



LORENTZ VIOLATION IN QUANTUM FIELD THEORY

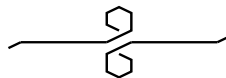
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Abstract

There are hints coming from some scenarios of modern String and Quantum Gravity theories that Lorentz invariance may not be an exact symmetry of Nature. The study of Lorentz violating theories therefore provides an insight into ultraviolet physics. We employ the Effective Field Theory technique to study the most general extension of the Standard Model and its Supersymmetric modifications with Lorentz-violating interactions of mass dimension five. We provide a complete classification of the interactions in these theories and determine a typical experimental sensitivity to the size of Lorentz violation. A detailed study of the operators that induce *CPT*-odd Electric Dipole Moments is performed and it is shown that they yield an independent constraint on Lorentz violating physics. We provide an application of Lorentz violating physics to the problem of generation of baryon asymmetry of the universe. A scenario of Leptogenesis driven by *CPT*-odd interactions is considered and confronted with experimental constraints on Lorentz-violating physics.

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Chapter 1

Introduction

IN 1887, when A. Michelson and E. Morley performed their experiment [1], the era of Special Relativity started. The complicated history of the development of Special Relativity was caused by different interpretations of the results of the experiment. Nevertheless, all interpretations exploited one simple, albeit strange, fact of the permanence of the speed of light in their foundation: the absence of a preferred reference frame, as one would say now. Neither G. FitzGerald, H. Lorentz, H. Poincaré, A. Einstein, nor A. Michelson and E. Morley realized how far-sighted and fundamental their conclusions would be. Nor could they predict that

a hundred years later the scientific community would come back to the very questions of the fundamental aspects of Special Relativity, in search for new properties of space and time.

There are reasons to do so. Not in order to put the brilliance of century-old discoveries under doubt, for at those energies available for the past hundred years, the properties of nature were indistinguishable from being Lorentz invariant. But to extend out beyond the range of validity of Special Relativity where new laws could be at power. Indeed, many believe that ultraviolet physics may be described by String theory. It is well known that some compactification limits of String theory and M-theories predict [2, 3] that space-time might be of a non-commutative character at very small distances. This property inevitably carries violation of Lorentz invariance. In certain scenarios, Loop Quantum Gravity predicts modifications of dispersion relations for particles at the Planck scale, in a Lorentz-noninvariant way [4]. There are also hints that Lorentz symmetry may be broken coming from cosmological observations. The simplest example is the Cosmic Microwave Background, which defines a preferred frame (Earth, Solar system and the Galaxy are all moving with respect to this frame). If there is a background vector or a tensor which is non-trivially coupled to the Standard Model, it then will manifest itself in the Lagrangian as a preferred direction violating Lorentz invariance. Dark energy could provide such a background. If represented by quintessence, a slowly varying scalar field, its evolution in time also

defines a preferred frame (which actually matches the CMB frame), and thus could be observed as effective Lorentz violation. These considerations encourage the search for departures from Special Relativity in the form of violation of Lorentz invariance at the fundamental level.

One of the earliest attempts to violate Lorentz invariance was to modify dispersion relations for particles, the idea which dates back to the 1960's [5]. The simplest way to modify the dispersion relations is to introduce terms of higher order in momentum:

$$E^2 = m^2 + p^2 + \frac{\eta}{M} \cdot p^3 + \frac{\kappa}{M^2} \cdot p^4 + \dots . \quad (1.1)$$

Here η and κ are the parameters of violation. Speculatively, if one ascribes violation of Lorentz invariance to Quantum Gravity, the scale M then is associated with the Planck scale $M \sim M_{\text{Pl}} \sim 10^{19}$ GeV. These relations can be modified for all sorts of particles — photons, electrons, protons or any other species pertinent to the model at hand (generically, all particles could have corrections to their dispersion relations, but the more “non-familiar” the particle, the more difficult it is to probe for effects). However, until recently such modifications were introduced in a completely *ad hoc* manner, without any connection with Quantum Field Theory (QFT). The emergence of a wide scale separation and a small parameter E/M (which for $M \sim M_{\text{Pl}}$ is much smaller than one) calls for application of an *Effective* Field Theory description. The

idea of applying the Effective Field Theory (EFT) to Lorentz violation is not very old, and it constitutes the main subject of this dissertation. It should be noted, however, that Lorentz violation via EFT is one of the examples of Quantum Field Theory in external backgrounds [6,7]. Lorentz violation (LV) is more generic than background field theory in the sense that it allows fairly arbitrary interactions on the Lagrangian level. It is special, however, in the sense that the LV background is not a gauge field and is typically constant.

One type of LV theories, an example of which is Quantum Gravity, is where the presence of the minimal length scale is a property of the theory. Since Lorentz invariance does not assume any dimensionful scale parameters for the space-time, it must be broken by the minimal length. An alternative type of LV theories is where Lorentz invariance breaking is realized dynamically. The main problem of this direction is to achieve condensation of a vector or a tensor field. This problem seems to be tractable in condensed matter physics, *e.g.* via a non-zero chemical potential, but it seems rather unfeasible for providing a Lorentz-breaking behavior to quantum field theory at the fundamental level. There are a number of candidate theories which Lorentz symmetry but none of them have a satisfactory ultraviolet (UV) description. To mention a few, there is the “Einstein-Aether theory”, which uses a Lagrange multiplier to “condense” a vector field on the equations of motion [8]. Another popular example is “ghost condensation” [9], where a scalar field is

condensed to a configuration, and the space-time derivative of this configuration plays the rôle of a Lorentz-breaking parameter. The latter theory cannot be fundamental, since it involves higher-dimensional interactions. However, it has not been proven that there exists a renormalizable UV completion of the ghost condensation theory. Quite similar to this problem is the aspect of writing an LV theory consistent with General Relativity [9–11]. One is bound to introduce new degrees of freedom even at the effective theory level, and provide a mechanism which condenses the new fields or suppresses their fast oscillations. In general, the dynamical breaking of Lorentz invariance and its coexistence with the General Relativity possess the well-known problems of ultraviolet completion and the presence of instabilities. We will not discuss these approaches here, as we concentrate on the Effective Field Theory description of LV.

The most powerful feature of the Effective Field Theory approach to Lorentz violation is that it can universally describe the low-energy theory where the Lorentz invariance is broken, without making any assumptions about its UV limits. In this way, one can generate the modified dispersion relations as in (1.1), and reproduce the consequences of space-time non-commutativity or the low-energy interactions caused by the ghost condensate, if desired.

The basic idea of EFT was motivated by Wilson’s approach to renormalization of a divergent theory [12–16]. The Effective Field Theory method allows one to pass

from a theory defined at one (presumably very high) energy scale Λ_{UV} to an effective theory describing dynamics at a lower energy scale by integrating out the degrees of freedom with momenta between these two scales. The high energy theory (the *UV* or *microscopic theory*) induces various kinds of interactions in the low energy effective Lagrangian. These interactions obtain a natural hierarchy in accord with their mass dimension — the higher the dimension, the less important the operator is. The usual terms in a Lagrangian in four dimensions have mass dimension four, so that the action is dimensionless. Operators of other mass dimensions thus can only enter with a dimensionful parameter so as to form an operator of dimension four:

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi + c_3 M \bar{\phi}\phi\varphi + \frac{c_5}{M}\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu} + \dots$$

Here c_3 and c_5 are dimensionless coefficients of order one, which are called the *Wilson coefficients*. In the low-energy effective Lagrangian, there is a natural candidate for such a dimensionful parameter — namely, the UV scale Λ_{UV} . This way, the operators of dimension less than four are enhanced by the ultraviolet scale and are called the *relevant* operators, while those of dimension greater than four are suppressed by that scale and are called the *irrelevant*¹ operators. The operators of dimension exactly four are called the *marginal* operators.

¹The actual meaning of the terms “relevant” and “irrelevant” is slightly deeper [12].

It is very commonly accepted that the Standard Model (SM) of fundamental interactions is an effective theory. There are a number of hints coming both from experiment (*e.g.* neutrino oscillations, existence of dark matter and dark energy) and from theory (*e.g.* gauge hierarchy problem, unification of fundamental forces, *etc.*) which indicate that there is New Physics governing at some high energy scale which we do not understand. Consequently, there have been numerous candidate theories proposed as extensions of the Standard Model in various directions. To mention just a few: there is a whole spectrum of phenomenological theories generated by string theory, there are a number of theories of quantum gravity, models with large extra dimensions, non-commutative extensions, supersymmetric models, and many more. Some of them predict that Lorentz invariance must be broken. Lorentz violation is therefore a tool which helps one study such theories and determine whether they are phenomenologically acceptable. One can make both quantitative and qualitative phenomenological statements and conclusions about Lorentz violation using EFT, without specifying whether the UV theory that sources LV is loop quantum gravity, string theory, or something else. Since UV dynamics is not known (or not specified), generically one has to admit all kinds of terms satisfying certain symmetry constraints into the effective Lagrangian accompanied by appropriate (but indeterminate) coefficients. The symmetry constraints are normally the requirements of gauge invariance (as the UV theory is not expected to break it), and in certain cases, invariance under

discrete symmetries.

Let us give some examples of how fundamental symmetries can provide probes of New Physics. Schematically, the Lagrangian of the Standard Model can be written as

$$\mathcal{L}_{\text{SM}} = - \sum_i \frac{1}{4g_i^2} F_i^2 + \sum_f \bar{\psi}_f \not{D} \psi_f + (\mathcal{D}_\mu H)^2 - \sum_{f,g} y_{fg} \bar{\psi}_f \psi_g H - V(H) ,$$

where F_i enumerates the gauge fields, ψ_f runs through all lepton and quark species of the Standard Model, H is the Higgs field, the term with y_{fg} symbolizes the Yukawa couplings and $V(H)$ is the Higgs potential. This theory is being verified in current accelerators up to the scale of 200-300 GeV (LHC will be able to probe for up to a few TeV). It is clear that if New Physics takes place at the Planck scale $M_{\text{Pl}} \sim 10^{19}$ GeV or at GUT scale $\Lambda_{\text{GUT}} \sim 10^{15}$ GeV, it will never be directly accessible to accelerator experiments.

Now assume that some unknown New Physics takes place at a high scale Λ . Integrating out the degrees of freedom with momenta higher than 200 GeV leads to an effective theory which has the degrees of freedom of the Standard Model. Much like 4-fermion interactions appear in the Lagrangian after the weak bosons are integrated out, one may naturally assume that the Standard Model Lagrangian can similarly

contain higher-dimensional interactions

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{n=1}^{\infty} \lambda_n \frac{1}{\Lambda^n} \mathcal{O}^{4+n} . \quad (1.2)$$

Here the coefficients λ_n are the Wilson coefficients of the operators, which run with the scale. The massive parameter Λ enters as the only dimensionful parameter associated with the New Physics. An example of such interactions can be

$$\mathcal{O} = (\bar{\psi}_1 \gamma_\mu \psi_1) (\bar{\psi}_2 \gamma^\mu \psi_2) . \quad (1.3)$$

Virtually arbitrary operators should be included in the Lagrangian (1.2), subject only to the invariance under gauge and other basic symmetries which must be respected. Having a higher-dimensional operator, such as (1.3) of dimension six, the scale Λ at which one can probe for new physics greatly depends on the symmetry properties of the operator itself. In particular, the operator may violate some of the discrete symmetries.

One of the important examples of higher dimensional operators breaking discrete symmetries is the neutrino mass operator of dimension five

$$\mathcal{O} = Y_{ij} H^\dagger L_i^\alpha H^\dagger L_{j\alpha} .$$

This operator is compatible with all symmetries of the Standard Model (except the lepton number) and is very likely to be the source of the observed neutrino oscillations. Therefore, neutrino oscillations can provide a window into physics at the scales of up to 10^{15} GeV.

In general, the more “fundamental” the broken symmetry, the higher the scale which can be probed:

- \mathcal{O} violates CP :

$$\mathcal{O} = \bar{e} i \gamma_5 e \cdot \bar{q} q, \quad \text{dim } 6 \quad \rightarrow \quad \Lambda \sim 10^6 \text{ GeV},$$

- \mathcal{O} violates flavor symmetry:

$$\mathcal{O} = \bar{s} \gamma^\mu d \cdot \bar{s} \gamma_\mu d, \quad \text{dim } 6 \quad \rightarrow \quad \Lambda \sim 10^7 \text{ GeV},$$

- \mathcal{O} violates baryon + lepton number:

$$\mathcal{O} = \epsilon_{abc} u_a^T C \gamma_5 d_b \cdot \bar{e}^+ u_c, \quad \text{dim } 6 \quad \rightarrow \quad \Lambda \sim 10^{16} \text{ GeV}.$$

In a Lorentz-violating theory, one assumes that the new interactions are coupled to vector or tensor backgrounds $c_n^{\mu\nu\dots}$

$$\mathcal{L}_{\text{LV}} = \mathcal{L}_{\text{SM}} + \sum_n c_n^{\mu\nu\dots} \frac{\mathcal{O}_{\mu\nu\dots}^n}{\Lambda^n}. \quad (1.4)$$

The Lorentz symmetry is more fundamental and there are no intrinsic parameters at hand to relate them to Λ , except for probably Λ_{GUT} or M_{Pl} . In this way, as we will see, effective Lorentz-violating theories can provide probes for physics at and beyond

the Planck scale.

One of the most illustrative examples of a generic Lorentz violating effective field theory is the Standard Model Extension (SME) by Don Colladay and V. Alan Kostelecky [17]. In this model, the authors introduced Lorentz violation at the dimension three and four levels into the Standard Model. The Lorentz violation was realized in the most general way, *i.e.* the only requirement to LV operators was that they retain the whole $SU_C(3) \otimes SU_L(2) \otimes U(1)$ gauge invariance of the Standard Model. Otherwise, they are arbitrary operators of mass dimension three and four coupled to external vectors or tensors, *e.g.*:

$$\mathcal{L}_{\text{SME}} \supset - (a_L)_\mu \bar{L} \gamma^\mu L - (a_R)_\mu \bar{R} \gamma^\mu R + \frac{1}{2} i (c_L)_{\mu\nu} \bar{L} \gamma^\mu D^\nu L + \frac{1}{2} i (c_R)_{\mu\nu} \bar{R} \gamma^\mu D^\nu R, \quad (1.5)$$

where L is the left-handed lepton doublet, R is the right-handed electron, D^ν the covariant derivative and $a_{L,R}$ and $c_{L,R}$ the LV background tensors. Aside of these aspects, generality also means that the operators of the matter sector considered in [17] are flavor-nondiagonal, *i.e.* they are matrices in the flavor space:

$$(a_L)_\mu \bar{L} \gamma^\mu L = (a_L)_\mu^{AB} \bar{L}^A \gamma^\mu L^B .$$

In the Quantum Electrodynamics subsector, Lorentz violation of this kind leads to observable effects, the absence of which has put strong constraints on the LV parameters

of SME. In particular, SME admits the Chern-Simons term, the phenomenological implications of which were first discussed in [18]:

$$\mathcal{L}_{\text{Chern-Simons}} = \frac{1}{2} (k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu} A^\lambda F^{\mu\nu} .$$

In Quantum Electrodynamics, this term causes vacuum birefringence of the electromagnetic radiation. Geomagnetic and cosmological observations allow one to constrain the magnitude of this term quite severely.

The work [17] was one of the pioneering attempts to introduce Lorentz symmetry breaking in the whole Standard Model. The Standard Model Extension might seem to be no less consistent as a field theory than the Standard Model itself; that is, it does not introduce extra theoretical problems. However, the SME does have difficulties when one tries to embed it into a curved space — unlike the Standard Model, which can easily be written in the context of General Relativity. But more importantly, it appears that Lorentz violation at dimension three and four level has particular problems with experiment, and there need to be very strong arguments why the LV parameters of SME should be so unnaturally small. The latter problems are more acute from the EFT point of view, since one would expect that the dimension three operators must be enhanced by a power of the ultraviolet scale, rather than miraculously suppressed.

Smallness of the magnitude of Lorentz background tensors comes about naturally in theories with irrelevant LV interactions, *i.e.* those of mass dimension greater than four. Although there seems to be a problem of renormalizability, one should keep in mind that they are *effective* field theories, and as such are regularized by their natural cutoff Λ_{UV} which limits the momenta running in the loops. There are only a few studies of Lorentz violation due to higher dimensional operators, and we now briefly review some of them.

As we mentioned before, Lorentz invariance is intrinsically broken in non-commutative theories [19–21], which originate from non-commutative geometry [22]. In such theories, the space-time coordinates are defined to be *operators* which satisfy certain commutation relations. A typical example occurs when the operators commute into a constant antisymmetric tensor:

$$[\hat{x}_\mu, \hat{x}_\nu] = \frac{i\theta_{\mu\nu}}{\Lambda_{\text{NC}}},$$

where $\theta_{\mu\nu}$ is a tensor structure of order one, and Λ_{NC} defines the scale of noncommutative physics. The Seiberg-Witten map allows the reflection of a noncommutative theory into a theory on an ordinary space-time, as a series in the order parameter $\theta_{\mu\nu}/\Lambda_{\text{NC}}$. Each term of the series is an operator coupled to $\theta_{\mu\nu}/\Lambda_{\text{NC}}$ and thus effectively an LV interaction. The scale of non-commutativity is constrained at the level

$\Lambda_{UV} > 5 \times 10^{14}$ GeV.

In [23] Myers and Pospelov have extended Quantum Electrodynamics with LV interactions of mass dimension five. The main purpose of the theory is to induce modifications of dispersion relations for fundamental particles, within QED, by introducing a number of LV interactions. The work [23] also specifies a thorough list of requirements a LV effective theory should meet. In brief, when writing a number of LV operators, one must ensure that they are all gauge invariant and Lorentz invariant, except for their coupling to the external LV background. They should not be reducible to lower dimension operators on the equations of motion (otherwise, at tree level, they would effectively have lower mass dimension and should be considered within a different appropriate theory). Similarly, if one is not worrying about non-perturbative effects, no LV operators can be a total derivative. Results from low-energy anisotropy searches allowed the authors to infer constraints of the order of 10^{-24} to 10^{-25} GeV⁻¹ on Lorentz violation at dimension five level.

Another interesting example can be found by *supersymmetrization* of a theory. As was recently discovered [24], supersymmetry may serve as a good selection rule for LV operators. When taken together with gauge invariance, supersymmetry appears to prohibit all dimension three and four LV operators. It also has a useful property of redefining the ultraviolet scale to that of supersymmetry breaking, as will be discussed later. A Lorentz violating extension of supersymmetric QED will be addressed in

Chapter 4.

We emphasize that the Effective Field Theory approach is very efficient in terms of phenomenology of LV. If present in nature, the violation of Lorentz invariance cannot comprise a large effect at low energies, and in fact has been strongly bounded. LV interactions are corrections to the Standard Model Lagrangian and therefore can be easily linked to specific phenomena implied by their presence. There are numerous experiments that have been set up to test these phenomena, and more are expected. Let us provide some familiar examples where Lorentz violation has been investigated.

First, there are a number of Earth-bound experiments which search for an effective anisotropy of space. One type of experiments is a “comagnetometer” or “clock-comparison” experiment which measures and compares frequencies of oscillations of two different particles in a magnetic field [25,26]. If these two particles have spin couplings to the LV background which do not scale exactly as their magnetic g -factors, they would exhibit a LV difference of the precession frequencies which *cannot* be caused by fluctuations of the magnetic field. The works [26] and [25] obtain similar limits on the LV frequency shift at the level of 100 nHz which translates into a limit of 10^{-31} GeV on certain combinations of LV parameters in Eq. (1.5).

Another type of experiment searching for deviations from Lorentz invariance is a “torsion pendulum” experiment, which reaches the topmost sensitivity to Lorentz violation in the electron spin sector [27–29]. This experiment is looking for a “free”

torque of a magnetized torsion pendulum mounted on a fiber in a magneto-isolated space. If the LV background couples to the particle's and (as a consequence) the atomic spin, the pendulum would obtain a non-zero torque. The experiment sets the best limit on Lorentz violation in the electron sector at the level of 10^{-28} GeV.

There are a number of astrophysical observations which significantly bound various Lorentz violating parameters. The most noticeable effect which strongly constrains the LV parameter η [30] follows directly from the dispersion relations in Eq. (1.1). Particles with modified dispersion relations can have a maximum achievable cut-off frequency of synchrotron emission [30], or they can emit Čerenkov radiation in vacuum. For the former effect, the observations of synchrotron emission of a high frequency coming from Crab Nebula constrain the parameter $\eta > -10^{-7}$ from below [30], assuming M to be the Planck scale. For the latter effect, it results in energy loss by energetic particles. Indeed, the dispersion relations (1.1) allow the processes $p \rightarrow p\gamma\gamma$, $p \rightarrow \bar{\nu}\nu$, *etc* for the protons in the cosmic rays. If these reactions were open, the cosmic rays would not have been observed. From mere existence of ultra-high energy ($E \sim 10^{20}$ eV) cosmic rays one can put the most stringent constraints [31]: $\eta < 10^{-15}$.

The third class of phenomena has to do with polarizational effects of photon radiation over cosmological distances due to the modification of dispersion relations [32]. For the photon, the corrections to the dispersion relations resulting from LV

operators of dimension five are predicted to be of the opposite sign for left- and right-circularly polarized photons:

$$E^2 = p^2 \pm \frac{\xi}{M_{\text{Pl}}} \cdot p^3 + \dots, \quad (1.6)$$

where ξ is a new parameter. This modification causes different polarizations of waves to propagate with different velocities, inducing rotation of the polarization plane of the radiation, *i.e.* vacuum birefringence. For a source located over a cosmological distance, this might create quite a noticeable rotation of the polarization plane. The same photon dispersion relations also lead to dependence of the speed of light propagation on its wavelength, detectably smearing a signal from a distant object. The work [33] uses data on polarization of radiation from distant galaxies to place a limit of 10^{-5} on the parameter ξ .

Looking towards the future, the existing experiments are improving. It is expected that the accuracy of the torsion pendulum and clock comparison experiments can be increased by several orders of magnitude. A new experimental technique is prepared at Princeton University, which allows for the most precise measurements of a magnetic field with the use of a self-compensated atomic comagnetometer, and can also be deployed for the search of LV [34]. Space-based experiments can also bring further advancement of tests of Lorentz violation [35, 36].

1.1 Overview of Research

We have given an introduction to the concepts of Lorentz violation and described a few models exploiting the Effective Field Theory approach to LV. This method still leaves a number of questions which immediately follow from these models, but have not been considered in the literature. In the rest of this manuscript we apply the EFT method to the problems which we found important and compelling in the course of this PhD research. In this section we briefly outline the essence of each of the problems.

Lorentz violation in the Standard Model. At present, within the Standard Model, LV operators of dimension three and four have been thoroughly studied and constrained. At the level of higher-dimensional operators, a complete generic Effective Field Theory description of Lorentz violation in the Standard Model, together with its predictions for observations, is desired, as only some particular subsectors of the Standard Model have been studied in the literature. We address this issue in Chapter 2, where we classify all LV interactions of mass dimension five and provide basic analysis on phenomenological properties of these operators, giving plausible experimental constraints on their magnitude.

Aside from the task of classification of the existing operators, there is also a problem of dimensional transmutation in LV theories. Obviously, higher dimensional operators can induce lower dimensional ones via loop effects. This poses a severe nat-

urality problem for these higher dimensional operators, for the lower dimensional interactions are virtually excluded by experiment. This problem is very acute in the EFT approach to LV in the Standard Model, since among the numerous LV interactions in such a model, some will inevitably be found that mix with the operators of mass dimension three, for example. The existence of dimension three operators would mean the breakdown of the EFT. There is no general or universal solution to this problem. A useful approach, however, is to enforce conservation of certain symmetries, at least at loop order, such as T -invariance or supersymmetry. A particularly feasible approach is the facilitation of Lorentz-irreducible tensor structures, which significantly reduces and restricts the set of dangerous LV interactions. Within this approach, as we will see, the dangerous interactions take a very specific form. Effective Field Theory therefore imposes certain restrictions on the fundamental LV theory with respect to these operators.

Electric Dipole Moments as LV Interactions. A somewhat deeper insight in the phenomenology of the LV extension of the Standard Model can be given by studying those sectors of this model that predict sharp observational signatures. Of particular importance are those LV interactions which induce Electric Dipole Moments (EDMs) of fundamental particles, nuclei and atoms. The search for permanent EDMs has fundamental value for the consistency of the Standard Model of interactions. Therefore, if a non-zero Electric Dipole Moment is observed at some time, one would need

to be able to correctly interpret such results. A hypothetically observed EDM would then be a composite of two sources: a conventional CP -odd term and a LV CPT -odd contribution. While an observation of an EDM would always signify a departure from the Standard Model, one has to be able to discern between these two contributions to make statements in favour of conventional EDM or LV EDM models. In Chapter 3 we investigate the particular sector of the LV Standard Model which introduces CPT -odd contributions to EDMs and examine feasible techniques of distinguishing such contributions in experiment. We show that a combination of EDM measurements performed on different systems (neutron, deuteron, heavy atoms) would allow to determine the true origin of the effect, CPT -odd or CPT -even.

Lorentz violation in Supersymmetric Theories. It has recently been shown [24] that supersymmetry may serve as a very powerful selection rule in LV theories, capable of solving the naturalness problem. Lorentz violation compatible with supersymmetry starts with operators of dimension five. Supersymmetry greatly restricts the number of allowed LV interactions and brings them to a very specific form. In Chapter 4 we study dimension five and six LV operators in the theory of Supersymmetric Quantum Electrodynamics, analyze their properties at the quantum level, and describe the observational consequences of Lorentz violation in this theory. In addition, we will see that LV interactions do not produce destabilizing quantum effects such as D-term anomalies and gauge anomalies. We will pursue the long standing problem of the

possible emergence of a Chern-Simons term, and a definite answer will be given that the these term cannot appear in supersymmetric theories at the quantum level, even when supersymmetry is softly broken. With all these properties taken into account, the problem turns into a correct effective theory.

CPT-odd equilibrium baryogenesis. One of the most favorable motivations for studying Lorentz violation is the possibility of *CPT*-odd equilibrium baryogenesis. The idea that the Baryon Asymmetry of the Universe (BAU) can be generated in thermodynamic equilibrium by *CPT*-odd interactions has already been partially explored [37], [38]. Furthermore, in recent years, there has been considerable progress in understanding baryon- and lepton-number violating processes in the Early Universe induced by sphalerons. In models involving heavy right-handed neutrinos, additional information comes from measurements of neutrino oscillations. However, there is no convincing theory of *CPT*-odd baryogenesis which can link the size of higher-dimensional LV operators to the observed baryon asymmetry.

In Chapter 5 we construct a Leptogenesis scenario where the Universe first develops a lepton number asymmetry via a combination of *CPT*-odd interactions and dimension five operators that induce the majorana mass for neutrinos. The asymmetry then propagates into the baryonic sector via sphaleron transitions. This type of scenario is superior to the direct *CPT*-baryogenesis, since the initial asymmetry is seeded at significantly higher temperatures. We will determine the strength of *CPT*-

odd interactions capable of producing the observed value of BAU and compare it with the observational tests of Lorentz invariance. As we will see, the combination of low-energy constraints and astrophysical data will strongly disfavor the obtained range of Lorentz violation. We will discuss whether (and with what certainty) one can make a statement that the whole series of relevant CPT -odd Leptogenesis mechanisms are overconstrained by these estimates.

Chapter 2

Dimension 5 Operators in the Standard Model

In this chapter we give a complete account of LV operators of dimension five in Quantum Electrodynamics and in the Standard Model. As a basis for our construction we use the approach taken in [23], which defines the meaning of true LV operators in EFT. We, however, try to make our LV theory as generic as possible and allow Lorentz-violating operators to take any tensor form. It is easy to see then that a theory with LV interactions of mass dimension five admits a more diverse set of operators (and as a consequence more LV backgrounds $C^{\mu\nu\dots}$) than one has at dimension three and four levels. Since there is a significant freedom in the choice of LV spurions, we take the approach that each of them represents an irreducible Lorentz tensor structure,

which significantly facilitates the analysis of loop effects, and often protects dimension five operators from transmuting into lower dimensions when the quantum effects are taken into account. The most convenient way to achieve this is to apply the Young tableaux method. Thus, we augment the requirements specified in [23] by demanding that LV spurions be irreducible tensors under the Lorentz group transformations. In total, these conditions look as follows: an LV operator of specific dimension must be

- gauge invariant
- Lorentz invariant, after contraction with a background tensor
- not reducible to lower dimension operators by the equations of motion (EOM)
- not reducible to a total derivative
- coupled to an irreducible background tensor.

One can show that operators built in this manner can be subdivided into three main groups. The first group includes the “unprotected operators”, *i.e.* those which can generate lower-dimensional interactions by developing quadratic divergencies. Such operators are dangerous, and as a rule, severely constrained by strong limits on lower dimensional operators multiplied by the square of the UV scale. The second group is the UV-enhanced operators, which induce modifications of the dispersion relations that grow with a particle’s energy. These operators induce new testable LV

signatures in laboratory experiments and in astrophysics, and are severely constrained by both. The last group is formed by “soft LV interactions” which are protected from developing quadratic divergencies at loop level and do not significantly modify the propagation of energetic particles in the UV. Typically, such operators are constrained by the laboratory searches of spatial anisotropy.

We start with building an LV extension of the theory of Quantum Electrodynamics (QED). The latter is one of the most popular testing grounds for LV [23, 30, 32, 39] and many generic features of an LV extension of a theory can be captured already at the level of QED. Furthermore, the detailed study of LV QED facilitates a smooth transition to the Standard Model, to which we proceed immediately after.

2.1 Dimension V Operators in QED

In essence, we build a generalization of Myers-Pospelov electrodynamics introduced in [23]. The Lagrangian of QED was modified by adding a number of LV operators which were generated by an absolutely symmetric 3-rank irreducible tensor background. Originally, the choice of symmetric tensors was motivated by the fact that LV operators can modify the dispersion relations, and also that they do not induce dangerous quadratic divergencies. Our intention is to classify *all* dimension five operators in Quantum Electrodynamics, and thus the list of the external LV tensors will necessarily be expanded. Generic operators will produce new non-minimal interac-

tions between the electron and the photon. The LV extension of the photon sector of QED appears to be the most simple, whereas the matter sector shows a rich structure of LV terms.

