

Pedagogical Content Knowledge in an Educational Context (PCK-EC)

by

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B.A., Universidad Sur Colombiana, 1995
M.Sc., University of Puerto Rico, 2003

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of the Requirements for the Degree of

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Supervisory Committee

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Abstract

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The Pedagogical Content Knowledge in an Educational Context (PCK-EC) model is proposed as a framework to support teachers, coaches and researchers in the examination of teacher knowledge within a specific context and with a particular focus. This framework combines the theoretical and practical aspects represented by five dimensions of teachers' attitudes and teachers' knowledge (technology, learners' cognition, subject matter, pedagogy) within an educational context that includes curricular, technological, social, cultural, and teaching - learning contexts.

Two case studies were used to examine the utility of the proposed PCK-EC model. Data collected included: semi-structured initial and final interviews; teacher's journals of reflection (completed after teaching each lesson); direct observations during lessons; observations from video recordings of lessons; transcripts from initial and final interviews; and other collected documents in regards to the educational context. The interpretive repertoires method allowed us to identify and characterize groups of themes in each dimension of teachers' attitudes and knowledge, and supported inter-relationships between themes.

The PCK-EC was useful to support a deep description of a collection of themes by using different sources of data. Analysis of each one of these collections of themes allowed us to understand teachers' PCK-EC and provided

insights about how different technological tools might affect teachers' attitudes and their knowledge.

The dimensions of teachers' attitudes and knowledge are not isolated, but rather they are inter-related during teaching practice. It is possible to recognize inter-relationships (outgoing and incoming) between themes (within and across dimensions). It is suggested that the frequency of the outgoing and incoming inter-relationships found between themes might give us an average weight for each of the dimensions of the PCK-EC and this could represent teachers' attitudes and knowledge used during teaching practice.

The collection of themes identified might be useful as a tool to support teachers as they explore their attitudes and their knowledge needed for teaching a specific topic with the use of technological tools, and may provide coaches with an effective mechanism to support the identification of an individuals' PCK and development needs.

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List of Acronyms

AFI	Alex's Final Interview
AII	Alex's Initial Interview
AJR	Alex's Journal of Reflections
AP	Advanced Placement
APO	Alex's Protocol of Observations
AVO	Alex's Video Observations
BC	British Columbia
BCTF	British Columbia Teachers' Federation
CCSSM	Common Core State Standards for Mathematics
EC	Educational Context
GVRD	Greater Vancouver Regional District
ICT	Information and Communication Technologies
KLC	Knowledge of Learners' Cognition
KP	Knowledge of Pedagogy
KSM	Knowledge of Subject Matter
KT	Knowledge of Technology
MFI	Mary's Final Interview
MII	Mary's Initial Interview
MJR	Mary's Journal of Reflections
MPO	Mary's Protocol of Observations
MVO	Mary's Video Observations
NCTM	National Council of Teachers of Mathematics
PCK	Pedagogical Content Knowledge
PCK-EC	Pedagogical Content Knowledge in an Educational Context
TPACK	Technological Pedagogical And Content Knowledge
TPCK	Technological Pedagogical Content Knowledge
TT	Technological tools
WNCP	Western and Northern Canadian Protocol

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“Prayer can transform everything. Pray and have faith.
Then you will experience the miracles of God. Thanks God”

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Dedication

To my family for their love and unconditional support

Introduction

The field of mathematics education research “is aimed to study the factors affecting the teaching and learning of mathematics and to develop programs to improve the teaching of learning mathematics” (Godino, Batanero & Font, 2007, p. 127). The changes and development of teaching and learning mathematics could be examined using the approach of the didactical triangle (Figure 1) proposed by Steinbring (1998), in which its vertices are: the mathematics knowledge, the students and the teachers (as cited in Steinbring, 2011).

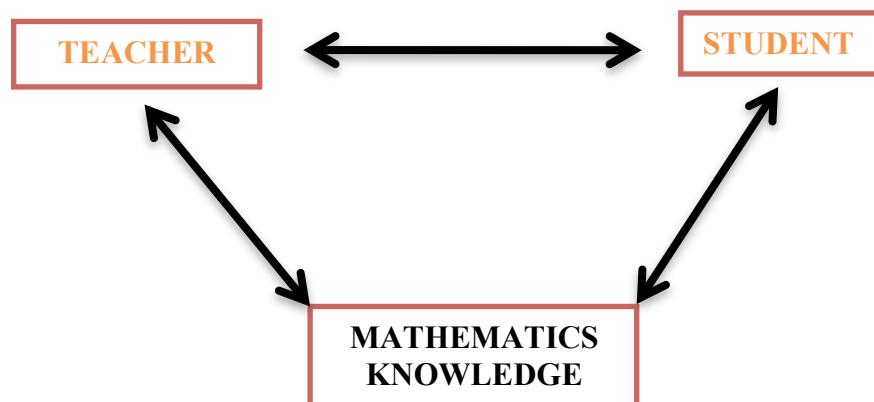


Figure 1. Didactical triangle. Adapted from “Changed views on mathematical knowledge in the course of didactical theory development: independent corpus of scientific knowledge or result of social constructions?” by H. Steinbring, 2011. In: T. Rowland & K. Ruthven (Eds.), *Mathematical knowledge in teaching*, Mathematics Education Library 50, p. 44.

The interrelationship between the components of this triangle raises several complex questions in regards to the teaching-learning process including what is the mathematical knowledge that students need to learn, how students learn mathematics, what is the mathematical knowledge that teachers need to know for teaching, how do teachers deal with mathematics teaching? And so on. In order to answer those questions, educational research has emphasized students’ learning process, the teachers’ teaching process, and more recently the teacher’s knowledge for teaching. Researchers interested in improving the teaching of mathematics redirected additional attention to investigate the knowledge that teachers need to know for teaching. Shulman (1986) proposed to

develop a theoretical framework to understand what knowledge was needed for teaching, and suggested three categories of content knowledge: “(a) subject matter content knowledge, (b) pedagogical content knowledge, and (c) curricular knowledge (p. 9), and later included four more categories (Shulman, 1987).

