Weighted Downlink Beamforming Algorithm in Mobile Communication

by

Himika Rahman
BSc, Bangladesh University of Professionals, 2010

A Project Submitted in Partial Fulfillment
of the Requirements for the Degree of

MASTER OF ENGINEERING

in the Department of Electrical and Computer Engineering

Supervisory Committee

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Abstract

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In recent times, an immense increase in traffic has been experienced by wireless communication systems, due to a significant growth of number of users and the development of new high bit rate applications. It is expected that soon this trend will be established. This challenging scenario involves not only a well-established market of cellular systems, but also a field of emerging wireless technologies, such as WiMAX (Worldwide Interoperability for Microwave Access) for wireless metropolitan area networks, and Wi-Fi (Wireless Fidelity) for wireless local-area networks, mobile ad-hoc networks and wireless mesh networks. In order to satisfy the increasing demand of network capacity, the exploitation of spatial domain of communication channel using multiple antenna systems can be a critical improvement for enhancing the spectral efficiency of wireless systems. The combination of antenna arrays and beamforming algorithms can suppress the undesired sources and receive the signals incoming from the desired ones. Downlink beamforming is a well-known technique to reduce strong interferences effectively induced by high data rate users. Different weighted beamforming methods for downlink transmission in cellular mobile communication systems using an antenna array at the base station are presented. The techniques are based on estimation of beam patterns in terms of the desired, and the interference signals at the base transceiver station using different weighted beamforming algorithms. Beam-patterns for a special case of three mobiles, uniform linear array at the base station are obtained using two well-known algorithms.

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Acknowledgments

I would like to express my honest gratitude to my supervisor Dr. Pan Agathoklis for his invaluable guidance, patience, and persistent support towards my Masters of Engineering project. His assistance directed me in research, design and implementation of my project. Besides my supervisor, I would like to thank Dr. Dale Shpak for sharing expertise; valuable suggestions and encouragements extended to me during this research and serving my supervisory committee.

I would also like to extend my thanks to my friends Chamira Edussooriya, Farnoosh Talaei for their support during my study. At last, I am deeply thankful to my parents, parent-in-laws and husband for their endless compassion and support who played an outstanding role in completion of my graduate studies.

Dedication

I would like to dedicate my report to my parents (Dr. Hamidur Rahman, Dr. Bilkis Begum), my parent-in-laws (Dr. Abu Hanif Sheikh, Shahina Pervin) and my beloved husband (Ehtesham Al Hanif) for their endless love, support and encouragement in all stages of my life.

Chapter 1: Introduction

The rapid growth of mobile users and the increasing amount of next-generation data services requirement has made the issues of growing capability and performance improvement for wireless communication systems progressively critical [1], [2].

In recent years, interference cancelation through beamforming [3] has been recognized as one of the most promising and cost-effective techniques in order to raise the capacity and carrier efficiency of a wireless communication system. To reduce wireless interference, the 5th generation Wi-Fi standard 802.11ac adopts an efficient technology, named as beamforming. In early 1960's, the performance of sensor arrays was developed in the field of sonar [4]–[8], radar [9]–[12] and seismic signal processing [13]–[18] (Seismic waves are energy waves that travel through the Earth's layers as a consequence of an earthquake, explosion, or a volcano that gives low-frequency audio power). In downlink mobile communication, one of the most accomplishing goals is to reject noise and interference (multiple access interferences, MAI) in the downlink through downlink beamforming to improve beam output signal-to-noise ratio (SNR) [19].

Wide band and narrow band direct-sequence code-division multiple-access (DS-CDMA) is a potential radio access technique for mobile communication systems because of its elasticity to carry a diversity of voices, videos, and data services. These services will require higher data rates and higher received signal power levels, consequently, create a larger amount of interference among users for wide-band DS-CDMA. In this case, high system capacity will be obtained if the interference levels are reduced efficiently.

Spatial-division multiple access (SDMA) [20]–[22] allows to have more antenna elements, which are set at the base station to facilitate, receive and transmit data from and to the desired user and to restrain interferences. Uplink and downlink beamforming are one of the major operations in SDMA. Both Uplink and downlink beamforming consists of uplink and downlink beamforming weight generation. But the major difference is uplink beamforming includes uplink signal demultiplexing, where downlink beamforming includes downlink signal multiplexing. Hypothetically, in both cases, the related link channel responses are important to generate the appropriate beamforming weights.

Research showed that uplink beamforming is easier for implementation than downlink beamforming since the antenna array is generally set at the base station. In addition, the uplink capacity of DS-CDMA systems can be increased by power control and joint beamforming. Maximal ratio combining (MRC) beamforming criteria and power-based power control scheme and are stated in [23]; whereas in [24], minimum mean-square-error (MMSE) beamforming approaches and signal-to-interference ratio (SIR) based power controls are projected. Compared to single antenna scenarios both the methods can considerably improve the whole system capacity. In reality, improvement of the overall system capacity is also desirable by increasing the downlink capacity. In general, downlink performance is even more significant for the next-generation communication systems, including wireless internet, video-on-demand, and multimedia services and so on.

However, possible lack of direct measurement of downlink channel responses at the base station is a high concern for downlink beamforming problem, particularly for frequency-division duplex (FDD) systems. Therefore, probing-feedback approach [25] is a conceptually simple method for downlink channel estimation, but it is only applicable in environments that are slow in time, and may require absolute remodel of uplink and downlink protocols. Another numerical method is based upon the use of direction-of-arrival (DOA) information, which depends on uplink received signals [26],[27].

In wireless communication systems, subscribers are usually spatially separated, and the use of antenna arrays makes it possible to track down the direction-of-arrival (DOA) of each signal and locate the position of a subscriber. The direction of arrival (DOA) of signals from different mobile stations (MSs) in uplink are estimated by the conventional generalized eigenvalue based beamforming techniques [28]–[31]. Based on the position information, the spatial separation can be exploited through beamforming to multiplex the channel in the spatial dimension in addition to the frequency and time dimensions to receive and transmit signals in a directional approach. Therefore, the effect of CCI can be reduced. Moreover, by using beamforming, the capacity, carrier efficiency, and coverage of a wireless communication system can be significantly improved.

Both uplink and downlink signals pass through one station to other due to the same scatter surrounding the mobile and the base station. As a result, the DOAs of the uplink signals might be the only considerable stable parameters for downlink channel estimation. DOA-based approaches utilize the received uplink signals to compute the DOAs of the target user first; then downlink channel responses are constructed using known relations between uplink and downlink steering vectors.

After that, lack of efficient downlink beamforming algorithm is considered as other issues that are complicating a downlink beamforming problem, even though the downlink channel responses are available. Particularly, it is often believed that complicated multivariable optimization problem solving is required to obtain the optimal downlink beamforming weights. One such method is MRC method that sets the downlink channel responses as the downlink beamforming weights, and keeps the main beam of the downlink beam pattern toward the desired user. However, MRC-based downlink beamforming may fail to provide sufficient system capacity to match its uplink counterpart, when uplink uses MMSE beamforming, specifically, for the cases when lower data rate users are spatially closed to higher data rate users.

The use of beamforming for interference rejection is especially attractive in the third-generation (3G) and future wireless communication systems where capacity, carrier efficiency, and coverage are the most important issues. Wideband code-division multiple-access (WCDMA) and code-division multiple access-2000 (CDMA 2000) standards are the 3G standards that are designed to provide the pilot channels, which are required for fast and accurate DOA estimation and beamforming. In the universal mobile telecommunications system (UMTS), the dedicated physical control channel (DPCCH) in the uplink is used to transmit pilot symbols at each mobile station (MS) and user-specific beamforming allows the generation of individual beams at the base station (BS) for each MS without any limitations of selecting beamforming weight vectors [32]. In the downlink, each beam is related to a unique secondary common pilot channel (S-CPICH); therefore, MSs can use it to distinguish the signal coherently. In CDMA 2000 systems, a user-specific pilot signal is available for both uplink and downlink that can be used as a reference signal for DOA estimation and adaptive beamforming [33], [34].

Recently, a virtual uplink beamforming and a power control technique (V-UBPCT) is proposed by Rashid-Farroki et al. to obtain downlink beamforming weights for SDMA [35], that requires simple computations similar to real uplink beamforming. The achievement of V-UBPCT relies on the combination of power control and downlink beamforming. For this reason, the multi-variable optimization algorithm in [36] is not a self-completed algorithm. However, the limitation of V-UBPCT algorithm [35] is that it cannot be applied to multimedia DS-CDMA systems directly due to the following reasons:

I. In DS-CDMA systems, different users may entail different quality of service (QoS), such as data rates, which should be taken into consideration while designing the optimal downlink beamforming weights.

II. In DS-CDMA systems, when the downlink problem is transformed into a virtual uplink one, due to the existence of interfinger interference (IFI), the generated virtual uplink problem does not correspond to a pure uplink situation. In particular, the presence of IFI makes downlink beamforming problem way more complex.

Therefore, the purpose of this project is to analysis the performance of different beamforming scheme for DS-CDMA systems such as: Minimum Variance Distortion-less Response (MVDR), Linear Covariance Minimum Variance (LCMV) beamforming algorithms and compare their outcome in terms of reducing the effect of interference signals. Simulation Results show that both methods lead to maximum antenna gains in the directions of the desired MS, but LCMV method is more efficient in minimizing the interference effect than the MVDR method.

1.1 Previous Work

1.1.1 DOA Estimation

The DOA of signals is required in most smart antenna techniques where signals are transmitted and received in a directional manner. The performance of these methods relies heavily on the accurate estimation of the DOA of each signal. Various methods for DOA estimation have been proposed [37]–[48] in the past several decades. The most commonly used among these techniques are estimations of signal parameters via rotational invariance technique (ESPRIT) [41]–[43], multiple signal classification (MUSIC) [39], [40], and their differences [44], [45]. For DOA estimation, the signal incident over ULA of dipoles can be modeled as,

$$x(t) = A(\theta)s(t) + n(t)$$

where, $A(\theta)$ is a steering vector ($m \times p$) (p uniform plane waves incident on an array of m elements). $s(t)$ is a signal vector $(p \times 1)$, and $n(t)$ is a noise vector $(m \times 1)$. DOAs are contained in $(p \times 1)$ parameter vector θ. Also, signal received at array elements is stored in $x(t)$ and given by the following equation:

$$x(t) = \begin{bmatrix} x_1(t_1) & x_1(t_2) & \dots & x_1(t_{samples}) \\ x_2(t_1) & x_2(t_2) & \dots & x_2(t_{samples}) \\ \dots & \dots & \dots & \dots \\ x_m(t_1) & x_m(t_2) & \dots & x_m(t_{samples}) \end{bmatrix}$$

where, $t_1, t_2, \ldots, t_{samples}$ are the instances at which signal is sampled. Further, it is assumed that $m > p$ (more antennas than signals). DOA was estimated using different algorithms both based on spectral estimation techniques and subspace methods. Block diagram of signal processing for FM DOA estimation is shown in Figure 1. DOA algorithms can basically be divided into two main categories [49]:

(i) Spectral and

(ii) Subspace estimations.

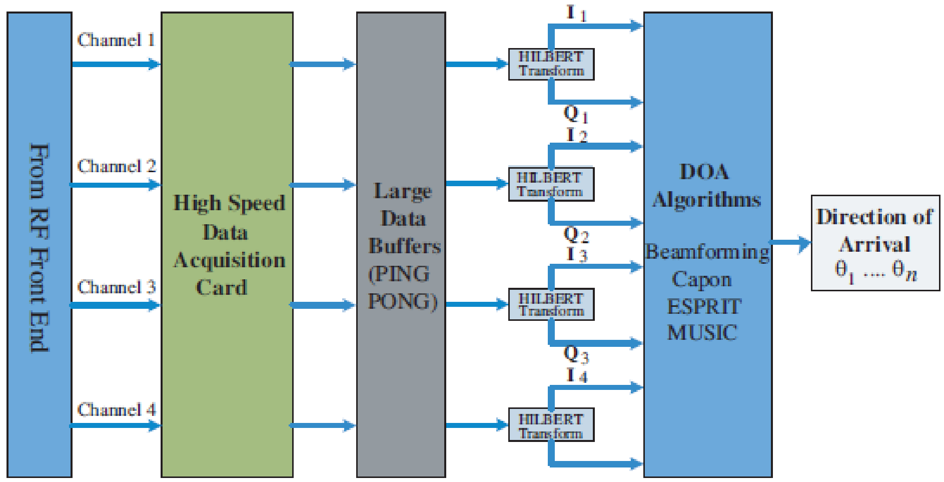

Figure 1 Signal processing block diagram

Capon's Algorithm

Capon's minimum variance method attempts to overcome the poor resolution and noise resolving problems associated with classical beamforming [50]. The technique uses some of the degrees of freedom to form a beam in the desired direction while simultaneously using the remaining degrees of freedom to form nulls in the direction of interfering signals. Capon weighting vector can be given as [50]

$$w_{CAP} = \frac{R'^{-1} a(\theta)}{\sqrt{a^H(\theta) a(\theta)}}$$

The output power of the array $P_{CAP}(\theta)$, as a function of DOA, is given by Capon's spatial spectrum.

$$P_{CAP}(\theta) = \frac{1}{a^H(\theta)R'^{-1}a(\theta)}$$

By computing and plotting the spectrum over the whole range of θ the DOA can be estimated by locating the peaks in the spectrum [50]. Capon's method requires the computation of a matrix inverse, which can be computationally expensive for large antenna arrays [50].

1.1.2 Single User Beamforming (SUB)

Spatially selective reception and transmission are consummate by using adaptive beamformer. A beamformer is a spatial filter with a narrow passband in a target direction that optimally combines received signals from different antenna elements in such a way to enhance signals arriving from a target source. The goal of SUB in the uplink of mobile communications is to maximize the power received from the target MS and at the same time minimize the received power from MSs other than the target one [19], [38]. In early SUB techniques, the beamforming weights for different MSs are optimized individually. Various optimality criteria have been proposed to obtain uplink SUB weights using optimal weight control. In [51], beamforming weights are selected in order to minimize the mean squared error (MSE) between the reference signal and the beamformer output signal. In [52], [53], the signal-to-interference noise ratio (SINR) of the signal is maximized at the beamformer output. The minimum variance (MV) criterion has been used in [54] to minimize the noise variance at the beamformer output, and the ML (Maximum Likelihood) rule have been used in [55] to obtain the beamforming weights.

Different beamforming algorithms have been developed, based upon these optimality criteria. Performance analysis of various beamforming schemes can be found in [38][56]–[58]. The computational complexity of beamforming algorithm based on different optimality criteria is discussed in [51]. The effect of receiver nonlinearity and random error on adaptive beamforming is analyzed in [59].