2.1.1 Purely Gauge Operators in QED

Dimension five LV interactions can admit LV backgrounds up to rank five. Higher ranks can appear only in combination with operators of dimension six or higher. In this particular sector, the method of Young tableaux is inferior to the method of simple enumeration: there are 26 numbered Young tableaux to consider, which in fact lead to only one LV operator.

It can be shown that a generic content of a gauge invariant tensor has to be bilinear in the field strength $F_{\mu\nu}$ and contain one extra derivative, which must be a covariant derivative in the case of a non-abelian field.

The only non-vanishing terms that satisfy these properties are

$$F_{\mu\nu}\partial^\nu\tilde{F}^{\mu\rho}, \quad F_{\mu\nu}\partial^\nu F^{\rho\sigma}, \quad F_{\mu\lambda}\partial_\nu\tilde{F}^{\rho\lambda} \quad \text{and} \quad F^{\mu\nu}\partial^\lambda F^{\rho\sigma}. \quad (2.1)$$

It can be easily seen that the first two terms are *reducible* on the equations of motion, and, in accord with our requirements should be ignored. Amongst the two structures left, $F_{\mu\lambda}\partial_\nu\tilde{F}^{\rho\lambda}$ and $F^{\mu\nu}\partial^\lambda F^{\rho\sigma}$, the first has been studied in [23] and shown to modify

the dispersion relations of the photon. It was shown in particular that this operator has to be contracted with an irreducible absolutely symmetric tensor,

$$C^{\mu\nu\rho} F_{\mu\lambda} \partial_\nu \tilde{F}_\rho{}^\lambda, \quad C^\mu{}_\mu{}^\rho = 0. \quad (2.2)$$

Conditions of absolute symmetry and irreducibility of the tensor $C^{\mu\nu\rho}$ follow from the requirement of independence of this operator of the lower-rank operators of (2.1), which is also a way of protection against the mixing with such operators at the loop level.

The last structure in (2.1), the five-index object $F^{\mu\nu} \partial^\lambda F^{\rho\sigma}$, upon a naive substitution into the equations of motion, seems to modify the dispersion relations in a manner similar to (2.2). However, that would be a misleading conclusion. As in the case of the 3-rd rank operator just discussed, one needs to separate it from all lower-rank interactions. In other words, one needs to subtract all possible $g^{\mu\nu}$ and $\epsilon^{\mu\nu\rho\sigma}$ traces of this term, and *then* substitute it into the equations of motion. It turns out that this operator is completely expressible in terms of its $\epsilon^{\mu\nu\rho\sigma}$ -trace, which coincides

with the operator $C^{\mu\nu\rho}$:

$$\begin{aligned}
F_{\mu\nu}\partial_\lambda F_{\rho\sigma} = & \\
& - \frac{1}{5} \epsilon_{\mu\nu\rho\chi} \tilde{F}^{\zeta\chi} \partial_\lambda F_{\zeta\sigma} + \frac{1}{5} \epsilon_{\mu\nu\sigma\chi} \tilde{F}^{\zeta\chi} \partial_\lambda F_{\zeta\rho} + \\
& + \frac{1}{5} \epsilon_{\rho\sigma\mu\chi} \tilde{F}^{\zeta\chi} \partial_\lambda F_{\zeta\nu} - \frac{1}{5} \epsilon_{\rho\sigma\nu\chi} \tilde{F}^{\zeta\chi} \partial_\lambda F_{\zeta\mu} - \\
& - \frac{1}{10} \epsilon_{\mu\lambda\rho\chi} \tilde{F}^{\zeta\chi} \partial_\nu F_{\zeta\sigma} + \frac{1}{10} \epsilon_{\nu\lambda\rho\chi} \tilde{F}^{\zeta\chi} \partial_\mu F_{\zeta\sigma} + \\
& + \frac{1}{10} \epsilon_{\mu\lambda\sigma\chi} \tilde{F}^{\zeta\chi} \partial_\nu F_{\zeta\rho} - \frac{1}{10} \epsilon_{\nu\lambda\sigma\chi} \tilde{F}^{\zeta\chi} \partial_\mu F_{\zeta\rho} .
\end{aligned}$$

This relation shows that it is not possible to bring the rank five operator to an irreducible form, and consequently there is no dimension 5 LV interaction contracted with an irreducible rank five tensor. We conclude that the only possible LV operator in QED is $C^{\mu\nu\rho}$. All these arguments straightforwardly extend to a non-abelian gauge field.

2.1.2 Matter Sector of QED

In contrast to what we have seen in the gauge sector, the LV terms in the matter sector of QED have much wider variety. The reason for that is because the operators can be formed both by using covariant derivatives \mathcal{D}_μ and by inserting gamma matrices.

In order to make the enumeration of operators more systematic, we use Young tableaux (see Appendix B). Omitting the details, we show the result for the LV

operators in the matter sector,

$$\begin{aligned}
\mathcal{L}_{\text{QED}}^{\text{matter}} = & \\
& [c_1^\mu \cdot \bar{\psi} \gamma^\lambda F_{\mu\lambda} \psi^+] + [c_2^\mu \cdot \bar{\psi} \gamma^\lambda \gamma^5 F_{\mu\lambda} \psi^-] \\
& + \tilde{c}_1^\mu \cdot \bar{\psi} \gamma^\lambda \tilde{F}_{\mu\lambda} \psi^+ + \tilde{c}_2^\mu \cdot \bar{\psi} \gamma^\lambda \gamma^5 \tilde{F}_{\mu\lambda} \psi^- \\
& + f_1^{\mu\nu} \cdot \bar{\psi} F_{\mu\nu} \psi^- + f_2^{\mu\nu} \cdot \bar{\psi} F_{\mu\nu} \gamma^5 \psi^- \\
& + h_1^{\mu\nu} \cdot \bar{\psi} \mathcal{D}_{(\mu} \mathcal{D}_{\nu)} \psi^+ + h_2^{\mu\nu} \cdot \bar{\psi} \mathcal{D}_{(\mu} \mathcal{D}_{\nu)} \gamma^5 \psi^+ \\
& + C_1^{\mu\nu\rho} \cdot \bar{\psi} \gamma_{(\mu} \mathcal{D}_\nu \mathcal{D}_{\rho)} \psi^- + C_2^{\mu\nu\rho} \cdot \bar{\psi} \gamma_{(\mu} \gamma^5 \mathcal{D}_\nu \mathcal{D}_{\rho)} \psi^+ \\
& + D_1^{\mu\nu\rho} \cdot \bar{\psi} \gamma_{(\mu} F_{\rho)\nu} \psi^+ + D_2^{\mu\nu\rho} \cdot \bar{\psi} \gamma_{(\mu} F_{\rho)\nu} \gamma^5 \psi^- \\
& + E_1^{\mu\nu\rho\lambda} \cdot \bar{\psi} \sigma_{\mu\nu} \mathcal{D}_{(\rho} \mathcal{D}_{\lambda)} \psi^- + E_2^{\mu\nu\rho\lambda} \cdot \bar{\psi} \sigma_{\mu(\lambda} F_{\rho)(\nu)} \psi^+ + E_3^{\mu\nu\rho\lambda} \cdot \bar{\psi} \sigma_{\mu[\nu} F_{\rho](\lambda)} \psi^+ \\
& + E_4^{\mu\nu\rho\lambda} \cdot \bar{\psi} (\sigma_{\mu[\nu} \mathcal{D}_{\rho]} \mathcal{D}_{(\lambda} - \sigma_{\nu](\mu} \mathcal{D}_{\lambda)} \mathcal{D}_{\rho]} + 2 \sigma_{\nu\rho} \mathcal{D}_{(\mu} \mathcal{D}_{\lambda)}) \psi^-.
\end{aligned} \tag{2.3}$$

Here we used the (square) round brackets to denote the (anti)symmetrization in the corresponding indices. The + and - superscripts in this formula refer to the parity of the corresponding LV term under charge conjugation. We stress again that all structures shown here assume their coefficients to be irreducible tensors of the corresponding rank. Square brackets over the first two operators, c_1^μ and c_2^μ , indicate that these two terms vanish upon the use of EOM, but we list them because they become nontrivial in the non-abelian case.

We would like to make a side note on the symmetrizations in the interactions

in (2.3) and in subsequent formulae. We take the field operators to have certain symmetries (dictated by the corresponding Young tableaux), while their Wilson coefficients are taken to be just traceless tensors. Equivalently, one could have cast all symmetrizations onto the Wilson coefficients, *e.g.* $E_1^{\mu\nu\rho\lambda}\bar{\psi}\sigma_{\mu\nu}\mathcal{D}_{(\rho}\mathcal{D}_{\lambda)}\psi \rightarrow E_1^{(\mu\nu)(\rho\lambda)}\bar{\psi}\sigma_{\mu\nu}\mathcal{D}_\rho\mathcal{D}_\lambda\psi$, or just imply $E_1^{\mu\nu\rho\lambda}$ to obey the corresponding symmetries: $E_1^{\mu\nu\rho\lambda}\bar{\psi}\sigma_{\mu\nu}\mathcal{D}_\rho\mathcal{D}_\lambda\psi$. We emphasize that this is only a matter of notation, and choose to expose the symmetry properties of tensors via explicit symmetrizations on the Lorentz indices of the field operators.

2.1.3 1-loop RG coefficients

If the violation of Lorentz invariance is a true UV phenomenon, one has to evolve the operators down to the IR scale, where the majority of tests are performed. For this purpose, we study the renormalization group (RG) equations for operators (2.2) and (2.3). The RG running brings about the change in the magnitude of Wilson coefficients and mixing of different operators.

Due to a rather large number of LV operators, one might expect that this mixings can be rather complicated. They are simplified, however, by two reasons, namely the discrete symmetries and irreducibility of the Lorentz tensors, which reduce this mixing to a minimum. The charge conjugation symmetry, which is an exact symmetry in QED, prevents the mixing of C -odd and C -even operators. The irreducibility of the

background tensors dictates that any tensor of higher rank will not mix with a tensor of a lower rank. Thus, only operators of the same rank can admix to each other.

A brief examination of (2.3) reveals that \tilde{c}_1^μ cannot mix with \tilde{c}_2^μ due to C -parity. Similarly, $f_{1,2}^{\mu\nu}$ cannot mix with $h_{1,2}^{\mu\nu}$, but they can mix within each other. At the level of rank three tensors, the photon operator (2.2) is even under charge conjugation, and therefore it can mix only with the $C_2^{\mu\nu\rho}$ operator.

As we have admitted generic tensor structures to the theory we need to ensure that the latter is free of quadratic divergencies. It is obvious that quadratically divergent operators must necessarily couple to a vector background, as there are no dimension three structures which would be simultaneously CPT -odd and contracted with a tensor background. In our list (2.3), only the \tilde{c}_1^μ term generates quadratically divergent corrections to LV dimension three operators. The result of explicit computation gives the following set of RG equations:

$$\begin{aligned}
 \frac{d}{dt} \tilde{c}_1^\mu &\stackrel{\text{formally}}{=} -\frac{13e^2}{96\pi^2} \tilde{c}_1^\mu & \frac{d}{dt} \tilde{c}_2^\mu &= \frac{e^2}{32\pi^2} \tilde{c}_2^\mu \\
 \frac{d}{dt} f_{1,2}^{\mu\nu} &= -\frac{7e^2}{24\pi^2} f_{1,2}^{\mu\nu} + i \frac{7e^2}{48\pi^2} \tilde{f}_{2,1}^{\mu\nu} & \frac{d}{dt} h_{1,2}^{\mu\nu} &= \frac{e^2}{6\pi^2} h_{1,2}^{\mu\nu} \\
 \frac{d}{dt} C_1^{\mu\nu\rho} &= \frac{25e^2}{48\pi^2} C_1^{\mu\nu\rho} & \frac{d}{dt} C_2^{\mu\nu\rho} &= \frac{25e^2}{48\pi^2} C_2^{\mu\nu\rho} - \frac{5e^2}{48\pi^2} C^{\mu\nu\rho} \\
 & & \frac{d}{dt} C^{\mu\nu\rho} &= \frac{e^2}{48\pi^2} C^{\mu\nu\rho} - \frac{e^2}{6\pi^2} C^{\mu\nu\rho} \\
 \frac{d}{dt} D_1^{\mu\nu\rho} &= -\frac{e^2}{16\pi^2} D_1^{\mu\nu\rho} & \frac{d}{dt} D_2^{\mu\nu\rho} &= \frac{5e^2}{48\pi^2} D_2^{\mu\nu\rho} \\
 \frac{d}{dt} E_1^{\kappa\mu\nu\rho} &= \frac{13e^2}{24\pi^2} E_1^{\kappa\mu\nu\rho} & \frac{d}{dt} E_3^{\kappa\mu\nu\rho} &= \frac{e^2}{3\pi^2} E_3^{\kappa\mu\nu\rho} \\
 \frac{d}{dt} E_2^{\kappa\mu\nu\rho} &= \frac{e^2}{12\pi^2} E_2^{\kappa\mu\nu\rho} & \frac{d}{dt} E_4^{\kappa\mu\nu\rho} &= \frac{e^2}{8\pi^2} E_4^{\kappa\mu\nu\rho} .
 \end{aligned}$$

Here $t = \log \mu$, and we have introduced $\tilde{f}_{1,2}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} (f_{1,2})_{\rho\sigma}$. As anticipated, most operators renormalize independently. It is also clear that one can easily form the linear combinations of LV interactions that are eigenvectors of one-loop RG equations.

2.2 Classification of Operators of Dimension V in the Standard Model

In the Standard Model, the set of LV operators is more complicated, due to the wider gauge group. Since the LV physics in our approach is associated with the UV scale, the LV operators must respect all the symmetries which are present at that scale.

Although the UV physics and its symmetries are not known, it is quite natural to require that LV interactions be invariant under $SU_C(3) \otimes SU_L(2) \otimes U(1)$. Clearly the existence of families causes coefficients of all LV interactions in the matter sector (2.3) to be matrices in the flavor space [40]. Furthermore, the presence of the Higgs sector creates new possibilities for LV interactions. However, there is one simplification arising from intrinsic chirality of SM spinors, which together with gauge invariance would essentially prohibit all $E^{\kappa\mu\nu\rho}$ operators at dimension five level. In the rest of this section, we present our results for LV operators in different sectors of the SM.

2.2.1 Operators in the Gauge Sector of the Standard Model

As in the QED case, the gauge sector is the simplest since we already know that the only possible LV gauge structure is (2.2). Thus we replicate this structure for the three gauge groups of the SM:

$$\mathcal{L}_{\text{SM}}^{\text{gauge}} = C_{U(1)}^{\mu\nu\rho} \cdot F_{\mu\lambda} \partial_\nu \tilde{F}_\rho{}^\lambda + C_{SU_L(2)}^{\mu\nu\rho} \cdot \text{tr} W_{\mu\lambda} \mathcal{D}_\nu \tilde{W}_\rho{}^\lambda + C_{SU_C(3)}^{\mu\nu\rho} \cdot \text{tr} G_{\mu\lambda} \mathcal{D}_\nu \tilde{G}_\rho{}^\lambda. \quad (2.4)$$

2.2.2 Matter Sector of the Standard Model

Although the matter sector of the Standard Model is more diverse than that of QED, the number of “types” of operators is smaller. Due to chirality of both leptons and quarks, the structures with an even number of γ -matrices in (2.3) are not $SU_L(2)$ -

gauge invariant. That greatly simplifies the structure of the LV Lagrangian, as one can only have operators with an odd number of gamma matrices. In the resulting Lagrangian we have to abandon the C -parity eigenstates, (2.3) and list operators using $V - A$ and $V + A$ combinations of Dirac matrices.

Since in QED

$$\mathcal{D}_{[\mu}\mathcal{D}_{\nu]} = ieF_{\mu\nu} , \quad (2.5)$$

the first two terms in (2.3) actually vanish on the equations of motion. However, with the exception of right-handed leptons, the covariant derivatives for SM field contain different gauge potentials. For example, for quarks one has

$$\mathcal{D}_{[\mu}\mathcal{D}_{\nu]} = iYg'F_{\mu\nu} + igW_{\mu\nu} + ig_3G_{\mu\nu} , \quad (2.6)$$

where Y is the hypercharge of the quark. The use of equations of motion allows one of the operators $\bar{Q}\gamma^\lambda F_{\mu\lambda}Q$, $\bar{Q}\gamma^\lambda W_{\mu\lambda}Q$ or $\bar{Q}\gamma^\lambda G_{\mu\lambda}Q$ to be expressed in terms of the other two, but one cannot eliminate such operators completely. Taking this into

account, in the quark sector one obtains the following LV interactions:

$$\begin{aligned}
 \mathcal{L}_{\text{SM}}^{\text{quark}} = & \\
 & c_{Q,1}^{\mu} \cdot \bar{Q} \gamma^{\lambda} F_{\mu\lambda} Q + c_{Q,3}^{\mu} \cdot \bar{Q} \gamma^{\lambda} W_{\mu\lambda} Q + \\
 & + c_u^{\mu} \cdot \bar{u} \gamma^{\lambda} F_{\mu\lambda} u + c_d^{\mu} \cdot \bar{d} \gamma^{\lambda} F_{\mu\lambda} d + \\
 & + \tilde{c}_{Q,1}^{\mu} \cdot \bar{Q} \gamma^{\lambda} \tilde{F}_{\mu\lambda} Q + \tilde{c}_{Q,2}^{\mu} \cdot \bar{Q} \gamma^{\lambda} \tilde{W}_{\mu\lambda} Q + \tilde{c}_{Q,3}^{\mu} \cdot \bar{Q} \gamma^{\lambda} \tilde{G}_{\mu\lambda} Q + \quad (2.7) \\
 & + \tilde{c}_{u,1}^{\mu} \cdot \bar{u} \gamma^{\lambda} \tilde{F}_{\mu\lambda} u + \tilde{c}_{u,3}^{\mu} \cdot \bar{u} \gamma^{\lambda} \tilde{G}_{\mu\lambda} u + \\
 & + \tilde{c}_{d,1}^{\mu} \cdot \bar{d} \gamma^{\lambda} \tilde{F}_{\mu\lambda} d + \tilde{c}_{d,3}^{\mu} \cdot \bar{d} \gamma^{\lambda} \tilde{G}_{\mu\lambda} d + \\
 & + C_Q^{\mu\nu\rho} \cdot \bar{Q} \gamma_{(\mu} \mathcal{D}_{\nu} \mathcal{D}_{\rho)} Q + C_u^{\mu\nu\rho} \cdot \bar{u} \gamma_{(\mu} \mathcal{D}_{\nu} \mathcal{D}_{\rho)} u + C_d^{\mu\nu\rho} \cdot \bar{d} \gamma_{(\mu} \mathcal{D}_{\nu} \mathcal{D}_{\rho)} d + \\
 & + D_{Q,1}^{\mu\nu\rho} \cdot \bar{Q} \gamma_{(\mu} F_{\rho)\nu} Q + D_{Q,2}^{\mu\nu\rho} \cdot \bar{Q} \gamma_{(\mu} W_{\rho)\nu} Q + D_{Q,3}^{\mu\nu\rho} \cdot \bar{Q} \gamma_{(\mu} G_{\rho)\nu} Q + \\
 & + D_{u,1}^{\mu\nu\rho} \cdot \bar{u} \gamma_{(\mu} F_{\rho)\nu} u + D_{u,3}^{\mu\nu\rho} \cdot \bar{u} \gamma_{(\mu} G_{\rho)\nu} u + \\
 & + D_{d,1}^{\mu\nu\rho} \cdot \bar{d} \gamma_{(\mu} F_{\rho)\nu} d + D_{d,3}^{\mu\nu\rho} \cdot \bar{d} \gamma_{(\mu} G_{\rho)\nu} d .
 \end{aligned}$$

Here all coefficients are assumed to be Hermitian matrices in the flavor space, *e.g.*

$$c_{Q,1}^{\mu} \cdot \bar{Q} \gamma^{\lambda} F_{\mu\lambda} Q \equiv (c_{Q,1}^{\mu})_{ik} \cdot \bar{Q}_i \gamma^{\lambda} F_{\mu\lambda} Q_k .$$

Similarly, LV interactions in the lepton sector of the Standard Model take the

form:

$$\begin{aligned}
 \mathcal{L}_{\text{SM}}^{\text{lepton}} = & \\
 & c_L^\mu \cdot \bar{L} \gamma^\lambda F_{\mu\lambda} L + \tilde{c}_{L,1}^\mu \cdot \bar{L} \gamma^\lambda \tilde{F}_{\mu\lambda} L + \tilde{c}_{L,2}^\mu \cdot \bar{L} \gamma^\lambda \tilde{W}_{\mu\lambda} L + \\
 & + \tilde{c}_\nu^\mu \cdot \bar{\psi}_\nu \gamma^\lambda \tilde{F}_{\mu\lambda} \psi_\nu + \tilde{c}_e^\mu \cdot \bar{\psi}_e \gamma^\lambda \tilde{F}_{\mu\lambda} \psi_e + \\
 & + C_L^{\mu\nu\rho} \cdot \bar{L} \gamma_{(\mu} \mathcal{D}_\nu \mathcal{D}_{\rho)} L + C_\nu^{\mu\nu\rho} \cdot \bar{\psi}_\nu \gamma_{(\mu} \mathcal{D}_\nu \mathcal{D}_{\rho)} \psi_\nu + C_e^{\mu\nu\rho} \cdot \bar{\psi}_e \gamma_{(\mu} \mathcal{D}_\nu \mathcal{D}_{\rho)} \psi_e + \\
 & + D_{L,1}^{\mu\nu\rho} \cdot \bar{L} \gamma_{(\mu} F_{\rho)\nu} L + D_{L,2}^{\mu\nu\rho} \cdot \bar{L} \gamma_{(\mu} W_{\rho)\nu} L + \\
 & + D_\nu^{\mu\nu\rho} \cdot \bar{\psi}_\nu \gamma_{(\mu} F_{\rho)\nu} \psi_\nu + D_e^{\mu\nu\rho} \cdot \bar{\psi}_e \gamma_{(\mu} F_{\rho)\nu} \psi_e .
 \end{aligned} \tag{2.8}$$

Here we have included the terms with the hypothetical right-handed neutrinos for the sake of completeness. As one can see, the absence of strong interactions for leptons makes (2.8) more compact compared to (2.7).

2.2.3 Higgs sector

The scalar sector of the SM in its minimal form contains one electroweak (EW) doublet, which also admits LV extensions. All LV operators with the use of the Higgs field can be further subdivided into two groups. The first are interactions built of the

Higgs field and derivatives:

$$\begin{aligned}
 \mathcal{L}_{\text{SM}}^{\text{Higgs-gauge}} = & \\
 & l^\mu \cdot i H^\dagger H \cdot H^\dagger \mathcal{D}_\mu H + \kappa^{\mu\nu\rho} \cdot i H^\dagger \mathcal{D}_{(\mu} \mathcal{D}_\nu \mathcal{D}_{\rho)} H + \\
 & + m_1^\mu \cdot i H^\dagger F_{\mu\lambda} \mathcal{D}^\lambda H + m_2^\mu \cdot i H^\dagger W_{\mu\lambda} \mathcal{D}^\lambda H + \text{h.c.} + \quad (2.9) \\
 & + \tilde{m}_1^\mu \cdot i H^\dagger \tilde{F}_{\mu\lambda} \mathcal{D}^\lambda H + \tilde{m}_2^\mu \cdot i H^\dagger \tilde{W}_{\mu\lambda} \mathcal{D}^\lambda H + \\
 & + n_1^{\mu\nu\rho} \cdot i H^\dagger F_{\nu(\mu} \mathcal{D}_{\rho)} H + n_2^{\mu\nu\rho} \cdot i H^\dagger W_{\nu(\mu} \mathcal{D}_{\rho)} H + \text{h.c.}
 \end{aligned}$$

The second group contains all possible LV extensions of interaction of Higgs field and fermions. This group is somewhat larger and includes higher-rank structures.

The following are the operators involving quarks:

$$\begin{aligned}
 \mathcal{L}_{\text{SM}}^{\text{Higgs-quark}} = & h_{QQ}^\mu \cdot \bar{Q} H \gamma_\mu H^\dagger Q + \\
 & + p_{QQ}^\mu \cdot \bar{Q} \gamma_\mu Q \cdot H^\dagger H + p_{uu}^\mu \cdot \bar{u} \gamma_\mu u \cdot H^\dagger H + p_{dd}^\mu \cdot \bar{d} \gamma_\mu d \cdot H^\dagger H \\
 & + q_{Qd}^{(1)\mu} \cdot \bar{Q} d \mathcal{D}_\mu H + q_{Qu}^{(1)\mu} \cdot \bar{Q} u \mathcal{D}_\mu \epsilon H^* + \text{h.c.} \quad (2.10) \\
 & + q_{Qd}^{(2)\nu} \cdot \bar{Q} \sigma^{\mu\nu} d \mathcal{D}_\nu H + q_{Qu}^{(2)\nu} \cdot \bar{Q} \sigma^{\mu\nu} u \mathcal{D}_\nu \epsilon H^* + \text{h.c.} \\
 & + r_{Qd}^{(1)\mu\nu\rho} \cdot \bar{Q} \mathcal{D}_{(\mu} \sigma_{\nu)\rho} d \cdot H + r_{Qd}^{(2)\mu\nu\rho} \cdot \bar{Q} \sigma_{\nu\rho} d \mathcal{D}_{(\mu} H + \text{h.c.} \\
 & + r_{Qu}^{(1)\mu\nu\rho} \cdot \bar{Q} \mathcal{D}_{(\mu} \sigma_{\nu)\rho} u \cdot \epsilon H^* + r_{Qu}^{(2)\mu\nu\rho} \cdot \bar{Q} \sigma_{\nu\rho} u \mathcal{D}_{(\mu} \epsilon H^* + \text{h.c.}
 \end{aligned}$$

where ϵH^* is the charge conjugate of the Higgs field. One also has the similar set of

operators for interaction of the Higgs with leptons:

$$\begin{aligned}
 \mathcal{L}_{\text{SM}}^{\text{Higgs-lepton}} &= h_{LL}^\mu \cdot \bar{L} H \gamma_\mu H^\dagger L + p_{LL}^\mu \cdot \bar{L} \gamma_\mu L \cdot H^\dagger H + p_{ee}^\mu \cdot \bar{e} \gamma_\mu e \cdot H^\dagger H \\
 &+ q_{Le}^{(1)\mu} \cdot \bar{L} e \mathcal{D}_\mu H + q_{Le}^{(2)\nu} \cdot \bar{L} \sigma^{\mu\nu} e \mathcal{D}_\nu H + \text{h.c.} \quad (2.11) \\
 &+ r_{Le}^{(1)\mu\nu\rho} \cdot \bar{L} \mathcal{D}_{(\mu} \sigma_{\nu)\rho} e \cdot H + r_{Le}^{(2)\mu\nu\rho} \cdot \bar{L} \sigma_{\nu)\rho} e \mathcal{D}_{(\mu} H + \text{h.c.} \\
 &+ \zeta^{\mu\nu} \cdot (H^\dagger L)^T \sigma_{\mu\nu} (H^\dagger L) + \text{h.c.}
 \end{aligned}$$

The last term in the Higgs-lepton sector, which couples to the matrix $\zeta^{\mu\nu}$, which is antisymmetric in the flavor space, is unusual. It is special in the sense that it does not have analogues in other sectors, or in lower dimensions, as it violates the lepton number by two units.

This completes the list of the dimension V Lorentz-violating operators in the Standard Model. The set of operators in the Standard Model appears to be much wider than that in QED due to the diversity of fields and interactions. Loop corrections are expected to intermix the operators in an even more complicated way. Clearly, there are many operators that give rise to dimension 3 LV interactions with quadratically divergent coefficients. Although, as indicated earlier, studying renormalization of interactions is beneficial for refining constraints on LV, we are not setting the goal to derive all one-loop RG equations similarly to what we have done in QED.

On the other hand, of particular interest are the rank three absolutely symmetric

operators $C_X^{\mu\nu\rho}$ and $\kappa^{\mu\nu\rho}$ that modify the dispersion relations for the SM particles. For these operators, we calculate the one-loop RG equations and present the results in Appendix A.

2.3 Overview of LV Phenomenology at Dimension V level

We now discuss typical limits on sensitivity to LV dimension 5 operators, which can be inferred from experimental tests of Lorentz symmetry in laboratory, astrophysical observations and data on neutrino oscillations. Given the abundance of non-minimal interactions we have derived in the last section, it would be useful to separate them in several classes and deduce a typical experimental sensitivity within each class.

Many of the constraints result from laboratory experiments or astrophysical observations at energies much lower than the weak scale. The Higgs boson, W and Z bosons and heavy SM fermions do not propagate at these energies and can be integrated out. Such integration at tree level provides new operators of higher mass dimensions which we are not considering here. One should keep in mind, however, that loop effects admix LV operators with heavy particles to the light quark and lepton LV operators of the same dimension. Such (typically one-loop) corrections include the logarithmic mixing under the RG running, as well as the finite threshold

corrections. Therefore, the bounds discussed below contain an intrinsic sensitivity to LV interactions involving Higgs and weak bosons.

For most phenomenological applications it is useful to rewrite the LV Lagrangian at the normalization scale of around 1 GeV, the borderline of applicability for the quark-gluon description. At this scale it is useful to abandon chiral fermions and combine the left- and right-handed fields into full Dirac spinors as well as split the $SU_L(2)$ doublets.

For practical reasons, one can also pass to the mass basis of the flavor matrices, as it facilitates the decoupling of heavy quarks:

$$\begin{aligned}
 c_Q^\mu, c_u^\mu &\rightarrow c_u^\dagger \Big|_{\text{below EW}} = \frac{1}{2} \left(W_u^\dagger c_u^\mu W_u + U_u^\dagger c_Q^\mu U_u \right) \\
 &\rightarrow c_{u,5}^\mu \Big|_{\text{below EW}} = \frac{1}{2} \left(W_u^\mu c_u^\mu W_u - U_u^\dagger c_Q^\mu U_u \right) \\
 &\dots\dots \\
 u_L &\rightarrow U_u u_L, \quad u_R \rightarrow W_u u_R, \quad d_L \rightarrow U_d d_L, \quad d_R \rightarrow W_d d_R.
 \end{aligned} \tag{2.12}$$

For consistency of the effective theory, we need to ensure that the operators that we have introduced do not transmute into lower dimensions, and thereby do not develop quadratic divergencies. We can formulate certain criteria to ensure that operators cannot induce lower dimensional interactions:

- *Tensor structure.* Since in the Standard Model there are no CPT -odd dimension

three operators of rank higher than one, any LV structure that is coupled to an irreducible tensor (which is not a vector) is unconditionally protected from developing quadratic divergencies.