The concept of Pedagogical Content Knowledge (PCK) has given us a way to represent and formulate teaching practice so that it can be understood and discussed (Shulman, 1986, p. 9). With it, we can examine “the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by students” (Shulman 1987, p. 15). The PCK model has been used as a seed by several scholars to generate new frameworks to examine the teaching of mathematics (Grossman, 1990; Fennema & Franke, 1992; Rowland, Huckstep & Thwaites, 2003; Ball, Thames & Phelps, 2008; Park & Oliver, 2008; Petrou & Goulding, 2011).

The great potential of new technological tools (the computer, blogging, internet, and others) for use in educational systems has increased the complexity of the teaching and learning situation and highlights the need to expand the PCK model. Pierson (2001) incorporated the concept of technological knowledge as one of the components of a model for teacher knowledge and suggested the technological – pedagogical - content knowledge as a reference for effective technology integration (p. 427). Koehler and Mishra (2008) continued developing the model to examine the use of technology in education, and proposed a framework based on the integration of technology, content and pedagogy called Technological Pedagogical And Content Knowledge (TPACK). Mishra, Koehler & Kereluik (2009) indicated that the TPACK could be considered an alternative framework that “emphasizes the role of teachers as decisions makers who design their own educational technology environment as needed” (p. 52). In this sense, teachers have the flexibility and capacity of adapting or repurposing different technological tools according to specific subject matter topics where the use of technological tools empowers teaching and learning through teachers’ strategies, planning and development of the TPACK.

The framework of TPACK has been also a starting point for other approaches that include the use of Information and Communication Technologies (ICT) as one of the

dimensions of teacher's knowledge needed for teaching (Angeli & Valanides, 2009), and the inclusion of TPACK in an educational context (Doering, Veletsianos, Scharber & Miller, 2009), among others. However, TPACK is considered an extension of the concept of PCK following the original research of Shulman (Koehler and Mishra, 2008). Hence, we could consider that there is no need for having a separate framework involving technology in education or TPACK, but only the PCK concept. Niess (2011) suggested "Conceivable, at some future point, the attention will be redirected to PCK as the knowledge that teachers need for teaching where digital technologies are included among the many other technological resources teachers have for teaching" (p. 307), and she highlights that differences between PCK and TPACK "may be less identifiable for the experienced teachers" (p. 311).

The existence of several frameworks for studying the teachers' knowledge for teaching constitutes a challenge for researchers as well as for educators. Hence, it is important to reframe the concept of PCK for providing a reference point not only to consider what kind of knowledge teachers need for teaching, but also how that knowledge could evolve under a dynamic environment that is guided not only for constant development of new digital technologies, but also for curricular changes, teaching and learning approaches, and social and cultural interactions. These aspects constitute an educational context, which guides teachers' theoretical and practical knowledge in the classroom community.

Stoilescu (2011), indicates two major limitations with TPACK: the fact that TPACK "did not offer the possibility to systematically take into account teachers' attitudes, opinions, philosophy, and paradigms for teaching" (p. 198), and "the lack of clarity in determining the level of integrating technology that a teacher displays in his or her classroom" (p. 199). Also, Doering et al. (2009) argued that the TPACK framework is static in regards to the teacher's knowledge, and they highlighted three limitations: (a) In teaching practices the knowledge possessed by the teacher is different to the knowledge used; (b) The knowledge that a teacher uses "depends on varied factors including the specific classroom culture, students characteristics, school and district policy, and numerous other factors that can neither be predicted nor accounted for a priori"; and (c) "Depending on the context of a situation and the various levels of knowledge a teacher

has, certain domains of knowledge may be used more than others” (p. 336). They concluded that “...context influences both teacher knowledge and practice; in turn, teacher knowledge influences practice and practice influences which types of knowledge are used more in the classroom” (Doering et al., 2009, p. 336).

As a consequence the existence of several frameworks for studying the teachers’ knowledge for teaching constitutes a challenge for researchers, coaches and educators as they attempt to choose the most appropriate model. Curricular requirements, teaching and learning approaches, social and cultural interactions and other factors constitute the educational context that interacts with the teacher’s PCK and these, along with the topic being addressed, define their educational practice. Hence, a new model is proposed to illuminate the dimensions of the pedagogical content knowledge needed for teaching a particular topic in an educational context.

The proposed model is called Pedagogical Content Knowledge in an Educational Context (PCK-EC). The PCK-EC model includes the following dimensions: teacher’s Attitudes, Knowledge of Technology, Knowledge of Learners’ cognition, Knowledge of the Subject matter (or content knowledge), and Knowledge of Pedagogy which it is situated within an educational context (see Figure 2). This model was mainly influenced by the model proposed by Fennema and Franke (1992).

The concept of pedagogical content knowledge in an educational context might help to integrate technology in terms of how and why it is used in the classroom community in a meaningful way, rather than the amount and types of technological tools that might be used for teaching. Identifying all of the components of teachers’ knowledge proposed in Figure 2, and how they are interconnected represents a challenge for the researcher, but it might provide insights for in-service and pre-service teachers to better incorporate technological tools in the teaching process.

Consequently, the purpose of this study is to explore the utility of a new framework called Pedagogical Content Knowledge in an Educational Context (PCK-EC). Specifically it aims to explore the teachers’ knowledge needed for teaching a specific mathematics topic with the teacher’s knowledge of technology as a focus.

