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Solving Computationally Hard Problems Using Quantum Computing

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Motivation

- Certain computationally solvable problems can be time-consuming due to their complexity. Quantum computers (QCs) have the potential to solve some of these problems faster by leveraging principles of quantum mechanics.
- One such hard problem is MaxCut, with various applications like portfolio optimization, drug discovery, and inventory management.
- This research dives into solving these problems on QC with introduction to how problems are encoded mathematically (QUBO); converted into quantum language (Hamiltonian) and solved using quantum algorithm (QAOA).
- The goal of research was to run MaxCut problem on different variants of a quantum algorithms to analyze results.

MaxCut Problem

- Problem: MaxCut seeks to split a graph into two sets to maximize the total weight of the edges cut.
- Input: A graph with vertices and weighted edges.
- Output: A division of vertices into two sets, maximizing the total weight of cut edges.

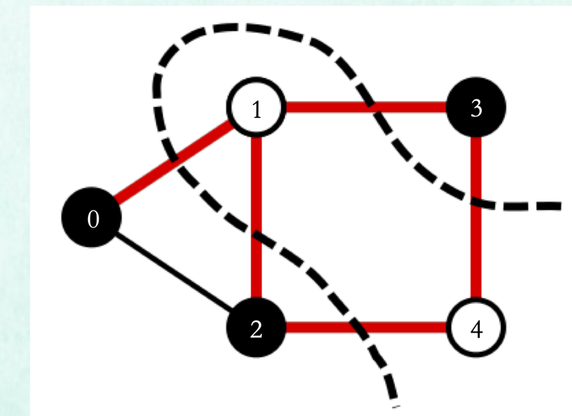


Figure 1. Maximum Cut [3]

Hamiltonian

- The Hamiltonian of a system represents the total energy of the system. They are used to map the problem's objective function into a quantum form, enabling quantum algorithms to run.

QUBO Formulation

- Quadratic Unconstrained Binary Optimizers (QUBO).
- Method used to solve optimization problems.
- Variables can only have two possible values: 0 or 1.
- MaxCut Example (Objective Function)

$$y = (x_1 + x_2 - 2x_1x_2) + (x_1 + x_3 - 2x_1x_3) + (x_2 + x_4 - 2x_2x_4) + (x_3 + x_4 - 2x_3x_4) + (x_3 + x_5 - 2x_3x_5) + (x_4 + x_5 - 2x_4x_5)$$

- Solving a QUBO gives solution set - Ref Figure 1
  - Vertices (1, 4) - Set 1 - White vertices
  - Vertices (2,3, 5) - Set 2 - Black Vertices

Quantum Adiabatic Theorem

- Imagine a quantum system starting in its calmest state; example a ball sitting at the bottom of a valley.
- If we change the system's conditions very gradually, the system won't get too excited, staying close to original state.