The objective of downlink SUB is to transmit maximum power to the target MS and meanwhile minimize it to other MSs sharing the same frequency channel. Conventional generalized eigenvalue-based SUB algorithms [26], [28]–[31], [60] have been generally used to obtain the weights for transmit beamforming adaptively. In [26], maximizing the downlink signal power to the target MS helps to obtain the beamforming weights relative to the total power radiated in the direction of other MSs and at the same time it keeps the antenna array gain constant in the direction of the desired MS. In [28]–[30], the power from the downlink signal to the target MS is maximized while keeping the total power to other MSs less than or equal to a given constant level. The optimality criterion used in [31] aims to transmit a given power to the desired MS and simultaneously minimize the power to other MSs. It can be shown that all these criteria are equivalent in the sense that they lead to the same direction of weight vector and; therefore, the same radiation pattern is obtained.

1.1.3 Multiuser Beamforming (MUB)

Though, the SUB algorithms are computationally simple, but it provides suboptimal solutions to minimize the BS transmitted power. Recently, a more dominant approach has been proposed, namely, MUB [60]. In the MUB approach, the beamforming weights for all MSs are mutually optimized. For the uplink, MUB is formulated as an optimization problem where the weight vectors for different MSs are mutually optimized at the BS to satisfy given SINR conditions at the BS and, concurrently, minimizes the total power transmitted from all the MSs.

Both the uplink and downlink MUB formulates non-convex with quadratic constraints optimization problems. In [24], an iterative algorithm is developed to solve the optimal uplink MUB. Two different algorithms have been developed for downlink MUB, that is called, duality-based [35], [42] and semi-definite programming (SDP) [61]–[63]. Initially, the duality between uplink and downlink was presented in [35] where it is shown that optimal downlink weight vectors can be obtained through the application of a virtual uplink system. An optimal MUB algorithm is developed based on this duality, to obtain iteratively the downlink optimal weight vectors. As the SINR requirements become more severe, the early version of the duality-based algorithm [35] tends to converge more slowly. The uplink and downlink duality is further discussed in [64], [65] and a novel duality-based MUB algorithm is projected in [66] where numerous stopping criteria are anticipated to improve the convergence behavior of the iterative algorithm. The SDP based MUB algorithm is described in [61]–[63]. In this algorithm, the optimal MUB optimization problem is transformed into an SDP optimization problem after Lagrangian relaxation [67]. Therefore, from the optimal solution of the SDP problem the weighting vectors are determined. The solution of SDP based MUB required complex computations and requires longer simulation time as like increasing number of antenna elements.

1.1.4 Outline of the Report

Chapter 2 contains literature review such as an introduction to antenna systems and preliminary studies on antenna response, SUB, and MUB for mobile communications. Chapter 3 provides the main body of the system model and comparison of the MVDR and LCMV beamforming technique.

In chapter 4, the comparison of the beam-patterns obtained using two different beamforming algorithms are presented. This is done for a special case of three mobiles and a base station with a uniform linear array (ULA) with varying number of antennas.

Finally, Chapter 5 describes conclusion, application, and future work.

Chapter 2: Literature Review

In this chapter, the fundamental ideas of beamforming are going to present, which are relevant to this project. The chapter includes background knowledge, concepts, and terminology about radio propagation, and then different antenna systems in wireless communications. Finally, beamforming configurations, DOA estimation, SUB, and MUB for wireless communications are established.

2.1 Radio Propagation

Wireless communication systems function in radio environments such as urban areas, mountains, forests, and plains, etc. Depending on the radio environment; a wireless radio channel can embrace a line-of-sight signal path or multipath that is severely obstructed by surrounding buildings, foliage, and mountains; consequently, the power of a transmitted signal is attenuated. Therefore, the received signal power is smaller than the transmit power. This is known as path loss. In this section, the small-scale fading and large-scale path loss due to multipath and Doppler spread will be discussed.

2.1.1 Large-Scale Path Loss

Large-scale path loss describes the variation of the average power of a received signal with respect to the distance between the receiver and the transmitter. The term 'large-scale' refers to the small fluctuation of the normal power of the received signal during the time that the transmitted signal travels a long distance relative to the carrier wavelength. It has been found that there is a normal power drop of a received signal in proportion to the logarithmic distance between the receiver and the transmitter in either indoor or outdoor environments. If the average large-scale path loss $\overline{p}(l)$ in dB, then a line-of-sight path for a random receiver-transmitter separation can be expressed as [68]:

$$\overline{p}(l) = \overline{p}(l_0) + 10n\log\left(\frac{l}{l_0}\right) \qquad \text{Eqn. (2.1)}$$

where, l the distance between the receiver and transmitter is, l_0 is the reference distance that is calculated from the measurements near the transmitter, and n stands as the path loss exponent that specifies the rate at which the path loss increases with distance.

On the other hand, if the signal paths are not a line-of-sight, the obstructions surrounding the transmitter will reflect the transmitted signal that initiates statistical variability to the average power of a received signal, known as shadowing effect. Considering the shadowing effect, the path loss is a random variable having a log-normal distribution [68]. A general expression of the average large-scale path loss is given by

$$\overline{p}_L(l) = \overline{p}(l_0) + 10n\log\left(\frac{l}{l_0}\right) + x_\sigma \qquad \text{Eqn. (2.2)}$$

where, x_σ is a zero-mean Gaussian distributed random variable (dB) with σ as standard deviation [69].

2.1.2 Small-Scale Fading

When a transmitted signal propagates through a wireless channel, it may suffer from small-scale fading. Depending on the channel, the bandwidth of the transmitted signal, and the velocity of an MS, Doppler spread, multipath delay, and led to small-scale fading

[70]. Relative to large-scale path loss, small-scale fading causes the received signal to change rapidly while the signal travels through a short distance.

2.1.3 Multipath Fading

In mobile communication systems, the transmitted signal reflects by nearby obstructions and travels through multiple paths to the receiver. Since different paths have different propagation delays and losses, the received signal will be a combination of several time-delayed versions of transmitted signal. This leads to frequency-selective or flat fading. A transmitted signal will suffer flat fading if the mobile radio channel has fixed amplitude and linear phase response over the bandwidth of the transmitted signal. In such a case, the signal bandwidth is narrower compared to coherence bandwidth of the mobile radio channel. When root-mean-square path delay is smaller than a signal symbol period, and the spectral characteristic of the transmitted signal is unchanged after propagating through the mobile radio channel, then the channel is known as a flat fading channel. Moreover, flat fading channel gain varies with time due to the multipath effect that causes amplitude oscillations in the received signal. The Rayleigh distribution is considered as one of the most commonly used amplitude distributions for a flat fading channel. The probability density function of Rayleigh's distribution is given by [71].

$$p(m) = \begin{cases} \frac{m}{\sigma^2} e^{\frac{-m^2}{2\sigma^2}} & 0 \le m \le \infty \\ 0 & otherwise \end{cases} \qquad \text{Eqn. (2.3)}$$

where, m represents the envelope amplitude of the received signal, and σ^2 is the power of the multipath signal before envelope detection.

If the coherence bandwidth is lower than the signal bandwidth of the mobile radio channel, then the channel is frequency-selective, and transmitted signal will undergo frequency-selective fading. In such a case, the spectral characteristic of transmitted signal is no longer preserved at the receiver that causes time-varying distortion. The amplitude and phase of the received signal change rapidly with time. A frequently used multipath fading method is the two-ray Rayleigh fading. The impulse response of a two-ray model is given by [72].

$$h_b(t) = \alpha_1 e^{j\phi_1}\delta(t) + \alpha_2 e^{j\phi_2}\delta(t-\tau) \qquad \text{Eqn. (2.4)}$$

where, α_1 and α_2 are two independent variables with Rayleigh's distribution; ϕ_1 and ϕ_2 are two independent variables with a uniform distribution over [0, 2π], and τ is the time delay between the two rays.

2.1.4 Doppler Fading

If an MS is in motion, the transmitted signal will undergo Doppler fading due to the Doppler shift. A Doppler fading channel is characterized by two important parameters, coherence time and Doppler spread. The Doppler spread with maximum Doppler shift can evaluate the spectral change of the Doppler fading channel. In the time domain, the coherence time of the Doppler spread is doubled. It is a measure of the duration over which the channel remains unchanged.

If the bandwidth of a transmitted signal is greater than the Doppler spread, a symbol period would be greater than the channel coherence time. In such a case, the channel is static during several symbol periods and thus the transmitted signal will undergo slow fading. In contrast, if the signal bandwidth is lower than the Doppler spread, a symbol period will be greater than the time of channel coherence. In this context, the channel behaves like a fast fading channel those changes in a symbol period. Details of modeling and simulation of the Doppler fading can be found in [72], [73].

2.2 Antenna Systems in Wireless Communications

There are four categories of antennas have been used for mobile communications, which is omnidirectional, sectored [74], switched beam antennas [75], and adaptive antenna arrays.

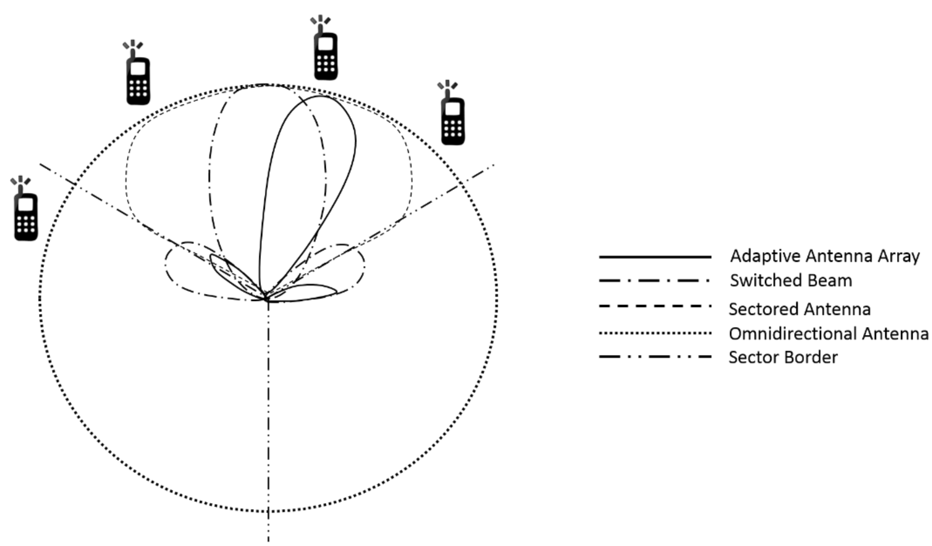

Figure 2 Beam patterns of different antenna systems

An omnidirectional antenna has a circular beam pattern with a uniform gain in all directions as shown in Figure 2. Using an omnidirectional antenna, signals will be uniformly transmitted and received in all directions. In the uplink of CDMA communication systems, an omnidirectional antenna at the BS will receive the signal of interest (SOI) from the target MS along with co-channel interference caused by all other

MSs in the service area. As a result, high transmitted power is required at the MS to satisfy the SINR requirement at the BS. In the downlink, an omnidirectional antenna will uniformly radiate power in all directions. Since the target MS receives signals at only one place at a time, most of the energy is wasted. In addition, an omnidirectional antenna causes co-channel interference to other MSs and BSs that are using the same frequency channel.

Cell sectorization has been widely used to increase the capacity of mobile communication systems; for example, global system for mobile communications (GSM) and the interim standard 95 (IS-95) CDMA communication systems [74], [76]. In these systems, each cell is divided into three or more sectors, and the same number of directional sector antennas is deployed at the BS. The sectored antenna uses one fixed beam in a sector as shown in Figure 2. Signals are transmitted and received through the beam covering only one sector other than the whole cell like an omnidirectional antenna. In this way, both uplink and downlink co-channel interference becomes limited to one sector and the influence of co-channel interference over system performance is minimized. This leads to an increase in cell capacity and reduction in the transmitted power at the BSs and MSs. If the radiation pattern of a sector is ideal without overlapping, then a cell with N sectored antennas should have approximately N times more capacity than a cell with an omnidirectional antenna.

A switched-beam antenna forms several fixed narrow beams. Although they cannot be steered to follow an MS but the best beam that leads to the highest SINR is selected to communicate with it. This further reduces the effect of co-channel interference relative to the sectored antenna systems. It has been shown in [77] that in general downlink interference can be reduced by approximately 6 dB by installing an eight-beam antenna system with a 120^0 sector configuration.

In contrast to an omnidirectional antenna, or a sectored antenna, or a switched beam antenna, an adaptive antenna array combines a digital signal processor and an antenna array in order to receive and transmit signals in a directional manner. It tracks the movement of an MS and the change to the radio environment, dynamically adjusts a narrow beam towards the MS, and simultaneously minimizes the power of CCI. In the uplink, a beam can be steered towards a direction such that the received power of the SOI at BS is maximized, and the power of co-channel interference from other MSs is minimized. In downlink, a beam pattern at the BS is chosen to maximize the signal power received by the target MS and simultaneously to minimize the power at the other MS. Using an adaptive antenna array, the co-channel interference to other MSs and BSs is reduced along with both transmitter, and receivers are power efficient. As a result, it will improve system capacity, frequency efficiency, and coverage [78].

2.3 Antenna Response Vector

An antenna array consists of a number of antenna elements that are distributed in a particular pattern. For higher user capacity in 3G wireless networks, beamforming in the smart antenna is known as a promising technology that reduces CCI effectively [79]. Smart Antennas, are also called as adaptive array antennas or multiple antennas, which are utilized to maximize the efficiency of a digital wireless communication system. Typically, it performs effectively with the diversity effect at the transceiver of a wireless system that acts as a source or a destination. The term diversity effect denotes the reception and transmission of multiple radio frequencies that are applied to reduce the errors during data communication. Therefore, it also increases the speed of data transfer between the source and the destination. The special antenna array is already well established in the wireless communication systems because of their usage in signal processing algorithms that can simply locate various wireless targets, including mobiles. In addition, it can be used to evaluate the beamforming vectors and the DOA of a signal [79].

When the signal bandwidth is much smaller than the carrier frequency, then it is called band-limited signals. If an antenna array response vector of an M-element antenna array with arbitrary configuration is assumed, then it can be written as bellow:

$$a(\theta,\psi) = \begin{vmatrix} A_1(f_c) \\ A_2(f_c)e^{-j2\pi f_c \tau_2(\theta,\psi)} \\ \vdots \\ A_M(f_c)e^{-j2\pi f_c \tau_M(\theta,\psi)} \end{vmatrix} \qquad \text{Eqn. (2.5)}$$

where, $A_M(f_c)$ represents the amplitude response at the $m-th$ antenna element for the carrier which has frequency f_c, τ_M is the delay of the signal impinging on the $m-th$ antenna element relative to that on the first antenna element which is the reference one, and θ, ψ are the azimuth and elevation angles, respectively.

After down-converting to baseband, the signal received from an M-element arbitrary antenna array for a single signal source can be represented by the M-dimension vector as

$$u(t) = s(t)a(\theta,\psi) + n(t) \qquad \text{Eqn. (2.6)}$$

where, $s(t)$ and $n(t)$ represents the signal of interest (SOI) and the background noise, respectively.