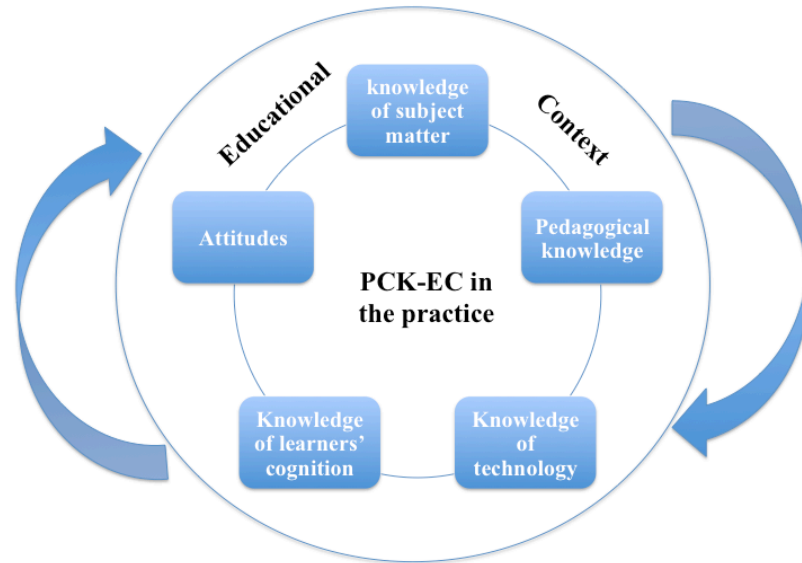


Figure 2. Theoretical Framework of Pedagogical content knowledge in Educational Context (PCK-EC) for teaching a specific mathematics topic.

In particular the research is designed as a multiple case study and the research questions that will be attempted during this study are:

1. How effective is the proposed PCK-EC model (Figure 2) in supporting the examination of teacher practice and providing insights to the pedagogical content knowledge?
2. How do different Technological Tools (TT) mediate or affect teachers' PCK-EC when mathematics teachers are teaching a particular topic?

Literature Review

The initial segment of this literature review focuses on research specifically pertaining to Pedagogical Content Knowledge (PCK) and some models that have been proposed to support the understanding of PCK including Technological and Pedagogical Content Knowledge (TPACK). The purpose of presenting these models is to illustrate how models are constructed through an iterative process in an ongoing attempt to better represent the dimensions of teachers' knowledge needed for teaching. This section will conclude with a new proposed framework for studying teachers' knowledge needed for teaching a specific mathematics topic in the presence of technology. The second section of the literature review expands on experiences and ideas of previous researchers who conducted qualitative studies of PCK and TPACK.

Understanding Pedagogical Content Knowledge

The concept of pedagogical content knowledge (PCK) is directly related with teachers' knowledge in the practice of teaching a specific topic. Shulman (1986, 1987) identified the components of teacher's knowledge needed for teaching and proposed a theoretical framework based on seven different categories of teacher knowledge for effective teaching. He conceived four categories in regards to general aspects of teacher knowledge including general pedagogical knowledge, knowledge of learners' characteristics, knowledge of educational context, and knowledge of educational purposes and values. The other three categories constitute the knowledge base of teaching and they were identified as content knowledge or subject matter, curriculum knowledge, and pedagogical content knowledge (Shulman, 1986; Angeli & Valanides, 2009; Petrou & Goulding, 2011).

The content knowledge or subject matter "refers to the amount and organization of knowledge *per se* in the mind of the teacher" (Shulman, 1986, p. 9). The content knowledge requires that the teacher knows the subject matter in regards to the facts or concepts of a domain and also understands of its organising structure (Shulman, 1986; Petrou & Goulding, 2011).

According to Shulman (1986) curriculum knowledge is:

Represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (p. 10).

The Pedagogical Content Knowledge (PCK) as a new concept developed by Shulman (1986) “includes the content specific representations, examples and applications that teachers use in order to make subject matter comprehensible to students together with the strategies that teachers use in order to overcome their students’ difficulties” (Petrou & Goulding, 2011, p. 12). The PCK also “includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons” (Shulman, 1986, p.9). Shulman’s framework of PCK (1986, 1987) involves the way in which content, pedagogy, and knowledge of learners are “blended into an understanding about how particular topics to be taught are represented and adapted to learners’ characteristics, interests, and abilities” (Angeli & Valanides, 2009).

Shulman’s conceptualisation of PCK has been criticized because it considers teacher knowledge as something transmissible, and learners as passive receptors for obtaining this knowledge. Authors such as Grossman, 1990; Fennema & Franke, 1992; Cochran, Derutier & King, 1993; and Bullough, 2001, recognized these issues and proposed different PCK models that address the dynamic and interactive nature of teacher knowledge, teaching contexts, beliefs about the purpose for teaching particular topics, and environmental contexts of learning. Indeed, the concept of PCK has been further interpreted as “engendering a variety of meanings” (Park & Oliver, 2008, p. 262) and there are several definitions about what constitutes the PCK. Most of the concepts of PCK are consistent and maintain as a core foundation Shulman’s conceptualization, i. e. knowledge of subject matter, knowledge of pedagogy, and knowledge of learners’ conceptions and content-related difficulties. Finally, it is important to remember that

PCK is specific to the teaching of particular topics, and it is developed in classroom practice (Angeli & Valanides, 2009).

Models for the conceptualization of PCK in mathematics education

Situating Shulman's perspective of PCK in the teaching of mathematics we see that a teacher's knowledge of the subject matter and knowledge of pedagogy alone are insufficient but need to be integrated (Petrou & Goulding, 2011). This inclusive perspective enables us to approach PCK in mathematics education as combining the need to know about mathematics with good pedagogical practice and other crucial understandings needed for teaching. This broadens the question of what knowledge could be required for teaching. It is important to clarify that the concepts of PCK have been developed in general and some of them have been adapted specifically to the teaching of mathematics. A review of some of the evolving frameworks for conceptualizing PCK in mathematics, such as, Fennema and Franke (1992); Ball et al. (2008); Rowland et al. (2003); and Petrou and Goulding (2011), elucidates how they are "not inconsistent; rather they build on each other". This evolutionary characteristic is what makes it possible to further develop and improve on the PCK foundation. This evolutionary process invites further efforts to develop and improve on the PCK.

