SIMULATION PROGRAMS FOR ECONOMIC ANALYSIS

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CONTENTS

PREFACE

The computer programs outlined below are designed to supplement a modern course in economic principles. The purpose of the programs is to provide a basis for systematic independent study outside of the classroom, and to illustrate the use of simulation models in analysis of economic problems and economic policy. Both goals can be served by examination of many concrere examples, using the computer as a "scratch pad" to eliminate any burden of numerical calculation.

In macroeconomic analysis, two broad questions of policy are usually distinguished. On the one hand is the traditional question of employment or stabilization policy, based primarily on analysis of effective demand and capacity utilization. On the other hand there is the newer issue of growth policy, based primarily on analysis of potential output and the required level of expenditures for expansion of productive capacity. (Questions of income distribution will arise in both cases, of course, but are better studied with the aid of more detailed and less highly aggregated models than those set out here.)

Problems of inflation policy and of balance of payments equilibrium can be fully understood only by looking to the interactions of these two factors - effective demand and potential output - and thus these problems require rather elaborate analytical tools. The simulation model is one such tool that is becoming more widely used.

These computer programs attempt to simulate aggregate features of an evolving economy, and to illustrate how such simulation techniques may be used in evaluating economic policy in a dynamic setting.

The first set of programs (the Supply Models) develop the idea of potential output, or the supply side, of the national product flow. Beginning with the computation of aggregate output as determined by available resources and the production decisions of individual firms or enterprises, the models go on to study the evolution of productive capacity and potential output as the economy's endowments of productive factors grow.

The second set of programs (the Demand Models) deal with the demand side of the flow of incomes and expenditures. Proceeding through successively more detailed specification, they arrive at a fully-specified (though still small) model of effective demand, in the Keynesian tradition, and taking into account the four major components of national income.

Bringing the two sides together in the third set of programs (the Link Models) completes the construction of a workable policy model similar in principle to many already in use for evaluation of proposed economic policies.

Each program presents the student with a problem in macroeconomic analysis arising from some question of economic policy, or some interpretation of observed economic behaviour, and advances a simple model to deal with the problem. The solution is presented in the form of a graph or a table of the main variables, frequently in summary tables of the type used in national accounting.

The intent of the programs is two-fold: to illustrate how macroeconomic theory may be viewed as an attempt to simplify and summarize in an analytical model the essential relationships inherent in complex economic structures, and to illustrate how such models may be used in evaluation of economic policy.

The programs are intended to be used by the student at any point in his study. The text in this present document provides all necessary operating instructions, and outlines the setting and significance of individual programs as the student works through them. By collecting his own "hard copy" from the teletype terminal as he proceeds through the programs, the student can construct a summary outline of the topics together with an unlimited number of worked examples.

It should be clearly understood, however, that this document is intended to supplement classroom discussion or a textbook, not to replace them. Our explanations of the programs are confined to specification of the model to be studied, and some brief elaboration of the motives for following a particular approach to the analysis. Detailed explanation of economic relationships is not attempted, nor is any critical assessment of the models employed; the purpose of these programs is simply to put into the hands of the student the computational power to illustrate lectures or textbooks for himself, and to learn from experience, through solution of numerical examples on his own or in collaboration with fellow students.

The present program set has evolved over four years of discussion and trial. A first version was used in the Economics course developed by Sterling Institute for the U.S. Naval Academy, and the continuing encouragement of Sterling Livingston has been important in the subsequent development of this work. Partly as a result of that Naval Academy experience, and partly through collaboration with an imaginative undergraduate student, the basic ideas and organization of the programs were re-examined, and the whole set re-written from scratch. Some of the revised models were used in a summer course at the University of British Columbia, and again, in a small way, in an undergraduate course at the University of Toronto. On the basis of these trials the programs were again extensively revised and new items added. Needless to say, the set has not yet reached its steady state; a number of new programs are already being prepared. But the present package does seem to form a coherent and effective whole, and will continue to provide the basic analytical structure even as new applications or extensions are added.

For much of the elegance and efficiency of these programs, credit must go to the undergraduate student mentioned above. Michael Wolfson first became involved in this work as a programmer while he was a second year student at the University of Toronto, and followed it through three subsequent years of a collaboration I think we both found fruitful. Since he is, at the time of this writing, on his way to India during a break in his graduate program in economics at Cambridge University, it is safe for me to acknowledge here the extent of his contribution to this joint effort. If I could hope that these programs might encourage some equally imaginative and thoughtful undergraduates to become actively interested in applied policy analysis of this sort, I would judge the effort sunk in them to have been thoroughly worthwhile. Certainly there is no shortage of challenging, entertaining, and (I think) relevant work to be tackled in extending and improving such models. We offer the present version as a starting point.

A.R. Dobell

Ottawa, August 1971: Preface revised August 1972.

THE SUPPLY MODELS

Introduction

Production of goods demands the services of productive agents - inputs of managerial effort and labour services or use of equipment and structures being two particular examples. Because there is a limit to how much labour service can be provided by the community, and to how much use can be extracted from existing plant and equipment, there is a ceiling to the level of output the community can produce at any one time. Within that limit, however, the rate at which goods and services are generated depends on the rate of utilization of the available labour supply and the available capital stock. The output rate attained when these resources are being utilized at the maximum feasible and sustainable rate will be called the potential output or full employment output attainable with the given level of resources.

Of course output will generally not be produced unless a demand is anticipated for it. Hence the potential rate of output need not be attained if anticipated aggregate demand falls short of potential output, and then unemployment of labour and capital may result. This problem is analyzed in the Demand Models to be studied later.

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These supply models attempt to estimate what level of output the economy could potentially attain with various endowments of available resources. They thus suggest a target level of employment at which the fiscal and monetary policies analyzed in the demand models could be directed. In conventional national income analysis, this target level is frequently referred to simply as the full employment level of output, or national income at full employment.

Thus these supply models focus on the uses of the community's resources in production, on the transformation of inputs into outputs, and thus on the supply of goods and services. (Models of aggregate demand, by contrast, focus on the uses of income and the consequent demands for goods and services.) There are several sides to this question of the use of resources in production. Decisions on these uses are made by individual production units (firms or enterprises), so it is necessary to consider when such enterprises may be led to employ factors, when they may be led to adopt different combinations of inputs, and when they may be led to allocate resources to different mixes of outputs. In addition to these decisions of individual production units, it is necessary to consider the overall growth of productive factors, and the general progress of technology.

In a sense these supply models are nearer to the heart of classical economics than are national income models. Supply models focus on the allocation of scarce resources to production of alternative outputs, and thus deal with the fundamental economizing problem of any community - wise use of its produced and natural resources, including its human resources. It would seem natural that these resources should not be left unutilized in any rational community decisions. But because these decisions are, in effect, delegated to individual production units each producing to anticipated demand under the free play of market forces, it is possible to observe aggregate supply below potential, and under-utilization of available resources, including unemployment of labour willing and able to work. Part of the explanation for this phenomenon is to be found in the input decisions of firms, studied in Supply Model 1, and in the conditions governing the supply of factors, studied in Supply Model 2.

Supply Model 1 confines itself to study of the individual production unit, and is thus an exercise in micro-economics. It deals with an analysis in which both capital and labour inputs are freely variable, and have to be determined simultaneously according to some criterion for selecting inputs. In particular, the program compares how inputs and outputs might be determined if enterprises were managed by committees of workers who hired capital goods and divided the surplus revenue from production among themselves, with the case where decisions are made by managers hiring labour and capital on behalf of shareholders who divide the profit among themselves.

In Supply Model 2, the concept of potential output is introduced, and an aggregate supply curve is derived under the assumption that capital inputs (equipment and structures) are fixed while labour inputs are variable up to a maximum representing full employment. Actual labour input is assumed to be determined by each individual enterprise so as to yield maximum profits. The aggregate supply curve so derived could be described as a "short-run capitalist supply curve" - "short-run" because of the assumption that capital inputs (and the maximum labour input) are fixed, and "capitalist" because it corresponds to the case of owners of capital stock hiring labour as a variable input and retaining the surplus after payment of wages. Supply Model 2 focuses on the effects, on this short-run aggregate supply curve, of pressures in the labour market to impose a minimum money wage below which labour will not work. (If the minimum wage is zero, then the short-run aggregate supply curve is simply a vertical line at the level of output corresponding to full employment of labour and capital.)

Supply Model 3 changes the focus from a concern with the input mix or the level of utilization of factors to the study of an aggregate economy with growing endowments of productive factors. In the long-run, supplies of both factors vary, (though they are fixed at any instant) and thus aggregate output (at full employment) alters. The interesting question therefore is to know how output will evolve over time in response to different patterns of growth of capital and labour. Supply Model 3 looks at two possible paths of evolution of the capital stock, one associated with the "capitalist" firm, in which investment is related to profits and the expected rate of profit, the other associated with "socialist" organization, in which the overall saving and investment rate can be set by a central economic agency. The program traces some main features of the comparison in a simplified way.

Supply Models 4 and 5 each pick up one thread of this analysis. Supply Model 4 studies in more detail the pattern of growth when investment is determined by a single overall saving rate, and explores the effects of changes in this saving rate upon capital/labour ratios and consumption per capita. Supply Model 5, (based upon a larger model due to Professor Lester Thurow), on the other hand, traces the pattern of growth arising when investment is directly related to the profits and depreciation allowances of corporations,* and technical progress increases the effectiveness of labour and capital inputs.

* In the language of the literature on economic growth, then, Supply Model 4 has a saving function but no investment function, whereas Supply Model 5 has an investment function but no saving function.

Further issues in the analysis of the supply side could be explored with extensions of these models.

Supply Model 1:

Micro-Analysis for Two Institutional Forms

1. The Model

In these "Supply" models we are concerned with the growth of endowments of productive resources and the corresponding evolution of potential output. But just as a movie is an orderly sequence of snapshots, so a growth path in economics can be viewed as an orderly sequence of momentary patterns - snapshots of the economy at a point in time. In the same way that one may sometimes stop a movie to examine some manoeuvre frame-by-frame, it is sometimes helpful to study economic activity by looking at momentary patterns before examining their changes over time.

As well, there are times where one wishes to study the movements of a single player or individual rather than the pattern of a whole team or group simultaneously. Even though the general sweep of events has to be seen as a whole to appreciate emerging patterns, careful study of individual elements helps in following events as they unfold.

In the production side of economics the individual element is the producing unit or enterprise, charged with organizing the use of productive resources toward creation of outputs. Supply Model 1 enables you to study a single producing unit in order to see how input and output decisions may be formed.

The basic technological facts facing a production unit are described in the book of blueprints or "recipes" detailing how inputs can be transformed into outputs. One approach to describing production possibilities which is widely used in industry is linear programming; this technique deals with a limited number of individual production processes with detailed specification of the resources required and the outputs generated by each one. The procedure is illustrated, for a simple case, in the program LINMOD described in Appendix 1.*

A second approach is more traditional and involves summarizing production possibilities in a single smooth function, the so-called production function. This function describes how great an output flow can be generated by use of services of specified amounts of labour and capital. You may recall studying representations of such functions through graphs of "isoquants" or contours of equal output.

For a single firm purchasing inputs, then, there will be three primary concerns - the price to be had for the product (described by a demand curve for output), the amount of output obtainable from purchased services (described by the production function), and the cost to be paid for the purchased services (described by a cost curve). In this

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* The program SCHEMA described in Appendix 2 provides a convenient tool for study of somewhat larger linear programming problems.

analysis, it is assumed that the cost of raw materials is already subtracted from the value of output, so that the production function actually describes the generation of so-called "value-added" by factor services. This simplification causes no difficulty here, though it does in more detailed work.

In Supply Model 1 you are able to select different possible demand, production, and cost functions, and thus to study the decision problem confronting an individual production unit. Of course the behaviour of the unit will depend not only upon these data describing the problem, but also upon the goals of the management itself. If the firm owns no resources at all, but simply purchases services of capital and labour, its goal might be to maximize the residual remaining after all costs of inputs have been deducted from revenue. If the firm owned all the capital goods used, but purchased labour, its goal might be to make the residue after the wage bill is subtracted from revenue as large as possible. On the other hand, a workers' committee managing an enterprise in their own interests, and hiring the services of state-owned capital, would perhaps seek the maximum surplus per worker after payment of rentals for capital services. Since the usual institutional arrangement has management acting on behalf of shareholders (the owners of the capital equipment, which is fixed at any one time), they are usually assumed to hire labour up to the point where profit after payment of wages is a maximum. But in some countries - Yugoslavia is the best-known example - workers' committees or co-operatives do play managerial roles, and their production decisions would presumably be directed at choosing the input and output levels that yield maximum surplus per worker. For each desired level of output, then, workers' committees hiring capital would select different levels of capital and labour services than would management acting on behalf of shareholders. The path of capital and labour inputs employed in order to yield different levels of outputs is referred to as the expansion path for the enterprise; the above discussion thus suggests that the expansion path is different for a firm operated in the immediate interests of its own workers (owners of labour) than it would be for a firm operated in the immediate interests of its own shareholders (owners of capital). The second part of Supply Model 1 displays these computations.

Supply Model 1 thus focusses on two issues - first, the choice (in the long-run) as to which productive resources to use in generating output, and second, a comparison of the production decisions associated with different methods of organizing production. Since different institutional forms may pursue different goals, the input mix or capital intensity or degree of automation in production may be different for

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each, and thus the distribution of income to owners of productive resources may be different as well, even with full utilization of all resources.

2. Detailed Model Structure

The model is based upon the three <u>key relationships</u> mentioned above - the demand, production, and cost functions. These are represented as follows:

$P = P_o + P_1Q$	(linear demand function)
$Q = CK^AL^B$	(Cobb-Douglas production function)
$C = W.L + R_1.K$	(linear cost-function based on fixed wage and rental rates)

In addition one can compute several <u>derived quantities</u>, as follows:

$R = P.Q$	(revenue)
$PI = R - W.L$	(short-run profit with given capital stocks)
$S = R - R_1K$	(surplus)

Then the rate of profit can be expressed as Π/K, and the per-capita surplus as S/L. This latter quantity is labelled PC in the program.

The <u>parameter values</u> stored in the program are as follows:

$Po = 25.$	(the vertical intercept in the demand function)
$P_1 = - 25$	(slope of demand function)

C = 2.	(transformation coefficient in the production function)
A = .3	(exponent in the production function - the elasticity of output with respect to capital input - dimensionless)
B = .7	(elasticity of output with respect to labour input)
W = .60	(wage rate for labour services)
R1 = .15	(rental rate for capital services)

The symbols for variables computed within the program (endogenous variables) are as follows:

K - input of capital services - varies in 7 steps from KMIN to KMAX

L - input of labour services - varies in 5 steps from LMIN to LMAX

P, Q, C, R, PC are also computed as indicated in the equations given above.

Restrictions on parameter values:

$0 < A < 1$ $0 < B < 1$	$0 < W$	$0 < KMIN < KMAX$
$0 < C$	$0 < R_1$	$0 < LMIN < LMAX$
$P_1 \leq 0 \leq Po$		$0 < QMAX$

3. Running the Program

The program for Supply Model 1 runs itself as soon as initial parameters are set. Output consists of two tables, the first displaying output, revenue, and cost possibilities for different combinations of inputs, the second tabulating the expansion path of the enterprise under either of the two institutional arrangements described in Part 1. The program permits computation of either or both tables for as many different demand, production, or cost functions as you wish. Otherwise the output from the program is self-explanatory.

Table 1 thus maps out production possibilities associated with different input combinations, and shows the profile of optimal input combinations for each target output level. Both computations take input prices to be fixed, but suppose input quantities to be freely variable.

4. Suggested Experiments

1. From the first table, identify (approximately) contours of equal cost, equal revenue, equal per-capita surplus. Account for the observed shapes and slopes of these curves. (You may wish to run three times with different parameter values so as to have a separate table for each one of these curves. Note also that by setting Po = 1, Pl = 0, you force revenue to coincide with output and you can then easily study the isoquants or contours of the productive function).
2. For a particular entry in the first table (that is, for a particular selection of capital input K and labour input L), verify the values computed for cost, revenue, and per-capita surplus.

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3. From the second table, plot, on a graph of L against K, the expansion path for the capitalist firm as compared to the cooperative.
4. By running several times with different values for W and/or R_1, explore the effect on the firm's input decisions of changes in factor costs. Explain why the firm changes its input mix in the way it does in response to factor price change.
5. From the second table find the output level yielding maximum profit. By changing the parameters of the demand curve, explore the effects of changing demand upon the firm's optimal output level.

5. Theoretical Exercises

1. a) given $F(K, L) = C K^A L^{1-A} = \bar{Q}$, A constant, what is the relation implied for required inputs of K & L? Draw a sketch in K-L space of the resulting curve ("isoquant").
   b) show that this curve is downward sloping.
   c) how does the curve change as $\bar{Q}$ increases?
2. Given revenue function $P(Q) = Po + P_1 Q$, wage rate W, a rental rate for capital R_1, a production function F(K, L), and a target level of real output $\bar{Q}$;
   a) what is the entrepreneur's profit if he hires L units of labour and K units of capital; what is

the associated input cost?

b) sketch a diagram in K,L space of the set of inputs that yield output $\bar{Q}$ for $F(K,L) = CK^{A}L^{1-A}$; and the set of inputs whose cost is C - where C is the minimum cost of inputs necessary to attain $\bar{Q}$.

c) what is the optimum level of K,L to attain the output $\bar{Q}$ found by maximizing profit in the expression for part a)? How is this consistent with the diagram for b)?

3. Given revenue for $P(Q) = Po + P_1Q$, rental rate Rl, production function F(K,L), and a target level of real output $\bar{Q}$, suppose a group of workers formed a co-operative in which they rented capital and supplied their own labour.

a) if the workers divide gross profits equally among themselves, what is the expression for each worker's share (assuming one unit of labour is equal to one man-year)?

b) how many workers should rent how much capital to attain maximum workers' shares while meeting the target of $\bar{Q}$ units of output?

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SUPPLY

PROGRAM DESIGN BY A. R. [illegible] FOR STERLING INSTITUTE
PROGRAM WRITTEN BY [illegible]

THIS PROGRAM CONTAINS MODELS OF AGGREGATE
SUPPLY AND GROWTH. THE MODELS ARE:
1. MICROANALYSIS
2. SHORT RUN KEYNESIAN SUPPLY
3. A GROWTH MODEL
4. SOLOW-S GROWTH MODEL
5. THURON-S GROWTH MODEL

(CONSULT YOUR WORKBOOK FOR THE EQUATION STRUCTURE
AND PARAMETER NAMES USED BY THE MODELS.)

WHICH MODEL DO YOU WISH TO USE ?
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SUPPLY MODEL 1. MICROANALYSIS
(CONSULT YOUR WORKBOOK FOR SUGGESTED EXERCISES)

INITIAL PARAMETER VALUES:

P0	=	25.00	QMAX=		100.00
P1	=	-0.25	LMIN=		1.00
C0	=	2.00	KMIN=		1.00
A	=	0.30	LMAX=		20.00
B	=	0.60	KMAX=		200.00
R1	=	0.15	B	=	.7

DEMAND: $P = P0 + P1 \cdot Q$
PROD'N: $Q = C0 \cdot K^{A} L^{B}$
W = wage rate
R1 = Rental rate

DO YOU WISH TO CHANGE ANY PARAMETER VALUES ?
NO

DO YOU WISH TO SEE A TABLE OF REVENUE, COST, AND PER CAPITA
SHARES FOR THE SPECIFIED PRODUCTION FUNCTION ?

L,K		1.00	34.17	67.33	100.50	133.67	166.83	200.00
	C	12.1	17.1	22.1	27.1	32.0	37.0	42.0
20.0	R	540.3	622.7	610.7	569.3	517.6	461.4	402.3
	PC	17.0	30.9	30.0	27.7	24.9	21.8	18.6
	C	9.3	14.3	19.2	24.2	29.2	34.2	39.1
15.2	R	291.4	593.9	625.6	621.5	607.0	585.8	560.9
	PC	19.1	38.6	40.2	39.3	38.5	36.8	34.8
	C	6.4	11.4	16.4	21.4	26.3	31.3	36.3
10.5	R	232.4	524.2	580.6	606.3	618.9	624.1	624.8
	PC	22.1	49.4	54.3	56.3	57.0	57.1	56.7
	C	3.5	8.6	13.5	18.5	23.5	28.5	33.4
5.7	R	158.5	391.4	456.8	494.2	520.5	540.2	555.7
	PC	27.5	67.7	77.7	83.3	87.0	89.5	91.4
	C	0.7	5.7	10.7	15.7	20.6	25.5	30.6
1.0	R	49.0	135.9	154.3	163.5	193.3	210.5	221.0
	PC	48.8	130.8	154.2	156.4	178.2	185.[illegible]	191.0

Construct iso-cost, iso-revenue, iso-shares contours.

(Table shows K increasing along top, i.e. across rows – L increasing up the columns.)

← analogous to isoquant map – no optimizing behaviour.

DO YOU WISH TO SEE A TABLE OF COMPARATIVE FACTOR
ALLOCATIONS ALONG THE EXPANSION PATH ?
YES

CAPITALIST						COOPERATIVE		
C	R	PI	KC	LC	Q	[illegible]	[illegible]	[illegible]
3.6	225.0	221.4	7.3	4.3	10.0	0.7	450.0	215.7
7.3	400.0	392.7	14.6	8.5	20.0	1.5	[illegible]	163.1
10.9	525.0	514.1	21.9	12.8	30.0	2.4	1050.0	151.3
14.6	600.0	585.4	29.2	17.0	40.0	3.5	1200.0	121.4
18.2	625.0	606.8	36.5	21.3	50.0	4.7	1250.0	93.0
21.9	600.0	578.1	43.8	25.5	60.0	6.2	1200.0	68.0
25.5	525.0	499.5	51.0	29.8	70.0	8.1	1050.0	45.1
29.2	400.0	370.8	58.3	34.0	80.0	11.1	800.0	25.3
32.8	225.0	192.2	65.6	38.3	90.0	16.[illegible]	450.0	9.4

TOTAL REVENUE IS NO LONGER POSITIVE AT Q = 100.0

← optimizing behaviour assumed in selection of input mix to generate indicated output.

A RUN WITH DIFFERENT INITIAL PARAMETER VALUES ?
YES

(TYPE: 'PARAM = VALUE' UNTIL 'DONE')
P1=0

P0=1

A=.5

DONE

DO YOU WISH TO SEE A TABLE OF REVENUE, COST, AND PER CAPITA
SHARES FOR THE SPECIFIED PRODUCTION FUNCTION ?
YES

L,K		1.00	34.17	67.33	100.50	133.67	166.83	200.00
	C	12.1	17.1	22.1	27.1	32.0	37.0	42.0

etc.

Supply Model 2:

Short-Run Keynesian Supply

1. The Model

From Supply Model 1 you have seen that a profit maximizing enterprise with fixed capital stocks would be led to demand more labour the lower is the real wage. If one carries this relationship over from the behaviour of a single firm to that for the production sector as a whole, the demand for labour is seen to be a downward-sloping function of the level of wages (supposing product prices fixed). Then instead of asking how much labour would be employed at a given wage, one could ask what wage would be required before enterprises would be led to demand all available labour. Thus if labour were supplied at any wage - if there were no limit to how low a wage labourers would accept - then this analysis could tell us what wage realizes full employment of all available labour. However, unlike machines, people may attach conditions to supplying their labour. Supply Model 2 takes us from the microeconomic model of the production enterprises (where the demand for labour is determined) to the macro-economic model of employment for an aggregate economy at a given moment, with a fixed capital stock.

As in Supply Model 1, the underlying economic ideas are straightforward. The firm is taken to own a fixed stock of capital in the form of buildings, machines, land, and so on. By employing labour to work with this fixed stock of durable capital goods, the firm is enabled to produce output. The terms on which this output can be produced - that is, the conditions of production - are described by a production function showing how much output can be produced with given inputs. The key assumption as to the way firms behave is that additional labour will be hired as long as the contribution of an additional worker to output and hence revenue is greater than the wage he demands. The key assumption as to the way workers behave is that, up to full employment, additional labour will offer itself for employment only so long as the money wage offered is greater than some conventional minimum. Below that minimum, no labour will be available. Finally, the key assumption about the way labour markets work is that if more labour is demanded than supplied, competition among employers will force up the money wage offered for labour. If more labour is supplied than demanded, competition among labourers may force down the money wage, but never below the conventional (or legal) minimum.

Do you see, then, that a key feature of this model is that individual enterprises are assumed to base their decision about employing labour on the level of the money wage in relation to the price of their product (on the so-called "real wage" - the ratio of the money wage to the money price of goods) whereas the decision of the labourer to supply services is assumed to be based on the level of the money wage independently of the price of goods? This difference in behaviour can account for the presence of unemployment, as Supply Model 2 illustrates.

The questions on which Model 2 sheds light therefore include the effect of the price of goods on employment, or of the minimum money wage on employment, or the effect of either upon the level of output (the aggregate supply curve) produced within the economy. The model shows that if the price of output is sufficiently high, or the floor money wage sufficiently low, firms will be led to employ all available labour, there will be full utilization of all resources, and output will be at its full employment, or capacity, level. The model thus has the purpose of introducing the notion of potential output, and at the same time illustrating one reason why the community may be led to leave some labour idle and produce less than potential output. In fact, do you see that this condition prevails when the demand for goods, as measured by the price producers anticipate for their product, is weak relative to the price labourers demand for their services? We could thus identify a situation with output below potential as one of too weak an aggregate demand for goods. But can this situation be considered one of involuntary unemployment?

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Yes, because if demand were a little stronger, so that the product price P were a little higher, producers would wish to hire additional labour, and available labour would be willing to work at the same wage as prevailed earlier.

2. Detailed Model Structure

Supply Model 2 may be summarized in the equations:

$$Q = C(KO)^{A} L^{1-A} \tag{1}$$

$$L_D = \left[\frac{C(KO)^{A} (1-A)}{W/P}\right]^{1/A} \tag{2}$$

$$L_S = \begin{cases} LF & \text{if } W > W_O \\ 0 & \text{if } W < W_O \end{cases} \tag{3}$$

$$L = \min (L_S, L_D) \tag{4}$$

Equation (1) represents the production function describing the flow of output Q attainable by fully utilizing the services of a fixed stock KO of capital equipment and a labour input L. Equation (2) represents the demand of a profit-maximizing enterprise for labour as an input to production when technical conditions are described by the function (1), with a product price P and a money wage rate W prevailing. Equation (3) expresses the conditions of labour supply, indicating that the entire labour force LF is available at a money wage not less than a prescribed minimum wage W_O, but that no labour will offer its services at any wage lower than the prescribed minimum. The final equation shows that actual labour input will be equal to the lesser of that demanded by managers and that offered by workers. It is assumed that when there is excess demand for labour the wage will be bid up, but that workers will accept no cut in money wages even in times of unemployment. The money wage, in other words, is assumed to be "sticky downwards".

Thus Supply Model 1 focuses on the production function as the principal determinant of potential output, with a "Keynesian" theory of labour market behaviour in which actual labour input depends both upon the money wage (as compared to a floor minimum wage) and the product price. "Full employment" prevails only when the product price is high enough that the demand for labour exceeds the available labour force at a wage in excess of the floor level.

