Error reachable set based stabilization of switched linear systems with bounded peak disturbances

Xinxin Shang, Songlin Zhuang, Tianyu Tan, & Yang Shi

2023

This article was originally published at:
https://doi.org/10.1016/j.jai.2023.06.001

Citation for this paper:

Shang, X., Zhuang, S., Tan, T., & Shi, Y. (2023). Error reachable set based stabilization of switched linear systems with bounded peak disturbances. *Journal of Automation and Intelligence, 2*(2)*,* 87-98. https://doi.org/10.1016/j.jai.2023.06.001.

Journal of Automation and Intelligence 2 (2023) 87–98

Contents lists available at ScienceDirect

Journal of Automation and Intelligence

journal homepage: www.keaipublishing.com/en/journals/journal-of-automation-and-intelligence/

Research article

Error reachable set based stabilization of switched linear systems with bounded peak disturbances

Xinxin Shang [a], Songlin Zhuang [b], Tianyu Tan [a], Yang Shi [a,*]

[a] Department of Mechanical Engineering, University of Victoria, Victoria, BC, Canada
[b] Yongjiang Laboratory, Ningbo, China

ARTICLE INFO

Keywords:
Bounded peak disturbances
Persistent dwell-time
Reachable set minimization
Stability and stabilization
Switched systems

ABSTRACT

This paper investigates the error reachable set based stabilization problem for a class of discrete-time switched linear systems with bounded peak disturbances under persistent dwell-time (PDT) constraint. A double-clock-dependent control scheme is presented that can split the disturbed switched system into a nominal system and an error system, and assign to each system a controller scheduled by a clock. A necessary and sufficient convex stability criterion is presented for the nominal system, and is further extended to the stabilization controller design with a nominal clock. In the presence of bounded peak disturbances, another stabilization controller with an error clock is developed for the error system, with the purpose of "minimizing" the reachable set of the error system by the ellipsoidal techniques. It is demonstrated that the disturbed system is also globally exponentially stable in the sense of converging to an over approximation of the reachable set of the error system, i.e., a union of a family of bounding ellipsoids, that can also be regarded as the cross section of a tube containing the trajectories of the disturbed system. Two numerical examples are provided to verify the effectiveness of the developed results.

1. Introduction

As a typical class of hybrid systems, switched systems have been well investigated from both the academic and industrial communities in the last decades. A switched system is composed of a collection of subsystems and a switching event that indicates the active subsystem mode at each time instant [1]. It has been widely applied to model many practical systems with multiple-mode characteristics, including systems with time delay [2,3], with intermittent failures [4,5], with external triggers [6,7], and with operating regions [8,9], etc. So far, the research on switched systems has attracted significant attention, e.g., stability analysis [10,11], controller synthesis [12,13], and filter design [14,15].

The switching events in switched systems can be classified into two categories: time-dependent versus state-dependent [1]. The time-dependent events are commonly described by signal sets with various time constraints. Three basic but important sets include dwell-time (DT) set, average dwell-time (ADT) set, and persistent dwell-time (PDT) set. The PDT set is of interest of this paper. It consists of non-switching intervals and arbitrary-switching intervals and is more flexible as it can reduce to DT and ADT sets under specific conditions [16]. In the framework of Lyapunov function, a number of effective results have been reported for the stability analysis of the switched systems with these three kinds of switching signals [17]. In [18,19], an improved method called the lifting technique is presented that can convert the well-known non-convex stability conditions [20] to equivalent convex ones for DT switched systems. This technique further facilitates the study of non-conservative stability criteria for switched systems under DT constraint [21] or even arbitrary switching [22]. Some attempts have also been performed to lift the existing non-convex stability conditions, e.g., [23,24], for ADT and PDT switched systems to convex conditions, but still fail to achieve the convexification of the conditions due to the complexity and exibility of these two switching signals [25]. Among others, the widely-used quasi-time-dependent (QTD) technique presents the QTD Lyapunov function approach. Its main idea is to confine the Lyapunov function to descend in the non-switching intervals, and have a bound in the arbitrary-switching intervals. Some tuning parameters are necessary to shape the Lyapunov function. As a result, the QTD technique can only obtain non-convex stability conditions in the form of bilinear matrix inequalities (BMIs) which are known to be NP-hard. Some other results about dynamics with PDT switching features also suffer from the nonconvexity [5,26,27]. Till now, to the best of authors' knowledge, necessary and sufficient convex stability conditions for PDT switched linear systems have not been addressed, which motivates the first interest of this paper.

As a crucial issue in control discipline, the stabilization problem of systems with disturbances has been broadly investigated [2,28–31]. For

* Corresponding author.
E-mail address: yshi@uvic.ca (Y. Shi).

https://doi.org/10.1016/j.jai.2023.06.001
Received 5 May 2023; Received in revised form 2 June 2023; Accepted 10 June 2023

the switched systems under various types of time constraints, lots of significant results are reported for the systems with energy-bounded disturbances [32–34], but relatively seldom for the amplitude-bounded disturbances. Recently, a study on characterizing the robust invariant set for the switched system with bounded additive disturbance under DT constraint has been carried out [35]. This method is further extended to the PDT case for the uniform tube based stabilization [36]. However, it only presents a method to compute the robust invariant set for PDT switched systems, and the so-called stabilization controller is unreasonably considered same as the linear quadratic optimal regulator. On the other hand, reachable set estimation problem for the switched systems has received increasing attention in recent years by the so-called ellipsoidal technique [37–40], which aims to determine a set of ellipsoids tightly containing the concerned reachable set. An important problem therein is to design a controller such that the reachable of the closed-loop system is over approximated by some bounding ellipsoids that are as small as possible [41,42]. However, available results only cover the cases of switched systems with arbitrary switching and dwell-time switching [40,42], and the controller synthesis to "minimize" the reachable set approximation for PDT switched systems has not been addressed. Thus, the second interest of this paper arises: How to stabilize the PDT switched system with bounded peak disturbances and simultaneously "minimize" the effect caused by the disturbances?

Based on the above considerations, this paper considers the error reachable set based stabilization problem of switched systems with bounded peak disturbances under PDT constraint. The main contributions of this paper are summarized as follows:

- We present a double-clock-dependent control scheme for PDT switched systems, that can split the disturbed switched system into a nominal system and an error system, and each system is assigned a controller scheduled by its own clock.
- We present a necessary and sufficient convex global exponential stability criterion for PDT switched systems, and accordingly design the stabilization controller for the nominal system.
- We present a virtual-sequence-dependent reachable set estimation method, and further design the stabilization controller to minimize the reachable set of the error system by ellipsoidal technique.
- The closed-loop switched systems integrated with the two controllers are proved to be globally exponentially stable in the sense of converging to a set which is also regarded as the cross section of a tube containing the practical trajectories of the disturbed system.

Organization: In Section 2, the considered problem is formulated, and the double-clock-dependent control scheme is introduced. In Section 3 we give the concept of switching sequence list with admissible concatenation list, which can convert the infinite number of admissible PDT switching sequences to a finite number of certain concatenations. Section 4 gives the nonconservative convex global exponential stability criterion, and accordingly the stabilization controller scheduled by a clock. Section 5 is devoted to the virtual-sequence-dependent reachable set estimation, and the controller design with the purpose of minimizing the reachable set. The stabilization scheme for PDT switched systems with bounded peak disturbances is also given. Two examples are provided in Section 6 to verify the effectiveness of the developed results. We conclude the paper in Section 7.

Notations: $\mathbb{R}^n$ denotes the n dimensional Euclidean space; $\mathbb{Z}_+$ denotes the set of non-negative integers; $\mathbb{Z}_{\geq t} := \{k \in \mathbb{Z}_+ \mid k \geq t\}$; $\mathbb{Z}_{[t_1,t_2]} := \{k \in \mathbb{Z}_+ \mid t_1 \leq k \leq t_2\}$. For a list $\mathcal{A}$, we use $|\mathcal{A}|$ to denote the list length. $\mathcal{A}_i$ denotes the ith element of $\mathcal{A}$, and $\mathcal{A}_{i,j}$ denotes the jth element of list $\mathcal{A}_i, i = 0, 1, \dots, |\mathcal{A}|-1, j = 0, 1, \dots, |\mathcal{A}_i|-1$. A^{T} denotes the transpose of A. The set of $n \times n$ symmetric positive definite matrices is denoted by $\mathbb{S}^n_{>0}$. $\|x\|_S = d(x, S)$ denotes the distance of a point $x \in \mathbb{R}^{n_x}$ from a set $S \subset \mathbb{R}^{n_x}$ and is defined by $d(x, S) := \inf_{\hat{x}}\{d(x, \hat{x}) \mid \hat{x} \in S\}$, where $d(x, \hat{x})$ denotes the distance between x and $\hat{x}$.

2. Preliminaries and problem formulation

Consider a class of discrete-time switched linear systems with bounded peak disturbances:

$$x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + D_{\sigma(k)}\omega(k) \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ and $u(k) \in \mathbb{R}^{n_u}$ are the system state and control input, respectively; $\omega(k) \in \mathbb{R}^{n_\omega}$ denotes the bounded peak disturbance satisfying:

$$\omega^{\mathrm{T}}(k)\omega(k) \leq \bar{\omega}^2, k \in \mathbb{Z}_+ \tag{2}$$

where $\bar{\omega} > 0$ is a known constant. The switching signal $\sigma(k) : \mathbb{Z}_+ \to \mathcal{I}_N := \{1, 2, \dots, N\}$ indicates the activated subsystem among N possible modes. The switching instants of subsystem modes are denoted by $k_s, s \in \mathbb{Z}_+$. It should be noted that another kind of disturbances widely discussed in literature are energy-bounded, which has the form of $\sum_{k=0}^{\infty}\omega^{\mathrm{T}}(k)\omega(k) \leq \bar{\omega}^2$. Compared to energy-bounded disturbances, bounded peak disturbances is a kind of uncertainties that persistently influence the system evolution over time, and it will not degrade with time.

Definition 1 (*[16]*). The set $S_{p-dwell}[\tau, T], \tau > 0, T \in [0, \infty]$ is a PDT signal set, if there is an infinite number of disjoint intervals of length no smaller than τ on which σ is constant, and consecutive intervals with this property are separated by no more than T. The constant τ is called the persistent dwell-time and T the period of persistence.

According to Definition 1, any admissible PDT switching sequences can be segmented into two kinds of intervals, denoted by τ-portion and T-portion, which correspond to the non-switching intervals and the arbitrary switching intervals [23,43], respectively. For example, consider the PDT signal set $S_{\mathrm{p-\ dwell}}[3, 3]$. Fig. 1(a) gives three switching sequences, where the first switching sequence is an admissible PDT switching sequence since the running time for each τ-portion is no less than τ, and for each T-portion is no more than T. The other two sequences are inadmissible because they violate the time constraints of T-portion and τ-portion, respectively.

To handle the bounded peak disturbances, we refer to the tube-based control methodology widely used in model predictive control (MPC) areas [44–46], and present a double-clock-dependent control scheme. It splits the disturbed system (1) into a nominal system and an error system. The nominal system of the disturbed system (1) is denoted by:

$$z(k+1) = A_{\sigma(k)}z(k) + B_{\sigma(k)}v(k) \tag{3}$$

where $z(k) \in \mathbb{R}^{n_x}$ and $v(k) \in \mathbb{R}^{n_u}$ are the nominal system state and control input, respectively. Define the system error $e(k) = x(k) - z(k)$, then by (1) and (3) we can obtain the error system of the disturbed system (1) in form of:

$$\begin{aligned} e(k+1) &= A_{\sigma(k)}e(k) + B_{\sigma(k)}(u(k) - v(k)) + D_{\sigma(k)}\omega(k) \\ e(0) &= x(0) - z(0) = 0 \end{aligned} \tag{4}$$

The main idea of the tube-based MPC is to control the trajectory of the nominal system (3) to converge to the origin by optimizing $v(k)$ at each sampling instant, and keep the trajectory of the error system (4) within a robust positive invariant set. However, in this paper, the concept of the invariant set is not suitable because:

- The invariant set requires all trajectories originating from the set always stay within the set, but in fact, the initial state of the error system (4) is $e(0) = 0$, so the invariant set contains unnecessary initial states.
- Generally, the invariant set is iteratively generated with a given controller gain, and the synthesis problem still yet remains open: How to design a controller such that the invariant set of the switched system is as "small" as possible under certain switching signals?