Fennema and Franke: model of mathematics teacher knowledge. The model proposed by Fennema and Franke (1992) introduces the concept of context as "the structure that defines the components of knowledge and beliefs" (p. 162), and highlights the idea that "within a given context, teachers' knowledge of content interacts with knowledge of pedagogy and students' cognitions and combines with beliefs to create a unique set of knowledge that drives classroom behaviour" (p. 162). Thus, the model includes teacher knowledge of the content of mathematics, knowledge of pedagogy, knowledge of students' cognitions, context-specific knowledge, and teachers' beliefs as it is illustrated in Figure 3.

Fennema and Franke (1992) also make relevant the interactive and dynamic nature of teacher knowledge, indicating that some components of mathematics teacher knowledge evolve through teaching. Thus, teaching becomes "a process within which

new knowledge is created” (Fennema & Franke, 1992, p. 162). They point out “knowledge can be and it is transformed through classroom interaction” (Fennema & Franke, 1992, p. 162) and “when the knowledge is transformed during instruction, that knowledge becomes tied to the context in which it was developed” (Fennema & Franke, 1992, pp. 162-163). They state that the future for research in mathematical knowledge is to develop methodologies that help to understand “the dynamic interaction between components of teacher knowledge and beliefs, the role they play, and how the roles differ as teachers differ in the knowledge and beliefs they possess” (Fennema & Franke, 1992, pp. 163).

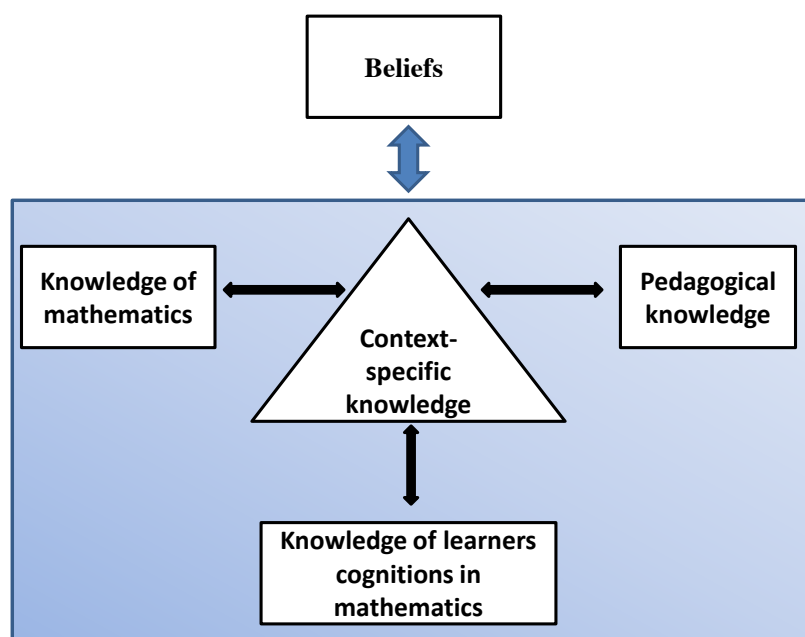


Figure 3. Teachers’ knowledge: Developing in context. Adapted from “Teachers’ knowledge and its impact” by E. Fennema, and L. M. Franke, 1992, In: D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*, New York, NY: Macmillan, p. 162.

Ball, Thames and Phelps: framework of Domain of Mathematics Knowledge for teaching: A practice-based theory of content knowledge for teaching. Ball et al. (2008), interested in conceptualizing the mathematical knowledge and skills needed by teachers, developed a practice-based theory of content knowledge for teaching based on Shulman’s ideas. They extended the conceptualization of content knowledge (CK) or subject matter knowledge (SMK) and pedagogical content knowledge (PCK).

The SMK was divided into two main categories: common content knowledge (CCK), and specialized content knowledge (SCK). The model includes horizon content knowledge (HCK) as a tentative category within SMK as it is represented in Figure 4.

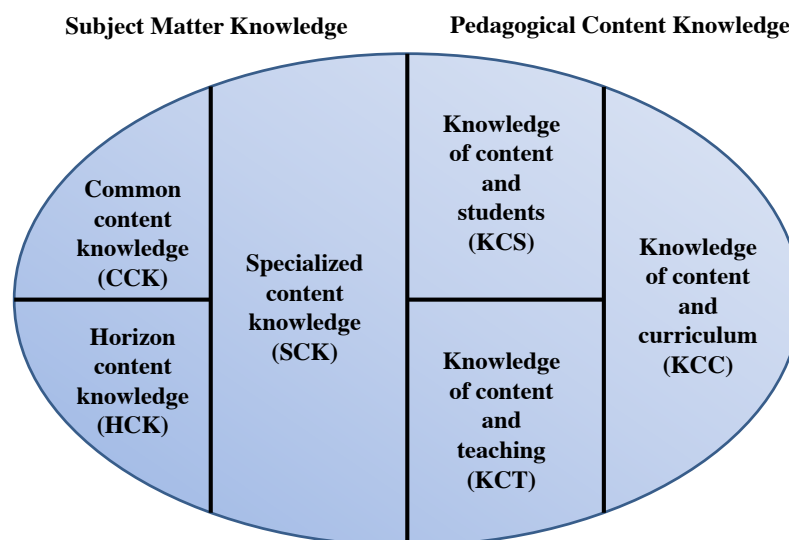


Figure 4. Domains of mathematical knowledge for teaching. Adapted from “Content knowledge for teaching: what makes it special?” by D. L. Ball, M. H. Thames, and G. Phelps, 2008, *Journal of Teacher Education*, 59 (5), p. 403.