The detailed mathematical structure of the model can be set out as follows:

(a) Equation Structure:

$Q = C \cdot KO^{A} \cdot L^{(1-A)}$	Cobb-Douglas production function
$W/P = \partial Q / \partial L$	Marginal productivity pricing for real wage
$W_f = P \cdot \partial Q(LO) / \partial L$	Definition of full employment wage

$L^d(W/P) = f(W/P)$		Labour demand derived from marginal productivity condition
$L(W/P) = \min\left[L^d(W/P), LO\right]$		Actual labour employed
$U = 100\cdot\left(LO - L(W/P)\right)/LO$		Resulting unemployment rate
$\overline{W} = \max\left[WO, W_f\right]$		Resulting money wage
$Q_f = C\cdot KO^A\cdot LO^{(1-A)}$		Full capacity output
$Q(P) = \min\left[C\cdot KO^A\cdot L(W/P)^{(1-A)}, Q_f\right]$		Derived aggregate supply curve
$= Q(P;C, KO, A, LO, WO)$		

(b) <u>Parameters, standard values, and units:</u>

A =	.3	Coefficient (elasticity of output with respect to capital input) in production function (dimensionless)
KO =	200.	Fixed flow of services of given capital stock (machine-years/year)
LO =	20.	Total available flow of labour services (man-years/year)
WO =	2.	Initial money wage (thousand dollars/man-year)
P =	1.	Product Price ($)
C =	2.	$\left(\dfrac{\text{Transformation coefficient goods}}{(\text{machine years})^{A}(\text{man-years})^{(1-A)}}\right)$

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(c) Restrictions:

$0 < A < 1$
$0 < KO$
$0 < LF$
$0 < W$
$0 < P$
$0 < C$.

3. Running the Program

As the attached sample run shows, the program is simple to run. Following input of your preferred parameter values, the program computes and graphs the supply and demand functions in the labour market and in the goods market, and determines the equilibrium configuration in each. Where excess demand for goods or labour prevails it permits the wage level to be bid up to restore equilibrium. Where demand for goods or labour is deficient, the program computes the resulting unemployment rate.

You may run the program as many times as you wish with different prices, wages, production conditions, or resource endowments.

4. Suggested Experiments with Supply Model 1

Supply Model 1 is designed to illustrate the concept of a "full employment" level of national income and to show how the existence of a minimum wage may lead to unemployment and a level of output less than potential. The Model may therefore be used to study:

1. The effect on actual employment and output of a change in the minimum wage rate W_o. This effect may be studied simply by running Supply Model 1 with different values for the parameter W_o, noting how employment and output vary from run to run. Thus one sees the reasoning underlying the argument that minimum wage legislation may increase unemployment rates, for example.
2. If it is not possible to bring about reductions in minimum wage levels, then an alternative may be to increase product prices, for example, through increases in the money supply. You may study this possibility by running Supply Model 1 with different values for the parameter P, again noting how income and employment levels respond.
3. The effect on the demand for labour output, or on employment of an increase in the capital stock KO or an increase in technological efficiency represented by an increase in the coefficient C may be observed simply by varying these parameters in repeated runs.
4. In a more elaborate analysis, you might study how the impact of any of the above changes might vary depending upon the parameter A, which itself depends upon production conditions.

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5. Theoretical Exercises and Questions for Discussion

1. Given a money price P for goods, a money wage W for labour services, and a production function, Q = F(K,L), suppose that production decisions (in this case hiring decisions) are made by a "capitalist" (or an enterprise) who owns fixed stock Ko of capital and operates to earn maximum profit after payment of wages.

   (a) What is the expression for the profit if L units of labour are hired?

   (b) How many units of labour should be hired to obtain maximum profit? Do you see that this computation should lead to a demand curve for labour inputs? (Note that such a demand curve is computed numerically in the program).

   (c) Does this procedure lead to a situation where the price of labour services is set by the marginal product of labour services?

   (d) What mathematical assumption as to the form of the production function are crucial in this discussion?

   (e) What economic assumptions are crucial to the analysis?

2. If the total available supply of labour services is LF, find an expression determining a full employment wage

W_f such that the enterprise will be led to employ exactly LF units of labour input. Is there any wage rate W which would lead to unemployment?

3. Derive the aggregate supply function for Supply Model 2. (The aggregate supply curve is graphed, and its equation printed, in the output from the program.)

4. Show that the concavity of the aggregate supply curve depends upon the parameter A, and determine the value of A for which the curve is concave.

5. a) Can you derive expressions to predict the signs of the following derivatives (where E = 1-U)?

$$\frac{\partial W_f}{\partial Ko} \qquad \frac{\partial W_f}{\partial LF} \qquad \frac{\partial W_f}{\partial A} \qquad \frac{\partial W_f}{\partial P}$$

$$\frac{\partial E}{\partial Ko} \qquad \frac{\partial E}{\partial LF} \qquad \frac{\partial E}{\partial A} \qquad \frac{\partial E}{\partial P}$$

$$\frac{\partial Qf}{\partial Ko} \qquad \frac{\partial Qf}{\partial LF} \qquad \frac{\partial Qf}{\partial A} \qquad \frac{\partial Qf}{\partial P}$$

   b) Verify your results with appropriate runs with the model.

6. Suppose that proposed Federal minimum wage legislation would increase the prevailing wage W_o. Find the amount by which the product price P must increase if full employment is to be maintained.

WHICH MODEL DO YOU WISH TO USE ?
2

SUPPLY MODEL 2. SHORT RUN CLASSICAL SUPPLY
(CONSULT YOUR HANDBOOK FOR SUGGESTED EXERCISES)

INITIAL PARAMETER VALUES:

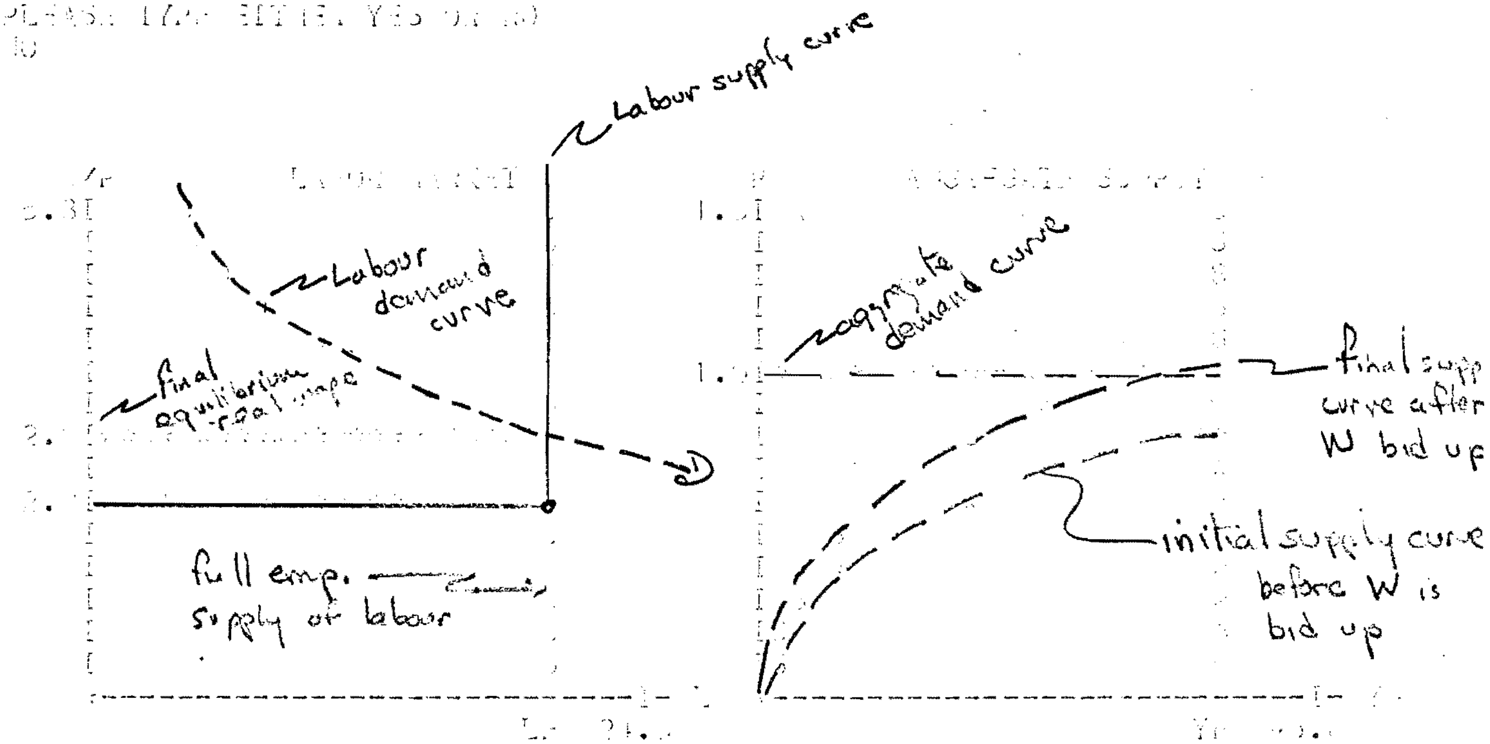
A	=	0.30	CO	=	2.00
KO	=	200.00	P	=	1.00
LO	=	20.00	WO	=	2.00

PROON: $Q = CO \cdot KO^{A} L^{1-A}$

LABOUR SUPPLY: $L(W) = LO$ if $W \geq WO$; $L(W) = 0$ if $W < WO$

DO YOU WISH TO CHANGE ANY PARAMETER VALUES ?
[illegible]

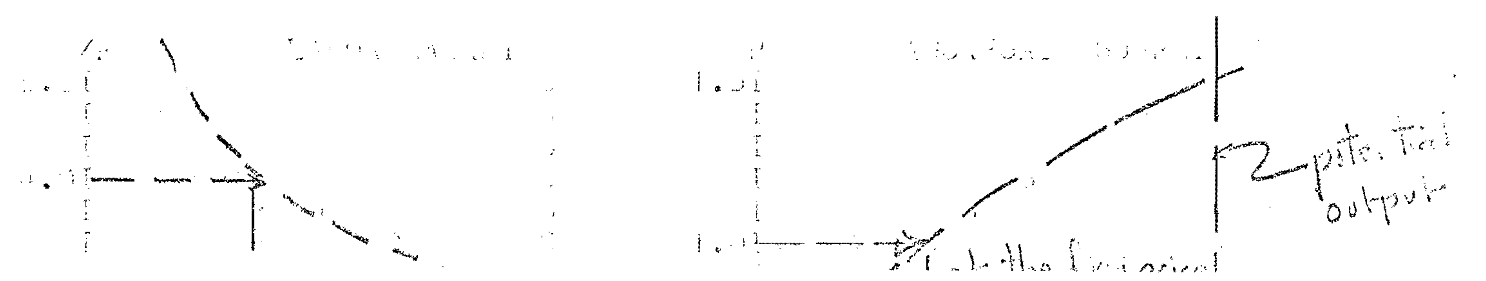
PLEASE TYPE EITHER YES OR NO
NO

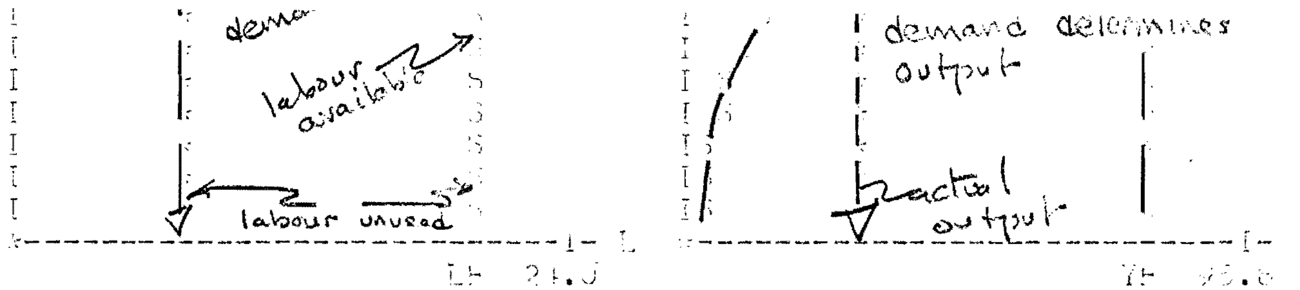


[illegible]

DO YOU WISH TO CHANGE ANY PARAMETER VALUES ?
YES

[illegible]






```
LABOUR SUPPLY:            LF = 2[illegible]
LABOUR DEMAND:            L  = [illegible]21.[illegible] * (W/P) ** ( -[illegible] )
AGGREGATE CAPACITY:       YF = 7[illegible]
AGGREGATE SUPPLY:         Y  = [illegible]77.05[illegible] * (P/[illegible]) ** ( [illegible]3)
UNEMPLOYMENT:             U  = [illegible]9.7[illegible]

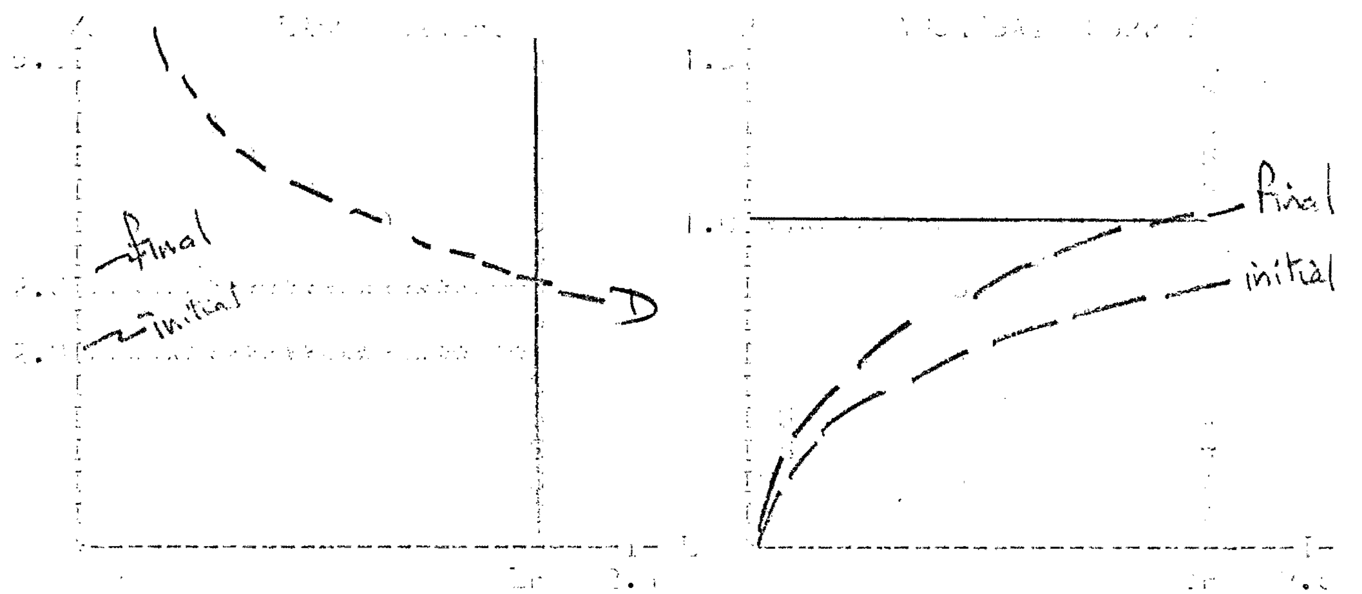
[illegible] WITH DIFFERENT INITIAL PARAMETER VALUES ?
YES

([illegible] 'PARAM = VALUE' UNTIL 'DONE').
[illegible]

L =?

[illegible]
[illegible]
[illegible]

[illegible]
```




```
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]

[illegible]
[illegible]

([illegible] = VALUE' [illegible])
[illegible]
```

Supply Model 3:

1. The Model

Supply Model 1 enabled you to investigate the demand by an individual firm for labour and capital when the rentals for each are given. Turning this relationship around, Supply Model 2 provided the facility to study the level of employment and output that would be attained in the aggregate by the whole production sector under particular labour supply conditions, with the capital stock fixed and fully utilized at any rentals rate. Now in Supply Model 3 it is supposed that no problems of unemployment exist, but rather that both wage rates and rental rates can be flexibly set at whatever level is required to bring about full utilization of the stocks available at any instant. Supply Model 3 then concentrates upon the problem of how these stocks of productive resources grow over time.

Thus, whereas both Supply Model 1 and Supply Model 2 emphasize decisions on the degree of utilization of available stocks, Supply Model 3 assumes full utilization at any moment, and emphasizes the decisions on how stocks are accumulated. The first of these decisions has to do with growth of the labour force; this decision is usually considered to be determined mainly by non-economic considerations - population size, birth rates, traditions of participation in

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education and in the labour force. The first additional equation of Supply Model 3 therefore is a labour force growth equation, expressing the assumption that labour force growth may be well approximated by a curve of geometric growth at a constant rate, independent of market considerations.

The second equation describes the accumulation of capital equipment, in a way that permits the assumption of a constant fraction of output saved and re-invested, or of a constant fraction of property income saved and re-invested. An equation of the first kind might be considered consistent with the accumulation plans of a centrally-organized economy, whereas an equation of the latter kind might be more descriptive of the process of accumulation in a system of corporate capitalism.

Supply Model 3 outputs only a simple table showing inputs, outputs, factor rewards, and the division of total product between consumption and investment along a growth path. (Supply Model 4 develops details of a growth path based on a fixed saving rate out of output, and Supply Model 5 illustrates a growth path based on reinvestment of internal funds of the corporation.) With Supply Model 3, therefore, one can follow the general features of growth under the thrust of any one of a variety of hypotheses about how capital is accumulated.

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2. Detailed Model Structure

(a) Equation Structure:

$$RHO = (1 - SIG) / SIG$$

$$Q_t = GAM \left[DEL \cdot K_t^{(-RHO)} + (1 - DEL) \cdot L_t^{(-RHO)} \right]^{-1/RHO} \quad \text{for } SIG \neq 1$$

$$= GAM \cdot K_t^{DEL} \cdot L_t^{(1-DEL)} \quad \text{for } SIG = 1$$

$$= F(K_t \; L_t)$$ (the production function)

$$W_t = \partial F(K_t, L_t) / \partial L_t$$ (the full-employment level of wages)

$$R_t = \partial F(K_t, L_t) / \partial K_t$$ (the full-employment level of rentals)

$$XX_t \begin{cases} = X1 & R_t > RR \\ X2 & R_t \leq RR \end{cases}$$ (the fraction of property income re-invested)

$$I_t = X3_t \cdot Q_t + XX \cdot R_t \cdot K_t$$ (the investment equation)

$$C_t = (Q_t - I_t) / L_t$$ (consumption per capita)

$$L_{t+1} = (1+GG) \cdot L_t; \quad L_0 = LO$$ (labour force growth equation)

$$K_{t+1} = (1-D) \cdot K_t + I_t; \quad K_0 = KO$$ (capital accumulation equation)

Note that program here (B3) computes $\rho = (\sigma-1)/\sigma$ internally

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(b) Parameters, standard values, and units:

GAM	= 2.	Scale factor	CES production function
DEL	= .3	Distribution parameter	CES production function
SIG	= .9	Elasticity of substitution	CES production function
RR	= .08	Money market interest rate	
GG	= .02	Population growth rate	
D	= .10	Rate of capital depreciation	
X1	= 0.	Fraction output for investment when return on capital high	
X2	= 0.	Fraction output for investment - low return on capital	
X3	= .1	Fraction investment out of total product	
KO	= 200.	Initial stock of capital at time t = 0	
LO	= 20.	Initial labour force at time t = 0	

(c) Restrictions:

$0 < GAM$	$0 < X1 < 1$
$0 < DEL < 1$	$0 < X2 < 1$
$0 < SIG$	$0 < X3 < 1$
$0 < RR$	$0 < KO$
$0 < GG$	$0 < LO$
$0 < D$	

3. Running the Program

As the attached printout reveals, running this computer program also is straightforward. Once parameter values are set, the program proceeds to tabulate values for the relevant variables along the growth path. Thus you can observe how the level of potential output (output at full employment) grows as factor endowments grow (or decays if endowments decrease). You can also study the way rewards for factor services change as the relative scarcity of one factor or the other changes. As you can see, instead of trying to use a single formula to describe capital and output growth, the model instead views this growth as just one result of a system of interacting processes. The model provides a formula for each of the assumed basic processes (production, investment, labour force growth), but it would clearly be a complex expression indeed to describe capital or output growth in a single formula. This situation is familiar to the economist: although he cannot know the details of the very complex relations governing growth and utilization of resources, in a real economy, he can hope to observe general patterns which can be interpreted. Supply Model 3 enables you to follow the same sort of procedure.

SIG = 1/(1+ [illegible])

RR = floor value of [illegible]

$L(t) = (1 + GG)\, L(t-1)$

$K(t) = K(t-1)(1-[illegible]) + I(t)$

$I(t) = X_3 \cdot Q_t + XX\, R_t K_t$

where $XX = \begin{cases} X1 & \text{if } R > RR \\ X2 & \text{if } R < RR \end{cases}$

growth path

[illegible]

/Ls(t)

SUPPLY MODEL 4:

Solow's Growth Model

1. The Model

The basic ideas for Solow's famous growth model have already been introduced in the previous discussion, and you can see how straightforward they are in concept, even though the detailed computations are messy. The advantage of a simulation program is that you can explore the basic ideas of a model without getting bogged down in burdensome computation, and that is certainly the case here.

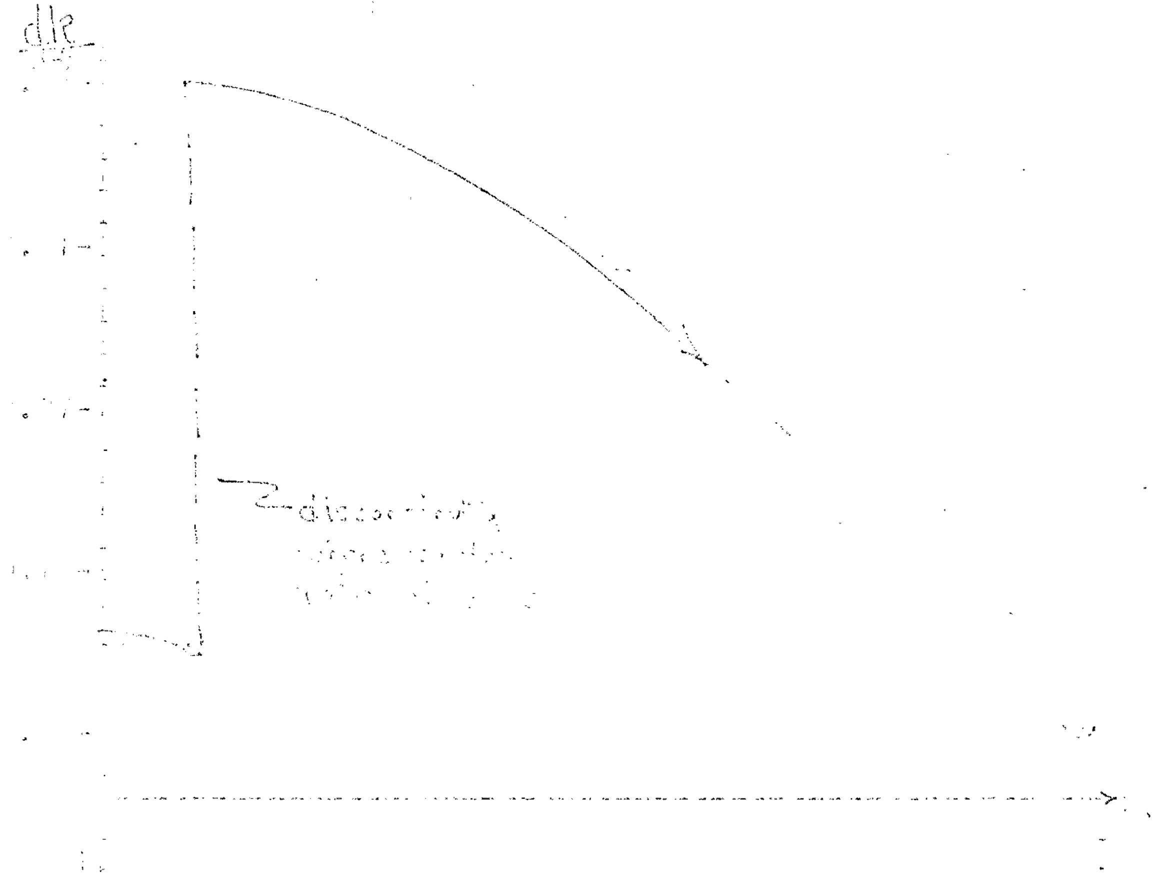
For this model it is assumed that at each instant full utilization of all available capital and labour prevails. As in Supply Models 2 and 3, the level of output is then completely determined (through a production function) by the available supply of resources. Of these resources, some are used for production of consumption goods designed to be used immediately, and the remainder are devoted to production of capital equipment to be used itself in future production activity. In the Solow model it is assumed that one can describe the flow of output in the form of capital equipment as a constant fraction of total potential output. Thus the accumulation of capital stocks is described by an equation which says that at each instant a specified fraction of output is saved and added to the existing stock to be used in later

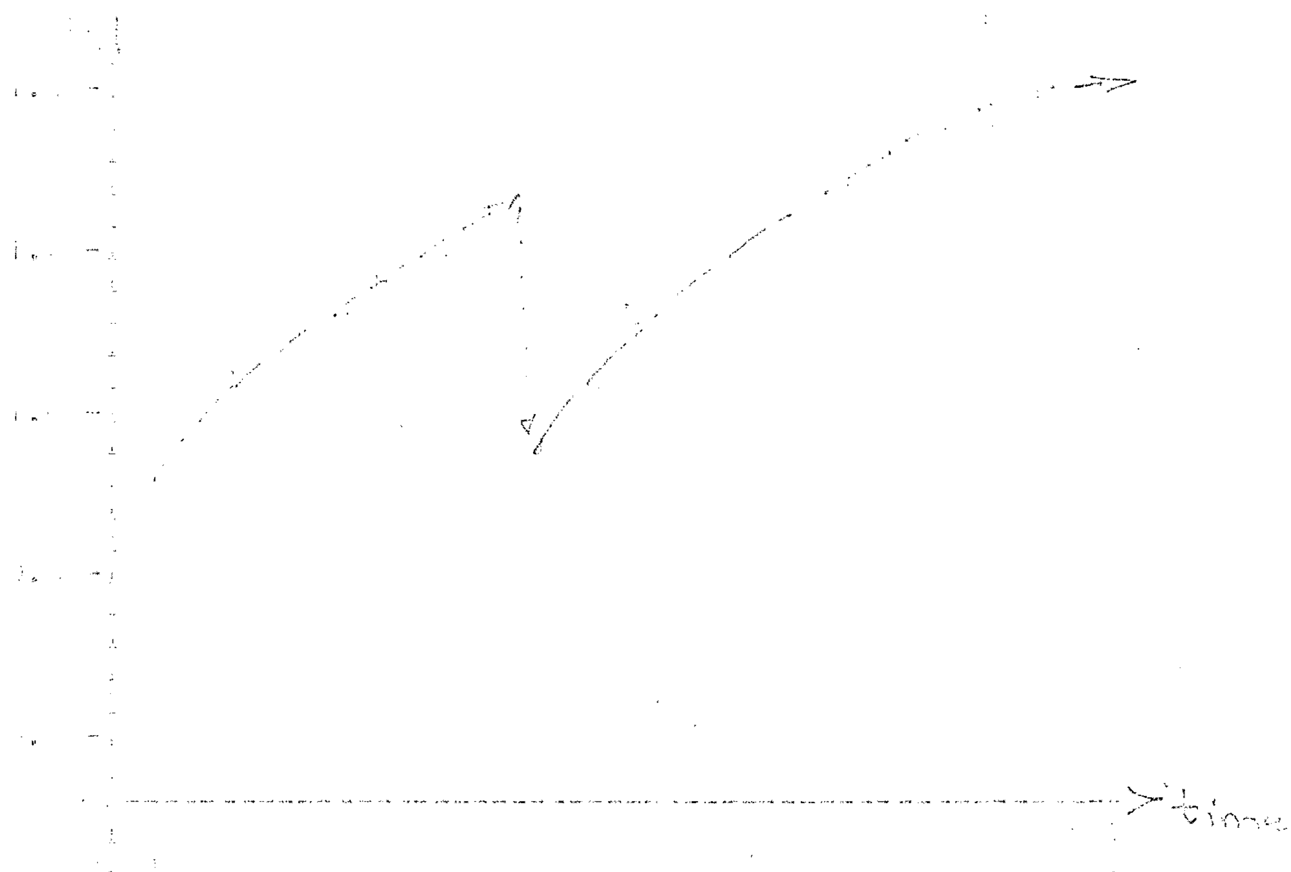
CES, with $\rho = (1-\sigma)/\sigma$ internally - different from B3 production.