- *Supersymmetry.* In the supersymmetric Standard Model, dimension three LV operators do not exist at all. Therefore, as long as the theory is considered above the supersymmetry breaking scale, those operators which fall into supermultiplets of the LV Minimal Supersymmetric Standard Model (MSSM), are protected (see Chapter 4). By cancellation of loop contributions due to superpartners, the quadratic divergencies turn into logarithmic ones if supersymmetry is exact. It turns out that there is only one such type of operators

$$\mathcal{L}_{\text{SUSY}} = \tilde{c}_{\text{SUSY},Q}^{\mu} \cdot \left(Y_Q g' \bar{Q} \gamma^{\lambda} \tilde{F}_{\mu\lambda} Q + g \bar{Q} \gamma^{\lambda} \tilde{W}_{\mu\lambda} Q + g_3 \bar{Q} \gamma^{\lambda} \tilde{G}_{\mu\lambda} Q \right) + \dots, \quad (2.13)$$

(here, Y_Q refers to the hypercharge of the left quark doublet) which, in the case of quarks, must form a certain linear combination to be part of a supermultiplet. Linear combinations orthogonal to the one above are not supersymmetric, and therefore not protected. When the supersymmetry is broken, the above operators are allowed to induce quadratic divergencies, which will be stabilized at the supersymmetry breaking scale.

- *T-invariance.* Since in the Standard Model one needs multiple loops to flip the T -parity of flavor-diagonal interactions, one can conclude that the operators which do not have dimension three counterparts with the same T -parity, are protected.
- *Lepton-number violation.* There are no dimension three LV operators compatible with the Standard Model which would violate lepton number. We know that there is only one $\Delta L = 2$ operator of dimension five — $\zeta^{\mu\nu}$, which therefore is protected against developing quadratic divergencies.

The operators for which the above criteria do not apply have no reason to be protected, and therefore will intermix with lower-dimensional interactions in a UV-sensitive way. We call such operators “unprotected”. Such interactions are dangerous and we will exclude them from our low energy effective theory. Using T -parity, it is easy to show that the dangerous operators in the quark and lepton sectors (Eqs. (2.7) and (2.8)) are the ones coupled to the dual field strengths

$$\begin{aligned}
 \mathcal{L}_{\text{SM}}^{\text{divgt}} &= \tilde{c}_{Q,1}^\mu \cdot \bar{Q} \gamma^\lambda \tilde{F}_{\mu\lambda} Q + \tilde{c}_{Q,3}^\mu \cdot \bar{Q} \gamma^\lambda \tilde{G}_{\mu\lambda} Q + \\
 &+ \tilde{c}_{q,1}^\mu \cdot \bar{q} \gamma^\lambda \tilde{F}_{\mu\lambda} q + \tilde{c}_{q,3}^\mu \cdot \bar{q} \gamma^\lambda \tilde{G}_{\mu\lambda} q \\
 &+ \tilde{c}_{L,1}^\mu \cdot \bar{L} \gamma^\lambda \tilde{F}_{\mu\lambda} L + \tilde{c}_e^\mu \cdot \bar{e} \gamma^\lambda \tilde{F}_{\mu\lambda} e + \mathcal{L}_{\text{Higgs}}^{\text{divgt}}, \quad (2.14)
 \end{aligned}$$

where we have abbreviated $q = u, d$. In the Higgs sector, the following operators in

Eqs. (2.9)-(2.11) are unprotected from transmuting into lower dimensional terms:

$$\begin{aligned}
 \mathcal{L}_{\text{Higgs}}^{\text{divgt}} &= l^\mu \cdot i H^\dagger H \cdot H^\dagger \mathcal{D}_\mu H + \tilde{m}_1^\mu \cdot i H^\dagger \tilde{F}_{\mu\lambda} \mathcal{D}^\lambda H + \tilde{m}_2^\mu \cdot i H^\dagger \tilde{W}_{\mu\lambda} \mathcal{D}^\lambda H + \\
 &+ h_{QQ}^\mu \cdot \bar{Q} H \gamma_\mu H^\dagger Q + h_{LL}^\mu \cdot \bar{L} H \gamma_\mu H^\dagger L + \tag{2.15} \\
 &+ p_{QQ}^\mu \cdot \bar{Q} \gamma_\mu Q \cdot H^\dagger H + p_{uu}^\mu \cdot \bar{u} \gamma_\mu u \cdot H^\dagger H + p_{dd}^\mu \cdot \bar{d} \gamma_\mu d \cdot H^\dagger H + \\
 &+ p_{LL}^\mu \cdot \bar{L} \gamma_\mu L \cdot H^\dagger H + p_{ee}^\mu \cdot \bar{e} \gamma_\mu e \cdot H^\dagger H + \\
 &+ q_{Qd}^{(2)\nu} \cdot \bar{Q} \sigma^{\mu\nu} d \mathcal{D}_\nu H + q_{Qu}^{(2)\nu} \cdot \bar{Q} \sigma^{\mu\nu} u \mathcal{D}_\nu \epsilon H^* + q_{Le}^{(2)\nu} \cdot \bar{L} \sigma^{\mu\nu} e \mathcal{D}_\nu H + \\
 &+ \text{h.c.}
 \end{aligned}$$

Using the quadratic divergence of the loop corrections generated by operators (2.14), one can estimate the strength of the naturalness constraints resulting from experimental limits on dimension 3 LV terms [31]:

$$|b^\mu| = (\text{loop factor}) \Lambda^2 |\tilde{\mathcal{C}}^\mu| \lesssim 10^{-29} \text{ GeV} . \tag{2.16}$$

Even in the very conservative assumption about the UV cutoff, *e.g.* $\Lambda = \Lambda_{\text{weak}}$, this limit would make futile any efforts of detecting interactions (2.14) directly. We again note that even though certain linear combinations (2.13) might be protected by supersymmetry, below the supersymmetry breaking scale they are unprotected and therefore subject to constraints (2.16).

From this moment, we concentrate only on UV-safe operators and list the following effective interactions in the quark sector at the scale of 1 GeV:

$$\begin{aligned}
 \mathcal{L}_{\text{SM } 1 \text{ GeV}}^{\text{quark}} &= c_q^\mu \cdot \bar{q} \gamma^\lambda F_{\mu\lambda} q + c_{q,5}^\mu \cdot \bar{q} \gamma^\lambda \gamma^5 F_{\mu\lambda} q \\
 &+ C_q^{\mu\nu\rho} \cdot \bar{q} \gamma_{(\mu} \mathcal{D}_\nu \mathcal{D}_\rho) q + C_{q,5}^{\mu\nu\rho} \cdot \bar{q} \gamma_{(\mu} \mathcal{D}_\nu \mathcal{D}_\rho) \gamma^5 q \\
 &+ D_q^{\mu\nu\rho} \cdot \bar{q} \gamma_{(\mu} F_{\rho)\nu} q + D_{q,5}^{\mu\nu\rho} \cdot \bar{q} \gamma_{(\mu} F_{\rho)\nu} \gamma^5 q \\
 &+ D_{qg}^{\mu\nu\rho} \cdot \bar{q} \gamma_{(\mu} G_{\rho)\nu} q + D_{qg,5}^{\mu\nu\rho} \cdot \bar{q} \gamma_{(\mu} G_{\rho)\nu} \gamma^5 q ,
 \end{aligned} \tag{2.17}$$

and similarly in the lepton sector:

$$\begin{aligned}
 \mathcal{L}_{\text{SM } 1 \text{ GeV}}^{\text{lepton}} &= C_l^{\mu\nu\rho} \cdot \bar{\psi} \gamma_{(\mu} \mathcal{D}_\nu \mathcal{D}_\rho) \psi + C_{l,5}^{\mu\nu\rho} \cdot \bar{\psi} \gamma_{(\mu} \mathcal{D}_\nu \mathcal{D}_\rho) \gamma^5 \psi \\
 &+ D_l^{\mu\nu\rho} \cdot \bar{\psi} \gamma_{(\mu} F_{\rho)\nu} \psi + D_{l,5}^{\mu\nu\rho} \cdot \bar{\psi} \gamma_{(\mu} F_{\rho)\nu} \gamma^5 \psi .
 \end{aligned}$$

Passing to the gauge sector, we refer to Eq. (2.4). At low energies the LV Lagrangian in the gauge sector takes the form

$$\mathcal{L}_{\text{SM } 1 \text{ GeV}}^{\text{gauge}} = C_{\text{EM}}^{\mu\nu\rho} \cdot F_{\mu\lambda} \partial_\nu \tilde{F}_\rho{}^\lambda + C_{\text{SU}_C(3)}^{\mu\nu\rho} \cdot \text{tr} G_{\mu\lambda} \mathcal{D}_\nu \tilde{G}_\rho{}^\lambda . \tag{2.18}$$

Here, the electromagnetic operator $C_{\text{EM}}^{\mu\nu\rho}$ emerges as a linear combination of the LV

tensors from the $U(1)$ and $SU_L(2)$ sectors:

$$C_{\text{EM}}^{\mu\nu\rho}\Big|_{M_W} = C_{U(1)}^{\mu\nu\rho}\Big|_{M_W} \cos^2 \theta_W + C_{SU_L(2)}^{\mu\nu\rho}\Big|_{M_W} \sin^2 \theta_W .$$

In the Higgs sector, in Eqs. (2.9)-(2.11) many “protected” terms involve a space-time derivative acting on the Higgs field. Below the EW scale, where the Higgs does not propagate, such terms do not contribute. One is left with the following low-energy interactions:

$$\begin{aligned} \mathcal{L}_{\text{SM } 1 \text{ GeV}}^{\text{Higgs-induced}} &= \frac{v}{\sqrt{2}} r_q^{\mu\nu\rho} \cdot \bar{q} \mathcal{D}_{(\mu} \sigma_{\nu)\rho} q + \frac{v}{\sqrt{2}} r_\psi^{\mu\nu\rho} \cdot \bar{\psi} \mathcal{D}_{(\mu} \sigma_{\nu)\rho} \psi + \text{h.c.} \\ &+ \frac{v^2}{2} \varsigma_\nu^{\mu\nu} \cdot \nu^T \sigma_{\mu\nu} \nu + \text{h.c.} , \end{aligned} \quad (2.19)$$

where $q = u, d$ and $\psi = e, \nu$. The last operator in Eq. (2.19) violates the lepton number by two, and in the low-energy theory can only exist for neutrinos.

The interactions in (2.17)-(2.18) can be divided into two groups. The first group is formed by the operators which modify dispersion relations and grow with energy [23], which we call the *UV-enhanced operators*. The second group, correspondingly, hosts all other structures, which we designate as “soft” LV interactions (see Table 2.1). We now outline the main sources of constraints applicable to these groups of low-energy LV interactions.

Table 2.1: Typical constraints for dimension five operators

Operators	Typical constraints	Source of constraints
Unprotected operators		
$\tilde{c}_{Q,1}^\mu \tilde{c}_{Q,3}^\mu \tilde{c}_{q,1}^\mu \tilde{c}_{q,3}^\mu \tilde{c}_{L,1}^\mu \tilde{c}_\psi^\mu$	$\ll 10^{-31} \text{ GeV}^{-1}$	constraints on dim 3 operators
Operators growing with energy (UV-enhanced operators)		
$C_q^{\mu\nu\rho} C_{q,5}^{\mu\nu\rho} C_l^{\mu\nu\rho} C_{l,5}^{\mu\nu\rho} C_{\text{EM}}^{\mu\nu\rho}$	$\lesssim 10^{-33-34} \text{ GeV}^{-1}$	high energy cosmic rays
Soft LV interactions		
$c_{q,5}^\mu D_{q,5}^{\mu\nu\rho} D_{qq}^{\mu\nu\rho} D_q^{\mu\nu\rho} r_q^{\mu\nu\rho}$	$\lesssim 10^{-28-30} \text{ GeV}^{-1}$	nuclear spin precession
$c_{q,5}^\mu D_{q,5}^{\mu\nu\rho} D_{qq,5}^{\mu\nu\rho} c_{e,5}^\mu D_{e,5}^{\mu\nu\rho}$	$\lesssim 10^{-25} \text{ ecm}$	atomic and nuclear EDMs
$\Delta L = 2$ interaction		
$\zeta_\nu^{\mu\nu}$	$\lesssim 10^{-23-24} \text{ GeV}^{-1}$	data on neutrino oscillations

Ultra-high energy cosmic rays. The existence of high-energy cosmic rays of energies $E_{max} \sim 10^{11} \text{ GeV}$ puts stringent bounds on UV-enhanced operators in certain sectors of the Standard Model, depending on the relative magnitude of LV sources in these sectors [31]. Renormalization group equations (see Appendix A) then spread these limits on the other sectors. The UV-enhanced operators modify the dispersion relations of the particles, and this would allow the nucleons in the cosmic rays to emit photons or light leptons, and therefore efficiently lose all their energy before reaching the Earth. The fact of observation of high-energy cosmic rays sets typical constraints on UV-enhanced LV of the order of $10^{-33-34} \text{ GeV}^{-1}$. It would be fair to say that not

all “corners” of the parameter space of UV-enhanced operators are covered by these limits. For example, sufficiently strong LV in the up quark sector would allow protons in the cosmic rays to decay into Δ^{++} which could become stable at high energies [31].

Precision experiments. Astrophysical constraints are not applicable to soft LV interactions, since the latter do not modify propagation of particles. For this type of interactions the bounds from low-energy precision experiments are in order. The strongest limits occur when an operator induces the interaction of nuclear spin with the nuclear electric or chromomagnetic field. One finds that the operators $D_q^{\mu\nu\rho}$, $D_{qg}^{\mu\nu\rho}$ and $r_q^{\mu\nu\rho}$, when averaged over the nucleus give an effective interaction

$$\mathcal{L}_{\text{eff}} \propto \bar{N} \partial_{(\mu} \sigma_{\nu)\rho} N ,$$

multiplied by a coefficient $\sim \Lambda_{\text{QCD}}$ which can be estimated by a naive dimensional counting of the nucleon matrix element [41]. For a non-relativistic nucleus this induces the interaction of the nuclear spin with the external preferred directions. Known limits [25] on interaction of nuclear spin with external directions allow one to estimate typical constraints on operators

$$|D_{qg}^{0ik}|, |D_{qg,5}^{ijk}| < 10^{-30} \text{ GeV}^{-1}, \quad |r_q^{0ik}| < 10^{-31} \text{ GeV}^{-1} .$$

The limits on the interactions $D_q^{\mu\nu\rho}$, $D_{q,5}^{\mu\nu\rho}$ and c_q^μ are less strong by a factor of α

due to the suppression of the nuclear electric field relative to the chromomagnetic field strength. In constraining the leptonic operators $D_l^{\mu\nu\rho}$ and $r_e^{\mu\nu\rho}$ one loses the advantage of using the strong internal nuclear fields, and the corresponding bounds are weakened by the ratio of the characteristic atomic energy scale to the nuclear energy scale $p_{at}/p_{nucl} \sim \alpha m_e/\Lambda_{\text{QCD}}$.

Electric dipole moments. The operators $D_{q,5}^{\mu\nu\rho}$, $D_{qq,5}^{\mu\nu\rho}$ and $D_{l,5}^{\mu\nu\rho}$, written in terms of low-energy effective Hamiltonian possess the signature of Electric Dipole Moment interactions. Averaged over the nucleus, the first two induce nuclear EDMs, for which the existing limits can be used to constrain the amount of Lorentz violation (see Chapter 3 for details):

$$|c_{q,5}^0|, |D_{q,5}^{i0k}| \lesssim 10^{-12} \text{ GeV}^{-1} .$$

The electron operator $D_{l,5}^{\mu\nu\rho}$ applied to paramagnetic atoms induces the electric dipole moment of the atom, and this way a bound of the similar strength is obtained.

Neutrino phenomenology. The operator $\zeta_\nu^{\mu\nu} \cdot \nu^T \sigma_{\mu\nu} \nu$ is capable of changing the patterns of neutrino oscillations. Constraints from reactor and atmospheric neutrino oscillation data can be used [42] to limit various flavor components of $\zeta_\nu^{\mu\nu}$. In the best case scenario, the sensitivity of the neutrino oscillation experiments could provide the

limits at the level of

$$|\zeta_\nu^{\mu\nu}| \lesssim 10^{-23} \text{ to } 10^{-24} \text{ GeV}^{-1} .$$

We comment that the constraints on LV operators displayed in this section do not generically restrict all the components of corresponding LV tensors. However, a customary argument applies, that the “unobservable” parts of the tensors could induce detectable effects due to the Lorentz boost caused by the motion of the Earth relative to the Galaxy, and so cannot exceed the observable components by more than $O(10^3)$.

2.4 Discussion

In this chapter we have presented a systematic Lorentz-violating extension of Quantum Electrodynamics and the Standard Model with all possible dimension five operators. Quantum Electrodynamics is the simplest phenomenological example to consider. At the level of mass dimension five, QED admits a plethora of LV interactions, parametrized by background vectors and tensors, most of which are concentrated in the matter sector. We found the one-loop logarithmic renormalization group equations for the QED LV operators, and identified one operator that has quadratic sensitivity to the UV-cutoff.

Extending QED to the full Standard Model, we listed all possible LV interactions

satisfying the criteria of the effective field theory given in the beginning of this chapter. Certainly, the wider gauge group and the diversity of field content lead to a sufficiently broader set of LV structures than in QED, although the gauge sector remains very simple. However, one quickly runs into the problem of dimensional transmutation into dimension three operators via quadratic divergencies, which can easily invalidate the distinction between operators of different dimensions. One therefore is naturally lead to an additional requirement of the absence of uncontrollable divergencies in a consistent effective theory. We identify broad classes of LV operators that are protected against dimensional transmutations by a variety of different mechanisms. These mechanisms include the protection by the irreducibility of the LV spurion tensor structures, supersymmetry, T -invariance, and the lepton number conservation.

The protected operators of the effective low-energy theory can be divided into several qualitatively different groups. The first group of interactions directly affects the propagation of particles by introducing a frequency-dependent effective refraction index for the vacuum. We call such operators UV-enhanced, since they introduce corrections to the dispersion relations which grow with the energy. The bounds on UV-enhanced operators are well-known to be of the order 10^{-33} to 10^{-34} GeV^{-1} and come from astrophysical observations. Although these bounds are well studied, we calculate the renormalization group equations that govern the logarithmic evolution of these operators over the energy scales, which can be helpful in strengthening the

bounds on operators involving heavy fields.

All the other protected operators present in our effective theory do not grow with energies of propagating particles, and we call them soft LV interactions. In general strong astrophysical constraints are not applicable to such operators. Limits of the order 10^{-28} to 10^{-30} GeV^{-1} on several classes of such operators can be deduced from clock comparison types of laboratory experiments, and are still sufficiently tight. Among less constrained type of operators are those that violate lepton number by two units, and T -odd operators that are limited only by the electric dipole moment constraints.

The strength of the analysis performed in this project is in its generality. Indeed, if one day a theory of quantum gravity or any other UV-sensitive theory would reach the stage of predicting the low-energy LV phenomenology, such predictions can be readily compared with the set of operators derived in this study, and in addition can be tested for consistency with respect to their UV behavior at the loop level.

Chapter 3

EDMs as Probes for CPT

Invariance

In Chapter 2 we have seen that the LV extension of the Standard Model admits the existence of CPT -odd interactions which possess the signature of the Electric Dipole Moment. In this chapter, we discuss in detail how searches for permanent Electric Dipole Moments can be used to put strong constraints on such interactions, independent of other available tests.

Initially suggested as an accurate test of parity conservation in strong interactions [43], the electric dipole moments of neutrons and heavy atoms provide an important test of P and T symmetries [44–49]. A non-relativistic Hamiltonian for a neutral

particle of spin S can be written as the combination of two terms,

$$H = -\mu \mathbf{B} \cdot \frac{\mathbf{S}}{S} - d \mathbf{E} \cdot \frac{\mathbf{S}}{S}. \quad (3.1)$$

Under the reflection of spatial coordinates, $P(\mathbf{B} \cdot \mathbf{S}) = \mathbf{B} \cdot \mathbf{S}$, whereas $P(\mathbf{E} \cdot \mathbf{S}) = -\mathbf{E} \cdot \mathbf{S}$. Under time reflection, $T(\mathbf{B} \cdot \mathbf{S}) = \mathbf{B} \cdot \mathbf{S}$ and $T(\mathbf{E} \cdot \mathbf{S}) = -\mathbf{E} \cdot \mathbf{S}$. The presence of a non-zero d would therefore signify the existence of both parity and time-reversal violation. In a world with perfect CPT symmetry, an EDM interaction must then be created by a CP -odd operator, the simplest form for which is $\bar{\psi} \sigma^{\mu\nu} F_{\mu\nu} \psi$. When CPT invariance is broken, one is allowed to include other operators which induce coupling of the particle's spin to the electric field.

If we take the simplest CPT -odd operator inducing EDM, leaving aside the issue of it possibly vanishing on-shell (see discussion below), the total electric dipole moment Lagrangian corresponding to Hamiltonian (3.1) can be represented as

$$\mathcal{L}_{\text{EDM}} = \frac{-i}{2} d_{\text{CP}} \bar{\psi} \sigma^{\mu\nu} F_{\mu\nu} \psi + d_{\text{CPT}} \bar{\psi} \gamma_\mu \gamma_5 \psi F_{\mu\nu} n^\nu, \quad (3.2)$$

where $d_{\text{CP}} + d_{\text{CPT}} = d$. Here n^μ is a unit length background vector breaking Lorentz (and thus CPT) invariance. Thus, quite generically, the nil result for the neutron EDM searches provides a constraint on the combination $d_{\text{CP}} + d_{\text{CPT}}$. Introducing an axial four-vector of spin \mathbf{a}^μ and four-velocity u^μ , we generalize (3.2) for a particle of

arbitrary spin:

$$\mathcal{L}_{\text{EDM}} = F_{\mu\nu} \mathbf{a}^\nu (d_{\text{CP}} u^\mu + d_{\text{CPT}} n^\mu). \quad (3.3)$$

Allowing for more complicated backgrounds, we notice that the *CPT*-odd EDM-type correlation may also result from interaction with irreducible tensor $D^{\mu\nu\rho}$, symmetric in $\nu\rho$: $F_{\mu\nu} \mathbf{a}^\rho D^{\mu\nu\rho}$, a fact that we already mentioned in Section 2.3. Now we analyze the structure of the *CPT*-odd and *CP*-even effective Lagrangian of type (3.2), deduce its consequences for the EDMs of neutrons and heavy atoms, and explore the possibility of distinguishing d_{CP} and d_{CPT} in experiment, should the non-zero EDMs be found.

3.1 *CPT*-odd, *CP*-even operators.

The operator d_{CPT} in (3.2) is odd under each of *C*, *P* and *T*, and so does qualify as an EDM interaction. Furthermore, we know that its low energy effective Hamiltonian has the right form (3.1). In general, this structure can be written for any species of the Standard Model, and $F_{\mu\nu}$ can be replaced with any appropriate gauge field strength. It is convenient to classify all such operators at the scale of 1 GeV, where only light quark fields, gluons, photons, electrons and muons are the remaining degrees of freedom, while weak bosons and heavy quarks are already decoupled. Taking a quark field ψ_q with the electric charge Q_q , and using the full equation of motion in

the electromagnetic and strong backgrounds,

$$iD_\mu \gamma^\mu \psi_q \equiv (i\partial_\mu - g_s t^a A_\mu^a - eQ_q A_\mu) \gamma^\mu \psi_q = m_q \psi_q, \quad (3.4)$$

we deduce an identity that relates gluon and photon-containing operators for quarks (*c.f.* discussion in Section 2.2.2)

$$\begin{aligned} \bar{\psi}_q (eQ_q F_{\mu\nu} + g_s t^a G_{\mu\nu}^a) \gamma^\nu \gamma_5 \psi_q &= -i \bar{\psi}_q [D_\mu, D_\nu \gamma^\nu \gamma_5] \psi_q \\ &= 2m_i \bar{\psi}_q D_\mu \gamma_5 \psi_q = m_q \bar{\psi}_q [D_\nu \gamma^\nu, \gamma_\mu \gamma_5] \psi_q = 0. \end{aligned} \quad (3.5)$$

Here $[,]$ is the commutator. Eq. (3.5) effectively reduces the number of independent quark operators, and we choose to eliminate $\bar{\psi}_q g_s t^a G_{\mu\nu}^a \gamma^\nu \gamma_5 \psi_q$ by expressing it via $\bar{\psi}_q eQ_q F_{\mu\nu} \gamma^\nu \gamma_5 \psi_q$. Remarkably, there is no *CPT*-odd, *CP*-even operators for Dirac particles that have only electromagnetic interactions, such as muons and electrons, because in this case Eq. (3.5) degenerates to an identity $\psi_e F_{\mu\nu} \gamma^\nu \gamma_5 \psi_e = 0$. It turns out that the vanishing of this effective operator is well known in the standard *CP*-odd EDM computations. The correction to the electron Hamiltonian created by operator $\bar{\psi}_e F_{0\nu} \gamma^\nu \gamma_5 \psi_e$ is proportional to the product of electric field and relativistic spin operator $\mathbf{\Sigma}$, $\mathbf{E}\mathbf{\Sigma}$. This product can be represented as a result of the commutator of another operator with the full Dirac Hamiltonian, $\mathbf{E}\mathbf{\Sigma} = (1/e)[\mathbf{\Sigma}\nabla, H]$. Therefore, the expectation value of $\mathbf{E}\mathbf{\Sigma}$ over any eigenstate of H is zero [47, 48], which is another

way of stating that $\bar{\psi}_e F_{0\nu} \gamma^\nu \gamma_5 \psi_e$ vanishes on shell.

Taking these identities into account, we write down the effective T , P , CPT -odd Lagrangian at 1 GeV scale in a remarkably simple form, that contains only three terms:

$$\mathcal{L}_{\text{CPT}} = \sum_{i=u,d,s} d_i^\mu \bar{q}_i \gamma^\lambda \gamma^5 F_{\lambda\mu} q_i. \quad (3.6)$$

This is a rather compact form compared to a usual CP -odd effective Lagrangian where a few dozens of terms have to be taken into account [49].

An important difference between CP -odd and CPT -odd EDMs comes from the $SU(2) \times U(1)$ properties of Eq. (3.6). CP -odd effects require helicity flip and thus correspond to dimension 6 operators above the electroweak scale, decoupling as $1/\Lambda_{\text{CP}}^2$ as the scale of CP violation Λ_{CP} gets larger. One can easily see that CPT -odd terms (3.6) correspond to genuine dimension 5 operators such as $\bar{q}_{R(L)} \gamma^\lambda \gamma^5 F_{\lambda\mu} q_{R(L)}$ and $\bar{q}_L \gamma^\lambda \gamma^5 \tau^a F_{\lambda\mu}^a q_L$ and do not require chirality flip. Consequently, CPT -odd physics decouples only linearly, $d_{\text{CPT}} \propto \Lambda_{\text{CPT}}^{-1}$. The combination of the present day limit on the neutron EDM with the linear decoupling property furnishes sensitivity to the scale of CPT violation as large as

$$\Lambda_{\text{CPT}} \sim (10^{11} - 10^{12}) \text{ GeV}. \quad (3.7)$$

Future generation experiments could potentially probe CPT -violating physics all the

way to the Planck scale, being limited only by the prediction of the Kobayashi-Maskawa (KM) model for the neutron EDM at the level of $10^{-31} - 10^{-33} ecm$.

3.2 Signatures of *CPT*-odd EDMs

There are three main groups of observable EDMs, which include EDMs of neutrons, diamagnetic atoms (Hg, Xe, etc.) and paramagnetic atoms (Tl, Cs, etc.). A rather simple structure of the *CPT*-odd effective Lagrangian helps to determine the dependence of these observables on different d_i^μ in (3.6).

The QCD calculations of conventional *CP*-odd EDMs [49] are very close to a constituent quark model prediction, $d_n \simeq \frac{4}{3}d_d - \frac{1}{3}d_u$, with the contribution of the *s*-quark being zero. In the *CPT*-odd case, we use matrix elements of the axial-vector charges of light quarks inside a nucleon, which can be obtained from the nucleon spin structure functions [50]. This way, to $\sim 20\%$ accuracy, we get

$$d_n \simeq 0.8d_d^0 - 0.4d_u^0 - 0.1d_s^0. \quad (3.8)$$

Using $|d_n| < 3 \times 10^{-26} ecm$ [44–46] and barring significant cancellation between the constituents, we conclude that *CPT*-odd EDMs of light quarks are limited at $O(10^{-25} ecm)$.

The measurements of EDMs of diamagnetic atoms are usually quite competi-

tive with d_n due to color EDM contributions to the *CP*-odd pion-nucleon coupling constant $\bar{g}_{\pi NN}$ [47–49]. As we already noted, interactions (3.6) preserve quark chirality, and involve a photon field, thus leading to a strong suppression of $\bar{g}_{\pi NN}(d_q^\mu)$, which makes the *T*-odd pion exchange ineffective. Consequently, the EDM of the diamagnetic atoms are induced by the EDMs of the valence nucleons. For the most important case of mercury EDM [51], we have

$$\begin{aligned} d_{\text{Hg}} &\simeq -5 \times 10^{-4}(d_n + 0.1d_p) \\ &\simeq -5 \times 10^{-4}(0.74d_d^0 - 0.32d_u^0 - 0.11d_s^0), \end{aligned} \quad (3.9)$$

and an approximate relation $d_{\text{Hg}}/d_n \sim -5 \times 10^{-4}$ could be interpreted as a signal consistent with *CPT* violation should the nonzero d_{Hg} and d_n be found. Due to the absence of *CPT*-odd electron EDM operator, EDMs of paramagnetic atoms are predicted to be extremely suppressed.