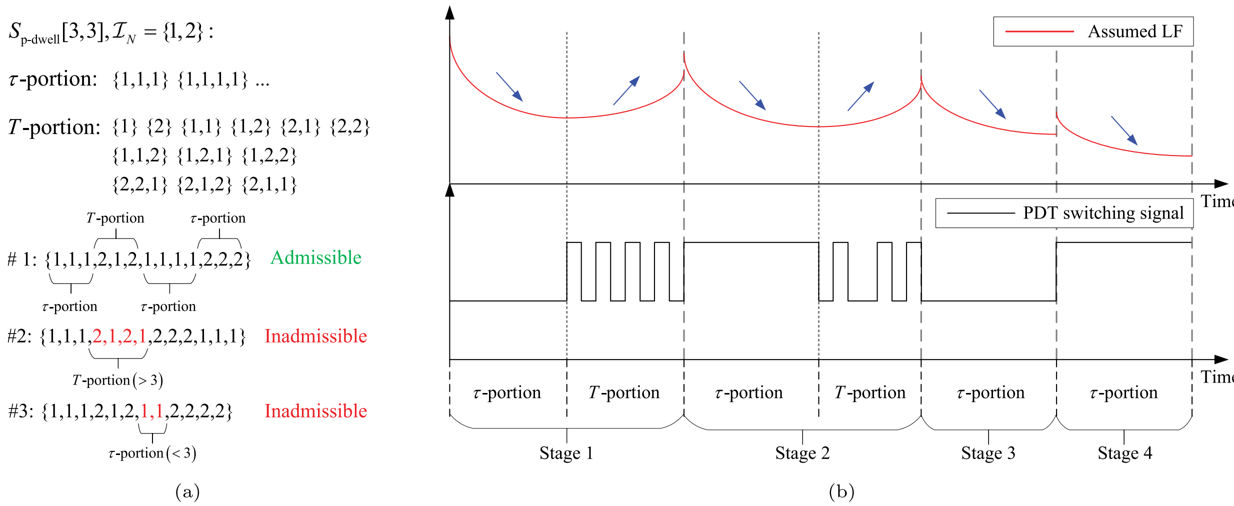


Fig. 1. Illustration of PDT signal set and the widely-used framework of stage partition. (a) τ-portions, T-portions, and three illustrative PDT switching sequences for $S_{\text{p-dwell}}[3,3]$ with $\mathcal{I}_N = \{1,2\}$. (b) Evolutions of Lyapunov function in the framework of stage partition. "LF" is short for Lyapunov function. Blue arrows indicate the variation trend of the assumed LF.

For the purpose of "minimizing" the effect of $\omega(k)$ in the system(1), we replace the invariant set by the reachable set of the error system (4), which is defined by:

$$\mathcal{R}_e := \left\{ e(k) \in \mathbb{R}^{n_x} \mid e(k), e(0) \text{ satisfy } (4), k \in \mathbb{Z}_+ \right\} \tag{5}$$

and employ ellipsoidal techniques to estimate the reachable set [6, 7]. The estimated reachable set $\hat{\mathcal{R}}_e$ satisfies $\mathcal{R}_e \subseteq \hat{\mathcal{R}}_e$, and is denoted by a union of some bounding ellipsoids in form of:

$$\mathcal{E}(P) := \left\{ e(k) \in \mathbb{R}^{n_x} \mid e^{\mathrm{T}}(k) P e(k) \le 1, P \in \mathbb{S}^{n_x}_{>0} \right\}. \tag{6}$$

The main advantages of using ellipsoidal techniques include: (1) Attenuation of bounded peak disturbances can be turned to an optimization problem of minimizing the reachable set, which is mathematically tractable, (2) By embedding our non-conservative stability conditions in Theorem 1, the reachable set can be further minimized compared to other existing methods, and (3) By introducing the virtual-sequence-dependent control scheme, operations on the error system do not affect the nominal system.

In the framework of double-clock-dependent control scheme, we use the following controllers for the systems (1), (3), and (4):

$$u(k) = v(k) + g(k) \tag{7}$$

$$v(k) = F_{\sigma(k)}\left(\theta_z(k)\right) z(k) \tag{8}$$

$$g(k) = K_{\sigma(k)}\left(\theta_e(k)\right) e(k) \tag{9}$$

where $F_{\sigma(k)}\left(\theta_z(k)\right)$ and $K_{\sigma(k)}\left(\theta_e(k)\right)$ are the controller gains for the nominal system (3) and the error system (4), respectively. $\theta_z(k)$ and $\theta_e(k)$ are online scheduled controller clocks which can be simply calculated by

$$\theta_z(k) = \begin{cases} k - k_s, & k \in \left[k_s, k_s + \tau_z\right) \\ \tau_z, & k \in \left[k_s + \tau_z, k_{s+1}\right) \end{cases} \tag{10}$$

and

$$\theta_e(k) = \begin{cases} k - k_s, & k \in \left[k_s, k_s + \tau_e\right) \\ \tau_e, & k \in \left[k_s + \tau_e, k_{s+1}\right) \end{cases} \tag{11}$$

where τ_z and τ_e denote persistent dwell time to ensure the stability of the system (3), and minimize the reachable set of the system (4), respectively.

Definition 2. The equilibrium $z = 0$ of the system (3) is said to be globally exponentially stable (GES) with a decay rate $\mu > 0$ if $\|z(k)\| \le c e^{-\mu(k-k_0)} \left\| z\left(k_0\right) \right\|$ holds for any initial condition $z\left(k_0\right) \in \mathbb{R}^{n_x}$, any $k \ge k_0$ and a constant $c > 0$.

Consider the composite system (1) and (3) with the composite state $(x(k), z(k))$ [45]. Define its norm by $\|(x(k), z(k))\| := \|x(k)\| + \|z(k)\|$.

Definition 3. A set $\mathcal{R}$ is said to be GES for the composite system (1) and (3) with a decay rate $\mu > 0$, if $\|(x(k), z(k))\|_{\mathcal{R}} \le c e^{-\mu(k-k_0)} \| \left(x\left(k_0\right), z\left(k_0\right)\right) \|_{\mathcal{R}}$ for any composite initial state $\left(x\left(k_0\right), z\left(k_0\right)\right)$, any $k \ge k_0$ and a constant $c > 0$.

Then, for a given period of persistence T, the objectives of this paper include:

- Design the controller gains $F_{\sigma(k)}\left(\theta_z(k)\right)$ in (8) with minimal τ_z such that the closed-loop nominal system is GES.
- Design the controller gains $K_{\sigma(k)}\left(\theta_e(k)\right)$ in (9) with minimal τ_e such that the reachable set $\mathcal{R}_e$ bounded by $\hat{\mathcal{R}}_e$ is as small as possible.
- Determine a set $\mathcal{R}$ and admissible switching signal sets for the closed-loop composite system (1) and (3) such that it is GES in the sense of Definition 3.

3. Switching sequence list with admissible concatenation list

So far, to the best of authors' knowledge, all existing results about PDT switched systems are based on a same framework, named "stage partition" [5,23,26,27,34,43]. In this framework, PDT switching sequences are segmented to stages, and each stage consists of a τ-portion, or a τ-portion followed by a T-portion. The Lyapunov function under this framework follows three rules:

- decreases in the τ-portion;
- has a bound in the T-portion;
- decreases at the starting instant of each stage.

Fig. 1(b) illustrates the evolution of the Lyapunov function under the framework of the stage partition. Some tuning parameters, like the descending rate α and jump bound μ, have to be introduced to ensure the system stability as the above rules, which yields that the conditions about, whatever stability analysis or control synthesis, are in the form of bilinear matrix inequalities (BMIs). The main reason lies in that the stage partition may generate uncertain intervals, e.g., τ-portion contains an infinite number of switching sequences, and the finite switching sequences in the T-portion are not fully used under this framework. To solve this problem and obtain nonconservative convex stability conditions, we need to develop a novel approach that can represent the infinite number of admissible PDT switching sequences

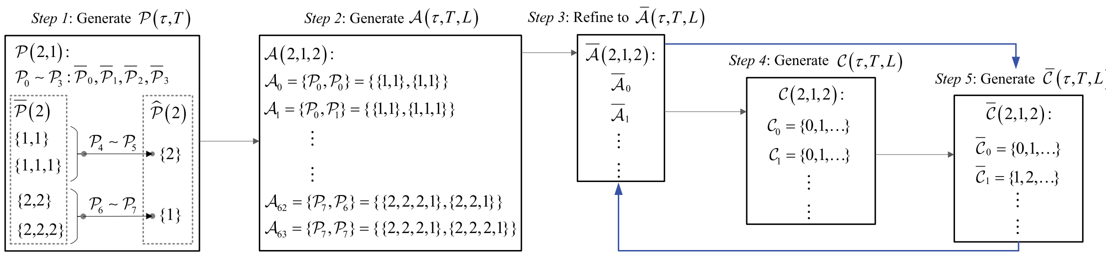


Fig. 2. Procedures of generating switching sequence list and its concatenation list for a given PDT signal set $S_{\text{p- dwell}}[\tau, T]$.

by a finite number of certain switching subsequences. To address this, some novel concepts are given as follows.

Definition 4. A switching sequence list $\mathcal{A} := \{\mathcal{A}_0, \mathcal{A}_1, \ldots, \mathcal{A}_{M-1}\}$ is a list that consists of M certain switching subsequences, $M \in \mathbb{Z}_{\geq 1}$. The concatenation list $\mathcal{C} := \{\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_{M-1}\}$ of $\mathcal{A}$ is a list where $\mathcal{C}_i$ is an index list that consists of all the indices of admissible elements in $\mathcal{A}$ which can follow $\mathcal{A}_i, i \in \mathbb{Z}_{[0,M-1]}$. Here, the "admissible" means the switching sequence $\{\mathcal{A}_i, \mathcal{A}_j\}, j \in \mathcal{C}_i$ is an admissible PDT switching sequence.

Example 1. Consider a PDT signal set $S_{p-dwell}[2,1]$ with $\mathcal{I}_N = \{1,2\}$. We can construct a switching sequence list $\mathcal{A} = \{\mathcal{A}_0, \mathcal{A}_1\} = \{\{1,1,1\},\{2,1\}\}$, and assume its concatenation list $\mathcal{C}$ as $\mathcal{C} = \{\mathcal{C}_0, \mathcal{C}_1\} = \{\{0,1\},\{0,1\}\}$.

Definition 5. A PDT signal set $S_{p-dwell}[\tau, T]$ is said to be equivalent to a switching sequence list $\mathcal{A}$ with its concatenation list $\mathcal{C}$, if it follows:

- Any admissible PDT switching sequences can be denoted by concatenations of elements in $\mathcal{A}$ with $\mathcal{C}$, e.g., $\{\mathcal{A}_i, \mathcal{A}_j, \mathcal{A}_k, \ldots\}$, where $j \in \mathcal{C}_i, k \in \mathcal{C}_j, \forall \mathcal{A}_i \in \mathcal{A}$.
- Any elements of $\mathcal{A}$ with concatenation list $\mathcal{C}$, denoted by $\{\mathcal{A}_i, \mathcal{A}_j, \mathcal{A}_k, \ldots\}$, can generate an admissible PDT switching sequence, where $j \in \mathcal{C}_i, k \in \mathcal{C}_j, \forall \mathcal{A}_i \in \mathcal{A}$.
- If $\mathcal{A}_i \in \mathcal{A}$, then $\mathcal{A}_j \in \mathcal{A}, \forall j \in \mathcal{C}_i, \forall i \in \mathbb{Z}_{[0,|\mathcal{A}|-1]}$.
- "$\mathcal{A}$ with $\mathcal{C}$ " means that any concatenations of elements in $\mathcal{A}$ comply with the concatenation list $\mathcal{C}$.

In Example 1, it can be checked that the switching sequence list $\mathcal{A}$ with its concatenation list $\mathcal{C}$ is not equivalent to $S_{\text{p-dwell}}[2,1]$ because: (i) $\{2,2,1\}$ is an admissible PDT switching sequence but cannot be denoted by $\mathcal{A}$ with $\mathcal{C}$, (ii) $\{\mathcal{A}_1, \mathcal{A}_1\} = \{\{2,1\},\{2,1\}\}$ is a switching sequence generated by $\mathcal{A}$ with $\mathcal{C}$ (because $1 \in \mathcal{C}_1$), but it is not an admissible PDT switching sequence due to the violation of constraints in the T-portion. Thus, a problem to be addressed in this section arises: How to construct a switching sequence list $\mathcal{A}$ with its concatenation list $\mathcal{C}$ that is equivalent to a given PDT signal set $S_{\text{p-dwell}}[\tau, T]$? If this problem can be solved, then we can only focus on a finite number of certain switching subsequences for further convex conditions about stability analysis and control synthesis.

Next, we will give a detailed procedures of generating $\mathcal{A}$ with $\mathcal{C}$ (equivalent to $S_{\text{p-dwell}}[\tau, T]$) by the following five steps. Fig. 2 summarizes the process.

Step 1: We first give the concept of PDT primary sequence list, denoted by $\mathcal{P}(\tau, T)$, for a given $S_{\text{p-dwell}}[\tau, T]$. To represent all admissible sequences in τ-portion, we define a sequence list $\bar{\mathcal{P}}(\tau) := \{\bar{\mathcal{P}}_0, \ldots, \bar{\mathcal{P}}_{N\tau-1}\}$ which is composed of sequences $\{i, i, \ldots, i\}$ with length of $\mathbb{Z}_{[\tau,2\tau-1]}, i \in \mathcal{I}_N$. It can be easily checked that any admissible τ-portion sequences can be constructed by concatenating elements in $\bar{\mathcal{P}}(\tau)$. Moreover, we define a sequence list $\hat{\mathcal{P}}(T) := \{\hat{\mathcal{P}}_0, \hat{\mathcal{P}}_1, \ldots, \hat{\mathcal{P}}_T\}$ which contains all admissible sequences in T-portion. The "admissible" here indicates the length of $\hat{\mathcal{P}}_j, j \in \mathbb{Z}_{[0,T]}$, is no more than T, and the length of consecutive same subsystem modes in $\hat{\mathcal{P}}_j$ is less than τ. Then, the PDT primary sequence list is defined as $\mathcal{P}(\tau, T) := \{\mathcal{P}_0, \mathcal{P}_1, \ldots, \mathcal{P}_{M-1}\}$ where elements from $\mathcal{P}_0$ to $\mathcal{P}_{N\tau-1}$ are same to $\bar{\mathcal{P}}(\tau)$ elements, and each element from $\mathcal{P}_{N\tau}$ to $\mathcal{P}_{M-1}$ is constructed by one element in $\bar{\mathcal{P}}(\tau)$ (denoted by $\bar{\mathcal{P}}_i$) followed by one element in $\hat{\mathcal{P}}(T)$ (denoted by $\hat{\mathcal{P}}_j$), and the subsystem mode in $\bar{\mathcal{P}}_i$ is different with the first mode in the concatenated $\hat{\mathcal{P}}_j$. Fig. 2 also gives an example of PDT primary sequences for $S_{\text{p-dwell}}[2,1]$. It can be checked that $S_{\text{p-dwell}}[2,1]$ is equivalent to $\mathcal{P}(2,1)$ with the concatenation list $\mathcal{C} = \{\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_7\}$ where $\mathcal{C}_i = \{0, 1, \ldots, 7\}, i \in \mathbb{Z}_{[0,7]}$.