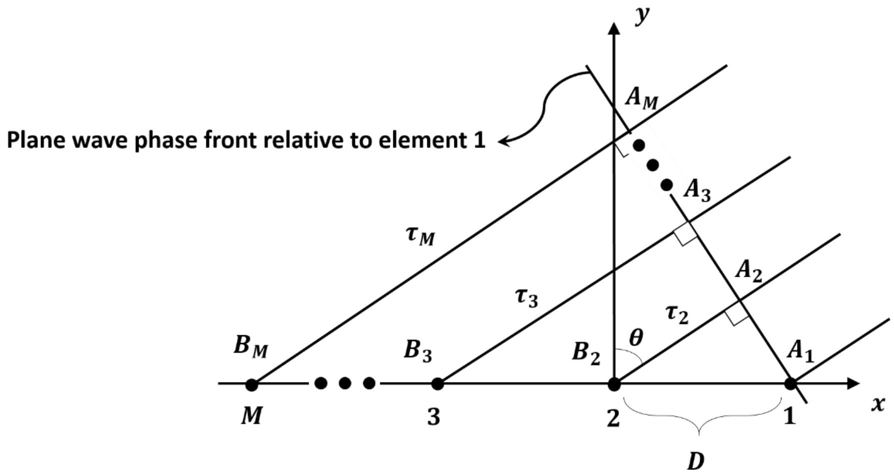

Figure 3 Inter-element signal delay of a uniform linear antenna array

Again, consider an M-element uniform linear antenna array (ULA) along the *x*-axis with isotropic antenna element spacing of D as illustrated in Figure 3. If it is assumed that a plane wave, that is $,\psi = 0$, carrying a baseband signal arrives at the ULA in the horizontal plane at an azimuth angle θ, the delay of the signal received at the *m-th* antenna element is given by

$$\tau_M(\theta,0) = \tau_M(\theta) = \frac{(m-1)D\sin\theta}{c} \quad \text{Eqn. (2.7)}$$

where, c is the speed of light. Since all the elements are isotropic and have the same amplitude response, without sacrificing generality, it can be assumed that $A_m(f_c) = 1$ for $m = 1,2,....,M$. The antenna response vector in (2.5) is then simplified as

$$a(\theta) = [1..z(\theta)...z(\theta)^{M-1}]^T \quad \text{Eqn. (2.8)}$$

where, $z(\theta) = e^{-j2\pi D\sin\theta/\lambda_c}$ and λ_C is the carrier wavelength.

2.4 DS-CDMA System

In DS-CDMA transmission, the user data sequence is multiplied by a binary data sequence [80]. The duration of an element within code is called "chip time". Whereas, the ratio in between chip time and the user symbol time is referred to spread factor.

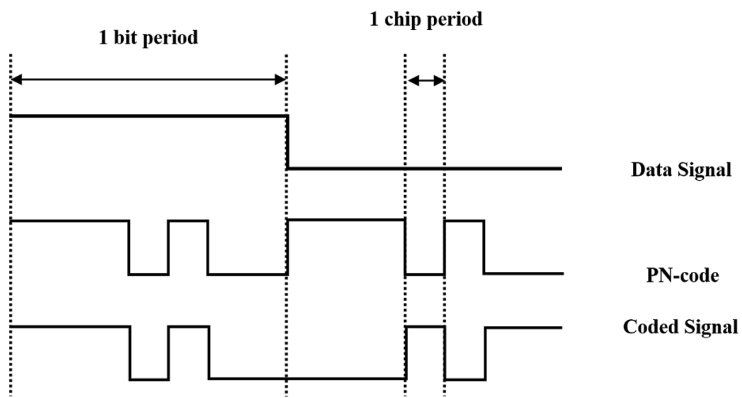

Figure 4 DS-CDMA system

Figure 4 shows the basic idea of a DS-CDMA system. It has been considered as a practical scheme to reduce multiple access interferences in quasi-synchronous transmission. This method allows the reduction of multiple access interferences by transferring the orthogonal characteristics of signals into the frequency domain where the orthogonality is robust relative to chip offsets among the spreading codes of the different users. Thus, the multi-carrier technique results in frequency non-selective fading in the sub-channels, because of the narrower bandwidth.

2.5 Uplink Beamforming

In the uplink of wireless communication systems, the signal arrives at an antenna array at the BS consists of the signal components from the target MS through multiple paths and co-channel interference from MSs other than the target one. The effect of co-channel interference can be reduced through the use of uplink beamforming. In this section, a number of existing uplink SUB and MUB algorithms for wireless communication systems will be discussed.

2.5.1 Uplink Signal Model

Consider an M-element antenna array deployed at a BS assuming that there are L_k paths for the $k-th$ MS. After down-converting to baseband, the received signal $x_{k,l}$ corresponding to the $l-th$ path of MS k is given by

$$x_{k,l}(t) = \sqrt{p_k}\beta_{k,l}s_k(t-\tau_{k,l})a(\theta_{k,l}) \qquad \text{Eqn. (2.9)}$$

where p_k is the transmitted power by the *k-th* MS, $\beta_{k,l}$ is the complex channel response for the *l-th* path, $s_k(t)$ is the normalized transmitted signal, $\tau_{k,l}$ is the path delay, $\theta_{k,l}$ is the DOA of the *l-th* path of MS k, and $a(\theta_{k,l})$ is the antenna array response vector.

If the number of MSs in a BS service area is *K*, then received signal at the antenna array can be represented by the M-dimensional vector

$$x(t) = \sum_{k=1}^{K}\sum_{l=1}^{L_K} x_{k,l}(t) + n(t) \qquad \text{Eqn. (2.10)}$$

where, $n(t)$ denotes M-dimensional complex noise vector with zero mean covariance.

$$E[n(t_1)n(t_2)] = \sigma_n{}^2\delta(t_1 - t_2)I \qquad \text{Eqn. (2.11)}$$

Given the SOI $x_{u,v}(t)$ from the path of MS u, the multiple access interference (MAI) comprises of signal from other paths and MSs, and the received signal in (2.10) may be re-written as

$$x(t) = x_{u,v(t)} + x_I(t) \qquad \text{Eqn. (2.12)}$$

where,

$$x_I(t) = \sum_{\substack{l=1 \\ l \neq v}}^{L_V} x_{u,l}(t) + \sum_{\substack{k=1 \\ k \neq u}}^{K} \sum_{l=1}^{L_K} x_{k,l}(t) + n(t) \qquad \text{Eqn. (2.13)}$$

denotes the interference plus noise.

2.5.2 Uplink Single User Beamforming

In the uplink SUB of a mobile communication system, the SOI arriving at the antenna array at the BS rarely has the same DOA as the interfering components and thus the SOI can be spatially resolved from the received signal by passing it and rejecting the interference at the beamformer. This can be achieved by selecting beamforming weights to obtain a high antenna gain in the direction of SOI and a small gain in the directions of interfering components. The simplified schematic diagram of a typical uplink per-path-per-beamformer SUB system is illustrated in Figure 5.

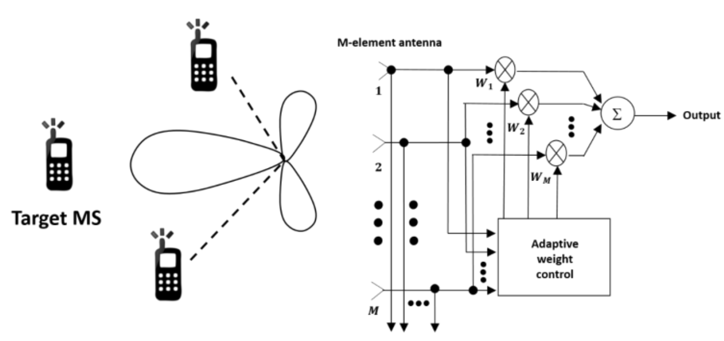

Figure 5 Uplink per-path- per-beamformer SUB

The down-converted baseband signals from different antenna elements are optimally combined using the beamforming weights to form a main beam towards the target MS and nulls towards other MS.

In several previous studies, various criteria have been recommended for the estimation of beamforming weights.

(a) Maximum Signal-to-Interference-plus-Noise Ratio

One commonly used optimality criterion for uplink beamforming at the BS is to select a weight vector so as to get the highest SINR at the output of the beamformer [52], [53]. If $w_{u,v}$ denotes the BS weight vector for the SOI arriving from path v of MS u, the maximum SINR optimality criterion is given by

$$\underset{w_{u,v}}{\min imize} \frac{w_{u,v}{}^{H} R_{u,v} w_{u,v}}{w_{u,v}{}^{H} R_{I} w_{u,v}} \qquad \text{Eqn. (2.14)}$$

where,

$$R_{u,v} = x_{u,v} x_{u,v}{}^{H} \qquad \text{Eqn. (2.15)}$$

and

$$R_{I} = x_{I} x_{I}{}^{H} \qquad \text{Eqn. (2.16)}$$

are respectively the correlation matrices of the SOI and the interferences. The optimum weight vector of a maximum SINR beamformer can be derived as

$$w_{u,v}{}^{SINR} = \zeta R_{I}{}^{-1} a_{u,v} \qquad \text{Eqn. (2.17)}$$

where, ζ can be any nonzero constant.

(b) Minimum Mean-Squared Error

In [51], beamforming weights are considered to minimize the mean-squared error (MSE) between the signal at the beamformer output and the reference signal is as following

$$\underset{w_{u,v}}{\min imize}\, E[\| x^H(t)w_{u,v} - r_u(t)\|^2] \qquad \text{Eqn. (2.18)}$$

where, E [.] represents the expectation of [.] and r_u is the reference signal which can be a pilot signal, a decision-directed signal, or an estimate of the desired signal. The optimum weight vector of an MMSE beamformer can be obtained in closed-form [59] as

$$w_{u,v}{}^{MMSE} = \frac{E(r_u{}^2)}{1 + E(r_u{}^2)a_{u,v}{}^H R_I{}^{-1} a_{u,v}} R_I{}^{-1} a_{u,v} \qquad \text{Eqn. (2.19)}$$

(c) Minimum-Variance Distortionless response

When the DOA of the SOI is known at the receiver, the minimum variance distortionless response (MVDR) beamformer [81] can be used. The weight vector for an MVDR beamformer is chosen such that the signal power at the beamformer output is minimized, and simultaneously the amplitude and phase responses of the beamformer in the direction of the SOI satisfy the condition $w_{u,v}{}^H a_{u,v} = 1$. The optimization problem is formulated as

$$\underset{w_{u,v}}{\min imize}\, w_{u,v}{}^H R w_{u,v}$$

$$\text{subject to } w_{u,v}{}^H a_{u,v} = 1 \qquad \text{Eqn. (2.20)}$$

and its optimum solution can be obtained as

$$w_{u,v}{}^{MV} = \frac{1}{a_{u,v}{}^{H} R_I{}^{-1} a_{u,v}} R_I{}^{-1} a_{u,v} \qquad \text{Eqn. (2.21)}$$

Many algorithms have been developed to adjust the weight vector adaptively based on above optimality criterion. Among them, recursive least square (RLS), sample matrix inversion (SMI) and least mean square (LMS) algorithms are the most commonly used algorithms. A performance analysis of these beamforming algorithms can be found in [38].

2.5.3 Uplink Multiuser Beamforming

In the uplink of a wireless communication system, MUB can be formulated as an optimization problem where the weighting vectors at the BS for various MSs are combinely optimized to fulfil the pre-specified SINR requirements at the BS and, simultaneously, the total power transmitted from all MSs is minimized within the service.

If w_k is the BS weight vector for MS *k*, the uplink MUB optimization problem can be written as follows:

$$\underset{p,w}{\min imize} \sum_{k=1}^{K} p_k$$

$$\text{Subject to } \frac{p_k w_k{}^H R_k w_k}{\sum_{\substack{j=1 \\ j\neq k}}^{K} p_j w_k{}^H R_j w_k + \sigma^2 w_k{}^H w_k} \geq \gamma_k \qquad \text{Eqn. (2.22)}$$

For k = 1, 2, ..., K

where, $p=(p_1..p_2...p_k)^T$ is the transmitted power vector, and σ^2 is the noise variance at the BS, γ_k is the required minimum SINR for the uplink signal received from MS *k*, and correlation matrix R_k of the signal from MS *k* is given by

$$R_k = \sum_{l=1}^{L_K} E[|\beta_{k,l}|^2]a(\theta_{k,l})a^H(\theta_{k,l}) \qquad \text{Eqn. (2.23)}$$

It can be shown that all the constraints in (2.22) must be active at the optimum solution [45]. Thus, the inequality in (2.22) can be replaced by equality. The constraints in (2.22) can be written in matrix form as

$$p = D_w F_w p + u_w$$

where,

$$D_W = diag[\frac{\gamma_1}{{w_1}^H R_1 w_1}\frac{\gamma_2}{{w_2}^H R_2 w_2}\cdots\frac{\gamma_K}{{w_K}^H R_K w_K}] \qquad \text{Eqn. (2.24)}$$

$$[F_w]_{ij} = \begin{cases} =0 & for \quad j=i \\ ={w_i}^H R_j w_i & for \quad j \neq i \end{cases} \qquad \text{Eqn. (2.25)}$$

$$u_w = [\frac{\gamma_1 \|w_1\|^2}{{w_1}^H R_1 w_1}\frac{\gamma_2 \|w_2\|^2}{{w_2}^H R_2 w_2}\cdots\frac{\gamma_K \|w_k\|^2}{{w_K}^H R_K w_K}] \qquad \text{Eqn. (2.26)}$$

Based on the observation that for a given transmitted power vector, the optimum solution of BS weight vector for an MS is the one that maximizes the SINR. An iterative algorithm was developed in [24] to solve the optimization problem in (2.22). It has been shown that using this iterative algorithm, the sequence (p^n) and (w_k^n) for $n=1,\ldots,2\ldots,N$

produced will converge to the optimum solution starting from an arbitrary initial power vector $\left(p^0\right)$. This algorithm is summarized as follows.

Iterative Algorithm for Uplink Multiuser Beamforming

Step 1: Initialize power vector, p^0

Step 2: Compute the weight vector

$$w_k{}^n = \arg\max_{w_k} \frac{p_k w_k{}^H R_k w_k}{\sum_{\substack{j=1 \\ j\neq k}}^{K} p_j w_k{}^H R_j w_k + \sigma^2 w_k{}^H w_k}$$

For *k = l, 2...., K*

Step 3: Update $D_w(n)$, $F_w(n)$ and $u_w(n)$ using (2.24), (2.25), and (2.26) respectively

Step 4: Update the uplink power vector

$$p^{n+1} = D_w(n)F(n)p^n + u_w(n)$$

Step 5: If the sequence of power vectors { p^n } converges, output solutions $p = p^n$ and $w_k = w_k{}^n$ for *k=1,2,.....K*, and stop. Otherwise, set *n=n+1* and repeat from Step 2

2.6 Downlink Beamforming

In downlink of a wireless communication system, the signal transmitted to the target MS through a BS antenna array will be received by other MSs that share the same frequency channel within the service area, which leads to co-channel interference. The effect of co-channel interference on system performance can be reduced by using downlink beamforming. Here, a number of existing downlink SUB and MUB algorithms for wireless communication systems will be discussed.