- *Common content knowledge (CCK)* refers to mathematical knowledge and skills that are not specific to teaching. It could be an “individual’s ability to calculate an answer and to solve mathematical problems correctly” (Petrou & Goulding, 2011, p. 15) in a variety of settings.
- *Specialized content knowledge (SCK)* as the main focus of the model, proposes and includes the mathematical knowledge and skills that teachers need to use for teaching effectively in the classroom (Ball et al., 2008; Petrou & Goulding, 2011).
- *Horizon content knowledge (HCK)* “includes teachers’ awareness of how the mathematical topics covered in previous years in schools are related to curriculum topics addressed in the subsequent years in schools” (Petrou & Goulding, 2011, p. 15). Ball et al. (2008) consider this category as provisional, pointing out “we are not yet sure whether this may be a part of our category of knowledge content and teaching or whether it may run across the several categories or be a category in its own right” (p. 403).

The other development that Ball et al. (2008) suggest to the conceptualization of Shulman's PCK is to include three categories: Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT) and Knowledge of Content and Curriculum (KCC) as it is represented in figure 4.

- *Knowledge of Content and Students* (KCS) refers to “knowledge that combines knowledge about students and knowing about mathematics” (Ball et al., 2008, p. 401). “This means that teachers must be able to anticipate students’ difficulties and obstacles hear and respond appropriately to students’ thinking, and choose appropriate examples and representations while teaching. Both in planning and teaching, teachers must show awareness of students’ conceptions and misconceptions about a mathematics topic” (Petrou & Goulding, 2011, p. 16).
- *Knowledge of Content and Teaching* (KCT) “combines knowing about teaching and knowing about mathematics. Many of the mathematical tasks of teaching require a mathematical knowledge of design and instruction” (Ball et al., 2008, p. 401). In the process of teaching, teachers need to know how to guide activities, exercises, and mathematical representations. Also, they need to know how to manage students’ mathematical ideas to clarify or emphasize mathematical aspects (Petrou & Goulding, 2011).

Ball et al. (2008) do not conceptualize explicitly what is understood as knowledge of content and curriculum or if KCS and KCT categories represent the content and curriculum knowledge. Even though they follow Shulman's ideas and some of the categories are similar, it is not consistent with the conceptualization of PCK as a unique body of knowledge. Ball et al. (2008) do not distinguish the relationship between the two core domains: subject matter knowledge (SMK) and pedagogical content knowledge (PCK). Are they different forms of knowledge needed for teaching mathematics? Or conversely, do we need to see them as a unique body of mathematics knowledge needed for teaching?

The framework of Ball et al. (2008) is considered relevant because they made progress in identifying the relationship between teachers' content knowledge and their

students' achievement and they contributed the development of a series of multiple choice items that could be used to measure mathematical knowledge for teaching (Petrou & Goulding, 2011).

Rowland, Huckstep, and Thwaites: The Knowledge Quarter framework. The Knowledge Quarter is a theoretical framework based on Shulman (1986) and responds to Fennema and Franke's (1992) suggestion of identifying the way different components of teachers' knowledge are integrated in the practice of teaching (Petrou & Goulding, 2011). This theoretical framework is the result of investigating mathematical content knowledge of pre-service elementary school teachers in England and Wales and it "can be used as a tool for classifying ways in which the pre-service teachers' SMK and PCK come into play in the classroom" (Petrou & Goulding, 2011, p. 18). The Knowledge Quartet includes four categories: Foundation, Transformation, Connection and Contingency.

- *Foundation*: represents "trainees' knowledge, beliefs and understanding acquired in the academy, in preparation (intentionally or otherwise) for their role in the classroom" (Rowland et al. 2003, p. 97). Then, this category illustrates the combination of teachers' knowledge and understanding of content, pedagogy and beliefs about mathematics teaching.
- *Transformation*: "concerns knowledge-in-action as demonstrated both in planning to teach and in the act of teaching itself" (Rowland et al. 2003, p. 98). This includes the way teachers give explanations, use examples and make representations of mathematical concepts and topics during the teaching practice (Petrou & Goulding, 2011).
- *Connection*: "binds together certain choices and decisions that are made for the more or less discrete parts of mathematical content" (Rowland et al. 2003, p. 98). "It also includes the sequencing of topics of instruction within and between lessons" (Rowland et al. 2003, p. 98).
- *Contingency*: "concerns classroom events that are almost impossible to plan for" (Rowland et al. 2003, p. 98).

The categories above describe specifically different teachers' situations and the way teachers' respond to those situations during their teaching practices. As a

consequence the knowledge quarter model elaborates teachers' knowledge as a functional model in mathematics teaching while the previous models emphasize structural aspects of teachers' knowledge. The advantage of this model is that it allows identifying interactions between the elements considered as part of teachers' knowledge for teaching. However, the framework of the knowledge quarter model does not consider curriculum knowledge in regards to the use of instructional materials in teaching (Petrou & Goulding, 2011).

Petrou and Goulding Model. Petrou and Goulding (2011) make relevant the conceptualization of curriculum knowledge by Shulman (1986) pointing out that that it is vital in “understanding what teachers need to know in order to teach mathematics effectively” (pp. 21-22), and propose three categories that include curriculum knowledge as a main category, as well as subject matter and pedagogical content knowledge.