Step 2: Define $\mathcal{A}(\tau, T, L) := \{\mathcal{A}_0, \ldots, \mathcal{A}_{M^L-1}\}$ as the augmented PDT primary sequence list, which contains all possible combinations of $L\mathcal{P}(\tau, T)$ elements. Fig. 2 shows the augmentation operation from $\mathcal{P}(2,1)$ to $\mathcal{A}(2,1,2)$ by illustratively letting $L = 2$. It is concluded that $S_{\text{p-dwell}}[2,1]$ is also equivalent to $\mathcal{A}(2,1,2)$ with the concatenation list $\mathcal{C} = \{\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_{63}\}$ where $\mathcal{C}_i = \{0, 1, \ldots, 63\}, i \in \mathbb{Z}_{[0,63]}$.

Step 3: Note that $\mathcal{A}(\tau, T, L)$ may contain some same elements (for example, $\mathcal{A}_1 = \{\mathcal{P}_0, \mathcal{P}_1\}$ is same to $\mathcal{A}_9 = \{\mathcal{P}_1, \mathcal{P}_0\}$). Thus, we define $\bar{\mathcal{A}}(\tau, T, L)$ as the refined PDT sequence list, which is generated by removing the redundant same elements in $\mathcal{A}(\tau, T, L)$. The equivalence between $S_{\text{p-dwell}}[2,1]$ and $\bar{\mathcal{A}}(2,1,2)$ with the concatenation list $\mathcal{C} = \{\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_{|\bar{\mathcal{A}}(2,1,2)|-1}\}$ is also guaranteed, where $\mathcal{C}_i = \{0, 1, \ldots, |\bar{\mathcal{A}}(2,1,2)| - 1\}, i \in \mathcal{I}_L := \mathbb{Z}_{[0,|\bar{\mathcal{A}}(2,1,2)|-1]}$.

Step 4: Although Step 3 generates equivalent $\bar{\mathcal{A}}(\tau, T, L)$ with $\mathcal{C}$ to $S_{\text{p-dwell}}[\tau, T]$, there may exist different concatenations corresponding to same switching sequences. For example, there may exist the case of $\{\bar{\mathcal{A}}_{i_1}, \bar{\mathcal{A}}_{j_1}\} = \{\bar{\mathcal{A}}_{i_2}, \bar{\mathcal{A}}_{j_2}\}$ where $i_1, i_2 \in \mathcal{I}_L, j_1 \in \mathcal{C}_{i_1}, j_2 \in \mathcal{C}_{i_2}$. To simplify the concatenations lists, we only maintain the concatenations as the following cases (here we denote the concatenation in form of $\{\bar{\mathcal{A}}_i, \bar{\mathcal{A}}_j\}, \forall i, j \in \mathcal{I}_L$) :

(i) If $\bar{\mathcal{A}}_{i,\text{ last}} \neq \bar{\mathcal{A}}_{j,\text{first}}$, then this concatenation is kept, where $\bar{\mathcal{A}}_{i,\text{ last}}$ denotes the last subsystem mode in $\bar{\mathcal{A}}_i$, and $\bar{\mathcal{A}}_{j,\text{ first}}$ denotes the first subsystem mode in $\bar{\mathcal{A}}_j$.

(ii) If $\bar{\mathcal{A}}_{i,\text{ last}} = \bar{\mathcal{A}}_{j,\text{ first}}$, but the last $\mathcal{P}(\tau, T)$ element in $\bar{\mathcal{A}}_i$ belongs to $\bar{\mathcal{P}}(\tau)$, then the concatenation is kept.

(iii) If $\bar{\mathcal{A}}_{i,\text{ last}} = \bar{\mathcal{A}}_{j,\text{ first}}$, and the last $\mathcal{P}(\tau, T)$ element in $\bar{\mathcal{A}}_i$ does not belong to $\bar{\mathcal{P}}(\tau)$, then this concatenation is discarded. The reason lies in that, in this case, we can move the consecutive same subsystem modes at the end of $\bar{\mathcal{A}}_i$ to the front of $\bar{\mathcal{A}}_j$, which yields to $\bar{\mathcal{A}}_{i,\text{ last}} \neq \bar{\mathcal{A}}_{j,\text{ first}}$, so this case is converted to case i).

We use $\mathcal{C}(\tau, T, L)$ to denote the concatenation list under the above three cases. As in fact $\bar{\mathcal{A}}(\tau, T, L)$ with $\mathcal{C}(\tau, T, L)$ can also cover all admissible PDT switching sequences, the equivalence to $S_{\text{p- dwell}}[\tau, T]$ still holds.

Step 5: Note that there may still exist different concatenations corresponding to same switching sequences in case (ii), such as $\{\bar{\mathcal{A}}_{i_1}, \bar{\mathcal{A}}_{j_1}\} = \{\{1,1\},\{1,1,1,2\}\}$, and $\{\bar{\mathcal{A}}_{i_2}, \bar{\mathcal{A}}_{j_2}\} = \{\{1,1,1\},\{1,1,2\}\}$, where $i_1, i_2 \in \mathcal{I}_L$, $j_1 \in \mathcal{C}_{i_1}, j_2 \in \mathcal{C}_{i_2}$. Note that here $\bar{\mathcal{A}}_{i_1}, \bar{\mathcal{A}}_{i_2} \in \bar{\mathcal{P}}$ due to $\bar{\mathcal{P}}_0 = \{1,1\}, \bar{\mathcal{P}}_1 = \{1,1,1\}$, and we assume $L = 1$ for simplification. So we only keep one concatenation for the above case, and use $\bar{\mathcal{C}}(\tau, T, L)$ to represent the concatenation list of $\bar{\mathcal{A}}(\tau, T, L)$ after removal.

Thus, according to the above five steps, we can generate a switching sequence list $\bar{\mathcal{A}}(\tau, T, L)$ with the concatenation list $\bar{\mathcal{C}}(\tau, T, L)$, which is equivalent to any given PDT signal set $S_{\text{p-dwell}}[\tau,T]$. In summary, any admissible PDT switching sequences can be represented by the circle marked by blue arrows in Fig. 2. The sequence starts from one element $\bar{\mathcal{A}}_i, i \in \mathcal{I}_L$, in $\bar{\mathcal{A}}(\tau,T,L)$, then taking one index j in $\bar{\mathcal{C}}_i$, and evolves as the sequence indicated by the element $\bar{\mathcal{A}}_j$. The above process repeats to generate an infinite admissible PDT switching sequence.

For any admissible PDT switching sequences, denoted by $\{\bar{\mathcal{A}}_i, \bar{\mathcal{A}}_j, \ldots\}, i \in \mathcal{I}_L, j \in \bar{\mathcal{C}}_i$, without loss of generality, we define the switching instants of $\bar{\mathcal{A}}(\tau,T,L)$ elements by $\bar{k}_s, s \in \mathbb{Z}_{\pm}$, and use the symbol $\hat{\sigma}(k)$ to indicate the activated $\bar{\mathcal{A}}(\tau,T,L)$ elements at time k.

4. Stabilization of nominal systems

This section aims to design the stabilization controller(8) for the nominal system (3). According to the concepts of the switching sequence list and the concatenation list introduced in Section 3, we first give a nonconservative convex stability criterion for PDT switched systems.

Theorem 1. *Consider the switched system (3). The following two statements are equivalent.*

(a) The switched system (3) is GES under PDT constraint for a given $S_{p-dwell}[\tau_z,T]$.

(b) There exist a scalar $L \in \mathbb{Z}_{\geq 1}$ and a symmetric matrix sequence $R_i(p) \in \mathbb{S}^{n_x}_{>0}, i \in \bar{\mathcal{I}}_L, p \in \mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|]}$, such that $\forall i \in \mathcal{I}_L, j \in \bar{\mathcal{C}}_i$

b. (i) $\forall i \in \mathcal{I}_L, p \in \mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|-1]}$,

$$A^T_{\bar{\mathcal{A}}_{i,p}} R_i(p+1) A_{\bar{\mathcal{A}}_{i,p}} - R_i(p) \prec 0 \tag{12}$$

b.(ii) $\forall i \in \mathcal{I}_L, j \in \bar{\mathcal{C}}_i$

$$R_j(0) - R_i\left(|\bar{\mathcal{A}}_i|\right) \prec 0 \tag{13}$$

Proof. Proof of (a) ⇒ (b). For any arbitrarily chosen $R_i\left(|\bar{\mathcal{A}}_i|\right) \in \mathbb{S}^{n_x}_{>0}, i \in \mathcal{I}_L$, we can define a matrix sequence $W_i(p) \in \mathbb{S}^{n_x}_{>0}, p \in \mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|-1]}$, and let

$$R_i(p) := W_i(p) + A^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}} R_i(p+1) A_{\bar{\mathcal{A}}_{i,p}} \tag{14}$$

which ensures the condition (12). This further gives rises to $R_i(0) = \bar{W}_i(0) + \left(\prod_{k=0}^{|\bar{\mathcal{A}}_i|-1} A_{\bar{\mathcal{A}}_{i,k}}\right)^{\mathrm{T}} R_i\left(|\bar{\mathcal{A}}_i|\right)\left(\prod_{k=0}^{|\bar{\mathcal{A}}_i|-1} A_{\bar{\mathcal{A}}_{i,k}}\right)$ where

$$\bar{W}_i(0) := \sum_{k=0}^{|\bar{\mathcal{A}}_i|-1}\left(\prod_{p=0}^{k-1} A_{\bar{\mathcal{A}}_{i,p}}\right)^{\mathrm{T}} W_i(k)\left(\prod_{p=0}^{k-1} A_{\bar{\mathcal{A}}_{i,p}}\right) \tag{15}$$

Since the switched system (3) is GES, there must exist a decay rate $\mu > 0$, and a scalar $L \in \mathbb{Z}_{\geq 1}$ to regulate the considered switching sequence length, such that $\left\|z\left(\bar{k}_1\right)\right\| := \left\|\Phi\left(\bar{k}_1,k_0\right) z\left(k_0\right)\right\| \leq ce^{-\mu(\bar{k}_1-k_0)}\left\|z\left(k_0\right)\right\|$, where $c > 0$ is a constant, $\bar{k}_1 = |\bar{\mathcal{A}}_i|, \forall i \in \mathcal{I}_L$, and

$$\Phi\left(\bar{k}_1,k_0\right) := \prod_{p=0}^{|\bar{\mathcal{A}}_i|-1} A_{\bar{\mathcal{A}}_{i,p}} = A_{\bar{\mathcal{A}}_{i,|\bar{\mathcal{A}}_i|-1}} \cdots A_{\bar{\mathcal{A}}_{i,0}}$$

Thus, we can get that $\lim_{L\to\infty}\Phi\left(\bar{k}_1,k_0\right) = 0$ due to $\bar{k}_1 - k_0 \to \infty$. For arbitrarily chosen $R_j\left(\left|\bar{\mathcal{A}}_j\right|\right) \in \mathbb{S}^{n_x}_{>0}$, $\forall j \in \mathcal{I}_L$, there exists a scalar $\varepsilon > 0$ such that

$$\lim_{L\to\infty}\Phi^{\mathrm{T}}\left(\bar{k}_1,k_0\right) R_i\left(|\bar{\mathcal{A}}_i|\right)\Phi\left(\bar{k}_1,k_0\right) - R_j\left(\left|\bar{\mathcal{A}}_j\right|\right) \prec -\varepsilon I$$

Moreover, there exists a scalar $L^* \in \mathbb{Z}_+$such that for any $L > L^*$, it holds that

$$\Phi^{\mathrm{T}}\left(\bar{k}_1,k_0\right) R_i\left(|\bar{\mathcal{A}}_i|\right)\Phi\left(\bar{k}_1,k_0\right) - R_j\left(\left|\bar{\mathcal{A}}_j\right|\right) \prec -\varepsilon I \tag{16}$$

Considering (16) together with (14), we know that

$$R_i(0) - R_j\left(\left|\bar{\mathcal{A}}_j\right|\right) \prec -\varepsilon I + \bar{W}_i(0) \tag{17}$$

holds for any $i,j \in \mathcal{I}_L$. Since $j \in \bar{\mathcal{C}}_i \subseteq \mathcal{I}_L$, $R_j(0) - R_i\left(|\bar{\mathcal{A}}_i|\right) \prec -\varepsilon I + \bar{W}_j(0)$ also holds for $i \in \mathcal{I}_L$, $j \in \bar{\mathcal{C}}_i$. Since the norm of $\bar{W}_j(0)$ can be adjusted sufficiently small by the matrix sequences $W_j(p)$, $R_j(0) - R_i\left(|\bar{\mathcal{A}}_i|\right) \prec 0$ is ensured.

