1971, found that while the forward rate had some predictive ability it was, in fact, no better than the forecast ability of the naive view that interest rates would not change. Fama (1976) also tested the predictive power of the forward against a martingale model, which predicts that a future interest rate will be the same as the current rate, and that any error will be purely random. His data was also based on U.S. Treasury Bills but covered the period 1953 to 1974. Overall, he found that the forecast ability of the forward rate could not perform as well as a martingale model.

1.3.2 Predictive Power of the Slope of the Yield Curve

Shiller, Campbell & Schoenholtz (1983) tested whether the slope of the yield curve provided a good forecast of future interests using U.S. data for a 25-year period beginning in 1955. They found, however, that the hypothesis of the predictive power of the slope of the yield curve was not supported by the evidence. In particular, there were extended periods of poor forecast performance and large forecast errors throughout.

Mankiw (1986) undertook a similar analysis, but expanded the study to be cross-sectional. He did this by testing the slope of the yield curve hypothesis using data for four different countries: the United States, Canada, the United Kingdom and Germany. He hoped that by interpreting divergent national experiences, the general applicability of the hypothesis would be put to the test. These results were also disappointing. For instance, his work revealed that a steeply-sloped yield curve could successfully forecast a rise in short-term rates on occasion, but it failed to forecast a rise in long-term rates: "In fact, to the extent that the slope of the yield curve forecasts long rates at all, it does so in the direction opposite to the one predicted by the theory." This phenomena, observed much earlier by MacCaulay in 1938, was also observed by Shiller, et al.

All of this suggests that the slope of the yield curve has, in the historical context, proven to be no better in forecasting future interest rate movements than has the forward rate.

1.3.3 Time Series Forecasting

The lack of success of both of these basic hypotheses (forward rate and slope of the yield curve) has led to an important development in the study of interest rate forecasting; this is the increasingly sophisticated attempts to determine how past movements in interest rates can be used to formulate models which will provide predictive ability about future interest rate movements. In a sense, this approach was referred to earlier with respect to Fama's testing of the martingale model. The martingale, however, considers only one past observation.

In an alternative test form, Hamburger & Platt (1975) estimated several equations using a variety of lag structures, including up to a two-year lag on quarterly data (e.g. eight lags). In this case, they tested for serial correlation in changes in the

3-month Treasury Bill rate to determine whether there existed a systematic relationship between current rates and past rates which, in turn, would help formulate expectations about future rates. While the results of their analyses were not significant, the testing for serial correlation in past observations has become a fundamental component of recent research on the subject of forecasting future interest rates.

Another form of estimation, which was employed by Nelson (1970), was on the basis of a technique developed by Box and Jenkins (1970) known as ARIMA modelling. This process also considers one or more lagged observations of a univariate time series. Nelson had some success in using this process, described in detail in Appendix IV, to achieve useful results.

1.4 Premiums in Interest Rates

As described, the empirical research has had mixed results with respect to the ability of the forward rate and the slope of the yield curve to predict future interest rates and the evidence is inconclusive. A major refinement to the basic hypothesis is therefore introduced by suggesting that the forward rate contains not only the market's expectation of future interest rates, but also a premium. A premium can be incorporated into tests of both the forward rate and the slope of the yield curve. In terms of the forward rate, Eq. [2] would be adjusted to

$$[6] \quad {}_{t+1}F_{1,t} = E_t[{}_{t+1}r_1] + P_t$$

where

P_t is the current premium.

In terms of the slope of the yield curve, Eq. [5] would be adjusted to

$$[7] \quad {}_t r_2 = E_t\,[{}_{t+1}r_1] + P_t.$$

Testing for the existence of a premium has important implications for the theoretical models outlined in Section 1.2. For example, the existence of a premium is inconsistent with the pure expectations hypothesis, in which interest rates are assumed to reflect only expectations of future rates. The expectations model would therefore suggest that the premium is zero.

1.4.1 Term (Constant) Premium

The liquidity preference model, described earlier as attributable to Hicks, supports the theory of the term premium. The concept of a term premium is based on the suggestion that long-term maturities carry a premium over shorter-term maturities to compensate investors for risk associated with uncertain returns caused by possible capital losses on longer-term securities.

Under the liquidity preference model, the premium increases monotonically over term; that is, the longer the term, the higher the premium. For any given term, however, the premium would be constant over time. The liquidity preference model therefore suggests that the premium would be greater than zero, positive and constant over time.

1.4.2 Variable Premium Based on Interest Rates

Several authors have suggested that the premium will vary over time. Studies assessing what factors go into the composition of that premium, as well as how it can be modelled or estimated, proliferate throughout the literature.

One of the most conceptually appealing is that the premium will vary depending on the level of current interest. There are, however, two diametrically opposite points of view on this issue.

Keynes (1930), for example, believed that interest rates would tend toward a norm; that is, if interest rates were unusually high or low, they would trend back toward their norm. If an investor believes this to be the case, his premium would be inversely related to the current level of interest rates.

Kessel (1965), on the other hand, suggested the relationship between the premium and interest rate levels is positive. His explanation is that short-term bonds can be considered to be money substitutes and that the premium will therefore rise with the interest rate level to reflect the higher opportunity cost of holding money.

Mankiw (1986) postulated that the premium could be related to volatility in interest rates. This would suggest that the premium varies over time -- specifically that it would increase during periods of large movements in interest rates, since a longer-term investment (more than one period) is riskier in more volatile markets. Assuming investors are generally risk-adverse, as they are normally viewed to be, then during high volatility periods they would have to be induced into longer-term maturities by the compensation of a larger premium. When Mankiw tested this hypothesis by regression, however, it was not supported by his data.

In addition, while Shiller, et al., (1983) found interest rate volatility to be useful in identifying the term premium, the overall ability of the slope of the yield curve in predicting future interest rates was very little changed by the inclusion of the premium.

Fama (1976), on the other hand, had some success in associating the premium with interest rate uncertainty and found a meaningful relationship, especially for longer maturities. It might be argued that his analysis was more successful because of the term of the instruments he investigated. Fama worked with varying term Treasury Bills, but only to a 6-month term, while both Shiller, et al., and Mankiw used 30-year bonds. This possible explanation loses some credibility, however, when one considers that Park (1982) replicated Fama's United States' analysis with Canadian data, to no meaningful end.[4] More significantly, Park's analysis not only concluded that the premium in the forward rate is constant, but also that it is close to zero.

Given the controversy and varying results of interest rate volatility as an explanatory measure of a term premium, this subject is one which will be explored in more depth in Section 3.

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[4]Strict comparisons are not possible because the two data sets were not directly comparable. Park used the last observation in each month of a weekly auction, while Fama used the last trading day observation of each month.

1.4.3 Variable Premium Based on Exogenous Economic Data

Mankiw (1986) tested several other hypotheses. For example, he suggested that an investor may prefer a diversified portfolio to hedge against risk. He further postulated than long-term bonds could provide hedges against changes in other market returns and tested against consumption patterns and stock market returns, as well as postulated and tested for premiums based on changes in asset supply -- all without success.

Shiller (1983) postulated that the premium could be related to money supply announcements. These announcements provide some insight into both the strength of the domestic economy and how the Federal Reserve may be attempting to effect interest rate adjustments. He also points out, however, that it can be nearly impossible to interpret money supply figures because they can give different signals depending on the stage of the economic cycle, the Federal Reserve's announced money growth target ranges and the velocity of money. For example, when the economy is weak and an announcement of a large money supply increase is made, money-market players will assume the Federal Reserve is embarking on a loose monetary policy and interest rates will be expected to fall. On the other hand, a large money supply increase during a strong economic period may lead to fears of inflation and consequently higher interest rates. Thus, the same money supply announcement could have diametrically opposite results, depending on the economic and financial environment. In any event, Shiller's analysis indicated that incorporating money supply announcements did not improve the predictive power of the slope of the yield curve.

Hamburger & Platt (1975), on the other hand, found that inclusion of money supply announcements in their regression results did, in fact, improve the predictive power of the forward rate.

Nelson (1970) attempted to relate the premium in forward rates to the level of interest rates and business confidence. While his empirical evidence supported his hypothesis, the more noteworthy element of his study was his relatively novel approach to the problem of determining market expectations of future rates. The use of ARIMA modelling, such as was used by Nelson, will form a substantial part of later sections of this paper.

1.5 Summary of Special Considerations

The foregoing has illustrated that the evidence with respect to the predictive ability of information contained in the yield curve has been mixed and, in some cases, an analysis has proven successful for one author but not for another. In the latter case, there are a number of possible explanations.

For example, an experience in one country may not be directly transferable to another country. Regulatory processes and market practices differ from country to country with respect to banking and interest rate policy. These can distort not only developments in interest rate patterns, but also market expectations.

In addition, a study which uses commercial interest rates may not be comparable to an analysis using government (e.g. Treasury Bill) rates.[5] Commercial

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[5]For example, Modigliani and Sutch (1966) and Nelson (Footnote Continued)

rates depend not only on the overall interest rate environment, but also on such factors as "credit scares" and liquidity concerns. When failures of banks and near-bank financial institutions are relatively widespread, as has been the case in recent years in Canada and the United States, investors normally exhibit a "flight to quality" and seek to invest in government securities rather than private-sector commercial paper. In such an instance, commercial paper rates rise, while Treasury Bill rates fall. Similarly, in periods of extreme interest rate volatility, investors often prefer to position themselves in more liquid government securities. This will enable them to quickly reverse a position should interest rates make a sudden move. Again, commercial rates and government rates may move in opposite directions -- or, if in the same direction, by different degrees.[6]

(Footnote Continued)
(1970) used commercial rates, while Park (1982, Fama (1976, 1984) and Shiller, Campbell and Schoenholtz (1982) used government Treasury Bill or bond yields.

[6]As an illustration, 3-month Canadian corporate commercial paper rates in Canada moved in the opposite direction to 3-month Canada Treasury Bill rates on 25 out of a possible 382 occasions (6.5 per cent), based on Thursday
(Footnote Continued)

Another possible explanation is that some studies define six months as the longer-term rate, while other studies use 30-year bond rates. Analyses which differ in this way will clearly not be comparable.[7] Six-month rates, while at the longer-term end of money market securities, are nonetheless short-term instruments. History has shown that money market rates and longer-term bond rates frequently move in different magnitudes and occasionally even in different directions. For example, based on weekly observations, the 30-day rate has moved in a different direction than the 20-year rate on 67 of a possible 187 occasions between January 1, 1985 and August 2, 1988. It is typically suggested that this occurs because money market rates are driven by central bank intervention and exchange rates, while bond rates are driven more by long-term prospects for economic growth and government fiscal positions. Given the different driving forces behind these

(Footnote Continued)
observations from October 1980 to August 1988 provided by the Canadian investment dealer ScotiaMcLeod Inc.

[7]This will be especially true for very long terms. Malkiel and Kane (1969), for example, found that expectations-based models were not supported by the evidence beyond a 15-month time horizon.

rates, it is not surprising that they behave differently and that analyses will not be directly comparable.[8]

Finally, structural changes over time may exert different effects on the determination of interest rates. For example, periods of price control or a change from a fixed to floating exchange rate could have a dramatic (and previously unexpected) effect on interest rates.[9]

Notwithstanding these possible qualifiers which could help explain the wide divergence of empirical findings, it must be admitted that the evidence to date has been disappointing with respect to the

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[8]The expectations hypothesis is not consistent with long- and short-term rates moving in different directions unless the divergence is the result of a change in a time variable premium. Given that history shows these rates do, in fact, sometimes move in opposite directions, this provides support for the concept of a time variable premium.

[9]An example of such an occurrence is with respect to the evidence as it pertains to the extrapolative-regressive interest rate expectations formation hypothesis of Modigliani and Sutch (1966). Empirical estimates yielded evidence consistent with their hypothesis until the mid-1970s, but not beyond.

predictive ability of forward rates or the slope of the yield curve.