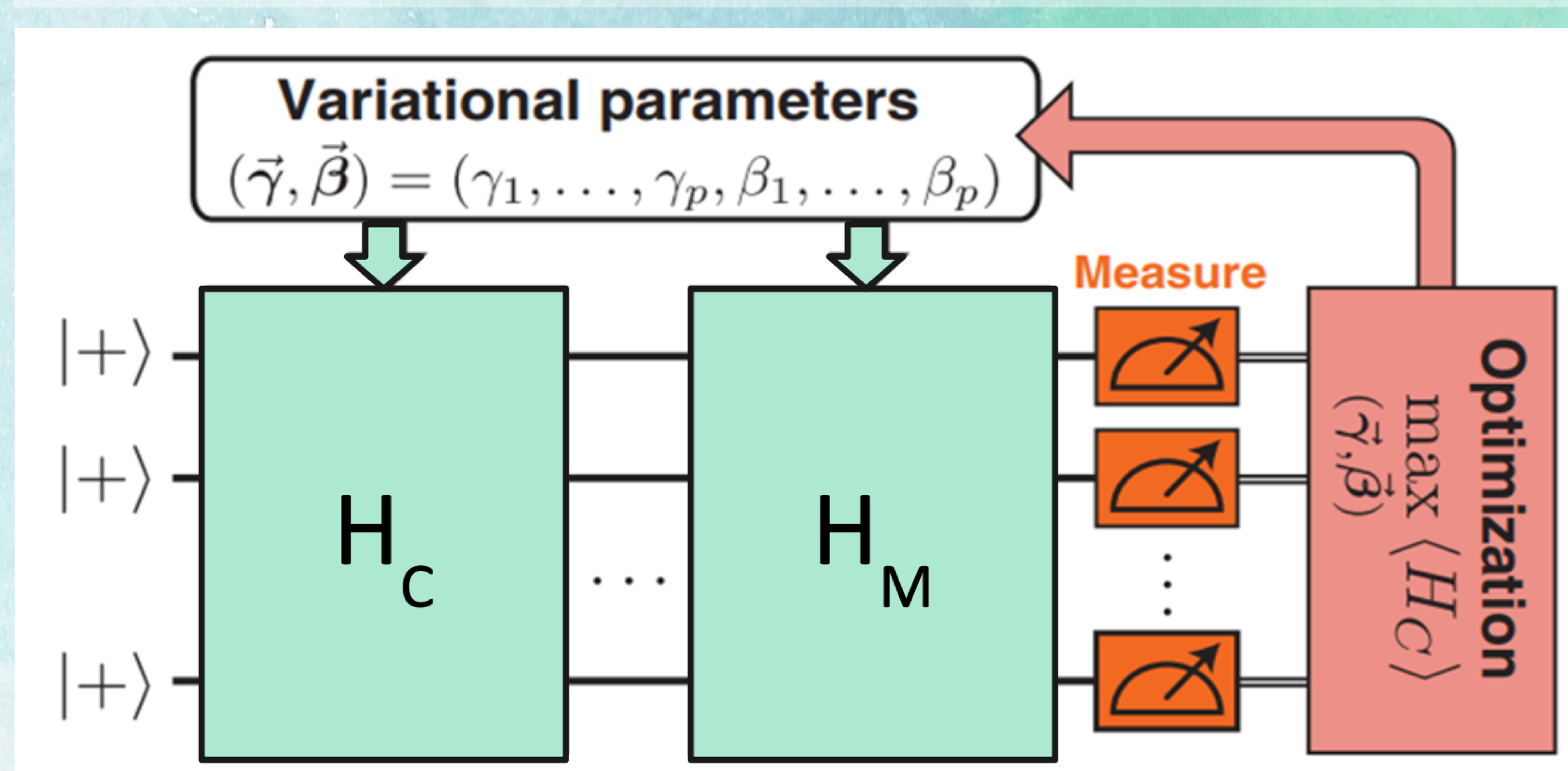
Quantum Approximate Optimization Algorithm

- QAOA encodes an optimization problem into a quantum circuit, preparing an initial quantum state that represents potential solutions [2].
- Uses classical optimization techniques to adjust parameters in the quantum circuit, aiming to find the optimal solution to the encoded problem.

Quantum Alternating Operator Ansatz

- Constructs a quantum circuit, known as the Ansatz, by alternating between single- and two-qubit operators, which encode the problem's objective function [1].
- Iteratively refines and optimizes parameters in the Ansatz circuit using classical optimization algorithms, aiming to find the set of parameters that minimize the cost function.

QAOA Circuit



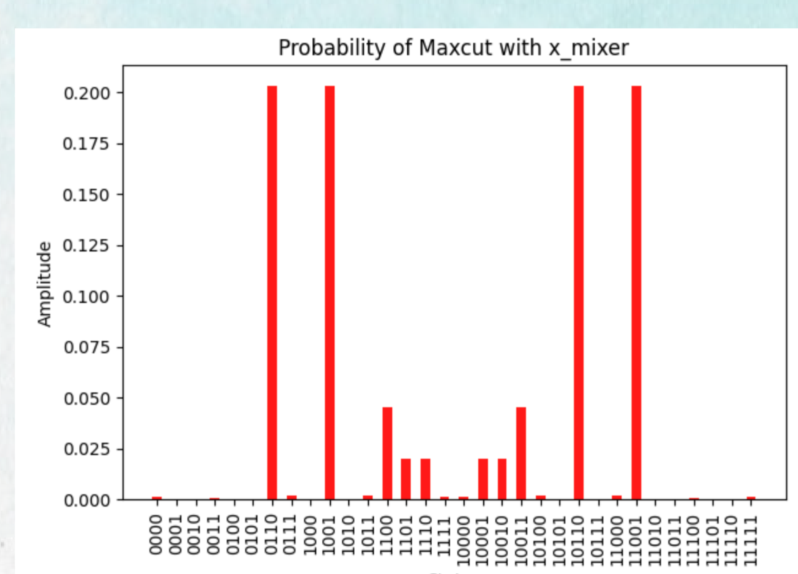
- Hc - Cost Hamiltonian
- H_m - Mixer Hamiltonian
- Optimization - Classical Optimizers
- |+⟩ - Superposition of |0⟩ and |1⟩
- α and β - parameters for hamiltonians.

