UNCERTAINTY AND THE CONSERVATION OR DESTRUCTION OF NATURAL FORESTS AND WILDERNESS

by

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1. INTRODUCTION

In the last few years much attention has been focused on the adverse effects of man's activities on the world he inhabits. In the eyes of many the major threat to the continuation of life on the planet is no longer that of thermonuclear war but rather one brought about by the by-and-large peaceful economic activities of man. As the century draws to a close the converging threats of global climate change, the destruction of the ozone layer, the unchecked growth in world population and the loss of biodiversity loom ominously as portents of an ecological crisis unprecedented in human history.

No single economic activity has drawn so much obloquy for it's adverse environmental effects as that of the destruction of natural forests. This activity has been blamed for aggravating the greenhouse effect, for destroying multitudes of plant and animal species, for the desertification of formerly fertile lands, for soil erosion, for flooding, for the silting of rivers and estuaries and, no doubt, for many other things besides. This is not a new phenomenon. Plato in the fourth century BC, described the results of logging on the hills around Athens.[1] "What now remains compared with what existed is like the skeleton of a sick man, all the fat and soft earth wasted away and only the bare framework of the land being left." The destruction Plato was describing was on a relatively local scale. By the time of the fall of the Roman Empire, the environmental damage attributable to logging was on a much larger scale, and possibly was one of the causes of that fall. For example, the Roman provinces of North Africa, which had formerly been the bread basket of the empire, producing half a million tons of grain a year and boasting some six hundred cities, were reduced to a hot barren desert, the harbours of its once bustling ports silted up and abandoned, victims of the ancient world's attitude to Nature and its insatiable demand for wood. Wood was virtually the only known fuel; it was indispensable in the construction of buildings, furniture, chariots and ships. The ancient world's dependence on wood was perhaps even greater than the modern world's dependence on fossil fuels. And to the Romans the natural world was inexhaustible; they saw no reason for not extracting from it as much as they wanted. Indeed the state gave legal title to undeveloped land to anyone who cleared it of forest. The essential role of the forests in maintaining the ecology of the region were not understood.

The destruction of the forests which once ringed the Mediterranean was complete by the time of the decline of the Venetian Empire. The demand for wood, especially for building ships, was as great as in Roman times. For example it has been estimated that the fleets fighting in the Battle of Lepanto in 1571 had been constructed at the expense of the destruction of over a quarter of a million mature trees. By the seventeenth century, after the centres of power, especially maritime power, had moved to western Europe, much of that territory had been denuded of trees. Unlike the Mediterranean where soil erosion and the grazing of goats prevented the reestablishment of forests, in Western Europe some afforestation was possible. John Clare, a courtier in the reign of Charles II, published in 1664 his classic textbook *Sylva* exhorting landowners to create artificial forest plantations and today virtually all of Britain's woodland consists of replanted forests.

Today most of the natural forests of Europe and eastern North America have been cleared and converted into agricultural land, while the tropical rainforests of Central America, Amazonia and Southeast Asia are rapidly being destroyed. As the scale of logging operations has increased so have the environmental consequences. It has been suggested that the serious monsoon flooding along the Ganges in India and Bangladesh in recent years is a result of large scale deforestation in the Himalayan watershed some hundreds of miles away. In Panama deforestation has been blamed for a continuing drop in rainfall and the consequent drop in the water level of the lakes feeding the Panama Canal. It has also been blamed for silting of the Canal itself. Perhaps most serious of all, the destruction of natural forests, especially tropical rainforest, has been blamed for between ten and thirty percent of the carbon released annually into the atmosphere. Not only does the destruction of forest cause a one-time increase in atmospheric carbon, it also removes an important carbon sink, which otherwise could be relied upon to absorb atmospheric carbon on an ongoing basis.

Another global effect of deforestation is the irreversible loss of genetic resources. Tropical forests are incredibly rich in their diversity of species. A single hectare of Malaysian jungle may contain more than 800 species of woody plants -- about as many as in all of North America -- while between a half and three quarters of all animal species live in tropical forests, including ninety percent of all insects.[2] The rate of loss of species is staggering. Writing about a decade ago Myers (1979, 1983) estimated that species were being destroyed at the rate of "more than one a day" with the possibility that over the last quarter of this century it could average "over 100 extinctions a day." He estimates that by the year 2000 as many as a million species of the current five to ten million species could have been annihilated.

What are the reasons behind this continued deforestation? In many ways, in the poorer, tropical countries at least, they are not that different to those prevalent in the Ancient World -- an ethos based on the notion of Nature being infinite, and the imperatives for ever more wood and ever more land driven by a burgeoning population. For millions of poor villagers around the world firewood is the only source of fuel. It is estimated that[3] worldwide fifty percent of trees cut down annually are for fuel use. Usually in poor countries sources of wood are common property resources and so there is no incentive for the transfer of land to fuel-producing forest. With a growing population inexorably the forests are destroyed often with horrific consequences such as those which have occurred in Ethiopia and Haiti.

With respect to land, similar laws to those of Rome which gave title to the clearer of forest land continue to operate. Such laws helped to clear the forests of North America and are now operative in Brazil and many other countries. In addition there are often other indirect incentives and subsidies for clearing the forest. For example Brazil's tax law provides credits for livestock projects, encouraging the landless poor north to the new frontiers in Amazonia.

No doubt there has always been conflict between those who want to log or clear forest land, and those who want to preserve it, but in recent years this conflict has greatly intensified resulting in a polarization of the positions of the advocacy groups and an excessive politicization of the issues. Part of the reason for this is that there appears to be little common ground for rational discourse between the opposing factions, each basing its claims on different beliefs and principles. The arguments put forward by the proponents of logging are usually expressed in economic terms -- the wealth generated for society, the number of jobs created or preserved, the increase in the supply of land for food production and the consequent value of its product, etc. To the North American logging industry and its supporters, forest land is a natural resource like any other, which can be converted into wealth for the benefit of the industry and indeed for all of society. To governments of tropical Third World countries forests represent not only a source of timber wealth, but also a potential source of agricultural land. The conversion of forest land to agricultural land is seen as a step on the road to economic development, one taken by industrialized countries early on in their development.[4]

Those who oppose the logging of natural forest usually eschew economic arguments, basing their claims instead on philosophical or moral grounds.[5] To many environmentalists and recreational users virgin forest and wilderness are irreplaceable natural treasures, valuable for their own sake, and worthy of considerable expense and effort for their preservation. Many scientists, especially those in the biological sciences, have lent support to the conservationists pointing out the important role forests play in preserving genetic diversity and in maintaining environmental and ecological balances.

It would appear at first sight that many of these aspects defy quantification and therefore render economic analysis as being of little help in decision-making with respect to conservation etc. However if, as Samuelson [1975] describes it, economics is the study of "how men and society end up choosing, with or without the use of money, to employ scarce productive resources which could have alternative uses. . .", then it would appear that the questions facing society and indeed all of mankind, with respect to conservation or otherwise of natural forests, fall squarely within the realm of economic inquiry, and indeed in recent years economists have begun to grapple with some of the difficulties involved in environmental decision making.

In principle at least the normative theory of welfare economics (see Mäler [1985] for a discussion of welfare economics and the environment) and the applied methodology of cost-benefit analysis provide a framework for addressing such decision problems. Nonetheless there are many difficulties in applying cost-benefit analysis to decision problems such as those concerning the fate of natural forests and wilderness. Not least of these is the evaluation of the non-marketed services which standing forest provides. However this issue has been addressed in other areas of environmental regulation with, in some jurisdictions, cost-benefit analysis now being a mandated requirement for policy decisions. For example in the United States an

Executive Order of 1981 requires (for regulations concerning pollution control) "the measurement in monetary terms of all the beneficial and adverse effects of the proposed regulations and stipulates that, unless otherwise precluded by law, regulatory policies should be designed to maximize net economic benefits."[6] It would indeed be a big step forward in forest and wilderness management if such criteria were applied to harvesting decisions. This is patently not the case at present. Environmental costs, although often enormous, are typically considered as externalities and are often ignored. Furthermore even many of the direct costs are not properly assessed or allocated. For example in North America the costs of building access roads and preparing logging sites are usually paid out of the public purse. Recently the U.S. Forest Service sold logging rights on the Tongass rain forest of Alaska for two percent of what it spent in preparations for logging.[7] It was claimed in 1989 that each year the American taxpayer was providing $100 million to pay for the harvesting of timber too cheap to cover its costs.[8]

Also considered as externalities are the costs associated with the disappearance of a valuable natural asset. In addition to the "user cost" in its technical sense there is the cost associated with the termination of the flow of amenity benefits from the forest on it's destruction. The amenity benefits which tropical forest provide include the means to a livelihood for forest dwellers through hunting and gathering of fruit, nuts, etc., in addition to the value to mankind of the forest as a repository of wildlife and genetic diversity and as a carbon store. These benefits are also part of the amenity services of temperate zone forests. In addition temperate zone forests have a value as sites for recreational and touristic activities. It has been suggested that the benefits through tourism and recreation alone exceed those of logging in forests managed by the U.S. Forest Service,[9] and that the value of the output of fruit, latex etc.

in tropical rainforest can exceed by a factor of three that of land used for cattle pasture.[10]

If these benefits and costs were correctly allocated, it is clear that an application of cost-benefit methodology would lead to a great reduction in the rate at which natural forests are being destroyed. Nonetheless there are serious problems with the application of cost-benefit analysis. The difficulty in assessing the value of non-marketed amenity services has been touched upon above, but methods such as those of contingent choice have been developed for handling such questions (see *e,g,* Freeman [1985] for a discussion of such methods)[11]. Another difficulty with cost-benefit analysis is that it does not deal with distributional and equity issues. Decisions involving the fate of forests involve re-distribution of benefits. But cost-benefit analysis which seeks to maximize overall utility is unable to deal with problems such as how to find a tradeoff between the needs of a poor villager for firewood and that of mankind for preserving biodiversity and keeping carbon locked up; or between the benefits of preserving endangered species with those of maintaining jobs and viable communities in areas traditionally dependent on logging. Furthermore there is the problem of intergenerational equity. How does one strike a balance between the benefits that can be enjoyed by people living now with the rights or needs of generations yet unborn? These are undoubtedly serious issues, which impose limits on the applicability of cost-benefit analysis to problems of this type. As Kneese and Schutze [1985] suggest, public policy questions involving environmental issues are probably best dealt with by using cost-benefit analysis, but constrained by libertarian and egalitarian considerations.