2.6.1 Downlink Signal Model

After down-converting to baseband, the downlink signal $x^s_{k,l}(t)$ received from the $l-th$ path at MS *k* is given by

$$x^s_{k,l}(t) = \beta_{k,l}\sqrt{\tilde{p}_k}\, s_k(t-\tau_{k,l})a^H(\theta_{k,l})w_k \qquad \text{Eqn. (2.27)}$$

where $\beta_{k,l}$ is the complex channel response for the *l-th* path of MS k, $\tilde{p}_k$ is the BS transmitted power for the downlink signal to MS *k*, s_k (t) is the normalized transmitted signal to MS *k*, $\tau_{k,l}$ is the path delay, w_k is the BS weighting vector of MS *k*, $\theta_{k,l}$ is the is the direction of departure of the *l-th* from the BS antenna array to MS *k*, and $a(\theta_{k,l})$ is the M-dimension BS antenna array response vector. The CCI, $x_{k,l}{}^{l_i}(t)$ is received via the *l-th* path at the target MS *k* caused by the downlink signal to MS *i* can be symbolized by

$$x_{k,l}{}^{l_i}(t) = \beta_{k,l}\sqrt{\tilde{p}_i}\, s_i(t-\tau_{k,l})a^H(\theta_{k,l})w_i \qquad \text{Eqn. (2.28)}$$

The received signal at MS *k* includes target downlink signal and CCI caused by other MSs. If the number of MSs in a BS service area is *K* and the number of dominant paths from the BS to MS k is L_k, then the received signal at MS *k* can be represented by

$$x_k(t) = \sum_{l=1}^{L_k} x^s{}_{k,l}(t) + \sum_{\substack{i=1 \\ i\neq k}}^{K} \sum_{l=1}^{L_k} x^{l_i}{}_{k,l}(t) + n_k(t) \qquad \text{Eqn. (2.29)}$$

where, $n_k(t)$ is the noise that is assumed to have zero mean and covariance $\sigma^2{}_k$.

2.6.2 Downlink Single-User Beamforming

In the downlink of a mobile communication system, the aim of SUB is to deliberate the transmitted power in the direction of the target MS and minimize it in the direction of other MSs. The block diagram of a downlink per-user-per-beamformer SUB system is shown in Figure 6. Signal s_u to the target MS u is first split into M signal components corresponding to M antenna elements. Then the M signal components are weighted by the beamforming weights that determine the beam pattern of the antenna array at BS. Lastly, the weighting factors of signal components are transmitted from their corresponding antenna element.

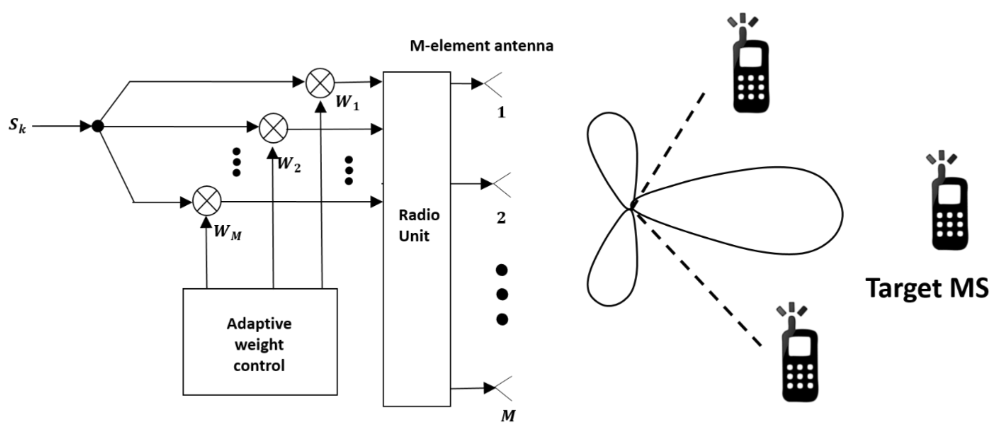

Figure 6 Downlink per-user-per-beamformer SUB

Conventional generalized eigenvalue-based beamforming algorithms for downlink lead to similar beam pattern; therefore, it has the same normalized weight vector $\hat{w}_u$. If the number of signal paths to target MS u is L_u and the channel gain $|\beta_{u,l}|$ for *l-th* path is known at the transmitter, $\hat{w}_u$ can be determined by solving the optimization problem:

$$\underset{\hat{w}_u}{\max imize} = \frac{\hat{w}_u{}^H \left(\sum_{l=1}^{L_u} R_{u,l} \right) \hat{w}_u}{\hat{w}_u{}^H \left(\sum_{\substack{k=1 \\ k \neq u}}^{K} \sum_{l=1}^{L_k} R_{k,l} + \sigma^2 I \right) \hat{w}_u} \qquad \text{Eqn. (2.30)}$$

where, $\sigma^2 I$ is introduced to increase the algorithm robustness to channel uncertainties [60], and

$$\begin{aligned} R_{u,l} &= E\left[|\, \beta_{u,l} \,|^2 \right] a_{u,l} a_{u,l}{}^H \\ R_{k,l} &= E\left[|\, \beta_{k,l} \,|^2 \right] a_{k,l} a_{k,l}{}^H \end{aligned} \qquad \text{Eqn. (2.31)}$$

are the spatial correlation matrices.

The optimum solution of the normalized eigenvector in (2.30) is the eigenvector corresponding to the largest eigenvalues of generalized eigenvalues problem [82] given by

$$\left(\sum_{l=1}^{L_u} R_{u,l} \right) \hat{w}_u = \lambda_{\max} \left(\sum_{\substack{k=1 \\ k \neq u}}^{K} \sum_{l=1}^{L_k} R_{k,l} + \sigma^2 I \right) \hat{w}_u \qquad \text{Eqn. (2.32)}$$

2.6.3 Downlink Multi-User Beamforming

In downlink of a wireless communication system, the optimum downlink weight vectors can be evaluated by reducing the total transmitted power from the BS so that a given SINR specification at each MS can be achieved. Figure 7 shows the MUB system with an *M*-element antenna array that is deployed at the BS whereas an omnidirectional antenna having a unit gain is deployed at each MS. Signal s_k to MS k is first divided into M signals with p antenna elements, which are then weighted by the beamforming weights.

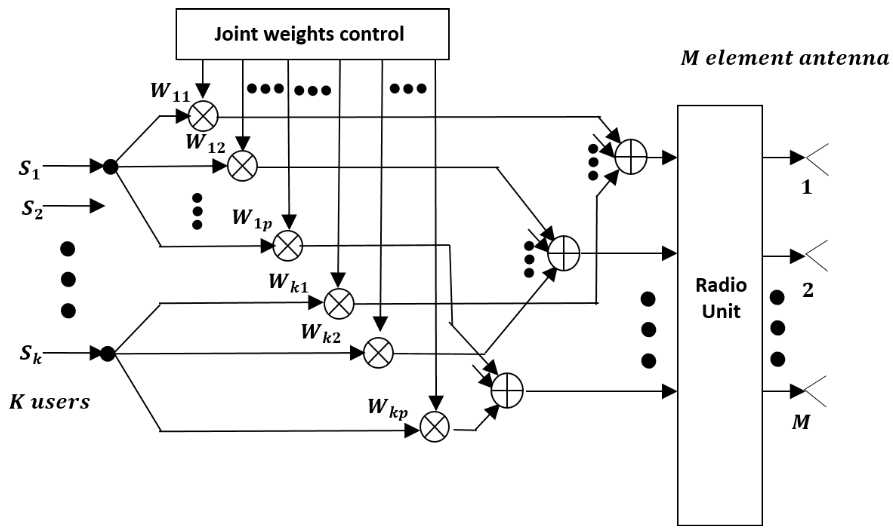

Figure 7 Block diagram of downlink MUB at BS

The beamforming weights, which determine the BS radiation pattern and downlink signal power are corresponding to different MSs, are jointly determined based on the provided channel specification. Then, the weighted signal components to different MSs are combined and transmitted from each antenna element.

The downlink MUB optimization problem can be formed as:

$$\min_{\tilde{p}, w} imize \sum_{k=1}^{K} \left[\tilde{p}_k w_k^{\ H} w_k \right]$$

$$\text{Subject to } \frac{\tilde{p}_k w_k^{\ H} R_k w_k}{\sum_{\substack{j=1 \\ j \neq k}}^{K} \tilde{p}_j w_j^{\ H} R_k w_j + \sigma_k^2} \geq \gamma_k \qquad \text{Eqn. (2.33)}$$

For k = 1, 2, ..., K

where,

$$R_k = \sum_{l=1}^{L_k} E\left[\| \beta_{k,l} |^2 \right] a(\theta_{k,l}) a^H(\theta_{k,l}) \qquad \text{Eqn. (2.34)}$$

is the correlation matrix of the downlink signal to MS k ; $\tilde{p} = [\tilde{p}_1 \ldots \tilde{p}_2 \ldots \tilde{p}_k]^T$ is the downlink transmitted power vector, σ^2 and γ_k are the noise variance and the required minimum SINR at MS k, respectively.

In the downlink, signal correlation matrix R_k , for $k = 1,\ldots 2,\ldots K$ needs to be evaluated to resolve the MUB problem. Moreover, TDD (Time-division duplexing) systems where the uplink and downlink channel are reciprocal, R_k can be determined by estimating uplink channel [35]. On the other hand, the frequency channels are used in the downlink, and uplinks are different in FDD (Frequency division duplexing) systems. Moreover, R_k can be estimated via feedback signaling [25], [66]. Based on the estimated R_k , two classes of algorithms are established to solve the optimization problem in (2.33), namely, duality-based [35], [66] and semi-definite programming (SDP) based [61]–[63] MUB algorithms.

(a) Duality-Based Downlink Multiuser Beamforming

The duality between the downlink and uplink MUB was primarily proposed in [35]. It has been recommended that the optimal downlink weight vectors can be achieved by the usage of a virtual uplink channel. Based on this duality, the optimal downlink weight vectors can be determined iteratively as in an uplink MUB optimization problem, all the constraints in (2.33) must be active at the optimum solution. The minimum downlink BS transmitted power is achieved when the SINR is equal to the minimum SINR. Thus, the constraints in (2.33) can be written in matrix form as

$$\tilde{p} = D_w (F_w)^T \tilde{p} + \tilde{u}_w \qquad \text{Eqn. (2.35)}$$

where $\tilde{u}_w$ is defined as

$$\tilde{u}_w = \left[\frac{\gamma_1 N_1}{w^H{}_1 R_1 w_1} \frac{\gamma_2 N_2}{w^H{}_2 R_2 w_2} \cdots \frac{\gamma_k \sigma_k^2}{w^H{}_K R_K w_K} \right]^T \qquad \text{Eqn. (2.36)}$$

Considering one BS in a service area, the iterative algorithm for the downlink MUB is summarized as follows.

Iterative Algorithm for Downlink Multiuser Beamforming

Step 1: Initialize uplink power vector p^0 and downlink power vector $\tilde{p}^0$

Step 2: Compute the virtual uplink weight vector

$$w_k{}^n = \arg \max_{w_k} \frac{p_k w_k{}^H R_k w_k}{\sum_{\substack{j=1 \\ j \neq k}}^{K} p_j w_k{}^H R_j w_k + \sigma^2 w_k{}^H w_k}$$

For $k = l, \ldots 2, \ldots K$

Step 3**:** Update $D_w(n)$, $F_w(n)$ $u_w(n)$ and $\tilde{u}_w(n)$ using (2.24), (2.25), (2.26) and (2.36) respectively

Step 4: Update the uplink power vector

$$p^{n+1} = D_w(n) F(n) p^n + u_w(n) \qquad \text{Eqn. (2.37)}$$

Step 5: Update downlink power vector

$$\tilde{p}^{n+1} = D_w(n) F^T(n) \tilde{p}^n + \tilde{u}_w(n) \qquad \text{Eqn. (2.38)}$$

Step 6: If the sequence of power vectors converges, $\{\tilde{p}^n\}$ output solutions $\tilde{p} = \tilde{p}^n$ and $w_k = w_k{}^n$ for $k=1, 2,....,K$ and stop. Otherwise, set $n=n+1$ and repeat from Step 2

(b) SDP-Based Downlink Multiuser Beamforming

In [61]–[63], the SDP-based MUB algorithm has been discussed where the optimal MUB optimization problem is relaxed into an SDP optimization problem after Lagrangian relaxation [67]. After that, the weighting vectors are calculated based on the optimal solution of the SDP problem.

For simplicity, and without considering the loss of generality, the BS transmitted power p_k can be merged with the BS weight vector w_k for $k = 1,\ldots 2,\ldots K$. The optimization problem in (2.33) can be rewritten as

$$\underset{w}{\min imize} \sum_{k=1}^{K} \left[w_k{}^H w_k \right]$$

$$\text{Subject to } \frac{w_k{}^H R_k w_k}{\sum_{\substack{j=1 \\ j \neq k}}^{K} w_j{}^H R_k w_j + \sigma_k^2} \geq \gamma_k \qquad \text{Eqn. (2.39)}$$

for *k=1,2,......K*

If we define $W = w_k w_k{}^H$ and let, $tr[.]$ denote the trace of a matrix. After relaxing the constraint rank $(W_k) = 1$ for *k=1, 2... K*, the optimization problem in (2.39) can be reformulated as the SDP optimization problem

$$\min_{w} \sum_{k=1}^{K} tr[W_k]$$

$$\text{Subject to } tr[R_k W_k] - \gamma_k \sum_{\substack{j=1 \\ j \neq k}}^{K} tr[R_k W_j] - \sigma_k^2 \gamma_k \geq 0 \qquad \text{Eqn. (2.40)}$$

$$W_k = W_k^H$$

$$W_k \geq 0$$

for $k=1, 2... K$.

Based on the solution W_k for $k=1,2,...,K$ of the optimization problem in (2.40), w_k can be calculated as

$$w_k = \sqrt{\mu_k} q_k \text{ for } k=1,2,......,K \quad \text{Eqn. (2.41)}$$

where, q_k is the eigenvector associated with the non-zero eigenvalue μ_k of matrix W_k.The solution of the SDP problem after the Lagrangian relaxation in (2.40) cannot guarantee the constraint rank $(W_k)=1$ for $k=1,2,.....,K$ being satisfied and, therefore, it may not lead to an optimum solution to the original MUB problem in (2.39). However, in practice, these degenerate cases almost never occur and if the algorithm offers a high-rank solution, a small perturbation can be added to the correlation matrixes that will make the problem have a rank one solution [46].