An unambiguous separation of *CP*-odd and *CPT*-odd EDM terms in (3.3) may come from measuring the difference of their relativistic effects. The *CP*-odd EDM interacts with the magnetic field and leads to the precession of the spin relative to $[\mathbf{B} \times \mathbf{v}]$, while the *CPT*-odd component does not contribute to the precession for a particle on a circular orbit. Thus, the experimental proposal of measuring deuteron EDM in the storage ring [52] would in principle have capabilities of separating the two

Coefficient	Operator	\parallel	C	P	T
a^0	$\bar{\psi}\gamma_0\psi$	\parallel	-	+	+
b^0	$\bar{\psi}\gamma_0\gamma_5\psi$	\parallel	+	-	+
d^0	$F_{\lambda 0}\bar{\psi}\gamma^\lambda\gamma^5\psi$	\parallel	-	-	-

Table 3.1: C , P , T properties of dimension three and EDM-type LV operators effects, as perpendicular \mathbf{B} and \mathbf{E} would be employed in the experimental set-up. In practice, the signal of spin precession due to the CPT -odd EDM is not exactly zero but suppressed by the deuteron anomaly, $|a_D| = 0.143$, because of the $|\mathbf{E}| = |a_D\mathbf{B}|$ choice [52]. The suppression of the deuteron d_{CPT} signal measured in the storage ring relative to d_n is opposite to the case of d_{CP} where an enhancement of $d_D/d_n \sim 5$ is expected [53] due to the CP -odd pion exchange.

3.3 Naturalness

In Chapter 2 we have learnt that some of the operators of mass dimension five are dangerous, since they are not protected from transmuting into the operators of mass dimension three. Corresponding dimension three operators can be easily listed [40],

$$\mathcal{L}_3 = - \sum \bar{\psi}(a^\mu\gamma_\mu + b^\mu\gamma_\mu\gamma_5)\psi, \quad (3.10)$$

with a_μ and b_μ being the respective Lorentz/CPT violating couplings. Table 3.1

states the result that we already know from Chapter 2: the EDM operators d^μ are protected against transmutation to a_μ and b^μ to a high loop order because of their difference in CP . Thus, only loops with intrinsic CP violation can convert d^μ into a^μ or b^μ . In the SM this is rather difficult to achieve, as the violation of CP symmetry in the flavour-conserving channel happens minimum at three loops, and is further suppressed by the Kobayashi-Maskawa mixing angles and quark Yukawa couplings. A crude estimate of dimension three operators resulting from multi-loop CP -violating corrections gives an admittedly imprecise prediction for a light quark,

$$a^\mu, b^\mu \sim d^\mu (10^{-20} - 10^{-18}) \times \text{GeV}^2. \quad (3.11)$$

This provides sensitivity to d_μ up to $10^{-12} \text{ GeV}^{-1}$, which is essentially the same sensitivity as (3.7). Therefore a detectable signal from the CPT -odd EDMs induced by a vector background would likely come accompanied by b^μ , which could be searched for via *e.g.* sidereal modulation of spin precession frequencies [54]. A difference of down and strange a_μ terms can be searched for with the neutral K mesons producing a typical bound on $|a_s^0 - a_d^0|$ of order $\sim 10^{-19} - 10^{-20} \text{ GeV}$. Through the loop effects, this amounts to sensitivity to d_q^0 terms on the order of 10^{-5} GeV^{-1} , which is significantly less sensitive than (3.7).

3.4 Tensor backgrounds

What if the nature of CPT -violation is so intricate as to give rise to an external rank-three tensorial background $D^{\mu\nu\rho}$? In this case the T , P and CPT odd interaction $F_{\mu\nu}\alpha^\rho D^{\mu\nu\rho}$ induces the EDM-like signatures via an anisotropic effective Hamiltonian for the spin:

$$H = -\mu\mathbf{B} \cdot \frac{\mathbf{S}}{S} - \mathcal{D}^{ij}E_i \cdot \frac{S_j}{S}. \quad (3.12)$$

Here \mathcal{D}^{ij} is the traceless symmetric tensor with spatial components, $\mathcal{D}^{ik} = \mathcal{D}^{i[0k]} + \mathcal{D}^{k[0i]}$. The tensor interaction in (3.12) creates a correction to the spin precession frequency proportional to $E_i B_k \mathcal{D}^{ik}$ which changes sign under the reversal of the electric field. The effect averages to zero if the orientation of parallel \mathbf{E} and \mathbf{B} fields is randomly changing relative to the external tensor \mathcal{D}^{ik} due to its tracelessness. However, in EDM experiments such averaging is not done. Therefore, $E_i B_k \mathcal{D}^{ik}$ gives an EDM signature, which in addition changes during the day because of the change of the orientation of a laboratory relative to \mathcal{D}^{ik} if, of course, the frame that breaks Lorentz invariance is not related to the Earth itself. Generically, one expects 12 and 24 hour modulations of the EDM signal due to the CPT -odd tensor background. The structure of operators leading to (3.12) is more complex than in the vector case. In particular, the electron operator, $\bar{\psi}_e F_{\mu\nu} \gamma_\rho \gamma_5 \psi_e D^{\mu\nu\rho}$ does not vanish, and leads to the EDMs of a paramagnetic atom, albeit with the matrix element suppressed by a factor

of ~ 10 relative to the CP -odd case. As in the vector case, the EDMs of diamagnetic atoms are induced by the EDMs of valence nucleons. Finally, tensor backgrounds are protected against transmutation to lower dimensional operators.

We conclude this chapter by pointing out that EDMs put stringent limits on a new type of CPT -odd CP -even interactions independently from other tests of Lorentz invariance and CPT . The scale of CPT -breaking probed by current versions of EDM experiments is as high as 10^{12} GeV. The unambiguous separation of CPT -odd and CP -odd effects would require EDM experiments with antiparticles, which might be a formidable challenge. Instead, we point out the main pattern in EDM observables consistent with CPT violation: nuclear and atomic EDMs will be induced by the EDMs of neutrons and protons, while electron EDM and T -odd nuclear forces are largely ineffective in the CPT -odd case.

Chapter 4

Lorentz Violating Supersymmetric Quantum Electrodynamics

In this chapter we analyze in detail LV operators in supersymmetric quantum electrodynamics (SQED), prove the absence of the naturalness problem in the LV sector, and derive phenomenological constraints on the LV parameters in SQED. Following Ref. [24], we parametrize all dimension five operators of LV SQED by three vector N^μ , N_+^μ and N_-^μ and one irreducible rank three tensor $T^{\mu\nu\lambda}$ backgrounds. These vector and tensor backgrounds enter in the LV operators composed of a vector superfield (containing photons and photinos) and chiral superfields (corresponding to left- and right-handed (s)electrons), respectively. We introduce these CPT-violating operators in the superfield formalism, and then derive their component form. We

also classify dimension six CPT-conserving LV operators in superspace notations. We observe that by using the equations of motion some parts of dimension five operators can be reduced to dimension three LV operators. The relation between them $[LV]_{\text{dim } 3} \sim m_e^2 [LV]_{\text{dim } 5}$ is controlled by the electron mass m_e .

The main emphasis of this study is investigate the quantum effects. We show that even in the presence of supersymmetric LV operators, no destabilizing quadratically divergent D-terms ever arise. We prove that gauge anomalies are not affected by the presence of these LV operators. This analysis essentially implies that the Chern-Simons (CS) term $k_\mu \epsilon^{\mu\nu\kappa\lambda} A_\nu \partial_\kappa A_\lambda$ cannot arise from quantum corrections. We derive the renormalization group evolution for the LV operators, showing explicitly that only the logarithmic divergences arise in the limit of exact supersymmetry (SUSY). We solve the one-loop renormalization group equations (RGE's) to obtain the low-energy values of LV parameters in terms of their values at the UV scale M . Then we investigate the consequences of SUSY breaking for LV operators by introducing the soft-breaking masses for superpartners of electrons. Dimension three LV operators can now be induced by dimensional transmutation: $[LV]_{\text{dim } 3} \sim m_{\text{soft}}^2 [LV]_{\text{dim } 5}$. Although a loop effect, this constitutes a dramatic enhancement over the case with unbroken SUSY, as $m_{\text{soft}}^2/m_e^2 > 10^{10}$. One might expect that similar quantum corrections could induce the CS term once SUSY is broken. However, our analysis rules out this possibility.

We investigate phenomenological consequences of LV in the framework of softly-broken SQED. The strongest constraints on the LV parameters are due to the (non) observation of anomalous spin precession around directions defined by the LV background vectors N^μ , N_+^μ and N_-^μ . Another constraint comes from the comparison of the anomalous magnetic moments of electrons and positrons. It is important to note that all constraints obtained in this chapter are laboratory constraints, as astrophysical and cosmological searches of LV are not sensitive to LV effects in SQED.

4.1 LV operators in SQED

Supersymmetric Quantum Electrodynamics is described by two chiral superfields Φ_+ and Φ_- , that are oppositely charged under a U(1) gauge superfield V :

$$\begin{aligned} \mathcal{L}_{SQED} = & \int d^4\theta \left(\bar{\Phi}_+ e^{2eV} \Phi_+ + \bar{\Phi}_- e^{-2eV} \Phi_- \right) \\ & + \int d^2\theta \left(\frac{1}{4} WW + m_e \Phi_- \Phi_+ \right) + \int d^2\bar{\theta} \left(\frac{1}{4} \bar{W} \bar{W} + \bar{m}_e \bar{\Phi}_+ \bar{\Phi}_- \right). \end{aligned} \quad (4.1)$$

Here $W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V$ is the super gauge invariant expression for the field strength. Throughout this chapter, we use predominantly Wess and Bagger notations [55]. The fermionic components of superfields Φ_+ and Φ_- correspond to the left-handed electron and right-handed charge-conjugated electron fields. With a slight abuse of the language, we call them the electron and positron superfields, or just the electron

and the positron for brevity. We define the charge of electron as $e = -|e|$. Finally, m_e denotes the (complex) electron mass.

LV extensions of SQED can be constructed as a set of effective operators containing the superfields Φ_- , Φ_+ , gauge covariant derivatives ∇_α , $\bar{\nabla}_{\dot{\alpha}}$ and *arbitrary* constant tensor coefficients with Lorentz indices that specify the breakdown of Lorentz invariance [24]. For constructing LV operators, we use the same rules as in Chapter 2. Within the context of SQED, however, we impose additional requirements related to supersymmetry. Specifically, we require that all LV operators be

- supersymmetric,
- local super gauge invariant with chiral gauge parameters,
- have local component expressions.

Let us explain these conditions in more detail.

First of all, by having supersymmetry we mean that the sub-algebra

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \quad (4.2)$$

of the $N = 1$ super Poincaré algebra remains unbroken. (LV Theories with higher amounts of SUSY coming from extra dimensions have also been investigated [56–58]).

If we assume that the breaking of the Lorentz symmetry is *spontaneous*, we are

guaranteed that $\sigma_{\alpha\dot{\alpha}}^{\mu}$ represent the standard Pauli matrices. However, if the breaking of Lorentz symmetry is *explicit* from the outset of the theory, these objects are simply structure coefficients parameterizing this supersymmetry algebra. (We do not pursue this possibility further here. Possible modifications of superalgebra by LV parameters have been discussed in Ref. [59, 60].) This assumption allows us to perform our analysis using conventional superspace.

The requirement of having a local component expression allows for a conventional effective field theory interpretation of the Lagrangians that we obtain. However, the locality of the component Lagrangian does not necessarily imply that the superspace expression of a given Lagrangian appears local¹. For example, the electron mass term can be written in a seemingly non-local way $\int d^4\theta m_e \Phi_- D^2 / (-4\Box) \Phi_+ + \text{h.c.}$ Finally, we require that LV operators preserve the standard local super gauge transformations

$$\Phi_{\pm} \rightarrow e^{\mp 2e\Lambda} \Phi_{\pm}, \quad \bar{\Phi}_{\pm} \rightarrow e^{\mp 2e\bar{\Lambda}} \bar{\Phi}_{\pm}, \quad V \rightarrow \Lambda + \bar{\Lambda}, \quad (4.3)$$

with a chiral parameter Λ . In particular, we do not allow for non-local or non-chiral extensions of the gauge transformations that seem to be required by non-commutative SUSY [61, 62].

As was shown in [24], these conditions combined impose strong restrictions on

¹We would like to thank N. Arkani-Hamed for this comment.

the number of LV terms of a specific mass dimension one can construct: no dimension three or four LV operators can be written down within the context of the MSSM. Here we do not repeat all the arguments leading to this general claim, but simply illustrate the underlying philosophy by showing that the CS term $k_\mu \epsilon^{\mu\nu\kappa\lambda} A_\nu \partial_\kappa A_\lambda$ does not have a SUSY extension satisfying all three conditions stated above.

The CS term is a dimension three operator that is bilinear in the gauge field and proportional to an external vector. Therefore the local superspace extension of it can be represented as

$$\mathcal{L}_{\text{SCS}}^{\text{local}} = \frac{1}{2} k^\mu \int d^4\theta \bar{\sigma}_\mu^{\dot{\alpha}\alpha} V [D_\alpha, \bar{D}_{\dot{\alpha}}] V = k_\mu \left(-\epsilon^{\mu\lambda\nu\rho} A_\lambda \partial_\nu A_\rho + 2 A^\mu D + 2 \bar{\lambda} \bar{\sigma}^\mu \lambda \right). \quad (4.4)$$

This is the only possible structure, as the insertion of an anti-commutator $\{D_\alpha, \bar{D}_{\dot{\alpha}}\}$ immediately gives rise to a total spacetime derivative. The component expression shows that this operator indeed contains the CS term, which is gauge invariant up to a total derivative. However, the SUSY extension as a whole is not super gauge invariant:

$$\delta \mathcal{L}_{\text{SCS}}^{\text{local}} = 2i k^\mu \int d^4\theta V \partial_\mu (\bar{\Lambda} - \Lambda). \quad (4.5)$$

Notice that this statement is independent of the Wess-Zumino gauge, and that even under the restriction of gauge invariance under ordinary U(1) transformations ($\Lambda = i\alpha$) the supersymmetric extension (and the $A^\mu D$ term in particular) of the CS term

fails to be gauge invariant.

These arguments do not show that it is impossible to construct a super gauge invariant extension of the CS term. Indeed, by inserting the transversal projector $P_V = D^\alpha \bar{D}^2 D_\alpha / (-8\Box)$ we obtain a manifestly super gauge invariant expression

$$\mathcal{L}_{\text{SCS}}^{\text{non-local}} = \frac{1}{2} k^\mu \int d^4\theta \bar{\sigma}_\mu^{\dot{\alpha}\alpha} V P_V [D_\alpha, \bar{D}_{\dot{\alpha}}] V = k^\mu \int d^4\theta \bar{W} \bar{\sigma}_\mu \frac{1}{\Box} W. \quad (4.6)$$

This expression clearly appears to be non-local in superspace, but the CS term itself is still local. In fact, because the true CS term in (4.4) already was gauge invariant, the insertion of P_V did not affect it at all. However, other terms in the component expression of (4.6) are non-local because they contain $1/\Box$ explicitly. Hence, as asserted, the CS term does not allow for a SUSY extension that is super gauge invariant and that has a local component expression. Additional discussion of LV due to a CS term in supersymmetric theories can be found in Refs. [63, 64].

4.1.1 CPT-violating dimension five LV operators

There are only three different types of LV operators satisfying the above requirements in SQED at the dimension five level. In this subsection we give their superfield expressions, while their component forms can be found in Section 4.4. The first type

is the electron and positron superfield operators

$$\mathcal{L}_{\text{LV}}^{\text{matter}} = \frac{1}{M} \int d^4\theta \left\{ N_+^\mu \bar{\Phi}_+ e^{2eV} i \nabla_\mu \Phi_+ + N_-^\mu \bar{\Phi}_- e^{-2eV} i \nabla_\mu \Phi_- \right\}, \quad (4.7)$$

which are parameterized by two external real vectors N_\pm^μ . The super gauge covariant spacetime derivative $\nabla_\mu = -\frac{i}{4} \bar{\sigma}_\mu^{\dot{\alpha}\alpha} \{ \nabla_\alpha, \bar{\nabla}_{\dot{\alpha}} \}$ is defined in terms of the super gauge covariant derivatives ∇_α and $\bar{\nabla}_{\dot{\alpha}}$. Their precise form depends on the super gauge transformation properties of the object that they act on. For example, we define

$$\nabla_\alpha \Psi_\pm = e^{\mp 2eV} D_\alpha (e^{\pm 2eV} \Psi_\pm), \quad \bar{\nabla}_{\dot{\alpha}} \Psi_\pm = \bar{D}_{\dot{\alpha}} \Psi_\pm, \quad (4.8)$$

for generic superfields Ψ_\pm , that have the same gauge transformations as the (chiral) superfields Φ_\pm , see (4.3).

For the photon super multiplet we can construct two independent operators. The first operator is parameterized by a real vector N^μ . We can give a Kähler-like representation of this vector operator as

$$\mathcal{L}_{\text{LV dim 5}}^{\text{gauge (V)}} = \frac{1}{M} \int d^4\theta N^\kappa \bar{W} \bar{\sigma}_\kappa W. \quad (4.9)$$

Using a superspace identity, this operator can also be written as a superpotential-like

term

$$\mathcal{L}_{\text{LV dim 5}}^{\text{gauge (V)}} = -\frac{N_\kappa}{2M} \epsilon^{\kappa\lambda\mu\nu} \left(\int d^2\theta W \sigma_{\mu\nu} \partial_\lambda W + \int d^2\bar{\theta} \bar{W} \bar{\sigma}_{\mu\nu} \partial_\lambda \bar{W} \right). \quad (4.10)$$

The most general LV superpotential-like term takes the form

$$\mathcal{L}_{\text{LV dim 5}}^{\text{gauge (T)}} = \frac{1}{4M} \int d^2\theta T^{\lambda\mu\nu} W \sigma_{\mu\nu} \partial_\lambda W + \frac{1}{4M} \int d^2\theta \bar{T}^{\lambda\mu\nu} \bar{W} \bar{\sigma}_{\mu\nu} \partial_\lambda \bar{W}. \quad (4.11)$$

In principle this operator is parameterized by a complex rank-three tensor $T^{\lambda\mu\nu}$, antisymmetric in the last two indices (μ, ν) due to its contraction with $\sigma_{\mu\nu}$. Notice, that $\sigma_{\mu\nu}$ acts as a projector on the imaginary self-dual part of the tensor since $\frac{1}{2} i \epsilon_{\mu\nu}{}^{\rho\sigma} \sigma_{\rho\sigma} = \sigma_{\mu\nu}$. This implies that we may take $T^{\lambda\mu\nu}$ real. From Chapter 2 we know that we should take it irreducible

$$T_\mu{}^{\mu\rho} = 0, \quad \epsilon_{\kappa\lambda\rho\sigma} T^{\lambda\rho\sigma} = 0. \quad (4.12)$$

The trace of $\int d^2\theta W \sigma^{\mu\nu} \partial_\lambda W + \int d^2\theta \bar{W} \bar{\sigma}^{\mu\nu} \partial_\lambda \bar{W}$ is trivially zero, while the $\epsilon^{\kappa\mu\nu\lambda}$ -trace is entirely accounted for by (4.10), so no other operators occur when expanding $T^{\lambda\mu\nu}$ in irreducibles.

In a non-Abelian theory, operator (4.11) cannot exist because the W_α 's are not gauge invariant but only gauge covariant. Thus, to maintain gauge invariance, any

derivative acting on W_α has to be replaced by a corresponding super gauge derivative. In particular, a non-Abelian generalization of (4.11) would have to contain the covariant derivative ∇_μ . But then the integrand would not be chiral, as

$$\bar{D}_{\dot{\gamma}} \nabla_\mu W_\alpha = -ie (\epsilon \bar{\sigma}_\mu)_{\dot{\gamma}\beta} (W_\alpha W_\beta + W_\beta W_\alpha) \neq 0. \quad (4.13)$$

Therefore, in the non-Abelian case one cannot write down superpotential-like LV terms for gauge multiplets, and only the Kähler-like terms (4.9) are allowed.

We have listed all possible dimension five operators in SQED framework. These results have been reported before in the MSSM setting [24]. All operators of dimension 5, listed in this section, break CPT invariance. The CPT-conserving LV operators start at dimension 6 level. For the matter of completeness, we now classify all dimension six LV operators compatible with SQED. However, our main analysis of quantum loop effects and observational implications of LV will concentrate on the dimension five operators (4.7), (4.9) and (4.11).

4.1.2 CPT-conserving dimension six LV operators

Let us start by considering possible superpotential-like terms. To obtain dimension six operators in the Lagrangian density, one has to consider the superpotential at dimension five level, *i.e.* two dimensions higher than the standard mass term $m_e \Phi_- \Phi_+$.

Because of the chirality condition of the superpotential, gauge invariance and the absence of fermionic LV backgrounds, all possible terms have to be built out of the (dimension two) operator $\Phi_+\Phi_-$ and the (dimension three) operator $W_\alpha W_\beta$, with possible derivative insertions in the latter. Omitting all Lorentz-preserving terms in the superpotential, we arrive at the following LV operator at dimension six level,

$$\mathcal{L}_{\text{LV dim 6}}^{\text{super}} = \frac{1}{M^2} \int d^2\theta S^{\mu\nu} W \partial_\mu \partial_\nu W + \text{h.c.} . \quad (4.14)$$

The dimensionless matrix S is symmetric: $S^{\mu\nu} = S^{\nu\mu}$. All other possible operators would involve $W \sigma_{\mu\nu} W$ which vanishes for a single U(1). As mentioned before, the superpotential term (4.14) can be represented as an integral over the full superspace by factoring out $-\frac{1}{4}\overline{D}^2$. This can be done in various ways leading to seemingly different expressions for these operators. Since the superpotential expression above defines these operators uniquely, there is no need to give full superspace representations of these operators here.

Aside from the operator (4.14), we can construct gauge invariant LV operators from the (dimension two) building blocks $\overline{\Phi}_\pm e^{\pm 2eV} \Phi_\pm$, $\Phi_- \Phi_+$, $\overline{\Phi}_- \overline{\Phi}_+$, $D_\alpha W_\beta$ and $\overline{D}_{\dot{\alpha}} \overline{W}_{\dot{\beta}}$ with possible gauge covariant derivatives inserted. From the identity

$[\nabla_\mu, \nabla_\nu]\Phi_\pm = \pm e (\epsilon^T \sigma)^{\alpha\beta} \nabla_\alpha (W_\beta \Phi_\pm)$ we infer that

$$\int d^4\theta \bar{\Phi}_\pm e^{\pm 2eV} [\nabla_\mu, \nabla_\nu]\Phi_\pm = \int d^4\theta \Phi_- [\nabla_\mu, \nabla_\nu]\Phi_+ = 0. \quad (4.15)$$

Moreover, full superspace integrals of $\Phi_- \Phi_+ D_\alpha W_\beta$, $\Phi_- \Phi_+ \bar{D}_\alpha \bar{W}_\beta$ and their conjugates vanish as well. Therefore, the most general (genuine Kähler and non-reducible to superpotential) dimension six LV matter Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{\text{LV dim 6}}^{\text{matter}} &= \frac{1}{M^2} \int d^4\theta \left[\bar{\Phi}_\pm e^{\pm 2eV} \Phi_\pm \left(A_\pm^{\mu\nu} D_{\sigma\mu\nu} W + \bar{A}_\pm^{\mu\nu} \bar{D}_{\bar{\sigma}\mu\nu} \bar{W} \right) \right. \\ &\quad \left. + S_\pm^{\mu\nu} \bar{\Phi}_\pm e^{\pm 2eV} \{\nabla_\mu, \nabla_\nu\} \Phi_\pm + Z^{\mu\nu} \Phi_- \{\nabla_\mu, \nabla_\nu\} \Phi_+ + \bar{Z}^{\mu\nu} \bar{\Phi}_- \{\nabla_\mu, \nabla_\nu\} \bar{\Phi}_+ \right], \end{aligned} \quad (4.16)$$

where $S_\pm^{\mu\nu}$ are real symmetric traceless matrices, $Z^{\mu\nu}$ is a complex symmetric traceless matrix and $\bar{Z}^{\mu\nu}$ is its complex conjugate.

In this section, we do not give the explicit component expressions of these supersymmetric operators, but it is not hard to see that operators like $F_{\mu\rho} F_{\nu\sigma} F^{\rho\sigma}$ or $F_{\rho\sigma} F^{\rho\sigma} F_{\mu\nu}$, do not arise. This might seem surprising, since such terms do appear in investigations of non-commutative SUSY models, and SQED in particular [61, 62]. However, there is no inconsistency: as pointed out in [62] the Seiberg-Witten map for non-commutative supersymmetric gauge theories cannot simultaneously have local and chiral gauge transformations and be invariant under conventional supersymmetry.

In our construction we have insisted on these three principles. Thus our framework is more restrictive and does not allow for the operators cubic in the electromagnetic field strength.

4.2 Quantum corrections in the presence of LV

4.2.1 Absence of a LV induced D-term

In this subsection we want to show that the dimension five LV operators discussed in section 4.1.1 do not lead to dangerous power law divergences in SQED. Before we enter this analysis, we would like to emphasize why this is an important issue.

One of the main reasons why supersymmetry is conventionally introduced is that supersymmetric theories are free of destabilizing quadratic divergences. There is of course one well-known exception to this assertion, the D -term of a U(1) vector multiplet, which is in principle quadratically divergent at one loop. However, in any supersymmetric theory that is free of anomalies the coefficient in front of the D term vanishes identically. The introduction of higher dimensional LV operators could upset the fine balance of the cancellation of the D -term, reintroducing a quadratic divergence. We will now show that such destabilizing effects do not arise.

To begin this investigation, we specify the relevant Feynman rules. The matter LV operators (4.7) can be decomposed into a modification of the kinetic term of the

Diagrammatically this may be represented as

$$\begin{array}{c} \text{wavy line} \\ \circlearrowleft \\ \text{---} \circ \end{array} = \begin{array}{c} \text{wavy line} \\ \text{---} \rightarrow \end{array} . \quad (4.21)$$

This shows that for a single insertion of the interactions (4.18) and (4.19) only the latter survives, leading to logarithmic renormalization of the dimension five LV operators, which will be studied in the next subsection. Another immediate consequence of (4.21) is that LV does not modify the cancellation of the Fayet-Iliopoulos (FI) D -term at one loop. Indeed, the $\overline{D}_\alpha D_\alpha V$ -proportional interaction gives a total derivative in the superspace when Φ_\pm fields are integrated out, and thus vanishes. The linear in V part of interaction (4.18) could induce the D -term via the tadpole diagrams obtained by closing the chiral loop in the diagrams above. However, cancellation property (4.21) reduces the tadpole with LV to a standard tadpole diagram of the Lorentz-preserving case,

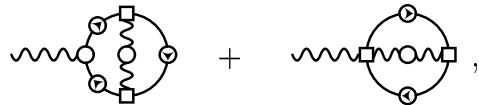
$$\begin{array}{c} \text{wavy line} \\ \circlearrowleft \\ \text{---} \oplus \end{array} = \begin{array}{c} \text{wavy line} \\ \text{---} \circlearrowright \end{array} = 0 , \quad (4.22)$$

where the last diagram gives a vanishing D -term when both Φ_+ and Φ_- loops are taken into account. More generically, the D -term will vanish for any chiral field content, provided that the sum of all charges of chiral fields is zero. Hence, to first order in the LV parameters, no extra quadratic divergences are introduced into SQED

by LV interactions.

The situation becomes more complicated if we go to higher orders in the LV parameters and to higher loop orders: the arguments presented above are sufficient to prove that to all orders no quadratic divergences arise, as long as we ignore the second interaction structure (4.19). At two-loop level vertex (4.19) introduces additional factors of $\bar{D}_{\dot{\alpha}}D_{\alpha}$ into diagrams, and thereby raises the degree of divergence of a diagram by one, as $\{\bar{D}_{\dot{\alpha}}, D_{\alpha}\} = -2i\sigma_{\alpha\dot{\alpha}}^{\mu}\partial_{\mu}$. Unlike in the one-loop calculation of the FI tadpole, the $\bar{D}_{\dot{\alpha}}D_{\alpha}$ derivatives may now act inside the diagrams, and hence still can lead to a potential power-like divergence. In addition, each of the internal propagator lines may be dressed with multiple LV insertions.


Even though the cancellation property we relied upon at one loop, Eq. (4.21), does not apply here, luckily, one can show that at two (and higher) loops the effects of all such possible insertions still cancel. The proof of this statement is similar to the proof in a standard Lorentz preserving U(1) theory [65]. At two loops, there are two types of diagrams,



$$(4.23)$$

where the vertices with boxes denote either regular gauge interactions or the ones given in (4.19) with the derivatives $\bar{D}_{\dot{\alpha}}D_{\alpha}$ acting on the internal gauge lines. Using

the diagrammatic result (4.21), the vertex of the first diagram with the external V line and one adjacent chiral line can be turned into an ordinary Lorentz-preserving combination. By partial integration on the internal gauge line all (LV) operators can be moved away as far as possible from the vertex with the external gauge multiplet. After these manipulations the diagrams can be represented pictorially as



$$(4.24)$$

After some straightforward algebra involving \overline{D}^2 and D^2 along the chiral field propagators, one can show that the ordinary chiral line in the first diagram of (4.24) can be reduced to a delta function in the superspace. This makes both diagrams in (4.24) identical in structure, but with opposite signs. Thus, we observe that these diagrams indeed cancel, and no FI D -term arises even at two (or higher) loop level.

4.2.2 Absence of LV induced gauge anomalies and of the Chern-Simons term

It is well known that anomalies put severe restrictions on the matter spectrum of particle physics models. One may wonder whether LV might lead to new anomalies. Should this happen, either the LV vectors must be restricted by stringent conditions that ensure the anomaly cancellation, or gauge non-invariant terms would have to be

included to the classical action in order to cancel the gauge variation of the effective action obtained by integrating out the fermions. In the SUSY LV context the supersymmetric extension of the CS term would be a possible term that could cancel new anomalies. We will show now that LV terms at dimension five do not modify the chiral anomaly. As a consequence, there are no further restrictions on the LV vectors and the local gauge non-invariant SUSY CS term (4.4) is not admissible. In addition to this indirect anomaly argument against the CS term, we show that it is not generated by explicitly computing relevant diagrams.