Apart from the difficulties discussed above with respect to cost-benefit methods, there is another technical problem on which we wish to focus in this paper. It concerns the inadequacy of naive cost-benefit methods when actions taken are *irreversible* and are taken in the face of future *uncertainty*. As an example consider the problem of deciding whether a particular area of natural forest should be logged or not. A naive application of cost-benefit analysis would involve evaluating the expected present value of benefits through logging (sale of timber, plus future rents from the land), and the expected present value of costs (logging costs, environmental costs and amenity benefits foregone), and would prescribe logging if the expected present value of benefits exceeded that of costs. It is easily shown that this procedure is not optimal in the sense of maximizing the expected present value of benefits net of costs. The reason is that it ignores the essential *dynamic* or temporal nature of the problem. If a decision is made not to log this year, it leaves open the option of logging next year or at some other future date. However future benefits and costs are uncertain, because timber prices, the demand for amenity services, etc. are uncertain. One way around this difficulty is to use *expected values* for future benefits or costs, and then choose the date now or in the future for logging, which maximizes the benefits net of costs. This is known as a *certainty-equivalence* procedure. If applied in a *feedback manner* (*i.e.* with the expected values of future benefits and costs being updated as new information arrives), it will provide a decision rule which, dependent upon current information on timber prices, amenity values etc., will prescribe either logging or not logging at each point in time. Even this rule however is not optimal, because it ignores the essentially *stochastic* nature of the decision problem. The *optimal* decision rule will take into account *all possible future values of timber and amenity services etc., with their associated probabilities,* and not just the *expected values* of these quantities (as does certainty-equivalence).

How can the optimal decision rule be obtained? One answer is through the use of *stochastic dynamic programming* (S.D.P.) (see *e.g.* Ross [1983]). In the example discussed above where there are only two actions possible at each time (log or do not log), the problem reduces to one of *optimal stopping* (see *e.g.*, Brock, Rothschild & Stiglitz [1988]). In more general cases where logging or development of part of the forest area is possible, the problem does not so reduce, but S.D.P. can still be used to obtain a solution.

The shortcomings of naive cost-benefit analysis for irreversible decisions made in the face of future uncertainty, were first identified by Arrow and Fisher [1974] and Henry [1974]. They used simple two-period and three-period models, with linear net benefit structures, to demonstrate the phenomenon. They identified a value (termed *quasi-option value*) associated with not taking an irreversible action. By not taking the irreversible action, one still has the option of taking it at a later date, when more information is available. Conrad [1980] showed that quasi-option value is equivalent to the expected value of information conditional on the irreversible decision not being taken immediately.

In this article the sub-optimality of the cost-benefit methodology in its naive form, and in its more refined certainty-equivalence form are discussed for two models for the decision problem concerning the conservation of destruction of natural forest and wilderness. In both models the risk-neutral policy which maximizes the expected present value of benefits net of costs is obtained, and it is shown how the naive and certainty-equivalence versions of cost-benefit methodology lead to premature destruction of the forest. These results are entirely in accord with those of Arrow and Fisher [1974] and Henry [1974] etc. However the models used are more detailed and realistic than the simple models used by these and other authors. Both are in continuous time with an infinite time horizon.

The first model (Section 2) considers in isolation a stand of old-growth forest from which amenity benefits flow, but which is vulnerable at all times to the possibility of random catastrophic destruction. It is assumed that current timber values and amenity values are known, but that future values are not known with certainty. It is assumed that for each the growth rate is made up of a fixed and a random component. Optimal stopping methods are used to determine the decision rule which maximizes expected net present value. It is shown how optimally the expected benefits must exceed the expected costs by a factor, bigger than one, before harvesting should take place. This factor can be quite large (in numerical calculations given, it is shown that it could quite easily be as high as 13), and increases with increased uncertainty. It follows that naive cost-benefit analysis could be seriously sub-optimal. The same is true for the certainty-equivalence method. Numerical calculations are given to indicate the degree of sub-optimality both in terms of the reduction in expected present value and in the expected survival time of the forest stand.

The second model (Section 3) allows for partial harvesting or clearing of the forest or wilderness land, and assumes that the flow of benefits from wilderness land and 'developed' land depend on the quantities of each. While the current values of these flows are assumed known, the future values are assumed to be uncertain. The possibility of unforeseen benefits from the wilderness land, such as those resulting from technological breakthroughs, utilizing the genetic resources of the wilderness, is also included in the model. The dynamic programming equation is used to find an analytic form for the feedback rule which maximizes expected net present value. It is shown that optimally development takes place down to a level at which the *marginal* expected present value of developed land exceeds that of wilderness land by a factor again bigger than one, and which increases with increasing future uncertainty. The sub-optimality of the naive cost-benefit procedure and the certainty-equivalence procedure are again discussed.

The results of the analysis of the two models are entirely consistent, and mathematically remarkably similar. The two models complement each other. While the first model does not permit partial harvesting, it does allow for uncertainty in future timber prices and does include the possibility of random catastrophic destruction. It is perhaps more appropriate for a stand of old-growth forest in western North America. The second model ignores uncertainty in timber benefits and harvest costs but does include flows of benefits from both forest land and developed land which are size-dependent and which are subject to future uncertainty. This model is perhaps more appropriate for tropical forests, which can be cleared for agriculture.

Neither model is meant to be an operational model for use by forest or land managers. The purpose for which they are used is to investigate analytically how uncertainty coupled with irreversibility interact in conservation-development decisions concerning natural forests and wilderness, and in particular to demonstrate how uncertainty should lead to policies which are much more conservative than would be indicated by simple cost-benefit analysis.

2. *A SINGLE-STAND MODEL FOR THE HARVEST-CONSERVATION DECISION PROBLEM*[12]

Consider an area of old-growth forest which can be either conserved *in toto* or clear-cut harvested. Clearly the decision to conserve is reversible, in that the forest can be harvested at a later date, while the decision to harvest is irreversible. Suppose that if the forest is harvested at time t a net revenue $V(t)$ is realized. This includes the revenue from the sale of timber net of harvest costs,[13] plus the value of the bare land on which future rotations may possibly be grown. We shall assume that $V(t)$ is an observable stochastic process following *geometric Brownian motion,* governed by the stochastic differential equation

$$\frac{dV}{V} = b\,dt + \sigma_1 dw_1 \tag{1}$$

where b is a drift parameter, $\{w_1(t)\}$ is a standard Wiener process, and σ_1^2 is a variance parameter. Thus we assume that *the proportional change in net value over time* dt *is a constant* $b\,dt$ *plus or minus a random component* $\sigma_1 dw_1(t)$. Large values of the parameter σ_1 will correspond to a large amount of uncertainty in future timber values. If harvest and replanting costs are neglected, $V(t)$ should be proportional to the current price of timber, since both the revenues from the sale of timber and the land value will then be proportional to timber price (see Clarke and Reed, 1989). The assumption that a commodity price follows geometric Brownian motion is very reasonable. A similar assumption for asset prices lies at the heart of much of the recent literature in financial economics.

The realization of the immediate benefit $V(t)$ resulting from harvesting at time t is offset by the foregoing of the option to harvest at a later date when timber prices might conceivably be higher, as well as by the foregoing of the flow of amenity services from the old-growth forest at all times beyond t.

Suppose that the flow of amenity services at time t has social valuation $A(t)$. It should be emphasized that $A(t)$ is a flow and has units such as dollars per month. It represents the rent that society is willing to pay to preserve the old-growth forest. To reflect the fact that future valuations of amenity services are uncertain, we shall assume that $A(t)$ is an observable stochastic process with proportional changes comprising fixed and random components. Thus we shall assume it follows geometric Brownian motion

$$(2) \qquad \frac{dA}{A} = a\,dt + \sigma_2 dw_2.$$

As before, a and σ_2^2 are drift (mean growth rate) and variance (uncertainty) parameters while $\{w_2(t)\}$ is another standard Wiener process. To reflect the fact that future changes in amenity value and timber value may be correlated, we shall assume that the white-noise processes $\{dw_1(t)\}$ and $\{dw_2(t)\}$ have correlation ρ. Positive values of ρ will reflect a positive association between fluctuations in timber and amenity values, and negative values of ρ a negative association.

Note that the assumption that the stochastic process $\{A(t)\}$ is observable is something of a simplification since societal valuations of amenity services can be determined only through sampling methods and thus will contain an element of sampling error. Similarly, even though current timber prices may be known exactly, the net revenue $V(t)$ to be realized through an immediate clear-cut harvest of the old growth cannot be known exactly prior to the harvest. For example, harvesting costs can only be estimated and the volume of usable timber is usually known only via sampling methods. However, uncertainty in future values of $V(t)$ and $A(t)$ is much greater than uncertainty concerning current values. Abstracting from the former source of uncertainty captures the essence of the problem of when, if ever, a clear-cut harvest should take place.