2.7 Conclusion

The background knowledge, concepts, and terminology that are necessary for the development of weighted beamforming have been reviewed. Specifically, the basic models of antenna arrays have been introduced and several important beamforming techniques using antenna arrays, i.e., the SUB and the MUB for both uplink and downlink in wireless communication systems, have been described.

Chapter 3: Methodology

In this chapter, different beamforming methods will be discussed including the difference in weighting vector calculation. Direction of arrival (DOA) estimation was not taken into consideration in this study. Therefore, the DOA's of the desired and the interference signals were assumed during simulation.

Beamforming can be implemented through various conventional beamforming methods; such as Minimum Variance Distortionless Response (MVDR) beamforming, and Linear Constrained Minimum Variance (LCMV) beamforming [3]. Furthermore, the MVDR beamforming is also known as optimum beamforming. In MVDR beamforming, an antenna pattern is formed to maximize the output signal-to-interference-plus-noise ratio while maintaining a constant gain in the direction of the desired signal [83]. The LCMV beamforming is developed from MVDR beamforming with additional linear constraints to improve its robustness[84]. This algorithm can be implemented by placing nulls in the directions of interferers when multiple interferers are considered [84], [85]–[87]. To cancel or null all the interferers using LCMV beamforming, the number of antenna elements are required to go beyond the number of nulls by one [88]. In a practical beamforming system, the number of interferers can be larger than the number of antenna elements, and there might be some residual interferers in the wireless system [88]. A new weighted downlink SUB algorithm was proposed [69]. This work is followed by the previous work.

As discussed earlier in Chapter 2, the objective of conventional generalized eigenvalues based SUB algorithm is to adjust the beamforming weights adaptively so that the transmitted power from the BS is concentrated in the direction of the target MS, and its interference with the downlink signals to other MSs is also reduced. In this chapter, the methodology of different weighted beamforming algorithms such as: general conventional downlink SUB algorithm, MVDR beamforming algorithm, LCMV beamforming algorithm and the new weighted beamforming algorithm are discussed in terms of the calculation of the weight vector.

The organization of the chapter is as follows: Conventional Downlink SUB Algorithms is presented in sec 3.1. MVDR and LCMV algorithms are discussed in sec 3.2 and 3.3 respectively. Finally, the weighted downlink SUB algorithm is presented in sec. 3.4.

3.1 Conventional Downlink SUB Algorithms

As mentioned earlier, after down-converting to baseband, the downlink signal $x^{s}_{k,l}(t)$ received from the $l-th$ path at MS *k* can be given by

$$x^{s}_{k,l}(t) = \beta_{k,l}\sqrt{\tilde{p}_k}\, s_k(t-\tau_{k,l})a^{H}(\theta_{k,l})w_k$$

where, $\beta_{k,l}$ is the complex channel response for the $l-th$ path of MS $k, \tilde{p}_k$ is the BS transmitted power for the downlink signal to MS *k*, $s_k(t)$ is the normalized transmitted signal to MS *k*, $\tau_{k,l}$ is the path delay, w_k is the BS beamforming weight vector of MS *k*, $\theta_{k,l}$ is the direction of departure of the *l-th* from the BS antenna array to MS *k*, and $a(\theta_{k,l})$ is the M-dimension BS antenna array response vector. The co-channel interference $x_{k,l}^{\,l_i}(t)$ received through the *l-th* path at the target MS *k* caused by the downlink signal to MS *i* can be represented by

$$x_{k,l}{}^{l_i}(t) = \beta_{k,l}\sqrt{\tilde{p}_i}\, s_i(t-\tau_{k,l})a^H(\theta_{k,l})w_i$$

The received signal at MS *k* consists of desired downlink signal as well as CCI caused by the downlink signals to other MSs. If the number of MSs in a BS service area is K and the number of dominant paths from the BS to MS *k* is L_k, then the received signal at MS *k* can be represented by

$$x_k(t) = \sum_{l=1}^{L_k} x^s{}_{k,l}(t) + \sum_{\substack{i=1 \\ i\neq k}}^{K}\sum_{l=1}^{L_k} x^{l_i}{}_{k,l}(t) + n_k(t)$$

where, $n_k(t)$ is the noise that is assumed to have zero mean and covariance $\sigma^2{}_k$.

In chapter 2, conventional downlink SUB algorithms [26]–[30] were introduced. In this section, the performance of these algorithms will be further discussed. Assuming that MS u is the target user, and then the optimization problem in (2.30) can be rewritten as

$$\underset{w_u}{\max imize}\left(\frac{p_{u,u}}{\sum\limits_{\substack{j=1 \\ j\neq u}}^{K} p_{j,u}}\right) \qquad \text{Eqn. (3. 1)}$$

where, $p_{u,u}$ and $p_{j,u}$ depend on the information available about the downlink channel gain. If the BS operates in 'open-loop' [89], [90], full information about the downlink channel may not be available. The downlink SUB is based on the DOAs obtained from the uplink. In such a case, $p_{u,u}$ is the BS transmitted signal power in the direction of MS *u*, and $p_{j,u}$ is the power of the co-channel interference radiated in the direction of MS *j* caused by the downlink signal to MS u. If the BS operates in 'closed-loop' [25], [90], the downlink channel gain and direction of departure will be estimated. In such a case, $p_{u,u}$ is the received downlink signal power at MS u, and $p_{j,u}$ is the received power of the co-channel interference at MS j caused by the downlink signal to MS u.

It should be noted that, using (3.1), the beamforming weight vector is derived independently of the required downlink signal power and, therefore, it provides only the direction for the optimal weight vector. The optimization problem in (3.1) can be combined with the required downlink signal power p_u determined from the downlink power control, in order to obtain the beamforming weight vector w_u.Based on the above discussion, the optimization problem for the conventional generalized eigenvalue-based beamforming algorithms can be formulated as

$$\underset{w_u}{\max imize}\left(\frac{p_{u,u}}{\sum_{\substack{j=1\\ j\neq u}}^{K} p_{j,u}}\right) \qquad \text{Eqn. (3. 2)}$$

$$\text{Subject to } p_{u,u} = p_u$$

3.2 MVDR Beamforming Algorithm

The MVDR algorithm is able to suppress the interference but with high value in SNR. At the same time, the MVDR algorithm depends on the steering vectors that depend on the incident angle of the received signal from the element of the array antenna. Thereby, the principle of the MVDR beamformer is to find the optimal weights that give improved performance. The direction of the desired signal must be known and the output power subjected to a unity gain constraint in the direction of the desired signal must be minimized. The array output is given by

$$y = w^H x \qquad \text{Eqn. (3. 3)}$$

Therefore the output power is as follows:

$$p = E\{|y|^2\} = E\{w^H x x^H w\} = w^H E\{x x^H w\} = w^H R \qquad \text{Eqn. (3. 4)}$$

where, the covariance matrix (R) should be (M, 1) for the received signal x and H is the Hermitian transpose.

The optimum weights are selected to minimize the array output power p_{MVDR} while maintaining unity gain in the look direction $a(\theta)$, which is the steering vector of the desired signal. Then MVDR adaptive algorithm can be written as follows:

$$\begin{gathered} \min_w \{w^H R w\} \\ \text{Subject to } w^H a(\theta) = 1 \end{gathered} \qquad \text{Eqn. (3. 5)}$$

The steering vector $a(\theta)$ is given by

$$a(\theta) = \begin{bmatrix} 1 \\ \exp\{j\frac{2\pi}{\lambda}(\sin\theta_i)d\} \\ \exp\{j\frac{2\pi}{\lambda}(\sin\theta_i)(m-1)d\} \end{bmatrix} \qquad \text{Eqn. (3. 6)}$$

where, d is the space between the elements of the antenna, θ_i is the desired angle, and m is the number of antenna elements. The optimization weight vectors can be acquired:

$$w_{MVDR} = \frac{R^{-1}a(\theta)}{a^H(\theta)R^{-1}a(\theta)} \qquad \text{Eqn. (3. 7)}$$

Consequently, the beamformer weights are selected based on the minimum mean value of output power according to the number of users within the coverage area while maintaining a unity response in the desired direction. Nevertheless, the restraint ensures that the signal passes through the beamformer undistorted. Therefore, the output signal gain is similar to the look at the direction of source gain. The total noise, including interferences and uncorrelated noise, is then reduced by the minimization process. Notably, the minimization of the total output noise, while consistently maintaining the output signal gain, is the same as maximizing the output SINR.

3.3 LCMV Beamforming Algorithm

This type of beamformer imposes more constraints on the beamformer characteristics in cases where an unexpected change of the working conditions takes place such as changing the supposed angle of arrival where the desired signal arrives from a different angle. For this purpose, some linear constrains can be imposed to control the behavior of the beamformer. If we consider a scenario while few antenna elements are implemented at a mobile station, and large antenna arrays are installed at a base station, then receive beamforming at a base station for each desired signal can be implemented independently without affecting the performance of other links [35], [91]. If a ULA is considered where λ is the carrier wave length, and θ is an arrival angle of incident waves then in the ULA system, a combining network connects an array of antenna elements and generates a beam pattern [84].

$$G = \left| W^H V(\theta) \right|^2 \qquad \text{Eqn. (3. 8)}$$

where θ is the azimuth angle, W^H is a (1 × M) vector, considers as weight vector for antenna elements, M is the number of antenna elements, and $V(\theta)$ is an (M × 1) vector works as an array manifold vector.

$$V(\theta) = \left[e^{-j\left(\frac{M-1}{2}-0\right)\pi\cos(\theta)}, \ldots\ldots, e^{j\left(\frac{M-1}{2}-M+1\right)\pi\cos(\theta)} \right]^T \qquad \text{Eqn. (3. 9)}$$

The beam could be steered to the desired direction by varying θ. For conventional beamforming, the weight factor is

$$W_c = \frac{1}{M} V(\theta) \qquad \text{Eqn. (3. 10)}$$

and the beam pattern can be simplified as [81],

$$G(\psi, \theta) = \left| \frac{\sin(dM\pi(\cos\theta - \cos\psi))}{M \sin(dM\pi(\cos\theta - \cos\psi))} \right|^2 \qquad \text{Eqn. (3. 11)}$$

where ψ is the elevation angle and the weight factor can be expressed as [84],

$$W_{LCMV}^H = g^H [C^H S_n^{-1} C]^{-1} C^H S_n^{-1} \qquad \text{Eqn. (3. 12)}$$

where S_n is a noise spectral matrix which characterize both noise and interference, C is an (M × M_c) matrix, and g^H is a (1 × M_c) vector. Considering M_c is a linear constraint for LCMV beamforming, the constraint equation is

$$W_{LCMV}^H C = g^H \qquad \text{Eqn. (3. 13)}$$

The LCMV beamforming in [85]–[87] assumes null constraints. When the number of interferers is larger than the number of antenna elements, the maximum number of nulls that the LCMV beamformer can generate is (M – 1), and the number of constraints M_c = M. Thus we have

$$C_{LCMV} = [V(\theta)\ \ V(\theta_1)\ \ V(\theta_2) \ldots\ V(\theta_{M-1})] \qquad \text{Eqn. (3. 14)}$$

And

$$g^H_{LCMV} = [1\ \ 0\ 0\ .\ .\ .\ 0\ \ ] \qquad \text{Eqn. (3. 15)}$$

where, θ_i is the arrival angle of interferer *i*.

3.4 New Weighted Downlink SUB Algorithm

Considering a scenario where the downlink signals are transmitted from the BS to the MS and the power of the signals vary due to power control or multiple-bit-rate services. Downlink signals to different MSs possess different resistance to co-channel interference (CCI) caused by the downlink signals to other MSs. Conventional generalized eigenvalue-based beamforming algorithms [26]–[30], as formulated in (3.2), are equivalent to minimize the power of the interference to other MSs for a given power level p_u by solving the optimization problem [92]

$$\underset{w_u}{\max imize}\left(\sum_{\substack{j=1\\ j\neq u}}^{K} p_{j,u}\right) \qquad \text{Eqn. (3. 16)}$$

$$\text{Subject to}\ \ p_{u,u} = p_u$$

The fact that downlink signals to different MSs have different resistance to co-channel interference is not considered in these algorithms. When the power of the downlink signal to the target MS *u*, $p_{u,u}$ is high, the signal will interfere with the downlink signals to other MSs, particularly with those that have low power. Using the conventional algorithms, an

MS with low power downlink signal may receive the same amount of interference as an MS whose downlink signal has the high power. As a result, the received SIR (Signal-to-interference ratio) of an MS with low power downlink signal may be far less than that of an MS whose downlink signal has the high power. To consider the resistance to the co-channel interference, a weighted downlink SUB algorithm is proposed [69] to obtain the beamforming weight vector w_u for the target MS *u*. Given a power level p_u the objective of this algorithm is to minimize the relative power of the interference caused by the downlink signal to the target MS *u*. The optimization problem is thus written as

$$\underset{w_u}{\max imize}\left(\sum_{\substack{j=1\\ j\neq u}}^{K}\frac{p_{j,u}}{p_j}\right) \qquad \text{Eqn. (3. 17)}$$

$$\text{Subject to } p_{u,u}=p_u$$

Eq. (3.4) can be rewritten as

$$\underset{w_u}{\max imize}\left(\frac{p_{u,u}}{\sum_{\substack{j=1\\ j\neq u}}^{K}\alpha_{j,u}\,p_{j,u}}\right) \qquad \text{Eqn. (3. 18)}$$

$$\text{Subject to } p_{u,u}=p_u$$

where, the weighting coefficient $\alpha_{j,u}$ is given by [69]

$$\alpha_{j,u}=\frac{p_u}{p_j} \qquad \text{Eqn. (3.19)}$$

and can be interpreted as the relative interference strength (RIS) of the downlink signal to target MS *u* on the downlink signal to MS j.

As mentioned before, the expressions for p_j, $p_{u,u}$ and $p_{j,u}$ depend on the information available regarding the downlink channel gain. If such information is not available, and the downlink SUB is based on the DOAs obtained from the uplink, and then p_j is the BS transmitted signal power in the direction of MS *j*. In this case, $p_{u,u}$ is the BS transmitted signal power in the direction of the target MS u, and $p_{j,u}$ is the power of the co-channel interference radiated in the direction of MS *j* caused by the downlink signal to the target MS u. They are given by

$$p_{u,u} = w_u^H \left(\sum_{l=1}^{L_u} R_{u,l} \right) w_u \qquad \text{Eqn. (3. 20)}$$

and

$$p_{j,u} = w_u^H \left(\sum_{l=1}^{L_j} R_{j,l} \right) w_u \qquad \text{Eqn. (3. 21)}$$

and matrices $R_{u,l}$ and $R_{j,l}$ are given by

$$R_{u,l} = a_{u,l} a_{u,l}^{\ H}$$

and

$$R_{j,l} = a_{j,l} a_{j,l}^{\ H}$$

respectively.