The virtually unanimous finding that interest rates cannot be predicted well with reference to the forward rate or the slope of the yield curve might lead one to wonder why these theories have not been discarded. They would appear to have such conceptual merit, however, that there is evidently a great reluctance to do so. To use an analogy of Shiller, Campbell and Schoenholtz (1983):

> It is uncanny how resistant superficially appealing theories in economics are to contrary evidence. We are reminded of the Tom and Jerry cartoons [where, Tom, the villainous] cat may be buried under a ton of boulders ... or flattened by a steamroller. Yet seconds later he is up again plotting his evil deeds. (p. 175)

Another important consideration is that test results may suggest that rejection of a theory is appropriate, but the theory cannot be rejected because of the joint hypothesis problem. This problem arises because often in testing an hypothesis, certain assumptions and measurement techniques must be incorporated into the analysis. In a sense, then, most tests assess the joint hypothesis that both the theory and the specification of the test method are appropriate. Consequently, if test results are not favourable, we typically cannot be sure whether the underlying hypothesis has failed or whether the method of testing the hypothesis has been insufficient. This is one of the most difficult problems encountered in empirical research and one which will arise throughout this thesis.

Notwithstanding the difficulties presented by the foregoing, this paper will attempt to evaluate the predictive power of information contained in the yield curve for the purpose of forecasting future interest rates. I will test the experience in the Canadian context using 91- and 182-day Treasury Bill yields to forecast future 91-date rates.

In doing so, I will be replicating some of the tests tried by other authors as described earlier in this section. The nature of the short-term government sector market will simplify a number of conditions which might otherwise be problematic. For example, because all of the data is Canadian, no biases are introduced by differing regulatory or institutional practices in different countries. In addition, Canadian Treasury Bills are all of the same credit, so the analysis is not undermined by the introduction of securities carrying different risks. Because Treasury Bills are issued as discount notes, it is not necessary to adjust for varying interest coupon levels as was required, for example, by Shiller, et al., (1983) in their duration-adjusted analysis. Another important factor is that because the analysis uses debt instruments to a maximum length of 6 months, it falls well within the bounds identified by Malkiel and Kane (1969) whose empirical research on error learning suggests that the expectations hypothesis is plausible only for short-time horizons.

In summary, the paper first tests the predictive power of the forward rate and the slope of the yield curve. It then examines whether there is evidence of a premium built into the forward rate and/or the slope. Where there is evidence that a premium exists, I will then attempt to estimate it, both in terms of its magnitude and its variability over time. In addition, I will compare the results of using ARIMA modelling to estimate interest rate expectations to more conventional estimation techniques. The final step will be to re-estimate the predictive power of the forward rate and the slope after an adjustment is made for the premium. If the premium exists and has been accurately quantified, the predictive power of the forward rate and the slope should be improved.

2.0 Forecast Ability of Information Contained in the Yield Curve

2.1 Introduction

This section describes more fully how information will be drawn from the yield curve, both in terms of the forward rate and the slope of the yield curve. It will then test the forecast ability of the forward rate and the slope of the yield curve using regression techniques. In addition, it will use time series analysis to forecast future interest rates. Finally, it will summarize the results of these analyses.

2.2 Yield Curve, Definitions and Data

2.2.1 Yield Curve

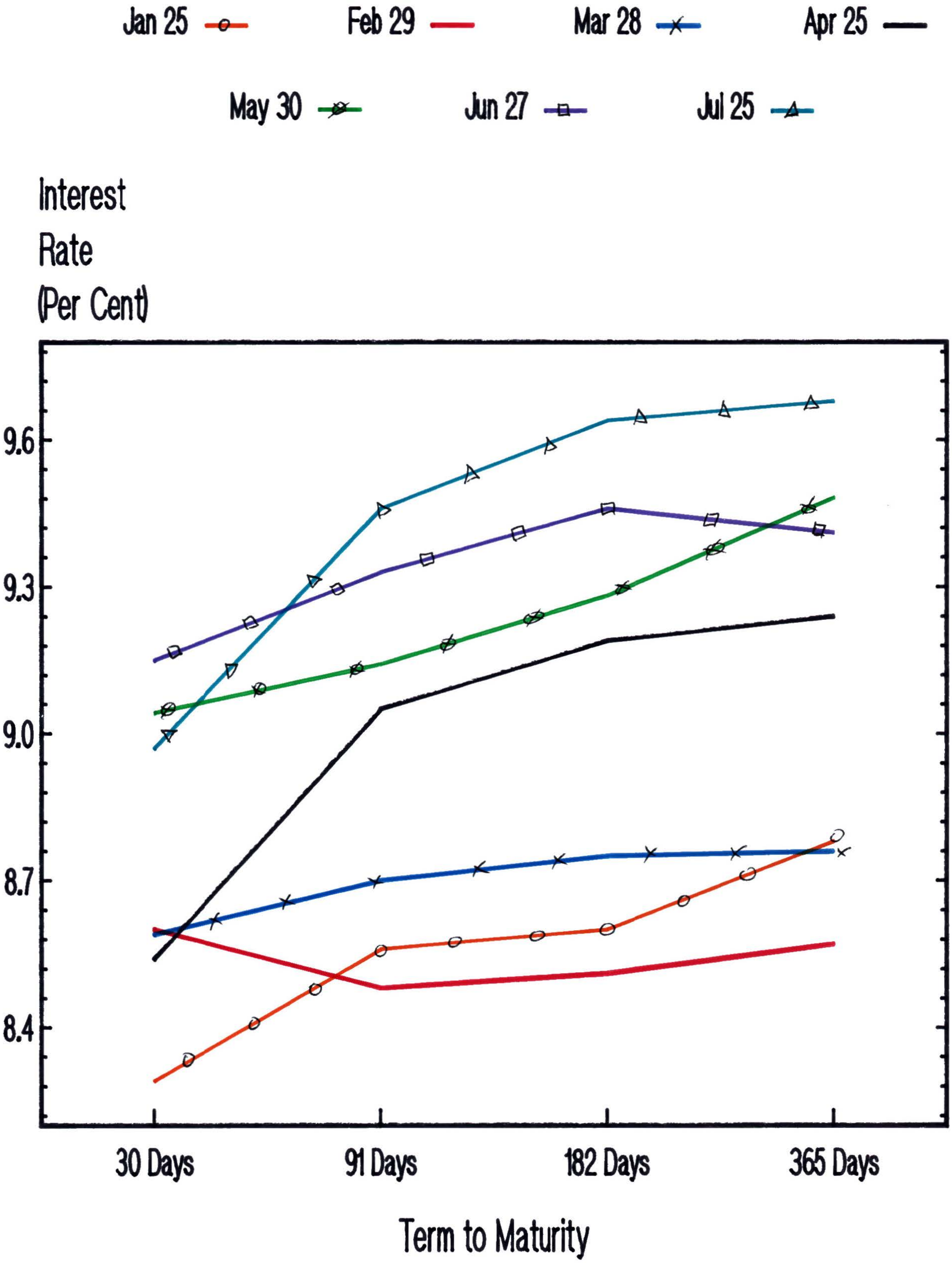
Section 1 of the paper introduced the concept of the yield curve and outlined that it can be best described graphically, as shown in Figure 2.

The vertical axis marks the yield (or interest rate) associated with a particular security, while the term to maturity of the security is represented on the horizontal axis. The plot as a whole therefore

Figure 2

SAMPLE YIELD CURVES

Canada Treasury Bills - 1988



represents interest rates applicable in the market-place across the entire spectrum of maturities (although Figure 2 is truncated at one year).

Figure 2 illustrates a number of yield curves representing the interest rates associated with a series of Treasury Bills of varying maturities for several different dates. The dates are the last Monday of each month between January and July 1988, inclusive. Several characteristics can be observed from the relationships between the curves.

First, all the yield curves but two are positively-sloped; that is, the longer the maturity, the higher the yield. The curve for February 29 is negatively-sloped at the short maturities and the June 27 curve is negatively-sloped at the longer-end of the maturities. On balance, however, the curves are positively-sloped, which is representative of a typical yield curve.

Second, interest rates for various terms tend to move simultaneously in the same direction. There is little crossing of the yield curves which implies that as short-term rates rise, for example, so do longer rates. This is consistent with the expectations hypothesis. (Where long- and short-term rates move in opposite directions, the occurrence can only be explained in terms of the expectations hypothesis by a time-varying premium.)

Third, the slopes of the yield curves can differ dramatically. For example, the April 25 yield curve is very steeply sloped, rising 73 basis points over the 1 - 12 month maturity spectrum. By contrast, the March 28 curve is very flat, rising only 17 basis points. The slope of the yield curve has important implications for the evaluation of expectations of future interest rates, as will be described in the next section.

2.2.2 Slope of the Yield Curve

Information contained in the yield curve is used in two ways in this paper. The first is simply with respect to the slope of the curve. This terminology refers to the difference in yield between two securities of different maturities; where the longer maturity has a higher yield, the yield curve is positively-sloped -- where lower, it is negatively-sloped.

For example, (the middle section of) the slope of the yield curve would be mathematically represented as

$$_tR_6 - {}_tR_3$$

where

$_tR_6$ represents the current 6-month rate and
$_tR_3$ represents the current 3-month rate.

Using the July 25 yield curve as an example, the 6-month yield of 9.64 per cent is higher than the 3-month yield of 9.46 per cent and the yield curve is positively sloped by the difference of 18 basis points.

The slope of the yield curve has implications insofar as the pure expectations hypothesis is concerned. As described in Section 1.3, the hypothesis suggests that information contained in the yield curve can be used to estimate the market's expectation of future interest rates. In simple terms, if the yield curve is positively-sloped, interest rates are expected to rise -- and the steeper the slope of the yield, the larger the expected rise. The converse would, of course, be true for a negatively-sloped curve. Consequently, a steep positively-sloped curve portends a large increase in rates, a flat curve suggests they will not change, and a negatively-sloped curve suggests they will fall.

In the case discussed above, the pure expectations hypothesis, when tied to the slope of the yield curve hypothesis, suggests that the market expects interest rates to rise by 18 basis points.

2.2.3 Forward Rate

A second commonly used concept (and one which will be used throughout this thesis) is that of the forward rate of interest implicit in the yield curve. The forward rate is the interest rate on a 1-period security applicable to period 2 which, when combined with the current return (spot rate) on a 1-period security, will result in the same return as a current

2-period security. It can be calculated using the following formula:

$$(1 + {}_tR_3)\,(1 + {}_{t+3}F_{3,t}) = (1 + {}_tR_6)^2$$

which can be redefined in terms of the forward rate as

$${}_{t+3}F_{3,t} = \frac{(1 + {}_tR_6)^2}{(1 + {}_tR_3)} - 1$$

where

> ${}_{t+3}F_{3,t}$ is the current (t) implied 3-month forward rate applicable to the 3-month period beginning three months hence (t+3).

In simple terms, the current 3-month rate, compounded by the forward rate, will be equivalent to the current 6-month rate. (An example is provided in Appendix II.[10])

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[10]The slope of the yield curve and the forward rate are closely related algebraically. Their relationship is outlined in Appendix III.

As was described with respect to the slope of the yield curve, the forward rate can be used with the pure expectations hypothesis to explain the market's expectation of future interest rates. For example, a forward rate which is significantly higher than the current rate indicates the market holds a view that interest rates are about to rise sharply. The reverse would be true if the forward rate were lower.

2.2.4 Data

The securities I have chosen to use in this study are the 91-day and 182-day Government of Canada Treasury Bills, based on the regular Thursday auction of the Bank of Canada.[11] There are two reasons for this choice. First, the 91- and 182-day markets are large and liquid and, as such, will be relatively free of market imperfections. The other major Treasury Bill market, which is in the one-year term, is much smaller. For example, the average weekly Treasury Bill auction for the first five months of 1988 totalled $2.3 billion and $1.2 billion for 3- and

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[11] All data is shown in full in Appendix I.