The fact that a harvest does not take place does not guarantee the survival of the forest forever. There is always the possibility that it may be destroyed by some natural catastrophe such as fire, tempest or pest infestation. In assessing the total value of amenity services obtained from the forest before a scheduled clear-cut harvest, this fact must be taken into account.

If there were to be no harvest, amenity services would accrue up until the time at which catastrophic destruction occurred, if indeed it ever did. Thus the expected present value of amenity services, using an *instantaneous discount rate,* δ, would be[14]

(3)
$$\bar{A}_f(0) = E\left\{\int_0^{\tau} e^{-\delta t} A(t) dt\right\}$$

where τ is a random variable denoting the time of destruction (τ would be infinity if destruction never occurs). The expectation in (3) is taken with respect to both τ and the stochastic process $\{A(t)\}$. The integral can be re-expressed as

(4)
$$\bar{A}_f(0) = E\left\{\int_0^{\infty} e^{-\delta t} A(t) S(t|0)\, dt\right\}$$

where $S(t|0)$ is the *survivor function* denoting the probability that the stand survives catastrophic destruction until time t, given that it is 'alive' at time 0.

The survivor function is determined by the *hazard-rate*. Any functional form could be used here (see *e.g.* Thompson, 1988). For simplicity, and in absence of any direct indication to the contrary, we shall assume a constant hazard rate λ, *i.e.* assume that the hazard of catastrophic destruction is time-independent. In this case the survivor function can be written as

$$S(t|0) = e^{-\lambda t}$$

and the expectation in (4) can be written as

(5) $$\bar{A}_f(0) = E\left\{\int_0^\infty e^{-(\delta+\lambda)t} A(t)\,dt\right\}$$

which can be evaluated (after solving (2) using the Itô integral -- see Reed [1992] for details) to give the expected present value, at time zero, of amenity services as

(6) $$\bar{A}_f(0) = \frac{A(0)}{\delta + \lambda - a}.$$

More generally we define $A_f(t)$ *as the expected present value (at time t) of future amenity services conditional on the stand not being destroyed at time* t, *and on the value* $A(t)$ *of the amenity flow at that time.* Specifically

Note that the presence of a constant risk of destruction affects the evaluation of future amenity services in the same way as the addition of a premium λ to the discount rate δ.

(7)
$$\bar{A}_f(t) = E\left\{\int_t^{\infty} e^{-\delta(z-t)} A(z)S(z|t)\,dz \,|A(t)\right\}$$
$$= \frac{A(t)}{\delta + \lambda - a.}$$

2.1 *The Cost-Benefit Prescription*

A naive cost benefit analysis would compare the immediate net benefits through harvesting, $V(0)$, with the cost (6) of amenity benefits foregone and would prescribe a harvest if the benefits exceeded the cost *i.e.* if

(8)
$$V(0) - \bar{A}_f(0) > 0.$$

If this condition were not met, cost-benefit analysis would prescribe no harvest. If this procedure were applied in an ongoing fashion one would arrive at a *harvest (stopping) rule* which prescribed a harvest at time t if and only if

(9)
$$V(t) > \bar{A}_f(t).$$

We shall show that *this cost-benefit rule is considerably sub-optimal from the point of view of maximizing the expected net benefit from timber harvest and amenity services, and that it unambiguously prescribes premature harvests.*

2.2 *The Optimal Harvest Rule*

To determine the optimal harvest rule, consider the *expected present value* (E.P.V.) of benefits if a harvest is scheduled to take place at time T. They include the E.P.V. of timber benefit

(10) $$e^{-\delta T} S(T|0)\, V(T) \;=\; e^{-(\delta+\lambda)T}\, V(T)$$

and the E.P.V. of amenity benefits

(11) $$E\left\{\int_0^T e^{-\delta t} A(t)\, S(t|0)\, dt\right\} = E\left\{\int_0^\infty e^{-(\delta+\lambda)t} A(t)\, dt\right\} - E\left\{\int_T^\infty e^{-(\delta+\lambda)t} A(t)\, dt\right\}$$
$$= \bar{A}_f(0) - e^{-(\lambda+\delta)T} \bar{A}_f(T).$$

Thus the net E.P.V. of a harvest scheduled at time T is

(12) $$e^{-(\lambda+\delta)T}\, V(T) \;-\; e^{-(\lambda+\delta)T} \bar{A}_f(T) \;+\; \bar{A}_f(0).$$

Note that to ensure convergence of the integrals in (11) we require the condition

(13) $$\delta + \lambda - a > 0.$$

We shall assume that this condition, and in addition

(14) $$\delta + \lambda - b > 0,$$

holds. If either of these conditions is not met it will be optimal to never harvest the forest.[15] Thus sufficient conditions for *conservation in perpetuity* to be optimal are that $\delta + \lambda < a$ *or* $\delta + \lambda < b$. If the former is met, amenity services are growing faster in expectation than the risk-adjusted discount rate and it therefore pays to conserve forever. If the latter condition is met, timber values are growing faster in expectation than the risk-adjusted discount rate, and so at all times it will pay to postpone a clear-cut harvest.[16]

The problem of determining an optimal harvest rule can be expressed as that of finding a *stopping time* T to maximize the total expected present value:

(15)
$$e^{-(\delta+\lambda)T}\left[V(T) - \bar{A}_f(T)\right] + \bar{A}_f(0).$$

This can be formulated as a problem of *optimal stopping* (see *e.g.* Brock, Rothschild and Stiglitz, 1988) with *intrinsic value* function

(16)
$$R(V,A) = V(T) - \bar{A}_f(T) = V(T) - \frac{A(T)}{\Delta - a}$$

and the objective of maximizing the expected present value

(17)
$$E\{e^{-\Delta T} R(V(T),A(T))\}$$

where $\Delta = \delta + \lambda$ is the *risk-adjusted discount rate.* (Note that (16) omits the last term on the right-hand side of (15) which is a constant independent of T, $A(T)$ and $R(T)$.) Note also that in general a stopping time T may depend on past and current values of the bivariate stochastic process $\{V(t), A(t)\}$, and that the expectation in (17) depends on the initial values (V_0, A_0) of this process.

It is shown in Reed [1992] that the *optimal stopping rule* involves a harvest at time t if and only if[17]

(18)
$$V(t) \geq \left(\frac{1+\theta}{\theta}\right)\bar{A}_f(t)$$

where θ is the *positive* root of the characteristic equation

(19) $$\frac{1}{2}\sigma^2\theta^2 + \left(b - a + \frac{1}{2}\sigma^2\right)\theta + (b - \Delta) = 0$$

and σ^2 is the variance of the $\left\{\ell n\left(\frac{V(t)}{A(t)}\right)\right\}$ stochastic process, *i.e.*

(20) $$\sigma^2 = \sigma_1^2 - 2\rho\,\sigma_1\sigma_2 + \sigma_2^2.$$

Compare the optimal rule (18) with the cost benefit rule (9). Since θ is positive it is clear that *the optimal rule is more conservative than the cost-benefit rule, i.e.* optimally one harvests if and only if the net timber revenues ("benefits") exceed the E.P.V. of revenues foregone ("costs") by a factor

(21) $$q^* = \left(1 + \frac{1}{\theta}\right)$$

which is bigger than one.

It is fairly straightforward (Reed [1992]) to show that this critical factor q^* *increases* with increases in

(a) the expected growth rate of timber values, b,

(b) the uncertainty in the growth of timber values σ_1,

(c) the uncertainty in the growth of amenity values σ_2.

Conversely q^* *decreases* with increases in

(d) the expected growth rate of amenity values, a,

(e) the discount rate δ,

(f) the hazard rate λ,

(g) the correlation ρ between amenity service values and timber values.

Table 1 displays the critical q^* for various values of the parameters a, b, δ and the total variance parameter σ^2. In all cases the hazard rate was set at $\lambda = .005$ per year corresponding approximately to a 1 in 200 chance of destruction in any given year. The blank entries (where a or b exceed $\Delta = \delta + \lambda$) correspond to cases where it is never optimal to harvest. Note how the critical ratio q^* is large when the difference $b - a$ in expected growth rates for timber values and amenity value is large, and how q^* increases with the variance σ^2. For example with $a = 0$, $b = 0.05$, $\delta = .05$ and $\sigma^2 = .02$ the benefits (the current timber value) would have to exceed the costs (expected present value of amenity services foregone) by a factor of more than 13 before, optimally, a harvest would be undertaken.

With σ^2 set at 0.02 one could be fairly confident ($\simeq$ 95% confidence) that within one year the ratio of timber value to amenity flow $V(t)/A(t)$ would lie within between 0.75 and 1.33 times its initial value;[18] with $\sigma^2 = .01$ the corresponding range would be 0.82 and 1.22 times the initial value. Such fluctuation seems well within the range of possibility.

2.3 *Sub-optimality of the Cost-Benefit Procedure*

The degree of sub-optimality of the cost-benefit procedure can be measured in a number of ways. The most natural (since the optimal rule maximizes E.P.V.) is to look at the reduction in E.P.V. through using the cost-benefit rather than the optimal rule.