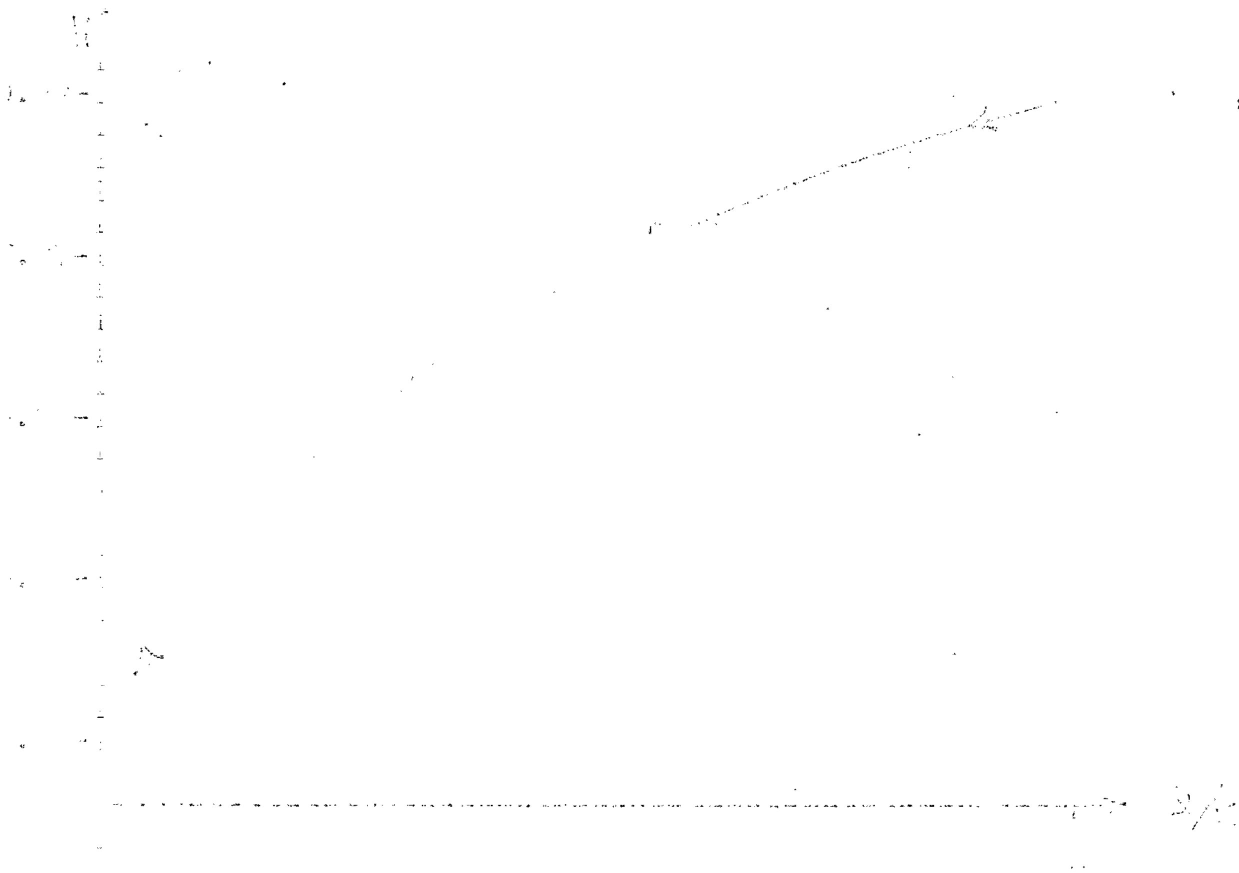
The growth of the labour force is described by the same kind of simple equation as in Supply Model 3, that is, continuing growth and a constant proportional rate.

Thus, from given starting values for stocks of labour and capital, output is determined. And when output is determined the flow of output saved and added to the stock of capital goods is determined, so that the subsequent stock of capital can be calculated. The additions to the labour force are known, so that the subsequent labour force can also be calculated. Thus the model bootstraps itself along from period to period, generating its own future from the specified starting position and the prescribed laws of accumulation.

(The key assumption in the whole story is that all resources are fully employed at each instant. The advanced student may recognize that, for this assumption to be valid, it is necessary that wages and rental rates be sufficiently flexible, and that owners of capital equipment be prepared to purchase all output saved and diverted from consumption. It is the possible failure of both these conditions that gives rise to "Keynesian" theories of aggregate economics, and leads to the Demand Models we shall study later. For now we suppose that full employment does prevail over the longer run described by growth models, and then the Solow model provides one description of the process of economic growth and development.)




time



SUPPLY MODEL 5:

Thurow's Growth Model

1. The Model

Sustained economic growth makes possible rising family incomes, improved provision for the poor, aged, and the handicapped, and a generally rising quality of life. It therefore ranks high in the goals of almost any government. Sustained economic growth, however, is impossible without growth in the size or quality of the labour force and in the quantity and efficiency of the stock of machines and structures available for use in production. In studying problems of economic growth, and in evaluating policies for growth, it is natural that economists should focus attention on factors affecting the labour force and the capital stock.

The present program carries out a simulation of the supply side of a simplified economy - that is, it traces the growth of the labour force, estimates the increased efficiency due to more highly educated labour, projects the decline in number of man-hours worked per year as the work week shortens and holidays increase in number, and arrives at an estimate of how much labour could be available for production. Similarly the program traces the potential growth of capital services, taking account of the effect of technological improvements, by estimating desired investment expenditures of firms. This in turn depends strongly on the corporate profits tax, depreciation allowances, and other similar issues. Hence the model attempts to project a fairly complete statement of sources and uses of funds in the corporate sector.

This growth model is only a portion of a more complete policy planning model developed by Professor Lester Thurow of MIT. His complete model includes the growth model described here, a demand model similar to those set out in the next section, and an income transfer mechanism distributing the revenue from production back to the spending units which determine the expenditure plans which in turn become the demand and revenue flows to production of the next period.

Even though Supply Model 5 is only a portion of this larger model, it is of interest here because it illustrates well the kinds of extensions necessary to turn our illustrative theoretical models into practical planning tools. The general approach and structure of the model are identical to the two previous models we have studied; to make the model sufficiently realistic, however, several complications have to be taken into account. The equations describing the accumulation of equipment and structures are kept separate, and are modified to take into account the fact that technological progress makes new equipment more productive than existing equipment; the equations describing the investment decision of the firm are modified to reflect the importance of the corporation's own funds (as compared to funds borrowed on bond markets or raised by issuing shares); the funds available to the corporation in turn are seen to depend upon depreciation allowances and corporate taxes; the production function itself includes terms reflecting general increases in productivity; and finally, the growth of labour inputs is seen to depend upon changes in hours worked per year (and upon measures of increasing labour productivity) as well as on participation rates and population growth.

The detailed model structure set out in the next session is probably too complex to be fully understood by the average beginning student (or the average government official charged with responsibilities in economic affairs). But it is important that you see the general approach of the model, and recognize that the underlying ideas are the same as in the simpler models we have already studied extensively. So the intuition that you have developed in analyzing the behaviour of the earlier models will help you in comprehending this more complex structure.

As in all our Supply Models, the basic relationship is the representation of production possibilities by a production function showing the output potentially attainable with specified inputs of capital and labour services. This calculation is set out in the equation block marked V in the next section.

Because of technical progress, however, one wishes to measure capital and labour inputs in efficiency units, where more modern equipment and more skilled labour are assigned greater weight. The equation block labelled I describes the growth of labour in efficiency units; this growth depends upon population growth, man-hours, and a rising productivity index. The equation block labelled IV describes the growth of the capital stock measured in efficiency units; this growth is seen to depend upon investment expenditures, described in equation block III. Expenditures on equipment and structures are partly determined, in turn, by the internal funds of the corporation described in equation block II. Blocks II, III, IV, and V must be solved as a non-linear simultaneous system; when that is done, the physical stocks of capital, trend terms, and productivity indexes in block VI can be easily computed and up-dated for the calculations of the next period.

So you see, even though the detailed structure of the model is complex, its general approach is already familiar to us. By introducing some scale parameters we can convey the effect of altering depreciation allowances, corporate tax rates, or the participation rate (in this model, the fraction of the total population employed in the private sector), and we can also investigate the effect of adopting a different target rate of unemployment.

2. Detailed Model Structure

I

$$P_t = P_{t-1}\,(1.013) \qquad P_{1963} = 194.6$$

$$EMP = (100. - \underline{UNEM})/100.$$

$$MH_t = 2301.5 - 11.229\cdot\underline{UNEM} - 15.98\cdot TMH_t + 11.552\cdot T57_t$$

$$LO1_t = MH_t\cdot EMP\cdot\underline{PRAT}\cdot P_t\cdot YIELD\ 1_t/114526.$$

II

$$CPCDA_t = 12.8977 - 1.5649\cdot\underline{UNEM}^t.2045\cdot Y_t\cdot DFGNP_t$$

$$CDAK_t = (-10.993 + .06636\cdot K_{t-1})\cdot DEFI_t\cdot\underline{DEPR}$$

$$CPTFD_t = 2.1276 + .3543\cdot\underline{CTAX}\cdot(CPCDA_t - CDAK_t)$$

$$IFC_t = (CPCDA_t - CPTFD_t)/DEFI_t \qquad (IFC_{1963} = 72.3$$

III

$$IS_t = -2.79 + .0194\cdot Y_t + 32.153\cdot IFC_{t-1}/K_{t-1} + .5146\cdot IS_{t-1} \qquad (IS_{1963} = 21.2)$$

$$IE_t = -6.19 + .0451\cdot Y_t - .0455\cdot KE_{t-1} + .312\cdot IFC_t + 116.22\cdot IFC_{t-1}/(K_{t-1}\cdot U_t) + .3369\cdot IE_{t-1} \qquad (IE_{1963} = 43.8)$$

IV

$$KO4_t = KO4_{t-1} + (IE_t + IS_t)\cdot YIELD\ 1 - D\cdot(KE_{t-1}\cdot YLDDE + KS_{t-1}\cdot YLDDS) \qquad (KO4_{1963} = 2277.1)$$

V

$$INDKO4_t = KO4_t\cdot E\ /370.37$$

$$PROD2_t = .6048 - .000269\cdot\underline{UNEM}^2 + .01167\cdot TPROD_t + .8304\cdot\ln(LO1_t/INDKO4_t)$$

$$Y_t = (e^{PROD2_t})\cdot 100.\cdot INDKO4_t$$

$$
\text{VI} \begin{cases}
KE_t = KE_{t-1} \cdot (1-D) + IE & KE_{1963} = 326.2 \\
KS_t = KS_{t-1} \cdot (1-D) + IS & KS_{1963} = 373.4 \\
K_t = KE_t + KS_t & K_{1963} = 699.6 \\
 & \\
TPROD_{t+1} = TPROD_t + 1. & TPROD_{1964} = 38. \\
TMH_{t+1} = TMH_t + 1. & TMH_{1964} = 21. \\
T57_{t+1} = T57_t + 1. & T57_{1964} = 10. \\
 & \\
YIELD1_{t+1} = YIELD1_t \cdot (1.01) & YIELD1_{1964} = 1.4451 \\
YIELDI_{t+1} = YIELDI_t \cdot (1.04) & YIELDI_{1964} = 4.2681 \\
 & \\
YLDDE_{t+1} = YLDDE_t \cdot (1.04) & YLDDE_{1964} = 2.8834 \\
YLDDS_{t+1} = YLDDS_t \cdot (1.04) & YLDDS_{1964} = 1.3159 \\
 & \\
DFGNP_{t+1} = DFGNP_t \cdot (1.03) & DFGNP_{1964} = 1.113 \\
DEFI_{t+1} = DEFI_t \cdot (1.03) & DEFI_{1964} = 1.102
\end{cases}
$$

3. Running the Model

The underlined parameters (UNEM, PRAT, DEPR, and CTAX) are taken to be the only instruments available in this model. You may set these instruments at desired values at the beginning of a run, and thereafter the program tabulates the essential features of the path of potential output (the output that could be achieved if the target rate of unemployment is always realized).

CTAX and DEPR are initially set at unity. Changing the first has the effect of scaling the historically observed corporate tax rate by the new value of CTAX, while changing the second has the effect of scaling up historically observed depreciation allowances by the new value of DEPR. (Setting both to a value of 2, for example, would have the effect of doubling both the corporate tax rate and the level of depreciation allowances).

The effect of changing participation rates can be explored by altering the parameter PRAT. Although the rate of labour forc participation is not usually considered an instrument of policy, it may be interesting to explore what would be the effect on potential output of changing work habits - more women in the labour force, people staying in school longer, or similar trends.

Finally, the notion of potential output entails some notion of full utilization of the labour force. If one's standard of what constitutes full utilization alters, one's expectation of potentially attainable output alters as well. The effect can be analyzed by changing the target unemployment rate UNEM.

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Introductory Comments to the Demand Models

A basic contribution of the modern analysis of national income determination is the observation that various economic agents establish their spending decisions independently. As a result, unless there are smoothly working markets reconciling the expenditure decisions of different agents, there is no guarantee that the economy will be operating at a level that makes full use of all available resources of equipment and man-power. It thus becomes a problem to calculate the equilibrium level of national income - that is, to determine the level of national income at which all decision-makers are spending exactly as much as they planned to do and desire to do in light of the income or revenue they receive.

It should be easy to see why this determination is important. We imagine each economic decision-maker as looking ahead a little and, in effect, setting his budget on desired expenditure. If, as each agent attempts to carry out his expenditure plans, it turns out that the total of desired spending is different from the prevailing level of output, then the level of output will tend to change. Only when the level of output exactly matches the total of desired spending by all sectors can we say that we have found a sustainable equilibrium level of national income and output.

A simple model which illustrates the working of the system is worthwhile. This model imagines that production plans can always be set so as to match the spending plans of consumers and firms. Thus, towards the end of one time period - call it a year - consumers look ahead a little to estimate their income in the coming year, and then decide (in the light of their estimate of income) how much they should plan to spend on consumption in the coming year. Similarly, firms look ahead to forecast their needs for equipment, and then decide upon their investment expenditures. If we look only at these two classes of expenditure, the sum of these decisions constitutes the total demand for goods to be produced in the coming year. We assume that firms can forecast this total demand quite accurately, and can therefore set production plans so as to yield an output equal to desired total expenditure. Thus effective demand in the coming period determines the rate of planned and realized output in the period.

The question is, is this realized output rate equal to what households and firms estimated it would be? If not, they will likely revise their expenditure plans, and income levels will change again. Do you see that income will keep changing in this way until it reaches the level expected by firms and households, and then need not change further? We call such a sustainable level of income an equilibrium level and we then seek to determine that equilibrium level of income on the basis of some theory about the way consumption and investment decisions are made. We want to know what level of income could be maintained from period to period in the light of individual spending plans, so that we can estimate whether that equilibrium level of income is adequate to provide for full utilization of available resources of manpower and equipment.

Do you see, then, that our motive in going through this exercise may be an implicit goal of full utilization - a full employment target? The reasoning is that a well-functioning economy should not allow resources to lie idle, factories to be shut, or men to be unable to find jobs. Given this implicit target, we break the analysis into two steps:

i) determine what equilibrium level of income results from given expenditure decisions (this is a problem in economic analysis - the determination of national income);

ii) determine means to influence some expenditure decisions so as to make total demand consistent with full utilization of resources (this is a problem in economic policy).

The first step is thus to calculate the equilibrium level of national income on the basis of expenditure decisions. Extensive discussion of this analysis may be found in many standard texts - for example, Lipsey and Steiner, pp. , Samuelson, pp. .*

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*

The following models illustrate several features of the analysis of national income based solely on expenditure decisions. The general approach is to observe that (by definition) aggregate demand is the sum of desired expenditure on final goods and services by all agents in the domestic economy (or demanding goods and services from the domestic economy). Classifying all demands as coming either from the household sector (consumption demand), the business sector (investment demand), the government sector (government demand for goods and services), or the rest-of-the-world sector (external demand), one may express the definition of aggregate demand as

$$Y_t^d = C_t^d + I_t^d + G_t^d + (X_t^d - M_t^d)$$

where C_t^d, I_t^d, G_t^d, and X_t^d represent consumption demand, investment demand, government demand, and external demand, respectively, and M_t^d represents that portion of domestic demand which is directed toward foreign suppliers rather than domestic producers. Y_t^d thus represents the total demand for final goods and services to be provided by domestic producers.

Producers, of course, do not know total demand beforehand, and actual output may be different from that desired by buyers. However, it may be expected that producers attempt to forecast their expected market, and to adjust their output toward that expected level.

In order to concentrate on the effects of expenditure decisions, we ignore possible lages in adjusting output, assuming instead that producers are invariably successful in forecasting demand, so that actual output is always equal to aggregate demand in each period.* Then we may take actual output to be the sum of desired expenditures, and our task of explaining the determination of actual output reduces to that of explaining the components of aggregate demand.

The first model, as mentioned above, concentrates on the system in which government and foreign demands are ignored, as are imports of goods and services, and investment decisions are taken as exogenous to the model. In the second model a more sensible representation of investment decisions is introduced. The third model is concerned with the consequences of the government's participation in the economy as a purchaser of goods and services (with expenditures financed by tax levies), and as the agency controlling interest rates which in turn affect

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* With only minor extension of our models, it is possible to analyze the behaviour of a system in which the adjustment of output to actual demand may be slow, so that we write

$$Y_t = Y_{t-1} + \lambda \left[Y_t^d - Y_{t-1} \right]$$

where $0 \leq \lambda \leq 1$. This relationship signifies that actual output this period begins from the base level given by actual output last period, and adjusts up or down according to whether demand this period exceeds that base level or falls short of it. For values of λ close to unity, actual output responds very quickly to changes in desired output, while for values of λ close to zero, the adjustment process is sluggish. However, for simplicity in this present series of models we assume $\lambda = 1$.

investment expenditures. The fourth and fifth models return for a more detailed examination of the effects of tax policy, recognizing that tax revenues depend upon output levels (and hence upon individual decisions) as well as upon government decisions. The sixth model adds relationships describing the foreign sector (exports and imports) to the model. Finally, the last model takes account of the role of the money market in determining rates of interest, and thus completes the specification of a standard income determination model including monetary considerations.

This set of models therefore provides examples to illustrate a number of concepts important in elementary (and not so elementary) national income analysis. By building progressively from the simplest structure, it illustrates the essential unity of the separate exercises, but at the same time provides (through the simulation structure) the opportunity to deal with several effects simultaneously.

The key analytical features are few:

1) a <u>definition</u> of aggregate demand:

$$Y_t^d = C_t^d + I_t^d + G_t^d + (X_t^d - M_t^d)$$

2) a <u>hypothesis</u> as to determination of actual output in light of aggregate demand:

$$Y_t = Y_{t-1} + \lambda \left[Y_t^d - Y_{t-1} \right]$$

3) a behavioural hypothesis as to formation of expectations

$$\hat{Y}_t = f(Y_{t-1}, Y_{t-2}, \ldots, Y_{t-n})$$

4) behavioural hypotheses as to the determination of each expenditure component (and tax revenues), each to be reduced ultimately to a dependence only on present or past levels of income, or interest rates.

The model thus reduces to a set of difference equations (or ultimately one difference equation involving output Y_t and its lagged values), which can be analyzed to explain growth and fluctuations in national income. The following models illustrate particular features which have been of lasting concern in national income analysis.

DEMAND MODEL 1:

The Investment Multiplier

1. The Model

Let us begin our analysis of national income determination by investigating a simplified model which deals with only two components of national income and expenditure, namely, the decisions of households (in the aggregate) to spend for consumption, and the decisions of firms (in total) to spend to acquire new capital equipment. In studying this model we ignore any questions arising from foreign trade or from government expenditure and taxation. Let us denote the consumption expenditure of households by the letter C, and the investment expenditure of firms by the letter I.

The main feature of economic life which the economist attempts to capture in this kind of model is that firms and households have very different motives for spending, and the resources they have to spend are also different. It is argued that household spending on consumption is strongly influenced by the income of the household, and thus, in the aggregate, by national income, while the investment of firms tends to be less influenced by levels of income and more influenced by other considerations (outside this present model). Under these circumstances, the level of investment expenditure can be seen to be a rather crucial determinant of the level of income.

The reasoning here is that the actual level of income and output must keep changing until it reaches an equilibrium level where the planned consumption of households and the planned investment of firms just exactly balances the prevailing level of income. Since the investment decision is taken as determined independently, while consumption expenditures and income levels adjust until they are mutually consistent, it makes sense to consider that the prescribed level of investment determines the associated equilibrium level of income. The present task is to analyze the association in more detail.

Clearly the exact nature of the association depends upon the precise way that consumption decisions depend upon income. An equation attempting to describe the determination of planned consumption expenditures is known as a consumption function; with different consumption functions there will be different relations between the level of investment and the associated level of income.

Let us imagine, for example, that firms decide upon a fixed level of investment they wish to maintain. Independently of this decision, households decide upon the desired level of consumption expenditure as a linear function of their expected income, taking their observed aggregate income last year as their best guess as to the income level they may expect in the current year.

This computer program calls for you to experiment with such a model by trying different consumption functions and

EXPENDITURE MODEL 3. MEI AND THE MULTIPLIER MODEL.
(CONSULT YOUR WORKBOOK FOR SUGGESTED EXERCISES)

INITIAL PARAMETER VALUES:

GAM =	0.500	CA =	15.000
DEL =	5.000	B1 =	0.600
R =	7.000		

$$C_t = CA + B1 \cdot Y_{t-1}$$

$$I_t = (GAM - DEL \cdot R/100.) \cdot Y_t$$

$$Y_t = C_t + I_t$$

DO YOU WISH TO CHANGE ANY INITIAL VALUES ?
NO

(MEI ≡ MARGINAL EFFICIENCY OF INVESTMENT)

INITIAL EQUILIBRIUM
GROSS NATIONAL EXPENDITURE

CONSUMPTION	(PCE)	51.0	— $\bar{C} = C_0$ OR C_{1970}
GOVERNMENT EXPENDITURE	(GPGS)	0.0	
INVESTMENT		9.0	— $\bar{I} = I_0$ I_{1970}
EXPORTS		0.0	
LESS IMPORTS		0.0	
GROSS NATIONAL EXPENDITURE		60.0	— $\bar{Y} = Y_0$ Y_{1970}

PARAMETERS OK ?
YES ← If the initial equilibrium tabulated by the program is unacceptable, it is possible to change the parameters either of the consumption function (CA, B1) or the investment function (GAM, DEL).

INITIAL INSTRUMENT VALUE:
R = 7.000 — 7%/yr.

WHAT NEW VALUE DO YOU WISH FOR THE INSTRUMENT ?
(TYPE: 'PARAM = VALUE' UNTIL 'DONE')
R = 8.5 — 8.5%/yr ← Program indicates available instruments and asks you to enter desired changes in the [illegible] of the [illegible]

DONE

IMPACT DISEQUILIBRIUM
GROSS NATIONAL EXPENDITURE

different levels of investment expenditure, and observing the process of adjustment of income to an equilibrium level where both households and firms are realizing their planned expenditure, and income does not change. By repeated trials, the relationship between parameters of the consumption function and the effect of changes in investment will become clear.

2. Equation Structure:

$C_t = CA + Bl \cdot \hat{Y}_t$	consumption function
$I_t = \overline{IA}$	investment function
$Y_t = C_t + I_t$	national expenditure identity
$\hat{Y}_t = Y_{t-1}$	expectations hypothesis

Parameters, standard initial values, and units:

$CA = 8.$	autonomous consumption (billions \$)
$Bl = .7$	marginal propensity to consume (out of income) (fraction)
$\overline{IA} = 8.$	autonomous investment (billion \$)

Restrictions:

$CA + \overline{IA} > 0$

(autonomous expenditure must be positive)

Instrument: $\overline{IA}$

Restrictions:

$|\overline{IA} - \overline{\overline{IA}}| > 0; \; CA + IA > 0$

Where $\overline{\overline{IA}}$ is the new level of investment.

These equations express the assumptions that

1) planned consumption expenditure is determined by the level of income expected;
2) planned investment expenditure is constant, independent of income (a control variable);
3) desired expenditure (aggregate demand) consists of only two components, desired consumption and desired investment, and actual income and output always equals desired expenditure;
4) the expected level of income is estimated as equal to the observed income of the last period.

We also take the time periods to be one calendar year in length.

3. Running the programs

In this model there are three structural parameters - CA_2 (the level of autonomous consumption expenditure), B1 (the marginal propensity to consume) and IA (the level of investment expenditure - of which the last is treated for the moment as an instrument of economic policy, controlled by the central government. The model assigns initial values to these parameters, and the first question the program asks is whether you wish to change any of these values. If not, the program computes the initial equilibrium level of national income, and its composition between investment and consumption expenditures, and tabulates the result in a standard accounting form. The program asks whether, in light of this table, the initial values appear satisfactory; if not, the program accepts new values from the console.