To prove the claim that there are no new gauge anomalies we closely follow the computation of the covariant anomaly presented in refs. [66, 67] using the techniques developed by Fujikawa and Konishi [68, 69]. We consider the classical LV chiral multiplet action

$$S = \int d^8z \bar{\Phi} e^V (1 + iN^\mu \nabla_\mu) \Phi, \quad (4.25)$$

with the gauge covariant super derivatives $\nabla_\mu = -\frac{i}{4} \bar{\sigma}_\mu^{\alpha\dot{\alpha}} \{\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}\}$, $\bar{\nabla}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}}$ and $\nabla_\alpha = e^{-V} D_\alpha e^V$. The variation of the effective action, obtained by integrating out the chiral multiplet Φ , under a chiral gauge transformation ($\delta\Lambda \neq 0, \delta\bar{\Lambda} = 0$) is given by

$$\delta_\Lambda \Gamma(V) = \langle \delta_\Lambda S \rangle = \left\langle \int d^8z \bar{\Phi} e^V (1 + iN^\mu \nabla_\mu) (\delta\Lambda \Phi) \right\rangle. \quad (4.26)$$

This expression is regularized by inserting the operator $\exp(\square_+/M^2)$, where \square_+ is

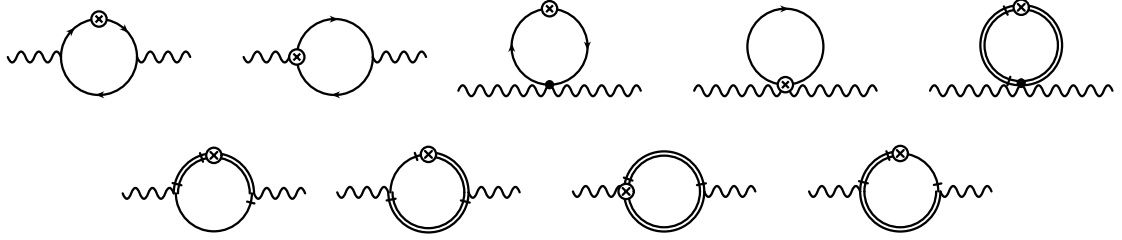


Figure 4.1: LV diagrams in massive SQED. Solid line denotes the chiral field propagator, wiggled line represents the external gauge superfield legs. The crossed circle denotes an insertion of the LV operator (4.7). Double lines represent chirality-flipping propagators $\langle \Phi \Phi \rangle$ and $\langle \bar{\Phi} \bar{\Phi} \rangle$. Bars denote the $\bar{\Phi}$ ends of the propagators. Only N_+^μ operator is included in this figure, the N_-^μ operator generates the same set of diagrams.

the covariant d'Alembertian that preserves chirality, and M is a regulator mass which will be taken $M \rightarrow \infty$ at the end of the computation. To evaluate the regularized amplitude we determine the propagator in the background field V :

$$\langle \Phi_2 \bar{\Phi}_1 e_1^V \rangle = i \left(1 + iN^\mu \frac{1}{16\Box_+} \bar{\nabla}^2 \nabla^2 \nabla_\mu \right)_2^{-1} \left(\frac{1}{16\Box_+} \bar{\nabla}^2 \nabla^2 \right)_2 \delta_{21}^8. \quad (4.27)$$

Here the subscripts 1 and 2 indicate that the corresponding expression is a function (or derivative) of the superspace coordinates z_1 and z_2 . By inserting this propagator in the variation of the effective action (4.26), one can show that the LV factors exactly cancel out, and the anomaly reduces to the standard one without any LV. Notice that in this derivation we have not used any properties of the operator $iN^\mu \nabla_\mu$ except that it is gauge covariant, therefore this argument shows that no kinetic modification ever leads to new anomaly constraints.

Because the gauge anomaly is the same as in the Lorentz preserving theory (and therefore absent in LV SQED), we conclude that no CS term can be generated by quantum effects. The reason is that the local version of the SUSY CS (4.4) is not gauge invariant, see (4.5). The gauge invariant version of the SUSY CS (4.6) is in its turn nonlocal. Since only local and gauge-invariant counterterms can arise in a non-anomalous quantum field theory, neither version of super-CS can get induced. This result can be confirmed by a direct loop computation: the diagrams of Fig. 4.1 can potentially generate the SUSY CS term (4.4), but an explicit calculation reveals that all these contributions cancel for both N_+^μ and N_-^μ backgrounds, even when the electron mass m_e is retained. The absence of CS term induced by quantum effects (4.4) can be understood as a SUSY version of the no-go theorem of Coleman and Glashow [32]. In section 4.3.2 we show that the CS term is not induced by the soft supersymmetry breaking either.

4.2.3 RGE evolution of dimension five LV operators in SQED

Again we are assuming that the operators (4.7), (4.9) and (4.11) are generated at the UV scale M , while all experimental limits are obtained at much lower energy scales. Therefore, in order to derive meaningful experimental constraints on parameters of LV SQED, we have to evolve the LV operators down to the low-energy scale. Furthermore, we know that SUSY is broken, and the operators of dimension five will

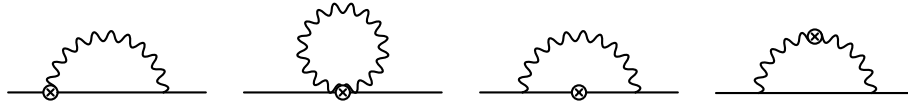


Figure 4.2: One-loop corrections to the matter multiplet operators (4.7). The notations are the same as in Fig. 4.1. Wiggled line represents the gauge superfield propagator, and the crossed circle is an insertion of the LV operators (4.7) or (4.9).

source dimension three LV operators via SUSY breaking, leading to tight bounds on LV parameters of the model. In this section, we derive and solve the renormalization group equations for dimension five LV operators assuming unbroken SUSY. In the next section we include the effects of soft breaking and calculate resulting dimension three operators.

We work in the linear approximation in LV parameters, and neglect all terms that involve higher powers of $1/M$. The running of the LV operators (4.7), (4.9) and (4.11) is, in part, a consequence of the wave function renormalization of various superfields induced by standard SQED one-loop diagrams. We do not give them explicitly here, but we take their effects into account in the resulting RGE's. For the logarithmic running of the LV parameters above the supersymmetric threshold, we can ignore soft breaking masses and electron mass inside loops. At one loop, this means that loop diagrams with internal lines of Φ_+ and Φ_- can be calculated independently.

The renormalization of the electron/positron LV operators (4.7) is induced by the diagrams shown in Fig. 4.2. The first two diagrams involve the interactions (4.19). Notice that the seagull diagram vanishes because the photon superfield loop contains

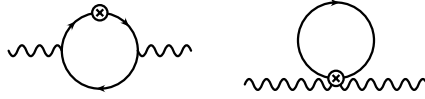


Figure 4.3: One-loop corrections to the gauge LV operator (4.9) using the same pictorial notation as in Fig. 4.2.

only two super covariant derivatives. The last diagram is induced by the operator (4.9). Loops with a single insertion of tensor interaction (4.11) vanish identically, as there are no operators in the chiral sector that can couple to $T^{\mu\nu\lambda}$.

In the gauge sector we find that the renormalization of the tensor LV gauge operator (4.11) is absent. Indeed, since we work in the first order in LV, this operator cannot receive any corrections from operators that depend on vector backgrounds. The renormalization of LV gauge operator with N^μ (4.9) is given by the diagrams shown in Fig. 4.3. Again, one can use the cancellation property (4.21) to observe that the vertex in the second diagram of Fig. 4.3 is given by (4.19) only. The combination of these gauge self-energy diagrams is only logarithmically divergent, which gives another reason why dimension three LV CS term is not generated by loop effects in this approximation.

After a straightforward calculation of logarithmically divergent parts of diagrams in Figs. 4.2 and 4.3, and inclusion of wave function renormalization effects, we arrive

at the renormalization group equation (RGE) for the LV parameters:

$$\mu \frac{\partial}{\partial \mu} \begin{pmatrix} N^\nu \\ N_+^\nu \\ N_-^\nu \\ T^{\mu\nu\rho} \end{pmatrix} = \frac{\alpha}{2\pi} \begin{pmatrix} 2 & -1 & -1 & 0 \\ -6 & 3 & 0 & 0 \\ -6 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} N^\nu \\ N_+^\nu \\ N_-^\nu \\ T^{\mu\nu\rho} \end{pmatrix}, \quad (4.28)$$

As usual, $\alpha = e^2/(4\pi)$ denotes the fine structure constant. The (1,1) and (4,4) elements of the matrix in (4.28) are equal and result only from the renormalization of wave functions. The electron and positron LV parameters N_\pm^μ both give and receive equal contributions to and from the vector LV parameter N^μ . This explains why the pairs of matrix elements (1,2) and (1,3), (2,2) and (3,3), and (2,1) and (3,1) are equal.

It will prove useful to introduce the following combinations of LV parameters that couple to operators of definite parity:

$$N_V^\mu = \frac{N_+^\mu - N_-^\mu}{2}, \quad N_A^\mu = \frac{N_+^\mu + N_-^\mu}{2}. \quad (4.29)$$

N_V^μ and N_A^μ are the charge conjugation odd and even combinations, respectively. In general, vector backgrounds do not need to have the same orientation in Minkowski space, and the off-diagonal elements of the renormalization group coefficients in (4.28) mix them, resulting in changes of their directions. By diagonalizing (4.28) we identify

a set of eigenvectors

$$N_1^\mu = N_V^\mu, \quad N_2^\mu = 3N^\mu - 2N_A^\mu, \quad N_3^\mu = 2N^\mu + N_A^\mu, \quad (4.30)$$

that evolve under the RGE independently; each of them may change its size but not its direction. N_V^μ renormalizes independently because it is the only combination that is odd under the charge conjugation. In this basis, the RGE's and their solutions are given by

$$\mu \frac{\partial}{\partial \mu} N_i^\nu = \lambda_i \frac{\alpha}{2\pi} N_i^\nu \quad \Rightarrow \quad N_i^\nu(\mu) = \left(\frac{\alpha(\mu)}{\alpha(M)} \right)^{\frac{\lambda_i}{2}} N_i^\nu(M), \quad (4.31)$$

where the eigenvalues read $\lambda_i = (\lambda_1, \lambda_2, \lambda_3) = (3, 6, -1)$. To obtain these solutions we have used the standard SQED beta function $\mu \frac{\partial}{\partial \mu} \alpha = \frac{1}{\pi} \alpha^2$.

Within the SQED framework, the renormalization effects of these LV parameters are small: even if we take $\mu = m_s \approx 1$ TeV and $M = M_{\text{Pl}} \approx 10^{19}$ GeV, the running affects the LV parameters by only about 10%. In other words, the linearized version of (4.31)

$$N_i^\nu(m_s) \simeq \left(1 - \frac{\lambda_i \alpha}{2\pi} \log(M/m_s) \right) N_i^\nu(M) \quad (4.32)$$

gives a good approximation to the exact answer. The same conclusion holds for the running of the irreducible tensor $T^{\lambda\mu\nu}$. Despite it may look as though a 10 percent

level change in N_i^μ is insignificant, one should keep in mind, however, that in a more realistic framework of MSSM the number of charged degrees of freedom running inside the loops is significantly larger than in SQED, which would lead to appreciable changes in LV parameters between the Planck and the weak scales. Nevertheless, the main numerical change in the actual size of observable LV effects will result from soft SUSY breaking, as will be discussed in the next section.

4.3 Induced dimension three operators by soft SUSY breaking

Once SUSY is broken, dimension three LV operators can be induced with coefficients controlled by the soft-breaking mass scale. Following the usual approach (see *e.g.* Ref. [55]), we introduce spurion superfields $(\theta^2, \bar{\theta}^2)$ in superspace expressions. We only consider soft SUSY breaking in the matter sector. We ignore other soft-breaking terms, including a gaugino mass, which can be motivated by the most common MSSM scenarios. Generically, we can assume that parity is broken, so that the scalar partners of left- and right-handed electrons have different masses.

The possible soft SUSY breaking masses of the electron and positron can be

written as

$$\mathcal{L}_{SB} = - \int d^2\theta \theta^2 (m_s^0)^2 \Phi_+ \Phi_- + \text{h.c.} - \int d^4\theta \theta^2 \bar{\theta}^2 \left((m_s^+)^2 \bar{\Phi}_+ \Phi_+ + (m_s^-)^2 \bar{\Phi}_- \Phi_- \right), \quad (4.33)$$

where m_s^\pm are real and m_s^0 a complex masses. To make parity violation manifest in the SUSY breaking (4.33) we introduce

$$\Delta m^2 = (m_s^+)^2 - (m_s^-)^2, \quad (m_s^\pm)^2 = m_s^2 \pm \frac{\Delta m^2}{2}. \quad (4.34)$$

The parity conserving scenario is obtained in the limit $\Delta m^2 \rightarrow 0$. Throughout the paper we assume that Δm^2 is somewhat smaller but not necessarily much smaller than m_s^2 , and that the values of the soft-breaking parameters are such that scalar electrons do not develop vacuum expectation values. Viewing SQED as a subset of MSSM, we can also neglect $(m_s^0)^2$, $(m_s^0)^2 \sim O(m_s m_e) \ll m_s^2$.

Once SUSY is broken via (4.33) in the Lorentz-conserving sector, it will be communicated to the LV sector via loop corrections or on the equations of motion (EOM's), resulting in LV operators of dimension three.

We start by listing all such operators in components, essentially extending the existing QED parametrization [54] to the SQED field content. In the matter sector

these operators are

$$\begin{aligned}
 \mathcal{L}_{\text{SB LV dim 3}}^{\text{matter}} &= 2i\tilde{A}_+^\mu \bar{z}_+ \mathcal{D}_\mu z_+ + 2i\tilde{A}_-^\mu \bar{z}_- \mathcal{D}_\mu z_- + i\tilde{C}^\mu z_- \mathcal{D}_\mu z_+ \quad (4.35) \\
 &+ \tilde{B}_+^\mu \bar{\psi}_+ \bar{\sigma}_\mu \psi_+ + \tilde{B}_-^\mu \bar{\psi}_- \bar{\sigma}_\mu \psi_- + \tilde{D}^{\mu\nu} \psi_- \sigma_{\mu\nu} \psi_+ .
 \end{aligned}$$

In superfield notation they can be expressed as:

$$\begin{aligned}
 \mathcal{L}_{\text{LV dim 3}}^{\text{matter SB}} &= \int d^4\theta \theta^2 \bar{\theta}^2 \left[2i\tilde{A}_+^\mu \bar{\Phi}_+ \nabla_\mu \Phi_+ - 2i\tilde{A}_-^\mu \bar{\Phi}_- \nabla_\mu \Phi_- \right. \\
 &+ \frac{1}{2}(\tilde{C}^\mu \bar{\Phi}_- \nabla_\mu \Phi_+ + \text{h.c.}) + \frac{1}{2}\tilde{B}_+^\mu \bar{\nabla} \bar{\Phi}_+ \bar{\sigma}_\mu \nabla \Phi_+ + \frac{1}{2}\tilde{B}_-^\mu \bar{\nabla} \bar{\Phi}_- \bar{\sigma}_\mu \nabla \Phi_- \\
 &\left. + \frac{1}{2}(\tilde{D}^{\mu\nu} \nabla \bar{\Phi}_- \sigma_{\mu\nu} \nabla \Phi_+ + \text{h.c.}) \right] . \quad (4.36)
 \end{aligned}$$

The superfield expressions (4.36) for the operators (4.35) are not unique. One can use alternative spurion insertions inside gauge-invariant supersymmetric LV operators [24]. However, at the component level these expressions will reduce to linear combinations of the operators given in (4.36).

In the gauge sector there are only two LV operators of dimension three:

$$\mathcal{L}_{\text{SB LV dim 3}}^{\text{gauge}} = \tilde{E}_\mu \epsilon^{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma + \tilde{F}_\mu \lambda \sigma^\mu \bar{\lambda} , \quad (4.37)$$

which can be rewritten in a superfield form using the CS superfield [70]:

$$\begin{aligned} \mathcal{L}_{\text{LV dim 3}}^{\text{gauge}} = & \int d^4\theta \left((\tilde{F}_\mu - \tilde{E}_\mu) \theta^A W_{\sigma^\mu} \overline{W} + \right. \\ & \left. + \tilde{E}_\mu \theta \sigma^\mu \bar{\theta} \{ D^\alpha (V W_\alpha) + \overline{D}_{\dot{\alpha}} V \overline{W}^{\dot{\alpha}} \} \right). \end{aligned} \quad (4.38)$$

4.3.1 Operators in the matter sector

We now turn to the discussion of possible mechanisms that transmute dimension five SUSY LV operators into dimension three LV operators. There are two generic ways this may occur, at tree level via reduction over the EOM and via loop effects,

$$\begin{aligned} [LV]_{\text{dim 5}} & \xrightarrow{\text{EOM}} (m_s^2 + m_e^2) [LV]_{\text{dim 3}}, \quad \text{for selectrons,} \\ [LV]_{\text{dim 5}} & \xrightarrow{1 \text{ loop}} m_s^2 [LV]_{\text{dim 3}}, \quad \text{for fermions and vector bosons.} \end{aligned}$$

The tensor operator (4.11) does not mix with dimension three operators in any order in SUSY breaking, because there is simply no dimension three operator that can couple to $T^{\mu\nu\lambda}$. Below we discuss in detail how dimension three operators are generated by tree level and loop effects.

The soft supersymmetry breaking (4.33) affects LV interactions for left- and right-handed selectrons already at tree level. The masses of scalar particles are lifted with respect to the masses of the electron and positron. This alters sfermions' equations

of motion, leading to an *enhancement* of certain dimension three operators. Ignoring Δm^2 for a moment, one can easily show that the combination of LV operators (4.7) and SUSY breaking (4.33) leads to the following dimension three LV operator

$$\mathcal{L}_{\text{particle}}^{\text{EOM}} = \frac{N_V^\mu}{M} 2i (m_e^2 + m_s^2) \left\{ \bar{z}_+ \mathcal{D}_\mu z_+ - \bar{z}_- \mathcal{D}_\mu z_- \right\}, \quad (4.39)$$

effectively generating the \tilde{A}_\pm^μ -terms (4.35),

$$\tilde{A}_\pm^\mu = \pm 2 \frac{N_V^\mu}{M} (m_e^2 + m_s^2). \quad (4.40)$$

However, we will not be interested in these particular operators due to current impossibility to study experimentally the superpartner sector. In the matter sector only those operators involving electrons and positrons are important for phenomenology. For the same reason, we ignore the possible appearance of the operators proportional to \tilde{C}^μ , and in the gauge sector we will only be interested in the CS term that might be induced for photons.

At one loop level, the transmission of SUSY breaking to the LV sector of chiral fermions and gauge bosons may indeed be possible. We start with one loop effects in the matter sector. It is sufficient for our purposes to consider the running of dimension three operators within the interval of momenta $m_s \ll |p_{\text{loop}}| \ll M$, and retain only the contributions enhanced by a large $\log(M/m_s)$, neglecting possible

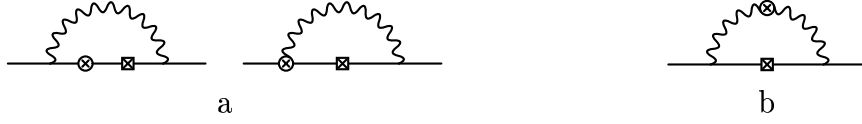


Figure 4.4: Diagrams generating dimension three LV operators for electrons and positrons due to soft supersymmetry breaking and dimension five SUSY LV operators in gauge and matter sectors. In Figs. 4.4a and 4.4b the inserted operators are (4.7) and (4.9), respectively. Finally, the box with a cross denotes the insertion of the SUSY breaking operator (4.33).

threshold corrections. To this accuracy, the soft breaking parameters inside loops can be treated as perturbations and inserted on internal lines of diagrams from Fig. 4.2. In Figs. 4.4a and 4.4b we have inserted the dimension five SUSY LV operators (4.7) and (4.9), respectively.

Our one loop RGE analysis concentrates on the induced dimension three operators \tilde{B}_μ . Besides the contributions from diagrams in Figs. 4.4a, 4.4b the complete set of RGE includes the one loop running of the operator \tilde{B}_μ itself, and its mixing with \tilde{A}_μ . The relevant set of RGE's includes:

$$\begin{aligned} \mu \frac{d\tilde{A}_+^\nu}{d\mu} &= \frac{\alpha}{\pi} \left(\tilde{A}_+^\nu - \tilde{B}_+^\nu \right), \\ \mu \frac{d\tilde{B}_+^\nu}{d\mu} &= \frac{\alpha}{2\pi} \left(\tilde{B}_+^\nu - \tilde{A}_+^\nu + 3 \frac{(m_s^+)^2}{M} N^\nu - 2 \frac{(m_s^+)^2}{M} N_+^\nu \right). \end{aligned} \quad (4.41)$$

Here we quote only the results for Φ_+ components; the extension to Φ_- follows upon some simple substitutions. The requirement of exact SUSY at UV scale M translates into the RGE boundary conditions: $\tilde{A}^\mu \Big|_M = \tilde{B}^\mu \Big|_M = 0$. In addition, the full set of

equations include the RGE's for the soft breaking masses,

$$\mu \frac{dm_s^2}{d\mu} = \frac{\alpha}{4\pi} m_s^2, \quad (4.42)$$

and RGE's for the dimension five SUSY LV operators (4.28).

Exact solutions of these RGE's are not warranted for our purposes. Instead, we use the same approximation as in (4.32) together with $\alpha/\pi \log(M/m_s) < 1$, to obtain the solution

$$\tilde{B}^{\pm\nu}(m_s) = \frac{\alpha}{\pi} \log(M/m_s) \frac{(m_s^\pm)^2}{M} \left\{ \frac{3}{2} N^\nu(M) - N_\pm^\nu(M) \right\}, \quad (4.43)$$

for \tilde{B}_μ^\pm in the leading $\alpha \log$ approximation.

4.3.2 Operators in the gauge sector. Chern-Simons term.

The absence of optical activity effects caused by the CS term has been checked over cosmological distances, providing a very sensitive probe of $k_\mu \epsilon^{\mu\nu\kappa\lambda} A_\nu \partial_\kappa A_\lambda$ (see *e.g.* Ref. [39] and references therein). The limit on k_μ is about the present Hubble expansion rate, and is ten orders of magnitude better than the level of sensitivity for the best terrestrial experiments searching for dimension three LV parameters. Not surprisingly, the issue of a CS term generated by radiative corrections from other LV interactions has drawn a lot of interest [32, 71–74], exhibiting the whole range of

answers for k_μ being induced by b_μ (including zero). A no-go theorem by Coleman and Glashow [32] indicates the absence of the radiatively generated CS term. If suitably rephrased, it states that the CS term cannot be induced to first order by gauge invariant LV interactions. In section 4.2.2 we have extended this theorem to the exact SUSY LV interactions.

We would like to argue that below the soft SUSY breaking scale the CS term also cannot be generated. Indeed, the CS interaction can only be generated by a fermion running in the loop, as a bosonic loop cannot produce $\epsilon^{\mu\nu\rho\sigma}$ entering the expression for the CS term. However, the SUSY breaking terms (4.33) only provide masses for the bosonic components of chiral superfields and thus only affect the scalar parts of the diagrams, which are incapable of inducing the CS interaction. (The possibility of a soft gaugino mass is not relevant because diagrams that could induce the CS term only include chiral matter fermions, not gauginos.) In particular, the no-go theorem by Coleman and Glashow [32] for QED is re-obtained by sending the soft masses to infinity.

We have confirmed this result by a direct calculation in the presence of the soft-breaking. The relevant diagrams, given in Fig. 4.5, are obtained by inserting the soft breaking interaction (4.33) into the diagrams shown in Fig. 4.3. As for the direct confirmation of the absence of the SUSY CS term in section 4.2.2, instead of calculating all possible terms, we have only concentrated on those structures that

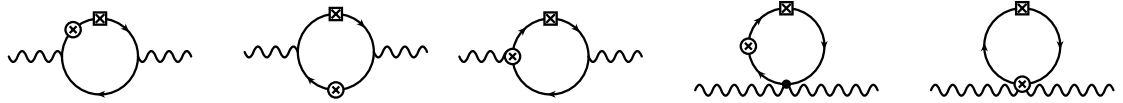


Figure 4.5: Dimension three one-loop contributions arising from the dimension five LV operators (4.7) and soft supersymmetry breaking.

can induce the CS term. Here, again, the vertex cancellation property (4.21) can be used quite effectively to mutually cancel contributions of particular diagrams. A straightforward calculation shows that all terms proportional to the CS interaction indeed cancel.

Note that this statement is only valid for the pure CS term $\epsilon^{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma$, while there is no evidence against the other possible operator in the photon sector, $\lambda \sigma^\mu \bar{\lambda}$. However, the presence/absence of the latter term is obviously not very relevant for phenomenological applications.

4.4 Phenomenology of LV SQED: LV observables and experimental limits

4.4.1 Component Expressions for LV operators

In order to derive phenomenological consequences of the LV operators, we need to obtain their component expressions. First we consider the matter operators (4.7).

The component form for the electron part is given by:

$$\begin{aligned}
 \mathcal{L}_{\text{LV dim } 5}^{\text{matter}(+)} &= \frac{N_+^\mu}{M} \left[i\bar{F}_+ \mathcal{D}_\mu F_+ + ie\bar{z}_+ D \mathcal{D}_\mu z_+ - ie \mathcal{D}_\mu(\bar{z}_+) D z_+ + \frac{1}{2} \bar{\psi}_+ \mathcal{D}_{(\mu} \mathcal{D}_{\nu)} \bar{\sigma}^\nu \psi_+ \right. \\
 &+ ie \frac{\sqrt{2}}{2} \left\{ \bar{\psi}_+ \bar{\sigma}_\mu \lambda F_+ - \bar{F}_+ \bar{\lambda} \bar{\sigma}_\mu \psi_+ \right\} + e^2 \bar{z}_+ \left\{ \lambda \sigma_\mu \bar{\lambda} - \bar{\lambda} \bar{\sigma}_\mu \lambda \right\} z_+ + \frac{1}{2} e \bar{\psi}_+ \bar{\sigma}_\mu D \psi_+ \\
 &- \sqrt{2} e \left\{ \mathcal{D}_\mu(\bar{\psi}_+) \bar{\lambda} z_+ + \bar{z}_+ \lambda \mathcal{D}_\mu \psi_+ \right\} - \frac{\sqrt{2}}{2} e \left\{ \bar{\psi}_+ \bar{\sigma}^\nu \sigma_\mu \bar{\lambda} \mathcal{D}_\nu z_+ + \mathcal{D}_\nu(\bar{z}_+) \lambda \sigma_\mu \bar{\sigma}^\nu \psi_+ \right\} \\
 &\left. - \frac{1}{4} e \bar{\psi}_+ \epsilon_\mu^{\nu\rho\sigma} F_{\rho\sigma} \bar{\sigma}_\nu \psi_+ + i \bar{z}_+ \mathcal{D}^\nu \mathcal{D}_\mu \mathcal{D}_\nu z_+ + \frac{1}{2} i e \mathcal{D}_\nu(\bar{z}_+) \epsilon_\mu^{\nu\rho\sigma} F_{\rho\sigma} z_+ \right], \quad (4.44)
 \end{aligned}$$

This component representation of dimension 5 LV operators allows for further reduction of several terms in (4.44) on the equations of motion. The result for this tedious but routine reduction is given in Appendix C. To facilitate phenomenological applications, we convert all Weyl spinors into Dirac/Majorana four component spinors:

$$\Psi = \begin{pmatrix} c\psi_+ \\ \psi_- \end{pmatrix}, \quad \text{and} \quad \lambda = \begin{pmatrix} c\lambda \\ \bar{\lambda} \end{pmatrix}. \quad (4.45)$$

Using notations (4.29) and (4.34), we present the dimension five LV operators (4.7), containing electron and photon fields, as:

$$\mathcal{L}_{\text{LV}}^{\text{matter}} = -\frac{N_A^\mu}{M} \frac{1}{2} e \bar{\Psi} \tilde{F}_{\mu\nu} \gamma_\nu \Psi - \frac{N_V^\mu}{M} \frac{1}{2} e \bar{\Psi} \tilde{F}_{\mu\nu} \gamma^\nu \gamma^5 \Psi + \frac{N_V^\mu}{M} m_e^2 \bar{\Psi} \gamma_\mu \Psi. \quad (4.46)$$

Using the same notation, the dimension three operators (4.43) can be rewritten as vector and axial-vector operators:

$$\begin{aligned} \mathcal{L}_{\text{LV dim 3}}^{\text{matter}} = & -\bar{\Psi}\gamma^\mu\Psi\left\{m_s^2 N_V^\mu + \frac{\Delta m^2}{2} N_A^\mu - \frac{3}{2}\frac{\Delta m^2}{2} N^\mu\right\}\frac{\alpha\log(M/m_s)}{\pi M} \\ & -\bar{\Psi}\gamma^\mu\gamma^5\Psi\left\{m_s^2 N_A^\mu + \frac{\Delta m^2}{2} N_V^\mu - \frac{3}{2}m_s^2 N^\mu\right\}\frac{\alpha\log(M/m_s)}{\pi M}. \end{aligned} \quad (4.47)$$

The next operator to consider in components is the photon operator (4.9):

$$\mathcal{L}_{\text{LV dim 5}}^{\text{gauge (K)}} = \frac{N^\mu}{M}\left(2\bar{\lambda}\gamma_\mu\Box\lambda + 2\lambda\partial_\mu\partial\bar{\lambda} - 2D\partial^\nu F_{\mu\nu} + \partial_\lambda F^{\lambda\nu}\tilde{F}_{\nu\mu}\right). \quad (4.48)$$

These components are reducible on the equations of motion, and only the last term in (4.48) leads to a contribution in the electron and photon sectors. By substituting $\partial_\lambda F^{\lambda\mu}$ with the electromagnetic current $J_{EM}^\mu = -e\bar{\Psi}\gamma^\mu\Psi$, we get an interaction term

$$\mathcal{L}_{\text{gauge (K)}}^{\text{EOM}} = e\bar{\Psi}N^\mu\gamma^\nu\tilde{F}_{\mu\nu}\Psi, \quad (4.49)$$

which has the same form as the second term in (4.46). Notice, however, that this coincidence holds only within QED, as in the full MSSM J_{EM}^μ will also have other (*i.e.* hadronic) contributions. Finally, the tensor operator (4.11) has the following

component expression

$$\mathcal{L}_{\text{LV dim 5}}^{\text{gauge (T)}} = \frac{2T_{\mu\nu\rho}}{M} \left(F^{\nu\lambda} \partial^\mu F^\rho{}_\lambda - D \partial^\mu \tilde{F}^{\nu\rho} - \bar{\lambda} \partial^\mu \partial_\sigma \bar{\sigma}_{\tau\lambda} \epsilon^{\sigma\tau\nu\rho} \right). \quad (4.50)$$

It can be reduced on the equations of motion by using integration by parts and Jacobi identities. Applying the equations of motion to the pure electromagnetic field strength term in (4.50), we obtain in the electron-photon sector of QED

$$\mathcal{L}_{\text{gauge (T)}}^{\text{EOM}} = 2e T_{\mu\nu\rho} \bar{\Psi} \gamma^\mu F^{\nu\rho} \Psi. \quad (4.51)$$

Confirming the general conclusion of [24], we observe that none of the LV operators give corrections to the EOM that grow at high energies.