6-month Bills, respectively, while the average auction size for 1-year Treasury Bills was only $0.5 billion. Consequently, trading in the 1-year Treasury Bill market is much thinner and is less representative of a true market than a market of larger size with active trading.

A second major reason is that 1-year Treasury Bills were not issued on a basis that would fit the terms of this thesis (i.e. issued at least every 13 weeks) until the 1980s. Prior to that time, 1-year Treasury Bills were issued on an ad hoc basis, usually (but not always) every four weeks.

The more important rationale for excluding the longer-term Treasury Bill market is that the expectations model is less defensible, the longer the term. This was documented by Malkiel and Kane (1969) who found during their research on error-learning that the expectations model is plausible only for short time horizons. Their research supported the hypothesis for a 3-month horizon, but suggested that it is of decreasing plausibility the longer the period, with little support for periods greater than one year.

For the three reasons cited above, an analysis based on 3- and 6-month interest rates is much more likely to be meaningful, given both practical and theoretical considerations.

Because Treasury Bills are issued on a discount basis and because compounding is not taken into account, it was also necessary to transform the data to adjust for these factors.[12] I have used a 182-day basis since:

i) the longest period in the analysis is 182 days, and

ii) it approximates the U.S. and Canadian bond standard (which is a semi-annual basis).

Observations are spaced 13 weeks apart. The choice of successive 13-week (91-day) periods is to avoid spurious correlation. Any more frequent calculations

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[12] The necessary calculations are shown in Appendix II.

would overlap time periods and lead to (built-in) serial correlation.

The choice of 13-week data is somewhat unusual. Most studies in this area rely on monthly data, typically using the last observation of every third month to approximate 13-week data.[13] It must be assumed that this method is used because of data restrictions, rather than by choice, because monthly data contains rounding error (albeit small). This occurs because not every 3-month interval will have 91 days. Hence, on occasion the monthly series will contain an extra observation and bias the results. I was able to avoid this problem by obtaining from the Bank of Canada a weekly series of 3- and 6-month Treasury Bill rates, and by then drawing data spaced exactly 13 weeks apart.[14]

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[13]Park, in his 1982 analysis, used this approach with Canadian data.

[14]On statutory holidays, the Bank of Canada moves its Treasury Bill auction from Thursday to Wednesday. Except for these rare occasions, the data points are exactly 91 days apart. On these occasions, the data is spaced 90 days for the period in question and 92 days for the subsequent period.

The data used in this study covers the period January 1960 to March 1988. This is (almost) the longest period possible, since 6-month Treasury Bills were first offered in 1959. The data covers 114 periods and is shown in full in Appendix I.

2.3 Quantitative Tests

2.3.1 Forecast Ability of the Forward Rate

The first step in the analysis of forward rates as predictors is to set the stage in terms of the conceptual hypothesis. In its most simple form, the hypothesis suggests that the forward rate of interest implicit in the yield curve is an accurate reflector of the market's expectation of future interest rates. The most basic form of this hypothesis was shown in Eq. [2], and is recast here using variable names related to the data as

$$[8] \quad {}_{t+3}F_{3,t} = E_t[{}_{t+3}R_3]$$

Moreover, assuming an efficient market, one would also expect the following to hold

$$[9] \quad {}_{t+3}R_3 = E_t[{}_{t+3}R_3] + \epsilon_t$$

where

ϵ_t is the error term which in essence reflects forecasting error.[15]

By substitution of Eq. [8] into Eq. [9], we obtain

$$[10] \quad {}_{t+3}R_3 = {}_{t+3}F_{3,t} + \epsilon_t$$

To test the predictive ability of the forward rate, a regression of the above equation can be estimated in the following form:

$$[11] \quad {}_{t+3}R_3 = \alpha_0 + \alpha_1 \; {}_{t+3}F_{3,t}$$

---

[15]This specification assumes that the error term has an expected value of zero and that the errors are uncorrelated in a statistical sense.

The hypothesis would predict that all of the variation in the future rate would be explained by the forward rate. If the hypothesis holds, we would therefore expect α_0 to have a value of zero and α_1 to have a value of one. In addition, because the errors terms are uncorrelated, we would expect tests for serial correlation in the errors to fail.

The results of the regression are outlined below. R^2 refers to the adjusted coefficient of determination, DW is the Durbin-Watson statistic and the figures shown in parentheses are the t-statistics.

$$[12] \quad {}_{t+3}R_3 = -.15 + 1.03\ {}_{t+3}F_{3,t} \qquad R^2 = .8853$$
$$(-0.5) \quad (28.4) \qquad DW = 1.86$$

The estimated coefficients are approximately what the expectations model suggested they should be and the relatively high Durbin-Watson statistic suggests that the required condition of no serial correlation in the error terms has been upheld. In addition, the high coefficient of determination, the lack of significance for the intercept coefficient and the highly significant t-statistic for the coefficient associated with the independent variable all suggest support for the hypothesis. In fact, a t-test for the hypothesis that the independent variable coefficient is equal to one yielded a t-statistic of (0.93), indicating further support for the hypothesis.

In contrast with the above result, however, a wide body of literature has pointed out that the current interest rate is as good as or better at forecasting future interest rates than any other predictor.[16] For this reason, the regression was recalculated with the current rate replacing the forward rate on the RHS of the equation, as shown below.

$$[13] \quad {}_{t+3}R_3 = \underset{(2.0)}{.57} + \underset{(28.5)}{.93}\,{}_tR_3 \qquad R^2 = .8867 \quad DW = 1.77$$

The test results were similar to those in Eq. [12], with virtually the same R^2 (.8867) and an estimated coefficient for the independent variable which is also close to one, as expected.

[16] See, for example, Hamburger and Platt (1975), Fama (1976), and Pesando (1986).

These remarkably similar results[17] suggest that the forward rate is no better than the current rate at predicting future interest rates. To further investigate this possibility, a regression was conducted with both the forward and current rates assumed to be explanatory variables.

If the forward rate has little to add to the predictive power of the current rate, we would expect the t-statistic for the forward rate to fall. In addition, an F-statistic can be used to test whether any improvement to the regression results after inclusion of the forward rate in the RHS of Eq. [13] is statistically significant.

$$[14] \quad {}_{t+3}R_3 = .19 + .49 \; {}_{t+3}F_{3,t} + .50 \; {}_tR_3 \qquad R^2 = .8899$$
$$(0.6) \qquad (2.0) \qquad (2.3)$$

The estimation results shown above add credence to the view that the forward has little to add to the current rate in improving its predictive ability.

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[17]The similarity in regression results parallels that found by Hamburger and Platt (1975) in their investigation which used U.S. data.

The statistical significance of the forward rate has been greatly diminished, although it does pass the significance test at the 95 per cent confidence level.[18] In addition, the F-statistic of 1.9, which tests the added value of the forward rate as an explanatory variable, was well below the critical value for significance at the 95 per cent confidence level.[19,20] Based on these results, we are likely to conclude that the forward rate has little independent predictive ability with respect to future interest rates and that most of the explanatory power lies in the current rate. In fact, we might conclude that one can do just as well holding the view that rates in the future will be the same as they are today.

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[18]These results differ only slightly from those discovered by other authors. Hamburger and Platt (1975), when trying this approach, found the same close relationship between the regressions using first $_tR_3$ and then $_{t+3}F_{3,t}$ on the RHS of the equation. When both were included on the RHS, the results were similar to this analysis (e.g. $R^2 = 0.85$) but the t-statistic for the forward rate (1.20) was shy of significance.

[19]The F-statistic result implies we cannot reject the null hypothesis that the forward rate is not statistically significant.

[20]A discussion of the forecast errors associated with this model follows in Section 2.5.

(This view manifests itself in the martingale model which will be discussed more thoroughly in Section 2.4.1.) It could also lead to the conclusion that the predictive value in the forward rate is as much a function of serial correlation in interest rates as anything else. (This possibility will be discussed in more detail in Section 2.4.4 under ARIMA modelling, of which the martingale model is one example.)

2.3.2 Forecast Ability of the Slope of the Yield Curve

As described in Section 1.3.2, the expectations hypothesis can be described in terms of the slope of the yield curve. To test the slope hypothesis, we assume that the slope of the yield curve provides information about expectations of future interest rates. This requires a test of whether the future 3-month rate is related to the current differential between the 3- and 6- month rates.

A test of the slope hypothesis can be accomplished with reference to Eq. [5] which has been restated here in terms of the data related to this thesis as

$$[15] \quad {}_tR_6 = E_t[{}_{t+3}R_3]$$

By substitution of Eq. [15] into Eq. [9], we obtain

$$[16] \quad {}_{t+3}R_3 = \alpha_0 + \alpha_1 \, {}_tR_6 + \epsilon_t$$

These equations are similar in form to those used in testing the forward rate and, in fact, we would expect similar results to those outlined in Section 2.3.1 because the slope of the yield curve and the forward rate are algebraically closely related. (This close mathematical relationship is evidenced in Appendix III.) Just as in the previous section, we would expect α_0 and α_1 to have values of zero and one, respectively, and the error terms to have an expected value of zero and to be uncorrelated.

The test results of Eq. [16] are shown below, as are the results of a repetition of a test performed in Section 2.3.1.

$$[17] \quad {}_{t+3}R_3 = \underset{(0.6)}{.18} + \underset{(29.2)}{.99} \; {}_tR_6 \qquad R^2 = .8910 \quad DW = 1.84$$

$$[18] \quad {}_{t+3}R_3 = .19 + .97\,{}_tR_6 + .02\,{}_tR_3 \quad R^2 = .8899$$
$$(0.6) \quad (2.0) \quad (0.0)$$

These results add credence to the view that the slope of the yield curve hypothesis and the forward rate hypothesis are closely related. For example, the coefficients, t-statistics, coefficient of determination and Durbin Watson statistics are virtually identical. For Eqs. [12] and [18], the coefficients of determination are exactly the same, as are the coefficients for the constant.

Given the close algebraic relationship between the forward rate forecast and the slope of the yield curve forecast, it is not surprising that the results are similar. What may be considered surprising is the degree of their similarity. Here, for example, the results are so similar that it could be concluded one formula could serve as an adequate proxy for the other.[21]

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[21]The suggestion that the forward and the slope may be able to serve as proxies for one another is strengthened by the results of a regression of the 6-month rate (which is the forecast created by the slope hypothesis) on the forward. The R^2 in this test is .9939, indicating a very (Footnote Continued)

2.4 Time Series Forecasts

The analysis outlined in Section 2.3 relied on regression equations as the appropriate testing technique. An alternative method to the regression approach is offered by time series forecasting. Time series analysis relies on current and past observations of a particular time series to make forecasts about future values of the series.

2.4.1 Martingale Model

The finding earlier that the current rate appears to be as good as or better in predicting future interest rates as is the forward rate manifests itself in the martingale model. The martingale model is a specific form of time series analysis, since it considers only the last observation in making a forecast about the future.

(Footnote Continued)
close correlation. (The estimated coefficients are shown in Appendix III.)

The martingale model is a broadly defined random walk model; that is, a random walk process without restrictions respecting independence and identical distribution of the observations in the series. In short, it suggests that the conditional expectation of the n+1th observation equals the nth value. Continuing the form of notation used in this thesis, it would be shown as

$$[19]\quad E({}_{t+3}R_3 \;/\; {}_{t}R_3,\ {}_{t-3}R_3,\ {}_{t-6}R_3 \ \cdots\ ) = {}_{t}R_3$$

Simply put, it states that knowledge of all historical 3-month rates is unnecessary because the expected value for the next-period 3-month rate is equal to the current 3-month rate.