Assuming that acceptable initial values are established, the program then observes that the level of investment expenditure might under some circumstances be influenced by economic policy, and might therefore be considered an instrument through which national income is altered. The program invites you to study the effects of a change in investment by altering the value assigned to the instrument IA. As a result of the postulated change the investment expenditure, national income is altered, and consumers are no longer in equilibrium at the new level of income. Their efforts to adjust consumption expenditures in response to the change lead to a sequence of successive adjustments, which the program follows step-by-step. If this adjustment process approaches a new equilibrium configuration, the program computes and tabulates that also. Comparing the new equilibrium with the old, one thus has an impression of the effect of the change in investment expenditures.

Economists frequently wish to summarize the results of such calculations, and they do so by calculating a <u>multiplier</u> - the equilibrium multiplier for national income on investment expenditure. This multiplier is computed as the ratio of the change in investment - that is, as $\frac{\overline{Y}\text{ (new)} - \overline{Y}\text{ (old)}}{\text{IA(new)} - \text{IA(old)}}$. For a linear model (and <u>only</u> for a linear model) this multiplier is a pure number independent of initial and terminal values for any of the variables.

As an exercise, you could run the model several times with different changes in the instrument IA, verifying that for any prescribed value B1, the multiplier can always be estimated directly as 1/(1-B1).

As you can see from the attached printout, running this model is very straightforward. Having called the program "DEMAND" as instructed by your lecturer or tutor, you will find that the teletype prints a preliminary identification and then asks which expenditure model you wish to use. You respond by typing 1, and the program prints the parameter values for the stored model structure. This procedure is for your convenience: you need input values only for particular parameters in which you are interested. If these parameter values lead to a suitable initial equilibrium configuration, you may then proceed directly to study how a change in investment expenditure can lead to an amplified (or "multiplied") change in national income and consumption, and you can thus study how government expenditures on public investment, or government policies to encourage private investment, may be used as a tool to stimulate (or restrain) the economy. Indeed, as above, you may wish to summarize this relationship in a so-called "equilibrium multiplier" (for income on investment) given by the ratio of the induced change in income to the imposed change in investment expenditures.

4. Suggested Exercises

Before going to the teletype to run the model, you may wish to prepare some experiments designed to help you estimate answers to questions such as the following:

1. Does the value of the multiplier depend upon the size of the change in the policy variable I? Does the resulting equilibrium income?
2. Does the value of the multiplier depend upon the value of the marginal propensity to consume, B? (You could, for example, run several trials with different values of B, plot the resulting multipliers against the value of B, and attempt to fit a simple curve to the resulting points.) Does the resulting equilibrium income? Can you describe how?
3. Does the value of the multiplier depend upon the value selected for CA?
4. Does the multiplier depend upon the initial equilibrium taken as a starting point?

The actual use of the model in attempting to answer such questions is indicated by the comments and instructions entered on the sample output which follows.

DEMAND MODEL 2:

1. The Model

The standard investment multiplier analysis set out in Demand Model 1 points up the importance of investment decisions, and the magnified impact of these decisions upon national income. But as a model of economic policy it is somewhat limited; it would apply more appropriately to economies - Eastern Europe, the Soviet Union, India - where there is some state control of industrial investment decisions. But in a mixed market economy such as the U.S. or Canada, investment decisions are not centrally controlled. Instead these decisions, and hence the overall level of investment, reflect a balancing of private costs and benefits by individual managers. Demand Model 2 augments the analysis of Demand Model 1 in order to take account of this feature.

In Demand Model 2, in other words, investment expenditure cannot be considered directly an instrument of of economic policy. This is not to say that investment decisions cannot be influenced by economic policy - there is some central control of interest rates, which may be supposed to influence business investment decisions - but only that the strength of this central economic policy will depend on the extent to which it can be transmitted through private decisions.

Control over interest rates is accomplished through a variety of techniques grouped under the general heading of monetary policy. We shall look into some aspects of these techniques later on (see Demand Model 7), but for now we may assume that central economic agencies have the power to establish the level of interest rates entering the investment decisions of firms. Since, in their investment decisions, firms presumably take account of the interest rate as a measure of the cost to them of acquiring funds for equipment purchases (or of the cost to them of using their own funds for this purpose rather than for lending), we expect that at higher rates of interest firms may spend less on investment. The essential feature of Demand Model 2 therefore is that it reflects the impact of monetary policy operating through private investment decisions.

Thus, the important question to investigate is how the power of monetary policy will depend upon properties of the investment function describing private investment decisions, or, in other words, how the multiplier for national income on interest rates may vary with the parameters of the model (or with the interest rate itself).

Notice that the only instrument of economic policy in this model is the rate of interest. Economic authorities cannot control household spending directly: households formulate their own spending plans as described by the consumption function.

Nor can investment spending by firms be controlled directly: private firms set their expenditure plans in the light of expected income levels and the prevailing rate of interest. Only by altering the rate of interest can the central government affect investment spending, and thus income levels. (Thus we see that the previous model is correct enough, but not complete since it leaves out the step linking investment expenditures to the interest rate.)

This program therefore asks for a trial change in the rate of interest. It computes the national income statement for the new situation resulting immediately after the change in interest rate. You should verify that neither households nor firms would find the new situation an equilibrium state, since their actual spending is different from what they would plan to spend in light of the observed level of income. Both households and firms therefore attempt to adjust spending, and national income changes as a result. The program graphs this adjustment process, tabulating a final equilibrium income statement as well.

Notice that it no longer makes sense to try to compute an investment multiplier in this model, because investment cannot change all by itself in this model. Investment changes only if the interest rate or income changes. In fact, the only thing that can change independently in this model is the interest rate. It comes from outside, so to speak, and everything else follows it. Thus we could calculate a multiplier which shows how much change in income we would get for a prescribed change in the rate of interest. We can think of performing that experiment a number of times with our little simulation models to obtain an estimate of this multiplier, which we might call the multiplier for income of the rate of interest.

If we do perform this experiment many times with the same consumption and investment functions, and different rates of interest, we will notice something that did not occur in Demand Model 1: we will notice that in this model the multiplier is not always the same. Even though both the consumption function and the investment function remain unchanged, the multiplier is different depending upon what interest rates we try.* Thus we cannot speak of "the" multiplier in this model, because there are many possible values. Yet with a simulation model we can always estimate the multipliers for the cases which are relevant and of interest in studying economic policy. The Appendix to this section sets out a more detailed analysis of the properties of this model.

* The reason for this is that the model is not completely linear; it has the interest rate multiplying the level of income in the investment function. You can check that Program A1, which has unique multipliers, is linear.

Equation Structure:

$C_t = CA + B1 \cdot \hat{Y}_t$		consumption function
$I_t = (GAM - DEL \cdot \overline{R}) \cdot Y_{t-1}$		investment function
$Y_t = C_t + I_t$		national expenditure identity
$\hat{Y}_t = Y_{t-1}$		expectations hypothesis

Parameters, standard initial values, and units:

CA =	8.	autonomous consumption (billions $/year)
B1 =	.7	marginal propensity to consume out of income (fraction)
GAM =	.5	direct marginal propensity to invest (fraction)
DEL =	.05	non-linear propensity to invest as a proportion of interest rate · income (fraction - year)
$\overline{R}$ =	7.	interest rate (%/year)

Restrictions:

$CA > 0$ (autonomous expenditure must be positive)

Instrument:

$\overline{R}$

Restrictions:

$|\overline{R} - \overline{\overline{R}}| > 0$

Running Demand Model 2

The attached printout shows that running Demand Model 2 is very little different from using Demand Model 1. As before, the program indicates its stored initial values, and permits you to change them if you wish. Having tabulated the initial equilibrium, it then selects the available instruments or control variables - in this case only the interest rate - and indicates the current value for that variable. Having accepted your desired new value for this instrument, the program tabulates the immediate impact of a change from the old value to your new value, the resulting adjustment process set in motion by that change (if you wish to see it), and then the new equilibrium value associated with the new instrument value you selected. You may repeat this process with as many new values for the policy variable as you wish, holding the model structure otherwise unchanged, and then you may repeat the whole exercise with as many new model structures (obtained by changing some or all of the parameter values) as you wish.

By running through this computation a few times with only the values for R changing, you may learn something of how the strength of monetary policy (as measured by the equilibrium multiplier) may vary with the prevailing level of interest rates. As well, by running with different selected values for the parameter DEL - which you can see measures the influence of interest rate changes on investment decisions - you can learn something of possible weaknesses of monetary policy as a tool for stimulating national income. Indeed one interesting exercise would be to plot, on a piece of graph paper, the values of the multiplier obtained at various rates of interest, holding the parameter DEL fixed at some value. Plotting the multiplier on the vertical axis and the interest rate on the horizontal, you thus obtain a scatter of points through which you could fit a smooth curve. Repeating the process on the same piece of graph paper, but with other values for the parameter DEL, you finish up with several curves, each one corresponding to a single value for the parameter DEL. If you can then account for the shape of these curves and their relation to one another, you will have learned quite a lot about the way that monetary policy is supposed to work, at least in simple Keynesian models.

```
(CONSULT YOUR WORKBOOK FOR SUGGESTED EXERCISES)

INITIAL PARAMETER VALUES:
   CA  =   15.000     IA  =   12.000
   B1  =    0.600

DO YOU WISH TO CHANGE ANY INITIAL VALUES ?
YNO

PLEASE TYPE EITHER YES OR NO
NO

        INITIAL EQUILIBRIUM
        GROSS NATIONAL EXPENDITURE
-----------------------------------------
CONSUMPTION                 (PCE)     55.5
GOVERNMENT EXPENDITURE (GPGS)          0.0
INVESTMENT                            12.0
EXPORTS                                0.0
LESS IMPORTS                           0.0
                                  --------
GROSS NATIONAL EXPENDITURE            67.5

PARAMETERS OK ?
YES

INITIAL INSTRUMENT VALUE:
  IA  =   12.000

WHAT NEW VALUE DO YOU WISH FOR THE INSTRUMENT ?
(TYPE: 'PARAM = VALUE', UNTIL 'DONE')
IA = 10.

DONE

        IMPACT DISEQUILIBRIUM
        GROSS NATIONAL EXPENDITURE
-----------------------------------------
CONSUMPTION                 (PCE)     55.5
GOVERNMENT EXPENDITURE (GPGS)          0.0
INVESTMENT                            10.0
EXPORTS                                0.0
LESS IMPORTS                           0.0
                                  --------
GROSS NATIONAL EXPENDITURE            65.5

DO YOU WISH TO SEE THE ADJUSTMENT PROCESS ?
YES

     *  10.0      12.0 *  52.5      55.5 *  62.5     67.5 *
   I *  INVESTMENT     *  CONSUMPTION     *  EXPENDITURE    *  MULT
------------------------------------------------------------------------
1969 *               I *                C *               Y*    *
1970 *               I *                C *               Y*    *
1971 * I               *                C *        Y       *   1.000
1972 * I               *       C          *     Y          *   1.600
1973 * I               *     C            *   Y            *   1.960
1974 * I               *    C             *  Y             *   2.176
1975 * I               *   C              * Y              *   2.306
1976 * I               *  C               *Y               *   2.383
1977 * I               * C                *Y               *   2.430
1978 * I               *C                 *Y               *   2.458
THE SYSTEM CONVERGES ASYMPTOTICALLY T...

        FINAL EQUILIBRIUM
        GROSS NATIONAL EXPENDITURE
-----------------------------------------
CONSUMPTION                 (PCE)     52.5
GOVERNMENT EXPENDITURE (GPGS)          [illegible]
INVESTMENT                            10.0
EXPORTS                                [illegible]
LESS IMPORTS                           [illegible]
                                  --------
GROSS NATIONAL EXPENDITURE            [illegible]
```

RECALL FROM WORKBOOK:

$C_t = CA + B1 \cdot Y_{t-1}$

$I_t = IA$

$Y_t = C_t + I_t$

At this point you may change the structure of the model — the closed parameters — to suit yourself.

$\bar{C} = C_0$ OR C_{1970}

$\bar{I} = I_0$ OR $\bar{I}_{1970}$

$\bar{Y} = Y_0$ OR Y_{1970}

If for some reason the initial equilibrium is implausible or unsuitable, you may adjust it here. Program will then print the corresponding new equilibrium.

Program indicates available instruments and initial values for each.

Input your desired value for the instrument variable.

C_{1971}

I_{1971}

Y_{1971}

Program then traces the first impact of the change to the new value for the instrument variable ...

and the resulting adjustment process ...

$\bar{C}$, $\bar{I}$, $\bar{Y}$

and finally — if appropriate — the new equilibrium values. You may then return to alter the instrument variable as many times as you like, keeping the structure of the model unchanged, or you may ...

APPENDIX 5

DEMAND MODELS 4 AND 5: GOVERNMENT TAX AND EXPENDITURE POLICY

1. The Model in General (A5)

$$C_t = CA + B1\ (\hat{Y}_t - \hat{T}_t) \quad [5.1]$$

$$I_t = IA + B2 \cdot \hat{Y}_t \quad [5.2]$$

$$G_t = \overline{\overline{GA}} \quad [5.3]$$

$$T_t = \overline{\overline{TA}} + (\overline{\overline{B4}}/100)\ \hat{Y}_t \quad (\text{N.B. } \overline{\overline{B4}} = 0 \text{ in A4}) \quad [5.4]$$

$$Y_t = C_t + I_t + G_t \quad [5.5]$$

$$\hat{Y}_t = Y_{t-1} \quad [5.6]$$

$$\hat{T}_t = T_{t-1} \quad [5.7]$$

Model A3 was concerned with simple types of fiscal and monetary policy. In model A5 we assume away monetary policy, but we give a more realistic content to fiscal policy by introducing taxes into the model. The point is that government expenditure is a useful and powerful instrument of economic policy, but these expenditures must be financed in some way: issuing bonds, printing new dollars,or collecting taxes, or any combination of these.

Since there was no tax revenue in model A3, we implicitly assumed that government expenditure was financed by issuing bonds, printing paper-money, or both; implicit also, was the heroic assumption that these financing means have no effect whatsoever on the real sector of the economy described by our model. Model A4 is more realistic in allowing government to finance its expenditure with tax collections, although we still have to assume that any deficit (expenditure larger than tax revenue) is financed by bonds or new money, that surplus (tax revenue > expenditure) is disposed of by buying bonds or destroying money, and that these have no effect on our model. (You should realise, now, why the introduction of a money market is important in any realistic macroeconomic policy model.)

Notice that since we calculate the government surplus or deficit at anytime t as the difference between its level of expenditure (G_t) and its tax revenue (T_t), i.e.

$$D_t = T_t - G_t \qquad [5.8]$$

where $D_t < 0$ in case of a deficit and $D_t > 0$ for a surplus, G_t must include all kinds of government expenditures on both capital formation and current purchases of goods and services. I_t just represents, therefore, private investment expenditure.

The consumption function (eq. [5.1]) is now slightly different from the one you are used to: consumption at time t is still made up of an autonomous term CA, but is now a function of expected disposable income (expected income, $\hat{Y}_t$, minus expected tax payments, $\hat{T}_t$), and the propensity to consume B1 must now be interpreted as the propensity to consume out of expected disposable income. You may expect B1 > B, i.e. the propensity to consume out of disposable income greater than the propensity to consume out of national income. Taxes are assumed to be the burden of final consumers only in this model, so that their level has no influence on firms' investment expenditures.

The investment function (eq. [5.2]) is also different. As in models A2 and A3, investment will usually vary directly with the level of expected income: for a $1 increase in expected income, investment outlays will increase by B2 dollars, if they change at all ($B2 \geq 0$); investment is related to the rate of interest, monetary policy being ignored in model A4. And like in model A1, investment has an autonomous component, i.e. even if $\hat{Y}_t = 0$ or $B2 = 0$, an amount of IA dollars will be spent in any case on investment projects; here, however, IA cannot be used as a policy instrument like $\overline{\overline{IA}}$ in model A1.

While the level of government expenditure is assumed to be given by considerations outside this model, i.e. is exogeneously fixed at a certain level $\overline{\overline{GA}}$ (cf. eq. [5.3]), the level of taxes is made of both an exogeneously given autonomous component $\overline{\overline{TA}}$ and an endogeneous term $(\overline{\overline{B4}}/100)\ \hat{Y}_t$ where $(\overline{\overline{B4}}/100)$ is an exogeneously given tax rate say (8/100) = 8% (cf. eq. [5.4]). In the program for Demand Model 5, the level of government expenditure, $\overline{\overline{GA}}$ the autonomou component of tax revenue, $\overline{\overline{TA}}$, and the tax rate $\overline{\overline{B4}}$ are the three policy instruments of the model. (In the program for Demand Model 4, $\overline{\overline{B4}} = 0$ and $\overline{\overline{GA}}$ and only $\overline{\overline{TA}}$ can be used as policy instruments.

Expected income is equal to realized income of last period and the tax payment expected by consumers is the same as the actual tax payment of the previous period; these two expectations hypothesis are given by eq. [5.6] and [5.7]. eq. [5.5] is the usual flow equilibrium condition.

2. Comparative Statics and Dynamics of the Model in the General Case (A5)

Substituting eg. [5.3] into [5.5], [5.6] into [5.1] and [5.2], [5.7] and [5.4] into [5.1], we get a system of 3 equations in 3 endogeneous variables, C_t, I_t and Y_t:

$$C_t = CA + B1\ [(1 - \overline{\overline{B4}}/100)\ Y_{t-1} - \overline{\overline{TA}}] \qquad [5.9]$$

$$I_t = IA + B2 \cdot Y_{t-1} \qquad [5.10]$$

$$Y_t = C_t + I_t + \overline{\overline{GA}} \qquad [5.11]$$

where $\overline{\overline{B4}}$, $\overline{T}$ and $\overline{\overline{GA}}$ are exogeneously fixed policy instruments. For a \$1 increment in national income of last year, consumption will increase by $B1(1 - \overline{\overline{B4}}/100)$ dollars and investment by B2 dollars this year; the total propensity to spend out of national income is therefore $\overline{S} = [B1(1 - \overline{\overline{B4}}/100) + B2]$, given a tax rate $\overline{\overline{B4}}$

The dynamics of the model can be analysed with the difference equation of the system. Substituting eq. [5.9] and [5.10] into [5.11], we get:

$$Y_t - \overline{S}.Y_{t-1} = CA + IA + \overline{\overline{GA}} - B1.\overline{\overline{TA}} = \overline{\overline{AE}} \qquad [5.12]$$

the solution of which is

$$Y_t = \bar{Y} + (Y_o - \bar{Y})\,\bar{S}^t \qquad [5.13]$$

if we start from any arbitrary initial level of income Y_o. If we start from an equilibrium situation $\bar{E}$ ($\bar{Y}$, $\bar{C}$, $\bar{I}$, $\bar{T}$, $\overline{B4}$, $\overline{TA}$, $\overline{GA}$) and see what happens if either the tax rate is changed ($\Delta\overline{B4} = \overline{\overline{B4}} - \overline{B4}$), or autonomous tax revenue is increased ($\Delta\overline{TA} = \overline{\overline{TA}} - \overline{TA}$), or government expenditure is altered ($\Delta\overline{GA} = \overline{\overline{GA}} - \overline{GA}$), or if any combination of these three policy changes is used, definining a new (final) equilibrium $\bar{\bar{E}}$($\bar{\bar{Y}}$, $\bar{\bar{C}}$, $\bar{\bar{I}}$, $\bar{\bar{T}}$, $\overline{\overline{B4}}$, $\overline{\overline{TA}}$, $\overline{\overline{GA}}$), eq. [5.13] becomes

$$Y_t = \bar{\bar{Y}} - (\bar{Y} - \bar{\bar{Y}})\,\bar{\bar{S}}^t \qquad [5.14]$$

If $\overline{\overline{B4}} = \overline{B4}$, $\bar{\bar{S}}$ is of course equal to $\bar{S}$. Depending on the value of $\bar{\bar{S}}$, the final equilibrium will be reached (stable) or not (unstable). Notice that, like in models A2 and A3, economic policy may change the dynamic properties of the system by altering the tax rate, thus changing the value of the stability factor from $\bar{S}$ to $\bar{\bar{S}}$; now, however, it is fiscal policy instead of monetary policy that is bound to influence the stability of the system.

The initial equilibrium $\bar{Y}$ is found by setting $Y_{t-1} = Y_t = \hat{Y}_t$ in the system of equations [5.9] to [5.11], and solving the new timeless system.

$$C = CA + B1\,[(1 - \overline{B4}/100)Y - \overline{TA}] \qquad [5.15a]$$

$$I = TA + B2.Y \qquad [5.15b]$$

$$Y = C + I + \overline{GA} \qquad [5.15c]$$

for $Y = \bar{Y}$, $C = \bar{C}$ and $Y = \bar{Y}$ given the initial values $\overline{B4}$, $\overline{TA}$ and $\overline{GA}$ of our policy instruments. The solutions are:

$$\bar{Y} = \frac{CA + IA + \overline{GA} - B1 \cdot \overline{TA}}{1 - \bar{S}} = \frac{\overline{AE}}{1-\bar{S}} \qquad [5.16]$$

$$\bar{C} = \frac{(CA - B1 \cdot \overline{TA})(1 - \bar{S}) + B1(1 - \overline{B4})(\overline{AE})}{1 - \bar{S}} \qquad [5.17]$$

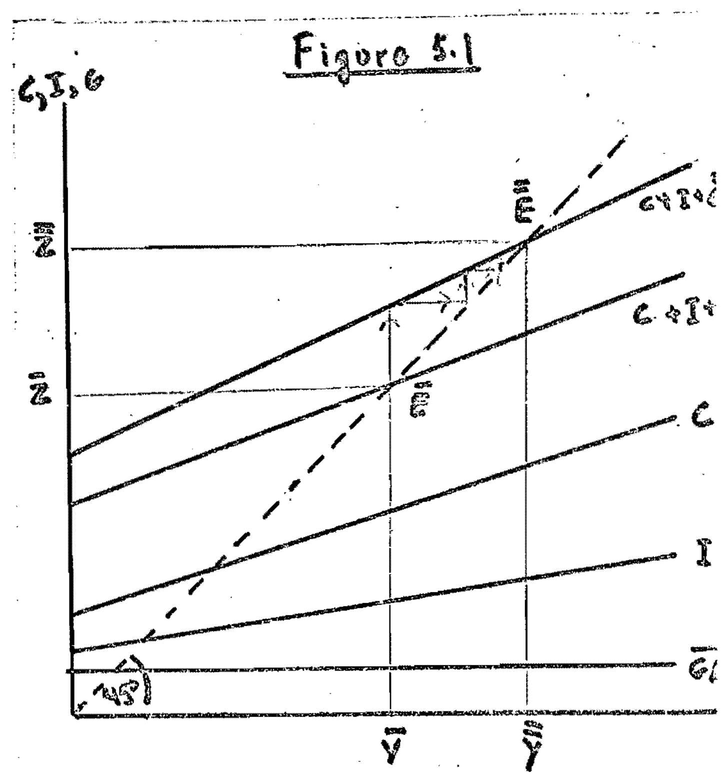
$$\bar{I} = \frac{IA(1 - \bar{S}) + B2 \cdot \overline{AE}}{1 - \bar{S}} \qquad [5.18]$$

Only if $\bar{S}<1$ is the initial equilibrium defined ($\bar{S} \neq 1$) and meaningful ($\bar{S} < 1$). After a change in policy, a final equilibrium $\bar{\bar{E}}$ is defined (if $\bar{\bar{S}} \neq 1$), where

$$\bar{\bar{Y}} = \frac{CA + IA + \overline{\overline{GA}} - B1 \cdot \overline{\overline{TA}}}{1 - \bar{\bar{S}}} = \frac{\overline{\overline{AE}}}{1-\bar{\bar{S}}} \qquad [5.19]$$

and similarly for $\bar{\bar{C}}$ and $\bar{\bar{I}}$. If only, say, the tax rate has been changed ($\bar{\bar{S}} \neq \bar{S}$), then $\overline{\overline{GA}} = \overline{GA}$ and $\overline{\overline{TA}} = \overline{TA}$. Depending on the value of $\bar{S}$ and $\bar{\bar{S}}$, $\bar{E}$ and $\bar{\bar{E}}$ will be defined or not and stable or not.

The model can be represented graphically in the usual way. On Fig. 5.1, the investment function I [5.15b] is plotted against national income Y, with a vertical intercept equal to IA and a slope of B2; the consumption function C has a vertical intercept equal to (CA - B2·$\overline{TA}$) and a slope B1(1 - $\overline{B4}$/100); the level of government expenditure is not dependent on the level of national income and is constant at $\overline{GA}$. Adding vertically these three schedules, you get the (C + I + $\overline{GA}$) line (slope equal to $\bar{S}$) showing aggregate demand as a function of Y. The intersection of this curve



with the 45° line define an equilibrium $\bar{E}$. If $\bar{\bar{GA}}$ and/or $\bar{\bar{TA}}$ and/or $\bar{\bar{B4}}$ is altered, the $(C + I + \bar{\bar{GA}})$ line shifts to $(C + I + \bar{\bar{\bar{GA}}})$, defining a final equilibrium $\bar{\bar{E}}$; a possible $(C + I + \bar{\bar{\bar{GA}}})$ line is drawn on Fig. 5.1 for the sake of illustration. Since $0 < \bar{\bar{S}} < 1$ in this case, you can show that $\bar{\bar{E}}$ is stable and will be approached asymptotically following changes in policy instruments.

If you start from a meaningful $\bar{E}$ $(\bar{S} < 1)$ and if $\bar{\bar{E}}$ is stable $(-1 < \bar{\bar{S}} < 1)$, the following equilibrium multipliers are defined: the equilibrium multiplier of income on government expenditure.

$$EM_{YG} = \left[\frac{\Delta\bar{Y}}{\Delta\bar{\bar{GA}}}\right]_{TA,\ B4\ constant} = \left[\frac{\bar{\bar{Y}}-\bar{Y}}{\bar{\bar{\bar{GA}}}-\bar{\bar{GA}}}\right]_{TA,\ B4\ constant}$$

$$= \frac{\partial\bar{Y}}{\partial\bar{\bar{GA}}} = \frac{1}{1 - \bar{S}} ; \qquad [5.20]$$

the multiplier of consumption on government expenditure

$$EM_{CG} = \left[\frac{\Delta C}{\Delta\bar{\bar{GA}}}\right]_{TA,\ B4\ constant} = \frac{\partial\bar{C}}{\partial\bar{\bar{GA}}} = \frac{B1(1-\bar{B4}/100)}{1 - \bar{S}}; \qquad [5.21]$$

the multiplier of investment on government expenditure

$$EM_{IG} = \left[\frac{\Delta\bar{I}}{\Delta\bar{\bar{GA}}}\right]_{TA,\ B4\ constant} = \frac{\partial\bar{I}}{\partial\bar{\bar{GA}}} = \frac{B2}{1 - \bar{S}} ; \qquad [5.22]$$

the multiplier of income on autonomous taxes

$$EM_{YT} = \left[\frac{\Delta\bar{Y}}{\Delta\bar{\bar{TA}}}\right]_{GA,\ B4\ constant} = \frac{\partial\bar{Y}}{\partial\bar{\bar{TA}}} = -\frac{B1}{1 - \bar{S}} ; \qquad [5.23]$$

the multiplier of consumption on autonomous taxes

$$EM_{CT} = \left[\frac{\Delta\bar{C}}{\Delta\bar{\bar{TA}}}\right]_{GA,\ B4\ constant} = \frac{\partial\bar{C}}{\partial\bar{\bar{TA}}} = -\frac{B1(1 - B2)}{1 - \bar{S}} ; \qquad [5.24]$$

the multiplier of investment on autonomous taxes

$$EM_{IT} = \left[\frac{\Delta\bar{I}}{\Delta\overline{TA}}\right]_{GA,\ B4\ constant} = \frac{\partial\bar{I}}{\partial\overline{TA}} = -\frac{B1.B2}{1-\bar{S}} \quad ; \qquad [5.25]$$

the multiplier of income on the tax rate

$$EM_{YB4} = \left[\frac{\Delta\bar{Y}}{\Delta\overline{B4}}\right]_{GA,\ TA\ constant} = \left[\frac{\bar{\bar{Y}} - \bar{Y}}{\bar{\bar{B4}} - \overline{B4}}\right]_{GA,\ IA\ constant}$$

$$= \frac{\overline{AE}\ (B1/100)}{(1-\bar{\bar{S}})(1-\bar{S})} \qquad [5.26]$$

(or in infinitesimal form)

$$\frac{\partial\bar{Y}}{\partial\overline{B4}} = \frac{\overline{AE}(B2/100)}{(1-\bar{S})^2} \quad ; \qquad [5.27]$$

the multiplier of consumption on the tax rate (EM_{CB4}) and the multiplier of investment on the tax rate (EM_{IB4}), - which you should derive as an exercise.