In cases where the critical ratio q^* of $V(t)/\bar{A}_f(t)$ is finite $(a,b < \Delta)$,[19] it is easy to compute the percentage loss in E.P.V. through using the cost-benefit rule. It depends on the initial values V_0 and A_0, or, what amount to the same thing, on the ratio $V_0/\bar{A}_f(0)$. If this is greater than or equal to q^* then both procedures prescribe an immediate harvest and there is no loss in E.P.V. If $1 < V_0/\bar{A}_f(0) < q^*$, then the cost-benefit procedure prescribes an immediate harvest while the optimal procedure does not. Finally if $V_0/\bar{A}_f(0) \leq 1$ then neither procedure prescribes a harvest. Formulas for the percentage loss in EPV are straightforward to obtain in the latter two cases. It can be shown that the percentage loss is maximum when $V_0/\bar{A}_f(0) = 1$ and that this maximum is given by the simple formula

$$\frac{100\theta^{\theta}}{\theta^{\theta} + (1+\theta)^{1+\theta}} \tag{23}$$

which is always $\leq 50\%$. Table 2 displays the maximum percentage loss in EPV for various values of the parameters a, b, σ^2 and δ. It can be seen it increases with increases in b and σ^2, and with decreases in a and δ. Thus qualitatively the maximum loss in EPV through using the cost-benefit procedure, behaves like the critical ratio q^*. Note especially that when uncertainty is large either in future growth of amenity values or of timber values (σ_1 or σ_2) the sub-optimality of the cost-benefit procedure can be considerable.

It has been noted above that the cost-benefit procedure will always lead to premature harvesting. Another way of characterizing the sub-optimality of this procedure is to examine for how much longer the old-growth forest would survive under the optimal harvesting rule. It can be shown from the results in Reed [1992] that the *expected* extra survival time using the optimal harvest rule is

(24) $$E(\tau_{ex}) = \frac{1}{\lambda}[1 - (q^*)^{-\phi}]$$

where ϕ is the *positive* root of

(25) $$\frac{1}{2}\sigma^2\phi^2 + \left(b - a + \frac{1}{2}\sigma^2\right) - \lambda = 0.$$

Table 3 presents values of this expected excess survival time when the hazard rate λ is set at 0.005. In the absence of any harvesting the expected survival time of the old-growth forest would be $1/\lambda = 200$ years. It can be seen that in all cases (when $a < \Delta$, $b < \Delta$), harvesting under the cost-benefit rule is considerably premature with the expected difference ranging from a low of 17 years to a high of 115 years. However unlike the maximum proportional loss in EPV through using the cost-benefit procedure (Table 2), the expected loss in survival time increases with increases in a and δ, but decreases with increases in σ^2. The greater the variability in amenity values $A(t)$ or timber values $V(t)$, the shorter the *expected* time to reach the critical barrier. In cases with no variability in both $A(t)$ and $V(t)$ ($\sigma^2 = 0$ -- deterministic model) the optimal rule collapses to the cost-benefit rule when $a \geq b$ (hence the zero loss in survival time), but not when $a < b$.

2.4 The Certainty-Equivalence Procedure

A more sophisticated application of cost-benefit methodology might recognize that the net difference between benefits (net timber revenues) and expected costs (amenity services foregone) could change over time and thus seek a harvest time to maximize the present value of the difference. The harvest rule that does this is of course the optimal rule discussed in 2.2. However in practice rather than using the methodology of stochastic dynamic programming and optimal stopping, such problems are frequently treated via the use of the *'certainty-equivalence'* (C.E.) procedure. This involves replacing random variables by their expected values and solving the resulting deterministic optimization problem. For some stochastic optimization problems this procedure provides a good approximation to the optimal solution.[20] However it is well-known (Arrow and Fisher [1974], Henry [1974], Conrad [1980]) that for problems involving *irreversibility* and *uncertainty* that the C.E. procedure can be considerably sub-optimal.

In the context of the model used here, the C.E. procedure can be shown [Reed, 1992] equivalent[21] to:

(a) the cost-benefit rule of Section 2.1 if $b \leq a$ *i.e.* if amenity values are growing in expectation as fast or faster than timber values;

(b) the rule which prescribes harvesting as soon as the ratio $V(t)/\bar{A}_f(t)$ of timber benefits to amenity services foregone reaches the barrier

(26) $$q_w = \frac{\Delta - a}{\Delta - b},$$

if $b > a$ *i.e.* if amenity values are growing in expectation slower than timber values.

In either case the C.E. procedure will lead to a premature harvest.

Table 4 displays the critical ratio $q_{C.E.}$ of $V(t)/\bar{A}_f(t)$ for the certainty-equivalence rule and the degree to which $q_{C.E.}$ is too small compared with the optimal q^*.

Table 5 displays the expected additional survival time through using the certainty-equivalence rather than the optimal rule. This is determined from the formula

(27) $$\frac{1}{\lambda}\left[1 - (q_{C.E.}/q^{*})^{\phi}\right].$$

It can be seen from comparing Tables 1 and 4 that in cases where $b > a$, the certainty equivalence barrier q_w is closer to the optimal barrier q^*, than the unit barrier of the cost-benefit rule. Also from Tables 3 and 5 one can see that the degree to which the certainty-equivalence rule prescribes premature harvesting in the case $b > a$ is less than that of the cost-benefit rule. Thus *in the (fairly unlikely) case of timber values having a higher mean growth rate than amenity service values, the certainty-equivalence procedure would provide a reasonable approximation to the optimal policy.* However *in the (much more likely) case of amenity service values having a higher mean growth rate than timber values the certainty-equivalence procedure would provide no improvement over the naive cost-benefit procedure.* It could prescribe considerably premature harvests and result in a large loss in expected present value.

3. A MODEL FOR CONSERVATION-DEVELOPMENT DECISIONS FOR A LARGE AREA OF WILDERNESS LAND, WHICH ALLOWS FOR PARTIAL DEVELOPMENT

A major shortcoming of the model discussed in Section 2 is that it treats a stand of old-growth forest in isolation, with the value of amenity services determined exogenously. Because old-growth forest, and wilderness in general, is an exhaustible resource one would expect its unit value (as an amenity) to increase as it becomes more scarce. This aspect, which was ignored in Section 2, will be incorporated in the model developed in this section, which will be for a large tract of wilderness land, which can be partially developed, rather than for a single stand. In addition to services provided by wilderness land, the model will include services from developed land (*e.g.* from agriculture). The social valuations of both types of services in the future will be considered uncertain. However for mathematical simplicity the revenues from harvesting (and/or costs of clearing the land) will be considered fixed and known. Also the possibility of catastrophic destruction will be ignored; on the other hand the model will include the possibility of unpredictable benefits arising from scientific or technological "breakthroughs" utilizing the genetic resources of the wilderness. The probability of such a benefit occurring will be assumed to depend on the quantity of wilderness remaining, reflecting the fact that the destruction of wilderness leads to reduction in biodiversity.

3.1 The Model[22]

Suppose that, at time t, a land development/conservation planner has to make development decisions on a partitionable homogeneous area of wilderness $X(t)$ and an area of developed land $1 - X(t)$. Assume that, apart from benefits associated with random breakthroughs, the instantaneous flow of utility accruing to the society when land is developed to this extent is described by the additively separable benefit function:

(28) $$W(t,X(t)) = \pi(t)P(1-X(t)) + \nu(t)Q(X(t))$$

where $\pi(t)$ and $\nu(t)$ are the *social valuations* placed on the services yielded by developed land and wilderness respectively; and $P(\cdot)$ and $Q(\cdot)$ measure the size of the *service flows* from the respective land resource assets. Assume that P and Q are twice continuously differentiable, non-decreasing concave functions with $P(0) = Q(0) = 0$. If $P'' < 0$, $Q'' < 0$ for some X then the marginal values will be non-constant and land-service-flows will be *size-dependent*; otherwise they are *size-independent* (*i.e.* $P'' \equiv Q'' \equiv 0$) and the welfare function is *linear* in $X(t)$.

To reflect future uncertainty in social valuations assume that $\{\nu(t)\}$ and $\{\pi(t)\}$ are observable stochastic processes which evolve through time as geometric Brownian motion processes governed by the stochastic differential equations

(29) $$d\nu/\nu = r_\nu + \sigma_\nu dw_\nu ; \; \nu(0) = \nu_0$$

(30) $$d\pi/\pi = r_\pi + \sigma_\pi dw_\pi ; \; \pi(0) = \pi_0$$

where r_ν and r_π are drift parameters representing constant upward or downward trends in relative valuations of wilderness and developed land and σ_ν^2, σ_π^2 denote the instantaneous variances in the relative valuation processes. The stochastic processes $\{w_v(t)\}$ and $\{w_\pi(t)\}$ are standard Wiener processes so that $\{dw_v(t)\}$ and $\{dw_{,\pi}(t)\}$ are white-noise processes. Thus social valuations are assumed to grow at a rate made up of a fixed component (r_v or r_π) and a random component ($\sigma_v\, dw_v$ or $\sigma_\pi\, dw_\pi$).

Consider now the development dynamics and assume that, at time t, land is being developed at the rate $u(t)$, and that this development is *irreversible*. Thus

(31) $$\frac{dX}{dt} = -u(t) \leq 0.$$

Suppose also $X(0) = 1$, so that initially the total area of land is in a wilderness state. Suppose also that the cost associated with developing at a rate $u(t)$ is

(32) $$cu(t)$$

where c is a positive[23] constant.

Finally assume that, apart from the socially valuable "consumption" service stream yielded by wilderness, it may also have a value in that the genetic pool contained therein may lead to scientific "breakthroughs" of social value. For example a wilderness species may yield an improved corn variety or a new drug. Suppose that such breakthroughs occur in a point process with intensity

(33) $$\lambda(t) = \lim_{\Delta \to 0} \left\{ P\left(\text{breakthrough in } (t, t+\Delta]/\Delta\right)\right\}$$

dependent on the area of wilderness remaining. Specifically assume that

(34) $$\lambda(t) = \psi\big(X(t)\big)$$

where ψ is twice differentiable, non-decreasing and concave with $\psi(0) = 0$. Thus the larger the area of wilderness land the higher the probability of a breakthrough at any time, but with diminishing returns operative.