Now we gather all operators of phenomenological interest of dimensions five and three, (4.47), (4.46), (4.49) and (4.51), in a single expression:

$$-\mathcal{L}_{\text{eff LV}} = \bar{\Psi} \gamma^\mu \left(a_\mu + b_\mu \gamma^5 + e c_\nu \tilde{F}^{\nu\mu} + e d_\nu \tilde{F}^{\nu\mu} \gamma^5 + e f_{\mu\rho\sigma} F^{\rho\sigma} \right) \Psi, \quad (4.52)$$

where we use the notations of [40] for the coefficients of dimension three operators. The Wilson coefficients in (4.52) are expressed in terms of the original LV parameters,

electromagnetic coupling constant, and soft breaking masses:

$$\begin{aligned}
 a^\mu &= -\frac{1}{M} m_e^2 N_V^\mu + \frac{\alpha \log(M/m_s)}{\pi M} \left\{ m_s^2 N_V^\mu + \frac{\Delta m^2}{2} N_A^\mu - \frac{3}{2} \frac{\Delta m^2}{2} N^\mu \right\}, \\
 b^\mu &= \frac{\alpha \log(M/m_s)}{\pi M} \left\{ m_s^2 N_A^\mu + \frac{\Delta m^2}{2} N_V^\mu - \frac{3}{2} m_s^2 N^\mu \right\}, \\
 c^\mu &= \frac{1}{M} \left\{ \frac{1}{2} N_A^\mu - N^\mu \right\}, \quad d^\mu = \frac{1}{M} \frac{N_V^\mu}{2}, \quad f^{\mu\nu\rho} = \frac{2}{M} T^{\mu\nu\rho}.
 \end{aligned} \tag{4.53}$$

The result is given in the leading $\alpha \log$ approximation, and with all LV parameters normalized at the SUSY threshold m_s . The operator proportional to a^μ does not lead to any physical effects as it can be totally absorbed into the kinetic term $-i \bar{\Psi} \not{\partial} \Psi$ via a phase redefinition, $\Psi(x) \rightarrow e^{i a^\mu x_\mu} \Psi(x)$. The rest of the operators lead to observable LV signatures.

4.4.2 Constraints on LV parameters of SQED

Now, we are prepared to extract observational consequences of LV SQED, and to impose constraints on the coefficients of the effective low-energy Lagrangian (4.52). To do that, we derive the non-relativistic effective Hamiltonian corresponding to (4.52) by splitting the external backgrounds into spatial and time-like components:

$$\begin{aligned}
 \mathcal{H}_{\text{eff}} &= \frac{\vec{p} \cdot \vec{d}}{m} + \vec{b} \cdot \vec{\sigma} + \left\{ \frac{e \vec{p}}{m}, [\vec{c} \times \vec{E}] - c^0 \vec{B} \right\} - e d^0 (\vec{B} \cdot \vec{\sigma}) \\
 &\quad + e \vec{d} \cdot [\vec{E} \times \vec{\sigma}] + \left\{ \frac{e p^k}{m}, 2 f_{k0l} E^l + f_{klm} \epsilon_{lmn} B^n \right\}.
 \end{aligned} \tag{4.54}$$

Here $\vec{p} = -i\hbar\vec{\partial}$ is the momentum operator and $\{.,.\}$ denotes the anti-commutator.

The tightest constraints come from the experiments searching for abnormal spin precession around external directions determined by the LV vectors (4.53). These experiments limit LV parameters for electrons and nucleons. The relevant parameter we should compare our estimates with is the energy shift due to LV effects $\Delta\omega_{LV}$. In LV SQED, the effects are mediated by dimension five operators, and therefore the strength of the constraints on combinations of M and LV backgrounds depends very sensitively on the energy scale μ relating $\Delta\omega_{LV}$ and M^{-1} , $\Delta\omega_{LV} \sim \mu^2 M^{-1}$. Our analysis shows that several possibilities for μ are possible: the soft breaking scale, the hadronic scale (*i.e.* Λ_{QCD}), electron mass, and finally, the energy scale given by an external magnetic or electric field.

LV electron spin precession

The soft breaking scale enters in the LV parameter b_μ , which is limited by torsion balance experiments searching for LV in the electron sector [28]. The sensitivity to the spatial part of the axial-vector coupling b^i achieved in these experiments is at the level better than $|b^i| < 10^{-28}$ GeV. This condition imposes a stringent constraint on the combination of the soft breaking masses, M , and LV parameters:

$$\frac{m_s^2}{(100 \text{ GeV})^2} \frac{10^{19} \text{ GeV}}{M} \left| N_A^i - \frac{3}{2} N^i + \delta_s N_V^i \right| < 10^{-12}, \quad (4.55)$$

where we normalize M to the Planck scale. We have introduced a dimensionless quantity $\delta_s = \Delta m^2/(2m_s^2)$ that parameterizes parity violation in the soft breaking sector. The lightest values for m_s not excluded by direct collider searches are slightly above 100 GeV, and therefore $|N_A^i - \frac{3}{2}N^i|$ is limited to be less than 10^{-12} .

LV nuclear spin precession

The next constraint uses the energy scale $\mu \sim \Lambda_{QCD}$ from hadron physics. In order to obtain it, we have to go beyond pure QED and include hadronic components in J_{EM}^μ (4.49). Then, as discussed earlier in [24], the LV SQED operator (4.9) gives rise to interaction of the spatial components of N^μ , electric field, and the spatial component of the hadronic current, $\mathcal{L} = M^{-1}\epsilon_{ijk}N^i E^j J_{EM}^k$. The average of such interaction inside the nucleus with spin I leads to the effective Hamiltonian $\mathcal{H}_{\text{eff}} = \kappa \vec{I} \cdot \vec{N}$. The strength κ of this interaction is given by a nuclear matrix element, which can be estimated as the product of the typical value of the electric field inside a heavy nucleus times the characteristic nucleon momentum. Combined with a 10^{-32} GeV level of sensitivity for $\Delta\omega_{LV}$ in the most advanced experiments [26, 75], this results in a stringent bound on $|N^i|$:

$$\kappa \sim \frac{e E p_{\text{nucl}}}{M m_p} \sim \frac{Z^{1/3} \text{fm}^{-3} \alpha}{M m_p} \Rightarrow \frac{10^{19} \text{ GeV}}{M} |N^i| < 10^{-9}. \quad (4.56)$$

Here m_p is the proton mass and $p_{\text{nucl}} \sim \text{fm}^{-1}$ is the typical nucleon momentum.

A more refined nuclear calculation can be done for mercury and xenon nuclei used in [26, 75] if needed.

LV precession of the angular momentum of a paramagnetic atom

If for some unexpected reasons the effective electron LV coupling b^μ is close to zero, the interaction term proportional to c^μ in (4.54) would still induce a coupling of the electron angular momentum j inside a paramagnetic atom to the spatial component of $N_A^\mu/2 - N^\mu$ with $\mathcal{H}_{\text{eff}} = \kappa j_i (N_A^i/2 - N^i)$. In this case, the characteristic scale connecting $\Delta\omega_{LV}$ with LV parameters is the typical momentum of atomic electrons, $\mu \sim p_{\text{atomic}} \sim \alpha m_e$. Apart from an overall coefficient, the atomic matrix element responsible for this interaction has the same strength as the usual spin-orbit interaction, resulting in the estimate of κ

$$\kappa \sim Z^2 \alpha^2 \frac{\alpha^2 m_e^2}{M} \quad \Rightarrow \quad \frac{10^{19} \text{ GeV}}{M} |N^i - N_A^i/2| < 10^{-2}. \quad (4.57)$$

CPT-odd anomalous magnetic moment of electrons and positrons

The limits explored so far do not use the fact that LV operators of dimension five break CPT, whereas the experiments [26, 75] are done with normal matter. Some other experiments explicitly compare properties of electrons and positrons, and can

therefore be used to constrain LV CPT-odd operators. For example, a d^0 -proportional term in (4.54) induces an interaction between the electron spin with a magnetic field, and thus contributes to the anomalous magnetic moment of electrons and positrons. The different g -factors for electrons and positrons are limited at 10^{-12} level: $|g_e - g_{\bar{e}}| < 8 \times 10^{-12}$ [76].

The interaction Hamiltonian for electrons and positrons, corrected by the CPT-odd d^0 -proportional interaction, takes the form:

$$\mathcal{H}_{\text{eff}}^e = -e d^0 \frac{\vec{B} \cdot \vec{S}}{S} - |\mu| \frac{\vec{B} \cdot \vec{S}}{S}, \quad \mathcal{H}_{\text{eff}}^{\bar{e}} = -e d^0 \frac{\vec{B} \cdot \vec{S}}{S} + |\mu| \frac{\vec{B} \cdot \vec{S}}{S}. \quad (4.58)$$

This gives a bound on the time-like component of N_V^μ :

$$\frac{m_e}{M} |N_V^0| < 2 \times 10^{-12} \quad \Rightarrow \quad \frac{10^{19} \text{ GeV}}{M} |N_V^0| < 10^{10}. \quad (4.59)$$

Obviously, this limit is inferior to those derived from the searches of the breakdown of rotational invariance [26, 75].

It is interesting to note that the CPT-violating correction to the magnetic moments of electrons and positrons arises in LV SQED even when SUSY is unbroken. At first sight this seems to be at odds with the Ferrara-Remiddi theorem which forbids emergence of the anomalous magnetic moment of the electron in the exact SUSY limit [77]: the anomalous magnetic moment of the electron, $e(4m)^{-1} \bar{\Psi} \sigma_{\mu\nu} \Psi F^{\mu\nu}$,

should appear as the highest component of some superfield, but no such supermultiplet exists [77]. However, one of the assumptions leading to this result is Lorentz invariance, therefore when SQED is extended by LV operators, the anomalous magnetic moment, $eM^{-1}N_-^\mu \bar{\Psi} \tilde{F}_{\mu\nu} \gamma^\nu \gamma^5 \Psi$, does arise as the highest component of a superfield operator, namely (4.7).

Consequences for some dimension five operators in LV SQED

The two most stringent limits, (4.55) and (4.56), are sensitive to different linear combinations of N^i and N_A^i vectors, thus imposing similar strength constraints on N^i and N_A^i separately. In order to impose a constraint on N_V^i , one has to make further assumptions about δ_s . In the full MSSM scenario (as opposed to its SQED subset), parity is broken above the weak scale. Hence, a δ_s at a percent level or larger arises from radiative corrections even if the boundary conditions at M are parity conserving, *i.e.* $m_s^+(M) = m_s^-(M)$. This provides a sensitivity to N_V^i at the level of 10^{-10} . The time-like components of vectors N_\pm^μ and N^μ are also constrained: the motion of the Earth and the solar system introduces a dependence of the laboratory frame on the velocity relative to the fixed vector backgrounds. Therefore, a non-zero N^0 would "mix" with N^i at $O(v/c) \sim 10^{-3} - 10^{-4}$ level. As a result, $10^{-6} - 10^{-8}$ level constraints can be imposed on N_V^0 , N_A^0 and N^0 .

LV induced by $T^{\mu\nu\rho}$ (4.11) is also subject to experimental constraints. For ex-

ample, a three dimensional vector $f^k = \epsilon_{ijk} T^{i0j}$, obtained from the tensor $T^{\mu\nu\rho}$, leads essentially to the same effects as vector c^k , and is therefore subject to bounds analogous to (4.56) and (4.57). Other components of $T^{\mu\nu\rho}$ can be limited using their contributions to f^k caused by the earth motion effects.

Absence of Planck-scale bounds on dimension six LV operators

Finally, we would like to assert that limits on dimension six operators are *not* able to rule out LV modifications at the M_{Pl}^{-2} level. Many of the operators listed in (4.14) and (4.16) contain antisymmetric tensors. After the inclusion of SUSY breaking, such terms can mix with the $m_e \bar{\Psi} \sigma_{\mu\nu} \Psi$ operator, leading to LV spin precession of the electron. Assuming that the sizes of the dimensionless tensors in (4.14) and (4.16) are $O(1)$, one can estimate the sensitivity to M via the dimension six LV operators: $M^2 \sim m_e m_s^2 / (10^{-28} \text{ GeV})$. This translates into a bound of $M \sim 10^{14} \text{ GeV}$ for $m_s \sim 100 \text{ GeV}$, which is lower than the Planck scale. On the other hand, we notice that in the framework of the LV MSSM the sensitivity to M via dimension six LV operators will be higher, when observables in the quark sector are employed. Indeed, we expect m_e to be replaced by m_q , and 10^{-28} GeV by 10^{-32} GeV , as experiments searching for anomalous spin precession of nucleons are intrinsically more precise. In this case the sensitivity to M would get close to the Planck scale, and future increase of the experimental sensitivity may probe this type of models. Although undoubtedly

very interesting, more detailed study of the observational consequences of dimension six LV operators goes beyond the scope of our survey.

4.5 Discussion

We have constructed a dimension five LV extension of SQED, as a subset of the full LV MSSM. The LV modifications are power-suppressed by the UV scale M and decouple in the limit of $M \rightarrow \infty$. In the leading order in the inverse UV scale, $O(M^{-1})$, dimension five LV operators can be coupled to two types of LV backgrounds. There are three background vectors N^μ , N_+^μ and N_-^μ , as well as an irreducible rank three tensor $T^{\mu\nu\lambda}$ (antisymmetric in $\nu\lambda$). The corresponding LV operators are all CPT-odd. At the dimension six level LV operators are CPT-even; their classification has been given in this chapter.

We have explored quantum effects in the presence of the LV terms. We have shown that no D -term is induced and the anomaly cancellation condition is not altered by the presence of LV in the limit of exact SUSY. The RGE's for LV operators of dimension five were derived in the limits of exact and softly-broken SUSY. Once SUSY is broken, dimension three operators can be generated. The transmutation of dimension five LV operators into dimension three is controlled by the scale of soft SUSY breaking. This alleviates the LV naturalness problem, because the potentially problematic quadratic divergences are stabilized at the SUSY breakdown scale. In

order to obtain phenomenologically applicable formulas, we broke SUSY by introducing scalar electron masses, and calculated the resulting effective LV Lagrangian for electrons. A dimension three operator for photons, the CS term, is not generated at the loop level. It is remarkable that none of the LV operators, considered in this chapter, lead to high-energy modifications of dispersion relations. Therefore, none of the stringent astrophysics-derived limits on LV parameters [30, 33] apply to LV SQED.

We have obtained explicit component expressions for LV interactions generated by vector and tensor backgrounds, which allowed us to derive observational consequences of LV in SQED. Using the results of high-precision searches for LV spin interactions, we derived stringent limits on some linear combinations of LV parameters. The most stringent results were obtained from a one-loop induced coupling between the electron axial vector current and some combination of the background vectors N^μ , N_+^μ and N_-^μ . The strongest bound was obtained from the absence of anomalous spin precession for electrons, which is checked at a level better than 10^{-28} GeV by torsion balance experiments [28]. Assuming that the UV scale is of the same order of magnitude as the Planck scale, we were able to constrain one linear combination of N^i , N_+^i and N_-^i at the level better than $O(10^{-12})$. Conversely, if we insist that $N^i \sim O(1)$, such experiments provide a sensitivity to the LV ultraviolet scale which is more than ten orders of magnitude *larger* than the Planck scale. Other precision experiments [26, 75]

provide stringent constraints on different linear combinations of the LV vector and tensor backgrounds.

The existence of strong constraints on LV at dimension five level (with or without SUSY), poses a serious challenge for theories that predict LV at $1/M_{\text{Pl}}$ level. Therefore, either such theories are ruled out, or they require abandoning an effective field theory description of LV. (The latter does not seem as a reasonable alternative to us.) However, it might be that dimension five operators are forbidden by some additional symmetry reasons, such as *e.g.* CPT. At the next order, $O(M^{-2})$, Planck suppressed LV effects are not excluded. The best constraints may come close [31], but are applicable only to operators that modify high-energy dispersion relations. The classification of dimension six LV operators in SQED has showed that they couple to symmetric or antisymmetric two-index tensor backgrounds. As we discussed at length for dimension five LV operators, similarly, some of dimension six operators will transmute into dimension four operators due to quantum effects in the presence of the soft-breaking terms. The scale of the transmutation is controlled by the SUSY breakdown scale, which gives an estimate for the size of LV backgrounds at dimension four as $m_s^2/M^2 \sim 10^{-32}$ for $m_s \sim 1$ TeV and $M \sim 10^{19}$ GeV. This prediction comes close to the experimental sensitivity to such operators, and therefore deserves further studies in the framework of LV MSSM.

Chapter 5

CPT-odd Leptogenesis

Since the seminal paper by Sakharov [78], it is well known that the baryon asymmetry of the Universe (BAU) can be generated dynamically, through the combination of baryon number violating processes, C and CP violation, and the departure from thermal equilibrium. It turns out that the Standard Model has all necessary ingredients for this to happen. Notably, the $B + L$ number is violated by the high-temperature sphaleron processes [79], [80]. However, the existing amount of CP -violation combined with tight constraints on the Higgs sector, prevent efficient baryogenesis in the SM. Thus, BAU presents a formidable hint on physics beyond SM, and motivates new experimental searches for the extended electroweak sector and new sources of CP violation. A potential solution to this problem was introduced in the form of leptogenesis [81]. According to this mechanism, the Universe first acquires a lepton

number, or equivalently an initial $B - L$, via non-equilibrium decays of right-handed neutrinos. In this mechanism, the decays of right-handed neutrinos must fall out of equilibrium before the sphaleron processes become fast. Sphaleron processes then transfer the lepton asymmetry to the baryon sector.

It has also been known for some time that CPT -odd perturbations can effectively replace two of Sakharov's conditions for baryogenesis: violation of CP invariance and the deviation from thermal equilibrium [37]. Indeed, a CPT -odd shift in the "mass" of a SM fermion (e.g. top quark [82]), Δm_{CPT} would serve as an effective chemical shift between baryons and antibaryons above the scale of the electroweak phase transition. It is easy to see that a $\Delta m_{CPT}/m_t \sim O(10^{-6})$ effect for top quark would be required to generate the observed asymmetry [82]. Unfortunately, at the level of the Lagrangian, it is impossible to define a consistent " CPT -odd mass" without breaking Lorentz invariance. CPT -odd mass would have to be identified with dimension three Lorentz-noninvariant operators [54]. Given the strength of constraints on lower-dimensional CPT /Lorentz noninvariant operators [40], one has to conclude that lower dimensional operators cannot be a source of the observed baryon asymmetry.

The problem of CPT -odd baryogenesis was readdressed in [38] and recently in [83, 84]. In [38] and [83] higher-dimensional CPT -odd operators were suggested as a source for baryon asymmetry, among other options. Suppose that a dimension five operator that shifts the dispersion relations of baryons relative to antibaryons is

added on top of the SM. Let us further assume that initial value for $B - L$ is zero. Then in the temperature range from 10^{10} to 10^2 GeV where the sphaleron processes are in thermal equilibrium, the resulting baryon asymmetry will be determined by the amount of CPT violation in the theory. If CPT -violating interactions are given by a dimension five operator parametrized by $1/\Lambda_{CPT}$, the inverse energy scale of CPT violation, the resulting baryon asymmetry at the sphaleron freeze-out ($T \sim M_W$) will be given by

$$Y_b = \frac{\Delta b}{s} \sim \frac{T}{\Lambda_{CPT}} \sim \frac{M_W}{\Lambda_{CPT}}, \quad (5.1)$$

where s is the entropy. It is clear then that $\Lambda_{CPT} < 10^{12}$ GeV will be required to produce an observable asymmetry. Given the fact that both low-energy data and astrophysical constraints limit a typical scale Λ_{CPT} to be higher than the Planck scale, such scenario is completely ruled out.

In this dissertation we explore the idea of the CPT -odd leptogenesis that is capable of enhancing estimate (5.1) by many orders of magnitude. The main feature of any leptogenesis scenario is the use of the lepton number non-conservation at high temperatures that results in a non-vanishing $B - L$ number that is preserved by sphaleron processes [81]. One of the advantages of leptogenesis is that the most natural way of mediating the lepton-violating processes is through heavy majorana neutrinos, which also supply masses to the light neutrinos via the see-saw mechanism. We assume that the mass M_R of heavy right-handed neutrinos is large enough so that

they have decayed at the temperatures of leptogenesis, but they still mediate lepton number violating processes off-shell. These processes keep leptons in equilibrium until the temperature decreases to the point where the Hubble rate Γ_H begins to dominate over the lepton-violation rate Γ_L . In the assumption that Yukawa couplings are on the order of one, this moment in Universe's history can be determined as

$$\Gamma_L \propto \frac{T^3}{M_R^2} \sim \Gamma_H \propto \frac{T^2}{M_{\text{Pl}}},$$

which gives an estimate for the temperature of the freeze-out for the $B - L$ number:

$$T_R \propto \frac{M_R^2}{M_{\text{Pl}}}.$$

Therefore, in the scenarios of CPT -odd leptogenesis, one obtains the asymmetry which freezes out at $T = T_R$ rather than at $T = M_W$:

$$Y_{l(b)} \sim \frac{M_R^2}{M_{\text{Pl}}\Lambda_{CPT}}.$$

Obviously, for $M_R \sim 10^{15}$ GeV one gets a great enhancement by $T_R/M_W \sim 10^9$ over the CPT baryogenesis scenarios (5.1) where $B - L$ is zero.

In this chapter we explore the CPT -odd leptogenesis scenario, determine the required strength of the CPT -violating operators, and compare it with the existing

laboratory and astrophysics constraints. For the reasons explained above, we concentrate on CPT -odd interactions of mass dimension five. We introduce CPT -odd operators into the fermion sector of the Standard Model [23] (see also Chapter 2):

$$\mathcal{L}_{LV} = \sum_{i=L,E,Q,U,D} \eta_i^{\mu\nu\rho} \cdot \bar{\psi}_i \gamma_\mu \mathcal{D}_\nu \mathcal{D}_\rho \psi_i , \quad (5.2)$$

which cause an asymmetric shift of the dispersion relations for fermions and antifermions. Here $\eta_i^{\mu\nu\rho}$ is a symmetric irreducible Lorentz violating spurion field, that can depend on the type of the SM fermions, and the summation extends over all fields that carry the lepton or baryon number. The transmutation to the lower-dimensional operators is protected by the irreducibility condition, $\eta^{\mu\nu} = 0$. A zeroth component of $\eta_i^{\mu\nu\rho}$, $\eta_i^{000} \equiv \eta_i$ in the reference frame where the primordial plasma is at rest provides an asymmetric shift in the dispersion relations for particles and antiparticles. This way, positive η_{lepton} creates a surplus of antileptons over leptons in equilibrium which is maintained when the rate for the lepton number violating processes is faster than the Hubble expansion. It is notable that such CPT -odd perturbations already allow for potential leptogenesis with one flavor of heavy majorana neutrinos, whereas conventional leptogenesis requires at least two flavors [81]. In the rest of this chapter, we examine closely the kinetic equations for the $L(B)$ -violating processes when CPT -odd shifts (5.2), lepton-number violation and sphaleron processes are taken into account.

We adjust the coupling constants η in (5.2) in such a way as to produce the observed value of the baryon asymmetry and compare the results with the existing limits on Lorentz violating interactions. We argue that the combination of bounds on LV from observations of high-energy cosmic rays [31] and the low-energy clock comparison experiments render the *CPT*-odd leptogenesis scenario fine-tuned for models with operators of mass dimension five (5.2), but allow it for higher-dimensional operators.

5.1 Reaction rates and Boltzmann equations

To demonstrate how the *CPT*-odd leptogenesis works we consider a model with only one heavy majorana neutrino. Its off-shell exchange mediates lepton number violating processes that freeze out at the temperatures well below M_R . At $T > T_R$ these processes maintain the equilibrium value for the lepton number asymmetry. In this section we calculate the rate of the lepton number violating processes and include it in the Boltzmann equations together with the sphaleron rate.

The mass term Lagrangian for heavy neutrinos reads as

$$\mathcal{L}_m = -\frac{1}{2}M_R \bar{N}_M N_M + h_a \cdot \bar{L}_a H N_M + h_a^\dagger \cdot \bar{N}_M H^\dagger L_a, \quad (5.3)$$

where N_M are singlet majorana neutrinos and h_a are the Yukawa couplings. We

switch to Weyl spinors for convenience, in which the Lagrangian can be rewritten as

$$\mathcal{L}_m = -\frac{1}{2} M_R (NN + \overline{NN}) + h_a \cdot \overline{L}_a \overline{N} H + h_a^\dagger \cdot H^\dagger N L_a, \quad (5.4)$$

where index a runs over three different generations, and

$$N_M = \begin{pmatrix} N_\alpha \\ \overline{N}^{\dot{\alpha}} \end{pmatrix}.$$

Integrating out the heavy neutrinos, one obtains an effective lepton number violating vertex:

$$\mathcal{L}_{\text{eff}} = \frac{Y_{ij}^\nu}{2 M_R} H^\dagger L_i^\alpha H^\dagger L_{j\alpha} + \text{h.c.}, \quad (5.5)$$

where $Y_{ab}^\nu = h_a^\dagger h_b^\dagger$. Substituting the vacuum expectation value for the Higgs field in (5.5) creates a majorana mass term for light neutrinos. This interaction induces lepton number violating processes which determine the lepton asymmetry until the lepton freeze-out. Alternatively, we could step by the stage with the heavy right-handed neutrinos and postulate (5.5) as a starting point in our analysis while taking Y_{ab}^ν to be an arbitrary complex symmetric matrix.

Introduction of the CPT -odd interactions (5.2), as we know, leads to the modification of dispersion relations for the SM leptons and antileptons. Taking lepton

doublets, we neglect the mass terms and find

$$E_L(p) = |\vec{p}| + \eta_L \vec{p}^2, \quad E_{\bar{L}}(p) = |\vec{p}| - \eta_L \vec{p}^2. \quad (5.6)$$

Equation (5.6) leads to a shift in the equilibrium number density of leptons

$$n_L^{\text{eq}} = \frac{g_L}{\pi^2 \beta^3} \left(1 - \frac{12 \eta_L}{\beta} \right),$$

with the opposite sign of the shift for antileptons. Here g_L is the total number of the spin, gauge and flavor degrees of freedom associated with electroweak doublets L , and β is the inverse temperature. The difference,

$$n_i^{\text{eq}} - n_{\bar{i}}^{\text{eq}} = -24 \eta_i g_i (\pi^2 \beta^4)^{-1}, \quad (5.7)$$

where $i = L$ for now, represents an equilibrium lepton number induced by CPT violation in the lepton doublet sector. As stated in the beginning of this chapter, the final abundance can be roughly estimated by evaluating the equilibrium density at the temperature of the freeze-out. A more accurate answer, however, can be obtained by analyzing Boltzmann equations in the presence of sphaleron processes and lepton number violation.

There are two types of interactions induced by the effective Lepton-Higgs ver-

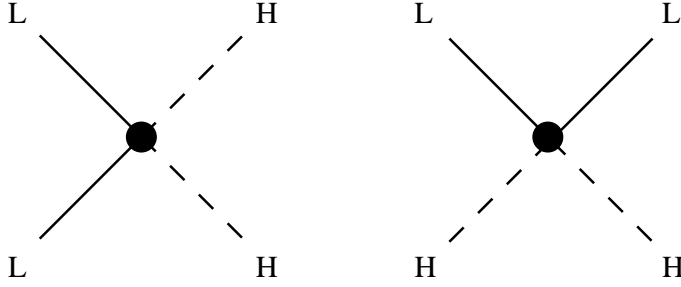
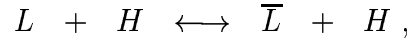
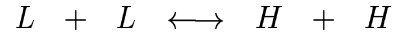


Figure 5.1: $\Delta L = 2$ processes generated by the effective vertex (5.5).

tex [85, 86], shown in Fig. 5.1. They generate the following processes relevant for leptogenesis:



with the same set of processes for antileptons. However, since the relevant part of the CPT -odd interactions is time reversal invariant, the amplitudes for direct and inverse processes are equal, and we therefore have only three different amplitudes, which we call A_{LL} , $A_{\bar{L}\bar{L}}$ and A_{LH} . Denoting the corresponding reaction rates (per unit volume)

by W_{LL} , $W_{\overline{LL}}$, W_{LH} and \overline{W}_{LH} , we have

$$\begin{aligned}
W_{LL} &= \int d\pi_p d\pi_q d\pi_k d\pi_r (2\pi)^4 \delta^4(p+q-k-r) |A_{LL}|^2 f_L^{\text{eq}}(p) f_L^{\text{eq}}(q) , \\
W_{\overline{LL}} &= \int d\pi_p d\pi_q d\pi_k d\pi_r (2\pi)^4 \delta^4(p+q-k-r) |A_{\overline{LL}}|^2 f_{\overline{L}}^{\text{eq}}(p) f_{\overline{L}}^{\text{eq}}(q) , \\
W_{LH} &= \int d\pi_p d\pi_q d\pi_k d\pi_r (2\pi)^4 \delta^4(p+q-k-r) |A_{LH}|^2 f_L^{\text{eq}}(p) f_H^{\text{eq}}(q) , \\
\overline{W}_{LH} &= \int d\pi_p d\pi_q d\pi_k d\pi_r (2\pi)^4 \delta^4(p+q-k-r) |A_{\overline{LH}}|^2 f_{\overline{L}}^{\text{eq}}(p) f_H^{\text{eq}}(q) ,
\end{aligned}$$

where $f_{L,H}^{\text{eq}}(p)$ are the equilibrium distribution functions for Higgs fields and lepton doublets. In a toy model where only the lepton doublets and Higgs fields are present one can immediately write the Boltzmann equations for the lepton number density as

$$\begin{aligned}
\left(\partial_t + 3\Gamma_H \right) n_L &= - 2 W_{LL} \left(\frac{n_L^2}{(n_L^{\text{eq}})^2} - 1 \right) - W_{L\overline{H}} \left(\frac{n_L}{n_L^{\text{eq}}} - \frac{n_{\overline{L}}}{n_{\overline{L}}^{\text{eq}}} \right) \\
\left(\partial_t + 3\Gamma_H \right) n_{\overline{L}} &= - 2 W_{\overline{LL}} \left(\frac{n_{\overline{L}}^2}{(n_{\overline{L}}^{\text{eq}})^2} - 1 \right) - \overline{W}_{L\overline{H}} \left(\frac{n_{\overline{L}}}{n_{\overline{L}}^{\text{eq}}} - \frac{n_L}{n_L^{\text{eq}}} \right) .
\end{aligned} \tag{5.8}$$

Here the Hubble rate is $\Gamma_H = 1.66 g_*^{1/2} T^2 / M_{\text{Pl}}$ in terms of the total effective number of degrees of freedom g_* . The factor of two in the right hand side of (5.8) reflects the fact that the LL processes change the number of leptons by two. An important thing to note is that even though we could have modified the dispersion relations for the Higgs field, its CPT -violating parameter would not enter the equations for the

lepton number density at tree level.