The forecasting capability of the forward rate relative to a martingale was discussed in Section 2.3.1. It can be tested further by emulating Fama's (1976) approach, which compared the characteristics of two sets of data; the summary data of the errors associated with the forward forecast relative to the errors associated with a martingale model.

Fama found that the standard deviation of the prediction errors associated with the martingale was less than the corresponding standard deviation of the forward rate minus the subsequently observed spot rate. He therefore concluded that the martingale model is better than forward rates at predicting future interest rates.

In this study, however, the results were not conclusive because the standard deviations of the two data sets were virtually identical, as shown in Table 1. As a result, we cannot conclude that the martingale model is superior. On the other hand, it certainly appears to do no worse in forecasting future interest rates, as was also shown in the regression test results of Section 2.3.1.[22]

Table 1 also suggests, however, that there may be serial correlation in the observations, given the

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[22] It is not surprising that the tests indicate the forward rate and the martingale model have approximately the same predictive power with respect to interest rates since this was the finding in Section 2.3.1. This section was using a different test but testing the same hypothesis. We would therefore be concerned only if the results were different.

Table 1

Summary Statistics of Forecast Errors
Associated With the Martingale Model and Forward Rates

	Mean	Standard Deviation	. . Autocorrelation Coefficients . .				
			ρ_1	ρ_2	ρ_3	ρ_4	ρ_5
${}_{t+3}F_{t,3} - {}_{t+3}R_3$	-.10	1.30	.11	-.10	.17	.01	.00
${}_{t}R_3 - {}_{t+3}R_3$	-.06	1.31	.09	-.19	.02	-.05	-.02

high value of second coefficient in the autocorrelation function. This suggests that there may be information contained in past observations, above and beyond that contained in the most recent observation. This possibly will therefore be assessed in the next section.

2.4.2 ARIMA Models

Box and Jenkins (1966) developed a process which has since become known as ARIMA modelling to make forecasts based on information contained in past observations. Unlike the martingale model which considers only one past observation, the general form of the ARIMA model may consider several lagged observations.

The ARIMA process, which is described in some detail in Appendix IV, uses lagged observations in three ways:

$AR_{(p)}$ = the autoregressive component, which suggests that a time series is generated by a weighted average of past observations going back "p" periods.

I = integrated, which states that the process can be integrated with differencing.

$MA_{(q)}$ = moving average, which suggests that the time series is generated by a weighted average of random disturbances going back "q" periods.

Hence, the acronym -- ARIMA.

The ARIMA process has three steps: identification, estimation and diagnostic checking. Appendix IV describes how the process was undertaken for the purposes of the data used in this thesis, and how an ARIMA model (1,1,1) -- with all three components (AR, I, and MA) -- was found to be most appropriate.

2.5 Analysis of Forecast Errors

Having identified a number of potential alternatives for the forecasting of interest rates using information contained in the yield curve, it is appropriate to compare the ability of each of the alternatives relative to the others. We can do this by using each model to make an "historical forecast"

-- that is, to estimate what each of the models would have predicted had it been used at the beginning of an historical period to estimate subsequent time periods.

For the purposes of this study, I chose four distinct periods for which to evaluate historical simulations, as follows:

Period 1: Observations 41 - 46 (December 1969 - March 1971), representing a period of rapidly changing interest rates. Over this period, 3-month Treasury Bill rates fell from 7.92 per cent to 3.05 per cent.

Period 2: Observations 70 - 75 (March 1977 - June 1978), representing a period of relatively stable interest rates. Over this period, 3-month rates varied from 7.18 per cent to 8.41 per cent.

Period 3: Observations 85 - 90 (December 1980 - March 1982), representing a period of high and volatile rates. Over this period, 3-month rates climbed to 20.92 per cent from 16.52 per cent, before declining again to 15.36 per cent.

Period 4: Observations 107 - 112 (June 1986 - September 1987), representing the most recent period for which a forecast could be made. (Although later observations are available, because some of the forecasts use lagged data, a forecast cannot be made right to the current period.) Over this period, 3-month rates fell from 8.86 per cent to 7.42 per cent and then rose again to 9.25 per cent.

Two tests were used to assess the ability of each of the models to forecast interest rates. The first, a variation of the mean per cent error, is calculated as the mean of the absolute value of the difference between the simulated variable and the actual value (i.e. the residual), in percentage terms. A model of "good fit" will have a low-valued mean per cent error.

$$MPE = \frac{\Sigma |Y^a - Y^s| \;/\; Y^a}{n}$$

where

Y^a is the actual value in the series.

Y^s is the simulated value forecast by the model.

n is the number of periods for which a forecast is made.

The second evaluation is based on Theil's inequality coefficient. The Theil coefficient measures the predictive performance of a model by evaluating the errors associated with the forecast on a scale which ranges from 0 to 1. If the coefficient (U) equals 0, the simulated values equal the actual values, and there are therefore no errors, so the model yields a perfect forecast. If, on the other hand U = 1, the model is as poor as could possibly be.

In addition, the Theil inequality coefficient can be used to disaggregate the errors into three components: errors associated with the means of the simulated and actual values; errors associated with their variance; and errors which can be considered unsystematic since they remain only after taking average values and variabilities into account. Because the proportions are scaled, they sum to one. We therefore hope that the first two components, denoted as U^m and U^s will be close to 0, while the latter, denoted as U^c, will be close to 1.[23]

A summary of the results of these tests is shown in Table 2. It compares the forecast errors associated with each of the four models discussed to this point: the forward, the slope, the martingale, and the ARIMA models. In addition, it provides an analysis of forecast errors for each of the four subperiods, and an average over those periods.

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[23]The Theil inequality coefficient should be used with caution since it is exceedingly sensitive to the choice of periods under examination. Appendix VI provides an example of the sensitivity discovered during this analysis. Further details of the Theil coefficient can be found in Pindyck and Rubinfeld (1981) page 364.

Table 2

Summary of Forecast Errors

	Theil Inequality Coefficient	Theil Coefficient of Non-Systematic Error	Mean Per Cent Error
Forward			
Period 1	.075	.079	.185
Period 2	.019	.672	.031
Period 3	.093	.716	.157
Period 4	.045	.879	.083
Average	**.058**	**.587**	**.114**
Slope			
Period 1	.080	.045	.201
Period 2	.023	.684	.039
Period 3	.079	.910	.136
Period 4	.043	.984	.081
Average	**.056**	**.656**	**.114**
Martingale			
Period 1	.087	.051	.216
Period 2	.029	.740	.047
Period 3	.075	.986	.117
Period 4	.045	.967	.079
Average	**.059**	**.686**	**.115**
ARIMA			
Period 1	.082	.055	.202
Period 2	.027	.793	.042
Period 3	.073	.989	.109
Period 4	.039	.917	.075
Average	**.055**	**.689**	**.107**

At this stage, there is little to differentiate between the four models in terms of their predictive power and the characteristics associated with their forecast errors. All models have relatively good fits, with the Theil inequality coefficients close to zero, the coefficient of non-systematic error relatively close to one, and low mean per cent errors.

The ARIMA model is, however, slightly better on all counts. It has the lowest Theil inequality coefficient, the highest measure of non-systematic error relative to total error, and the lowest mean per cent error. The forward and slope forecasts are "middle of the road", while the martingale model performance is the poorest.

3.0 Existence of a Premium

3.1 Introduction

The results of the analysis to this point are consistent with the findings in much of the literature -- there appears to be only marginal evidence that the forward rate and the slope have predictive power with respect to future interest rates. To this point, however, we have assumed that the forward rate and the slope contain only the market's expectation of future interest rates when, as described in Section 1.4, a strong theoretical case can be made that they also contain a premium. The effect of the premium on the forward rate can be represented as

$$[20] \quad {}_{t+3}F_{3,t} = E_t[{}_{t+3}R_3] + P_t$$

and on the slope

$$[21] \quad {}_tR_6 = E_t[{}_{t+3}R_3] + P_t$$

where P_t is the premium.

Clearly, if a premium exists it will bias the results of a forecast made on the basis of a forward rate or the slope if it has been assumed that they contain only the market's expectation of future rates. It is therefore important to assess the possibility of a premium carefully.

3.2 Constant or Time Variable Premium

One of the key questions in assessing the possibility of the existence of a premium is whether it will be constant or variable. On a theoretical basis, a case can be made for either. The Hicksian term premium model suggests a constant premium, where longer-term maturities carry a premium for risk, as described in Section 1.4.1. The premium would rise monotonically with the maturity, but would otherwise be constant (i.e. constant over time). If the premium were constant, it would be represented as α_0 in Eq. [11]. However, most theories suggest a premium which can vary, usually on the basis of interest rate levels or interest rate volatility.

The data collected for this thesis also point to the existence of a time variable premium. As mentioned earlier, the expectations hypothesis is not consistent with long- and short-term rates moving in different directions unless the divergence is attributable to variation in the premium. Given that short- and long-term rates do frequently move in opposite directions, this data suggests a time variable premium.[24]

3.3 Proxies for a Time Variable Premium

Based on information contained in the yield curve and past observations, a number of hypotheses exist which could explain variability in a premium. Typically, they are based on the suggestion that systematic variation in the premium would be the result of interest rate uncertainty. Fama (1976) had some success using uncertainty in interest rates to model the premium, and Park (1982) then sought to replicate Fama's findings, using a regression that will be used here, as follows:

$$[22] \quad {}_{t+3}F_{3,t} - {}_{t+3}R_3 = \alpha_0 + \alpha_1 U_t$$

[24]Section 1.5 described this to have occurred on 67 of a possible 187 occasions considered.

where

U_t is a measure of interest rate uncertainty.

3.3.1 The Current Rate

Several authors have suggested that the current interest rate acts as a good measure of the expected direction of interest rate movements. As described in Section 1.4.2, however, two theories exist: Keynes (1930) suggests a regressive formulation of interest rate expectation, which implies that the premium will be negatively correlated to interest rate levels, while Kessel (1965) suggests that it is positively related.

The results of the tests of the hypotheses using this thesis' data are shown below.

$$[23] \quad {}_{t+3}F_{3,t} - {}_{t+3}R_3 = \underset{(.77)}{.22} - \underset{(-1.25)}{.04U_t} \qquad R_2 = .0149$$

where

$U_t = {}_tR_3.$

The coefficient for the current interest rate is negative, indicating that to the extent there is a correlation between the premium and the level of current interest rates, it is negative and Keynes' view is supported. The t-statistic is not significant, however, so on balance neither hypothesis can be strongly supported and the results must be considered inconclusive.

3.3.2 Recent Trends in Interest Rates.

Another measure of uncertainty/risk could be based on recent trends in interest rates. This theory, known as the extrapolative expectations model, suggests that if interest rates have been rising in the near past, an investor will expect that rates will continue to rise and therefore expect a larger risk premium.

This hypothesis was tested by regression assuming uncertainty would be measured by the direction and degree of the interest rate change over the previous period. Unfortunately, as shown below, my results do not indicate significance.

$$[24] \quad {}_{t+3}F_{3,t} - {}_{t+3}R_3 = \underset{(-.77)}{-.10} - \underset{(-.78)}{.08}\, U_t \qquad R^2 = .0058$$

where

U_t measures the interest rate change during the last period.

Moreover, the t-statistics are not only insignificant, but they carry the wrong sign. Contrary to my theory, if interest rates are rising, the premium appears to shrink rather than grow!

3.3.3 Interest Rate Volatility

Another method of directly developing a proxy for interest rate uncertainty is to measure the degree of fluctuation in recent interest rate movements. This thesis will replicate one of several methods used by Park (1982), although none of his attempts found statistical significance. In this case, I postulate that interest rate uncertainty can be estimated by recent interest rate volatility, defined as the standard deviation of the last eight 3-month Treasury Bill observations, normalized for absolute levels by dividing by the 8-period mean.