If the expected present value of rewards from a breakthrough at any time t is $\bar{R}$ and $T_1, T_2, \cdots$ denote the random times at which breakthroughs occur then the *net expected present value of the services of land* when both development costs and breakthrough possibilities are accounted for is:

(35) $$J = E\left\{\int_0^\infty e^{-\delta t}\left[W(t, X(t)) - cu\right]dt + \sum_{i=1}^{\infty} \bar{R}e^{-\delta T_i}\right\}$$

where E is the expectation operator, δ is the instantaneous positive discount rate, and W is as given in (28).

Socially optimal development could be considered as that which maximizes (35) subject to (32), and the constraint

(36) $$u \geq 0$$

to reflect the irreversibility of wilderness development.

3.2 *The Cost-Benefit Prescription*

The cost-benefit procedure would prescribe, at any time, development up to the level which maximized the difference between the expected present value (E.P.V.) of benefits from developed land and the cost of development including the expected present value of wilderness benefits foregone, *i.e.* developing down to the level at which marginal expected benefits equalled marginal expected cost.

The marginal E.P.V. of benefits at level x at time zero is

(37) $$P'(1-x)E\left\{\int_0^\infty e^{-\delta t}\,\pi(t)\,dt\right\}$$

which (*c.f.* evaluation of (5)) can be evaluated (by first solving (30) using the Itô calculus), as

(38) $$P'(1-x)\,\frac{\pi_0}{\delta - r_\pi}\,.$$

The marginal E.P.V. of wilderness benefits foregone at level x is similarly[24]

(39) $$Q'(x)\,\frac{\nu_0}{\delta - r_\nu}$$

If the potential benefits of scientific "breakthroughs" are ignored (which is likely to be the case in a cost-benefit analysis since they are hard to quantify), the cost-benefit prescription would prescribe at time t development to the level at which x units of wilderness remain where x satisfies:

(40) $$P'(1-x)\,\frac{\pi(t)}{\delta - r_\pi} - Q'(x)\,\frac{\nu(t)}{\delta - r_\nu} - c = 0.$$

To include the benefits from random scientific breakthroughs the expectation

(41) $$E\left\{\sum_{i=1}^{\infty} e^{-\delta T_i}\right\}$$

occurring in (35) must be evaluated under the assumption that x is held fixed (so that $\lambda(t) \equiv \psi(x) \equiv$ constant). This is easily performed,[25] the result being simply

(42) $$\psi(x)/\delta.$$

The inclusion of these costs (foregone benefits) in the cost-benefit analysis would require (40) to be modified to the form

(43) $$P'(1-x)\frac{\pi(t)}{\delta - r_\pi} - Q'(x)\frac{\nu(t)}{\delta - r_\nu} - \frac{\bar{R}\psi'(x)}{\delta} - c = 0.$$

We shall show that *this procedure prescribes overdevelopment (compared to the optimal) and that the degree of overdevelopment increases with increased uncertainty in future values of the services from wilderness land or developed land.* As in the single-stand model of Section 2, it is the combination of uncertainty and irreversibility that leads to the sub-optimality of the cost-benefit procedure. Of course, if the potential benefits from scientific breakthroughs are ignored in the cost-benefit procedure (equation (40)) then the degree of overdevelopment will be even greater.

3.3 *The Optimal Development Rule*

The development rule which maximizes the E.P.V. of all benefits net of costs is found by solving the stochastic optimization problem of maximizing (35) subject to the state equations (29), (30) and (31) with the constraint (36). It has not been possible to analytically solve this problem in full generality. However a slightly simplified version has been solved, which is quite adequate for the purposes of demonstrating the sub-optimality of the cost-benefit procedure. This simplification involves removing the uncertainty of future valuations of one or the other of services from wilderness land or developed land. Thus we shall consider either

(A) *Wilderness valuation uncertainty* in which $\sigma_\nu > 0$ with $\sigma_\pi = 0$, or

(B) *Developed land valuation uncertainty* in which $\sigma_\pi > 0$ with $\sigma_\nu = 0$.

The method of solution used for the stochastic optimal control problem is *stochastic dynamic programming* which provides a second order partial differential equation for the value function (maximum of (35) regarded as a function of the initial values of x, π and ν). This equation can be solved analytically in the region for which optimally there is no development (where the optimal $u^* \equiv 0$); upon invoking the appropriate boundary conditions an explicit equation for the boundary of the no-development region can be found.[26] In the case (A) of uncertainty only in future wilderness valuation the optimal policy involves development if and only if

(44)
$$\frac{\pi(t)P'(1-x)}{\delta - r_\pi} - \frac{\bar{R}\psi'(x)}{\delta} - c > \left(\frac{1+\alpha}{\alpha}\right)\frac{\nu(t)Q'(x)}{\delta - r_\nu}$$

where α is the *positive* root of the quadratic equation:

(45)
$$\frac{1}{2}\sigma_\nu^2 z^2 - \left(r_\nu - r_\pi - \frac{1}{2}\sigma_\nu^2\right)z - (\delta - r_\pi) = 0.$$

If the above condition is met, the optimal policy prescribes developing to the level (of x) for which the left hand side of (44) is equal to the right hand side.

The corresponding condition for case (B) in which there is uncertainty only in developed land values the condition (44) is replaced by:

(46) $$\frac{\pi(t)P'(1-x)}{\delta - r_\pi} > \left(\frac{\beta}{\beta - 1}\right)\left[\frac{\nu(t)Q(x)}{\delta - r_\nu} - c - \frac{\bar{R}\psi'(x)}{\delta}\right]$$

where β is the *positive* root of

(47) $$\frac{1}{2}\sigma_\pi^2 + \left(r_\pi - r_\nu - \frac{1}{2}\sigma^2\right)z - (\delta - r_\nu) = 0.$$

It can easily be shown from the assumption that $\delta > r_\pi$ that the positive root β of (47) is bigger than one and hence that the factor $\beta/(\beta-1)$ in (46) above is also greater than one.

Conditions (44) and (46) have a simple interpretation in terms of marginal costs and benefits. Since in (44) $\alpha > 0$, it follows that for development to take place the marginal E.P.V. of services from developed land net of development costs and losses related to scientific breakthrough must exceed the marginal E.P.V. of wilderness services foregone by a factor $\left(\frac{1+\alpha}{\alpha}\right) > 1$. Condition (46) requires that the marginal E.P.V. of developed land must exceed the marginal loss (made up of the E.P.V. of wilderness services foregone, benefits from scientific breakthroughs foregone and development costs) by a factor $\left(\frac{\beta}{\beta-1}\right) > 1$. Thus in both cases the optimal development rule is seen to be more conservative than that which simple cost-benefit analysis would prescribe. The factors $\left(\frac{\alpha+1}{\alpha}\right)$ and $\left(\frac{\beta}{\beta-1}\right)$ indicate the degree to which the optimal policy is more conservative than that of cost-benefit analysis.

A third special case in which it is possible to obtain an analytic solution to the stochastic optimal control problem is:

(C) *Zero development costs and no benefits from scientific breakthroughs.*

$c = 0;\ \bar{R} = 0.$

In this case the optimal development policy involves development only when

(48)
$$\frac{\pi(t)P'(1-x)}{\delta - r_\pi} > \left(\frac{1+\gamma}{\gamma}\right)\frac{v(t)Q'(x)}{\delta - r_v}$$

where γ is the *positive* root of

(49)
$$\frac{1}{2}\bar{\sigma}^2 z^2 + \left(r_\pi - r_v - \frac{1}{2}\bar{\sigma}^2\right)z - (r_\pi - \delta) = 0$$

where

(50)
$$\bar{\sigma}^2 = \sigma_\pi^2 - 2\rho\,\sigma_\pi\sigma_v + \sigma_v^2$$

is the variance parameter for the stochastic process $\left\{\ell n\left(\frac{\pi(t)}{v(t)}\right)\right\}$ (ρ is the correlation between the valuation processes (29) and (30)). In this case development takes place only when the marginal E.P.V. of a unit of developed land (the left hand side of (48)) is bigger than the marginal E.P.V. of a unit of wilderness land by a factor $\left(\frac{1+\gamma}{\gamma}\right)$ bigger than 1.

3.2 *The Sub-Optimality of the Cost-Benefit Procedure*

The degree to which the cost-benefit procedure prescribes over-development is determined (on comparing (43) or (40) with (44), (46) and (48)) by the factors $(1+\alpha)/\alpha$, $\beta/(\beta-1)$ and $(1+\gamma)/\gamma$ in the three cases (A), (B) and (C).

Through examination of the characteristic equations (45), (47) and (49), it can be shown that

(i) α increases with σ_ν^2; (ii) β increases with σ_π^2; (iii) γ increases with $\bar{\sigma}^2$.

Thus *increases in the uncertainty of the future valuation of services from wilderness land or developed land, have the effect of making the optimal development rule more conservative, thereby making the cost-benefit procedure more sub-optimal.*

This result concurs with the results from the model of Section 2. Indeed the similarity of the difference between the optimal rule and the cost-benefit rule for the two models is striking. A comparison of (44), (46) and (48) with (18) shows that in each case the costs of development (in terms of future amenity benefits foregone, etc.) must be inflated by a factor bigger than one in order to make cost-benefit analysis yield the optimal harvest rule. In each case this factor[27] depends on the root of a quadratic equation dependent upon, and increasing with, a variance term reflecting the degree of future uncertainty.

3.4 *The Certainty-Equivalence Procedure*

An approach to the development problem which would recognize the dynamic irreversibility of the process but not the inherent uncertainty would be that of certainty-equivalence in which random variables would be replaced by their expected values and the resulting deterministic optimization problem solved by the methods of deterministic optimal control. The optimal control would then be applied in a feedback fashion.