QUBO To QAOA

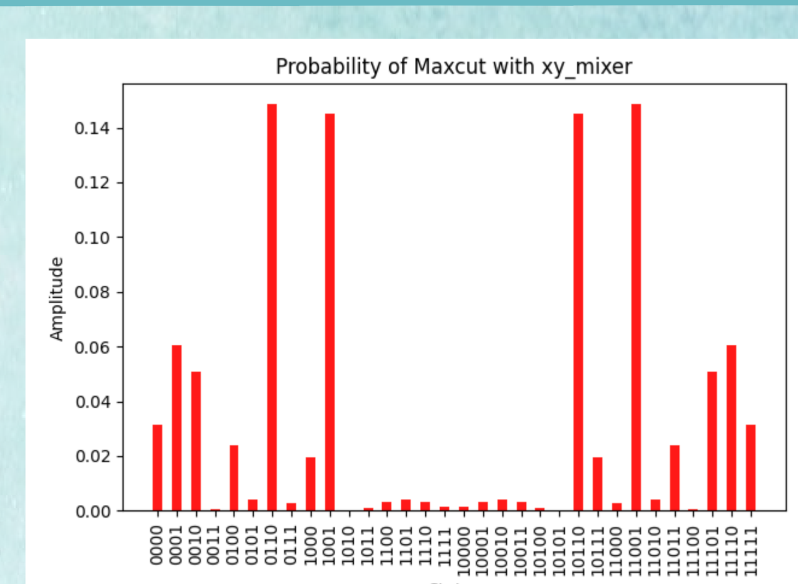
- QUBO problem is translated into a corresponding Hamiltonian, which is a quantum-compatible representation of the problem. Allows the problem to be encoded into the language of quantum mechanics.
- Hamiltonian is then encoded into a quantum circuit using the QAOA. Allows the problem to be represented as a series of quantum gates, making it amenable to quantum computation.
- Parameters of the QAOA circuit are optimized using classical optimization algorithms to minimize the expected energy of the Hamiltonian. It fine-tunes the quantum circuit to provide an accurate solution to the problem.

Results For Different Variants of QAOA

- Utilized Pennylane Software Development Kit (Xanadu) to simulate the results on quantum computer.
- Running the circuit with 6 layers of cost and mixer hamiltonians.
- **x_mixer**
  - Runs with Quantum Approximate Optimization Algorithm
  - Amplitude concentration for states 00110, 01001, 10110, and 11010 with higher amplitude distribution.
- **xy_mixer**
  - Runs with Quantum Alternating Operator Ansatz
  - Amplitude concentration for states 00110, 01001, 10110, and 11010 with lower amplitude distribution.



- Running in x_mixer
- Highest amplitude matches expected states.
- Less amplitude distribution between quantum states.



- Running in xy_mixer
- Highest amplitude matches expected states.
- More amplitude distribution between quantum states.

Future Work

- More in-depth analysis to understand differing results between variants of QAOA.
- Solving diet planning problem with similar quantum algorithms.

References

1. Hadfield, Stuart, et al. "From the quantum approximate optimization algorithm to a quantum alternating operator ansatz." Algorithms, vol. 12, no. 2, 12 Feb. 2019, p. 34, https://doi.org/10.3390/a12020034.
2. Farhi, Edward, Jeffrey Goldstone, and Sam Gutmann. "A quantum approximate optimization algorithm." arXiv preprint arXiv:1411.4028 (2014).
3. "Maximum Cut." Wikipedia, Wikimedia Foundation, 15 Jan. 2024, en.wikipedia.org/wiki/Maximum_cut.

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