My results were similar to those experienced by Park (1982): both α_0 and α_1 in the following equation were insignificant:[25]

$$[25] \quad {}_{t+3}F_{3,t} - {}_{t+3}R_3 = \underset{(.15)}{.03} - \underset{(-.70)}{.10}U_t \qquad R^2 = .0047$$

where

U_t is a measure of volatility as described above.

Moreover, α_1 again carried the wrong sign; that is, it was negative. Contrary to theory, this would

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[25]Specific comparisons cannot be made because Park does not publish his regression results. Rather, he merely states that α_0 and α_1 were insignificant.

suggest that the premium decreases when interest rate uncertainty increases!

3.3.4 Interest Rate Level and Volatility

A final method of calculating interest rate uncertainty was undertaken using a premium comprising a combination of interest rate uncertainty and the current level of interest rates. However, the results described below, while carrying the right signs, again are not significant.

$$[26] \quad {}_{t+3}F_{3,t} - {}_{t+3}R_3 = \underset{(.19)}{.08} + \underset{(.47)}{.82U_t} - \underset{(-1.25)}{.04_tR3}$$

While the results throughout Section 3.3 are not all one might hope them to be (the t-statistics are not significant), one cannot completely reject the hypotheses because of the problem of the joint hypothesis. As discussed in Section 1.5, the tests performed using Eqs. [23] to [27] are really testing two hypotheses: first, that the premium is a function of uncertainty, and, second, that the proxy for uncertainty is correct. Consequently, a failure in the test results does not necessarily mean that the hypothesis should be rejected. It could well mean that the measure for interest rate uncertainty is incorrectly specified.

3.4 ARIMA Modelling to Extract an Estimate of the Premium

An alternative method of estimating the premium was introduced by Nelson (1970) who sought to use an ARIMA model to extract the premium. The estimation process is quite straightforward. I outlined earlier that, where a premium exists, it will be as shown in Eq. [20] as

$${}_{t+3}F_{3,t} = E_t[{}_{t+3}R_3] + P_t$$

which can be rewritten as

$$[28] \quad P_t = {}_{t+3}F_{3,t} - E_t[{}_{t+3}R_3]$$

P_t can only be extracted, of course, if measures for ${}_{t+3}F_{3,t}$ (the forward rate) and $E_t[{}_{t+3}R_3]$ (the expected future rate) are available. The forward rate is, of course, observable -- and although $E_t[{}_{t+3}R_3]$ is not directly observable, it can be estimated. In this instance, we use an ARIMA model to perform the required estimation. Having estimated $E_t\ [{}_{t+3}R_3]$, Eq. [28] can then be used to derive the premium through simple subtraction.

Appendix IV outlines that ARIMA model (1,1,1) is most appropriate for the data used in this thesis. Therefore, ARIMA (1,1,1) is used to calculate the expected value of future interest rates and the premium is then directly derived.

Appendix VII shows the expected values of future interest rates ($E_t[{}_{t+3}R_3]$) and the premium, P_t, which has been derived. In addition, Figure 3 illustrates how the premium relates to the level of interest rates, while Figure 4 shows how the premium has behaved over time. No clear pattern emerges, although there may be some evidence that the premium has increased over time. Because there are only two large variances, however, these points could well be outliers and not representative of a trend.

3.5 Adjustment for a Premium

The final section of this paper will assess whether an adjustment for the premium can improve the

Figure 3

PREMIUM - INTEREST RATE RELATIONSHIP

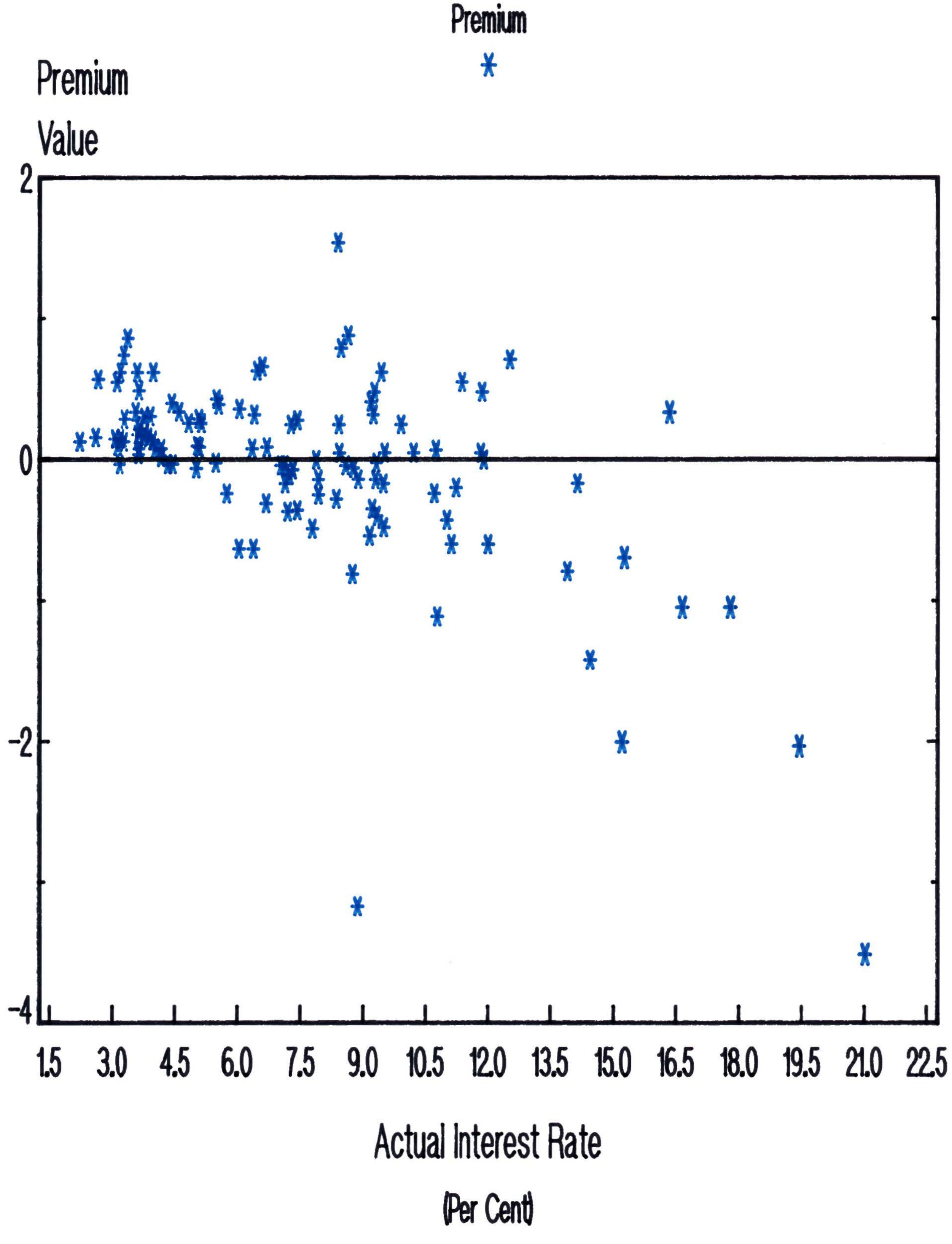
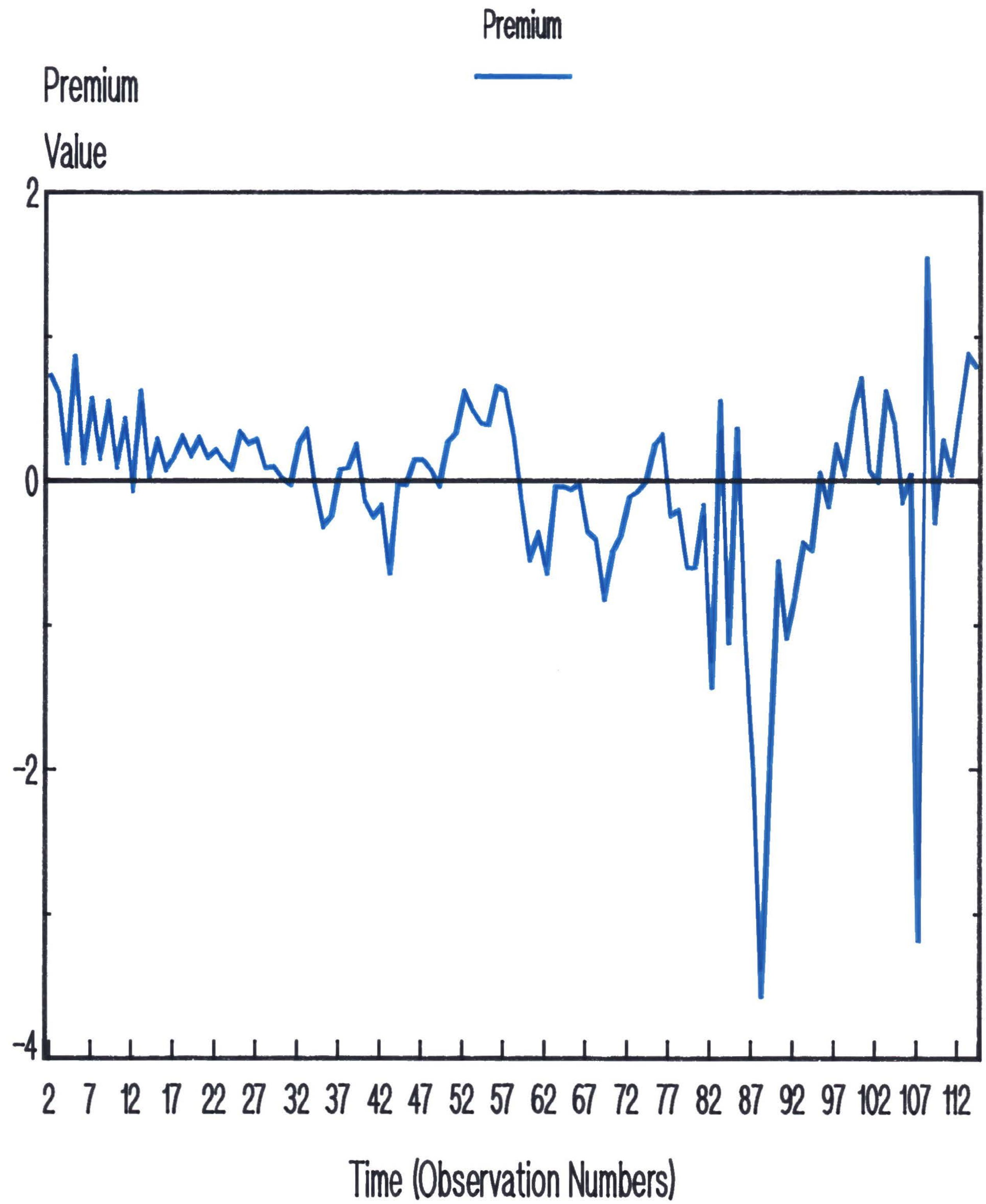


Figure 4

PREMIUM FORCASTS
ARIMA Model



predictive ability of the forward rate and the slope. This analysis is conducted by comparing the results of regression equations as outlined at the beginning of this paper (where premiums were not contemplated), with the same equations adjusted for a premium. The premium used will be that drawn from ARIMA model (1,1,1) as described in the previous section.

3.5.1 Forecast Ability of the Forward Rate Adjusted for a Premium

Section 2.3.1 opened with a simple approach to the basic question, "Are forward rates good predictors of future interest rates?" with the following results drawn from Eq. [12]

$$_{t+3}R_3 = -.15 + 1.03\ _{t+3}F_{3,t} \qquad R^2 = .8853$$
$$(-.50) \qquad (28.35)$$

We can now assess the usefulness of an adjustment for a premium by adding a proxy for the premium (based on the values drawn from the ARIMA model) to the RHS of the equation. The results were as follows:

$$[29]\quad _{t+3}R_3 = .27 + .97\ _{t+3}F_{3,t} - 0.61\ P_t \qquad R^2 = .8933$$
$$(.87) \qquad (23.8) \qquad (-2.9)$$

where

P_t is $_{t+3}F_{3,t} - E_t[_{t+3}R_3]$

and $E_t[_{+3}R_3]$ is that estimated by ARIMA (1,1,1).[26]

These results indicate there is some improvement in the forecasting power of the forward rate when there is an adjustment for a premium. The coefficient estimates are similar, but the R^2 has increased and the t-statistic for the premium is significant. Thus, adjustment for a premium improves the forecast ability of the forward rate.