Proof of (b) ⇒ (a) Define the Lyapunov function for the system (3) by:

$$V_{\hat{\sigma}(k)}(z(k)) := \sqrt{z^{\mathrm{T}}(k) R_{\hat{\sigma}(k)}\left(k-\bar{k}_s\right) z(k)} \tag{18}$$

where $k \in \left[\bar{k}_s, \bar{k}_{s+1}\right), s \in \mathbb{Z}_+$. Define $\delta_{\max}(\cdot)$ as the largest matrix singular value, and define $\delta_{\min}(\cdot)$ as the smallest matrix singular value. Moreover, define

$$\bar{\delta}_{\max} := \max_{i\in\mathcal{I}_L, p\in\mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|]}} \delta_{\max}\left(R_i(p)\right)$$

$$\bar{\delta}_{\min} := \min_{i\in\mathcal{I}_L, p\in\mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|]}} \delta_{\min}\left(R_i(p)\right)$$

Then we can get that

$$\sqrt{\bar{\delta}_{\min}}\|z(k)\| \leq V_{\hat{\sigma}(k)}(z(k)) \leq \sqrt{\bar{\delta}_{\max}}\|z(k)\| \tag{19}$$

$$\begin{aligned} & V_{\hat{\sigma}(k+1)}(z(k+1)) - V_{\hat{\sigma}(k)}(z(k)) \\ =& \sqrt{z^{\mathrm{T}}(k+1) R_{\hat{\sigma}(k+1)}\left(k+1-\bar{k}_s\right) z(k+1)} - \sqrt{z^{\mathrm{T}}(k) R_{\hat{\sigma}(k)}\left(k-\bar{k}_s\right) z(k)} \\ =& \frac{z^{\mathrm{T}}(k+1) R_{\hat{\sigma}(k+1)}\left(k+1-\bar{k}_s\right) z(k+1) - z^{\mathrm{T}}(k) R_{\hat{\sigma}(k)}\left(k-\bar{k}_s\right) z(k)}{\sqrt{z^{\mathrm{T}}(k+1) R_{\hat{\sigma}(k+1)}\left(k+1-\bar{k}_s\right) z(k+1)} + \sqrt{z^{\mathrm{T}}(k) R_{\hat{\sigma}(k)}\left(k-\bar{k}_s\right) z(k)}} \end{aligned} \tag{20}$$

According to (12), we can obtain (20), where $k \in \left[\bar{k}_s, \bar{k}_{s+1}-1\right)$, $\hat{\sigma}(k) = \hat{\sigma}(k+1) = i \in \mathcal{I}_L$ without loss of generality, and positive scalars $\varepsilon_1, \rho_1, \lambda_1$ are defined by

$$\varepsilon_1 := \min_{i\in\mathcal{I}_L, p\in[0,|\bar{\mathcal{A}}_i|-2]} \delta_{\min}\left(R_i(p) - A^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}} R_i(p+1) A_{\bar{\mathcal{A}}_{i,p}}\right)$$

$$\rho_1 := \max_{i\in\mathcal{I}_L, p\in[0,|\mathcal{A}_i|-2]} \delta_{\max}\left(A^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}} R_i(p+1) A_{\bar{\mathcal{A}}_{i,p}}\right)$$

$$\lambda_1 := \varepsilon_1 / \left(\sqrt{\rho_1} + \sqrt{\bar{\delta}_{\max}}\right)$$

It is seen that (20) indicates the decrease of Lyapunov function within each element $\bar{\mathcal{A}}_i, i \in \mathcal{I}_L$, in $\bar{\mathcal{A}}(\tau,T,L)$.

Next, consider the switching instants between two consecutive $\bar{\mathcal{A}}(\tau,T,L)$ elements. Let us define

$$\bar{V}_{\hat{\sigma}(\bar{k}_s)}\left(z\left(\bar{k}_{s+1}\right)\right) := \sqrt{z^{\mathrm{T}}\left(\bar{k}_{s+1}\right) R_{\hat{\sigma}(\bar{k}_s)}\left(|\bar{\mathcal{A}}_i|\right) z\left(\bar{k}_{s+1}\right)}$$

Then, from (12) we can also obtain that

$$\begin{aligned} & \bar{V}_{\hat{\sigma}(\bar{k}_s)}\left(z\left(\bar{k}_{s+1}\right)\right) - V_{\hat{\sigma}(\bar{k}_s)}\left(z\left(\bar{k}_{s+1}-1\right)\right) \\ =& \sqrt{z^{\mathrm{T}}\left(\bar{k}_{s+1}\right) R_{\hat{\sigma}(\bar{k}_s)}\left(\bar{k}_{s+1}-\bar{k}_s\right) z\left(\bar{k}_{s+1}\right)} \\ & - \sqrt{z^{\mathrm{T}}\left(\bar{k}_{s+1}-1\right) R_{\hat{\sigma}(\bar{k}_s)}\left(\bar{k}_{s+1}-\bar{k}_s-1\right) z\left(\bar{k}_{s+1}-1\right)} \\ <& -\frac{\varepsilon_2\left\|z\left(\bar{k}_{s+1}-1\right)\right\|^2}{\sqrt{\rho_2}\left\|z\left(\bar{k}_{s+1}-1\right)\right\| + \sqrt{\bar{\delta}_{\max}}\left\|z\left(\bar{k}_{s+1}-1\right)\right\|} \\ :=& -\lambda_2\left\|z\left(\bar{k}_{s+1}-1\right)\right\| \end{aligned} \tag{21}$$

where positive scalars $\varepsilon_2, \rho_2, \lambda_2$ are defined by

$$\begin{aligned} \varepsilon_2 = \min_{i\in\mathcal{I}_L} \delta_{\min}\Big(& R_i\left(|\bar{\mathcal{A}}_i|-1\right) \\ & - A^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,|\bar{\mathcal{A}}_i|-1}} R_i\left(|\bar{\mathcal{A}}_i|\right) A_{\bar{\mathcal{A}}_{i,|\bar{\mathcal{A}}_i|-1}}\Big) \end{aligned}$$

$$\rho_2 := \max_{i\in\mathcal{I}_L} \delta_{\max}\left(A^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,|\bar{\mathcal{A}}_i|-1}} R_i\left(|\bar{\mathcal{A}}_i|\right) A_{\bar{\mathcal{A}}_{i,|\bar{\mathcal{A}}_i|-1}}\right)$$

$$\lambda_2 := \varepsilon_2 / \left(\sqrt{\rho_2} + \sqrt{\bar{\delta}_{\max}}\right)$$

According to (13), we can get that

$$\begin{aligned}
&V_{\hat{\sigma}(\bar{k}_{s+1})}\left(z\left(\bar{k}_{s+1}\right)\right)-\bar{V}_{\hat{\sigma}(\bar{k}_s)}\left(z\left(\bar{k}_{s+1}\right)\right)\\
=&\sqrt{z^{\mathrm{T}}\left(\bar{k}_{s+1}\right)R_{\hat{\sigma}(\bar{k}_{s+1})}(0)z\left(\bar{k}_{s+1}\right)}\\
&-\sqrt{z^{\mathrm{T}}\left(\bar{k}_{s+1}\right)R_{\hat{\sigma}(\bar{k}_s)}\left(\bar{k}_{s+1}-\bar{k}_s\right)z\left(\bar{k}_{s+1}\right)}\\
<&-\frac{\varepsilon_3\left\|z\left(\bar{k}_{s+1}\right)\right\|^2}{2\sqrt{\bar{\delta}_{\max}}\left\|z\left(\bar{k}_{s+1}\right)\right\|}:=-\lambda_3\left\|z\left(\bar{k}_{s+1}\right)\right\|
\end{aligned}\tag{22}$$

where positive scalars ε_3 and λ_3 are defined by

$$\varepsilon_3:=\min_{i\in\mathcal{I}_L,j\in\bar{C}_i}\delta_{\min}\left(R_i\left(|\bar{\mathcal{A}}_i|\right)-R_j(0)\right)$$

$$\lambda_3:=\varepsilon_3/\left(2\sqrt{\bar{\delta}_{\max}}\right)$$

From (21) and (22), we know that

$$\begin{aligned}
&V_{\hat{\sigma}(\bar{k}_{s+1})}\left(z\left(\bar{k}_{s+1}\right)\right)-V_{\hat{\sigma}(\bar{k}_s)}\left(z\left(\bar{k}_{s+1}-1\right)\right)\\
=&-\lambda_2\left\|z\left(\bar{k}_{s+1}-1\right)\right\|-\lambda_3\left\|z\left(\bar{k}_{s+1}\right)\right\|\\
<&-\lambda_2\left\|z\left(\bar{k}_{s+1}-1\right)\right\|
\end{aligned}\tag{23}$$

According to (20) and (23), it holds that

$$\begin{aligned}
\Delta V(k):&=V_{\hat{\sigma}(k+1)}(z(k+1))-V_{\hat{\sigma}(k)}(z(k))\\
&<\begin{cases}-\lambda_1\|z(k)\| & k\in\left[\bar{k}_s,\bar{k}_{s+1}-1\right)\\ -\lambda_2\|z(k)\| & k=\bar{k}_{s+1}-1\end{cases}
\end{aligned}$$

which yields to

$$\Delta V(k)<-\lambda\|z(k)\|\tag{24}$$

where $\lambda:=\max\left(\lambda_1,\lambda_2,\sqrt{\bar{\delta}_{\max}}-\varepsilon\right)$ and $0<\varepsilon<\sqrt{\bar{\delta}_{\max}}$ is a constant.

According to (19) and (24), we can get that

$$V_{\hat{\sigma}(k+1)}(z(k+1))<\left(1-\frac{\lambda}{\sqrt{\bar{\delta}_{\max}}}\right)V_{\hat{\sigma}(k)}(z(k))$$

where $0<1-\frac{\lambda}{\sqrt{\bar{\delta}_{\max}}}<1$ is a constant. Moreover, we can get that

$$V_{\hat{\sigma}(k)}(z(k))<\left(1-\frac{\lambda}{\bar{\delta}_{\max}}\right)^kV_{\hat{\sigma}(0)}\left(z\left(k_0\right)\right)$$

which together with (19) generates

$$\begin{aligned}
\|z(k)\|&<\frac{1}{\sqrt{\bar{\delta}_{\min}}}V_{\hat{\sigma}(k)}(z(k))\\
&<\frac{1}{\sqrt{\bar{\delta}_{\min}}}\left(1-\frac{\lambda}{\sqrt{\bar{\delta}_{\max}}}\right)^{k-k_0}V_{\hat{\sigma}(k_0)}\left(z\left(k_0\right)\right)\\
&<\frac{\sqrt{\bar{\delta}_{\max}}}{\sqrt{\bar{\delta}_{\min}}}\left(1-\frac{\lambda}{\sqrt{\bar{\delta}_{\max}}}\right)^{k-k_0}\left\|z\left(k_0\right)\right\|\\
&:=ce^{-\mu(k-k_0)}\left\|z\left(k_0\right)\right\|
\end{aligned}$$

where $c:=\sqrt{\bar{\delta}_{\max}}/\sqrt{\bar{\delta}_{\min}}>0,\mu=-\ln\left(1-\frac{\lambda}{\sqrt{\bar{\delta}_{\max}}}\right)>0$. This completes the proof. □

Remark 1. Two techniques are developed in Theorem 1 to obtain the nonconservative global exponential stability conditions. The first one is the novel sequence segmentation technique. It cancels the parameters that overly shape the Lyapunov function in the existing literatures of PDT switched systems [10, 17, 32, 34, 44, 45], and thus realizes the convexification. The second technique is to present the augmented PDT primary sequence list, which gives the user an additional integer L with respect to the primary sequence list. Setting $L=1$, we actually require the Lyapunov function (18) to decrease at starting instants of $\mathcal{P}(\tau,T)$. Note that in this case, the stability conditions are still convex, and this is also an improvement compared to the existing nonconvex conditions.

Remark 2. The widely used QTD stability conditions, in the framework of stage partition, are nonconvex due to some tuning parameters, and are also sufficient to our conditions even if we set $L=1$. The strict proof can be drawn according to the linear version of Lemma 1 in [36] and Theorem 1 in this paper, and is omitted here.

Remark 3. As PDT signal set can cover ADT and DT signal sets, the nonconservative stability conditions in Theorem 1 can also be slightly modified for the switched linear systems under these two time constraints. For example, the nonconservative conditions for DT switched systems can be generated by setting $T=0$ in Theorem 1.

Remark 4. Theorem 1 gives a necessary and sufficient stability criterion for switched linear system (3). However, (12) and (13) correlate with the index i of the activated element in $\bar{\mathcal{A}}(\tau,T,L)$ and the activated concatenated element index j in $\bar{C}_i$, respectively. A problem rises that the system usually cannot know the switching sequences a priori as well as the indices i,j, and the control schedule clock $\theta_z(k)$. Thus, the general extension in the existing literatures from stability conditions in LMI form to mode-dependent or quasi-time-dependent controller design approach fails.