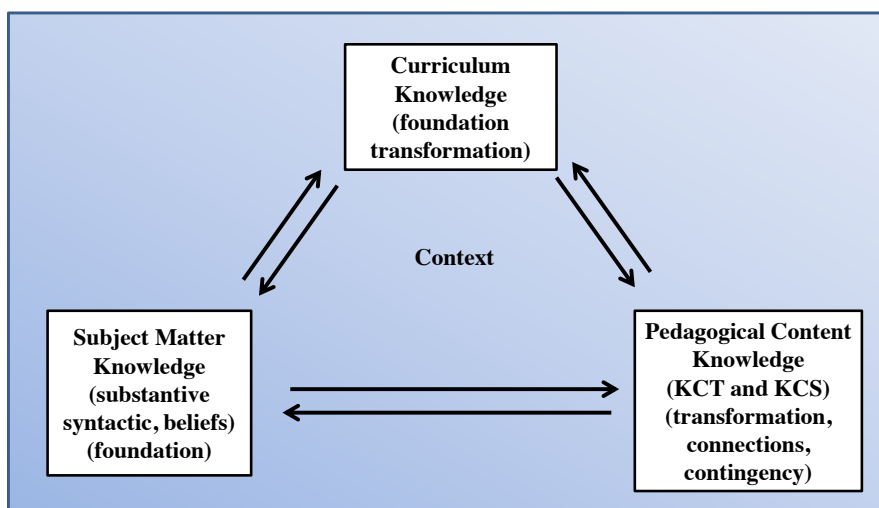


Figure 5. Synthesis of models on teacher mathematics knowledge. Adapted from “Conceptualising teachers’ mathematical knowledge in teaching” by M. Petrou, and M. Goulding, 2011. In: T. Rowland & K. Ruthven (Eds.), *Mathematical knowledge in teaching*, Mathematics Education Library, 50, p. 21.

The above four models of PCK, which have been developed for understanding mathematical knowledge required for teaching, still need to be reformulated considering

new changes in the teaching and learning process, and thus constructing further integrated frameworks.

Koehler and Mishra model of technological pedagogical content knowledge (TPCK). Koehler and Mishra (2008) extended the work of Shulman’s conceptualisation of PCK considering the influence of technology in education and added the category of technological knowledge as part of the knowledge needed for teaching with technology. They frame the dimension of technological pedagogical content knowledge based on the interaction between three main categories: content, pedagogy and technology. The interactions between each of these categories build the knowledge components needed to integrate technology in the classroom (see Figure 6). As a result, the Koehler & Mishra framework of TPCK involves an understanding of the pedagogical content knowledge, technological pedagogical knowledge, technological content knowledge and technological pedagogical content knowledge and represents “the intersections of knowledge about technology, content (content areas or subjects such as mathematics, science, or English), and pedagogy (specific instructional practices that are effective for teaching the subject)” (Kelly, 2008, p. 51).

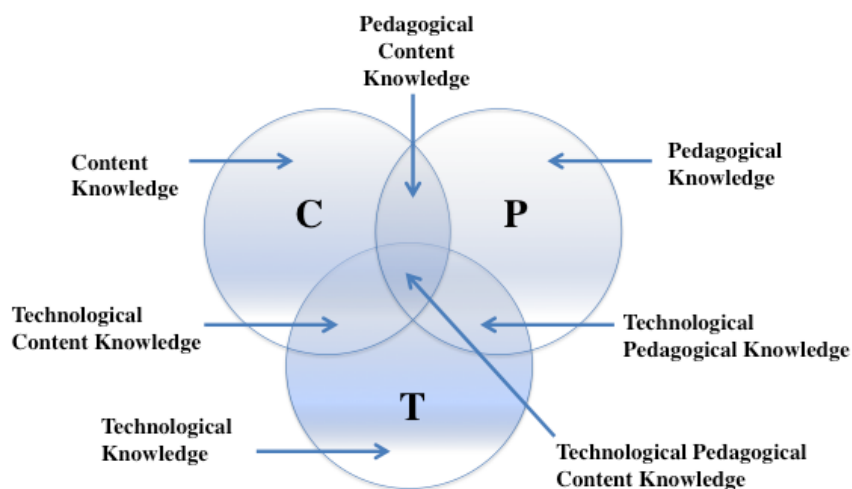


Figure 6. Technological Pedagogical Content Knowledge (TPCK). Adapted from “Tracing the development of teacher knowledge in a design seminar: integrating content, pedagogy and technology” by M. J. Koehler, P. Mishra, and K. Yahya, 2007, *Computers and education*, 49, p. 742.

It is important to clarify that Koehler and Mishra (2008) assume educational technology as “the sum of the tools, techniques, and collective knowledge applicable to education” including “analog technologies (e.g., chalkboard, pencil, and microscope) and digital technologies (e.g., the computer, blogging, and internet)” (Koehler & Mishra, 2008, p. 5). Also, Koehler and Mishra (2008) highlight the importance of recognizing affordances and constraints in particular technologies and they point out “technologies are neither neutral nor unbiased; rather, particular technologies have their own propensities, biases, and inherent attributes that make them more suitable for certain tasks than others” (Bromley, 1998; Bruce, 1993 as cited in Koehler & Mishra, 2008, p. 5). Another important consideration in the foundation of Koehler’s and Mishra’s work is the use of technology in a creative way to redefine existing tools with pedagogical purposes, and to avoid what has been called “functional fixedness”. This latter concept refers to “the manner in which the ideas we hold about an object’s function can inhibit our ability to use the object for a different function” (Birch, 1945; German & Barrett, 2005 as cited in Koehler & Mishra, 2008, p. 6).

The components of knowledge that frame the teachers’ knowledge for teaching using technology (Figure 6) are explained by Koehler and Mishra (2008) as follows:

- *Technological content knowledge (TCK)* implies an “understanding of the manner in which technology and content influence and constrain one another” (p. 16). Teachers need to be aware about possible effects of the technological tool in the content of the subject matter or vice versa. We might consider that this part of the framework could be the biggest challenge not just for teachers, but for the community of the subject matter which requires making judgement in regards to the kind of technology that “affords and constrains the types of content ideas that can be taught” (p. 16).
- *Technological pedagogical knowledge (TPK)* is related with teaching and learning changes when specific technologies are used. It is necessary to know “the pedagogical affordances and constraints of a variety of technological tools as they relate to disciplinary and developmentally appropriate pedagogical design and strategies (Koehler & Mishra, 2008, p. 16).

- *Pedagogical content knowledge (PCK)* allows “creative flexibility with available tools in order to repurpose them for specific pedagogical purposes” (Koehler & Mishra, 2008, p. 17).