In order to generalize equations (5.8) onto the full set of SM fields, we introduce the effective parameters of CPT violation in the lepton and baryon sectors:

$$\eta_l = \frac{g_L \eta_L + g_E \eta_E}{g_L + g_E}; \quad \eta_b = \frac{g_Q \eta_Q + g_U \eta_U + g_D \eta_D}{g_Q + g_U + g_D}, \quad (5.9)$$

where g_i is the corresponding number of degrees of freedom in each sector. These parameters enter (5.7) with $i = l, b$, and $g_l = g_L + g_E$, $g_b = g_Q + g_U + g_D$.

As already mentioned, one also has to include sphaleron processes, which affects one linear combination of baryon and lepton number densities. The main effect of sphalerons is to wash-out $B + L$, while keeping $B - L$ intact. Since the processes we consider occur far above the electroweak transition, the sphaleron rate has linear dependence on temperature [80, 87]. In the presence of CPT violation, the sphaleron contribution to the Boltzmann equation for n_l, n_b [80, 88, 89] should be modified for the presence of the equilibrium baryon and lepton numbers (5.7):

$$\partial_t (n_b + n_l) = -\Gamma_{\text{sph}} \left(n_b - n_b^{\text{eq}} + n_l - n_l^{\text{eq}} \right), \quad (5.10)$$

where

$$\Gamma_{\text{sph}} \simeq \omega T, \quad \text{with } \omega \simeq 10^{-5}.$$

Equation (5.10) implies that $B + L$ is washed out completely, and is somewhat simplified relative to the realistic case. A detailed analysis shows (see *e.g.* [90]) that the wash-out is only partial, with the final value of $B + L$ controlled by a nonzero $B - L$, but we will employ the naive evolution equation (5.10), arguing that the corrections to this equation are much smaller than the uncertainty with which ω is known.

Next we make a well-justified assumption of smallness of the chemical potentials,

$$\frac{n_i}{n_i^{\text{eq}}} = e^{\mu_i/T} \simeq 1 + \mu_i/T ,$$

which enables us to linearize the kinetic equations in μ_i . The kinetic equations for n_l take the following form:

$$\begin{aligned} \left(\partial_t + 3\Gamma_H \right) n_l &= - \left(4W_{LL} + 2W_{LH} \right) \mu_l/T - \omega T \left(\mu_l/T + \mu_b/T \right) \\ \left(\partial_t + 3\Gamma_H \right) n_{\bar{l}} &= \left(4\overline{W}_{\overline{LL}} + 2\overline{W}_{\overline{LH}} \right) \mu_l/T + \omega T \left(\mu_l/T + \mu_b/T \right) . \end{aligned} \tag{5.11}$$

For the (anti)baryons the kinetic equations are the same except that there are no contributions from the lepton number violating rates. A significant simplification comes from the smallness of the chemical potential. There are two possible sources for CPT -odd contributions to the reaction rates in (5.8): modified dispersion relations and CPT -odd modifications of thermal rates. The smallness of μ_i/T allows us to

neglect any CPT -odd effects in the reaction rates in the right hand side of (5.8), as they induce effects of the 2nd order in the CPT -violating parameter. Therefore, we take $W_{\overline{L}L} = W_{LL}$ and $\overline{W}_{LH} = W_{LH}$.

From the above equations we only need their difference, the actual lepton (baryon) asymmetry. For convenience, we express the equilibrium number density in terms of the unmodified number density $n_i^0 = g_i/\pi^2 \cdot T^3$

$$n_{i,\bar{i}}^{\text{eq}} = n_i^0 (1 \mp 12 \eta_i T) , \quad i = l, b .$$

The asymmetries Y_i then can be defined as

$$n_i - n_{\bar{i}} \equiv 2 n_i^0 \cdot Y_i , \quad Y_i = \mu_i/T - 12 \eta_i T .$$

We also introduce a dimensionless parameter γ , by factoring out the dimensionful parameters T^3/M_R^2 from the rate of lepton number violating processes,

$$4 W_{LL} + 2 W_{L\overline{H}} = \gamma \frac{T^6}{M_R^2} ,$$

so that γ scales as the fourth power of the neutrino Yukawa couplings or the sum of

the squares of the eigenvalues of Y_{ab}^ν :

$$\gamma = \frac{3}{2\pi^2} \left(\sum |h_a|^2 \right)^2 = \frac{3}{2\pi^2} \text{Tr} (Y_{\text{diag}}^\nu Y_{\text{diag}}^{\nu\dagger}). \quad (5.12)$$

Expressing equations (5.11) in terms of Y_i and changing variables from time to temperature, we get:

$$\begin{aligned} g_l \frac{d}{dT} Y_l &= \frac{0.6}{g_*^{1/2}} \frac{\omega M_{\text{Pl}}}{T^2} \left(g_l (Y_l + 12 \eta_l T) + g_b (Y_b + 12 \eta_b T) \right) \\ &+ \frac{0.6 \pi^2}{g_*^{1/2}} \frac{\gamma M_{\text{Pl}}}{M_R^2} \cdot (Y_l + 12 \eta_l T) \\ g_b \frac{d}{dT} Y_b &= \frac{0.6}{g_*^{1/2}} \frac{\omega M_{\text{Pl}}}{T^2} \left(g_l (Y_l + 12 \eta_l T) + g_b (Y_b + 12 \eta_b T) \right). \end{aligned} \quad (5.13)$$

The quantity of ultimate interest is the baryon asymmetry at the present time (normalized, e.g. on the photon number density, $n_\gamma = s/7.04$ [91]). Using $s = \frac{2\pi^2}{45} g_* T^3$, one can express the experimentally measured baryon to photon ratio via the asymmetry Y_b that enters (5.13),

$$\alpha_B = 7.04 \frac{45}{\pi^4} \frac{g_b}{g_*} Y_b \simeq 0.6 Y_b \equiv (6.1 \pm 0.3) \times 10^{-10}, \quad (5.14)$$

where we use $g_b = 18$ and $g_* = 106.75$.

Note, that in the limit when the rate of sphaleron processes is very small, $\Gamma_{\text{sph}} \ll \Gamma_L$ (Γ_L is the rate of the lepton number violating processes), one can solve the kinetic

equations exactly. Taking $\omega \rightarrow 0$ in (5.13), we have:

$$\frac{d}{dT} Y_l = \frac{\lambda M_{\text{Pl}}}{M_R^2} \left(Y_l + 12 \eta_l T \right) . \quad (5.15)$$

where we have introduced $\lambda = 0.6 \pi^2 (g_*^{1/2} g_l)^{-1} \gamma$. A solution that corresponds to n_l close to equilibrium value at $T \gg T_R$ has the following form:

$$Y_l = -12 \frac{\eta_l M_R^2}{\lambda M_{\text{Pl}}} - 12 \eta_l T , \quad (5.16)$$

which provides us with the expression for the lepton asymmetry:

$$Y_l^{\text{fr}} = -12 \frac{\eta_l M_R^2}{\lambda M_{\text{Pl}}} . \quad (5.17)$$

The inclusion of sphalerons will diffuse approximately half of the lepton number yield into the baryon number [80], so that Eq. (5.17) is also an estimate for the BAU.

5.2 The strength of *CPT* violation derived from BAU

In this section, we provide the numerical solutions to equations (5.13), determine the required strength of *CPT* violation and compare it with existing experimental

constraints.

To solve the system of kinetic equations, one has to add proper initial conditions. It is reasonable to impose these initial conditions at the temperatures where the essential part of leptogenesis begins, which we take to be $M_R = 10^{15}$ GeV:

$$\begin{aligned} Y_l|_{M_R} &= Y_l^{\text{eq}}, \\ Y_b|_{M_R} &= 0. \end{aligned} \tag{5.18}$$

At high temperatures leptons and antileptons were in thermal and chemical equilibrium, which had a nonzero value of the lepton number defined by η_l . This choice is quite sensible since the freeze-out temperature T_R suggested by neutrino masses is sufficiently smaller than M_R . As for baryons, we impose symmetric $n_b = n_{\bar{b}}$ conditions at high temperatures (10^{15} GeV), as there are no fast processes that would bring Y_b to the equilibrium position set by η_b .

Since we chose to fix M_R , the only free parameters left are η_i and η_b parametrizing the strength of *CPT* violation, and the neutrino Yukawa couplings. For the latter there is some natural range suggested by the oscillations among the light neutrino flavors. Introducing an “effective” neutrino mass that the kinetic equations (5.13) depend on,

$$m_\nu^{\text{eff}} \equiv \left(\sum m_{\nu_i}^2 \right)^{1/2} = \frac{v^2}{2M_R} \left(\sum |Y_{\text{diag}}^\nu|^2 \right)^{1/2}, \tag{5.19}$$

we notice that $(m_\nu^{\text{eff}})^2$ is larger than any of the individual Δm_{ij}^2 measured in the oscillations experiments. Thus, taking the largest of Δm_{ij}^2 , suggested by the oscillation of atmospheric neutrinos, $\sqrt{\Delta m_{\text{atm}}^2} \simeq 0.05$ eV [92] we find the following natural range for m_ν^{eff} :

$$0.05 \text{ eV} \leq m_\nu^{\text{eff}} \leq 0.65 \text{ eV} , \quad (5.20)$$

where the upper limit comes from the cosmological bound on the sum of neutrino masses [93]. Defining the freeze-out temperature via the relation $\Gamma_H(T_R) = \Gamma_L(T_R)$, one can translate (5.20) to the realistic range of T_R :

$$10^{12} \text{ GeV} < T_R < 10^{14} \text{ GeV} . \quad (5.21)$$

On the lower end of this range T_R overlaps with the sphaleron ignition temperature T_{sph} , which is estimated to be of the order 10^{12} GeV [94].

The final result of our analysis is a prediction for the strength of *CPT* violation in lepton and baryons sectors. Since equations (5.13) are linear in Y_i , it is sufficient to solve them numerically for two cases

$$\eta_l \neq 0, \quad \eta_b = 0 \quad \text{and} \quad \eta_l = 0, \quad \eta_b \neq 0 ,$$

and then using the experimental value of BAU, fix the values of η_l and η_b as functions

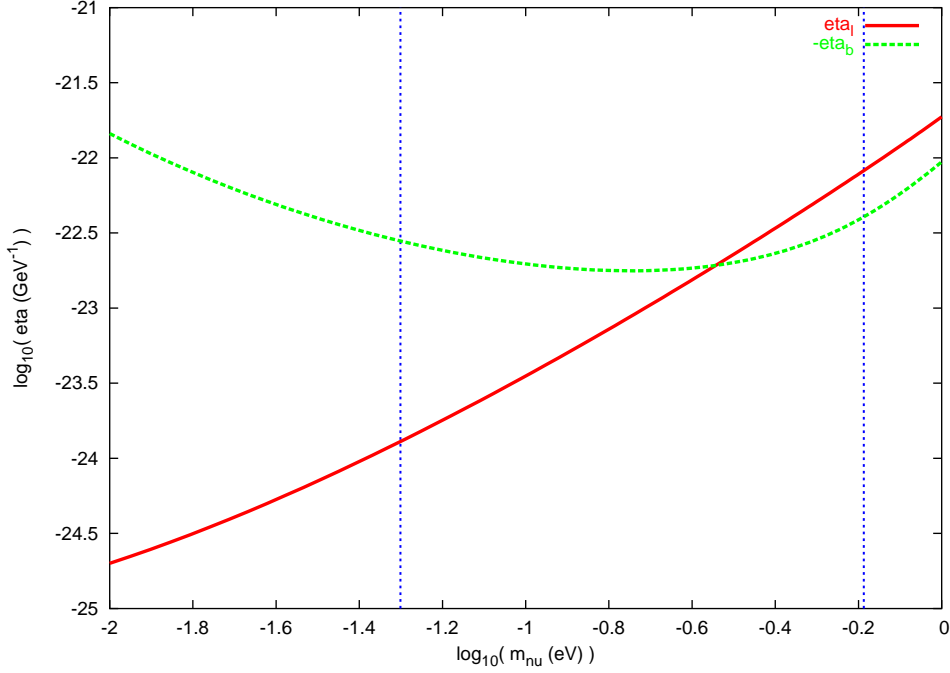


Figure 5.2: CPT -odd parameters η_l , η_b necessary to generate the observed BAU versus the effective neutrino mass. The left vertical line indicating the value of m_ν^{eff} suggested by the oscillation of atmospheric neutrinos and the right vertical line showing the cosmological upper limit on m_ν^{eff} [93], bound the phenomenologically viable domain of m_ν^{eff} .

of m_ν^{eff} .

Fig. 5.2 shows the resulting dependence of η_i on m_ν^{eff} within a phenomenologically viable range of m_ν^{eff} bounded by two vertical dashed lines. One notices that η_b does not change much in the “physical” region. For η_l -dominated scenario, in contrast, the increase of η_l with m_ν^{eff} is well pronounced. As expected, the lower mass m_ν^{eff} leads to a *higher* freeze-out temperature T_R , and thus lower m_ν^{eff} requires *lower* CPT violating parameter η_l to get an observed value of BAU. Also not surprisingly, η_l and η_b required to reproduce BAU in our scenario have opposite signs.

The lower end of the range (5.20) corresponds to a hierarchical scenario $m_1^2, m_2^2 \ll m_3^2$, with the tau-neutrino being the heaviest. The size of the CPT violation suggested by the CPT -odd leptogenesis in this case is found to be

$$\eta_l = 9 \times 10^{-25} \text{ GeV}^{-1}, \eta_b = 0 \quad \text{or} \quad \eta_l = 0, \eta_b = -1.5 \times 10^{-23} \text{ GeV}^{-1} .(5.22)$$

This is the main prediction of our analysis.

Figures 5.3 and 5.4 illustrate the case of $m_\nu^{\text{eff}} = 0.05 \text{ eV}$ in more detail, by showing the evolution of the baryon/lepton asymmetry as a function of temperature. When CPT violation is concentrated in the lepton sector, see Fig. 5.3, the lepton asymmetry follows the equilibrium value of the (lepton) asymmetry at high temperatures to freeze out below 10^{14} GeV . When CPT violation is given by η_b , the asymmetry Y_b starts from zero, overshoots the equilibrium curve just above 10^{13} GeV , to freeze out at lower temperatures.

5.3 Comparison with experimental constraints on CPT violation

Now we are ready to compare our predictions for CPT -violation (5.22) with the experimental constraints on it. The modification of dispersion relation by dimension

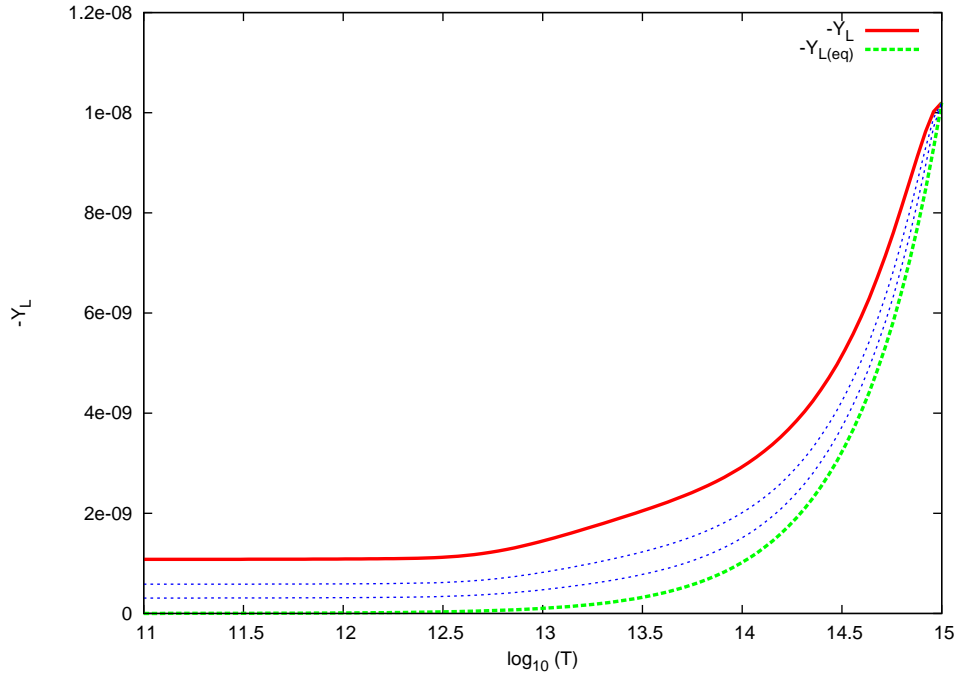


Figure 5.3: Lepton asymmetry (solid line) and equilibrium lepton asymmetry (dashed line) driven by CPT violation in the lepton sector for $m_\nu^{\text{eff}} = 0.05$ eV as function of temperature. The amount of CPT violation is fixed to $\eta_l = 9 \times 10^{-25}$ GeV^{-1} to yield the observed value of baryon asymmetry. The final low-temperature plateau of $-Y_l$ equals to the baryon asymmetry Y_b . The dotted lines correspond to $m_\nu^{\text{eff}} = 0.07$ eV and 0.10 eV, and demonstrate the approach to the equilibrium curve with the increase of mass m_ν^{eff} .

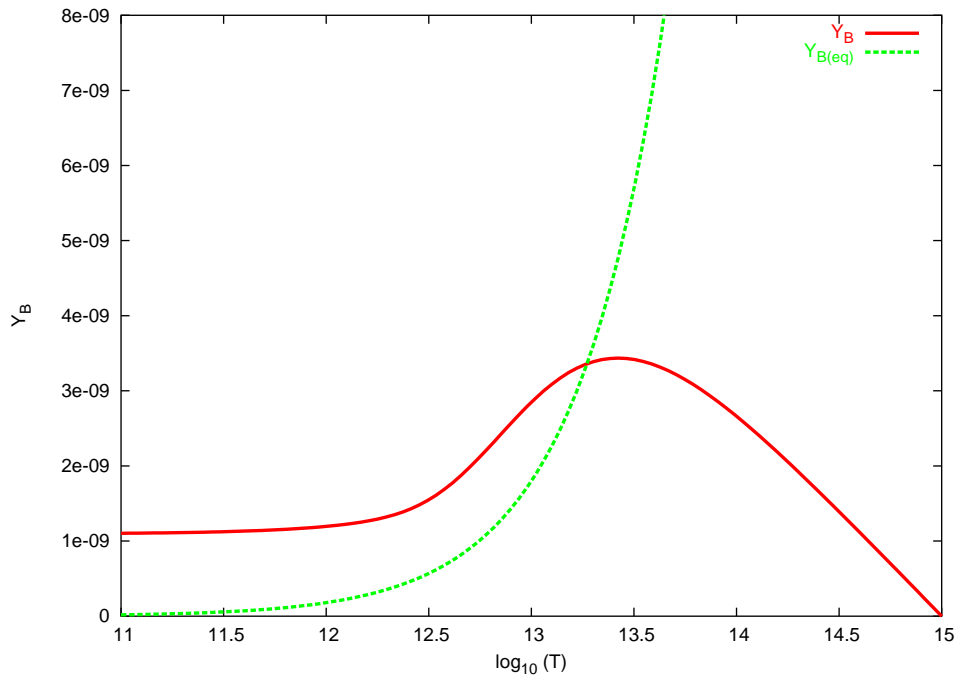


Figure 5.4: Baryon asymmetry Y_b and equilibrium baryon asymmetry vs temperature with CPT violation concentrated in the baryon sector. The parameters $m_\nu^{\text{eff}} = 0.05$ eV and $\eta_b = -1.5 \times 10^{-23} \text{ GeV}^{-1}$ are chosen to match the observed asymmetry.

five operators has been discussed at length in the literature. Below we list a set of relevant constraints on dimension five operators in the fermionic sector of the SM and comment on their applicability:

$$|\eta_d - \eta_Q - 0.5(\eta_u - \eta_Q)| < 10^{-27} \text{ GeV}^{-1}, \quad [23, 95]$$

$$|\eta_L|, |\eta_E| < 10^{-20} \text{ GeV}^{-1}, \quad [96]$$

$$|\eta_L|, |\eta_E| < 10^{-33} \text{ GeV}^{-1}. \quad [31]$$

The first constraint arises because the axial-vector-like combinations of η_i in the quark sector lead to the coupling of the nucleon spin to a preferred direction. In models where the preferred frame is associated with the rest frame of the cosmic microwave background, the net spin energy shift is proportional to the velocity of the lab frame $v \sim O(10^{-3})$, $\Lambda_{QCD}^2 \eta_i (v \cdot s)$, which is to be compared with the experimental sensitivity 10^{-31} GeV [26, 75]. The low-energy constraints on lepton operators are considerably weaker [23]. The strongest constraints on dimension five CPT -odd operators in the lepton sector come from considerations of $p \rightarrow pl\bar{l}$ processes that become energetically allowed and prevent acceleration of protons to energies of $\sim 10^{21} \text{ eV}$. It is important that constraints [31] are double-sided, which is the consequence of asymmetric modification of dispersion relation for leptons and antileptons (5.6).

The strength of CPT violation in the lepton sector derived from the baryon

asymmetry (5.22) is consistent with the astrophysical bounds on CPT -violating QED [96], but appears to be grossly inconsistent with [31]. In fact, typical constraints on dimension five operators [31] derived from the existence of the high-energy cosmic rays appear to destroy any hopes for the CPT -odd baryogenesis, even if the scale of T_R is pushed all the way up to the Planck scale. It is easy to see, however, that this is not the case. If CPT -odd sources in the quark sector dominate over the lepton sources by a factor of 20-30, strong constraints on CPT violation might be avoided. If, for example, among the CPT -odd sources the right-handed up quark has the largest modification of its dispersion relation, the energetically favored process $p \rightarrow \Delta^{++}\pi^-$ allows the ultra high-energy cosmic rays to exist in the form of Δ^{++} , an option which cannot be observationally ruled out [31]. It is very important to observe that the *negative* sign of η_U suggested by BAU (5.22) is exactly the sign of η_U needed for $p \rightarrow \Delta^{++}\pi^-$ to happen at high energies. Nevertheless, the required size of η_U , $\eta_U \sim -(10^{-23} - 10^{-22}) \text{ GeV}^{-1}$ appears to be in sharp conflict with the low-energy constraints [23, 95], and at least four orders of magnitude tuning for dimension five sources is needed. This consideration shows an important complementarity between the astrophysical bounds on Lorentz violation and low-energy searches of the breakdown of rotational invariance.

We can extend our analysis to theories where CPT violation comes from operators of dimension seven, nine, etc., should for some contrived reasons lower dimensional

operators be absent. We note that to sufficient accuracy, the resulting BAU will be determined by the equilibrium lepton asymmetry at the freeze-out time, $\eta^{(7)} T_R^3$, $\eta^{(9)} T_R^3$, where $\eta^{(n)}$ parametrize the strengths of the higher dimensional operators:

$$\mathcal{L} = \sum \eta_{\kappa\mu\dots\nu}^{(n)} \bar{\psi} \gamma^\kappa \mathcal{D}^\mu \dots \mathcal{D}^\nu \psi.$$

As before, the transmutation to lower-dimensional operators can be forbidden by the irreducibility of $\eta^{(n)}$ tensors.

The low-energy constraints on dimension seven and higher CPT -odd operators are totally irrelevant, as the possible influence on the nucleon spin is suppressed by extra power(s) of $(\Lambda_{QCD}/\Lambda_{CPT})^2$. The constraints coming from the propagation of the high-energy cosmic rays are harder to avoid, as their relative strength scales down as $(E_{max}/\Lambda_{CPT})^2$, where E_{max} is the maximal energy of the high-energy cosmic rays $E_{max} \sim 10^{12}$ GeV. In fact, since the decoupling temperature T_R can only be marginally larger than 10^{12} GeV, the CPT -violating sources of dimension seven in the lepton sector allowed by the cosmic rays would not be able to produce the required size of the baryon asymmetry. However, the same loophole with the stability of Δ^{++} at high-energies exists for the dimension seven operators, and the right-handed up-quark CPT violation at the level of

$$\eta_U^{(7)} = -[(10^{17} - 10^{18}) \text{ GeV}]^{-3} \tag{5.23}$$

results in the right magnitude of BAU while avoiding all experimental constraints on Lorentz and CPT violation.

5.4 Discussion

We have seen that the presence of CPT -odd interactions is theoretically capable of replacing *two* of Sakharov's conditions of baryogenesis: non-conservation of CP symmetry and departure from thermodynamical equilibrium. The reason for this is that non-zero lepton (or baryon) asymmetry can develop even in thermal equilibrium if the CPT -violating shifts of dispersion relations for particles and antiparticles and fermion number violating processes are operative at the same time [37]. In this chapter, we considered in detail the idea of leptogenesis driven by CPT -violating sources in the fermionic sector of the Standard Model. In this scenario, the generation of the $B - L$ number occurs at temperatures of about $10^{12} - 10^{14}$ GeV, which results in a huge enhancement of the asymmetry as compared to the CPT -odd electroweak baryogenesis scenario, where $B - L = 0$ and the equilibrium value for $B + L$ is maintained until the electroweak breaking, $T \sim 100$ GeV. Consequently, the CPT -odd leptogenesis requires only trans-Planckian size of CPT violation, $\eta_i \sim 10^{-22} - 10^{-24}$ GeV $^{-1}$.

We believe that this is the minimal level of CPT violation required to reproduce the observable asymmetry. Lower levels of CPT -breaking may generate BAU only at

the expense of raising the decoupling temperature for $B - L$ processes, to the range of the *e.g.* GUT scale. Models with such a high initial temperature possess very serious cosmological problems of their own related to the overproduction of dangerous relics (monopoles, gravitinos), and are difficult to incorporate into inflation.

The most natural models of CP -odd leptogenesis require two heavy neutrino singlets to work. We have shown that one species is perfectly sufficient for the CPT -odd scenario. In fact, one could take an even more conservative approach and associate the majorana masses of light neutrinos with the effective Lorentz-conserving interaction (5.5) without specifying its origin. The CPT -odd leptogenesis in this case will proceed exactly as described in this chapter, as long as (5.5) remains unsuppressed at high energies. As a consequence of the reduced heavy sector, the connection to the phenomenology of light neutrinos becomes more direct. As shown, the rate of the lepton-number violating processes is directly proportional to the sum of the mass squared of all light neutrino species.

Confronting the predicted size of CPT -violation with the existing experimental and astrophysical constraints we find that both the low-energy precision searches of preferred directions and the astrophysical constraints derived from the existence of charged high-energy cosmic rays put severe constraints on CPT -odd leptogenesis. The latter, being especially stringent, rules out a possibility of CPT -odd leptogenesis driven by η_l when $\eta_b = 0$. The inverse case, $\eta_l = 0$; $\eta_b \neq 0$ cannot be ruled out from the

astrophysical considerations, as the bounds would not apply if *e.g.* the CPT violation is concentrated in the right-handed up-quark sector. In this case, however, one should expect sizable effects in the clock comparison experiments. Current sensitivity to such operators is at the level of $10^{-27} \text{ GeV}^{-1}$, and thus would require at least four orders of magnitude fine-tuning to make (5.22) evade the bounds.

The CPT -odd interactions that modify dispersion relations represent a relatively small subset of dimension five CPT -odd interactions, see Chapter 2. Is it feasible that other operators could drive (baryo)leptogenesis while evading strong astrophysical and laboratory constraints? If physics responsible for CPT violation preserves supersymmetry, operators that modify dispersion relations are simply not allowed, (see Chapter 4 for details). Instead, a different class of CPT -odd operators may appear:

$$\bar{L}\gamma_\mu LH^\dagger H, \quad \bar{Q}\gamma_\mu QH^\dagger H, \quad \text{etc.} \quad (5.24)$$

When the lepton or baryon number is calculated in equilibrium, such operators will create an effective chemical potential that grows with temperature, $\mu \sim T^2\zeta$, where ζ parametrizes the strength of CPT violation. The easiest way to see that is to consider the thermal field theory correlator between the baryon/lepton number density and such CPT -odd operators. Inside a thermal loop, the Higgs field bilinear will produce T^2 , and the scaling of the effective chemical potential with temperature will be exactly the same as in the case of η_i operators. Although operators (5.24) do not

influence the propagation of the high-energy cosmic rays, they have a phenomenological "problem" of their own, as we discussed in Chapter 2. Inside loops such operators create quadratic divergencies and generate dimension three CPT -odd operators proportional to the square of the ultraviolet cutoff. In the most UV-protected case, the role of this cutoff is assumed by the supersymmetric soft-breaking scale. Still, the strength of typical constraints derived in Chapter 4 is on the order of $10^{-10} M_{\text{Pl}}^{-1}$, making the scenario driven by (5.24) fine-tuned below 1 ppm level. Finally, what if CPT -violation is concentrated in the heavy right-handed neutrino sector? Phenomenology of such model was addressed in [97], where it was shown that loop effects reintroduce CPT violation in the sector of charged leptons. Upon integrating out heavy neutrino fields, one produces operators similar to (5.24), and therefore such possibility is also fine-tuned.

Our main conclusion is that the natural levels of CPT /Lorentz violation suggested by the CPT -odd (lepto)baryogenesis scenario are $10^{-3} - 10^{-5}$ in the Planck mass units, which is well within the ranges already disfavored by the laboratory experiments and observations of the high-energy cosmic rays. Of course, our analysis assumes that the strength of the CPT -odd source was essentially the same in the early Universe and today. It is of course conceivable that dynamical effects could have been responsible for the CPT breaking at high temperatures, sourcing the baryogenesis, with relaxation of CPT -breaking sources to zero at the later stage [98].