Note that in Step 4 of procedures that generate $\bar{\mathcal{A}}(\tau,T,L)$ with $C(\tau,T,L)$, only two kinds of concatenations, denoted by $\left\{\bar{\mathcal{A}}_i,\bar{\mathcal{A}}_j\right\},i,j\in\mathcal{I}_L$, are kept under case (i) and case (ii). Among others, case (i) keeps the concatenation under $\bar{\mathcal{A}}_{i,\text{ last}}\neq\bar{\mathcal{A}}_{j,\text{first}}$, and in this case $\theta_z(k)$ is reset to 0 when $\bar{\mathcal{A}}_i$ switches to $\bar{\mathcal{A}}_j$; case (ii) keeps the concatenation under $\bar{\mathcal{A}}_{i,\text{ last}}=\bar{\mathcal{A}}_{j,\text{ first}}$ and the last $\mathcal{P}(\tau,T)$ element in $\bar{\mathcal{A}}_i$ belongs to $\bar{\mathcal{P}}(\tau)$. In this case, the last subsystem mode in $\bar{\mathcal{A}}_i$ has been activated for no less than τ_z at the switching instant from $\bar{\mathcal{A}}_i$ to $\bar{\mathcal{A}}_j$, so $\theta_z(k)$ is set to τ when $\bar{\mathcal{A}}_j$ starts.

Example 2. Consider a PDT signal set $S_{p-dwell}[3,2]$ with $\mathcal{I}_N=\{1,2,3\}$ and $L=1$. The two switching sequences $\left\{\bar{\mathcal{A}}_{i_1},\bar{\mathcal{A}}_{j_1}\right\}=\{\{1,1,1,2\},\{3,3,3\}\}$ and $\left\{\bar{\mathcal{A}}_{i_2},\bar{\mathcal{A}}_{j_2}\right\}=\{\{2,2,2\},\{2,2,2,1\}\}$ are admissible. Each inner brace indicates one element in $\bar{\mathcal{A}}(3,2,1)$. The clock $\theta_z(k)$, computed by (10), strikes as $\{\{0,1,2,0\},\{0,1,2\}\}$ and $\{\{0,1,2\},\{3,3,3,0\}\}$, respectively.

We can find out that the two admissible switching sequences in Example 2 are constructed corresponding to case (i) and case (ii), respectively. If we give controller design conditions that can cover all $\bar{\mathcal{A}}(\tau,T,L)$ elements with their concatenations indicated by $\bar{C}(\tau,T,L)$, the designed controller can also work for the infinite number of admissible PDT switching sequences. The controller gain $F_{\sigma(k)}\left(\theta_z(k)\right)$ in (8) depends on the activated subsystem mode $\sigma(k)$ and its schedule clock $\theta_z(k)$, where the former one can be denoted by the switching sequence list, such as $\bar{\mathcal{A}}_{i_1,p}=1,p=0,1,2$, and $\bar{\mathcal{A}}_{i_1,3}=2$ in Example 2. Thus, we aim to denote the online computed $\theta_z(k)$ by two offline calculated variables,

$$\bar{\theta}_z(i):=\left\{\bar{\theta}_z(i,0),\bar{\theta}_z(i,1),\ldots,\bar{\theta}_z\left(i,|\bar{\mathcal{A}}_i|-1\right)\right\}\tag{25}$$

$$\hat{\theta}_z(i):=\left\{\hat{\theta}_z(i,0),\hat{\theta}_z(i,1),\ldots,\hat{\theta}_z\left(i,\left|\hat{\mathcal{A}}_i\right|-1\right)\right\}\tag{26}$$

where

$$\bar{\theta}_z(i,p)=\min_{s\in\mathbb{Z}_+}\left(p-p_s,\tau_z\right)\tag{27}$$

and

$$\hat{\theta}_z(i,p)=\begin{cases}\tau_z, & s=0\\ \bar{\theta}_z(i,p), & s\in\mathbb{Z}_{\geq1}\end{cases}\tag{28}$$

where p_s denotes the position of the last switching instant of subsystem modes in $\bar{\mathcal{A}}_i$ before $p, s \in \mathbb{Z}_+, p \in \mathbb{Z}_{[0,|\mathcal{A}_i|-1]}$. It is seen that $\bar{\theta}_z(i)$ and $\hat{\theta}_z(i)$ cover all possible $\theta_z(k)$ for $\bar{\mathcal{A}}_i, i \in \mathcal{I}_L$. Consider the two switching sequences in Example 2. By (25) and (26), we can obtain

$$\bar{\theta}_z(i_1) = \{0,1,2,0\}, \bar{\theta}_z(j_1) = \{0,1,2\},$$
$$\bar{\theta}_z(i_2) = \{0,1,2\}, \bar{\theta}_z(j_2) = \{0,1,2,0\},$$
$$\hat{\theta}_z(i_1) = \{3,3,3,0\}, \hat{\theta}_z(j_1) = \{3,3,3\},$$
$$\hat{\theta}_z(i_2) = \{3,3,3\}, \hat{\theta}_z(j_2) = \{3,3,3,0\}.$$

It can be checked that $\theta_z(k)$ for the sequence $\{\bar{\mathcal{A}}_{i_1}, \bar{\mathcal{A}}_{j_1}\}$ is identical to $\{\bar{\theta}_z(i_1), \bar{\theta}_z(j_1)\}$, and $\theta_z(k)$ for the sequence $\{\bar{\mathcal{A}}_{i_2}, \bar{\mathcal{A}}_{j_2}\}$ is identical to $\{\bar{\theta}_z(i_2), \hat{\theta}_z(j_2)\}$. Thus, if the controller design conditions hold for all the sequences determined by $\bar{\mathcal{A}}(\tau, T, L)$ with $\bar{\mathcal{C}}(\tau, T, L)$, where $\theta_z(k)$ is set equal to $\bar{\theta}_z(i)$ or $\hat{\theta}_z(i)$ for each $\bar{\mathcal{A}}_i$, the designed controller also works for the infinite number of admissible PDT switching sequences. This is called "virtualsequence-dependent" controller design method.

Theorem 2. *Consider the switched system (3) with a given PDT signal set $S_{p-dwell}[\tau_z, T]$. Suppose that there exist a matrix sequence $S_i(p) \in \mathbb{S}^{n_x}_{>0}, i \in \mathcal{I}_L$, $p \in \mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|]}$, and matrix sequences $W_n(\theta), U_n(\theta)$, $n \in \mathcal{I}_N, \theta \in \mathbb{Z}_{[0,\tau_z]}$, such that*

(i) $\forall i \in \mathcal{I}_L, p \in \mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|-1]}, \theta = \bar{\theta}_z(i,p)$ or $\theta = \hat{\theta}_z(i,p)$,

$$\begin{bmatrix} -S_i(p+1) & A_{\bar{\mathcal{A}}_{i,p}} W_{\bar{\mathcal{A}}_{i,p}}(\theta) + B_{\bar{\mathcal{A}}_{i,p}} U_{\bar{\mathcal{A}}_{i,p}}(\theta) \\ \star & S_i(p) - W_{\bar{\mathcal{A}}_{i,p}}(\theta) - W^T_{\bar{\mathcal{A}}_{i,p}}(\theta) \end{bmatrix} \prec 0 \tag{29}$$

(ii) $\forall i \in \mathcal{I}_L, j \in \bar{\mathcal{C}}_i$

$$S_i(|\bar{\mathcal{A}}_i|) - S_j(0) \prec 0 \tag{30}$$

Then the nominal system (3) with controller (8) is GES. Moreover, the stabilizing controller gains are given by, $\forall n \in \mathcal{I}_N, \theta \in \mathbb{Z}_{[0,\tau_z]}$

$$F_n(\theta) = U_n(\theta) W_n^{-1}(\theta) \tag{31}$$

Proof. Due to the fact that $\left(S_i(p) - W_{\bar{\mathcal{A}}_{i,p}}(\theta)\right)^{\mathrm{T}} S_i^{-1}(p) \left(S_i(p) - W_{\bar{\mathcal{A}}_{i,p}}(\theta)\right) \geq 0$, one has $S_i(p) - W_{\bar{\mathcal{A}}_{i,p}}(\theta) - W^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}}(\theta) \geq -W^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}}(\theta) S_i^{-1}(p) W_{\bar{\mathcal{A}}_{i,p}}(\theta)$, where $\bar{\mathcal{A}}_{i,p}$ denotes the pth activated subsystem mode in $\bar{\mathcal{A}}_i$. Thus, (29) ensures that

$$\begin{bmatrix} -S_i(p+1) & A_{\bar{\mathcal{A}}_{i,p}} W_{\bar{\mathcal{A}}_{i,p}}(\theta) + B_{\bar{\mathcal{A}}_{i,p}} U_{\bar{\mathcal{A}}_{i,p}}(\theta) \\ \star & -W^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}}(\theta) S_i^{-1}(p) W_{\bar{\mathcal{A}}_{i,p}}(\theta) \end{bmatrix} \prec 0 \tag{32}$$

According to (31), we know that

$$F_n(\theta) W_n(\theta) = U_n(\theta)$$

where $n \in \mathcal{I}_N$, and $\theta \in \mathbb{Z}_{[0,\tau_z]}$. Then (32) is equivalent to

$$\begin{bmatrix} -S_i(p+1) & \bar{A}_{\bar{\mathcal{A}}_{i,p}}(\theta) W_{\bar{\mathcal{A}}_{i,p}}(\theta) \\ \star & -W^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}}(\theta) S_i^{-1}(p) W_{\bar{\mathcal{A}}_{i,p}}(\theta) \end{bmatrix} \prec 0 \tag{33}$$

where $\bar{A}_{\bar{\mathcal{A}}_{i,p}}(\theta) := A_{\bar{\mathcal{A}}_{i,p}} + B_{\bar{\mathcal{A}}_{i,p}} F_{\bar{\mathcal{A}}_{i,p}}(\theta)$.

Performing congruence transformations to (33) by diag $\{S_i^{-1}(p+1), W^{-\mathrm{T}}_{\mathcal{A}^{-\mathrm{T}}_{i,p}}(\theta)\}$, we can obtain

$$\begin{bmatrix} -S_i^{-1}(p+1) & S_i^{-1}(p+1)\bar{A}_{\bar{\mathcal{A}}_{i,p}}(\theta) \\ \star & -S_i^{-1}(p) \end{bmatrix} \prec 0$$

Letting $R_i(p) := S_i^{-1}(p)$ and $\bar{A}_{\bar{\mathcal{A}}_{i,p}}(\theta) := A_{\bar{\mathcal{A}}_{i,p}}$, we can get (12). Moreover, (30) ensures (13). By Theorem 1, the nominal switched system (3) with controller gains (31) is GES under PDT switching signal for $S_{\text{p-dwell}}[\tau, T]$. □

5. Systems with bounded peak disturbances

In this section, the estimated reachable set $\hat{\mathcal{R}}_e$ that contains the practical reachable set $\mathcal{R}_e$ of the error system (4) is designed as small as possible to minimize the effect caused by the bounded peak disturbances $\omega(k)$. A set $\mathcal{R}$ with admissible switching signal set is also determined to address the global exponential stability of the composite system (1) and (3) in the sense of Definition 3 . The following lemma introduces the basic idea of generating $\hat{\mathcal{R}}_e$ for PDT switched systems by ellipsoidal techniques.

Lemma 1. *Consider the switched system (4) with a given PDT signal set $S_{p-dwell}[\tau_e, T]$. If there exist a set of functions $V_i : \mathbb{R}^{n_x} \to \mathbb{R}_+$ satisfying $V_i(0) = 0$ and $V_i(x) > 0$, $\forall x \neq 0, \forall i \in \mathcal{I}_L$, and a scalar $0 < \alpha < 1$, such that*

(i) $\forall k \in [\bar{k}_s, \bar{k}_{s+1}), s \in \mathbb{Z}_+, \forall \hat{\sigma}(k) = i \in \mathcal{I}_L$

$$V_i(e(k+1)) - \alpha V_i(e(k)) - \frac{1-\alpha}{\bar{\omega}^2}\omega^T(k)\omega(k) < 0 \tag{34}$$

(ii) $\forall k = \bar{k}_{s+1}, s \in \mathbb{Z}_+, \forall \hat{\sigma}(\bar{k}_s) = i \in \mathcal{I}_L, \forall \hat{\sigma}(\bar{k}_{s+1}) = j \in \bar{\mathcal{C}}_i$

$$V_j(e(k)) - V_i(e(k)) < 0 \tag{35}$$

Then it holds that $V_{\hat{\sigma}(k)}(e(k)) < 1$ for alle (k_0) satisfying $V_{\hat{\sigma}(k_0)}(e(k_0)) < 1$.

Proof. According to (34), we can obtain that

$$\begin{aligned} V_i(e(k+1)) - \alpha V_i(e(k)) &< \frac{1-\alpha}{\bar{\omega}^2}\omega^{\mathrm{T}}(k)\omega(k) \\ &< 1-\alpha \end{aligned}$$

which further implies that

$$V_i(e(k+1)) - 1 < \alpha\left(V_i(e(k)) - 1\right) \tag{36}$$

holds for $k \in [\bar{k}_s, \bar{k}_{s+1})$. Moreover, (35) guarantees

$$V_j\left(e(\bar{k}_{s+1})\right) - 1 < V_i\left(e(\bar{k}_{s+1})\right) - 1 \tag{37}$$

Thus, for any $k \in \mathbb{Z}_+$, by (36) and (37), it follows that

$$\begin{aligned} V_{\hat{\sigma}(k)}(e(k)) - 1 &< V_{\hat{\sigma}(k-1)}(e(k-1)) - 1 \\ &< \cdots \\ &< V_{\hat{\sigma}(k_0)}\left(e(k_0)\right) - 1 < 0 \end{aligned}$$

due to $0 < \alpha < 1$. This completes the proof. □

Considering the error system (4) together with controllers (7) and (8), we can obtain the following closedloop error system:

$$e(k+1) = E_{\sigma(k)}\left(\theta_e(k)\right) e(k) + D_{\sigma(k)}\omega(k) \tag{38}$$

where $E_{\sigma(k)}(\theta_e(k)) := A_{\sigma(k)} + B_{\sigma(k)} K_{\sigma(k)}(\theta_e(k))$. The following theorem gives the linear version of Lemma 1 by employing the ellipsoidal techniques and multiple Lyapunov function approach [40–42].