If information regarding the downlink channel gain is available, then p_j is the received downlink signal power at MS *j*. In this case, $P_{u,u}$ is the received downlink signal power at the target MS *u*, and $P_{j,u}$ is the received power of the CCI at MS *j* caused by the downlink signal to MS *u*. Again they are given by

$$p_{u,u} = w_u^{\ H}\left(\sum_{l=1}^{L_u} R_{u,l}\right)w_u \qquad \text{Eqn. (3. 22)}$$

and

$$p_{j,u} = w_u^{\ H}\left(\sum_{l=1}^{L_j} R_{j,l}\right)w_u \qquad \text{Eqn. (3. 23)}$$

respectively, and matrices $R_{u,l}$ and $R_{j,l}$ are given by

$$R_{u,l} = E[|\beta_{u,l}|^2]a_{u,l}a_{u,l}^H$$

and

$$R_{j,l} = E[|\beta_{j,l}|^2]a_{j,l}a_{j,l}^H$$

respectively.

The optimization problem in (3.5) can be rewritten as

$$\underset{w_u}{\max imize} \frac{w_u^{\ H}\left(\sum_{l=1}^{L_U} R_{u,l}\right)w_u}{w_u^{\ H}\left[\sum_{\substack{j=1 \\ j\neq u}}^{K}\left(\alpha_{j,l}\sum_{l=1}^{L_j} R_{j,l}\right)+\sigma^2 I\right]w_u} \qquad \text{Eqn. (3. 24)}$$

$$\text{Subject to } w_u^{\ H}\left(\sum_{l=1}^{L_U} R_{u,l}\right)w_u = p_u$$

where, $\sigma^2 I$ is introduced to increase the robustness of this algorithm by integrating channel uncertainties [60]. Let $\hat{w}_u$ denote the normalized vector of w_u. It can be achieved by maximizing the design objective in (3.11).

The optimum solution is the eigenvector associated with the largest eigenvalues of the generalized eigenvalue problem [82] given by

$$\left(\sum_{l=1}^{L_U} R_{u,l}\right)\hat{w}_u = \lambda_{\max}\left(\sum_{\substack{j=1 \\ j\neq u}}^{K} \alpha_{j,u} \sum_{l=1}^{L_j} R_{j,l} + \sigma^2 I\right)\hat{w}_u$$

and w_u can be derived as

$$w_u = \left[\frac{p_u}{\hat{w}_u{}^H \left(\sum_{l=1}^{L_U} R_{u,l}\right)\hat{w}_u}\right]^{\frac{1}{2}} \hat{w}_u \qquad \text{Eqn. (3. 25)}$$

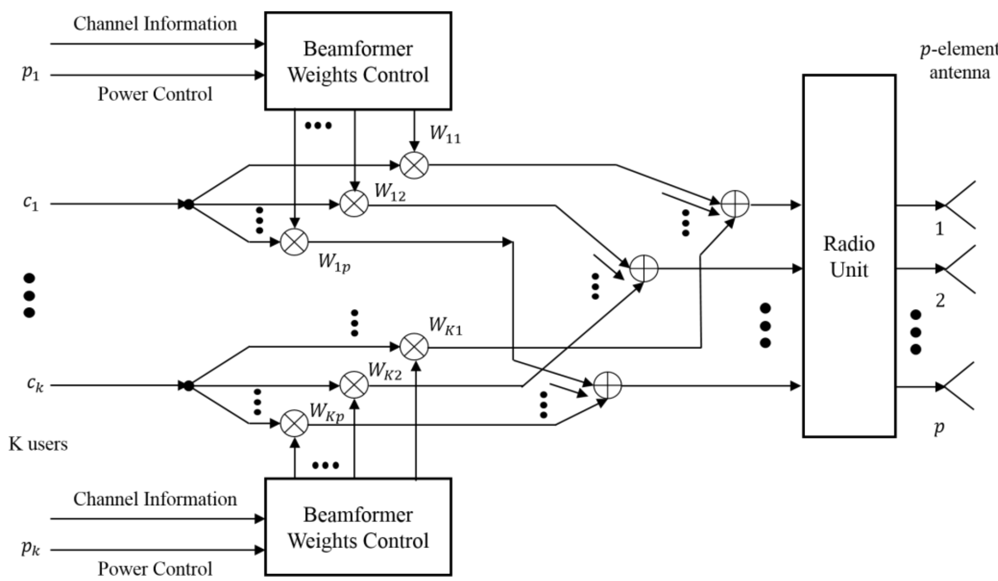

Figure 8 Block diagram of a downlink beamforming system at BTS

In this new weighted downlink SUB algorithm, the power of CCI in the direction of other MSs is weighted by using the RISs (3.6) of the downlink signal to the target MS. If there is any increase in the RIS of the downlink signal to the target MS *u* on the downlink signal to another MS, the low power is radiated in its direction. Thus, the power of co-channel interference in these directions is minimized, and the SIR at these MSs is increased.

3.5 Conclusion

In this chapter, the methodology of different beamforming algorithms has been discussed based on their optimal weighted vector calculation, which is the base for the simulation. The following chapter will justify the theory of MVDR and LCMV beamforming methods with the simulation results.

Chapter 4: Results and Discussion

In this chapter, simulation results are shown for the MVDR and LCMV beamforming algorithms. The result will be explained in the context of how the simulation works with the theory and will also include a comparison between the beam-patterns obtained using the two algorithms.

According to an earlier discussion in chapter 2, beamforming is a method in signal processing that is used in sensor arrays for directional signal reception and transmission. Adaptive or fixed receive and transmit beam pattern cause spatial selectivity. The beam pattern is formed by regulating the complex weights to the antenna elements so that the higher beam gain is achieved in the direction of interest [93]. Receive beamforming can raise the level of sensitivity in the direction of the target user by adjusting the phase and relative amplitude of the signal at each antenna element. In the direction of interferences sensitivity is reduced, thereby, increasing SINR in the direction of the desired user.

4.1 Simulation Setup

A uniform linear array of varying number of elements with interspacing of $d = \lambda / 2$ was considered for the design of beam patterns based on the MVDR and LCMV beamforming algorithms. The reason for choosing inter-element spacing $\lambda / 2$ (the Nyquist rate) [94] is to minimize the aliasing effect. For this experiment, noise power was set to 3 dB. The beamformers were modeled as a weighted sum of the signal of all antennas with the input signal being the signal of interest (SOI) or the desired mobile (MS A), other interference signals (MS B and MS C) and complex-valued Gaussian noise $n(t)$.The beamformer weights were normalized and denoted by w_1, w_2, w_3. The desired signal with two interfering signals received at the antenna array, considered having a carrier frequency of 1 GHz. The DOA for the target signal MS A was fixed at 0^0 while other interference signals MS B and C were considered respectively at $55^0, -40^0$.

Initially, the experiment was run by using seven-element antenna array. Furthermore, different numbers of antenna elements were used to show the directivity of the desired signal and effects of the beam pattern for both the MVDR and LCMV beamforming algorithms. MATLAB was used to conduct the simulations.

The following table shows the relevant parameters that are used while simulating the algorithms.

Table 1 Simulation Parameters for Beamforming

No of array element (#)	**Desired signal (MS A)**	**Interference signals (MS B, MS C)**
3	0^0	$55^0, -40^0$
5	0^0	$55^0, -40^0$
7	0^0	$55^0, -40^0$
15	0^0	$55^0, -40^0$
20	0^0	$55^0, -40^0$

In general, it is needed to determine the DOA for the desired signal and interferences. In the beamforming simulations, the DOA estimation was not taken into consideration but the DOAs of the desired and the interference signals were assumed to be known.

4.2 Receiving of signals in antenna array

In wireless transmission, antennas at the receiving end collect signals that are emitted by several sources with a different direction of arrival (DOA) as illustrated in Figure 9. Let, the desired signal arrives from θ_1 and interference signals arrive from $(\theta_2, ..., \theta_D)$ directions being received by an *M*-element array x_M with M potential weights w_M. Figure 9 shows how signals arrive in an M-element array with M potential weights w_1 w_2 . . . w_M.

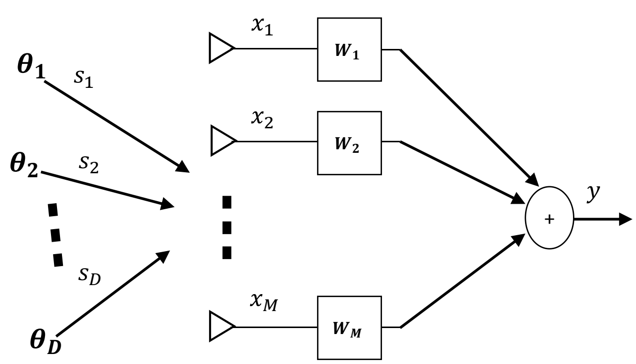

Figure 9 M element array with arriving signals

The weight vector gives a specific receiving pattern as a function of the direction of arrival (DOA). Linear arrays cannot scan in 3-D space, whereas planar arrays are able to scan the main beam in θ (azimuth angle) and ψ (elevation angle). In cases where DOA is not available, a general DOA estimation and interference rejection technique for a system are presented in Figure 10.

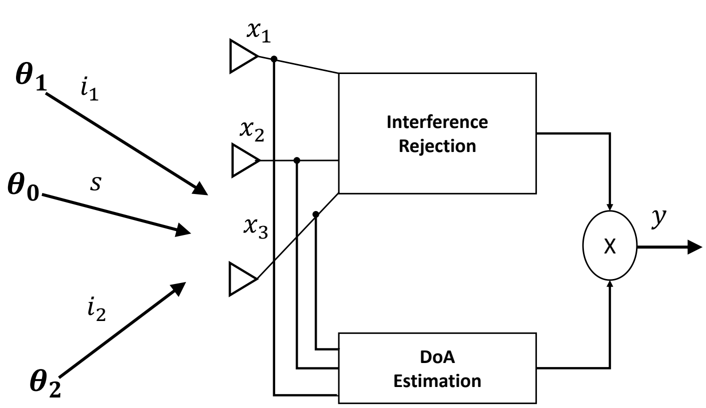

Figure 10 System with interference rejection and DOA estimation

As mentioned earlier, DOA of the desired (MS A at 0^0) and interference signals (MS B and MS C were at 55^0 and -40^0) were assumed for this experiment. The additive background noise is considered as complex-valued Gaussian noise.

The ULA minimum variance distortionless response (MVDR) estimates the spatial spectrum of incoming narrowband signals by scanning a region of broadside angles using a narrowband MVDR beamformer for the uniform linear array. The ULA MVDR algorithm can also be used to calculate the direction of arrival (DOA) of a specified number of signals by estimating peaks on the spectrum. The MVDR DOA estimator is also called the Capon DOA estimator. In this experiment, the DOA of MSs is assumed known, and DOA estimation is not needed.

4.3 Transmission of signals to the MS

Simulations were conducted to send a rectangular pulse as the desired signal from MS A, which is showed in Figure 11.

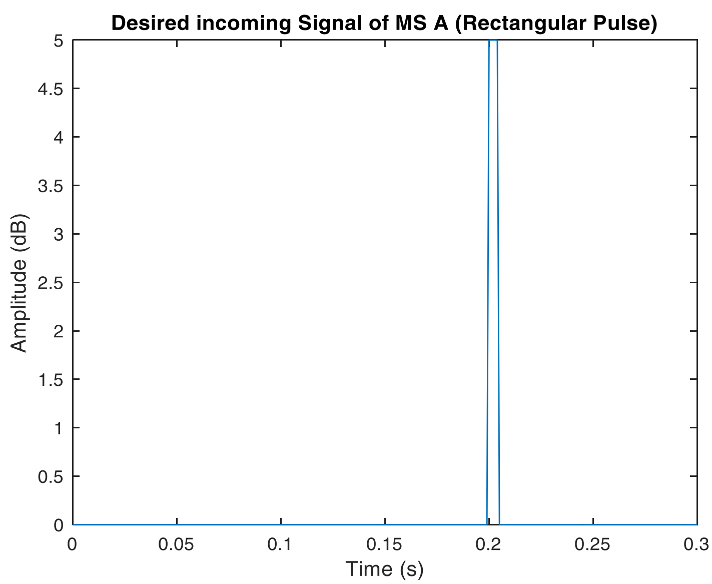

Figure 11 Magnitude of the desired signal (MS A)

Initially, a seven-element uniform linear array (ULA) was considered that is deployed at the base station. In uplink, different MS transmit signals to the BS from different DOA. In the downlink of mobile communication systems, the signal is transmitted to the target MS by the BS antenna array, and that signal is also received by other interference signals that share the same frequency channel in the service area. The signals received at the antennas include the desired signal (MS A) and additive zero-mean Gaussian noise during the propagation through the channel. Magnitudes of the first six received signals with noise at the seven array elements are shown in Figure 12. According to the figure, it is seen the magnitude of the original transmitted signal is still clearly visible despite the noise contamination.

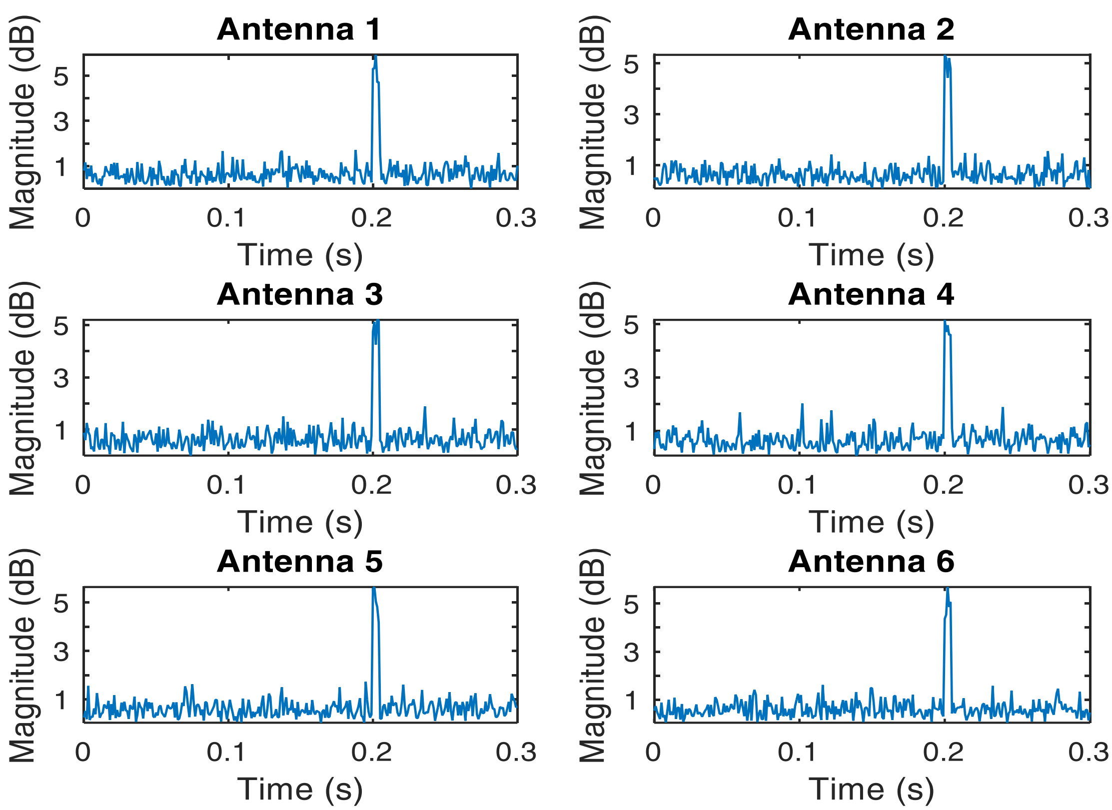

Figure 12 Magnitudes of received signals with noise

Furthermore, two random interference signals (MS B, MS C) were added with the desired signal (MS A) and additive complex valued zero-mean Gaussian noise. Magnitudes of the first six received pulses with noise and two interference signals at the 7-element antenna array are shown in Figure 13.