3.5.2 Forecast Ability of the Slope of the Yield Curve Adjusted for a Premium

As was undertaken in the previous section, a comparison can be made as to the forecast ability of

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[26] It could be argued that this formulation of the equation is not appropriate because the premium is already embedded in the forward rate. That is, the equation has been specified as $_{t+3}R_3 = f(_{t+3}F_{3,t}, P_t)$, but $P_t = f(_{t+3}R_3^{(expected)}, _{t+3}F_{3,t})$. A better specification would therefore be an appropriate topic for further study.

the slope of the yield curve both before and after an adjustment for the premium.

Eq. [18] provided the results of the forecast ability of the slope where there is no adjustment for a premium as

$$ {}_{t+3}R_3 = .18 + .99\,{}_tR_6 \qquad R^2 = .8910 $$
$$ (0.6) \qquad (29.2) $$

Adjustment for a premium based on ARIMA model (1,1,1) yielded the following

$$ [30] \quad {}_{t+3}R_3 = .37 + .96\,{}_tR_6 - .51\,P_t \qquad R^2 = .8910 $$
$$ (1.2) \qquad (24.4) \qquad (-1.4) $$

In this case, the inclusion of a premium provided no additional explanatory power, since the coefficients of determination were identical with and without a premium adjustment and the t-statistic for the premium is not significant.

4.0 Conclusion and Implications

4.1 Summary of Results

Table 3 provides a number of comparisons of the predictive ability of the forward rate and the slope of the yield curve both with and without adjustments for premiums. For comparison purposes, the martingale model has also been included.

Certain observations can be made about these results. First, the forward rate prediction can clearly be improved with the inclusion of a premium. Where a premium is included, the overall fit of the model is improved and the forecast error characteristics are "better". In addition, the t-statistic for the premium is significant. Clearly, the predictive ability of the forward rate can be improved by the inclusion of a premium.

Second, a forecast made on the basis of the slope of the yield curve is also improved when provision is made for a premium. The evidence is, however, not compelling. Although the coefficient of determination increases, the improvement is only

Table 3

Summary Statistics on the Predictive Power of the Forward Rate and the Slope, With and Without Premium Adjustments

	R^2	t-statistics		Theil Inequality Coefficient	Theil Coefficient of Non-Systematic Error	Mean Per Cent Error
Forward	.8853	28.4	F	.058	.587	.114
		-0.5	C			
Forward Adjusted By ARIMA	.8933	23.8	F	.056	.682	.113
		-2.9	P			
		0.8	C			
Slope	.8910	29.2	S	.056	.656	.114
		0.6	C			
Slope Adjusted By ARIMA	.8921	24.4	S	.057	.673	.116
		-1.4	P			
		1.2	C			
Martingale	.8867	28.5	F	.059	.686	.115
		2.0	C			

F = Forward
S = Slope
P = Premium
C = Constant

marginal. In addition, the forecast error characteristics are less supportive. Moreover, the t-statistic for the premium is not significant. It is difficult to argue that the slope of the yield curve forecast can be significantly improved by inclusion of a premium.

The most important finding, however, is that the results are not robust. Although the forward rate and the slope of the yield curve (when adjusted for a premium) have slightly better track records than the martingale model in forecasting interest rates, it is not clear that this improvement is meaningful. Certainly the overall fit of all of the models is similar, and no one model could be considered dramatically better than any other. It is therefore not clear that forecasts on the basis of the forward rate or the slope of the yield curve, even when adjusted for a premium, will be any better on a consistent basis than assuming interest rates will not change from current levels.

This finding is similar to that contained in much of the literature, as described in the early pages of this thesis. A great many authors have studied the forecast ability of forward rates and the slope of the yield curve in predicting future interest rates. Many others have sought to improve their findings by making an adjustment for a premium embedded in the forward rate or the slope. Few, however, have succeeded in finding a calculation for the premium which would consistently improve their forecast ability of the forward rate.

This study has shown that an adjustment for a premium can improve the predictive power of the forward rate and the slope of the yield curve, but not to a significant extent.

There are several possibilities which might explain this phenomenon. The first is the joint hypothesis problem alluded to earlier in this paper. In the tests undertaken here, we are testing not just market efficiency (i.e. that the market fully reflects all available information) and the underlying hypothesis of rational expectations, but also the specifications of the variables and equations used to test the hypotheses. Consequently, the poor findings in the paper could be attributed to a misspecification of the tests. One area for further study could therefore be with respect to a refinement or new definition for the premium.

Along these lines, it may not be possible to build all the myriad of factors that affect the market's view on future interest rates and, consequently, the premium, into a simple specification. It may be preferable to construct a structural model which would attempt to recreate the brain power and insightfulness of the market in developing a view on rates. This, of course, would not be an easy task. Notwithstanding the millions of dollars behind the efforts of the monetary authorities to create econometric models, Marcia Stigum, in her well-known book The Money Market, reports that a freedom-of-information suit by Ralph Nader once forced the Fed to make public an internal memo stating that the yield curve tracked interest rates better than its own statistical model.

A second possibility is that the monetary authorities deliberately attempt to make interest rates unpredictable. Mankiw and Miron (1986) describe that prior to the creation of the Federal Reserve in the United States in 1914, interest rates had a predictable seasonal pattern. After the creation of the Federal Reserve, the patterns were eliminated.

Mankiw (1988) has developed a theory of monetary policy that suggests the Federal Reserve finds it optimal to make interest rate changes unpredictable.[27] To the extent his theory is correct, it is not surprising that it is difficult to forecast interest rates given that the monetary authorities are deliberately taking action to make such forecasts inaccurate.[28]

This finding has important implications for those who might attempt to use the forward rate or the slope of the yield curve as an indication of future interest rates. It suggests that the monetary authorities

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[27]The theory is based on the concept that the interest rate can be viewed as a tax rate on holding money balances, since money does not earn interest. He then develops a theory which shows that the inefficiency of the tax is minimized by making short-term rates follow a random walk.

[28]Mankiw's theories are based on United States' experience. It is a widely held belief, however, that the Canadian experience, whether deliberate or not, tracks the United States' experience.

must be selective in their use of tools, since action aimed at short-term rates may not affect long-term rates as expected. It also suggests that borrowers and lenders, in making financial decisions, should find methods different from the slope and the forward rate to make interest rate forecasts.

Data

This appendix provides the data used in this thesis. Most of the data is available from the Bank of Canada's monthly review. Data from the early years, however, is unpublished and was obtained directly from the Bank of Canada.

Obs - is the number used to identify each observation.

Year, Mth, Day - represent the date of the observation.

R_3, R_6 - are the quoted 91-day and 182-day Canada Treasury Bill rates for the stated date, based on the Bank of Canada's regular Thursday auction.

Fwd - is the forward rate of interest implicit in the yield curve. Its derivation is shown in Appendix III.

R_3adj - is the 91-day rate adjusted to a semi-annual basis from its quoted quarterly basis. This calculation, which is described in Appendix II, facilitates a meaningful comparison between the 182-day Treasury Bill, rate which is quoted on a semi-annual basis, and the forward rate, which has also been calculated to a semi-annual basis.

Obs	Year	Mth	Day	R_3	R_3adj	R_6	Fwd
1	60	1	5	5.140	5.206	5.530	5.855
2	60	4	7	3.240	3.266	3.470	3.674
3	60	7	7	3.170	3.195	3.350	3.505
4	60	10	6	2.200	2.212	2.520	2.829
5	61	1	4	3.340	3.368	3.630	3.893
6	61	4	6	3.250	3.276	3.440	3.604
7	61	7	6	2.630	2.647	2.780	2.913
8	61	10	5	2.570	2.587	2.840	3.094
9	62	1	4	3.080	3.104	3.230	3.356
10	62	4	5	3.090	3.114	3.300	3.486
11	62	7	5	5.430	5.504	5.710	5.917
12	62	10	4	4.940	5.001	5.130	5.259
13	63	1	6	3.940	3.979	4.060	4.141
14	63	4	4	3.600	3.632	3.730	3.828
15	63	7	4	3.260	3.287	3.350	3.413

Obs	Year	Mth	Day	R_3	R_3adj	R_6	Fwd
16	63	10	30	3.610	3.643	3.760	3.878
17	64	1	2	3.740	3.775	3.940	4.105
18	64	4	2	3.870	3.907	4.020	4.133
19	64	7	2	3.580	3.612	3.740	3.868
20	64	10	2	3.730	3.765	3.890	4.015
21	64	12	31	3.820	3.856	3.960	4.064
22	65	4	2	3.620	3.653	3.730	3.807
23	65	7	2	3.930	3.969	4.050	4.131
24	65	10	1	4.130	4.173	4.350	4.528
25	65	12	31	4.540	4.592	4.770	4.949
26	66	3	31	5.060	5.124	5.300	5.476
27	66	6	29	5.000	5.062	5.100	5.138
28	66	9	29	5.010	5.073	5.150	5.227
29	66	12	29	4.960	5.022	5.030	5.038
30	67	3	30	4.130	4.173	4.110	4.047
31	67	6	29	4.280	4.326	4.520	4.715
32	67	9	28	4.760	4.817	5.010	5.204
33	67	12	28	5.950	6.039	6.130	6.222
34	68	3	28	6.980	7.102	6.980	6.858
35	68	6	27	6.560	6.668	6.510	6.353
36	68	9	26	5.660	5.740	5.750	5.760
37	68	12	24	6.240	6.337	6.470	6.603
38	69	3	27	6.580	6.688	6.800	6.912
39	69	6	27	7.130	7.257	7.260	7.263
40	69	9	25	7.770	7.921	7.820	7.719
41	69	12	23	7.770	7.921	7.840	7.759
42	70	3	25	7.000	7.122	6.760	6.399
43	70	6	25	5.940	6.028	5.980	5.932
44	70	9	24	5.410	5.483	5.470	5.457
45	70	12	23	4.390	4.438	4.450	4.462
46	71	3	25	3.030	3.053	3.080	3.107
47	71	6	23	3.170	3.195	3.300	3.405
48	71	9	23	3.980	4.020	4.050	4.080
49	71	12	23	3.150	3.175	3.230	3.285
50	72	3	23	3.590	3.622	3.900	4.179
51	72	6	22	3.530	3.561	3.810	4.059
52	72	9	21	3.570	3.602	3.910	4.219
53	72	12	21	3.610	3.643	3.820	3.998
54	73	3	22	4.370	4.418	4.710	5.003
55	73	6	21	5.480	5.555	5.930	6.306
56	73	9	20	6.480	6.585	6.970	7.356
57	73	12	20	6.360	6.461	6.580	6.699
58	74	3	21	6.290	6.389	6.360	6.331
59	74	6	20	8.680	8.868	8.790	8.712
60	74	9	19	8.940	9.140	8.870	8.601
61	74	12	19	7.290	7.423	7.050	6.678
62	75	3	21	6.270	6.368	6.320	6.272
63	75	6	20	6.900	7.019	7.090	7.161
64	75	9	18	8.390	8.566	8.620	8.674