Theorem 3. *Consider the switched system (4) with a given PDT signal set $S_{p-dwell}[\tau_e, T]$. If there exist matrices $O_i(p) \in \mathbb{S}^{n_x}_{>0}, i \in \mathcal{I}_L, p \in \mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|]}$, and a scalar $0 < \alpha < 1$, such that*

(i) $\forall i \in \mathcal{I}_L, p \in \mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|-1]}$,

$$\begin{bmatrix} -O_i(p+1) & O_i(p+1)E_{\bar{\mathcal{A}}_{i,p}}(\theta) & O_i(p+1)D_{\bar{\mathcal{A}}_{i,p}} \\ \star & -\alpha O_i(p) & 0 \\ \star & \star & -\frac{1-\alpha}{\bar{\omega}^2} I \end{bmatrix} \prec 0 \tag{39}$$

(ii) $\forall i \in \mathcal{I}_L, j \in \bar{\mathcal{C}}_i$,

$$O_j(0) - O_i(|\mathcal{A}_i|) \prec 0 \tag{40}$$

Then the reachable set of the error system (4) satisfies $\mathcal{R}_e \subseteq \hat{\mathcal{R}}_e$, and $\hat{\mathcal{R}}_e$ can be over approximated by

$$\hat{\mathcal{R}}_e := \bigcup_{i \in \mathcal{I}_L, p \in \mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|]}} \mathcal{E}\left(O_i(p)\right) \tag{41}$$

Proof. Define the Lyapunov function for the system (4) by

$$V_{\hat{\sigma}(k)}(e(k)) := e^{\mathrm{T}}(k)O_{\hat{\sigma}(k)}\left(k-\bar{k}_s\right)e(k)$$

where $k \in \left[\bar{k}_s, \bar{k}_{s+1}\right), s \in \mathbb{Z}_+$. By Schur complement, (39) guarantees

$$\Omega := \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \star & \Omega_{22} \end{bmatrix} \prec 0 \tag{42}$$

with $\Omega_{11} = E^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}}(\theta)O_i(p+1)E_{\bar{\mathcal{A}}_{i,p}}(\theta)-\alpha O_i(p)$, $\Omega_{12} = E^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}}(\theta)O_i(p+1)D_{\bar{\mathcal{A}}_{i,p}}$, and $\Omega_{22} = D^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}} \times O_i(p+1)D_{\bar{\mathcal{A}}_{i,p}} - \frac{1-\alpha}{\bar{\omega}^2}I$.

Let $\xi(k) := \left[e^{\mathrm{T}}(k)\omega^{\mathrm{T}}(k)\right]^{\mathrm{T}}$. From (42), we can get that

$$\begin{aligned}
&\xi^{\mathrm{T}}(k)\Omega\xi(k)\\
=&e^{\mathrm{T}}(k)\left(E^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}}(\theta)O_i(p+1)E_{\bar{\mathcal{A}}_{i,p}}(\theta)-\alpha O_i(p)\right)e(k)\\
&+2e^{\mathrm{T}}(k)\left(E^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}}(\theta)O_i(p+1)D_{\bar{\mathcal{A}}_{i,p}}\right)\omega(k)\\
&+\omega^{\mathrm{T}}(k)\left(D^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}}O_i(p+1)D_{\bar{\mathcal{A}}_{i,p}}-\frac{1-\alpha}{\bar{\omega}^2}I\right)\omega(k)\\
=&V_{\hat{\sigma}(k+1)}(e(k+1))-\alpha V_{\hat{\sigma}(k)}(e(k))-\frac{1-\alpha}{\bar{\omega}^2}\omega^{\mathrm{T}}(k)\omega(k)\\
<&0
\end{aligned}$$

which ensures (34). Moreover, (40) ensures (35).

$$\begin{aligned}
e(k) \in &\left\{e \mid e^{\mathrm{T}}O_i(p)e<1, i \in \mathcal{I}_L, p \in \mathbb{Z}_{[0,|\mathcal{A}_i|]}\right\}\\
:= &\bigcup_{i\in\mathcal{I}_L, p\in\mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|]}} \mathcal{E}\left(O_i(p)\right)
\end{aligned}$$

which is exactly (41). This completes the proof. □

Similar to the stabilization problem for the nominal system (3), the mismatch between the stability analysis and controller design for the error system (4) also exists. The stability conditions (39)–(40) depend on the index of the activated $\bar{\mathcal{A}}(\tau, T, L)$ elements, but the controller gain $K_{\sigma(k)}\left(\theta_e(k)\right)$ in (9) varies with the activated subsystem mode $\sigma(k)$ and the schedule clock $\theta_e(k)$. To solve this problem, here we also define two offline variables $\bar{\theta}_e(i)$ and $\hat{\theta}_e(i)$ by revising the subscripts "z" to " e " in (25)–(28).

Theorem 4. *Consider the switched system (4) with a given PDT signal set $S_{p\text{-dwell}}\left[\tau_e, T\right]$. If there exist a matrix sequence $Q_i(p) \in \mathbb{S}^{n_x^x}_{>0}, i \in \mathcal{I}_L, p \in \mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|]}$, matrix sequences $W_n(\theta), U_n(\theta), n \in \mathcal{I}_N, \theta \in \mathbb{Z}_{[0,\tau_e]}$, and a scalar $0<\alpha<1$, such that*

(i) $\forall i \in \mathcal{I}_L, p \in \mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|-1]}, \theta = \bar{\theta}_e(i,p)$ or $\theta = \hat{\theta}_e(i,p)$,

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ \star & \Psi_{22} & 0 \\ \star & \star & \Psi_{33} \end{bmatrix} \prec 0 \tag{43}$$

where $\Psi_{11} = -Q_i(p+1), \Psi_{12} = A_{\bar{\mathcal{A}}_{i,p}}W_{\bar{\mathcal{A}}_{i,p}}(\theta) + B_{\bar{\mathcal{A}}_{i,p}}U_{\bar{\mathcal{A}}_{i,p}}(\theta)$, $\Psi_{13} = D_{\bar{\mathcal{A}}_{i,p}}, \Psi_{22} = \alpha\left(Q_i(p) - W^T_{\mathcal{A}_{i,p}}(\theta) - W_{\bar{\mathcal{A}}_{i,p}}(\theta)\right)$, and $\Psi_{33} = -\frac{1-\alpha}{\bar{\omega}^2}I$.

(ii) $\forall i \in \mathcal{I}_L, j \in \bar{\mathcal{C}}_i$,

$$Q_i\left(|\bar{\mathcal{A}}_i|\right) - Q_j(0) \prec 0 \tag{44}$$

Then the reachable set of the error system (4) satisfies $\mathcal{R}_e \subseteq \hat{\mathcal{R}}_e$, and $\hat{\mathcal{R}}_e$ can be over approximated by

$$\hat{\mathcal{R}}_e := \bigcup_{i\in\mathcal{I}_L, p\in\mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|]}} \mathcal{E}\left(Q_i^{-1}(p)\right) \tag{45}$$

Moreover, the controller gains are given by, $\forall n \in \mathcal{I}_N$, $\theta \in \mathbb{Z}_{[0,\tau_e]}$,

$$K_n(\theta) = U_n(\theta)W_n^{-1}(\theta) \tag{46}$$

Proof. From (46), we can get that

$$U_n(\theta) = K_n(\theta)W_n(\theta)$$

holds, $\forall n \in \mathcal{I}_N, \theta \in \mathbb{Z}_{[0,\tau_e]}$. Then, (43) is equivalent to

$$Y_1 = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ \star & Y_{14} & 0 \\ \star & \star & Y_{15} \end{bmatrix} \prec 0 \tag{47}$$

where $Y_{11} = -Q_i(p+1), Y_{12} = E_{\bar{\mathcal{A}}_{i,p}}(\theta)W_{\bar{\mathcal{A}}_{i,p}}(\theta)$, $Y_{13} = D_{\bar{\mathcal{A}}_{i,p}}, Y_{14} = \alpha\left(Q_i(p) - W^{\mathrm{T}}_{\mathcal{A}_{i,p}}(\theta) - W_{\bar{\mathcal{A}}_{i,p}}(\theta)\right)$, and $Y_{15} = -\frac{1-\alpha}{\bar{\omega}^2}I$.

Due to the fact that $\left(Q_i(p) - W_{\bar{\mathcal{A}}_{i,p}}(\theta)\right)^{\mathrm{T}} Q_i^{-1}(p)\left(Q_i(p) - W_{\bar{\mathcal{A}}_{i,p}}(\theta)\right) \geq 0$, we know that $Q_i(p) - W_{\bar{\mathcal{A}}_{i,p}}(\theta) - W^{\mathrm{T}}_{\mathcal{A}_{i,p}}(\theta) \geq -W^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}}(\theta)Q_i^{-1}(p)W_{\bar{\mathcal{A}}_{i,p}}(\theta)$. Thus, from (47) we can get that

$$Y_2 = \begin{bmatrix} Y_{21} & Y_{22} & Y_{23} \\ \star & Y_{24} & 0 \\ \star & \star & Y_{25} \end{bmatrix} \prec 0 \tag{48}$$

where $Y_{21} = -Q_i(p+1), Y_{22} = E_{\bar{\mathcal{A}}_{i,p}}(\theta)W_{\bar{\mathcal{A}}_{i,p}}(\theta)$, $Y_{23} = D_{\bar{\mathcal{A}}_{i,p}}, Y_{24} = -\alpha W^{\mathrm{T}}_{\bar{\mathcal{A}}_{i,p}}(\theta)Q_i^{-1}(p)W_{\bar{\mathcal{A}}_{i,p}}(\theta)$, and $Y_{25} = -\frac{1-\alpha}{\bar{\omega}^2}I$.

Performing congruence transformations to (48) by diag $\left\{Q_i^{-1}(p+1), W^{-\mathrm{T}}_{\bar{\mathcal{A}}^{-\mathrm{T}}_{i,p}}(\theta), I\right\}$, we can obtain

$$Y_3 = \begin{bmatrix} Y_{31} & Y_{32} & Y_{33} \\ \star & Y_{34} & 0 \\ \star & \star & Y_{35} \end{bmatrix} \prec 0 \tag{49}$$

where $Y_{31} = -Q_i^{-1}(p+1), Y_{32} = Q_i^{-1}(p+1)E_{\bar{\mathcal{A}}_{i,p}}(\theta)$, $Y_{33} = Q_i^{-1}(p+1)D_{\bar{\mathcal{A}}_{i,p}}, Y_{34} = -\alpha Q_i^{-1}(p)$, and $Y_{35} = -\frac{1-\alpha}{\bar{\omega}^2}I$.

Letting $O_i(p) := Q_i^{-1}(p)$ and $E_{\bar{\mathcal{A}}_{i,p}} := E_{\bar{\mathcal{A}}_{i,p}}(\theta)$ in (49), we can get (39). Moreover, (44) ensures (40). By Theorem 3, $e(k) \in \hat{\mathcal{R}}_e$ which is given by (45). This completes the proof. □

Remark 5. The set $\hat{\mathcal{R}}_e$ needs to be minimized to resist the effect from the external disturbance. Based on Theorem 4, we use the following additional constraints to $\hat{\mathcal{R}}_e$:

$$Q_i(p) \prec \epsilon I, \epsilon > 0, p \in \mathbb{Z}_{[0,|\bar{\mathcal{A}}_i|]}, \forall i \in \mathcal{I}_L \tag{50}$$

This generates $\frac{1}{\epsilon}e^T(k)e(k) \leq e^T(k)Q_i^{-1}(p)e(k) \leq 1$. So it holds that $e(k) \in \hat{\mathcal{R}}_e \subseteq \mathcal{B}(0, \sqrt{\epsilon}) := \{e \in \mathbb{R}^{n_x} \mid \|e\| \leq \sqrt{\epsilon}\}, k \in \mathbb{Z}_+$.