The expected values of the $\nu(t)$ and $\pi(t)$ given initial values ν_0 and π_0 are

(51) $$E(\nu(t) \mid \nu_0) = \nu_0 e^{r_\nu t} \qquad E(\pi(t) \mid \pi_0) = \pi_0 e^{r_p t}.$$

The deterministic optimal control problem to be solved for the certainty-equivalence procedure is to maximize:

(52) $$\int_0^\infty e^{-\delta t}\left[\pi_0 e^{r_\pi t} P(1-x) + \nu_0 e^{r_\nu t} Q(x) + \bar{R}\psi(x) - c\right] dt$$

subject to

(53) $$\frac{dx}{dt} = -u; \qquad x(0) = 1$$

(54) $$u \geq 0.$$

This is a linear (in u) control problem and so it's solution can involve only components of bang-bang control and singular control. Here this means either control with $u = 0$, pulses with $u = \infty$ or control to stay on the singular path given by[28]

(55) $$\left(\frac{\delta - r_\pi}{\delta}\right)\left[\frac{\pi_0 e^{bt}}{\delta - r_\pi} P'(1-x)\right] = \left(\frac{\delta - r_\nu}{\delta}\right)\left[\frac{\nu_0 e^{r_\nu t}}{\delta - r_\nu} Q'(x)\right] + \frac{\bar{R}\psi'(x)}{\delta} + c.$$

If the singular path (55) is feasible the certainty equivalence procedure would involve initially developing down to the level x satisfying

(56) $$M_1\left[\frac{\pi_0}{\delta - r_\pi} P'(1-x)\right] = M_2\left[\frac{\nu_0}{\delta - r_\nu} Q'(x)\right] + \frac{\bar{R}\psi'(x)}{\delta} + c$$

where

(57) $$M_1 = \frac{\delta - r_\pi}{\delta} < 1 \text{ and } M_2 = \frac{\delta - r_\nu}{\delta} < 1.$$

Subsequently as the values of $\pi(t)$ and $\nu(t)$ evolved development would take place only when the left hand side of (56) (with π_0 replaced by $\pi(t)$) exceeded the left hand side (with ν_0 replaced by $\nu(t)$) and the development would be down to the level at which equality occurred.

However situations can arise in which it is not feasible for a trajectory of (53) to coincide with the singular path, because the latter is *increasing in* t. As an example consider the simplified case (C) in which $\bar{R} = c = 0$. In this case the singular path can be written

(58) $$\frac{P'(1-x)}{Q'(x)} = \frac{\nu_0}{\pi_0} e^{(r_\nu - r_\pi)t}.$$

The left hand side is increasing in x. If $r_\nu > r_\pi$ the right hand side will also be increasing in t. This implies that the singular path has x increasing with time. Clearly this is not feasible. In such cases the optimal control will be of bang-bang type *i.e.* it will involve pulses of instantaneous development $(u^* = \infty)$ and intervals of no development $(u^* \equiv 0)$. It can be shown that in fact the deterministic optimal solution involves initially developing down to the level x which solves

(59) $$\frac{\pi_0}{\delta - r_\pi} P'(1-x) = \frac{\nu_0}{\delta - r_\nu} Q'(x) + \frac{\bar{R}\psi'(x)}{\delta} + c$$

and subsequently staying there. If this solution is applied in a feedback fashion development will take place whenever the left hand side (with π_0 replaced by $\pi(t)$) exceeded the right hand side (with ν_0 replaced by $\nu(t)$). But this rule is just the cost-benefit development rule discussed above (equation (43)). Thus certainty-equivalence can yield either the cost-benefit rule or the rule (56) in which the E.P.V. of returns from developed and wilderness land are modified by the factors M_1 and M_2. This is analogous to the application of certainty-equivalence to the single-stand harvesting model of Section 2 in which certainty-equivalence leads to the cost-benefit rule, when amenity-values of old-growth forest are growing faster in expectation than timber values $(a > b)$, and another rule in the other case. Here we have that when amenity values of wilderness are growing faster in expectation then those of developed land $(r_\nu > r_\pi)$ certainty-equivalence can lead to the cost-benefit rule, but can lead to a different rule in the opposite case.[29]

3.5 *Size-Independent Land-Service Flows*

Consider the case in which land-service flows are size-independent *i.e.* $P'' \equiv Q'' \equiv 0$. Without loss of generality assume $P(1\text{-}x) \equiv 1 \equiv Q(x)$; and for simplicity assume $c = 0$ and $\psi(x) \equiv 0$ *i.e.* ignore benefits from scientific breakthrough and development costs. In this case (from (48)), it can be seen that the optimal policy involves no development until

(60) $$\frac{\pi(t)}{\delta - r_\pi} > \left(\frac{1+\gamma}{\gamma}\right)\frac{v(t)}{\delta - r_v}$$

at which time *all* of the land is developed.

This is essentially the same as the optimal stopping rule (18) of Section 2 (the only difference being that the immediate benefits from harvesting $V(t)$ are replaced by the E.P.V. of the developed land; and that the hazard of catastrophic destruction is not present, $\Delta = \delta$), since the equation defining γ (49) is identical to that defining θ (equation (19)).

Furthermore the certainty-equivalence procedures given by equations (56) and (59) reduce to those of Section 2 (see 2.4) in the two cases $r_\pi \geq r_v$ and $r_\pi < r_v$.

4. *FINAL REMARKS*

The purpose of this essay has been to demonstrate analytically that applying cost-benefit analysis methodology to decision problems concerning the conservation or destruction of natural forest or wilderness, can lead to premature destruction of the resource, if the inherent irreversibility and uncertainty of the system are ignored. While this observation is not new, it has to date apparently only been demonstrated using somewhat artificial two- or three-period models, which involve all or nothing harvest decisions. In contrast here we have analyzed two different models of greater realism which explicitly recognize uncertainty in either (a) future timber and amenity values along with the hazard of catastrophic destruction, or (b) size-dependent service flows from wilderness land and developed land, along with the possibility of unpredictable benefits resulting from scientific breakthroughs utilizing the genetic resources of the wilderness.

For each model the optimal feedback policy, which maximizes expected present value of benefits net of costs, has been determined. Also for each model a policy, based on cost-benefit ideas, which prescribes harvesting or development whenever the expected (marginal) present value of benefits exceeds that of costs, has been determined. We have referred to this as *the naive application of cost-benefit methodology*, and shown that it always leads to premature harvesting or development. It is naive because not only does it ignore the uncertainty inherent in the problem, but also because it ignores the temporal aspect, *i.e.* that if one does not harvest a unit of the forest now, or in any given period, there is always the option of doing so later on.

A more sophisticated procedure, which allows for temporal choice, but again ignores uncertainty is the *certainty-equivalence procedure*. Here uncertain future quantities are replaced by their expected values, and the resulting deterministic temporal decision problem is solved. However it is shown in the paper that under certain circumstances (essentially if the benefits from conservation are growing faster in expectation than those of development) this certainty-equivalence procedure reduces to the naive cost-benefit procedure. It seems likely that as societies develop, appreciation of environmental services will increase, and that amenity benefits from conservation will grow faster than the benefits from development. Thus application of certainty-equivalence will provide no improvement over naive cost-benefit methods.

To fully incorporate the temporal aspect (with the irreversibility of wilderness destruction) and the future uncertainty in the benefits from conservation and development, the problem must be treated as one of *stochastic optimal control*. Such problems can be solved, at least in principle, by stochastic dynamic programming. This has been the methodology used here to find the optimal development or harvesting rules (in the single-stand model of Section 2 we have used a characterization of the optimal stopping rule, which is derived from the dynamic programming equation). In all cases the optimal policy is very similar in form to that of the naive cost-benefit procedure, the difference being that the costs of development (or the benefits of conservation) are inflated by a factor bigger than one. This factor is shown to increase with increasing uncertainty in future valuations. Thus the more uncertainty there is in the future benefits from conservation or development, the more conservative the optimal policy will be, and furthermore the more sub-optimal the naive cost-benefit procedure. Since in reality there is considerable uncertainty as to the future values of the amenity services of both natural forest and developed land and also of timber products, it seems from the point of view of public policy that a high degree of conservatism should be employed when making irreversible decisions concerning the destruction of natural forest and wilderness.

It should be stressed that these results do *not* depend on risk aversion. In fact they have been derived assuming risk-neutrality with the conventional goal of maximizing expected present value of benefits, net of costs, both expressed in monetary terms.

One argument often put forward by the proponents of logging of old-growth forest is that if the forest is not harvested it will eventually be lost through fire or some other natural catastrophe (the "use it or lose it" argument). In the single-stand analysis of Section 2, this component is explicitly incorporated into the model. It is shown how the presence of risk does indeed make the optimal policy less conservative, acting like the addition of a premium to the discount rate. There are some cases (*e.g.* when the amenity services provided by the forest are growing in expectation faster than the discount rate) where the presence of the risk of destruction could change the optimal policy to one of never harvesting, to one of which involves harvesting whenever timber values are suitably high compared with amenity service values. However by no means does the presence of risk always indicate that prompt harvesting should be carried out.

In the body of this paper attention has been focused on one particular technical shortcoming of cost-benefit analysis, and it has been indicated how the correct application of stochastic optimal control methodology can overcome this difficulty. However as indicated in the Introduction, there are many other difficulties involved with cost-benefit analysis. Problems of measuring amenity service benefits and costs of environmental and ecological damage have been assumed away in this paper. This does not mean that they are not important -- they have simply been ignored in order to focus on the one issue under consideration. Indeed as has been indicated in the Introduction including the true costs of logging or clearing natural forest or wilderness would likely invalidate most of the destruction currently taking place, even if cost-benefit analysis were applied in a naive way. Using stochastic optimal control methods to handle future uncertainty would make for still more conservative policies.