You notice that the multipliers on government expenditure and on autonomous taxes are unique, but that the multipliers on the tax rate (eg. EM_{YB4}) are not: $\partial\bar{Y}/\partial\overline{B4}$ is approximately equal to $\Delta\bar{Y}/\Delta\overline{B4}$ only for very small changes in $\overline{B4}$. The problem is similar to the one we met with the multipliers on the rate of interest in Demand Models 2 and 3.

In both Demand Models 4 and 5, we assume $B2 \geq 0$, i.e. the level of investment either changes directly with the level of national income or does not change at all (I_t = IA). In Demand Model 4, $\overline{B4}$ is always equal to zero, and only autonomous taxes $\overline{TA}$ exist; Demand Model 5 is more general, and we then allow for a tax rate $\overline{B4}$. No restriction is put on the value of $\overline{B4}$, although it is obviously sensible to assume it non-negative.

(Notice that even in $\bar{\bar{B4}} < 0$, tax collections may still be positive, provided $\bar{\bar{TA}}$ is large enough; negative tax collections T_t could be though of as net transfer payments from government to individuals (e.g. welfare payments)). Total autonomous expenditure AE and autonomous investment IA are still assumed positive.

3. Demand Model 4

Program A4 corresponds to our general model (eq. [5.9] to [5.11]) with $\bar{\bar{B4}} = 0$, i.e. with only an autonomous component $\bar{\bar{TA}}$ for tax revenue. This is admittedly not very realistic: Parliament cannot legislate a change in tax revenue; it can only legislate change in tax rates without knowing what this will imply for tax collections (except by making economic analyses similar to what we are now doing). The model therefore becomes (Demand Model 4):

$$C_t = CA + B1\ (Y_{t-1} - \bar{\bar{TA}}) \qquad [5.28a]$$

$$I_t = IA + B2\ Y_{t-1} \qquad [5.29a]$$

$$Y_t = C_t + I_t + \bar{\bar{GA}} \qquad [5.30a]$$

The equilibrium values $\bar{Y}$, $\bar{C}$ and $\bar{I}$ can be found by setting $\bar{\bar{B4}} = 0$ in eq. [5.16], [5.17], [5.18] and [5.19]. Notice that a zero tax rate implies $\bar{S} = \bar{\bar{S}} = S = B1 + B2$, and the stability factor S cannot be changed by economic policy:

$$\bar{Y} = \frac{\bar{\bar{AE}}}{1 - S} \qquad [5.31]$$

Since we are left with only two policy instruments ($\bar{\bar{GA}}$, $\bar{\bar{TA}}$), only equilibrium multipliers on government expenditure and on autonomous taxes can be computed: these are given by setting $\bar{S} = S = (B1 + B2)$ in eq. [5.20] to [5.25]. Although much more straightforward, the comparative statics and dynamics of model A4 are basically the same as for the general case of model A5.

4. The Balanced Budget Multiplier in Model A4

When there is no tax rate, as in model A4, we can easily compute what is called a balanced budget multiplier: it indicates the change in an endogeneous variable ($C = \bar{C}$, $I = \bar{I}$, $Y = \bar{Y}$) brought about by a simultaneous \$1 increase in both $\bar{\bar{TA}}$ and $\bar{\bar{GA}}$. Three balanced-budget multipliers are defined in model A4: the balanced-budget multiplier on income (BBM_Y), the balanced-budget multiplier on consumption (BBM_C) and the balanced-budget multiplier on investment (BBM_I).

Balanced-budget multipliers can be either delayed or equilibrium multipliers, but we will only consider equilibrium BBM's here. The BBM_Y, for example, is defined as

$$BBM_Y = \left[\frac{\Delta\bar{Y}}{\Delta\bar{\bar{GA}}} \right]_{\Delta\bar{\bar{GA}} = \Delta\bar{\bar{TA}}} \qquad [5.32]$$

You should realize that a balanced-budget multiplier does not necessarily refer to a balanced budget! A balanced budget usually means $G_t = T_t$ and $D_t = (T_t - G_t) = 0$, i.e. government deficit is nil, there is no surplus or deficit. A balanced-budget multiplier does not imply no surplus or deficit, but only means that the deficit or surplus is kept constant from an initial equilibrium $\bar{E}$ to a final one $\bar{\bar{E}}$: $(\bar{TA} - \bar{GA}) = (\bar{\bar{TA}} - \bar{\bar{GA}})$. A more proper name for the balanced-budget multiplier would be a "constant-deficit multiplier". We shall, however, stick to the existing (and confusing) terminology by talking of "balanced-budget" multiplier. We only consider the case of the balanced-budget equilibrium multiplier on income (BBM_Y) in the following discussion.

Let's evaluate expression [5.32]. In general (no "ceteris paribus" assumption),

$$\frac{\Delta\bar{Y}}{\Delta\bar{GA}} = \frac{\bar{\bar{Y}} - \bar{Y}}{\bar{\bar{GA}} - \bar{GA}} = \frac{(\bar{\bar{GA}} - \bar{GA}) - B1\,(\bar{\bar{TA}} - \bar{TA})}{(1 - S)(\bar{\bar{GA}} - \bar{GA})} \qquad [5.33]$$

where, of course, $S = B1 + B2$; this result is derived simply by subtracting $\bar{Y}$ from $\bar{\bar{Y}}$ (cf eq. [5.31], [5.16] and [5.19]) and dividing by $(\bar{\bar{GA}} - \bar{GA})$ Eq. [5.33] indicates the change in equilibrium income following a \$1 change in government expenditure <u>and</u> any change in taxes $(\bar{\bar{TA}} - \bar{TA})$; in this case, as we expected, $\Delta Y/\Delta GA$ is not unique, but depends on the magnitude of both $(\bar{\bar{GA}} - \bar{GA})$ and $(\bar{\bar{TA}} - \bar{TA})$. If we keep taxes constant i.e. $(\bar{\bar{TA}} - \bar{TA}) = 0$, then, of course, eq. [5.33] reduces to $1/(1-S)$, which is the value of $EM_{YG} = \partial\bar{Y}/\partial\bar{GA}$ (taxes constant). Now, if taxes are increased (or decreased) by the same amount by which government expenditure is increased (or decreased), i.e. $(\bar{\bar{GA}} - \bar{GA}) = (\bar{\bar{TA}} - \bar{TA})$, expression [5.33] becomes

$$\left[\frac{\Delta\bar{Y}}{\Delta\overline{GA}}\right]_{\Delta\overline{GA} = \Delta\overline{TA}} = \frac{1 - B1}{1 - S} = \frac{1 - B1}{1-B1-B2} \qquad [5.34]$$

which is the value of BBM_Y, i.e. the change in equilibrium income brought about by a \$1 increase in both $\overline{TA}$ and $\overline{GA}$.

If investment is completely autonomous, i.e. $B2 = 0$ and $I_t = IA$, eq. [5.34] implies $BBM_Y = 1$: a change of both tax revenue and government expenditure by exactly the same amount (whatever the amount) will change the equilibrium level of income by exactly the increase in government expenditure. In other words, $B2 = 0$ implies $\Delta\bar{Y} = \Delta\overline{GA}$ when $\Delta\overline{TA} = \Delta\overline{GA}$. This is the famous so-called balanced-budget theorem in macroeconomics: an increase (or decrease) of taxes and government expenditure by the same amount, so as to keep the government surplus or deficit constant, is not neutral, but will increase (decrease) national income by the amount of the increase (decrease) in expenditure; in case of an increase of taxes and expenditure, a $\Delta\overline{GA} = \Delta\overline{TA}$ may therefore be inflationary.

It is important to observe, however, that if investment is sensitive to the level of income ($B2 > 0$), the balanced-budget multiplier on income will be greater than 1, because a positive B2 reduces the denominator of $(1-B1)/(1-S)$ in eq. [5.34]. An increase (decrease) in both government expenditure and taxes by the same amount will still be inflationary (deflationary), but by an amount now greater than $\Delta\overline{GA} = \Delta\overline{TA}$. The balanced budget multiplier is only equal to one when investment is insensitive to the level of national income.

This non-neutral character of a balanced increase of taxes and government expenditure is not intuitively immediately obvious: we would expect it to bring no change in $\bar{Y}$. But consider the phenomenon in the following way. Suppose income is at an equilibrium level $\bar{Y}$ in 1970 and you increase $\bar{GA}$ to $\bar{\bar{GA}}$ and $\bar{TA}$ to $\bar{\bar{TA}}$ from 1971 on. National income will be $\bar{Y} + (\bar{\bar{GA}} - \bar{GA})$ in 1971, because consumption will not have decreased yet $(C_{1971} = C_{1970} = \bar{C})$, being a function of expected income $(\hat{Y}_{1971} = Y_{1970} = \bar{Y})$; the increase of government expenditure immediately adds its full amount to the flow of income, while the tax increase does not immediately begins to subtract anything from national income. In 1972, the tax increase appears in consumption: $C_{1972} = CA + B1\ (Y_{1971} - \bar{\bar{TA}})$, but is cancelled by the fact that $Y_{1971} = Y_{1970} + \Delta\bar{GA}$, i.e. $C_{1972} = CA + B1\ (Y_{1970} - \bar{TA}) = C_{1971} \ldots$ Tax increases take more time to influence the flow of income than a corresponding increase in government expenditure, and by following the process of adjustment in 1973, 1974,...., you will observe that $BBM_Y \geq 1$.

As any simple equilibrium multiplier, a balanced-budget multiplier is only defined if you specify a $\Delta\bar{GA} = \Delta\bar{TA} \neq 0$ and if the final equilibrium $\bar{\bar{Y}}$ is stable, i.e. $-1 < S < 1$.

5. Policy Targets in Demand Model 4

Recall that, in general, two policy instruments enable you to hit two targets. As it was the case for model A3, however, two targets are inconsistent: instead of the combination $(\bar{\bar{Y}}^*, \bar{\bar{C}}^*)$, it is now the double target $(\bar{\bar{Y}}^*, \bar{\bar{I}}^*)$ that is in general impossible to hit. This is so because the investment function (cf. eq. [5.29b]) shows I as a function of Y only (IA and B2 are fixed parameters), so that no policy instrument can adjust in order that eq. [5.29b] be consistent with a given prescribed $\bar{\bar{Y}}^*$ and $\bar{\bar{I}}^*$. The double targets $(\bar{\bar{Y}}^*, \bar{\bar{C}}^*)$ and $(\bar{\bar{I}}^*, \bar{\bar{C}}^*)$ could however be set and reached by finding the appropriate values of $\bar{\bar{TA}}$ and $\bar{\bar{GA}}$.

The program, anyway, just allows you to set a target $\bar{\bar{Y}}^*$, starting from any initial equilibrium $\bar{Y} = Y_{1970}$, but you are able to aim at it with different policy instruments. Obviously, any target can be reached only if $-1 < S < 1$, i.e. if equilibrium is stable.

First, you can use only government expenditure to reach $\bar{\bar{Y}}^*$. Recalling from eq. [5.20] and our discussion of section 3 that the equilibrium multiplier of income on government expenditure (EM_{YG}) is equal to $1/(1-S)$ where $S = B1 + B2$, you can find the necessary change in government expenditure by solving for $\Delta\bar{\bar{GA}} = \bar{\bar{\bar{GA}}} - \bar{\bar{GA}}$ the following expression:

$$\bar{\bar{\bar{GA}}} - \bar{\bar{GA}} = \frac{\bar{\bar{Y}}^* - \bar{Y}}{EM_{YG}} \qquad [5.35]$$

Secondly, you can use only taxation to reach your target, keeping government expenditure constant at its initial level $\bar{\bar{GA}}$. Recalling that $EM_{YT} = -B2/(1-S)$, you find the required change in taxes as

$$\bar{\bar{\bar{TA}}} - \bar{\bar{TA}} = \frac{\bar{\bar{Y}}^* - \bar{Y}}{EM_{YT}} \qquad [5.36]$$

As a third way of reaching $\bar{\bar{Y}}^*$, you can make use of our discussion of the balanced-budget multiplier. In order to find the required equal change in both government expenditure and autonomous tax revenue, you calculate:

$$\bar{\bar{\bar{TA}}} - \bar{\bar{GA}} = \bar{\bar{\bar{GA}}} - \bar{\bar{GA}} = \frac{\bar{\bar{Y}}^* - \bar{Y}}{BBM_Y} \qquad [5.37]$$

If $B2 = 0$, you know that $BBM_Y = 1$, so that the required change in $\bar{\bar{GA}}$ and $\bar{\bar{TA}}$ will be equal to the required target change $\Delta\bar{Y}^* = \bar{\bar{Y}}^* - \bar{Y}$. In the general case when $B2 > 0$, $\Delta\bar{\bar{GA}} = \Delta\bar{\bar{TA}}$ will be smaller than $\Delta\bar{Y}^*$, since $BBM_Y > 1$ in this case.

Finally, you can try to reach $\bar{\bar{Y}}^*$ by changing both $\bar{\bar{GA}}$ and $\bar{\bar{TA}}$, but by different amounts. Using two instruments to reach a single target, you will observe that there is an infinite number of feasible combinations ($\Delta\bar{\bar{GA}}$, $\Delta\bar{\bar{TA}}$) required to attain $\bar{\bar{Y}}^*$. (In the balanced-budget case, $\bar{\bar{GA}}$ and $\bar{\bar{TA}}$ were changing together in such a way that $\Delta\bar{\bar{GA}} = \Delta\bar{\bar{TA}}$ always; ($\bar{\bar{CA}} - \bar{\bar{TA}}$) is then constant and should be considered as only one instrument.) This can be seen by multiplying both sides of eq. [5.33] by ($\bar{\bar{\bar{GA}}} - \bar{\bar{GA}}$) and prescribing a target value of $\bar{\bar{Y}}^*$ to $\bar{\bar{Y}}$; we get:

$$\bar{\bar{Y}}^* - \bar{Y} = \frac{(\bar{\bar{\bar{GA}}} - \bar{\bar{GA}}) - B1(\bar{\bar{\bar{TA}}} - \bar{\bar{TA}})}{(1 - S)} \qquad [5.38]$$

Expressed explicitly in terms of the required $\Delta\bar{\bar{GA}}$, eq. [5.38] yields:

$$\bar{\bar{\bar{GA}}} - \bar{\bar{GA}} = \frac{\bar{\bar{Y}}^* - \bar{Y}}{1 - S} + B1(\bar{\bar{\bar{TA}}} - \bar{\bar{TA}}) \qquad [5.38a]$$

For any given change in the level of taxes ($\bar{\bar{\bar{TA}}} - \bar{\bar{TA}}$), there exists a new change in government expenditure ($\bar{\bar{\bar{GA}}} - \bar{\bar{GA}}$) consistent with $\bar{\bar{Y}}^*$; an infinite number of combinations ($\Delta\bar{\bar{GA}}$, $\Delta\bar{\bar{TA}}$) are therefore possible.

By trying all these different policies to reach a certain income target, you will observe that although it is always technically possible to reach any $\bar{\bar{Y}}^*$, the distribution of national income between consumption, investment and government expenditure differs with the use of different economic policies. The level of consumption expenditure and of personal disposable income will be higher at $\bar{\bar{Y}}^*$, for example, when only a decrease in taxes has been used, than when only an increase of government expenditure has been chosen. When deciding which combination of tax and expenditure policies should be implemented, policy makers should always keep this in mind.

7. Policy Targets in Model A5

Three policy instruments are available in model A5, the ones of model A4 plus the tax rate $\overline{B4}$. You can check and should be able to explain why three targets $\bar{\bar{Y}}^*$, $\bar{\bar{C}}^*$ and $\bar{\bar{I}}^*$ are not feasible. (Hint: check from eq. [5.10] that $\bar{\bar{Y}}^*$ and $\bar{\bar{I}}^*$ will be inconsistent in general.) There is an infinite number of combinations ($\Delta\overline{GA}$, $\Delta\overline{TA}$, $\Delta\overline{B4}$) possible to hit a double target.

The program asks you to specify your target $\bar{\bar{Y}}^*$, and allows you to use other expenditure policy only, manipulation of autonomous taxes only or manipulation of the tax rate only to reach it. The multipliers defined by eq. [5.20], [5.23] and [5.26] enable you to find the required policy change without having to go through a trial-and-error process. Since EM_{YB4} is not unique, however, finding the required $\overline{\overline{B4}}$ involves solving system [5.15] or evaluating the complex expression given in Appendix 3; experiments with the simulation program (using as a starting point the approximative value of EM_{YB4} given by $\partial\bar{Y}/\partial\overline{B4}$) may be very useful in this case. Notice that an alteration of the tax rate from $\overline{B4}$ to $\overline{\overline{B4}}$ may change the stability properties of the equilibrium.

You can also try to attain $\bar{\bar{Y}}^*$ by any combination of the three available policies, but an infinite number of those combinations exist. You will observe once again that the composition of $\bar{\bar{Y}}^*$ depends on what policies are used.

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Toronto
Summer 70

DEMAND MODEL 6:

1. The Model

The primary lesson of Demand Model 6 involves import leakages and their impact on the multiplier process. The new feature of this model as compared with the five previous versions is the introduction of foreign trade and a concern for the balance of payments impacts of various policies. In order to focus discussion on questions of trade flows, we drop some of the detail in the previous investment equations; in particular, we drop discussion of the interest rate and its effect. In its place we introduce an equation describing expenditures by the domestic economy on imported goods and services, and expenditures by the rest of the world on goods and services produced in the domestic economy for export.

The export equation is very crude, but until price levels, inflation, and exchange rates are introduced into the discussion it is difficult to specify a satisfactory equation for export sales. For the present we suppose that the demand of foreigners for domestically produced goods can be described as a linear function of domestic output, in the following

$$X_t = XA + B3 \cdot Y_t$$

(Setting the parameter B3 to zero then expresses the assumption that export sales are fixed at a constant level set from outside the model.) The export demand so estimated forms a part of the aggregate demand for domestically produced output.

A more satisfactory equation can be estimated to describe the demand for imported goods. From input-output analyses, it is possible to compute import coefficients showing what fraction of each of the components of national expenditure is directed toward goods or services produced outside the country. The model permits you to specify these import propensities or import coefficients, one for each of the expenditure components (consumption, investment, government purchases, exports). The resulting demand for imported goods of course represents a reduction in demand for domestic output. The higher is the level of imports, the lower is aggregate demand.

A key point here is that the level of demand for imports may depend not only on the level of aggregate output, but also upon its composition. When we introduce imports in this way, therefore, the impact on demand of a dollar spent by the government may be different from the impact of a dollar spent by consumers (simply because different fractions of the two dollars may flow orders to foreign suppliers).

Once we have introduced foreign transactions, we raise the question of the general balance of trade, and possible deficits or surpluses in the balance of trade. The present model takes account of movements only in goods and services, and ignores capital flows and other movements in financial instruments.

The program thus assumes that any deficit on the current account of the balance of payments has to be financed by movements of official reserves - gold or dollars. When imports exceed exports, gold or dollars must flow out of the home country; when export earnings exceed payments for imports, gold or dollars flow into the home country and increase holdings of official reserves.

Apart from these additional equations and tables describing foreign trade, Demand Model 6 is identical in structure and operation to Demand Models 4 and 5.

2. Detailed Equation Structure

$C_t = CA \quad B1 \cdot \hat{D}_y$	consumption function
$I_t = \overline{IA}$	investment function
$G_t = \overline{GA}$	government expenditure function
$X_t = XA \quad B3 \cdot Y_{t-1}$	export function
$M_t = MA + E1 \cdot C_t + E2 \cdot I_t + E3 \cdot G_t + E_4 \cdot X_t$	import function
$Y_t = C_t + I_t + G_t + X_t - M_t$	national expenditure identity
$T_t = TA + B4 \cdot Y_t$	government tax function
$\hat{D}_t = Y_{t-1} - T_{t-1}$	disposable income expectations

Parameters, standard initial values, and units:

$CA =$	8.	Autonomous consumption (billions \$/year)
$B1 =$	.7	Marginal propensity to consume out of expected income (fraction)
$\overline{IA} =$	8.	Autonomous investment (billions \$/year)

$\overline{GA}$ =	12.	Autonomous government expenditure (billions $/year)
XA =	2.	Autonomous export expenditure demand (billions $/year)
B3 =	.07	Propensity to export out of last year's income (fraction)
MA =	0.	Autonomous import expenditure (billions $/year)
E1 =	.1	Marginal propensity to import for consumption
E2 =	.3	Marginal propensity to import for investment
E3 =	.05	Marginal propensity to import for government expenditure
E4 =	.05	Marginal propensity to import for exports
TA =	4.	Autonomous government expenditure (billions $/year)
B4 =	.1	Government tax rate (fraction)

Restrictions:

$$\left[(1-E1)(CA-B1\cdot TA) + (1-E2)\cdot\overline{IA} + (1-E3)\cdot\overline{GA} + (1-E4)XA - MA\right] > 0$$

Instruments:

$\overline{IA}$, $\overline{GA}$

Restrictions:

$$\left[(1-E1)(CA - B1\cdot TA) + (1-E2)\ \overline{\overline{IA}} + (1-E3)\cdot\overline{\overline{GA}} + (1-E4)\cdot XA-MA\right] > 0$$

$$|\overline{IA} - \overline{\overline{IA}}| + |\overline{GA} - \overline{\overline{GA}}| > 0$$

3. Running the Programs

This program operates in the same fashion as the earlier Demand models. To focus on the multiplier process, and the effect of import leakages on the size of investment or government expenditure multipliers, the program treats these two expenditure components as the only available instruments. (You may still explore consequences of tax policy by varying parameters TA and B4 if you wish, however.) In addition to the usual table of gross national expenditure, the program prints a summary balance of payments statement as well. Thus you are able to observe some consequences of domestic fiscal policy upon the foreign balance, and you can thus see how some aspects of general balance of payments problems arise.

The program permits you to alter either of these two fiscal instruments, follows the resulting adjustment process and graphs this if you wish, and computes certain relevant multipliers. You may attempt as many multiplier exercises, or as many runs with different instruments or parameter valves, as you wish.