To design $\hat{\mathcal{R}}_e$ as small as possible, an optimization problem can be formulated by adding (50) with (43)–(44) as follows:

$$\min \epsilon \text{ s.t. (50) and (43)–(44)} \tag{51}$$

Theorem 5. *Consider the composite system* (1) and (3). *Suppose that the controllers* (7) and (8) *exist for the nominal system* (3) *and the error system* (4) *with $S_{p\text{-dwell}}\left[\tau_z, T\right]$ and $S_{p\text{-dwell}}\left[\tau_e, T\right]$, respectively. Then the set $\mathcal{R} := \hat{\mathcal{R}}_e \times \{0\}$ is GES for the composite system* (1) and (3) *with $S_{p-dwell}\left[\max\left(\tau_z, \tau_e\right), T\right]$.*

Proof. If the controller (7) exists for the nominal system (3), then the closed-loop nominal system is GES by Theorem 2, so there must exist scalars $c>0, \mu>0$, such that

$$\|z(k)\| \leq ce^{-\mu(k-k_0)}\left\|z\left(k_0\right)\right\| \tag{52}$$

with $S_{\text{p-dwell}}\left[\tau_z, T\right]$. Moreover, we can get that

$$\begin{aligned}
\|x(k)\|_{\hat{\mathcal{R}}_e} &= d\left(z(k)+e(k), \hat{\mathcal{R}}_e\right)\\
&\leq d(z(k)+e(k), e(k))\\
&= \|z(k)\| \leq ce^{-\mu(k-k_0)}\left\|z\left(k_0\right)\right\|
\end{aligned} \tag{53}$$

where the first "' ≤ " holds due to $e(k) \in \mathcal{R}_e \subseteq \hat{\mathcal{R}}_e$. Then, it follows that

$$\begin{aligned}
\|(x(k), z(k))\|_{\mathcal{R}} &= \|x(k)\|_{\mathcal{R}_e} + \|z(k)\|_{\{0\}}\\
&\leq 2ce^{-\mu(k-k_0)}\left\|z\left(k_0\right)\right\|
\end{aligned}$$

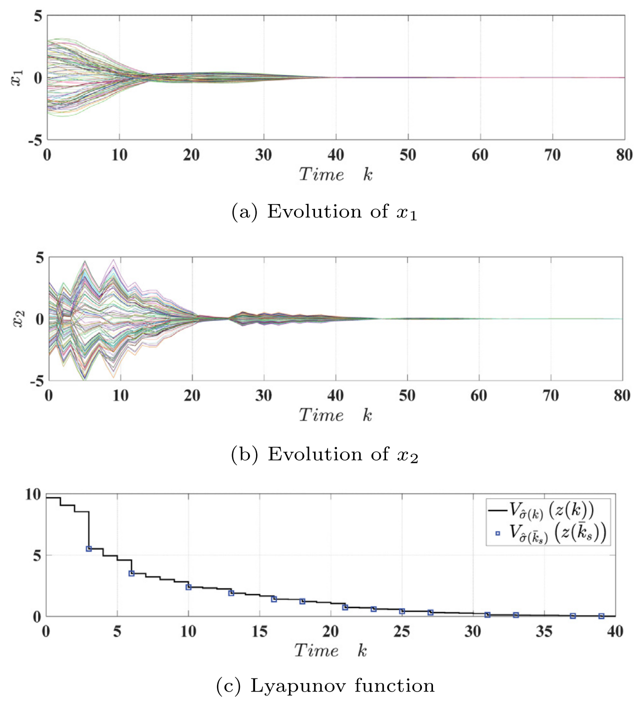


Fig. 3. Verification of the nonconservative global exponential stability conditions of the nominal system (3). (a)–(b) State trajectories with 100 randomly generated admissible PDT switching sequences. (c) The Lyapunov function $V_{\hat{\sigma}(k)}(z(k))$ along one sequence.

$$\leq 2ce^{-\mu(k-k_0)}\left(\left\|z\left(k_0\right)\right\|+\left\|x\left(k_0\right)\right\|_{\hat{\mathcal{R}}_e}\right)$$
$$=2ce^{-\mu(k-k_0)}\left\|\left(x\left(k_0\right),z\left(k_0\right)\right)\right\|_{\mathcal{R}}$$

if $S_{\text{p-dwell}}\left[\tau_e,T\right]$ and $S_{\text{p-dwell}}\left[\tau_z,T\right]$ hold. As (52) and (53) require the minimal persistent dwell-time τ for the composite system (1) and (3) satisfy $\tau\geq\tau_z$ and $\tau\geq\tau_e$, respectively, the set $\mathcal{R}$ is GES for the composite system (1) and (3) with $S_{\text{p-dwell}}\left[\max\left(\tau_z,\tau_e\right),T\right]$. □

6. Numerical example

In this section, two examples are demonstrate the validity of the obtained outcomes. The first example is to determine the admissible PDT signal set $S_{p-dwell}[\tau,T]$.

Example 3. Consider the following switched linear system with two subsystems given by:

$$A_1=\begin{bmatrix}0.9680 & 0.0760\\ -0.7599 & 0.8920\end{bmatrix}, A_2=\begin{bmatrix}0.9987 & 0.0684\\ -0.0068 & 0.7252\end{bmatrix}$$

Solving LMIs (12)–(13) in Theorem 1, we can find out that they are infeasible in the case of $\tau=T=L=1$, but is feasible in the case of $\tau=T=1, L\geq 2$. This demonstrates that the considered switched system is GES under arbitrary switching, which is obviously nonconservative in Example 3. However, it is always infeasible by solving the widely-used QTD stability conditions whatever the tuning parameters are. This also verifies Remark 2, i.e., the QTD stability conditions are sufficient to Theorem 1 even if we set $L=1$.

Figs. 3(a) and 3(b) show the system state trajectories with 100 randomly generated arbitrary switching sequences. The convergence demonstrates the system stability for $S_{\text{p-dwell}}[1,1]$. Fig. 3(c) shows one scenario of the 100 randomly generated cases, where the Lyapunov function (18) monotonously decreases with time instant k which is identical to (24).

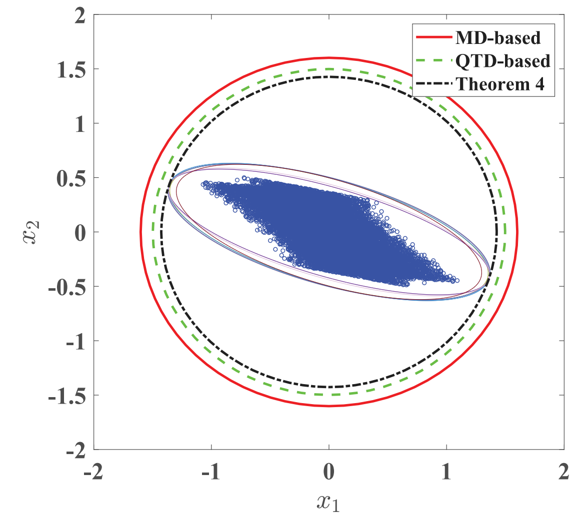


Fig. 4. Reachable set estimated by MD, QTD methods and Theorem 4. The union of all the ellipsoids is $\hat{\mathcal{R}}_e$. Blue circles denote the states of the closed-loop error system (4) with 50 randomly generated admissible PDT switching sequences.

Example 4. Consider the switched linear system (1) with the following parameters [42]:

$$A_1=\begin{bmatrix}1.5 & 1.2\\ 0.8 & -0.6\end{bmatrix}, A_2=\begin{bmatrix}-0.3 & 1.3\\ -1.1 & 1.2\end{bmatrix},$$
$$B_1=\begin{bmatrix}0.2 & 0.3\end{bmatrix}^T, B_2=\begin{bmatrix}0.1 & 0.7\end{bmatrix}^T,$$
$$D_1=\begin{bmatrix}0.2 & -0.4\end{bmatrix}^T, D_2=\begin{bmatrix}-0.5 & 0.4\end{bmatrix}^T, \bar{\omega}=1.$$

Our objective here is to design controller gains (8) and (9) and explore admissible $S_{p-dwell}[\tau,T]$ such that the composite system (1) and (3) is GES with a given set $\mathcal{R}:=\hat{\mathcal{R}}_e\times\{0\}$, where $\hat{\mathcal{R}}_e$ containing all trajectories of error system (4) is the optimized solution.

To address above problem, we split the disturbed system to the nominal and error systems in forms of (3) and (4). Since the admissible PDT signal set for the composite system (1) and (3) is $S_{p-dwell}\left[\max\left(\tau_z,\tau_e\right),T\right]$, and the stabilization conditions (43)–(44) for the error system are more conservative with tuning parameter α than conditions (29)–(30) for the nominal system, we will consider the error system first.

All the existing controller design conditions based on Lyapunov function approach for reachable set estimation, e.g., the mode-dependent (MD) and quasi-timedependent (QTD) ones, as well as Theorem 4 presented in this paper, are nonconvex and in the form of bilinear matrix inequalities (BMIs) [37–40,42]. Some efficient methods have been developed for BMIs optimization, such as iterative LMI approach or some numerical optimization algorithms like genetics algorithm, etc. As only one tuning parameter α is involved in Theorem 4, we use global searching algorithm here that repeatedly solves optimization problem (51) by changing α from 0.1 to 0.9 with searching step size of 0.1. For MD-based estimation approach, we use the same parameters listed in [42] which are optimized by genetics algorithm. For QTD-based estimation approach, we also use the global searching algorithm to explore the minimal ϵ by changing α from 0.1 to 0.9 and μ from 1.1 to 2.0 with searching step size of 0.1 [36,43]. Note that the error system in Example 4 can be stabilized with arbitrary switching sequences. We intentionally take this example here for comparison because the MD-based method is infeasible if the switched system cannot be stabilized under arbitrary switchings.

Fig. 4 shows three $\mathcal{B}(0,\sqrt{\epsilon})$ with ϵ optimized by MD, QTD methods and Theorem 4, respectively. Our presented reachable set estimation method is the least conservative as it has the smallest $\mathcal{B}(0,\sqrt{\epsilon})$. Moreover, all the trajectories of the error system (4) always stay within $\hat{\mathcal{R}}_e$

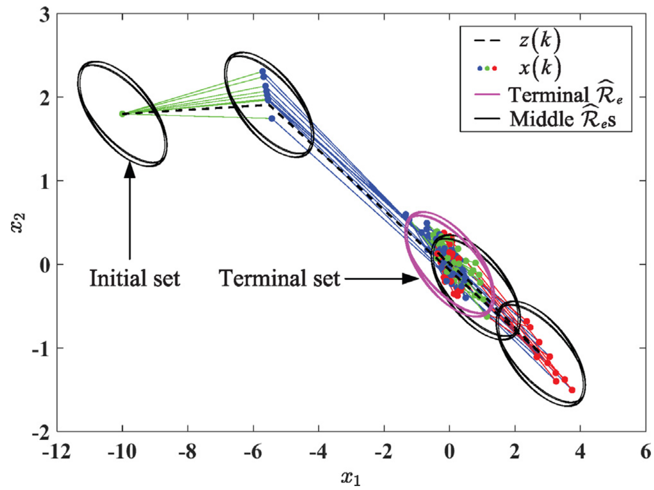


Fig. 5. Stabilization of switched linear system (1) with bounded peak disturbances by the double-clock-dependent control scheme.

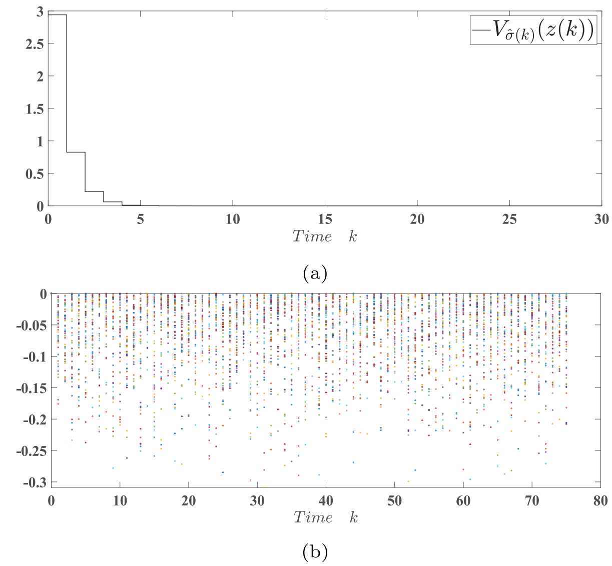


Fig. 6. Verification of Theorem 2 and Theorem 4. (a) Evolution of Lyapunov function (18) for the closed-loop nominal system. (b) Values of (34) and (35) with 50 randomly generated admissible PDT switching sequences.

also verifies the effectiveness of our proposed stabilization conditions in Theorem 4.

For the stabilization problem of the nominal system, we solve the LMI conditions (29)–(30), and further obtain the mode-dependent and clock-dependent controller (8) under arbitrary switching. Suppose that $x_0 = [-10, 1.8]^{\mathrm{T}}$, and consider an arbitrarily admissible PDT switching sequence. Fig. 5 shows the cluster of state trajectories of the disturbed system under 10 realizations. Three main observations and conclusions can be drawn from the figure:

(i) The nominal trajectory $z(k)$ converges to the origin. This demonstrates that the presented offline clocks $\bar{\theta}_z(i)$ and $\hat{\theta}_z(i)$ can cover all cases of the online clock $\theta_z(k)$, i.e., the mismatch problem between the stability analysis and mode-activation and clock-dependent controller design is solved.

(ii) All the disturbed system trajectories $x(k)$ always stay within the tube that centers at the nominal trajectory $z(k)$ with the cross section as $\hat{\mathcal{R}}_e$. This also verifies the effectiveness of the clock design among $\theta_e(k)$, $\bar{\theta}_e(i), \hat{\theta}_e(i)$. Moreover, this indicates that the doubleclock-dependent control scheme (the invariant set is replaced by the estimated reachable set of the error system compared to [34]) is effective.

(iii) All the state trajectories of the composite system converges to the set $\mathcal{R} := \hat{\mathcal{R}}_e \times \{0\}$ under the designed controllers (8), (9) and admissible switching signal set $S_{\text{p-dwell}}\left[\max\left(\tau_z, \tau_e\right), T\right]$. This solves the problem presented in Example 4.