Also distributional and equity issues have been ignored. Again this is not because they are not important, but simply because they are not amenable to analysis of this sort. To handle these issues other methods are required. At present there seems to be no consensus as to how this should be done (see Kneese and Schultze [1985] for a critique of cost-benefit analysis and a discussion of alternative environmental ethics and principles). Economists and social and moral philosophers are currently wrestling with these issues. Perhaps one day new principles will be established. In a sense one could view the current uncertainty over environmental criteria as being like the uncertainty discussed in this paper. It seems reasonable, because we don't know yet exactly what we want, to be yet more conservative with respect to irreversible decisions. It would indeed be a tragic irony if a consensus on environmental goals were reached after the environment had been so irreversibly degraded as to make these goals unachievable.

FOOTNOTES

1. Quoted in *The First Eden* by D. Attenborough [1987]. This attractive book, a companion to the television series of the same name, is the source of the information on the Mediterranean world in this and the following paragraph.

2. From the Economist Survey - The Environment, *The Economist*, no. 7618, vol. 312 [September 2, 1989].

3. Das Gupta. *The Control of Resources*, 1982.

4. Indeed in the 1960's and 1970's international development agencies such as the World Bank encouraged and financed many such deforestation projects.

5. Although land owners of old who wished to preserve woodland for their own private hunting and pleasure use would be more likely to justify their actions on *legal* grounds rather than ethical or moral ones. Nonetheless they would be unlikely to claim an *economic* justification.

6. Freeman, [1985].

7. The Future of Forests, *The Economist*, no. 7712, vol. 320 [June 22, 1991].

8. The Economist Survey - The Environment, *The Economist*, no. 7618, vol. 312 [September 2, 1989].

9. The Future of Forests. *The Economist*, no. 7712, vol. 320 [June 22, 1991].

10. From a study of the New York Botanical Gardens cited in the Economist Survey - The Environment, *The Economist* [September 2, 1989].

11. The recreational value of U.S. forests cited above was determined in this way.

12. The model and analysis in this section is from Reed [1992].

13. We are assuming here no external environmental costs. The inclusion of such would clearly reduce the net value V, thereby making cost-benefit and optimal policies more conservative.

14. We use the bar to indicate an expectation, and the argument 0 to indicate the present value at time zero of the flow of amenity benefits from time zero on. The subscript f is used to signify that $\bar{A}_f$ represents the present value of *future* amenity flows. Also $\bar{A}_f(0)$ represents the expected present values of amenity services *foregone* if a harvest

takes place at time 0.

15. In the technical parlance of optimal stopping the problem would not be 'stable' (see Ross [1983], p. 53).

16. In this case the stopping problem resembles the famous *St. Petersburg Paradox* (see *e.g.* Smith [1988], p. 23). While at each point in time it pays, in expectation, to defer the harvest, the eventual return in timber benefits will be zero since if the hazard rate λ is positive, then with probability one, the stand will eventually be destroyed.

17. An alternative way of specifying the optimal harvest rule is: harvest if and only if the ratio of the current timber value $V(t)$ to amenity value $A(t)$ exceeds some critical value, *i.e.* if and only if

$$\frac{V(t)}{A(t)} \geq C^* = \left(\frac{1+\theta}{\theta}\right) \cdot \frac{1}{\Delta - a}.$$

18. These calculations are based on a two standard deviation band for $\ell n(V(t)/A(t))$ after one year, which would exhibit a normal distribution with variance σ^2.

19. In the case when the mean growth rate in amenity values, a, exceeds the risk-adjusted discount rate Δ, the optimal E.P.V. will be infinite (the integrals in (5) and (7) do not converge). However the cost-benefit procedure will also never prescribe a harvest (since $\bar{A}_f(0) = \infty$) and so will also result in an infinite E.P.V. In the case when the mean growth rate in timber values, b, exceeds the risk-adjusted discount rate, Δ, (but $a < \Delta$), both the cost-benefit rule and the optimal rule will result in an E.P.V. of magnitude $\bar{A}_f(0)$, and so the percentage loss in E.P.V. is again zero. However both of these cases are somewhat special and stretch the model, perhaps beyond the limits of its credibility, and we shall henceforth ignore them.

20. *e.g.* for LQG (linear-quadratic-Gaussian) problems certainty-equivalence provides the optimal solution (see Whittle [1982]). See also Reed and Errico [1986] for an example in forest harvest scheduling.

21. This rule can be derived by simple calculus. Also the same rule arises as the limit of the optimal rule as $\sigma^2 \to 0$, since in that case, $\theta \to \frac{\Delta - b}{b - a}$ if $b > a$; and $\theta \to \infty$ if $b \leq a$. Also in case (b) the C.E. rule is equivalent to a stochastic version of the Wicksell (or Fisher) rule of capital theory which would prescribe harvesting when the intrinsic net revenue from harvesting $R(V,A)$ (equation (16) is growing in expectation at the risk-adjusted discount rate Δ (see Reed [1992]).

22. The model and optimization analysis in Section 3 is from Clarke and Reed [1990].

23. If there were immediate benefits *from the act of developing* (*e.g.* the sale of timber from forested land), then this could be included in the parameter c, which would then represent the cost of development net of immediate benefits. In this case c could be negative.

24. We shall assume $\delta > r_\nu$ and $\delta > r_\pi$. If the first condition is violated and not the second, the marginal E.P.V. of developed land is infinite while that of wilderness land is finite; this is reversed if the second condition is violated but not the first. In these cases both the optimal policy and the cost-benefit policy will lead either to total development or total conservation.

25. See *e.g.* Reed [1984] for a similar calculation.

26. See Clarke and Reed [1990] for details.

27. Henry [1974] and Hodge [1984] consider the use of a *bias factor* which will force a cost-benefit rule to yield the optimal policy. They consider only simple two-period models.

28. This can be established using standard methods (the Pontryagin Maximum Principle) -- see *e.g.* Kamien & Schwartz [1981, Section II.13].

29. Indeed when $\bar{R}$ and C are both set to zero the analogy is even stronger, for in this case the certainty-equivalence rule with $r_\pi > r_\nu$ leads (from (56)) to the rule which prescribes development only when

$$\left(\frac{\pi_0}{\delta - r_\pi}\right) P'(1-x) > \left(\frac{\delta - r_\nu}{\delta - r_\pi}\right)\left(\frac{\nu_0}{\delta - r_\nu}\right) Q'(x)$$

i.e. when the marginal expected benefits from development exceed the marginal expected benefits foregone by a factor $(\delta - r_n)/(\delta - r_\pi)$. This is analogous to the C.E. rule in Section 2, with $b > a$ which prescribes harvesting only when the timber benefits exceed the E.P.V. of amenity benefits foregone by the factor $(\Delta\text{-}a)/(\Delta\text{-}b)$.

REFERENCES

Attenborough, D. [1987]. *The First Eden: The Mediterranean World and Man.* Collins, London.

Arrow, K.J. & A.C. Fisher [1974]. Environmental preservation, uncertainty and irreversibility, *Quarterly Journal of Economics* **88**:312-319.

Brock, W.A., M. Rothschild & J. Stiglitz [1988]. Stochastic capital theory, in G. Feiwel (ed.) *Essays in Honour of Joan Robinson.* Cambridge University Press, Cambridge.

Clarke, H.R. & W.J. Reed [1989]. The tree-cutting problem in a stochastic environment, *J. Econ. Dyn. and Control* **13**:569-595.

__________ & _________ [1990]. Land development and wilderness conservation policies under uncertainty: A synthesis, *Nat. Res. Model.* **4**:11-37.

Conrad, J.M. [1980]. Quasi-option value and the expected value of information, *Quarterly Journal of Economics* **94**:813-820.

Dasgupta, P. [1982]. *The Control of Resources.* Basil Blackwell, Oxford.

Freeman, A.M. [1985]. Methods for assessing the benefits of environmental programs, in (A.V. Kneese and J.L. Sweeney, eds.) *Handbook of Natural Resource and Energy Economics*, North-Holland, Amsterdam.

Henry, C. [1974]. Investment decisions under uncertainty: The irreversibility effect, *American Economic Review* **64**:1006-1012.

Hodge, I. [1984]. Uncertainty, irreversibility and the loss of agricultural land, *Journal of Agricultural Economics* **35**:191-202.

Kamen, M.I. & N.L. Schwartz [1981]. *Dynamic Optimization: The calculus of variations and optimal control in economics and management.* North-Holland, New York.

Kneese, A.V. & W.D. Schulze [1985]. Ethics and environmental problems, in (A.V. Kneese and J.L. Sweeney, eds.) *Handbook of Natural Resource and Energy Economics*, North-Holland, Amsterdam.

Mäler, K-G. [1985]. Welfare economics and the environment, in (A.V. Kneese and J.L. Sweeney, eds.) *Handbook of Natural Resource and Energy Economics*, North-Holland, Amsterdam.

Myers, N. [1979]. *The Sinking Ark: A new look at the problem of disappearing species.* Pergamon Press, Oxford.

_________ [1983]. *A Wealth of Wild Species: Storehouse for human welfare.* Westview Press, Boulder, CO.

Reed, W.J. [1984]. The effects of the risk of fire on the optimal rotation of a forest, *J. Environ. Econ. & Manag.* **11**:180-190.

_________ [1992]. The decision to conserve or harvest old-growth forest, *Ecol. Econ.* (to appear).

_________ & D. Errico [1985]. Optimal harvest scheduling at the forest level in the presence of the risk of fire, *Can. J. For. Res.* **16**:266-278.