DEMAND MODEL 6 — FOREIGN SECTOR

```
   [illegible] = [illegible]     [illegible] = [illegible]
   [illegible] = 0.700           [illegible] = [illegible]
   [illegible] = [illegible]     [illegible] = [illegible]
   [illegible] = [illegible]     [illegible] = [illegible]
   [illegible] = [illegible]     [illegible] = [illegible]
   [illegible] = 0.100           [illegible]A = 0.000
   XA = 2.000

DO YOU WISH TO CHANGE ANY INITIAL VALUES ?
NO


      INITIAL EQUILIBRIUM
      GROSS NATIONAL EXPENDITURE
--------------------------------------------
CONSUMPTION                    (PCE)    [illegible]
GOVERNMENT EXPENDITURE (GPGS)           [illegible]
INVESTMENT                              [illegible]
EXPORTS                                 [illegible]
LESS IMPORTS                            [illegible]
                                        --------
GROSS NATIONAL EXPENDITURE              [illegible]

      [illegible]
--------------------------------------------
[illegible]                             [illegible]
[illegible]                             [illegible]
[illegible]                             [illegible]
                                        --------
[illegible]                             -1.[illegible]

PARAMETERS OK ?
YES


INITIAL EXOGENOUS VALUES
   I[illegible] = [illegible]     G[illegible] = [illegible]

[illegible] ?
([illegible] VALUES UNTIL [illegible])
[illegible]

DONE


      IMPACT DISEQUILIBRIUM
      GROSS NATIONAL EXPENDITURE
--------------------------------------------
CONSUMPTION                    (PCE)    [illegible]
GOVERNMENT EXPENDITURE (GPGS)           1[illegible].0
INVESTMENT                              [illegible]
EXPORTS                                 [illegible]
LESS IMPORTS                            [illegible].1
                                        --------
GROSS NATIONAL EXPENDITURE              [illegible].1


DO YOU WISH TO SEE THE ADJUSTMENT PROCESS ?
YES


[illegible] I(T), C(T), [illegible] Y(T) ?
```

```
 T *  INVESTMENT  *  CONSUMPTION  *  EXPENDITURE  *  [illegible]
----------------------------------------------------------------------
1969? * I          *                 *                   *     *
1970? * I          *                 *                   *     *
1971  * I          *                 *             1     *     *
1972  * I          *      C          *         Y         *     *
1973  * I          *          C      *            Y      *     *
1974  * I          *              C  *              Y    *     *
1975  * I          *               C *               Y   *     *
1976  * I          *                C*               Y   *     *
1977  * I          *                C*               Y   *     *
1978  * I          *                 C*               Y  *     *
```

A TABLE OF VALUES ?
YES

T	C(T)	I(T)	G(T)	T(T)?	[illegible]	[illegible]	Y(T)
1	[illegible].7	[illegible]	1[illegible].	1[illegible].	[illegible]	7.[illegible]	[illegible]
2	[illegible].7	.[illegible]	12.[illegible]	1[illegible].	[illegible]	7.[illegible]	[illegible]
3	[illegible].7	[illegible]	1[illegible].	1[illegible].	[illegible]	[illegible]	[illegible].1
4	[illegible].1	.[illegible]	1[illegible].	11.1	5.[illegible]	[illegible]	7[illegible].5
5	[illegible]2.[illegible]	.[illegible]	1[illegible].	11.[illegible]	5.[illegible]	.[illegible]	72.1
6	[illegible]	.[illegible]	1[illegible].	11.[illegible]	7.[illegible]	.[illegible]	[illegible]
7	[illegible]1.	.[illegible]	1[illegible].	11.[illegible]	7.1	.7	[illegible]
8	[illegible]1.[illegible]	.[illegible]	1[illegible].	11.[illegible]	7.[illegible]	.7	[illegible]
9	[illegible]1.	.[illegible]	1[illegible].	11.[illegible]	7.[illegible]	.7	7[illegible]
10	[illegible]2.	.[illegible]	1[illegible].	11.[illegible]	7.[illegible]	[illegible]	7[illegible]
11	[illegible]2.1	.[illegible]	1[illegible].	11.[illegible]	7.[illegible]	[illegible]	7[illegible]
12	[illegible]2.[illegible]	.[illegible]	1[illegible].	11.[illegible]	7.[illegible]	[illegible]	7[illegible]
13	[illegible]2.[illegible]	.[illegible]	1[illegible].	11.[illegible]	7.2	[illegible]	7[illegible].5
14	[illegible]2.[illegible]	.[illegible]	1[illegible].	11.[illegible]	7.2	[illegible]	7[illegible].7

THE SYSTEM CONVERGED ASYMPTOTICALLY TO

FINAL EQUILIBRIUM
GROSS NATIONAL EXPENDITURE

CONSUMPTION	(PCE)	52.[illegible]
GOVERNMENT EXPENDITURE (GPGS)		16.[illegible]
INVESTMENT		[illegible]
EXPORTS		7.2
LESS IMPORTS		[illegible]
GROSS NATIONAL EXPENDITURE		74.7

BALANCE OF PAYMENTS

EXPORTS	7.2
LESS IMPORTS	[illegible]
LONG TERM CAPITAL INFLOW	[illegible]
INCREASE IN OFFICIAL RESERVES	-1.6

MULTIPLIER EXERCISE ?
YES

PLEASE TYPE EITHER YES OR NO
YES

[illegible]

```
[illegible]. I [illegible] TO [illegible] 2.8[illegible] [illegible].
[illegible] IS JUST (1.-[illegible])/(1.-[illegible]).

[illegible]
[illegible] POLICY INSTRUMENTS.

WHAT TARGET LEVEL OF NATIONAL EXPENDITURE WOULD YOU LIKE TO TRY ?
100

TO ATTAIN THIS TARGET STARTING FROM THE INITIAL EQUILIBRIUM
ABOVE, HOW MUCH WOULD YOU HAVE TO CHANGE GOVERNMENT
SPENDING HOLDING INVESTMENT FIXED ?
[illegible]

NO, I [illegible] VALUE TO [illegible] 15.7[illegible].
RECALL THAT THE GOVERNMENT SPENDING MULTIPLIER IS 2.8[illegible].

HOW MUCH WOULD YOU HAVE TO CHANGE INVESTMENT
HOLDING GOVERNMENT SPENDING FIXED ?
1[illegible]

NO, I [illegible] VALUE TO [illegible] 1[illegible].[illegible]71.
RECALL THAT THE INVESTMENT MULTIPLIER IS 1.[illegible]1[illegible].

WOULD YOU LIKE TO TRY ANOTHER TARGET LEVEL OF NATIONAL EXPENDITURE ?
YES

WHAT TARGET LEVEL OF NATIONAL EXPENDITURE WOULD YOU LIKE TO TRY ?
80

TO ATTAIN THIS TARGET STARTING FROM THE INITIAL EQUILIBRIUM
ABOVE, HOW MUCH WOULD YOU HAVE TO CHANGE GOVERNMENT
SPENDING HOLDING INVESTMENT FIXED ?
-10

NO, I [illegible] VALUE TO [illegible] -1[illegible].[illegible]47.
RECALL THAT THE GOVERNMENT SPENDING MULTIPLIER IS 2.8[illegible].

HOW MUCH WOULD YOU HAVE TO CHANGE INVESTMENT
HOLDING GOVERNMENT SPENDING FIXED ?
-12

NO, I [illegible] VALUE TO [illegible] -17.[illegible]7[illegible].
RECALL THAT THE INVESTMENT MULTIPLIER IS 1.[illegible]10.

WOULD YOU LIKE TO TRY ANOTHER TARGET LEVEL OF NATIONAL EXPENDITURE ?
NO

A RUN WITH DIFFERENT CHANGES IN THE INSTRUMENTS ?
YES

INITIAL INSTRUMENT VALUES:
   IN =    [illegible].000    GN =    12.000

WHAT NEW VALUES DO YOU WISH FOR THE INSTRUMENTS ?
(TYPE 'PARAM = VALUE'  UNTIL  'DONE')
IN=15
```

DEMAND MODEL 7:

The previous models have added, step-by-step, detailed behavioural specification on other equations for consumption decisions, investment decisions, government expenditure and tax revenues, export and import decisions. The emphasis in the present model is upon a different issue - individuals' desires as to the cash balances they wish to hold, and the influence of these desires upon central control of interest rates. What the new part of this model attempts to explain is not an expenditure decision, but the determination of the interest rate. Whereas earlier it was simply assumed that the government could set the rate of interest as an instrument of economic policy, now it is recognized that interest rates are set through the trading of financial assets on money markets, and not by government decree, so that individual preferences will affect rates of interest, and may cushion their movements against the influence of economic policy-makers.

The effect of this model is therefore to add a money market model to the structure already developed in the earlier Demand Models. This money market model consists of demand for money equation (an equation describing the size of cash balances desired to be held in the hands of the public), a supply of money (assumed to be set by a central monetary authority), and an assumption that the rate of interest adjusts to bring about balance between desired balances and the existing stock.

The working of the model is therefore identical to that of the previous models, except that instead of the interest rate being taken as an instrument of economic policy, fixed at some desired level, it is determined through the interplay of demand and supply conditions on markets where money trades, not for tangible goods or services, but for other financial assets.

Within this market, for any given value of income, Y, it is assumed that only a single value of the interest rate, R, will bring about equality between money balances demand and money supplies available. The set of all pairs (Y, R) yielding such equality is conventionally called the "LM curve" set of (Y, R) pairs yielding equilibrium on the money market.

At the same time, for any given interest rate, R, there is a unique value of income, Y, which brings about balance between current aggregate demand and past income and expenditure, and so yields equilibrium on the product market. You have already studied the set of all such pairs (Y, R) under the name of the IS curve.

When we bring the equations for these two markets together, we see that there is, generally, only a single pair (Y, R) which brings about balance in both markets simultaneously. Thus, instead of being able (as in the previous models) to choose any arbitrary value of R and find the associated equilibrium level of income, we now find that when monetary conditions are taken into account, the rate of interest is determined within the model, as a reflection of relative supplies of and demands for money.

Equation Structure:

(L - Linear version; NL - non-linear version)

Money Markets	(NL)	$M_t^d = A \cdot \hat{Y}_t / (R_t - RMIN)$	real money demand
	(L)	$M_t^d = MDA + A1 \cdot \hat{Y}_t - A2 \cdot R_t$	
		$M_t^s = \overline{MS}/PA$	real money supply
		$M_t^s = M_t^d$	equilibrium condition
Goods Market		$C_t = CA + B1 \cdot (\hat{Y}_t - \hat{T}_t)$	consumption function
	(NL)	$I_t = (GAM - DEL \cdot R_t) \cdot \hat{Y}_t$	investment function
	(L)	$I_t = IA + B2 \cdot \hat{Y}_t - B3 \cdot R_t$	
		$G_t = \overline{GA}/PA$	real government expenditure
		$Y_t = C_t + I_t + G_t$	national expenditure identity
		$T_t = TA/PA + B4 \cdot Y_t$	real government tax function
		$\hat{Y}_t = Y_{t-1}$	
		$\hat{T}_t = T_{t-1}$	

Parameters, standard initial values, and units:

A	1.5	(constant)
RMIN =	3.	(constant)
MDA =	34.	autonomous real money demand (billions $)
A1 =	.2	coefficient of transactions demand (years)
A2 =	2.5	coefficient of speculative demand (billions $ - year)
PA =	1.	autonomous aggregate price index $\left(\frac{\text{money \$}}{\text{real \$}}\right)$
CA =	8.	autonomous consumption (billions $/year)
B1 =	.7	marginal propensity to consume out of income (fraction)
GAM =	.5	direct marginal propensity to invest (fraction)
DEL =	.05	non-linear propensity to invest as a proportion of interest rate (fraction - year)
IA =	8.	autonomous investment (billions $/year)
B2 =	.1	marginal propensity to invest out of last year's income (fraction)
B3 =	.5	propensity to export out of last year's income (fraction)
$\overline{GA}$ =	12.	autonomous government expenditure (billions $/year)
$\overline{TA}$ =	4.	autonomous government taxes (billions $/year)
$\overline{B4}$ =	.1	government tax rates (fraction)
$\overline{MS}$ =	30.	

Restrictions:

$$0 < \left[(CA+IA+B3\cdot MDA/A2)/PA+\overline{GA}+B3\cdot\overline{MS}/A2-B1\cdot\overline{TA}\right]$$

Linear version:

$$1-B1(1-\overline{B4})-B2 \neq 0; \quad A2 \neq 0; \quad PA \neq 0$$

Non-linear version:

$$PA \neq 0, \ \overline{MS} \neq 0, \ A \neq 0$$

$$(PA\cdot CA + \overline{GA} - B1\cdot\overline{TA}) > 0$$

```
[illegible] MODEL DO YOU WISH TO USE ?
7

MATHEMATICAL MODEL 7. [illegible] FUNCTION
(CONSULT YOUR [illegible] FOR [illegible])

DO YOU WISH TO RUN THE LINEAR VERSION ?
NO

INITIAL PARAMETER VALUES:
   CA   =   [illegible]       PA   =    1.000
   [illegible]   =    0.700       [illegible]    =    1.[illegible]00
   GA   =   12.000       GA[illegible] =    0.500
   TA   =    4.000       [illegible] =    0.[illegible]
   [illegible]   =    0.100       [illegible]=    [illegible]
   MS   =   30.000

DO YOU WISH TO CHANGE ANY INITIAL VALUES ?
N

        INITIAL EQUILIBRIUM
        GROSS NATIONAL EXPENDITURE
-----------------------------------------
CONSUMPTION                    (C)     55.0
GOVERNMENT [illegible] (GROSS)  12.0
INVESTMENT                             12.0
EXPORTS                                 0.0
LESS IMPORTS                            0.0
                                    --------
GROSS NATIONAL EXPENDITURE             79.0

PARAMETERS OK ?
YES

INITIAL INSTRUMENT VALUES:
   GA   =   12.000       [illegible]   =    0.100
   TA   =    4.000       MS   =   30.000

WHAT NEW VALUES DO YOU WISH FOR THE INSTRUMENTS ?
(TYPE 'PARAM = VALUE' UNTIL 'DONE')
MS=60

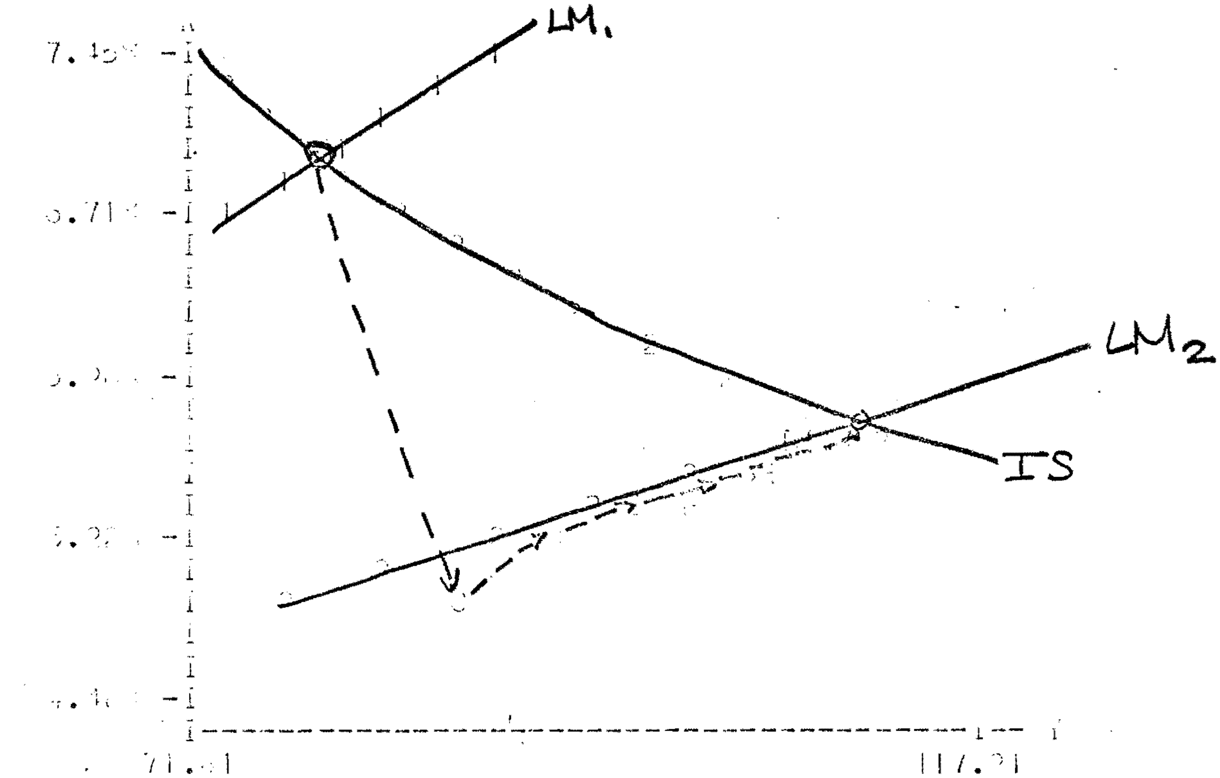
DONE

DO YOU WISH TO SEE A TABLE OF THE ADJUSTMENT PROCESS ?
YES
```

T	C(T)	I(T)	G(T)	[illegible](T)	R(T)	MS(T)	Y(T)
1	55.0	12.0	12.0	11.9	6.95	30.0	79.0
2	55.0	12.0	12.0	11.9	6.95	30.0	79.0
3	55.0	19.9	12.0	12.7	4.90	60.0	86.9
4	59.9	21.0	12.0	13.3	5.17	60.0	92.9
5	63.7	21.7	12.0	13.7	5.32	60.0	97.4
6	[illegible]5.6	22.2	12.0	14.1	5.[illegible]	60.0	100.[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

```
10 *  7[illegible] *  25.1 *  12.[illegible] *  1[illegible].7 *  [illegible] *  50.0 *  107.[illegible]
11 *  72.[illegible] *  25.[illegible] *  12.0 *  14.[illegible] *  5.5[illegible] *  50.0 *  10[illegible]
12 *  73.2 *  25.2 *  12.0 *  14.[illegible] *  5.70 *  50.0 *  108.4
13 *  73.[illegible] *  25.3 *  12.0 *  14.[illegible] *  5.71 *  50.0 *  10[illegible]
14 *  73.7 *  25.[illegible] *  12.[illegible] *  1[illegible] *  5.72 *  50.[illegible] *  10[illegible]

A GRAPH OF IS AND LM CURVES WITH THE ADJUSTMENT PATH ?
YES
```




```
INITIAL IS:  Y = 17.2[illegible]/( 0.05[illegible] + -0.1[illegible])
        LM:  Y = 2[illegible].00*[illegible] +-[illegible].[illegible])
FINAL   IS:  Y = 17.2[illegible]/( 0.05[illegible] + -0.13)
        LM:  Y = 4[illegible].[illegible] +120.0)

     FINAL EQUILIBRIUM
     GROSS NATIONAL EXPENDITURE
-----------------------------------
CONSUMPTION            (PCE)    74.2
GOVERNMENT EXPENDITURE (GPGS)   12.0
INVESTMENT                      [illegible]
LESS IMPORTS                    [illegible]
                              --------
GROSS NATIONAL EXPENDITURE     109.6

A RUN WITH DIFFERENT CHANGES IN THE INSTRUMENTS ?
YES

PLEASE TYPE EITHER YES OR NO
YES


INITIAL INSTRUMENT VALUES:
   G1  =   12.000   B4  =    0.100
   T1  =    4.000   B5  =   [illegible]

WHAT NEW VALUES DO YOU WISH FOR THE INSTRUMENTS ?
(TYPE 'PARAM = VALUE' UNTIL 'DONE')
B5=2[illegible]
```

THE LINK MODELS

Introduction

The three models in LINK bring together the discussion of potential supply (production decisions) illustrated in SUPPLY and the discussion of effective demand (expenditure and portfolio decisions) set out in DEMAND. Thus, we link the two halves of our earlier analysis, in a way which reminds one of the usual discussion of supply and demand in markets for a single consumption good. That is, we introduce the price of output, or the price level, explicitly into the analysis of potential supply and aggregate demand, and ask whether adjustments in this price level can serve to bring effective demand into balance with potential supply. In this way we complete our explanation of how the level of output and the level of resource utilization are determined; in the process we arrive at an explanation of the overall price level, and hence at some understanding of possible processes of price inflation.

To begin, we have to examine the concept of the "price level" briefly. Little difficulty attaches to the notion of the price of a particular commodity. But in macroeconomics we are, it will be recalled, talking about output as a measure of the flow of production of all final goods and services. The "price" attached to this flow is a questionable concept.

Essentially, the construction of a measure of the price level, or price to be attached to aggregate output, boils down to the selection of some bundle of specific commodities, and the creation of a weighted average price obtained by taking the price of each individual commodity, sealing by a fraction reflecting the importance of that commodity in the overall bundle of goods selected, and summing. The process of selecting the particular commodities, obtaining their prices, and establishing the weights to be attached to each, is fundamentally arbitrary; thus the resulting price index is also arbitrary, and its usefulness will depend on the purposes to which one puts it.

This is not the place to pursue such issues in detail. The interested student may consult references on price indexes; here we want only to emphasize that the price level is an arbitrary construct, and deserves little of the mystical or religious fervor it sometimes generates. In all the following, we shall discuss the behaviour of the price level in a simplified context where we will speak of "output" as if it consisted of a single commodity flow, and the "price level" were simply the price quoted (in dollars) for that commodity.

We begin by observing that all our earlier models of supply and demand depended upon the price level p as a parameter. Indeed, the final result of our supply models was an aggregate supply curve showing the output flow Q(P) that suppliers would be willing to produce under the given circumstances, as a function of the price p for the output good, while the results of our demand models could be summarized in an aggregate demand curve Y(p) showing the total final demand Y (measured in units of output per year) that all spending sectors taken together would generate at the quoted price p. It seems plausible that bringing these two curves together in a conventional way would show the output flow and price level at which the decisions of producers and customers are in balance. Then the resulting utilization rate is also determined by comparison of actual factor inputs with available factor supplies.

As with the earlier models, one's analysis will depend in part upon the time-scale of interest. A convenient simplification for expository purposes, adopted in the following discussion of the LINK models, runs as follows. With given demand and supply curves, the only adjustment problem relates to the adjustment of the price level to (or towards) its equilibrium value. We could imagine, therefore, a model based on a short time interval - say a "day" - which took both demand and supply curves as fixed, and simply described the process of price change as producers attempted to alter price quotations in response to observed discrepancies between demand and supply. Presumably the process of adjustment would involve cutting prices if supply were seen to exceed demand, and raising them if demand were to exceed supply; one could study many interesting features of various possible adjustment rules. Let us suppose, however, that the adjustment we adopt is such that over the course of the "day" the price level converges to the equilibrium value which brings aggregate demand and supply into balance.

Altering our time scale slightly, let us consider several "days" in succession. Each day the demand curve and the supply curve are both taken as fixed at the beginning of the day and unchanging throughout the day. Over the course of the day the price is adjusted until at the end it has been brought to its equilibrium level (for the given demand and supply curves). But you will recall that the demand curve itself depends upon the expectations of households or firms as to income prospects, and thus depends upon past values of income. Therefore the demand curve itself shifts as income adjusts during the course of the usual multiplier process we studied in the DEMAND models. Over several days - say over a "month" - these demand shifts might be significant. Thus if we wished to study the impact of the multiplier process upon output prices, we might find it convenient to construct a model, based on a monthly scale, in which the price is assumed to adjust instantly to its equilibrium value whenever the demand and supply curves are specified. (That is, in a model with a time scale based upon a month, it may be acceptable to ignore the details of price fluctuations within any one day.) The supply curve, however, can still be assumed unchanging. Then the adjustment process of interest is that of the (short-run) equilibrium price and output level as the various spending and re-spending decisions of the multiplier process work themselves out, with the demand curve shifting each period as a result.

Finally, if one adopts a perspective based upon a still larger time unit - say a year - a somewhat different structure suggests itself. At the risk of too much simplification, it might be appropriate to assume that over twelve "months" the multiplier process has pretty well worked itself out, so that national income has settled down to its equilibrium level, and the demand curve has therefore stopped shifting. Over the course of a year, however, growth of factor endowments may be significant, so that the aggregate supply curve may shift out. Thus a conceivable model on an annual scale might take price to be always at its equilibrium level (whenever the demand and supply curves are specified), and the demand curve to be the fully adjusted curve representing the equilibrium level of demand after the multiplier process is complete; then the adjustment process of interest is the effect of capacity growth upon equilibrium price and output levels.

Thus, in summary, there is a hierarchy of possible extreme models:

1. Daily model (price adjustment process):

    Supply curve: fixed and unchanging

    Demand curve: fixed and unchanging

    Price level: adjusted toward equilibrium as producers alter quotations.

    In the daily model, then, actual output may change in response to price changes as one moves up or down the fixed demand or supply curves, but the curves themselves do not shift.

2. Monthly model (multiplier process):

    Supply curve: fixed and unchanging

    Demand curve: shifts in response to changing lagged values of income (that is, in response to changing income expectations).

    Price level: assumed to be fully adjusted to bring the (changing) current demand into balance with the (fixed) aggregate supply.

    In the monthly model, therefore, output is always at a short-run equilibrium level because the price is assumed to adjust to the value that brings current demand into balance with supply, but that equilibrium level itself changes as one moves up or down the supply curve in response to shifts in the demand curve.

3. Annual model (growth of factor endowments):

Supply curve: shifts each year due to growth (or decline) of available factors.

Demand curve: assumed fully adjusted to an equilibrium curve

Price level: assumed fully adjusted to equilibrium values.

In the annual model, finally, output is always at a level which could be sustained if supply remained constant (because both price and demand are fully adjusted to a sustainable equilibrium), but the level of output (and hence price) shift over time because potential output changes.

In fact, of course, these models represent polar cases simplified for ease of exposition. Presumably the actual paths of income and price over time reflect a mix of all three processes (and others) going on at once. And it is relatively easy to model even such a mixed process with our simulation programs, although it is hard to analyze the resulting rather complex processes explicitly.

What we do is compromise somewhat. We ignore the actual process of adjustment from a disequilibrium price to an equilibrium value that balances supply and demand; the details of this process are usually reserved for study at the level of the advanced theory of general equilibrium, and are not crucial for the usual issues in macroeconomics.

Link Model 1 therefore represents a "monthly" model, in terms of the labels used above. It begins with a fixed supply, and with income and equilibrium. (Because of possible floors on money wages, there may of course be unemployment even in equilibrium as Supply Model 2 taught us.) In response to any imposed changes in policy, Link Model 1 follows the process of expenditure adjustments which result in shifting the demand curve, ultimately arriving at a sustainable equilibrium price and output level when expenditure adjustments have worked themselves out. (The model is actually a bit more complicated than this because the floor money wage may also get bid up after full employment is reached, but we need not bother with that issue here.)

Link Model 2 takes the foregoing structure and imposes upon it a process of factor growth which shifts the aggregate supply curve. It therefore represents an "annual" model which retains some of the structure of expenditure adjustment. Aggregate demand shifts as a result of policies which alter income, and thus set a multiplier process in motion, but potential output also changes as factor endowments evolve. By trying different values for the policy parameters of this model, one can study how fiscal or monetary policy may have to be structured to ensure full employment of a growing labour force, for example.

One final issue requires study. Both models 1 and 2 suppose that the "daily" process has in fact worked itself out, so that one always has short-run balance between demand and supply. But in fact the dynamics of the situation may be much more complicated: the process of price adjustment proceeds along with adjustment of demand, and may not be complete each period. This means that generally the system may be off both curves - that the realized output and price combination not only may not be at the intersection points of the previous diagrams, but may not lie on either curve. Because this situation is one of continuing (though changing) disequilibrium, there are few guidelines to follow; specification of adjustment processes in disequilibrium is highly arbitrary. Nevertheless, in Link Model 3 we develop one approach consistent with much contemporary discussion of problems of the "tradeoff". Instead of requiring that demand and supply be brought into balance each period, we impose a type of "Phillips-curve", or "tradeoff function" which relates the rate of price inflation to the level of resource utilization - in particular, to the unemployment rate. Although a simple diagramatic story is no longer possible, the model may turn out to be more realistic, and we can still study its behaviour by use of our simulation programs.

With these models, the linking of demand and supply considerations to determine national income and the price level is complete. If one is prepared to assume that price adjustment is fast, Link Model 1 lays out the conventional analysis, and Link Model 2 extends it to a situation of continuing growth in resources. If one believes that price adjustment is very imperfect, and that observed price and output paths represent a moving disequilibrium, Link Model 3 lays out the analysis for one case in which the requirement of demand/supply equilibrium is replaced by a disequilibrium condition linking inflation rates to unemployment. In either case, output and price are determined each period, as was required in the problem we set for ourselves.

With this last change, we take one more step away from specification of a model with equilibrium conditions derived from economic theory, and toward a descriptive dynamic model estimated from statistical observation. The structure of the analysis requires little change - beyond a disaggregation and fleshing out of the various behavioural relations in our model - in order to make the transition of full-blown econometric models such as the Brookings model, the Wharton model, or the MIT/FRB model in the U.S., and TRACE or RDX2 in Canada. But that is another story, which we cannot follow up here.

The main emphasis of these Link models is on policy problems. In earlier models we worked hard to understand the supply side, the production decisions, and the way they may reflect rigidities in labour markets as well as growth in underlying endowments of productive resources. We also worked hard in studying how expenditure decisions are inter-related through the circular flow of income and the multiplier process. It is impossible to keep all these pieces in the head at one time without some computational help, but with such help, we are in a position to study the impact of economic policies in situations where all these individual relationships are at work simultaneously. So the Link Models display a rich variety of possible policy options, some automatic or built-in (endogenous), some prescribed from outside (exogenous). The main pay-off to our long exercise in model-building lies in this opportunity we now have to try out imaginative policies in various mixed and compensating combinations. Try some yourself - experiment with a mix of tight fiscal policy and easy money, or a tax cut coupled with a more active spending response to unemployment. You will find that explaining all the various outcomes requires a close understanding of how various economic variables are related through each of the markets and relationships of our models. And when you have seen enough to become impatient at the limitations of these models, or have become dubious about the theoretical validity or factual accuracy of the relations employed here, you'll be ready to tackle the various working econometric models in use in businesses, governments, and universities around the country. Don't forget that they're all gross over-simplifications of a complex reality - but also don't forget that no one has ever comprehended a complex system except by such simplifications. You are in a position now to assess for yourself the validity and use - and the limitations - of such techniques, and to apply them to the task of bringing about better performance in economic systems.