To further verify the proposed design approach, we compute the values of the Lyapunov function $V_{\hat{\sigma}(k)}(z(k))$ of the closed-loop nominal system, as shown in Fig. 6(a). It monotonously decreases as the sampling time increases which verifies the effectiveness of Theorem 2. In addition, we also compute the values of (34) and (35) with 50 randomly generated admissible PDT switching sequences, as shown in Fig. 6(b). After we compute the controller (9) by Theorem 4 and optimization problem (51), we can accordingly obtain the closed-loop error system, which is further used to compute (34) and (35). All the values in Fig. 6(b) are smaller than 0, which also verifies the effectiveness of Theorem 4.

7. Conclusion

In this paper, we investigate the stabilization problem for the PDT switched systems with bounded peak disturbances based on the error reachable set minimization. We present a novel framework, called doubleclock-dependent control scheme, that separates the stabilization problem of the disturbed system into two stabilization problems for the nominal and the error systems, respectively. A nonconservative stability criterion is presented such that the nominal system is globally exponentially stable, and a nominal-clock-dependent controller is accordingly designed. For the error system, we present the virtual-sequence-dependent reachable set minimization conditions, and also achieves an errorclock-dependent controller. The disturbed system integrated with the double-clock-dependent controllers are also proved to be globally exponentially stable within a tube whose cross section can be regarded as the over approximation of the error system reachable set. Finally, the two numerical examples verify the validity of the presented theoretical findings.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Yang Shi reports financial support was provided by Natural Sciences and Engineering Research Council of Canada.

Data availability

No data was used for the research described in the article.

Acknowledgments

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

References

[1] D. Liberzon, Switching in Systems and Control, Vol. 190, Springer, 2003.
[2] Y. He, M. Wu, G.P. Liu, J.H. She, Output feedback stabilization for a discrete-time system with a time-varying delay, IEEE Trans. Automat. Control 53 (10) (2008) 2372–2377.
[3] R.M. Palhares, P.L. Peres, Robust filtering with guaranteed energy-to-peak performance—An LMI approach, Automatica 36 (6) (2000) 851–858.
[4] H. Dong, N. Hou, Z. Wang, Fault estimation for complex networks with randomly varying topologies and stochastic inner couplings, Automatica 112 (2020) 108734.
[5] D. Du, S. Xu, V. Cocquempot, Fault detection for nonlinear discrete-time switched systems with persistent dwell time, IEEE Trans. Fuzzy Syst. 26 (4) (2017) 2466–2474.

[6] J. Qiu, K. Sun, T. Wang, H. Gao, Observer-based fuzzy adaptive event-triggered control for pure-feedback nonlinear systems with prescribed performance, IEEE Trans. Fuzzy Syst. 27 (11) (2019) 2152–2162.
[7] P. Shi, H. Wang, C.C. Lim, Network-based event-triggered control for singular systems with quantizations, IEEE Trans. Ind. Electron. 63 (2) (2015) 1230–1238.
[8] M.V. Basin, P.C. Rodriguez-Ramirez, Sliding-mode filter design for linear systems with unmeasured states, IEEE Trans. Ind. Electron. 58 (8) (2010) 3616–3622.
[9] M. Basin, P. Rodriguez-Ramirez, Sliding mode filtering for stochastic systems with polynomial state and observation equations, J. Franklin Inst. B 351 (4) (2014) 2203–2217.
[10] L.I. Allerhand, U. Shaked, Robust stability and stabilization of linear switched systems with dwell time, IEEE Trans. Automat. Control 56 (2) (2010) 381–386.
[11] H. Lin, P.J. Antsaklis, Stability and stabilizability of switched linear systems: A survey of recent results, IEEE Trans. Automat. Control 54 (2) (2009) 308–322.
[12] H.R. Karimi, Robust delay-dependent H_∞ control of uncertain time-delay systems with mixed neutral, discrete, and distributed time-delays and Markovian switching parameters, IEEETrans. Circuits Syst.-I: Regular Pap. 58 (8) (2011) 1910–1923.
[13] J. Lam, Z. Shu, S. Xu, E.-K. Boukas, Robust control of descriptor discrete-time Markovian jump systems, Internat. J. Control 80 (3) (2007) 374–385.
[14] S. Xu, T. Chen, J. Lam, Robust H_∞ filtering for uncertain markovian jump systems with mode-dependent time delays, IEEE Trans. Automat. Control 48 (5) (2003) 900–907.
[15] S. Xu, J. Lam, X. Mao, Delay-dependent H_∞ control and filtering for uncertain Markovian jump systems with time-varying delays, IEEE Trans. Circuits Syst. I. Regul. Pap. 54 (9) (2007) 2070–2077.
[16] J.P. Hespanha, Uniform stability of switched linear systems: Extensions of Lasalle's invariance principle, IEEE Trans. Automat. Control 49 (4) (2004) 470–482.
[17] R. Yang, W.X. Zheng, H_∞ filtering for discrete-time 2-D switched systems: An extended average dwell time approach, Automatica 98 (2018) 302–313.
[18] C. Briat, Convex lifted conditions for robust l_2-stability analysis and l_2-stabilization of linear discrete-time switched systems with minimum dwell-time constraint, Automatica 50 (3) (2014) 976–983.
[19] C. Briat, Stability analysis and stabilization of stochastic linear impulsive, switched and sampled-data systems under dwell-time constraints, Automatica 74 (2016) 279–287.
[20] J.C. Geromel, P. Colaneri, Stability and stabilization of discrete time switched systems, Internat. J. Control 79 (07) (2006) 719–728.
[21] W. Xiang, H.D. Tran, T.T. Johnson, Nonconservative lifted convex conditions for stability of discrete-time switched systems under minimum dwell-time constraint, IEEE Trans. Automat. Control 64 (8) (2018) 3407–3414.
[22] W. Xiang, H.D. Tran, T.T. Johnson, Robust exponential stability and disturbance attenuation for discrete-time switched systems under arbitrary switching, IEEE Trans. Automat. Control 63 (5) (2017) 1450–1456.
[23] T.T. Han, S.S. Ge, T.H. Lee, Persistent dwell-time switched nonlinear systems: Variation paradigm and gauge design, IEEE Trans. Automat. Control 55 (2) (2009) 321–337.
[24] J.P. Hespanha, A.S. Morse, Stability of switched systems with average dwell-time, in: Proceedings of the 38th IEEE Conference on Decision and Control, Vol. 3, IEEE, 1999, pp. 2655–2660.
[25] Z. Fei, S. Shi, Z. Wang, L. Wu, Quasi-time-dependent output control for discrete-time switched system with mode-dependent average dwell time, IEEE Trans. Automat. Control 63 (8) (2017) 2647–2653.
[26] H. Shen, Z. Huang, X. Yang, Z. Wang, Quantized energy-to-peak state estimation for persistent dwell-time switched neural networks with packet dropouts, Nonlinear Dynam. 93 (2018) 2249–2262.
[27] S. Shi, Z. Fei, T. Wang, Y. Xu, Filtering for switched T–S fuzzy systems with persistent dwell time, IEEE Trans. Cybern. 49 (5) (2018) 1923–1931.
[28] H. Gao, T. Chen, J. Lam, A new delay system approach to network-based control, Automatica 44 (1) (2008) 39–52.
[29] H. Gao, X. Meng, T. Chen, Stabilization of networked control systems with a new delay characterization, IEEE Trans. Automat. Control 53 (9) (2008) 2142–2148.
[30] H.R. Karimi, A sliding mode approach to H_∞ synchronization of master–slave time-delay systems with Markovian jumping parameters and nonlinear uncertainties, J. Franklin Inst. B 349 (4) (2012) 1480–1496.
[31] R. Palhares, C. Campos, P.Y. Ekel, M. Leles, M. D'Angelo, Delay-dependent robust H_∞ control of uncertain linear systems with lumped delays, IEEE Proc. D 152 (1) (2005) 27–33.
[32] X.M. Sun, J. Zhao, D.J. Hill, Stability and l_2-gain analysis for switched delay systems: A delay-dependent method, Automatica 42 (10) (2006) 1769–1774.
[33] G. Zhai, B. Hu, K. Yasuda, A.N. Michel, Disturbance attenuation properties of time-controlled switched systems, J. Franklin Inst. B 338 (7) (2001) 765–779.
[34] J. Zhao, D.J. Hill, On stability, l_2-gain and H_∞ control for switched systems, Automatica 44 (5) (2008) 1220–1232.
[35] M. Dehghan, C.-J. Ong, Characterization and computation of disturbance invariant sets for constrained switched linear systems with dwell time restriction, Automatica 48 (9) (2012) 2175–2181.
[36] L. Zhang, S. Zhuang, P. Shi, Y. Zhu, Uniform tube based stabilization of switched linear systems with mode-dependent persistent dwell-time, IEEE Trans. Automat. Control 60 (11) (2015) 2994–2999.
[37] Z. Feng, J. Lam, On reachable set estimation of singular systems, Automatica 52 (2015) 146–153.
[38] Z. Feng, W.X. Zheng, On reachable set estimation of delay Markovian jump systems with partially known transition probabilities, J. Franklin Inst. B 353 (15) (2016) 3835–3856.
[39] J. Lam, B. Zhang, Y. Chen, S. Xu, Reachable set estimation for discrete-time linear systems with time delays, Internat. J. Robust Nonlinear Control 25 (2) (2015) 269–281.
[40] W. Xiang, H.D. Tran, T.T. Johnson, Output reachable set estimation for switched linear systems and its application in safety verification, IEEE Trans. Automat. Control 62 (10) (2017) 5380–5387.
[41] Y. Chen, J. Lam, Estimation and synthesis of reachable set for discrete-time periodic systems, Optim. Control Appl. Methods 37 (5) (2016) 885–901.
[42] Y. Chen, J. Lam, B. Zhang, Estimation and synthesis of reachable set for switched linear systems, Automatica 63 (2016) 122–132.
[43] L. Zhang, S. Zhuang, P. Shi, Non-weighted quasi-time-dependent H_∞ filtering for switched linear systems with persistent dwell-time, Automatica 54 (2015) 201–209.
[44] B. Kouvaritakis, M. Cannon, S.V. Raković, Q. Cheng, Explicit use of probabilistic distributions in linear predictive control, Automatica 46 (10) (2010) 1719–1724.
[45] D.Q. Mayne, M.M. Seron, S. Raković, Robust model predictive control of constrained linear systems with bounded disturbances, Automatica 41 (2) (2005) 219–224.
[46] K. Zhang, Y. Shi, Adaptive model predictive control for a class of constrained linear systems with parametric uncertainties, Automatica 117 (2020) 108974.

Xinxin Shang received the B.E. degree in automation and M. E. degree in control science and engineering from Harbin Institute of Technology, Harbin, China, in 2016 and 2018. She is currently working toward the Ph.D. degree at the Department of Mechanical Engineering, University of Victoria, Victoria, Canada.

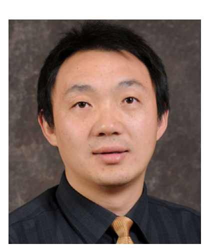
Her research interests include data-driven and hybrid model predictive control with applications.

Songlin Zhuang received the B.E. degree in automation and the Ph.D. degree in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2014 and 2019, respectively.

From 2019 to 2020, he was a Postdoctoral Fellow with the Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, ON, Canada. From 2020 to 2023, he was a Postdoctoral Fellow with the Department of Mechanical Engineering, University of Victoria, Victoria, BC, Canada. He is currently a Professor with Yongjiang Laboratory, Ningbo, China. His research interests include control theory and micromanipulation.

Tianyu Tan received the B.E. and M.Sc. degrees in Mechanical Engineering from Harbin Institute of Technology, Harbin, China, and University of Birmingham, Birmingham, UK, in 2015 and 2017, respectively. He is currently working toward the Ph.D. degree in Department of Mechanical Engineering, University of Victoria, Victoria, BC, Canada.

His current research interests include switched systems, networked control systems, model predictive control and their applications.



Yang Shi received the Ph.D. degree in electrical and computer engineering from the University of Alberta, Edmonton, AB, Canada, in 2005. From 2005 to 2009, he was an Assistant Professor and an Associate Professor in the Department of Mechanical Engineering, University of Saskatchewan, Saskatoon, SK, Canada. In 2009, he joined the University of Victoria, Victoria, BC, Canada, where he is currently a Professor in the Department of Mechanical Engineering. He was a Visiting Professor with the University of Tokyo, Tokyo, Japan, in 2013. His current research interests include networked and distributed systems, model predictive control (MPC), cyber-physical systems (CPS), robotics and mechatronics, autonomous systems (AUV and UAV), and energy system applications.

Prof. Shi received several teaching awards including the University of Saskatchewan Student Union Teaching Excellence Award in 2007, the Faculty of Engineering Teaching

Excellence Award in 2012. He was a recipient of the Craigdarroch Medal for Excellence in Research in 2015 at the University of Victoria, the 2017 IEEE Transactions on Fuzzy Systems Outstanding Paper Award, the JSPS Invitation Fellowship (short-term), the Humboldt Research Fellowship for Experienced Researchers in 2018, and the CSME Mechatronics Medal in 2023. He has been a member of the IEEE IES Administrative Committee since 2018 and is Vice President of IES (2022–2023). He served as Chair of IEEE IES Technical Committee on Industrial Cyber-Physical Systems during 2018–2022. He is the Co-Editor-in-Chief of IEEE Transactions on Industrial Electronics, and serves as an Associate Editor for Automatica, IEEE Transactions on Automatic Control, IEEE Transactions on Cybernetics, etc. He is a Fellow of IEEE, ASME, CSME, Engineering Institute of Canada (EIC), and Canadian Academy of (CAE); he is registered Professional Engineer in British Columbia, Canada.