Obs	Year	Mth	Day	R_3	R_3adj	R_6	Fwd
65	75	12	18	8.570	8.754	8.720	8.686
66	76	3	19	9.100	9.307	9.210	9.113
67	76	6	18	9.000	9.202	8.960	8.718
68	76	9	17	9.110	9.317	8.970	8.624
69	76	12	17	8.550	8.733	8.420	8.108
70	77	3	17	7.630	7.776	7.580	7.385
71	77	6	16	7.060	7.185	7.110	7.035
72	77	9	15	7.100	7.226	7.230	7.234
73	77	12	15	7.180	7.309	7.310	7.311
74	78	3	16	7.720	7.869	8.060	8.251
75	78	6	15	8.240	8.410	8.590	8.771
76	78	9	14	9.030	9.234	9.190	9.146
77	78	12	14	10.420	10.691	10.680	10.669
78	79	3	15	10.910	11.208	10.910	10.613
79	79	6	14	10.810	11.102	10.810	10.519
80	79	9	13	11.650	11.989	11.990	11.991
81	79	12	13	13.660	14.126	13.550	12.976
82	80	3	13	13.940	14.426	14.650	14.875
83	80	6	12	11.050	11.355	10.600	9.850
84	80	9	11	10.480	10.755	11.040	11.326
85	80	12	11	15.890	16.521	16.400	16.279
86	81	3	12	16.860	17.571	16.390	15.221
87	81	6	11	18.820	19.705	18.230	16.773
88	81	9	10	19.930	20.923	19.840	18.767
89	81	12	10	14.930	15.487	14.850	14.216
90	82	3	12	14.810	15.358	15.070	14.782
91	82	6	11	15.950	16.586	16.140	15.696
92	82	9	10	13.440	13.892	13.510	13.130
93	82	12	10	10.700	10.986	10.640	10.295
94	83	3	11	9.280	9.495	9.490	9.485
95	83	6	10	9.290	9.506	9.460	9.414
96	83	9	9	9.270	9.485	9.600	9.715
97	83	12	8	9.670	9.904	9.990	10.076
98	84	3	8	9.950	10.198	10.440	10.683
99	84	6	7	11.520	11.852	12.360	12.871
100	84	9	6	12.140	12.508	12.510	12.512
101	84	12	6	10.460	10.734	10.630	10.527
102	85	3	7	11.560	11.894	12.390	12.888
103	85	6	6	9.220	9.433	9.330	9.228
104	85	9	5	8.970	9.171	9.310	9.449
105	85	12	5	9.070	9.276	9.190	9.104
106	86	3	36	11.490	11.820	10.530	9.255
107	86	6	5	8.670	8.858	9.200	9.543
108	86	9	4	8.220	8.389	8.520	8.651
109	86	12	4	8.180	8.347	8.320	8.293
110	87	3	5	7.290	7.423	7.510	7.597
111	87	6	4	8.250	8.420	8.720	9.021
112	87	9	3	9.050	9.255	9.740	10.227
113	87	12	3	8.450	8.629	8.960	9.293
114	88	3	3	8.290	8.462	8.490	8.518

Calculation of the Forward Rate

Following is an example of the methodology used to calculate the forward interest rate, based on the first data points used in this study (shown in Appendix I), which are:

3 month quoted rate of 5.14%

6 month quoted rate of 5.53%

Adjustments must now be made to take into account the effects of compounding. Because 3-month T-Bills are quoted on a quarterly basis, to be truly comparable with a 6-month T-Bill quote, the yield must be adjusted to semi-annual compounding, as follows:

$$(1 + .0514/2)^2 = 1.0521 \text{ or } 5.21\%$$

The forward rate can now be calculated from the basic formula

$$_{t+3}F_{3,t} = \frac{(1 + R_6)^2}{1 + {}_tR_3} - 1$$

$$= \frac{(1.0553)^2}{1.0521} - 1$$

$$= .0585 \text{ or } 5.85\%$$

Consequently, the interest rates on a comparable basis for the purposes of this analysis are:

3-month adjusted	5.21%
3-month forward	5.85%
6-month	5.53%

Relationship Between the Forward Forecast and the Forecast of the Slope Hypothesis

This appendix highlights the close relationship between an interest rate forecast based on the forward rate and an interest rate forecast based on the slope of the yield curve.

The forecast provided by the forward rate, as outlined in Section 2.3.1 and Appendix II is calculated as

$$E_t[{}_{t+3}R_3] = {}_{t+3}F_{3,t} = \frac{(1 + {}_tR_6)^2}{(1 + {}_tR_3)} - 1$$

In this thesis, we test whether the forward rate is a good predictor of interest rates. The forecast is ${}_{t+3}F_{3,t}$.

We also test whether the slope of the yield curve can provide good predictions of future interest rates. The forecast provided by the slope, as outlined in Section 2.3.2 and Appendix II, is derived from

$${}_tR_6 - {}_tR_3 = E_t[{}_{t+3}R_3] - {}_tR_3$$

which is rewritten as

$$E_t[{}_{t+3}R_3] = {}_tR_6$$

Therefore

$$F^f = \frac{(1 + F^s)^2}{(1 + {}_tR_3)} - 1$$

where

F^f is the forecast prediction from the forward hypothesis

F^s is the forecast prediction from the slope hypothesis.

Clearly, the two forecasts are algebraically related.

Further evidence of their close relationship is illustrated by examination of the regression of the forward forecast on the slope forecast. The results are

$$F^f = \underset{(.59)}{.37} + \underset{(129.7)}{.95}F^s \qquad R^2 = .9939$$

Given a coefficient of determination of .9939, there can be no question of the close correlation between the two.

Description of the ARIMA Process

An ARIMA process is a time series analysis tool which can be used to make forecasts about future values of the time series. It uses information in three ways:

$AR_{(p)}$ = the autoregressive component, which suggests that a time series is generated by a weighted average of past observations going back "p" periods.

I = integrated, which states that the process can be integrated with differencing.

$MA_{(q)}$ = moving average, which suggests that the time series is generated by a weighted average of random disturbances going back "q" periods.

Hence, the acronym -- ARIMA.

The ARIMA technique requires that a time series be stationary and then seeks to identify and estimate it. The model is normally described as ARIMA (p,d,q) with the mathematical notation shown as

$$\phi_p (B) (1-B)^d y_t = \Phi_q (B) \epsilon_t$$

where B is the backshift operator; that is, the operator B imposes a 1-period lag on the variable each time it is applied.

The ARIMA process has three basic steps: identification, estimation and diagnostic checking. The identification process seeks to determine the underlying process, the estimation phase uses the specification identified to estimate the parameters of the time series, and the final phase examines the success of the estimation relative to the actual data.

The identification phase relies heavily on the autocorrelation and partial autocorrelation functions to determine the underlying autoregressive and moving average components. Before examination of these functions, however, the series must be stationary.

When a series is not stationary, this will be displayed by a trend in an observation plot of the data or by a failure of the autocorrelation function to damp quickly to zero. Differencing, "d" in (p,d,q) above, specifies the number of differences required to convert the data to a stationary series, where

y implies $d = 0$

Δy implies $d = 1$

$\Delta^2 y$ implies $d = 2$, etc.

Typically, a time series will be differenced as many times as are required to create a stationary series (where the autocorrelation function damps quickly to zero).

Once a stationary series has been achieved, the autoregressive and moving average components can be investigated. The autoregressive process suggests that the series is generated by a weighted average of past observations, going back "p" periods

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \delta + \epsilon_t$$

$$\text{or } y_t = \phi_p (B) + \delta + \epsilon_t$$

Generally, an autoregressive component will be demonstrated in an autocorrelation function by a sinusoidal pattern.

The moving average process suggests each observation is generated by a weighted average of random disturbances going back "q" periods, denoted as

$$y_t = \mu + \epsilon_t - \Phi_1\epsilon_{t-1} - \Phi_2\epsilon_{t-2} \cdots - \Phi_q\epsilon_{t-q}$$

$$y_t = \mu \Phi_q (B) \epsilon_t$$

A moving average process manifests itself in a sample autocorrelation function with non-zero values where the lag is less than or equal to "q" and near-zero values where the lag is greater than "q".

The general ARIMA process will therefore be to perform such differencing as is required to create a stationary series, then use the autocorrelation function of the differenced data to determine the likelihood of autoregressive (sinusoidal) or moving average (trailing to zero) components.

In this study, only a cursory glance at the autocorrelation function associated with the 3-month Treasury Bill rate was required to determine that the series was a candidate for differencing of one or more orders.[1] The autocorrelation function indicated clear positive serial correlation and, in fact, it was in excess of 20 lags before the function approached zero. One order of differencing was sufficient, however, to cause the series to become stationary and, in fact, a second differencing made no improvement. Hence, a first-order differencing seemed appropriate.

The autocorrelation function of the once-differenced series suggested either a random walk process (a martingale model), a moving average of order 2 (because only the second autocorrelation coefficient was significant) or possibly a mixed autoregressive - moving average process. For this reason, a few ARIMA models which had these characteristics were estimated -- and, once estimated, diagnostic checks performed.

Two diagnostic checks were performed. First, an analysis was undertaken of the residuals generated by the simulated series. To test whether the residuals were white noise, the Box and Pierce "Q-statistic" was used. Where the Q-statistic exceeds the value specified in a chi-square

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[1]Appendix V contains diagrams of the autocorrelation functions for the raw data, as well as for the once- and twice-differenced series.

table, we reject the hypothesis that there is no serial correlation in the residuals and assume that the model should be respecified.

The ARIMA models which passed the Box and Pierce test in this study were, in descending order, as follows:

ARIMA Model	Q-Statistic	Degrees of Freedom
(0,1,2)	4.6	10
(2,1,0)	6.1	10
(1,1,1)	6.2	10
(2,1,1)	8.0	9
(0,1,1)	10.1	11
(1,1,0)	10.4	11
(0,1,0)	11.2	11

Perhaps surprisingly, none of the several higher order models which were estimated passed the Q-statistic test. Also significantly, ARIMA model (0,2,0) did not pass, supporting the conclusion drawn at the start of this section that a first differencing is appropriate, but not a second differencing.