Chapter 6

Conclusion

The Effective Field Theory approach to Lorentz violation has been attracting especial interest of physicists in the last few years. One of its indisputable properties is the richness and specifics of phenomenology. In the literature, various problems and applications of LV have been considered, such as its implications for neutrino physics, Lorentz violation in theories with Gravity, dynamical breaking of Lorentz invariance, Lorentz violation in cosmological settings, implications from astrophysical observations, Lorentz violation in condensed matter physics and models with finite temperature, LV in brane physics and in Quantum Gravity, Lorentz violation in non-perturbative physics and in physics with extra dimensions, and a consideration of all sorts of predicted physical effects. One can say that Effective Field Theory provides a unifying framework for studying Lorentz violating physics in different phenomeno-

logical applications.

In this dissertation, we have presented a few problems involving LV physics, where we intensively used the EFT approach. In Chapter 2 we have considered the problem of classification of dimension 5 LV interactions in Quantum Electrodynamics and in the Standard Model. At this mass dimension, such a classification, together with basic phenomenological constraints is given for the first time. Let us list the main results of this project:

- We have obtained a list of all operators in QED consistent with EFT, identified the interactions which can transmute into lower dimensional operators, and computed the RG equations for those operators that are protected from such transmutation.
- We classified all LV interactions in the Standard Model, and divided them into three groups; the first group is the “unprotected” operators which suffer the above mentioned transmutation problem; the second group consists of “protected” operators which grow with the energy and therefore named “UV-enhanced”, where the constraints on UV-enhanced interactions come from astrophysical observations and reach the $10^{-33-34} \text{ GeV}^{-1}$ level; and the third group is comprised by all other operators which we call “soft LV interactions”, the constraints for which come from precision experiments and have a typical order of $10^{-28-30} \text{ GeV}^{-1}$.

We provided a similar classification of LV interactions in a supersymmetric extension of QED in Chapter 4. SQED has very favorable features from the point of view of Lorentz violation, one of which is the absence of lower-dimensional LV interactions. Thus, the operators in SQED are protected from dimensional transmutation by supersymmetry. Even if supersymmetry is broken, the quadratic divergencies which arise when dimension three terms are generated are stabilized at the SUSY breaking scale. The total number of LV operators in supersymmetric theories is greatly reduced compared with the Standard Model, and that allowed for a comprehensive analysis of loop effects. Listed below are the most important results of Chapter 4.

- We classified all dimension five and six operators compatible with SQED; we have confirmed that such interactions do not induce UV modifications of dispersion relations [24].
- We showed that LV interactions do not cause a D-term or gauge anomaly in SQED.
- We proved that a Chern-Simons term is absent in LV SQED due to the reason that it is not compatible with supersymmetry; even when supersymmetry is softly broken we have showed that it is absent at loop level.
- We derived the one loop effective action in the case of softly broken SUSY and obtained the observational constraints on the original LV operators; absence of

fine-tuning results in limits of the order $10^{-12}/M_{\text{Pl}}$ on dimension five operators.

In Chapter 3 we have considered the subsector of LV Standard Model which induces EDM-like interactions. At mass dimension five level, such interactions are odd under *CPT* conjugation and thus possess unique signatures which can be used to identify them in experiment. Analysis of these interactions is also valuable for providing independent constraints on Lorentz violation. Below are summarized the results of this part of our research:

- We classified all *CPT*-odd EDM-like interactions in LV Standard Model (*c.f.* Chapter 2), which includes tensorial interactions; we showed that the corresponding interactions in the right-handed lepton sector are suppressed.
- Using the known experimental limits from EDM searches, we constrained the scale of *CPT*-odd physics to be higher than 10^{12} GeV.
- We exhibited patterns which can help disentangle *CPT*-odd EDM contributions in EDM search experiments; the EDM of electron and Schiff moments of nuclei are expected to be suppressed relative to the neutron EDM.

In Chapter 5 we considered an application of LV physics to the problem of Leptogenesis. In this case, Lorentz violation seems to be an attractive candidate of solving the problem of BAU, by naturally having an asymmetry between baryons

and antibaryons (or particles and antiparticles in general). The simplest baryogenesis scenario, however, is ruled out by tight constraints on the baryon LV sector. A realization of CPT -odd leptogenesis has some direct advantages compared with baryogenesis. In this scenario, the lepton asymmetry of the universe is created in equilibrium by processes involving right-handed neutrinos, which violate lepton number, in conjunction with CPT -odd interactions which shift the equilibrium lepton number from zero. Lepton asymmetry is then transferred to the baryon sector by sphalerons. The main results of this project are:

- The resulting lepton (or baryon) asymmetry is determined by the freeze-out temperature of right-handed neutrino-mediated processes, which yields a great enhancement in comparison with CPT -odd baryogenesis realized at the electroweak scale.
- As a result, the operators of magnitude $\eta_i \sim 10^{-22-24} \text{ GeV}^{-1}$ could produce the observed amount of baryon asymmetry (where $i = l, b$ corresponds to LV in lepton and baryon sectors correspondingly)
- Current astrophysical constraints on η_i rule out the scenario where LV is concentrated in the lepton sector only ($\eta_l \neq 0, \eta_b = 0$); similar constraints for the baryon sector LV ($\eta_l = 0, \eta_b \neq 0$) are also strong, but they are not applicable if LV is concentrated in the up-quark sector only; in this case, table-top ex-

periments provide significant insight and show that a fine-tuning of about four orders of magnitude for the η_u operator is required.

Concluding, we have put upfront the task of *classification* as one of the most important. Indeed, all predictions of LV physics that are potentially observable are related to the Standard Model sector, if gravity is not taken into account. Classification of LV interactions can answer many questions, such as whether a given UV theory with broken Lorentz invariance is consistent with observations, or if it has problems even at the conceptual level (such as producing divergent operators). In this sense, the classifications of LV in the Standard Model and Supersymmetric theories presented in Chapters 2 and 4 are compelling, as they form a bridge between certain types of UV theories and phenomenology. We have provided a complete list of effective LV operators, *i.e.* a convenient basis that can be used for determining the phenomenological properties of a given theory, and even for obtaining the simplest experimental limits on its LV parameters. We have seen that recent experiments have a tendency to seriously constrain UV scenarios where Lorentz invariance is broken (the strength of constraints sometimes reaching $10^{-28-30} \text{ GeV}^{-1}$). This does not exclude them *per se*, but makes it hard to believe that they have a sensible effective IR limit. In the future, one expects the laboratory constraints to tighten, as the accuracy of experiment is improving.

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Appendix A

RG equations for Dimension Five operators which modify dispersion relations

We list one-loop RG equations for LV operators which modify propagation of particles, *i.e.* those which couple to absolutely symmetric tensors. Although the set of such interactions in the Standard Model is not diverse, the equations appear to be complicated. The notations for the operators are introduced in section 2.2.2. All operators bear three indices μ, ν and ρ which we omit for brevity.

In what follows, Y_X are the hypercharges of the corresponding particles; λ_X are Yukawa coupling matrices for species X ; g' , g and g_3 are correspondingly the U(1),

$SU_L(2)$ and $SU_C(3)$ gauge coupling constants; we introduce

$$\begin{aligned}\alpha_1 &= \frac{5/3 N_g + 1/8}{6\pi^2} \\ \alpha_2 &= -\frac{19 - 8N_g}{48\pi^2} \\ \alpha_3 &= -\frac{5 - 4/3 N_g}{8\pi^2},\end{aligned}$$

which are the gauge wavefunction renormalization coefficients for the Standard Model, where $N_g = 3$ is the number of generations; we also use the following notations:

$$\begin{aligned}N_W &= 2 \quad (\text{dim fund } SU(2)) & T_W &= \frac{N_W^2 - 1}{2N_W} \\ N_S &= 3 \quad (\text{dim fund } SU(3)) & T_S &= \frac{N_S^2 - 1}{2N_S}.\end{aligned}$$

Below we present the RG equations for UV-enhanced LV operators above the EW symmetry breaking scale. The Wilson coefficients are assumed to be flavor matrices

given in the gauge basis.

$$\begin{aligned}
\frac{d}{dt} C_L &= \frac{25}{48\pi^2} (g'^2 Y_L^2 + g^2 T_W) C_L + \frac{1}{32\pi^2} \{ \lambda_e \lambda_e^\dagger, C_L \} \\
&+ \frac{5g'^2}{48\pi^2} Y_L^2 C_U(1) \cdot \mathbf{1}_{\text{flavor}} + \frac{5g^2}{48\pi^2} T_W C_{\text{SU}_L(2)} \cdot \mathbf{1}_{\text{flavor}} \\
&- \frac{1}{96\pi^2} \lambda_e C_e \lambda_e^\dagger - \frac{1}{64\pi^2} \lambda_e \lambda_e^\dagger \cdot \kappa \\
\frac{d}{dt} C_Q &= \frac{25}{48\pi^2} (g'^2 Y_Q^2 + g^2 T_W + g_3^2 T_S) C_Q + \frac{1}{32\pi^2} \{ \lambda_d \lambda_d^\dagger + \lambda_u \lambda_u^\dagger, C_Q \} \\
&+ \frac{5g'^2}{48\pi^2} Y_Q^2 C_U(1) \cdot \mathbf{1}_{\text{flavor}} + \frac{5g^2}{48\pi^2} T_W C_{\text{SU}_L(2)} \cdot \mathbf{1}_{\text{flavor}} \\
&+ \frac{5g_3^2}{48\pi^2} T_S C_{\text{SU}_C(3)} \cdot \mathbf{1}_{\text{flavor}} \\
&- \frac{1}{96\pi^2} (\lambda_d C_d \lambda_d^\dagger + \lambda_u C_u \lambda_u^\dagger) - \frac{1}{64\pi^2} (\lambda_d \lambda_d^\dagger - \lambda_u \lambda_u^\dagger) \cdot \kappa \\
\frac{d}{dt} C_e &= \frac{25g'^2}{48\pi^2} Y_e^2 C_e + \frac{1}{16\pi^2} \{ \lambda_e^\dagger \lambda_e, C_e \} \\
&- \frac{5g'^2}{48\pi^2} Y_e^2 C_U(1) \cdot \mathbf{1}_{\text{flavor}} - \frac{1}{48\pi^2} \lambda_e^\dagger C_L \lambda_e + \frac{1}{32\pi^2} \lambda_e^\dagger \lambda_e \cdot \kappa \\
\frac{d}{dt} C_u &= \frac{25}{48\pi^2} (g'^2 Y_u^2 + g_3^2 T_S) C_u + \frac{1}{16\pi^2} \{ \lambda_u^\dagger \lambda_u, C_u \} \\
&- \frac{5g'^2}{48\pi^2} Y_u^2 C_U(1) \cdot \mathbf{1}_{\text{flavor}} - \frac{5g_3^2}{48\pi^2} T_S C_{\text{SU}_C(3)} \cdot \mathbf{1}_{\text{flavor}} \\
&- \frac{1}{48\pi^2} \lambda_u^\dagger C_Q \lambda_u - \frac{1}{32\pi^2} \lambda_u^\dagger \lambda_u \cdot \kappa \\
\frac{d}{dt} C_d &= \frac{25}{48\pi^2} (g'^2 Y_d^2 + g_3^2 T_S) C_d + \frac{1}{16\pi^2} \{ \lambda_d^\dagger \lambda_d, C_d \} \\
&- \frac{5g'^2}{48\pi^2} Y_d^2 C_U(1) \cdot \mathbf{1}_{\text{flavor}} - \frac{5g_3^2}{48\pi^2} T_S C_{\text{SU}_C(3)} \cdot \mathbf{1}_{\text{flavor}} \\
&- \frac{1}{48\pi^2} \lambda_d^\dagger C_Q \lambda_d + \frac{1}{32\pi^2} \lambda_d^\dagger \lambda_d \cdot \kappa
\end{aligned} \tag{A.1}$$

The analogous RG equations for the gauge LV operators take the form:

$$\begin{aligned}
\frac{d}{dt} C_{\text{U}(1)} &= -\frac{g'^2}{48\pi^2} \text{tr} \left(Y_L^2 C_L + N_S Y_Q^2 C_Q - Y_e^2 C_e - \right. \\
&\quad \left. - N_S Y_u^2 C_u - N_S Y_d^2 C_d \right) + \alpha_1 g'^2 C_{\text{U}(1)} \\
\frac{d}{dt} C_{\text{SU}_L(2)} &= -\frac{g^2}{192\pi^2} \text{tr} \left(C_L + N_S C_Q \right) + \left(\alpha_2 g^2 + \frac{7}{12\pi^2} N_W g^2 \right) \cdot C_{\text{SU}_L(2)} \\
\frac{d}{dt} C_{\text{SU}_C(3)} &= -\frac{g_3^2}{192\pi^2} \text{tr} \left(2C_Q - C_u - C_d \right) + \\
&\quad + \left(\alpha_3 g_3^2 + \frac{7}{12\pi^2} N_S g_3^2 \right) \cdot C_{\text{SU}_C(3)} \tag{A.2} \\
\frac{d}{dt} \kappa &= \frac{5}{12\pi^2} [(g')^2 Y_H^2 + g^2 T_W] \cdot \kappa \\
&+ \frac{1}{8\pi^2} \text{tr} \left(\lambda_e \lambda_e^\dagger + N_S \lambda_u \lambda_u^\dagger + N_S \lambda_d \lambda_d^\dagger \right) \cdot \kappa \\
&- \frac{1}{12\pi^2} \text{tr} \left(N_S \lambda_d^\dagger C_Q \lambda_d + \lambda_e^\dagger C_L \lambda_e - N_S \lambda_u^\dagger C_Q \lambda_u - \right. \\
&\quad \left. - N_S \lambda_d C_d \lambda_d^\dagger - \lambda_e C_e \lambda_e^\dagger + N_S \lambda_u C_u \lambda_u^\dagger \right) .
\end{aligned}$$

We observe that mixing of RG operators is quite noticeable between all sectors of the Standard Model.

The RG equations for UV-enhanced LV interactions below the EW scale read as:

$$\begin{aligned}
\frac{d}{dt} C_\nu &= 0 \\
\frac{d}{dt} C_{\nu,5} &= 0 \\
\frac{d}{dt} C_e &= \frac{25e^2}{48\pi^2} C_e \\
\frac{d}{dt} C_{e,5} &= \frac{25e^2}{48\pi^2} C_{e,5} + \frac{5e^2}{48\pi^2} C_{\text{EM}} \cdot \mathbf{1}_{\text{flavor}} \\
\frac{d}{dt} C_u &= \frac{25}{48\pi^2} (q_u^2 e^2 + T_S g_3^2) C_u \\
\frac{d}{dt} C_{u,5} &= \frac{25}{48\pi^2} (q_u^2 e^2 + T_S g_3^2) C_{u,5} + \frac{5}{48\pi^2} (q_u^2 e^2 C_{\text{EM}} + g_3^2 T_S C_{\text{SU}_C(3)}) \cdot \mathbf{1}_{\text{flavor}} \\
\frac{d}{dt} C_d &= \frac{25}{48\pi^2} (q_d^2 e^2 + T_S g_3^2) C_d \\
\frac{d}{dt} C_{d,5} &= \frac{25}{48\pi^2} (q_d^2 e^2 + T_S g_3^2) C_{d,5} + \frac{5}{48\pi^2} (q_d^2 e^2 C_{\text{EM}} + g_3^2 T_S C_{\text{SU}_C(3)}) \cdot \mathbf{1}_{\text{flavor}}
\end{aligned} \tag{A.3}$$

for matter operators, and

$$\begin{aligned}
\frac{d}{dt} C_{\text{EM}} &= \frac{e^2}{48\pi^2} \text{tr} \left(C_{e,5} + N_S q_u^2 C_{u,5} + N_S q_d^2 C_{d,5} \right) + e^2 \alpha_{\text{EM}} \cdot C_{\text{EM}} \\
\frac{d}{dt} C_{\text{SU}_C(3)} &= \frac{g_3^2}{96\pi^2} \text{tr} \left(C_{u,5} + C_{d,5} \right) + \left(\alpha_3 + \frac{7}{12\pi^2} N_S \right) g_3^2 \cdot C_{\text{SU}_C(3)} \tag{A.4}
\end{aligned}$$

for gauge LV interactions. Here the flavor matrices of the Wilson coefficients are given

in the mass basis. Below the EW scale we have done an obvious transition to

$$\frac{C_L + C_e}{2} \equiv C_e \Big|_{\text{EW}}, \quad \frac{C_L - C_e}{2} \equiv C_{e,5} \Big|_{\text{EW}}, \quad \text{etc.}$$

We have denoted the electric charges by q_X , and introduced

$$\alpha_{\text{EM}} = \frac{1}{6\pi^2} \times \sum_{\text{species}} q_i^2 ,$$

which is the wavefunction renormalization coefficient for the electromagnetic field.

Here the sum runs over all species existing at the given scale μ .

Although the RG mixing in Eqs. (A.3), (A.4) is not very considerable, we again emphasize that the operators effectively mix above the EW scale, see Eqs. (A.1), (A.2).

Appendix B

Young tableaux and irreducible tensors of the Lorentz group

To build irreducible tensors of an arbitrary rank one can use the Young tableaux. We describe here a recipe how to expand a tensor of a specific rank into its irreducible components¹. In the text we most extensively exploit rank three tensors, which we here now use as a non-trivial but rather simple example.

For a tensor of rank r one builds all possible numbered Young tableaux consisting of r boxes. For each tableau one builds an irreducible component by (anti)symmetrizing its indices as described below. After that, to make a component truly irreducible, one has to subtract from it all its $g^{\mu\nu}$ -traces.

¹For more details, the reader is referred to [99].

For each numbered diagram, one builds a tensor such that each number corresponds to an index (*e.g.* for $T^{\mu\nu\rho}$, one could identify $1 \rightarrow \mu$, $2 \rightarrow \nu$, $3 \rightarrow \rho$). Indices whose numbers form horizontal rows in the diagram are symmetrized. Indices which form vertical columns are antisymmetrized. Symmetrization always occurs with respect to the name of the index. Antisymmetrization is always done with respect to the position of the index (in this case the number not always corresponds to one and the same index).

As an illustration to what have been said, we build the diagrams for a tensor $T^{\mu\nu\rho}$. One finds four different Young diagrams which can be built out of three boxes:

$$\begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline \end{array}
 \quad
 \begin{array}{|c|} \hline 1 \\ \hline 2 \\ \hline 3 \\ \hline \end{array}
 \quad
 \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & \\ \hline \end{array}
 \quad
 \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & \\ \hline \end{array}
 \quad (B.1)$$

The first diagram corresponds to an absolutely symmetric component of the tensor:

$$\begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline \end{array}
 \longrightarrow
 S^{\mu\nu\rho} = T^{(\mu\nu\rho)} = T^{\mu\nu\rho} + T^{\nu\rho\mu} + T^{\rho\mu\nu} + T^{\mu\rho\nu} + T^{\rho\nu\mu} + T^{\nu\mu\rho} .$$

The second diagram is the absolutely antisymmetric component:

$$\begin{array}{|c|} \hline 1 \\ \hline 2 \\ \hline 3 \\ \hline \end{array}
 \longrightarrow
 A^{\mu\nu\rho} = T^{[\mu\nu\rho]} = T^{\mu\nu\rho} + T^{\nu\rho\mu} + T^{\rho\mu\nu} - T^{\mu\rho\nu} - T^{\rho\nu\mu} - T^{\nu\mu\rho} .$$

The two “corner” diagrams generate, correspondingly,

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & \\ \hline \end{array} \longrightarrow T_1^{\mu\nu\rho} = T^{\mu\nu\rho} - T^{\rho\nu\mu} + T^{\nu\mu\rho} - T^{\rho\mu\nu} ,$$

and

$$\begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & \\ \hline \end{array} \longrightarrow T_2^{\mu\nu\rho} = T^{\mu\nu\rho} - T^{\nu\mu\rho} + T^{\rho\nu\mu} - T^{\nu\rho\mu} .$$

All four components of (B.1) (weighed by appropriate coefficients) sum into the original tensor $T^{\mu\nu\rho}$:

$$T^{\mu\nu\rho} = \frac{1}{3!} \left(S^{\mu\nu\rho} + A^{\mu\nu\rho} + 2T_1^{\mu\nu\rho} + 2T_2^{\mu\nu\rho} \right) . \quad (\text{B.2})$$

The last step to perform is subtract from each component all traces obtained by contraction of any two indices which are not antisymmetrized (contraction of antisymmetrized indices is trivial). The solution can be sought by means of a tensor of a rank less by two:

$$T_{i(\text{irr})}^{\mu\nu\rho} = T_i^{\mu\nu\rho} - a_i^\rho g^{\mu\nu} + a_i^\mu g^{\rho\nu} + \dots , \quad (\text{B.3})$$

where $T_i^{\mu\nu\rho}$ is the i -th component obtained from the corresponding Young tableau. The trace part in the r.h.s. of (B.3) should possess the same symmetries as $T_i^{\mu\nu\rho}$ so as to promote these symmetries to the l.h.s. Contracting any two indices in equation

(B.3) and requiring the result to vanish one can obtain the explicit expression for the trace a_i^ρ . For the tensors listed in Eq. (B.2) one obtains:

$$\begin{aligned}
S_{(\text{irr})}^{\mu\nu\rho} &= S^{\mu\nu\rho} - \frac{1}{6} \left(b^\mu g^{\nu\rho} + b^\nu g^{\rho\mu} + b^\rho g^{\mu\nu} \right) , & b^\mu &= S^{\mu\lambda\lambda} , \\
A_{(\text{irr})}^{\mu\nu\rho} &= A^{\mu\nu\rho} , & & \\
T_{1(\text{irr})}^{\mu\nu\rho} &= T_1^{\mu\nu\rho} - \frac{1}{3} \left(a_{(1)}^{(\mu} g^{\nu)\rho} - 2 a_{(1)}^\rho g^{\mu\nu} \right) , & a_{(1)}^\mu &= T^{\mu\lambda\lambda} - T^{\lambda\lambda\mu} , \\
T_{2(\text{irr})}^{\mu\nu\rho} &= T_2^{\mu\nu\rho} - \frac{1}{3} \left(a_{(2)}^{(\mu} g^{\rho)\nu} - 2 a_{(2)}^\nu g^{\mu\rho} \right) , & a_{(2)}^\mu &= T^{\mu\lambda\lambda} - T^{\lambda\mu\lambda} .
\end{aligned} \tag{B.4}$$

These arguments are easily generalized to tensors of arbitrary ranks.

Appendix C

Reduction of chiral LV operators on equations of motion

The component expressions for LV terms in the chiral sector are given in (4.44). They can be transformed further by eliminating the auxiliary fields and higher derivatives via the equations of motion. Writing the result in terms of Dirac four-spinors, we get

the following rather lengthy expression:

$$\begin{aligned}
\mathcal{L}_{\text{LV}}^{\text{matter}} = & -\frac{N_A^\mu}{M} \frac{1}{4} e \bar{\Psi} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \gamma^\nu \Psi - \frac{N_V^\mu}{M} \frac{1}{4} e \bar{\Psi} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \gamma^\nu \gamma^5 \Psi + \\
& + \frac{N_+^\mu}{M} \left[\frac{1}{2} i e \mathcal{D}^\nu \bar{z}_+ \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} z_+ + \frac{1}{2} e \left(\bar{z}_+ F_{\mu\nu} \mathcal{D}^\nu z_+ + \mathcal{D}^\nu \bar{z}_+ F_{\mu\nu} z_+ \right) \right. \\
& \quad \left. - \frac{i}{2} e^2 \left(\mathcal{D}_\mu \bar{z}_+ z_+ - \bar{z}_+ \mathcal{D}_\mu z_+ \right) \left\{ z_- \bar{z}_- - \bar{z}_+ z_+ \right\} \right] + \\
& + \frac{N_-^\mu}{M} \left[-\frac{1}{2} i e z_- \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \mathcal{D}_\nu \bar{z}_- - \frac{1}{2} e \left(z_- F_{\mu\nu} \mathcal{D}^\nu \bar{z}_- + \mathcal{D}^\nu z_- F_{\mu\nu} \bar{z}_- \right) \right. \\
& \quad \left. - \frac{i}{2} e^2 \left(\mathcal{D}_\mu z_- \bar{z}_- - z_- \mathcal{D}_\mu \bar{z}_- \right) \left\{ z_- \bar{z}_- - \bar{z}_+ z_+ \right\} \right] - \\
& - \frac{N_+^\mu}{M} e^2 \bar{z}_+ \bar{\lambda} \gamma^\mu \gamma^5 \lambda z_+ - \frac{N_-^\mu}{M} e^2 z_- \bar{\lambda} \gamma^\mu \gamma^5 \lambda \bar{z}_- - \\
& - \frac{N_{+\mu}}{M} \frac{\sqrt{2}}{2} e \left(\bar{\Psi} \gamma^\nu \gamma^\mu P_R \lambda \mathcal{D}_\nu z_+ + \mathcal{D}_\nu \bar{z}_+ \bar{\lambda} \gamma^\mu \gamma^\nu P_L \Psi \right) + \\
& + \frac{N_{-\mu}}{M} \frac{\sqrt{2}}{2} e \left(\mathcal{D}_\nu z_- \bar{\lambda} \gamma^\mu \gamma^\nu P_R \Psi + \bar{\Psi} \gamma^\nu \gamma^\mu P_L \lambda \mathcal{D}_\nu \bar{z}_- \right) + \\
& + \frac{N_+^\mu}{M} \frac{\sqrt{2}}{2} e \left(\bar{\Psi} P_R \mathcal{D}_\mu \lambda z_+ + \bar{z}_+ \mathcal{D}_\mu \bar{\lambda} P_L \Psi \right) - \\
& - \frac{N_-^\mu}{M} \frac{\sqrt{2}}{2} e \left(z_- \mathcal{D}_\mu \bar{\lambda} P_R \Psi + \bar{\Psi} P_L \mathcal{D}_\mu \lambda \bar{z}_- \right) + \\
& + \frac{N_{A\mu}}{M} \frac{1}{2} e^2 \bar{\Psi} \gamma^\mu \Psi \left\{ z_- \bar{z}_- - \bar{z}_+ z_+ \right\} + \\
& + \frac{N_{V\mu}}{M} \frac{1}{2} e^2 \bar{\Psi} \gamma^\mu \gamma^5 \Psi \left\{ z_- \bar{z}_- - \bar{z}_+ z_+ \right\} - \\
& - \frac{N_{A\mu}}{M} \frac{\sqrt{2}}{2} i e \left(\bar{m}_e \bar{\Psi} \gamma^\mu P_L \lambda \bar{z}_- - m_e z_- \bar{\lambda} \gamma^\mu P_L \Psi \right) + \\
& + \frac{N_{A\mu}}{M} \frac{\sqrt{2}}{2} i e \left(m_e \bar{\Psi} \gamma^\mu P_R \lambda z_+ - \bar{m}_e \bar{z}_+ \bar{\lambda} \gamma^\mu P_R \Psi \right) + \\
& + \frac{N_{V\mu}}{M} 2i m_e \bar{m}_e \left(\bar{z}_+ \mathcal{D}_\mu z_+ + z_- \mathcal{D}_\mu \bar{z}_- \right) + \frac{N_{V\mu}}{M} m_e \bar{m}_e \bar{\Psi} \gamma^\mu \Psi .
\end{aligned} \tag{C.1}$$

The first, second and the last term in (C.1) enter the reduced Lagrangian (4.46) that involves only electrons, positrons and photons. A rather lengthy form of (C.1) and a large number of diagrams that these interactions can create underlines the superiority of the superfield method over the component calculations for all processes with momenta larger than m_s .

Appendix D

SUSY Conventions and Notations

Our notations for the superfield formalism are based on Wess & Bagger [55]. Covariant derivatives and hermitean conjugation are taken from [100] with a proper adaptation. We use the $(- + + +)$ metric signature. All spinor algebra definitions can be found in [55], and we list here only some minor conventional departures. Unlike in [55], we denote the space-time Lorentz indices by letters from the middle of the *Greek* alphabet: $v_\mu, \sigma_\nu, N^\rho$, etc, as it is normally accustomed in QFT. Spinor indices are taken, also as commonly accepted, from the beginning of the Greek alphabet: $\theta^\alpha, \epsilon_{\beta\gamma}, \bar{\psi}_\delta$. Spinor derivatives are designated as

$$\partial_\alpha = \frac{\partial}{\partial\theta^\alpha}, \quad \partial^\alpha = \epsilon^{\alpha\beta}\partial_\beta.$$

We use a notation with a slash in the case where a Lorentz vector is contracted with a σ -matrix, or a γ -matrix:

$$\not{v} = v^\mu \sigma_\mu, \quad \overline{\not{A}} = A^\mu \overline{\sigma}_\mu, \quad \not{n} = n^\mu \gamma_\mu.$$

For switching from Weyl to Dirac spinors we followed the notations of [101]. Weyl representation for Dirac spinors is the most appropriate in this case, where two Weyl spinors combine into one Dirac spinor:

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}, \quad \bar{\Psi} = \begin{pmatrix} \chi^\alpha \\ \bar{\xi}_{\dot{\alpha}} \end{pmatrix},$$

and the γ -matrices take the form

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

For complex conjugation, we use the notion of *hermitean conjugation* defined

in [100]. When translated into the Wess & Bagger notations, it implies

$$\begin{aligned}
(\psi_\alpha)^\dagger &= \bar{\psi}_{\dot{\alpha}} , & (\psi^\alpha)^\dagger &= \bar{\psi}^{\dot{\alpha}} , \\
\partial_\alpha^\dagger &= \bar{\partial}_{\dot{\alpha}} , & \partial_\mu^\dagger &= -\partial_\mu , \\
D_\alpha^\dagger &= -\bar{D}_{\dot{\alpha}} , & (\nabla_\alpha)^\dagger &= -\bar{\nabla}_{\dot{\alpha}} , \\
W_\alpha^\dagger &= \bar{W}_{\dot{\alpha}} .
\end{aligned}$$

Finally, the expansion of the chiral superfields of SQED in components is defined as

$$\Phi_\pm = z_\pm + \sqrt{2}\theta\psi_\pm + \theta^2 F_\pm ,$$

while the vector superfield in the Wess-Zumino gauge is given by

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D .$$