- *Technological pedagogical content knowledge (TPCK)* is the result of the interaction between the main core components, technology, content and pedagogy. TPCK is considered the base for effective teaching with technology and constitutes:

the representation of concepts using technologies; pedagogical techniques that use technologies in constructive way to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students’ prior knowledge and theories of epistemology; and knowledge of how technology can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones (Koehler & Mishra, 2008, p. 18).

The complexity of developing TPCK for teachers is evident because teachers need to confront several classroom situations where technology, content, and pedagogy are intertwined and “there is no single technological solution that applies for every teacher, every course, or every view of teaching” (Koehler & Mishra, 2008, p. 18). This is what Koehler & Mishra called the “wicked problem” and they argue that the solution consists in teachers’ flexibility and ability to explore the three elements and the interaction between them in specific contexts. This model has been criticized by Angeli and Valanides (2009) who argue that the TPCK “is too general because it does not deal explicitly with the role of tools affordances in learning” (p.157) and also it is questionable how the interactions between pedagogy, content, and technology affect the development of TPCK. They question the accepted hypothesis “that growth in any of the related constructs (i. e, content, technology, pedagogy) automatically contributes to growth in TPCK arguing that findings of their research show that “growth in the related construct does not automatically mean growth in TPCK” (Angeli & Valanides, 2009, p. 158). Angeli and Valanides (2009) point out:

Teachers educators need to explicitly teach how the unique features and affordances of a tool can be used to transform a specific content domain for specific learners and that teachers need to be explicitly taught about the interactions among technology, content, pedagogy and learners (p. 158).

Angeli and Valanides: model of Information and Communication Technologies (ICT-TPCK). Angeli and Valanides (2009) framework is based on the concept of TPCK formulated by Koehler and Mishra (2008) and restructures this model (ICT-TPCK) adding two new categories: knowledge of students and knowledge of the context within which learning take place. Also, they narrow the category of technology to Information and Communication Technology (ICT) to make specific the type of technology used in the model. Thus, the model is constituted by the intersection between five knowledge categories: pedagogy, ICT, content, learners and context as is represented in Figure 7.

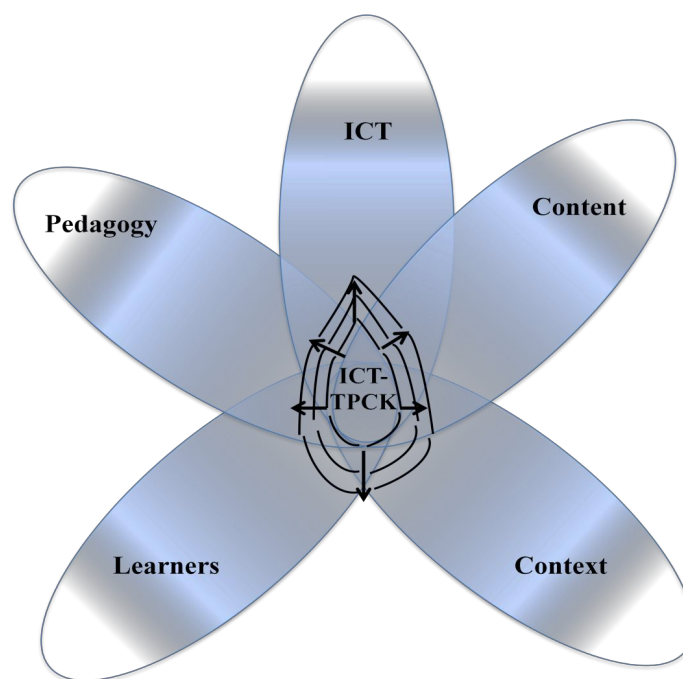


Figure 7. Model of ICT-TPCK. Adapted from “Epistemological and methodological issues for the conceptualizations, development and assessment of ICT-TPACK: advances in technological pedagogical content knowledge (TPCK)” by C. Angeli, and N. Valanides, 2009. *Computers and education*, 52, p.159.

The model ICT-TPCK is conceptualized by Angeli and Valanides (2009) as:

The ways knowledge about tools and their pedagogical affordances, pedagogy, content, learners, and context are synthesized into an understanding of how particular topics that are difficult to be understood by learners, or difficult to be represented by teachers, can be transformed and taught more effectively with ICT, in ways that signify the added value of technology (pp. 158-159).

Also, Angeli and Valanides (2009, p. 158) describe the components of the ICT-TPCK as follows:

- *Subject matter knowledge* includes an understanding of the fact and structures of a content domain.
- *Pedagogical knowledge* refers to broad principles and strategies of teaching, classroom management, and organization that are generic across different subject matter domains.
- *Knowledge of learners* refers to their characteristics and the preconceptions that they bring to a learning situation.
- *Knowledge of context* ranges from the working of the classrooms, to the educational values and goals, as well as their philosophical underpinnings in conjunction with teachers' epistemological beliefs about teaching and learning.
- *ICT knowledge* is defined as knowing how to operate a computer and knowing how to use a multitude of tools/software as well as troubleshoot in problematic situations.

In general, "ICT-TPCK is conceptualized as a unique body of knowledge that makes a teacher competent to design technology-enhanced learning" (Angeli & Valanides, 2009, p. 158).

Different frameworks have been developed to address the teacher knowledge needed for teaching with technology based on what is considered TPCK. The acronym TPCK was changed to Technological Pedagogical And Content Knowledge (TPACK, pronounced "tee-pack") at a meeting of the National Technology Leadership Institute in

September 2007 (Niess, 2011). The intention was to more readily draw attention “to the total package required for teaching—a package that integrates technology, pedagogy and content knowledge” (Niess, 2008; Thompson & Mishra, 2007 as cited in Niess, 2011, p. 301). TPACK as an extension of the earlier PCK framework contextualizes teachers’ knowledge as interactive and depicts the idea that it could be transformed according to the context in which teaching is developed and the teachers’ understanding is evolving. Then as Niess (2011) points out “research is needed to describe teachers’ learning trajectories in developing the knowledge, skills, and dispositions for incorporating new and emerging technologies as learning and teaching tools in various subject areas such that children’s knowledge is strengthened and enhanced” (p. 314).