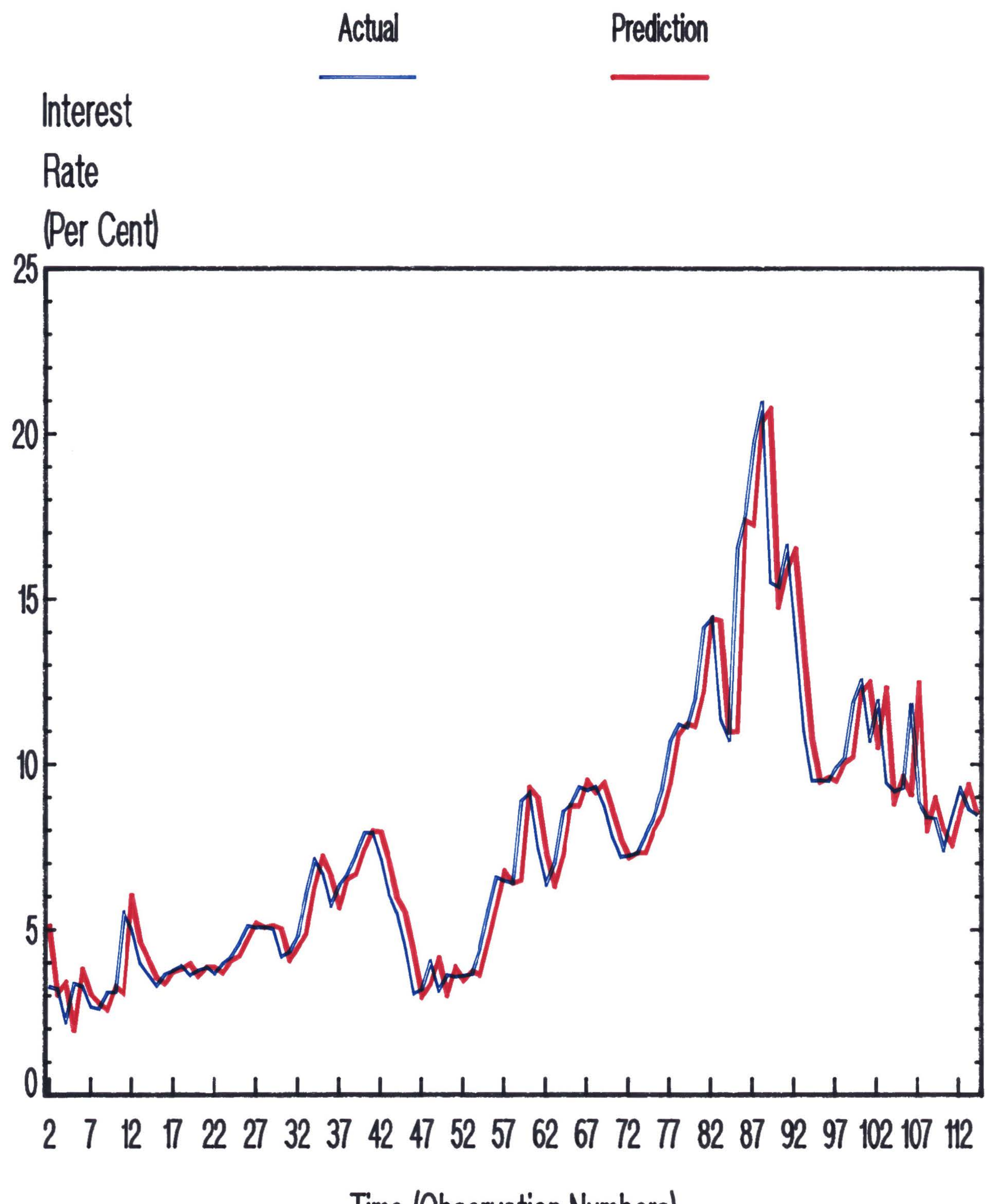
The second and more important diagnostic check considers whether the parameter estimates are significant. These results (shown below) indicate that only ARIMA model (1,1,1) can pass parameter significance tests. Consequently, it is this ARIMA model (1,1,1) which will be used to assess the viability of ARIMA modelling for the purposes of this thesis.

Model	Parameter	t-statistics
(0,1,2)	MA(1)	-1.3
	MA(2)	2.3
(2,1,0)	AR(1)	1.1
	AR(2)	-2.1
(1,1,1)	AR(1)	-2.1
	MA(1)	-3.5
(2,1,1)	AR(1)	10.4
	AR(2)	1.7
	MA(1)	116.1
(0,1,1)	MA(1)	-1.6
(1,1,0)	AR(1)	0.9
(0,1,0)	Not applicable.	

Appendix VII provides the predictions made by ARIMA model (1,1,1) and Figure 5 illustrates the actual values relative to the predicted values.

Figure 5

INTEREST RATE PREDICTIONS
ARIMA Model



Autocorrelation Functions

Following are graphic representations of autocorrelation functions for the raw data and the once- and twice-differenced series. The graphs illustrate that it was necessary to difference the data in order to convert it to a stationary series, but that a second differencing did not improve the characteristics of the autocorrelation function. It therefore appears that a once-differenced series is the most appropriate for this time series.

AUTOCORRELATION FUNCTION OF ORIGINAL SERIES

```
 1 0.94 .                 |RRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRR
 2 0.88 .                 |RRRRRRRRRRRRRRRRRRRRRRRRRRRRRR
 3 0.83 .                 |RRRRRRRRRRRRRRRRRRRRRRRRRRRR
 4 0.78 .                 |RRRRRRRRRRRRRRRRRRRRRRRRRR
 5 0.73 .                 |RRRRRRRRRRRRRRRRRRRRRRRRR
 6 0.69 .                 |RRRRRRRRRRRRRRRRRRRRRRR
 7 0.64 .                 |RRRRRRRRRRRRRRRRRRRRRR
 8 0.60 .                 |RRRRRRRRRRRRRRRRRRRR
 9 0.57 .                 |RRRRRRRRRRRRRRRRRRR
10 0.55 .                 |RRRRRRRRRRRRRRRRRRR
11 0.52 .                 |RRRRRRRRRRRRRRRRRR
12 0.49 .                 |RRRRRRRRRRRRRRRRR
13 0.46 .                 |RRRRRRRRRRRRRRRR
14 0.44 .                 |RRRRRRRRRRRRRRR
15 0.41 .                 |RRRRRRRRRRRRRR
16 0.37 .                 |RRRRRRRRRRRRR
17 0.36 .                 |RRRRRRRRRRRR
18 0.35 .                 |RRRRRRRRRRRR
19 0.34 .                 |RRRRRRRRRRRR
20 0.33 .                 |RRRRRRRRRRR
21 0.32 .                 |RRRRRRRRRRR
22 0.29 .                 |RRRRRRRRRR
23 0.26 .                 |RRRRRRRRR
24 0.24 .                 |RRRRRRRR
```

AUTOCORRELATION FUNCTION OF THE ONCE-DIFFERENCED SERIES

```
 1 0.09 .              |RRR
 2 -.19 .       RRRRRRR|
 3 0.02 .              |R
 4 -.05 .            RR|
 5 -.02 .             R|
 6 0.08 .              |RRR
 7 -.11 .          RRRR|
 8 -.16 .        RRRRRR|
 9 0.05 .              |RR
10 0.03 .              |R
11 0.01 .              |
12 -.03 .             R|
13 -.09 .           RRR|
14 0.08 .              |RRR
15 0.01 .              |
16 -.16 .        RRRRRR|
17 -.07 .            RR|
18 0.06 .              |RR
19 -.03 .             R|
20 -.02 .             R|
21 0.20 .              |RRRRRRR
22  .00 .              |
23 -.06 .            RR|
24 0.01 .              |
```

AUTOCORELATION FUNCTION OF THE TWICE-DIFFERENCED SERIES

```
 1 -.35 .  RRRRRRRRRRRR|
 2 -.25 .     RRRRRRRRR|
 3 0.15 .              |RRRRR
 4 -.06 .            RR|
 5 -.05 .            RR|
 6 0.19 .              |RRRRRR
 7 -.10 .           RRR|
 8 -.12 .          RRRR|
 9 0.11 .              |RRRR
10  .00 .              |
11 0.02 .              |R
12 0.01 .              |
13 -.12 .          RRRR|
14 0.12 .              |RRRR
15 0.06 .              |RR
16 -.15 .         RRRRR|
17 -.02 .             R|
18 0.12 .              |RRRR
19 -.06 .            RR|
20 -.10 .          RRRR|
21 0.22 .              |RRRRRRRR
22 -.07 .           RRR|
23 -.06 .            RR|
24 0.04 .              |R
```

Sensitivity of Theil Coefficients

This appendix is to highlight that Theil coefficients must be used with caution because they are highly sensitive to the particular data points over which they are calculated. As an example, the first OLS regression run for this thesis was

$$_{t+3}R_3 = a_0 + a_1 \; _{t+3}F_{3,t}$$

This regression yields the following estimates:

OBSERVATION	ESTIMATE	ACTUAL	DIFFERENCE
70	8.23	7.78	.45
71	7.48	7.19	.29
72	7.12	7.23	-.11
73	7.32	7.31	.01
74	7.41	7.87	-.46
75	8.38	8.41	.03

The Theil coefficient for the unsystematic error in the forecast is .38 (out of a possible 1.0) over the 71-76 range of observations, but .95 over the 70-75 range. The MEP, on the other hand, changed from only .026 to .030.

This is shown to demonstrate that the Theil figures are highly sensitive and must be viewed with caution.

ARIMA Predictions

This appendix provides the predictions calculated by ARIMA model (1,1,1) described in Appendix IV, and the premiums derived therefrom, as described in Section 3.4.

Observation	Actual	Prediction	Premium
2	3.27	5.12	0.74
3	3.19	3.05	0.62
4	2.21	3.37	0.13
5	3.37	1.97	0.86
6	3.28	3.76	0.13
7	2.65	3.03	0.57
8	2.59	2.75	0.16
9	3.10	2.55	0.55
10	3.11	3.26	0.10
11	5.50	3.05	0.43
12	5.00	5.98	-0.06
13	3.98	4.64	0.62
14	3.63	4.10	0.04
15	3.29	3.53	0.29
16	3.64	3.34	0.08
17	3.77	3.71	0.17
18	3.91	3.79	0.31
19	3.61	3.96	0.18
20	3.77	3.57	0.30
21	3.86	3.86	0.16
22	3.65	3.85	0.22
23	3.97	3.67	0.14
24	4.17	4.05	0.08
25	4.59	4.19	0.34
26	5.12	4.69	0.26
27	5.06	5.19	0.29
28	5.07	5.05	0.09
29	5.02	5.12	0.10
30	4.17	5.02	0.02
31	4.33	4.08	-0.03
32	4.82	4.46	0.26
33	6.04	4.85	0.36
34	7.10	6.26	-0.04
35	6.67	7.17	-0.31
36	5.74	6.59	-0.24

Observation	Actual	Prediction	Premium
37	6.34	5.68	0.08
38	6.69	6.52	0.09
39	7.26	6.66	0.25
40	7.92	7.41	-0.14
41	7.92	7.97	-0.25
42	7.12	7.93	-0.17
43	6.03	7.03	-0.63
44	5.48	5.96	-0.02
45	4.44	5.48	-0.03
46	3.05	4.31	0.15
47	3.19	2.96	0.15
48	4.02	3.32	0.08
49	3.17	4.11	-0.03
50	3.62	3.02	0.27
51	3.56	3.84	0.34
52	3.60	3.44	0.62
53	3.64	3.73	0.49
54	4.42	3.60	0.40
55	5.55	4.61	0.39
56	6.58	5.65	0.66
57	6.46	6.73	0.63
58	6.39	6.38	0.32
59	8.87	6.47	-0.14
60	9.14	9.26	-0.54
61	7.42	8.96	-0.36
62	6.37	7.31	-0.63
63	7.02	6.32	-0.04
64	8.57	7.20	-0.04
65	8.75	8.73	-0.06
66	9.31	8.71	-0.02
67	9.20	9.47	-0.35
68	9.32	9.12	-0.40
69	8.73	9.43	-0.81
70	7.78	8.60	-0.49
71	7.18	7.75	-0.37
72	7.23	7.15	-0.11
73	7.31	7.30	-0.07
74	7.87	7.31	0.00
75	8.41	8.00	0.25
76	9.23	8.45	0.32
77	10.69	9.38	-0.24
78	11.21	10.87	-0.20
79	11.10	11.21	-0.60
80	11.99	11.12	-0.60
81	14.13	12.16	-0.17
82	14.43	14.40	-1.42

Observation	Actual	Prediction	Premium
83	11.36	14.33	0.55
84	10.76	10.96	-1.11
85	16.52	10.97	0.36
86	17.57	17.36	-1.08
87	19.70	17.21	-1.99
88	20.92	20.34	-3.56
89	15.49	20.73	-1.97
90	15.36	14.77	-0.56
91	16.59	15.86	-1.08
92	13.89	16.48	-0.79
93	10.99	13.56	-0.43
94	9.49	10.78	-0.48
95	9.51	9.44	0.05
96	9.48	9.59	-0.17
97	9.90	9.47	0.25
98	10.20	10.03	0.05
99	11.85	10.20	0.48
100	12.51	12.16	0.71
101	10.73	12.45	0.07
102	11.89	10.53	0.00
103	9.43	12.26	0.62
104	9.17	8.82	0.41
105	9.28	9.59	-0.14
106	11.82	9.05	0.05
107	8.86	12.43	-3.17
108	8.39	8.00	1.54
109	8.35	8.94	-0.28
110	7.42	8.01	0.28
111	8.42	7.54	0.05
112	9.26	8.54	0.48
113	8.63	9.35	0.88
114	8.46	8.50	0.79

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