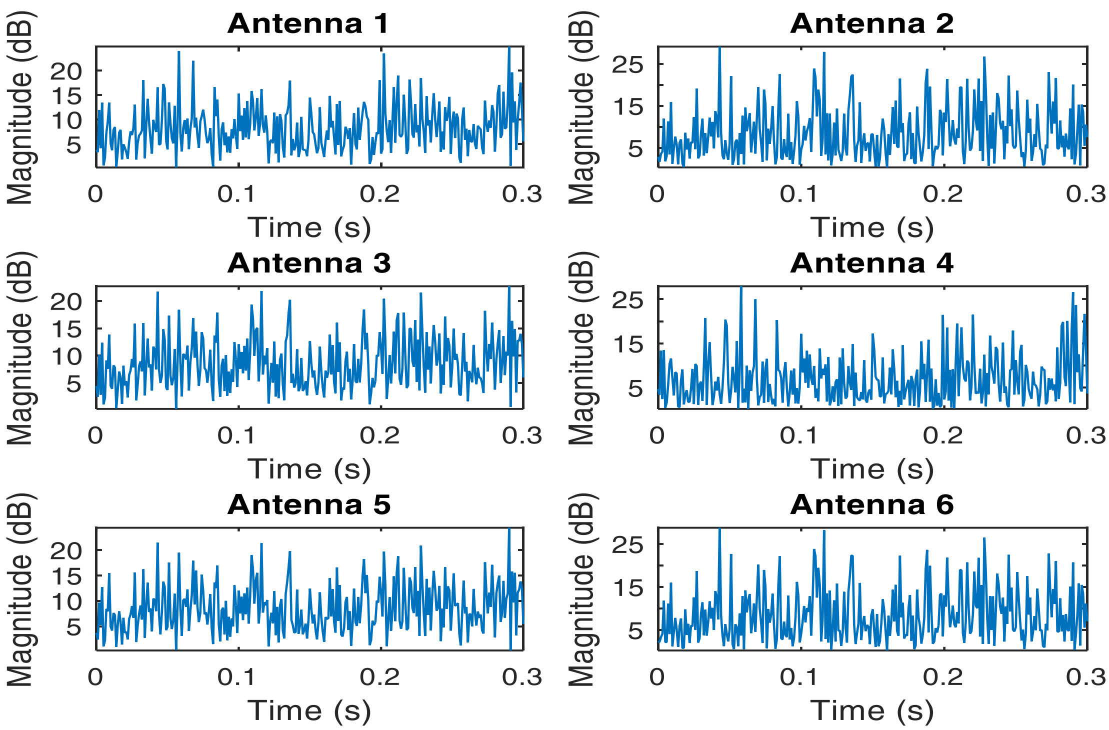

Figure 13 Magnitudes of signals with noise and interference

From the figure, it is seen that when the interference signals are added to the desired signals, it distorts the overall result, and the desired signal cannot be identified. To overcome the interference problem, two popular conventional weighted beamformers such as the MVDR and LCMV beamformer will be used. Both beamformers preserve the signal arriving along a desired direction while trying to suppress signals coming from other directions.

In the next section, it will be shown that using a beamformer the signal from MS A will be recovered, and the signals from MS B and MS C will be attenuated.

4.4 Simulation Results of MVDR Beamforming

If the desired signal (including noise and interference) goes through the MVDR beamformer, the required weight vectors are calculated adaptively. In this study, MATLAB “mvdrweights” function [95] was used to calculate the relevant weights of the signal components which are related to the Eqn. (3. 7).

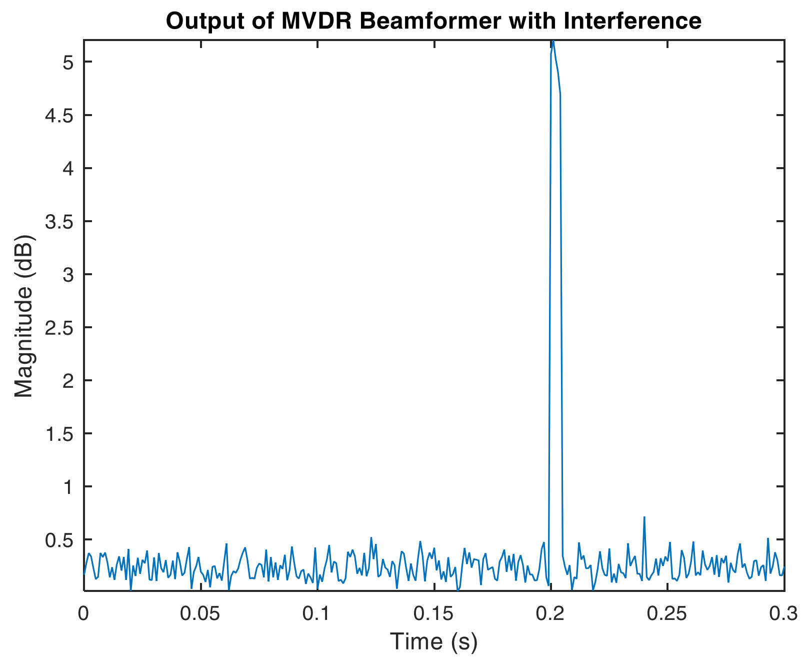

Figure 14 MVDR Beamformer output

Figure 14 shows the MVDR beamformer output result for desired signal (MS A). From the figure, it is seen that, while signals pass through the beamformer, even though it is in the presence of noise and two random interference signals from specific directions, the desired signal (MS A) can now be recovered and can be identified clearly, and it is getting the highest gain of 5 dB. At the same time, the interference signals (MS B and MS

C) are completely distorted and cannot be separated from the noise effect as the purpose of this algorithm is to reduce the gain in the direction of the interference signals.

As mentioned earlier, the desired signal (MS A) was considered at 0^0 and the two interference signals MSs B and C were fixed at 55^0, -40^0 respectively. For the design of beam patterns, the carrier frequency was taken to 1 GHz and linear array interspacing of $d = \lambda / 2$.

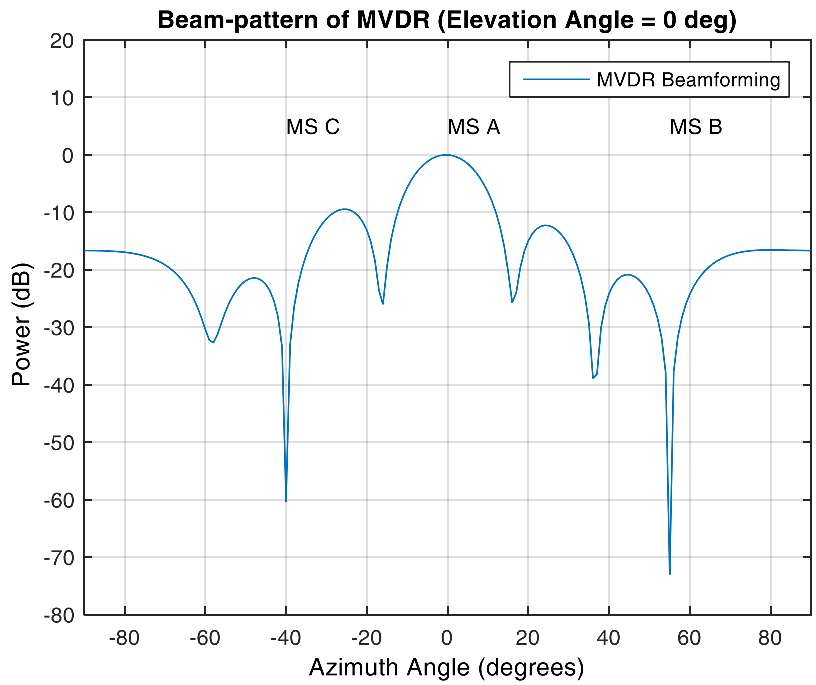

Figure 15 Beam pattern of MVDR Beamformer

The beam pattern of MVDR beamforming is showed in Figure 15. Form the Figure 15 it is observed that maximum gain is concentrated in the direction of target signal (MS A) while the gain is reduced in the direction of the interference signals (MS B and C). MVDR beamformer attenuates the interference signals coming from different directions than the desired signal.

Research has shown that, when DOA of the signal is known exactly, MVDR beamformer provides a distortionless response in the direction of the desired signal, while suppressing noise and interference. However, if there is uncertainty in DOA of the desired signal, the performance of MVDR beamformer is known to degrade severely. To improve interference rejection, another conventional beamforming technique is studied named LCMV beamforming.

4.5 Simulation Results of LCMV Beamforming

The LCMV beamforming algorithm considers the issue of reducing the effect of interference signals, and the objective is to improve the overall system performance by providing null gain in the direction of the interference signals while giving the maximum gain in the direction of the desired signal.

As MVDR beamforming, a seven-element array was considered for the LCMV beamforming algorithm and the noise power were assumed to 3 dB. As in the previous, the DOAs for the target MS A was assumed at 0^0 azimuth and 0^0 elevation whereas the DOA of the other MSs B and C were fixed at $55^0, -40^0$ respectively.

The weight vectors were adaptively adjusted in LCMV beamforming algorithm by using MATLAB “lcmvweights” function [96]. This specific function was used to determine the weighting coefficients by calculating spatial correlation matrices that are explained in Chapter 3 Eqn. (3. 12).

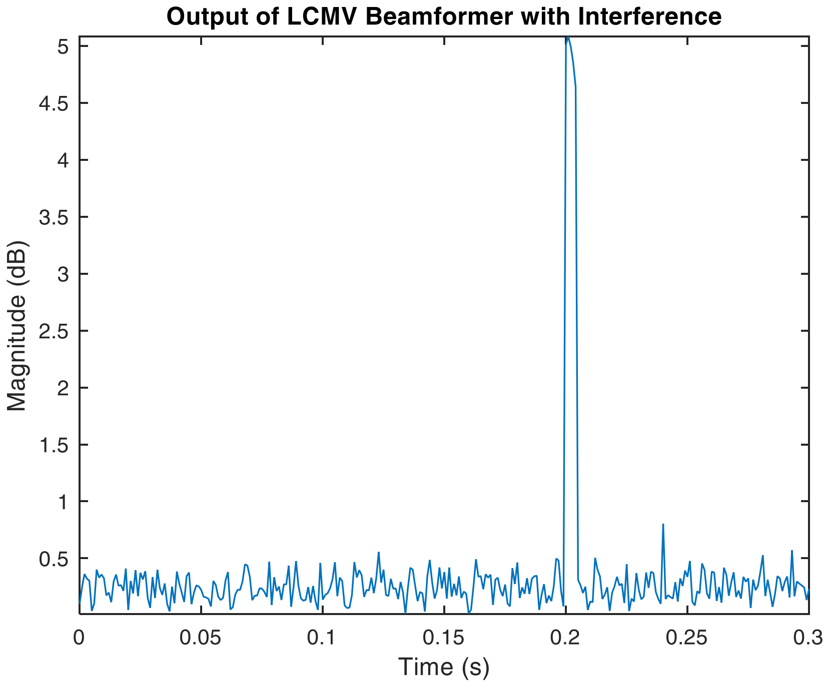

Figure 16 Signal output of LCMV Beamformer

In this simulation, the CollectPlaneWave method [97] from MATLAB is used to generate the signals $x_1, x_2, \ldots, x_M$ from the antennas of the array.

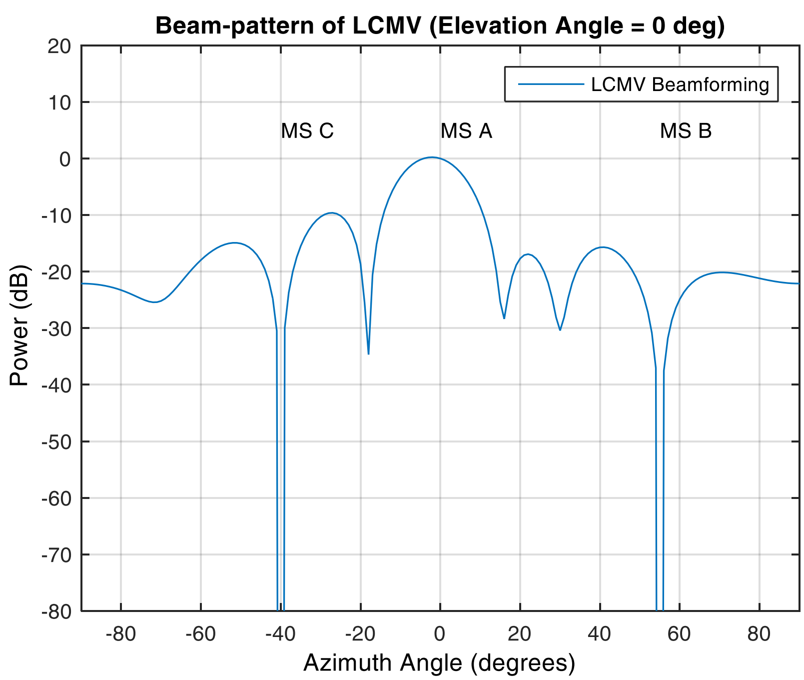

Figure 17 Beam pattern of LCMV Beamformer

The output of the LCMV beamformer is shown in Figure 16. From the Figure 16, the desired signal from MS A is clearly visible whereas the interference is attenuated. Both MVDR and LCMV give similar results in terms of the desired signal gain. The carrier frequency and the array inter-spacing were the same as before which is mentioned in sec 4.1. The beam pattern of LCMV beamforming is shown in Figure 17. This also shows that, the highest gain is acquired in the direction of the desired signal (MS A $= 0^o$) while the gain in the direction of interference signals (MS B= 55^o and MS C $= -40^o$) is nulled successfully. This indicates that the beam pattern obtained from LCMB beamforming will lead to rejection of strong interference components from both MS B and MS C.