LINK MODEL 1:

Classical Price Level Adjustment

1. The Model

As noted in the Introduction of this section, the basic idea of this model is simply to link a model of aggregate demand (expenditure decisions) with a model of aggregate supply (production decisions) in a consistant fashion. The traditional linkage mechanism in economics is accomplished through a price variable. In Link Model 1 the formation of expenditure decisions is described by the model familiar to you from Demand Model 7; you will recall that, for any specified value of the aggregate price level, the model established a corresponding value of aggregate demand. The higher the price level selected, the lower the corresponding value for aggregate demand. Similarly, the formation of production decisions is described by the same model as was used in Supply Model 2; you will recall that this model established a level of aggregate supply corresponding to any specified price level. The higher this price level, the greater the aggregate supply generated by producers, up to a maximum set by capacity or potential output.

The link between these two independent sets of decisions is accomplished very easily in Link Model 1, simply by determining the price level which brings about balance between output demanded by spending units and output supplied by producing units. Comparing the new price so determined with the old price previously prevailing yields the rate of price inflation. If the new price is too low to coax out a level of production great enough to employ all available labour, then unemployment also results.

Rates of unemployment and of price inflation are among the important indicators of aggregate economic performance. In order to achieve better performance in these directions, economic policy-makers may elect to alter government expenditure, tax rates, or the money supply. Link Model 1 permits you to specify any of these instruments in your attempt to arrive at more suitable policies (in the model world only!)

2. Detailed Model Structure

<u>Equation Structure</u>:

$$M^d_t = MDA + A1 \cdot \hat{Y}_t - A2 \cdot R_t$$

$$M^s_t = \overline{MS} / P_t$$

$$M^s_t = M^d_t$$

Demand 7 - Linear Version

$$C_t = CA + B1 \cdot (\hat{Y}_t - \hat{T}_t)$$

$$I_t = IA + B2 \cdot \hat{Y}_t - B3 \cdot R_t$$

$$G_t = \overline{GA} / P_t$$

$$Y_t = C_t + I_t + G_t$$

$$T_t = \overline{TA} / P_t + B4 \cdot Y_t$$

$$\hat{Y}_t = Y_{t-1}$$

$$\hat{T}_t = T_{t-1}$$

$$\Longrightarrow Y_t = f(Y_{t-1}, P_t, P_{t-1})$$

Supply 1

$$Q_t^f = C \cdot KO^A \cdot LO^{(1-A)}$$

$$W_t^f = P_t \cdot \partial Q_t^f / \partial LO$$

$$L_t^d = \cdot h(W_{t-1}/P\ ) \qquad \text{(see Supply 1)}$$

$$Q_t = \min\ [\overline{C} \cdot KO^A \cdot L^{d(1-A)}, Q_t^f]$$

$$W_t = \max\ [\overline{W}_{t-1}, W_t^f]; \; W_0 = WO$$

$$U_t = 100 \cdot (LO - \min\ [\overline{L}_t^d, LO])/LO$$

$$\Longrightarrow Q_t = g(P_t, W_{t-1}; KO, LO)$$

$$Y_t = Q_t \qquad PIR_t = 100 : (P_t - P_{t-1})/P_{t-1}$$

Parameters:

(same as Demand 7 and Supply 2)

Instruments:

$\overline{GA}$, $\overline{TA}$, $\overline{B4}$, $\overline{MS}$

Initial Restrictions:

$0 < WO$	$0 < A2$
$0 < C$	$0 < KO$
$0 < A \quad 1$	$0 < LO$

$B1 \cdot (1 - \overline{B4}) + B2 - B3 \cdot A1/A2 = 1 \Longrightarrow$ "Total propensity to spend out of income is 1 - Please try again"

$\overline{P} = 0$ or $\overline{Y} = 0$ or $f(P_t) = g(P_t)$ does not converge $\Longrightarrow$ "No plausible initial equilibrium is defined - please try again"

(Change Instruments):

$$|\overline{GA} - \overline{\overline{GA}}| + |\overline{TA} - \overline{\overline{TA}}| + |\overline{B4} - \overline{\overline{B4}}| + |\overline{MS} - \overline{\overline{MS}}| > 0$$

<u>Program Structure</u>:

→ Initial parameter values	←
Initial equilibrium	(not defined
→ Change instruments	
Adjustment path	(if it is defined)
Final equilibrium	" " " "
Graph	" " " "

3. Running the Program

The program begins, as usual, by printing the initial values for the parameters of the model - specifying the labour supply function, the production function, the consumption function, investment function, government expenditure function, tax function, money supply, money demand function, and resource endowments, respectively. By changing any of these initial values as desired, you may alter the specification of any one of these relationships.

When the initial values of the parameters are all set, the program computes and prints the static equilibrium configuration corresponding to the model with those values. If this initial equilibrium appears unsatisfactory, you have the opportunity to return to alter further parameter values.

When you are satisfied that the initial equilibrium is sensible, the program prints the initial values for the instrument variables - the policy variables assumed to be controlled by central economic policy decisions - and awaits input of some change in policy.

The program then tabulates the dynamic response of key variables in the model to your policy change, permitting you to follow the adjustment process step-by-step. It also offers to plot a graph of the shift in demand or supply curves and the resulting adjustment path. Finally, the program prints the equations for the aggregate demand and supply functions, and tabulates the new equilibrium to which the model converges as the multiplier process dies out.

In the table of the adjustment process, note that (as the program prints out) national income and its components are shown in current dollars not delfated by the price level. The rate of interest, the umenployment rate, and the rate of price inflation are all shown as percentages. On the graph of the adjustment process, the curve labelled SS is the aggregate supply curve (your work with Supply Model 2 will have explained why it is kinked), the curve labelled 11 is the initial demand curve, and that labelled 22 is the final demand curve. (The equations for these curves are also printed by the program under the graph.) The initial equilibrium point is labelled A, and successive points on the adjustment path are labelled B, C, D, and so on. Where successive points are too close together to permit printing of separate letters, the program skips some letters and prints the last of the letters pertaining to those points. Thus when adjustment is very fast, successive letters will be far apart on the graph; when adjustment is very slow, the letters will be close together on the graph and many may be skipped altogether.

This process may be repeated as many times as you wish, with further alternatives in instrument values or in parameter values, or both.

4. Suggested Experiments

The model permits you to explore various combinations of policies designed to achieve full employment of a growing labour force, with reasonable price stability. The range of experiments is large, and particular choices are up to you at this stage.

LINK MODEL 2:

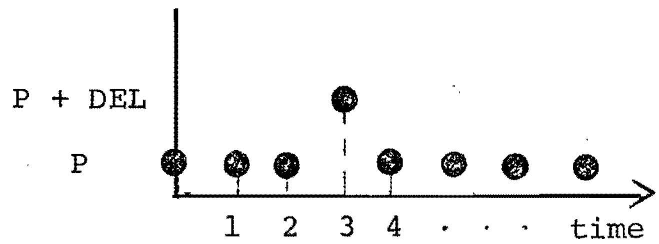
Long Run Labour Force Growth and Capital Accumulation

1. The Model

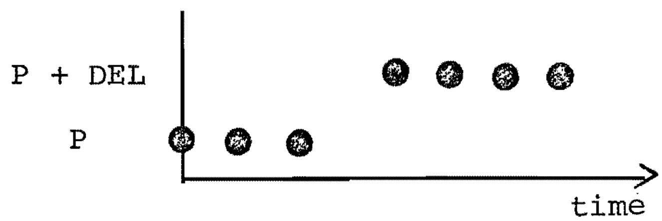
Link Model 2 is easy to describe because it is identical to Link Model 1 except in two respects. First, whereas in the previous model factor endowments were taken to be fixed, in the present model they are allowed to grow. This means that, as in Supply Models 3-5, we must have equations to describe how they grow. As in the supply models, the labour force growth equation describes a simple process of growth at a constant proportional rate (just like a colony of bacteria, or a bank balance under the forced compound interest). Unlike the supply models, the present model already has investment decisions determined from the expenditure equations, so the growth of capital stocks requires no new equations.

Second, the program permits much more elaborate specifications of economic policy moves accomplished through government expenditure decisions, tax changes, or the money supply. In the previous models, the emphasis was on understanding the behavioural structure, and the representation of possible policies was very simple: policy parameters or instruments could only be changed from one constant value to another. Now we allow a richer variety of changing policies. Any of four distinct classes of policy can be specified, as follows:

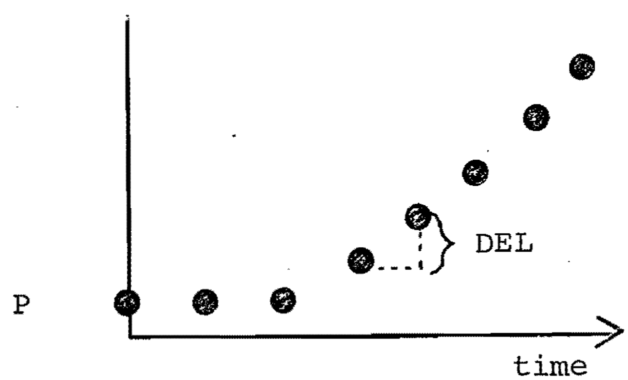
1. A "pulse" - as the diagram indicates, the policy parameter is kept unchanged for the first two periods, changed by rhe amount "DEL" for the third, and then reverts to the original level. This policy corresponds to a one-period change.



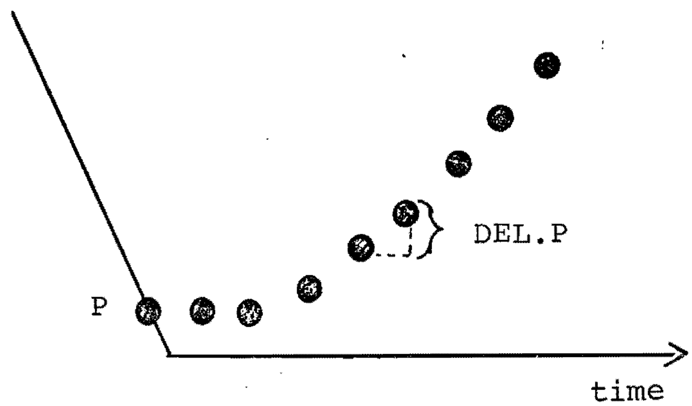
2. A "step"- this policy generates a permanent change from the original P to a new level P + DEL. It corresponds to a permanent move to a new plateau.



3. A "ramp" - this policy generates a regular change by an amount "DEL" in the third and all subsequent periods.



4. A "growth track" - this policy generates a regular change at a constant proportional rate in the third and all subsequent periods.



Any of these four forms may be prescribed for any of the four instrument values assumed in these models to be available to policy-makers. Thus quite elaborate and complex combinations of expenditure, tax, and monetary policy can be studied. Complete "game plans" can be settled in advance, with detailed schedules of future changes; rather than simply moves to different constant values for each instrument, this program permits expenditure, tax, and monetary instruments to vary over a 10 or 20-year horizon.

In addition, the operations of automatic mechanisms such as welfare payments schemes or unemployment insurance plans bring about some automatic, non-discretionary, policy changes. Some people argue that economic policy generally should be less discretionary than it is, and recommend that this be accomplished by establishing rules for how government expenditure, for example, should be changed to respond to the current performance of the economy as measured by indicators such as the unemployment rate or the rate of price inflation. The introduction of three additional policy parameters into Link Models 2 and 3 enables you to prescribe such automatic (endogenous) policy rules based upon target unemployment or inflation rates. The parameter G_1 specifies how real government expenditure should alter if the rate of price inflation changes by one percentage point, G2 specifies how expenditure should alter if the unemployment rate changes by one percentage point, and G3 serves as a target rate of unemployment. By experimenting with these parameters you may see whether you can find some combination of automatic rules which keeps the performance of this model economy satisfactory.

2. Detailed Model Structure

<u>Equation Structure:</u>

<u>Note</u> - No lags except for endogenous government policy

$$\left\{\begin{array}{l}
M_t^d = MDA + A1 \cdot Y_t - A2 \cdot R_t \\
M_t^s = \overline{MS}/P_t \\
M_t^d = M^s{}_t \\
C_t = CA + B1 \cdot (Y_t - T_t) \\
I_t = IA + B2 \cdot Y - B3 \cdot R_t \\
G_t = GA_t/P_t + G1 \cdot PIR_{t-1} + G2 \cdot (U_{t-1} - G3) \\
Y_t \quad C_t + I_t + G_t \\
T_t \quad TA_t/P_t + B4_t \cdot Y_t
\end{array}\right.$$

$$\left.\begin{array}{l} \overline{GA}_1 = GA \; ; \; \overline{GA}_t \\ TA_1 = TA \; ; \; \overline{TA}_t \\ B4_1 = B4 \; ; \; \overline{B4}_t \\ MS_1 = MS \; ; \; \overline{MS}_t \end{array}\right\} \text{ See Policy Description}$$

$$Q_t^f = C \cdot K_t{}^A \cdot L_t{}^{(1-A)}$$

$$W_t^t = P_t \cdot \partial Q_t^f / \partial L_t$$

$$L_t^d = h(W_{t-1}, P_t) \quad \text{(NOTE } W_{t-1} \text{ - Different time period)}$$

$$Q_t = \min \left[\bar{C} \cdot K_t{}^A \cdot L^d{}_t{}^{(1-A)}, \; Q^f{}_t \right]$$

$$W_t = \max \left[\bar{W}_{t-1}, \; W^f{}_t \right]; \; W_0 = WO$$

$$U_t = 100 \cdot L_t - \min \left[\bar{L}^d{}_t, \; L_t \right]) / L_t$$

$$Y_t = Q_t$$

$$PIR_t = 100 \cdot (P_t - P_{t-1}) / P_{t-1}$$

$$L_{t+1} = (1 + GG) \cdot L_t \; ; \qquad L_1 = LO$$

$$K_{t+1} = (1 - D) \cdot K_t + I_t \; ; \quad K_1 = KO$$

<u>Parameters</u>: same as Link 1, plus -

D = .1

GG = .02

G1 = 0. Real government expenditure response to 1% rate of price inflation

G2 = 0. Real government expenditure response to 1% deviation unemployment rate from target

G3 0. Target rate of unemployment (percent)

Restrictions:

$0 < WO$ $\qquad$ $0 < A2$

$0 < C$ $\qquad$ $0 < KO$

$0 < A < 1$ $\qquad$ $0 < LO$

$$\left.\begin{array}{l} B1\cdot(1 - \overline{B4}_t) + B2 - B3\cdot A1/A2 \neq 0 \\ Y_t > 0 \\ P_t > 0 \\ Y_t(P_t) = Q_t(P_t) \quad \text{converges} \end{array}\right\} \quad t = 0, 1, 2$$

If not: "No plausible adjustment path is defined. Please try different parameter values and/or policy specifications."

Policy Specifications:

Given DEL and X^{ε} { GA, TA, B4, MS }

1 - Pulse: $X_2=X_1$; $X_3=X_1+DEL$; $X_t=X_1$; $t=4, \ldots, 20$

2 - Shift: $X_2=X_1$; $X_t=X_1+DEL$; $3, \ldots, 20$

3 - Ramp: $X_2=X_1$; $X_t=X_{t-1}+DEL$; T $3, \ldots, 20$

4 - Growth: $X_2=X_1$; $X_t=X_{t-1}\cdot(1+DEL)$; t $3, \ldots, 20$

(No Policy: $X_t=X_1$; $t+2, \ldots, 20$)

Program Structure:

→ Initial parameter values

Policy specification

Initial equilibrium

First step of adjustment path $\qquad$ assume G1=G2=G3=0

(are they defined?)

Remainder of adjustment path

3. Running the Program

Again this program is straightforward to run. After printing the table of initial parameter values and accepting any desired changes, the program awaits your specification for any of the four instruments. For each, you may select one of the four possible shapes discussed earlier, or, if you do nothing, the program will assume that the instrument remains unchanged at the initial value printed in the table of initial parameter values. As the attached printout shows, you specify your desired policy by selecting the number from 1 to 4 corresponding to the desired shape, and specifying the desired increment or rate of growth in the instrument. When you have finished, the program tabulates the "game plan" that your policy specification entails over the whole time span of the model.

Thereafter the program tabulates the adjustment process, and plots a graph of expenditure components. (In this graph, the curve labelled G corresponds to government expenditure, the curve labelled I corresponds to the sum of government expenditure and investment expenditure, and the curve labelled Y corresponds to the total of government spending, investment, and consumption, which is national income in this simple closed model.)

As usual, the program permits you to repeat the computation as often as desired to explore alternative policies.

4. Suggested Experiments

These models are designed for experimenting with the effects of various macroeconomic policies. There are too many experiments of possible interest for one to list them all. But the standard questions would start with issues such as the following:

i) Increase the money supply MS by 10% from its initial value. Explain the reasons for the resulting changes in interest rates, income, and unemployment, and for the shift in the demand curve shown by the program.

ii) Increase government expenditure GA by 10% from its initial value, and again explain the resulting adjustment. Compare the impact of this fiscal policy with the monetary policy in (i), and explain why the resulting patterns of adjustment might be different in the two cases.

iii) Observe that reducing the tax parameter TA and increasing the marginal tax rate parameter B4 has the effect of making the tax system more "progressive". (Why?) Study the effect of such a change on the macroeconomic variables of this model. Determine the size of offsetting change in government expenditure required to compensate for this tax change and restore unemployment and inflation performance to their original values.

LINK MODEL 3:

The Tradeoff Function

1. The Model

A great deal of discussion is heard about the nature of the tradeoff between unemployment and inflation. Link Model 3 enables you to explore some of the issues involved in the debate.

The essential point is that the fast adjustment of prices to equilibrium levels that occurs in Link Models 1 and 2 may not occur. Some level of prices gets determined each period, but it need not be a level consistent with balance between output demanded and supplied. When demand exceeds supply, one might expect prices to move up quickly, and unemployment rates to be low, in an attempt to close the gap; but it need not be closed in a single period and, indeed, since other variables also are changing (as we saw in Link Models 1 and 2), such a gap might persist for a long time. Similarly, when demand is low relative to supply, unemployment will be high, and prices may not rise, or may even fall. Thus one observes an association between low rates of unemployment and high rates of price inflation. This relationship is referred to as a tradeoff function, or frequently as a "Phillips curve" in honour of the economist Bill Phillips, who first discussed such an association observed between unemployment rates and rates of change of wage levels. (Since the link between rates of change of wages and rates of change of prices is frequently very tight, the two sorts of tradeoff functions are often discussed interchangeably.)

Apart from the introduction of the tradeoff function, and the resulting elimination of the equilibrium condition on prices, Link Model 3 is identical to Link Model 2. Thus we finish our model construction exercise with an enlarged model which is not necessarily in equilibrium in any period, but which is formed from the same building blocks as earlier models we have studied. If you pursue work in macroeconomics, you will find the same building blocks also appearing in the same way in much larger econometric models of the United States, Canada, the U.K., Japan, the Netherlands, and many other countries. Many theoretical refinements and empirical extensions remain to be added, but the basic relations we have looked at constitute the foundation.

Equation Structure:

$$M_t^d = MDA + A1 \cdot Y_t - A2 \cdot R_t$$

$$M_t^s = \overline{MS}_t / P_t$$

$$M_t^d = M^s{}_t$$

$$C_t = CA + B1 \cdot (Y_t - T_t)$$

$$I_t = IA + B_2 \cdot Y_t - B3 \cdot R_t$$

$$G_t = \overline{GA}_t / P_t - G1 \cdot PIR_t + G2 \cdot (U_{t-1} - G3) + G4 \cdot (Q_{t-1} - Y_{t-1})$$

$$Y_t = C_t + I_t + G_t$$

$$T_t = \overline{TA}_t / P_t + B4 \cdot Y_t$$

$$\left.\begin{array}{lll} \overline{GA}_1 = GA; & & \overline{GA}_t \\ \overline{TA}_1 = TA; & & \overline{TA}_t \\ \overline{B4}_1 = B4; & & \overline{B4}_t \\ \overline{MS}_1 = MS; & & \overline{MS}_t \end{array}\right\} \quad \text{See policy specification}$$

$$Q_t = C \cdot K_t^A \cdot L_t^{(1-A)}$$

$$U_t = T3/(PIR_t - T1) + T2$$

$$Y_t = (1 - U_t/100) \cdot Q_t$$

$$PIR_t = 100 \cdot (P_t - P_{t-1})/P_{t-1}; \quad P1 = PO$$

$$K_{t+1} = (1 - D)\ K_t + I_t; \quad K_1 = KO$$

$$L_{t+1} = (L + GG)\ L_t; \quad L_1 = LO$$

<u>(Additional) Parameters (since Link 2)</u>:

(Delete WO)

G4 = 0. Real government expenditure response to $1 bill excess of capacity over demand

PO = 1. Initial price level

$$\left.\begin{array}{l} T1 = -2. \\ T2 = 1. \\ T3 = 10. \end{array}\right\} \quad \text{Parameters for hyperbolic Phillips curve}$$

Restrictions:

Same as Link 2

Policy:

Same as Link 2

Program Structure:

Same as Link 2

3. Running the Program

This program is run in the same way as Link Model 2, with the same range of policy options. The only additional feature is the introduction of the parameter G_4, which permits government expenditure to respond in an automatic way to disequilibrium between real effective demand and real aggregate supply, as well as to the rate of price inflation and the rate of unemployment.

4. Suggested Experiments

As in the previous model, the extent of possible experiments is limited only by your imagination. Although many aspects of policy problems are not taken into account in these models - notably problems of external trade and foreign exchange, or the whole range of issues associated with income distribution and transfer schemes - what remains should provide a challenge to your understanding of macroeconomic relations as well as an aid to analysis of conventional monetary and fiscal policies.

APPENDIX I

OPTIM

3-D Optimization Model with Generalized Quadratic Functions

The program solves the following NLP problem:

$$\left.\begin{aligned} \max \quad & Q(\chi_1, \chi_2, \chi_3) = \left[\sum_{i=1}^{3} \sum_{j=1}^{3} A_{ij} \, \chi_i^{B.G} \, \chi_j^{B.(1-G)} \right]^{1/B} \\ \text{subject to} \quad & \sum_{i=1}^{3} p_i \chi_i \leq y; \quad Q, \chi, p, y \geq 0 \end{aligned}\right\} \quad *$$

The program can be run in any of three ways:

1. Solve (*) given one value of (y, p_1, p_2, p_3); output on teletype.

2. Solve (*) for 11 different values of one of (y, p_1, p_2, p_3) e.g. if 2nd parameter is specified, 11 values of $(Q, \chi_1, \chi_2, \chi_3)$ are found as p_1 varies from .5 to 1.5 times its original value in uniform steps; output on teletype.

3. Solve (*) for N uniformly distributed observations of one of (y, p_1, p_2, p_3) for values between LL (Lower Limit) and UL (Upper Limit); includes multiplicative error term with standard deviation S1 for each of (y, p_1, p_2, p_3) (S1 $\geq$ and multiplicative error term with standard deviation S2 for each of $(\chi_1, \chi_2, \chi_3, Q)$ (S2 $\geq$ 0); output to disk in user specified file (name up to five alphanumeric characters - no '.DAT' extension in name specified, but resulting file

will have '.DAT'; for N lines where an observation is a set of values for $(x_1, x_2, x_3, Q, y, p_1, p_2, p_3)$ in '8F9.3' format.

[Note that the last five program parameters (S1, S2, N, LL, UL) are ignored in (1) and (2) above. The first fifteen program parameters refer exactly to the coefficients of (*).]

This program may therefore be used to illustrate the derivation of demand curves or Engel curves from the maximizing behaviour of the consumer, or the derivation of derived input demands or expansion paths from the cost minimizing behaviour of a producer. In addition, the data files generated (with errors added) from this computation may be used as input to a regression program to illustrate the accuracy or sensitivity of various estimation procedures to errors in equations or in variables.