Ross, S.M. [1983]. *Introduction to Stochastic Dynamic Programming.* Academic Press, New York.

Samuelson, P.A. & A. Scott [1975]. *Economics, Fourth Canadian Edition.* McGraw-Hill Ryerson Ltd., Toronto.

Smith, J.Q. [1988]. *Decision Analysis. A Bayesian approach.* Chapman and Hall, London.

Thompson, W.A. [1988]. *Point Process Models with Applications to Safety and Reliability.* Chapman and Hall, London.

Whittle, P. [1982]. *Optimization over Time. Dynamic programming and stochastic control, Vol. 1.* J. Wiley & Sons, Chichester.

		δ								
		.03			.05			.07		
		σ_z^2			σ_z^2			σ_z^2		
		0	.01	.02	0	.01	.02	0	.01	.02
$a = 0$	$b = 0$	1.00	1.46	1.70	1.00	1.35	1.53	1.00	1.29	1.44
	$b = .025$	3.50	4.16	4.77	1.83	2.15	2.40	1.50	1.73	1.91
	$b = .05$	--	--	--	11.00	12.09	13.16	3.00	3.24	3.51
$a = .025$	$b = 0$	1.00	1.19	1.36	1.00	1.17	1.31	1.00	1.16	1.28
	$b = .025$	1.00	2.00	2.62	1.00	1.50	1.77	1.00	1.37	1.55
	$b = .05$	--	--	--	6.00	7.16	8.27	2.00	1.32	2.62
$a = .05$	$b = 0$	--	--	--	1.00	1.10	1.20	1.00	1.10	1.19
	$b = .025$	--	--	--	1.00	1.19	1.38	1.00	1.17	1.32
	$b = .05$	--	--	--	1.00	2.62	3.73	1.00	1.55	1.86

TABLE 1. Values of the critical ratio, q^*, of timber value to expected present value of amenity benefits foregone. In all cases the hazard rate was $\lambda = .005$. Blank entries indicate that it is never optimal to harvest.

		δ = .03, σ^2			δ = .05, σ^2			δ = .07, σ^2		
		0	.01	.02	0	.01	.02	0	.01	.02
$a = 0$	$b = 0$	0	12.2	16.2	0	9.9	13.5	0	8.5	11.8
	$b = .025$	30.2	32.6	34.3	18.0	18.5	16.4	12.9	16.6	19.0
	$b = .05$	--	--	--	41.7	42.3	42.8	27.8	29.0	30.3
$a = .025$	$b = 0$	0	6.0	10.1	0	5.5	9.0	0	5.2	8.3
	$b = .025$	0	20.0	25.4	0	12.9	17.2	0	10.3	13.8
	$b = .05$	--	--	--	36.8	38.5	39.7	20.0	23.1	25.4
$a = .05$	$b = 0$	--	--	--	0	3.4	6.3	0	3.4	6.0
	$b = .025$	--	--	--	0	6.0	10.6	0	5.5	9.2
	$b = .05$	--	--	--	0	25.4	31.1	0	13.8	18.4

TABLE 2. Maximum percentage loss in E.P.V. through using the cost-benefit harvest rule, rather than the optimal one. In all cases the hazard rate was $\lambda = 0.005$. Blank entries correspond to situations in which it is never optimal to harvest. (See Footnote 19.)

		δ								
		.03 σ^2			.05 σ^2			.07 σ^2		
		0	.01	.02	0	.01	.02	0	.01	.02
$a = 0$	$b = 0$	0*	39.1	35.2	0*	33.8	28.8	0*	29.4	24.8
	$b = .025$	78.8	41.2	38.6	103.4	23.4	22.8	111.2	17.2	17.2
	$b = .05$	†	†	†	42.6	40.2	38.2	86.6	20.4	19.8
$a = .025$	$b = 0$	0*	103.6	84.6	0*	97.2	76.4	0*	92.2	70.8
	$b = .025$	0*	69.7	59.4	0*	44.4	37.6	0*	35.4	30.0
	$b = .05$	†	†	†	60.4	54.6	50.2	100.0	25.8	25.0
$a = .05$	$b = 0$	--	--	--	0*	115.4	104.6	0*	113.0	100.8
	$b = .025$	--	--	--	0*	105.6	87.2	0*	98.6	78.2
	$b = .05$	--	--	--	0*	89.6	76.4	0*	48.0	40.8

TABLE 3. Expected additional survival time (in years) through using the optimal harvest rule rather than the cost-benefit rule, when the hazard rate for catastrophic destruction is set at $\lambda = .005$. In the absence of any harvesting the expected life-time of the forest would be $1/\lambda = 200$ yrs. Cases marked with an asterisk (*) are for zero variance (deterministic model) with $a \geq b$. In such cases the optimal harvest rule collapses to the cost-benefit rule. However with zero variance and $a < b$ this is not the case (see Section 2.4). Dashes (--) occur in cases when both the cost-benefit rule and the optimal rule never prescribe a harvest $(a \geq \Delta)$, while daggers (†) correspond to cases $(b \geq \Delta, a < \Delta)$ where the cost-benefit rule prescribes a harvest while the optimal rule never does.

		δ								
		.03 σ_z^2			.05 σ_z^2			.07 σ_z^2		
		0	.01	.02	0	.01	.02	0	.01	.02
$a = 0$	$b = 0$	1.0 (0%)	1.0 (31.5%)	1.0 (41.2%	1.0 (0%)	1.0 (25.9%)	1.0 (34.6%)	1.0 (0%)	1.0 (22.5%)	1.0 (30.1%)
	$b = .025$	3.50 (0%)	3.50 (15.9%)	3.50 (26.6%)	1.83 (0%)	1.83 (14.9%)	1.83 (31.1%)	1.50 (0%)	1.50 (13.3%)	1.50 (21.5%)
	$b = .05$	1.17 (-)	1.17 (-)	1.17 (-)	11.00 (0%)	11.00 (9.0%)	11.00 (16.4%)	3.00 (0%)	3.00 (7.4%)	3.00 (14.5%)
$a = .025$	$b = 0$	1.0 (0%)	1.0 (16.0%)	1.0 (26.5%)	1.0 (0%)	1.0 (14.5%)	1.0 (23.7%)	1.0 (0%)	1.0 (13.8%)	1.0 (21.9%)
	$b = .025$	1.0 (0%)	1.0 (50.0%)	1.0 (61.8%)	1.0 (0%)	1.0 (33.3%)	1.0 (43.4%)	1.0 (0%)	1.0 (27.0%)	1.0 (35.5%)
	$b = .05$	1.08 (-)	1.08 (-)	1.08 (-)	6.00 (0%)	6.00 (16.2%)	6.00 (27.4%)	2.00 (0%)	2.00 (13.8%)	2.00 (23.7%)
$a = .05$	$b = 0$	1.0 (-)	1.0 (-)	1.0 (-)	1.0 (0%)	1.0 (9.1%)	1.0 (16.7%)	1.0 (0%)	1.0 (9.1%)	1.0 (16.0%)
	$b = .025$	1.0 (-)	1.0 (-)	1.0 (-)	1.0 (0%)	1.0 (16.0%)	1.0 (27.5%)	1.0 (0%)	1.0 (14.5%)	1.0 (24.2%)
	$b = .05$	1.0 (-)	1.0 (-)	1.0 (-)	1.0 (0%)	1.0 (61.8%)	1.0 (73.2%)	1.0 (0%)	1.0 (35.5%)	1.0 (46.3%)

TABLE 4. Values of the critical ratio $q_{C.E.}$, of timber value to expected present value of amenity benefits foregone for the certainty-equivalence harvest rule and in brackets the degree, r, to which $q_{C.E.}$ is too small compared with the optimal q^*; $r = 100(q^*-q_{C.E.})/q^*$. Dashes in brackets occur in cases when it is never optimal to harvest.

		δ = .03, σ^2 = 0	δ = .03, σ^2 = .01	δ = .03, σ^2 = .02	δ = .05, σ^2 = 0	δ = .05, σ^2 = .01	δ = .05, σ^2 = .02	δ = .07, σ^2 = 0	δ = .07, σ^2 = .01	δ = .07, σ^2 = .02
$a = 0$	$b = 0$	0	39.1	35.2	0	33.8	28.8	0	29.4	24.8
	$b = .025$	0	5.5	8.3	0	5.0	7.3	0	4.7	6.6
	$b = .05$	--	--	--	0	1.7	2.9	0	1.6	2.8
$a = .025$	$b = 0$	0	103.6	84.6	0	97.2	76.4	0	92.2	70.8
	$b = .025$	0	69.7	59.4	0	44.4	37.6	0	35.4	30.0
	$b = .05$	--	--	--	0	5.7	8.6	0	5.1	7.5
$a = .05$	$b = 0$	--	--	--	0	115.4	104.6	0	113.0	100.8
	$b = .025$	--	--	--	0	105.6	87.2	0	98.6	78.2
	$b = .05$	--	--	--	0	89.6	76.4	0	48.0	40.8

TABLE 5. Expected additional survival time through using the optimal harvest rule rather than the certainty-equivalence rule, when the hazard rate for catastrophic destruction is set at $\lambda = .005$. In cases where $a \geq b$ the entries coincide with those of Table 2, since in this case the certainty-equivalence rule coincides with the cost-benefit rule. Also in cases with zero variance the optimal rule reduces to the certainty-equivalence rule and so the additional survival time is zero. Dashes occur in cases where neither the certainty equivalence rule nor the optimal rule prescribe a harvest ($b \geq \Delta$ or $a \geq \Delta$).