4.6 Beam pattern comparison (MVDR and LCMV Beamforming)

Three MSs, namely, A, B, C were assumed earlier where MS A was considered as the desired signal and MS B and C were the two interference signals. Figure 18 shows the signal output comparison for MVDR and LCMV Beamformer.

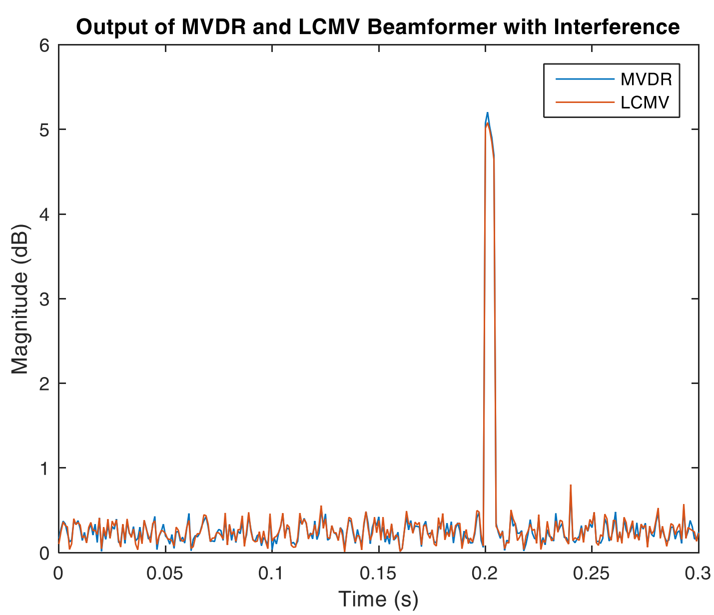

Figure 18 Signal Output comparison with MVDR and LCMV Beamformer

Figure 18 indicates that for both MVDR and LCMV beamforming techniques, the desired signals get the maximum amount gain, which is close to 5dB while interference signals MS B and MS C are attenuated. It can be said that both beamformers provide similar gain in the direction of the desired user as both MVDR and LCMV beamformer preserve the signal arriving along a desired direction, while trying to suppress signals coming from other directions.

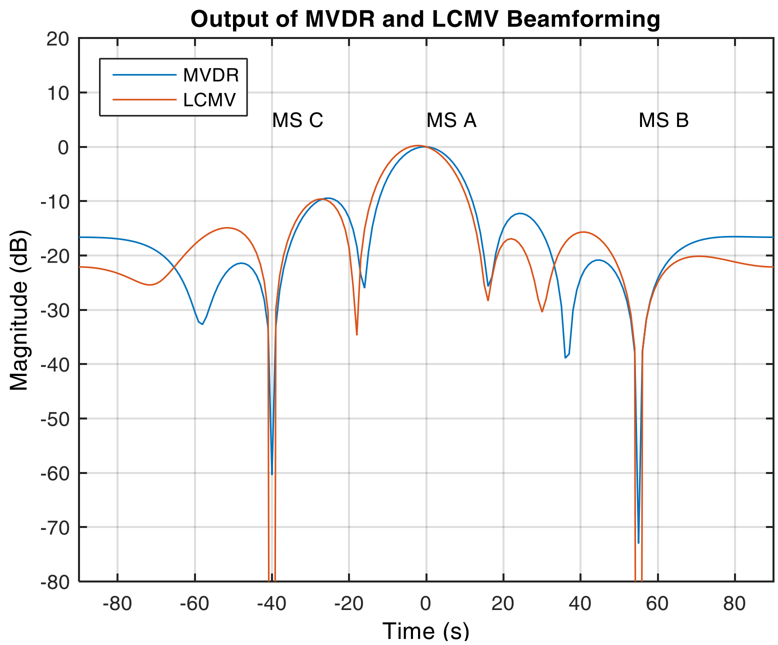

Figure 19 Beam pattern comparison for 7-element antenna

According to Figure 19, it is seen that both methods give a similar beam pattern while LCMV provides better performance as it nulls the gain in the direction of the interference signals and gives the maximum output to the desired signal. Whereas, MVDR beamforming technique cannot be null but only reduce the gain in the direction of the interference signals. The array gain using a different number of array element has been obtained. The DOA's of the signals is same as before (the desired signal (MS A) was considered at 0^0 and the two interference signals MSs B and C were fixed at $55^0, -40^0$

respectively). By changing the number of array elements in MVDR and LCMV has a significant impact on the beam pattern because increasing the number of antenna elements from 7 to 15 results in better directivity towards the desired signal. Therefore, the number of antenna elements plays a major role in interference suppression.

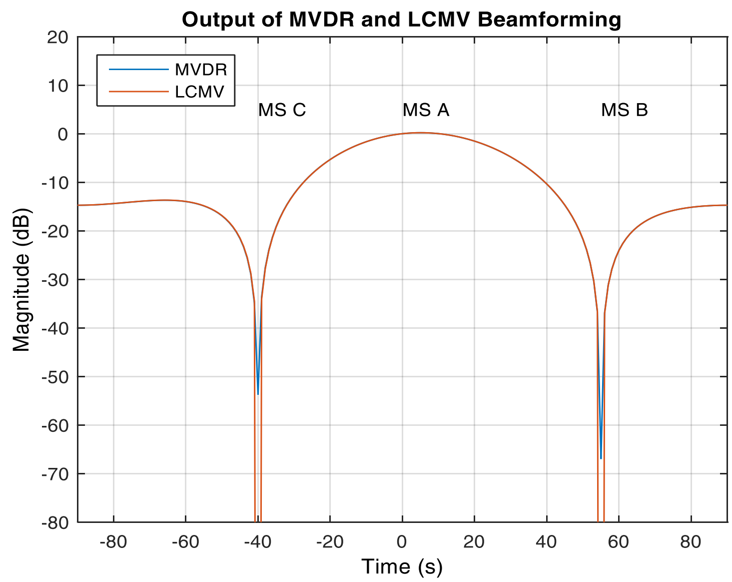

Figure 20 Beam pattern comparison for 3-element antenna

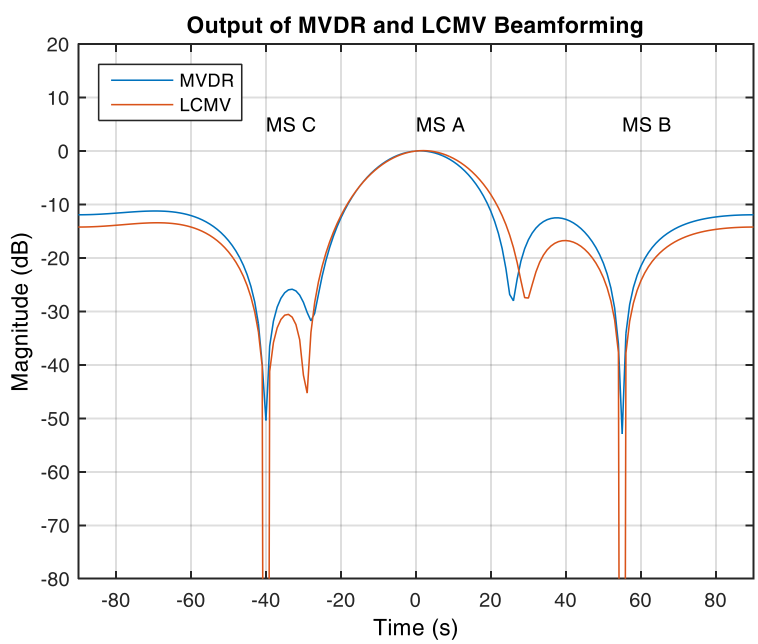

Figure 21 Beam pattern comparison for 5-element antenna

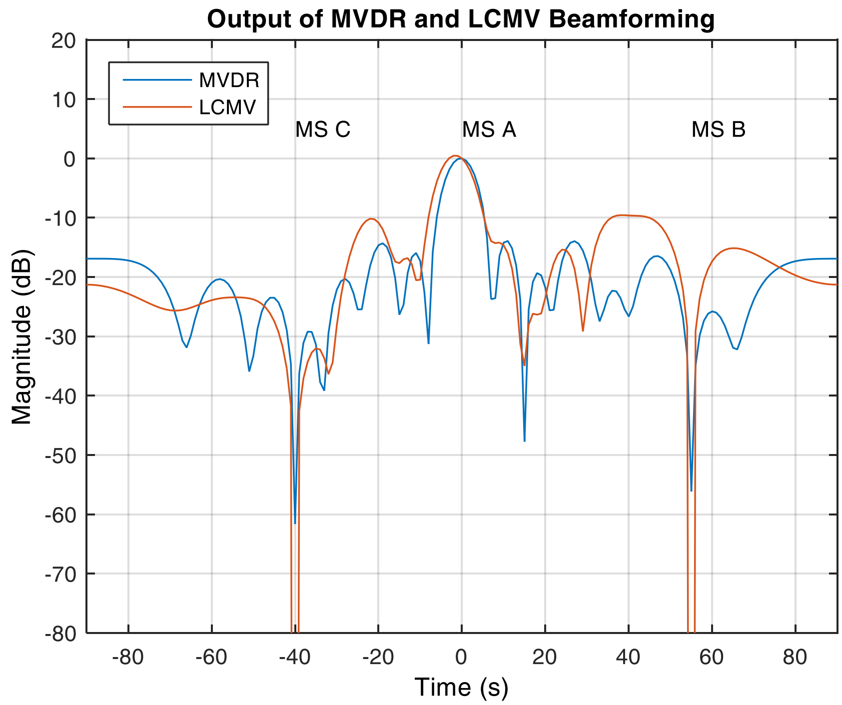

Figure 22 Beam pattern comparison for 15-element antenna

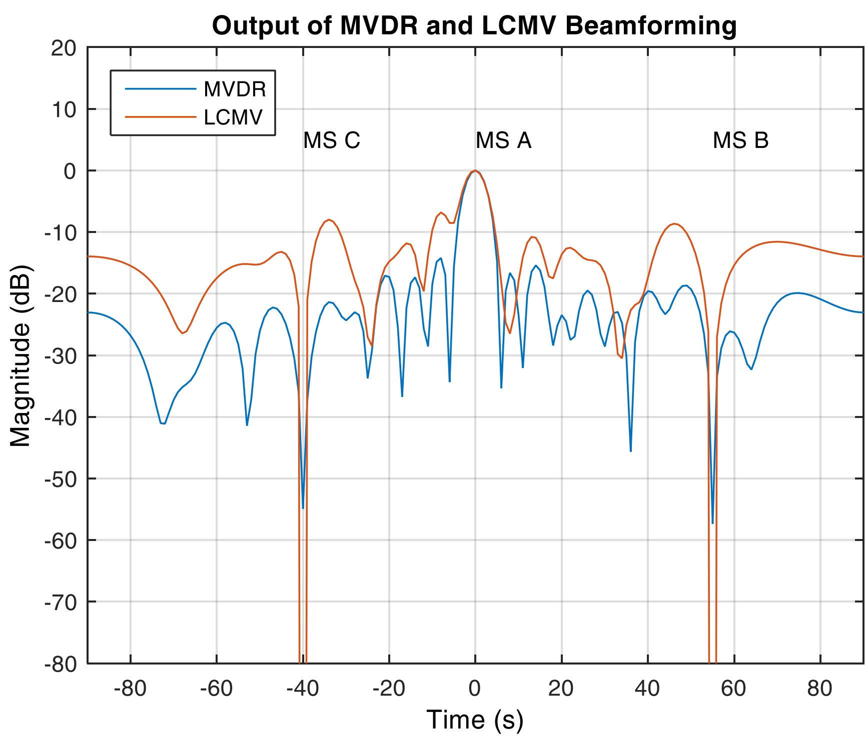

Figure 23 Beam pattern comparison for 20-element antenna

From the results, it can be seen that when the number of array element is increased, the gain is concentrated more precisely in the direction of the desired user. So the results can be summed up as: MVDR beamforming can concentrate the highest gain in the direction of target user but cannot null the interference signals. On the other hand, LCMV can null the interference signals successfully. So apparently, LCMV beamforming provides better results in terms of reducing the interference signals, and it can be widely used in many mobile operations.

4.7 Result Analysis and Conclusion

The purpose of this experiment is to illustrate the reduction of interference signals on the system performance by using MVDR and LCMV beamforming methods. The simulation results show that the beam pattern obtained for both the methods led to rejecting interference components MS B and MS C and getting the highest gain towards the desired signal MS A. The LCMV algorithm provides better performance for reducing the interference signals. By changing the number of elements from three to five, fifteen and twenty, it has been observed that the gain is concentrated more accurately in the direction of the desired user.

Chapter 5: Conclusions

This project presents two different weighted beamforming algorithms, which have gained importance in the wireless mobile communication system due to its ability to reduced co-channel and adjacent channel interferences. Depending on the beamforming technique such as the Minimum Variance Distortionless Response (MVDR) and the Linear Constraint Minimum Variance (LCMV), the beamforming weights are obtained. The objective is to minimize output variance or power subject to the response constraint. This has an effect of preserving the desired signal while minimizing contributions to the output from the interfering signals and noise arriving from directions other than the direction of interest. Both methods give a high-output gain in the desired direction, but require knowledge of the direction of all incoming sources which is difficult to obtain in practice.

It has been found that MVDR and LCMV beamformers share the same noise mitigation capabilities while they vary in suppressing interference. LCMV beamformer has higher interference suppression than MVDR beamforming method. On the other note, it has been shown that, increasing the number of antenna element in both MVDR and LCMV beamformers provides improved performance as the gain is concentrated more precisely in the direction of the desired signal. Finally, after comparing the beam pattern for different numbers of antenna elements, it was observed that LCMV beamforming provides better performance in the interference rejection. Beamforming has proven its benefits for the next-generation mobile system and plays a vital role in the next-generation mobile networks.

Both algorithms are considered as general enough to be utilized in a broad range of mobile communications scenarios (e.g., rural, urban, suburban, indoor). Both MVDR and LCMV beamforming methods can also be used in other applications such as speech enhancement and noise reduction.

5.1 Future Works

While doing the research, here are various exciting fields that caught my attention. Implementation of a new weighted algorithm is one of them. Beam pattern comparison between new weighted algorithm with the MVDR and LCMV algorithms would be an interesting subject as well as the impact of elective beamforming on the operations of a wireless system in terms of outage probability; it can be shown that how the new weighted algorithm is better than the other two algorithms. Again, this experiment can be done not assuming but using the DOA estimation technique for determining the DOAs for the desired and the interference signals that would be a great field to work further.

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