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LOADER 6K CORE
EXECUTION


3-D OPTIMIZATION MODEL WITH GENERALIZED QUADRATIC FUNCTION
BY A.R.DOBELL AND M.WOLFSON

(CONSULT WRITE-UP FOR PARAMETER
DESCRIPTIONS AND SUGGESTED EXERCISES)

   Y   =    1.00    A22 =    0.33
   P1  =    1.00    A23 =    0.00
   P2  =    1.00    A31 =    0.00
   P3  =    1.00    A32 =    0.00
   G   =    0.50    A33 =    0.33
   B   =    0.50    S1  =    0.00
   A11 =    0.33    S2  =    0.10
   A12 =    0.00    N   =   20.00
   A13 =    0.00    LL  =    1.00
   A21 =    0.00    UL  =    5.00

ANY PARAMETER CHANGES ?
YES

(TYPE: 'PARAM = VALUE'  UNTIL  'DONE')
N = 5

DONE


WHICH OF THE FIRST FOUR PARAMETERS WOULD
YOU LIKE TO ITERATE (TYPE 0 FOR NONE) ?
2


DO YOU WISH TO CREATE A DATA FILE ?
YES


WHAT FILE NAME ? TRIAL


A RUN WITH DIFFERENT INITIAL PARAMETER VALUES ?
NO

EXIT
↑C

.TI
4.83
24.82
KILO-CORE-SEC=311

.TY TRIAL.DAT
    0.121    0.422    0.350    0.269    1.000    1.743    1.000    1.000
    0.026    0.396    0.410    0.224    1.000    4.158    1.000    1.000
    0.035    0.427    0.420    0.297    1.000    3.728    1.000    1.000
    0.028    0.467    0.351    0.285    1.000    4.003    1.000    1.000
    0.143    0.345    0.410    0.299    1.000    1.483    1.000    1.000
```

```
3-D OPTIMIZATION MODEL WITH GENERALIZED QUADRATIC FUNCTIONS
BY A.R.DOWELL AND H.WOLFSON

(CONSULT WRITE-UP FOR PARAMETER
DESCRIPTIONS AND SUGGESTED EXERCISES)

   Y   =   1.00   A22 =   0.33
   P1  =   1.00   A23 =   0.00
   P2  =   1.00   A31 =   0.00
   P3  =   1.00   A32 =   0.00
   G   =   0.50   A33 =   0.33
   B   =   0.50   S1  =   0.00
   A11 =   0.33   S2  =   0.10
   A12 =   0.00   N   =  20.00
   A13 =   0.00   LL  =   1.00
   A21 =   0.00   UL  =   5.00

ANY PARAMETER CHANGES ?
NO


WHICH OF THE FIRST FOUR PARAMETERS WOULD
YOU LIKE TO ITERATE (TYPE 0 FOR NONE) ?
0

   X1     X2     X3     C      Y      P1     P2     P3
 0.333  0.333  0.333  0.327  1.000  1.000  1.000  1.000

A RUN WITH DIFFERENT INITIAL PARAMETER VALUES ?
YES

   Y   =   1.00   A22 =   0.33
   P1  =   1.00   A23 =   0.00
   P2  =   1.00   A31 =   0.00
   P3  =   1.00   A32 =   0.00
   G   =   0.50   A33 =   0.33
   B   =   0.50   S1  =   0.00
   A11 =   0.33   S2  =   0.10
   A12 =   0.00   N   =  20.00
   A13 =   0.00   LL  =   1.00
   A21 =   0.00   UL  =   5.00

ANY PARAMETER CHANGES ?
NO


WHICH OF THE FIRST FOUR PARAMETERS WOULD
YOU LIKE TO ITERATE (TYPE 0 FOR NONE) ?
4


DO YOU WISH TO CREATE A DATA FILE ?
NO

   X1     X2     X3     C      Y      P1     P2     P3
 0.250  0.250  1.000  0.436  1.000  1.000  1.000  0.500
 0.273  0.273  0.750  0.399  1.000  1.000  1.000  0.600
 0.292  0.292  0.595  0.373  1.000  1.000  1.000  0.700
 [illegible] 0.306  0.481  0.354  1.000  1.000  1.000  0.800
 0.321  0.321  0.397  0.339  1.000  1.000  1.000  0.900
 0.333  0.333  0.333  0.327  1.000  1.000  1.000  1.000
 0.344  0.344  0.284  0.317  1.000  1.000  1.000  1.100
```

Linear Programming Model and the Simplex Method

1. The Model: $\max \; p_1 Q_1 + p_2 Q_2 = \Pi$ subject to

$$a_{11} Q_1 + a_{12} Q_2 \geq C_1 \qquad (1)$$
$$a_{21} Q_1 + a_{22} Q_2 \geq C_2 \qquad (2)$$
$$a_{31} Q_1 + a_{32} Q_2 \geq C_3 \qquad (3)$$
$$Q_1 \geq C_4 \qquad (4)$$
$$Q_2 \geq C_5 \qquad (5)$$

With $Q_1, Q_2 \geq 0$ and $Q_3, \ldots, Q_7$ representing slack variables for constraints (1) - (5) respectively.

2. Program Structure: (see attached)

→ Display initial parameter values

Parameter changes?

Yes ⇒ type in changes

Initial tableau?

Yes ⇒ tableau printed

Iteration process?

Yes ⇒ sequence of tableaux resulting from successive pivot steps are printed

Final tableau?

Yes ⇒ it is printed

Graph of feasible region

Yes ⇒ it is printed

Another run — Yes → (back to Display initial parameter values)

No → Stop

3(a). <u>Tableau Format:</u>

a	b	'Q1'	'Q2'	'..'	'Q7'
'Qi$_1$'	Qi$_1$				
'Qi$_2$'	Qi$_2$				
'Qi$_3$'	Qi$_3$				
'Qi$_4$'	Qi$_4$				
'Qi$_5$'	Qi$_5$				
'Z'	Π		c		

a - column of names of current basis columns

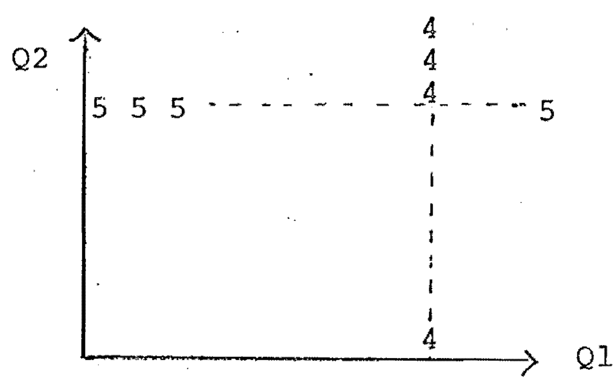
b - column of values of each of the Qi_j

c - row of profits associated with each Qi (positive if positive)

Π - the negative value of the criterion function

(c and Π: opposite of Chiang & Baumol)

3(b). <u>Graph Format:</u>



- Constraints represented by appropriate numbers
- Level curve of criterion represented by '*'

4. Suggested experiments

By varying the parameters of the revenue function, the sensitivity of the optimal solution can be explored, and the resulting path of outputs interpreted as a supply function. By varying the capacity constraints on fixed resource inputs, the sensitivity of the associated dual variable may be explored, and the results interpreted as similar to a demand for inputs (or marginal productivity) function.

References:

Baumol

Dosso

INITIAL PARAMETER VALUES:

A11	=	0.20	C2	=	6.90
A21	=	0.60	C3	=	3.40
A31	=	0.20	C4	=	8.50
A12	=	0.25	C5	=	10.00
A22	=	0.40	P1	=	3.20
A32	=	0.35	P2	=	2.70
C1	=	2.80			

DO YOU WISH TO CHANGE ANY PARAMETERS FOR EITHER THE TECHNOLOGY, THE CONSTRAINTS, OR THE PRICE VECTOR ?
YES

(TYPE: 'PARAM = VALUE' UNTIL 'DONE')
C5 = 9.5

A32 = .3

DONE

DO YOU WISH TO SEE THE INITIAL TABLEAU ?
YES

			Q1	Q2	Q3	Q4	Q5	Q6	Q7
Q3 *	2.80	*	0.20	0.25	1.00	0.00	0.00	0.00	0.00
Q4 *	6.90	*	0.60	0.40	0.00	1.00	0.00	0.00	0.00
Q5 *	3.40	*	0.20	0.30	0.00	0.00	1.00	0.00	0.00
Q6 *	8.50	*	1.00	0.00	0.00	0.00	0.00	1.00	0.00
PIVOT:		*	------						
Q7 *	9.50	*	0.00	1.00	0.00	0.00	0.00	0.00	1.00
Z *	0.00	*	3.20	2.70	0.00	0.00	0.00	0.00	0.00

DO YOU WISH TO SEE THE TABLEAUX OF THE ITERATION PROCESS ?
YES

TABLEAU 1

Q3 *	1.10	*	0.00	0.25	1.00	0.00	0.00	-0.20	0.00
PIVOT:		*		------					
Q4 *	1.80	*	0.00	0.40	0.00	1.00	0.00	-0.60	0.00
Q5 *	1.70	*	0.00	0.30	0.00	0.00	1.00	-0.20	0.00
Q1 *	8.50	*	1.00	0.00	0.00	0.00	0.00	1.00	0.00
Q7 *	9.50	*	0.00	1.00	0.00	0.00	0.00	0.00	1.00
Z *	-27.20	*	0.00	2.70	0.00	0.00	0.00	-3.20	0.00

DO YOU WISH TO SEE THE FINAL TABLEAU ?
YES

Q2 *	4.40	*	0.00	1.00	4.00	0.00	0.00	-0.80	0.00
Q4 *	0.04	*	0.00	0.00	-1.60	1.00	0.00	-0.28	0.00
Q5 *	0.38	*	0.00	0.00	-1.20	0.00	1.00	0.04	0.00
Q1 *	8.50	*	1.00	0.00	0.00	0.00	0.00	1.00	0.00
Q7 *	5.10	*	0.00	0.00	-4.00	0.00	0.00	0.80	1.00
Z *	-39.00	*	0.00	0.00	-10.80	0.00	0.00	-1.04	0.00

```
DO YOU WISH TO SEE A GRAPH OF FEASIBLE PRODUCTION POSSIBILITIES ?
YES

      C2
11.4 I3            *     2                     4
     I  13            *     2                  4
     I        13         *     2               4
     I555555555555555555*555555555555555555555555555
     I                1  8     *   2           4
     I                      1   3     *   2    4
     I                            1     3*   2 4
     I                                  1    *   2    4
     I                                      1     *32  4
     I                                            1    * 23 4
     I                                                  1 *2  3
     I                                                        1*24   3
     I                                                           *        3
     I                                                           4 *
     I                                                           4   2*1
     I                                                           4       *
     I                                                           4        2*
     I                                                           4
     I                                                           4
     I                                                           4
     I                                                           4
     *----------------------------------------------------I-- C1
                                                        10.2

WOULD YOU LIKE TO TRY ANOTHER RUN WITH DIFFERENT PARAMETERS ?
NO

EXIT
↑C
```

APPENDIX 2

SCHEMA PROGRAM OPERATION

Introduction

SCHEMA is an interactive program for the simultaneous solution of both the primal and the dual of a linear programming (LP) problem. The algorithm is based on a paper by Balinski*/. It is currently available on the Dataline system, Toronto (March 1972). The basic intention of the program is to provide a reasonably efficient method for solving small or medium size LP problems in the framework of highly flexible conversational program input and output.

Tableau Notation

The canonical form of LP problems is often represented as follows:

Primal	Dual
maximize cx	minimize $y'b$
subject to $Ax \leqq b$, $x \geqq 0$	subject to $y'A \geqq c$, $y \geqq 0$

where A is a matrix with m rows and n columns; x is an n dimensional column vector; c is an n dimensional row vector; and y and b are m dimensional column vectors. (´ denotes transpose.)

A more general representation of an LP problem can be expressed as follows:

---

* Balinski, (see reference on p. 13)

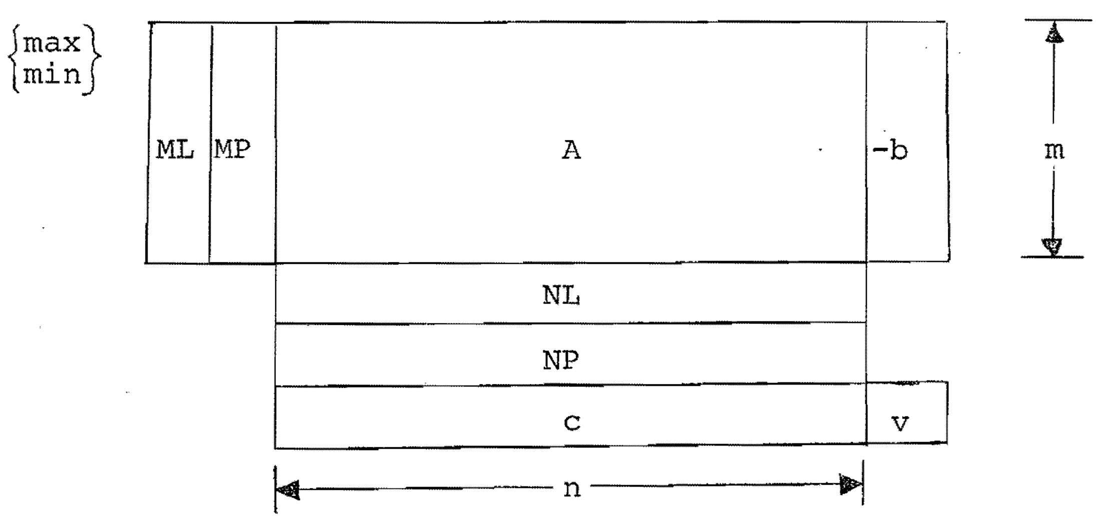
$$\left\{\begin{matrix}\text{maximize } \underline{\text{or}} \\ \text{minimize}\end{matrix}\right\} \quad cy \quad \text{subject to} \quad \begin{aligned} A_1 y - b_1 &= -s_1 \geqq 0 \\ A_2 y - b_2 &= -s_2 = 0 \\ A_3 y - b_3 &= -s_3 \leqq 0 \end{aligned}$$

$$\text{(P)} \qquad \text{and} \qquad \begin{aligned} & y_1 \geqq 0 \\ & y_2 \leqq 0 \\ & y_3 \ \text{unrestricted (free)} \end{aligned}$$

$$\text{where } y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}; \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}; \quad s = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}; \quad A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

and s is a vector of slack variables.

Any LP problem in the form (P) can be converted to the canonical primal (or dual) form above by appropriate manipulation. But SCHEMA will accept problems in the general form (P). It computes values for y and s from A, b and c as well as x and u, the dual variables and slacks, directly, by operating on the following tableau:



A, b and c are as indicated in (P). In addition, it is necessary to record

1. the type of problem - max or min
2. the constraint types - this is done in MP

   where $MP_i = \begin{cases} 0 & \text{for} \; = \\ 1 & \leq \\ 2 & \geq \end{cases}$

3. the variable types - this is done in NP

   where $NP_j = \begin{cases} -1 & \text{for free} \\ 1 & \text{non-negative} \\ 2 & \text{non-positive} \end{cases}$

4. the location of the variables and slacks during and after pivoting -

   initially $NL_j = j \qquad (j = 1,\ldots,n)$

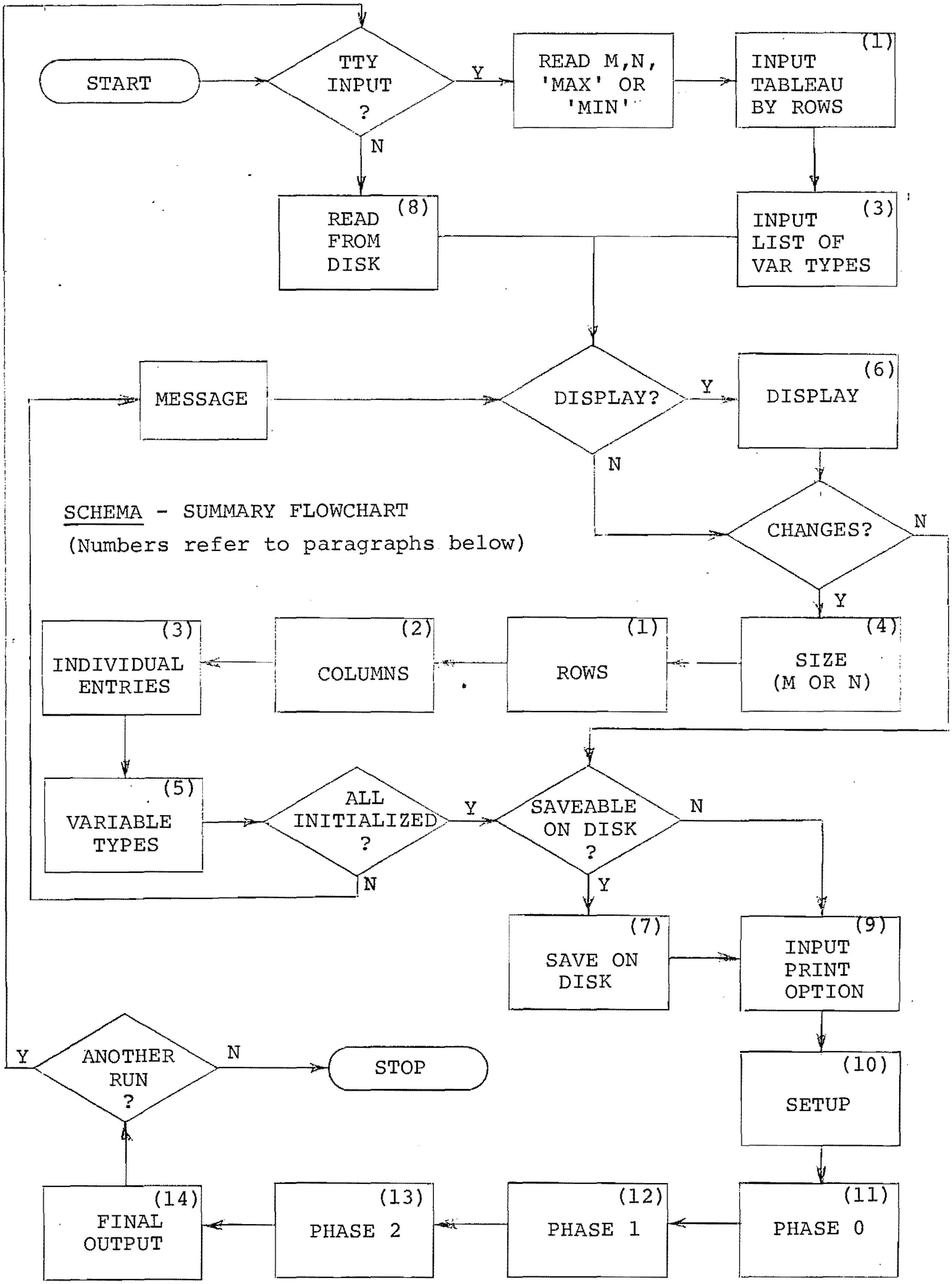
   and $ML_i = n + i \,. \qquad (i = 1,\ldots,m)$

5. the value of (P), i.e. the current value of the objective function is in $v = cy$.

   Initially, $v = 0$.

Note that the following labelling convention is used for the final output:

Primal Variables -	y_k	$k = 1,\ldots,n$
Primal Slacks -	y_k	$k = n+1,\ldots,n+m$
Dual Variables -	x_k	$k = 1,\ldots,m$
Dual Slacks -	x_k	$k = m+1,\ldots,m+n$



SCHEMA - SUMMARY FLOWCHART

(Numbers refer to paragraphs below)

Input Structure

In addition to short answer questions, the input block includes the following routines for adding to or modifying the tableau:

1. (Re)writing rows - A row, in the tableau format, represents either a constraint or the objective function ("c"). Since most of the information in the tableau is in rows, they are used as the basic unit of input. There are two types of rows, each precisely defined:

   (a) a constraint row consists of the row number i ($1 \leq i \leq m$), the constraint type $MP_i \varepsilon \{0,1,2\}$, n real numbers for the $i^{\underline{th}}$ row of matrix A, and one real number for the $i^{\underline{th}}$ element of "-b", the negative of the right hand side coefficient - thus two integers and n + 1 real numbers

   (b) the objective function row consists of the row number, which is always m + 1, followed by the n real numbers of the vector "c".

2. (Re)writing columns - A column, in the tableau format represents either A variable (Activity) or the right hand side (RHS) constraint coefficients. The information contained in columns is "dual" to that of rows and they are similarly defined:

   (a) a variable column consists of the variable number j ($1 \leq j \leq n$), the variable type $NP_j \varepsilon \{-1,1,2\}$, m real numbers for the j^{th} column of A, and one real number for the j^{th} element of "c", the objective function coefficient - thus two integers and m + 1 real numbers

(b) the constraint coefficient column consists of the column number, which is always n + 1, followed by the m real numbers of the vector "-b".

In fact, the same subroutine is used for (re)writing both rows and columns, although only rows can be used for the initial teletype input. Subsequent modification or initialization may be by either row or column.

3. Variable types - Aside from m, n, and max or min, the only other information required after row input is the list of variable types, the vector NP. These data are simply entered as a list of n integers, each of which must be -1, 1, or 2. This routine is used for initial teletype input and can be used for subsequent modification.

4. Changing size - Once a tableau has been input, its size can be changed (within the limits of the program - see programming appendix). If the new values of m and/or n are greater than the original values, the tableau is expanded by moving the "c" row down and/or the "-b" column to the right. A message will be printed reminding the user to initialize the newly created rows and columns by appropriate (re)writing. If the new values are less, the user is asked to enter a list of the

correct number of row or column numbers for those rows or columns to be deleted. The "c" row (m + 1) and the "-b" column (n + 1) can not be deleted.

5. Individual entries - The user can make changes in any element of A, b, c, MP, and NP as well as max or min. There are four subsections:

   (a) To make changes in A, b, or c, the "coefficients", a three-tuple is entered for each change:

$$\left\{\begin{array}{lll} a_{ij} & - & i,j,\ \text{NEW } a_{ij} \\ c_j & - & m+1,j,\ \text{NEW } c_j \\ -b_i & - & i,n+1,\ \text{NEW } -b_i \end{array}\right\} \quad (1 \le i \le m \ \&\ 1 \le j \le n)$$

   (Changes to $a_{m+1,n+1} = v$ are ignored since setup initializes v to zero.)

   (b) To make changes in MP, a pair of integers is entered for each change:

   MP_i - i, NEW MP_i ($1 \le i \le m$ & NEW $MP_i \varepsilon \{0,1,2\}$)

   (c) To make changes in NP, a pair of integers is entered for each change:

   NP_j - j, NEW NP_j ($1 \le j \le n$ & NEW $NP_j \varepsilon \{-1,1,2\}$)

   (d) max can be changed to min, and vice-versa.

There are four main types of input requested from the user at the teletype by SCHEMA:

(a) 'Yes' or 'No'
(b) a single integer number
(c) a string of numbers
(d) the name of a file on disk

Cases a. and b. generally involve a question ending in a question mark (?) or a request for information ending in a colon (:).

Case c. follows a request for inputting a string of numbers, and occurs in 1. through 5. above. At the beginning of the request, a double colon (::) is printed at the left margin. Numbers can then be typed separated by blanks or commas. A list of numbers can also be separated by carriage returns. If SCHEMA expects more numbers, after a carriage return, to complete the list, a colon asterisk (:*) is printed at the left margin. Note, lists in 4. and 5. above must go on only one line. If an arbitrary number of lists is expected (1., 2. and 5. above), inputting is terminated by typing the word 'DONE' immediately after a double colon. Multiple adjacent blanks are treated as one blank. Two adjacent commas are equivalent to two commas with a zero inbetween them. Real numbers do not need decimal points, but integers must not have decimal points. No plus (+) signs are allowed.

Case d. arises if a disk file is to be read or written. A proper filename has no more than five characters, the first of which must be alphabetic. (See - Dataline routines IFILE and OFILE.)

There are three other main sections in the input block:

6. Display - The user may request either that the entire tableau be printed or that individual rows and columns, requested one at a time, be printed. The tableau format does not, at this stage, include ML, NL and v, as does the final and intermediate tableau output. A value of -9 in NP or MP indicates that the corresponding row or column has not yet been initialized. Individual rows and columns are displayed using the same format as rows when the entire display is requested. Row m + 1 displays both the objective function and the variable types, while column n + 1 will display both the RHS constraint coefficients and the constraint types. Typing -1 terminates requests for row or column displays.

7. Disk output - Once the tableau has been properly initialized, and before computation begins, it is possible to save it in a disk file. Note that the process of computing the solution destroys the initial tableau. The format of the tableau on disk is as follows:

   1. 'MAX' or 'MIN'   n   m   - (A3,2I)
   2. Objective function row - n coefficients; ten per line (except last line); free format - (10F).
   3. Variable types - n integers; ten per line (except last line); each either -1, 1 or 2; free format - (10I).
   4. Constraints - m rows: each row consisting of one integer followed by n + 1 coefficients; ten numbers per line (except last line); free format - (I,9F) for first line of row, (10F) for remaining lines of row; integer at

beginning of row is constraint type (MP) and must be 0, 1, or 2.

8. Disk input - If a file has been created by SCHEMA or has the format indicated above, it can be input directly. Thus, a tableau can be initialized and saved, the solution computed, and the tableau re-initialized from disk and then modified for subsequent runs. The file must be in the user's disk area.

   Note - The suffix '.DAT' is appended to the file name entered when saving a tableau; it should be ignored when naming the file for input.

9. There are three print options available for the computation and final output phase:

   (1) = final values of primal and dual variables and slacks only

   (2) = (1) and the final tableau and one line of information for each pivot step

   (3) = (2) and the complete tableau at each pivot step.

Computation and Final Output

10. Setup - The tableau coming from the input block may require changes before the algorithm can be applied.

    (a) If this is a MAX problem, row m + 1 ("c") is multiplied by minus one.

    (b) If any constraint is of type 2 ($\geq$), that row is multiplied by minus one.

(c) If any variable is non-positive (type 2), its column is multiplied by minus one

(d) ML and NL are initialized; v is set to zero.

11. Phase 0 - The tableau may have free variables and/or equality constraints. If so, pivoting is necessary so that free variables become non-binding constraints and equality constraints (zero slacks) become zero variables. Thus, free variables will always be in the basis of the solution (see Balinski, p. 43).

12. Phase 1 - If the RHS column (originally "-b") is not non-positive, the tableau is not (yet) row feasible. Either pivoting is required or optimization of a sub-SCHEMA. (Thus, Phase 2 can be called before Phase 1 is completed.)

13. Phase 2 - the RHS column is now non-positive. To get an optimum, the objective row (originally "c") must be non-negative.

Note - Phase 1 and Phase 2 look only at those rows and columns associated with primal variables and slacks of type 1 and 2; free and zero variables and slacks are ignored.

14. Final Output - The following rules are used to assign final values to y and x from the optimized tableau: (recall page 3)

Let $k = ML_i$ $(1 \leq i \leq m)$ and $\ell = NL_j$ $(1 \leq j \leq n)$

(a) $1 \leq k \leq n$ - A variable is in the basis

$$y_k = \begin{cases} -b_i & \text{IF } MP_i \varepsilon \{-1,1\} \\ b_i & \text{IF } MP_k = 2 \end{cases}$$

$$x_{k+m} \equiv 0$$

(b) $n + 1 \leq k \leq n + m$ - A slack variable in the basis

$$y_k = \begin{cases} b_i & \text{IF } MP_i = 1 \\ -b_i & \text{IF } MP_i = 2 \end{cases}$$

$$x_{k-n} \equiv 0$$

(Note - MP_i is never 0 since equality constraints have non-zero shadow prices $(x_{k-n} \neq 0)$)

(c) $1 \leq \ell \leq n$ - A variable not in the basis

$$y_\ell \equiv 0$$

$$x_{\ell+m} = \begin{cases} c_j & \text{IF } NP_j = 1 \\ -c_j & \text{IF } NP_j = 2 \end{cases}$$

(Note - NP_i is never -1 since free variables are always in the basis)

(d) $n + 1 \leq \ell \leq n + m$ - A slack variable not in the basis

$$y_\ell \equiv 0$$

$$x_{\ell-n} = \begin{cases} c_j & \text{IF } NP_j \varepsilon \{0,1\} \\ -c_j & \text{IF } NP_j = 2 \end{cases}$$

Note that SCHEMA does not detect degeneracy.

REFERENCES

1. Balinski, M.L., "Notes on a Constructive Approach to Linear Programming", in Mathematics of the Decision Sciences, Part I, G.B. Dantzig and A.F. Veinott, Jr., eds., American Mathematical Society, Providence, Rhode Island, 1968.

2. Dataline Systems Ltd., User's Manual, Toronto

PROGRAMMING APPENDIX

The standard maximum size problem the SCHEMA can accept is sixteen rows and twenty columns. These maximum size limits can be altered by altering the main calling program, recompiling, and loading.

To use SCHEMA without changing the maximum size, the following instruction is used:

```
.RUN DSK SCHEMA[157,10]
```

You will need to know the password to run the program.

To alter the maximum size, the following procedure is used:

1. Alter the main calling program, SMAIN[157,10]
2. Compile the altered version
3. Load the necessary relocatable files into core - using LOAD instead of EX allows you to create your own save version of the altered program:

```
.LOAD YOURMAIN.REL,CODE.REL[157,10],SSUB1.REL[157,10],SSUB2.REL[157.10]
```

4. Create a SAVed version of the program for possible later use:

```
.SAVE DSK SCHEMA
```

5. Begin execution by typing ST (for STart).

A listing of the main program will reveal comment cards explaining the method of changing the maximum size. There are three lines that must be changed in a consistent manner - the dimension statement for arrays A, ML, MP, NL, and NP, and the assignment statements for the variables MMAX and NMAX. The three statements are generally correct if they can be represented:

```
DIMENSION A(m,n),ML(m),MP(m),NL(n),NP(n)
MMAX = (m-1)
NMAX = (n-1)
```

with $m + n \leq 101$. (If the size is any larger, some of the I2 formats will not be sufficient for the three-digit numbers being printed.)

M.C. Wolfson
April 1972