Conceptualization of PCK in an Educational Context (PCK-EC)

We have discussed several research models that approach the teachers’ knowledge needed for teaching, illustrating some frameworks of PCK and also TPACK. Also, we have illustrated the conceptualization of the categories that constitute those models and the way the PCK and TPACK is framed under each perspective. We could approach the PCK-EC following relevant aspects of the previous framework of PCK and TPACK to build a theoretical framework for studying the elements of teachers’ knowledge needed for teaching with technological tools (TT). The conceptualization of each one of the dimensions of teachers’ knowledge is based mainly on Fennema and Franke’s model of teachers’ knowledge (1992) and the functional framework “the Knowledge quarter” developed by Rowland and colleagues (Rowland, 2007; Rowland, 2005; Rowland et al. 2003) to support the complexity of researching how elements of teachers’ knowledge are integrated in the practice of teaching.

The pattern characterized in the majority of the models (PCK and TPACK) is represented explicitly in Fennema and Franke’s (1992) framework which includes three main dimensions: knowledge of subject matter or content (in this case, mathematics), pedagogical knowledge, and knowledge of learners’ cognitions in mathematics. Also, the identification of teachers’ knowledge as interactive and dynamic, as well as the presence of a context and teacher beliefs proposed by Fennema and Franke’s (1992) model is utilized in most of the frameworks studied.

Another aspect that we might consider relevant for conceptualizing the pedagogical content knowledge in an educational context (PCK-EC) is to include the knowledge of materials, resources or tools that could be used in teaching, which is presented in the form of technology by Koehler and Mishra's (2008) model and as the information communication technologies (ICT) by Angeli and Valanides' model (2009). Then, we could consider that it is not necessary to conceptualize TPACK as an extension of PCK just because technology or ICT is added as one of the elements necessary for teaching practices. Technology may include any instructional material used in an educational context. Indeed in the model of PCK proposed by Shulman (1986), he included implicitly instructional materials as part of the curricular knowledge in regards to the programs designed for teaching, and those instructional materials may fit in Koehler and Mishra's definition of educational technology as "the sum of the tools, techniques, and collective knowledge applicable to education, which includes both analog technologies (e.g., chalkboard, pencil, and microscope) and digital technologies (e.g., the computer, blogging, and internet)" (2008, p. 5). Keeping with this definition, one can say that technology could be another dimension of the teacher knowledge needed for teaching including in the PCK instead of viewing this as TPACK.

It is also important to examine the central and external elements of Fennema and Franke's model (1992), the "Context – specific knowledge" in which teachers work, and teachers' beliefs about mathematics as components round out their conceptualization of teachers' knowledge. In regards to the context, they point out; "the context is the structure that defines the elements of teachers' knowledge and beliefs" (Fennema & Franke, 1992, p. 162), and the context in which teachers work is considered as situated because teachers could adapt their teaching knowledge in different contexts according to the situations presented in any of the classrooms in which the teaching practice occurs. Interestingly, Fennema and Franke (1992) explicitly placed belief outside of their teacher knowledge model – although they acknowledge that it influences teaching practice.

Indeed, considering that any component of teacher knowledge needed for teaching is situated in an educational perspective, we might say that an Educational Context is the structure that influences the ways in which the dimensions of teachers' knowledge are integrated in the practice of teaching. These dimensions of teachers'

knowledge may be adjusted according to the way teachers use their Attitudes and Knowledge in a classroom.

Hence, it is propose a new framework called Pedagogical Content Knowledge in an Educational Context (PCK-EC) for understanding teachers' knowledge needed for teaching a specific mathematics topic as it is represented in Figure 2.

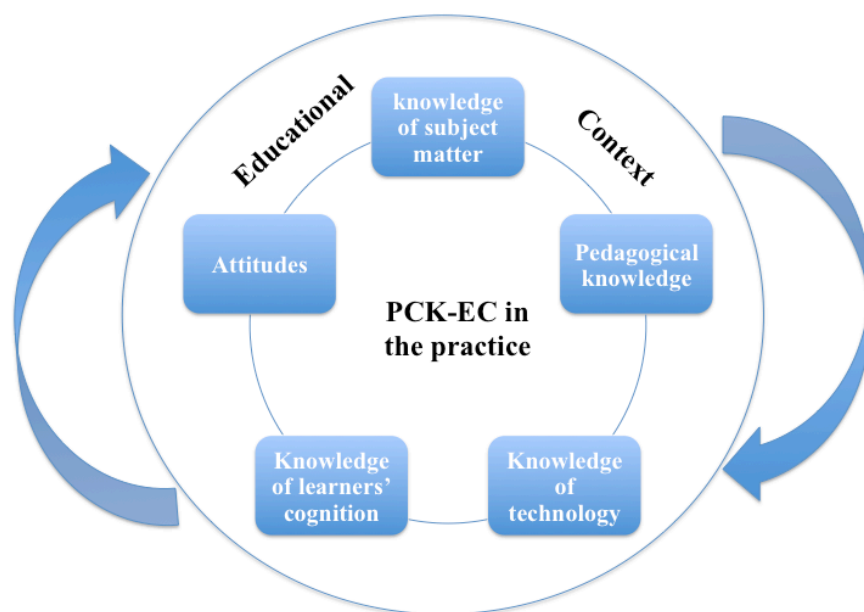


Figure 2. Theoretical Framework of Pedagogical Content Knowledge in an Educational Context (PCK-EC) for teaching a specific mathematics topic.

The PCK-EC includes five dimensions: teacher's Attitudes, Knowledge of Technology (KT), knowledge of Learners' cognition (KLC), Knowledge of Subject matter (KSM), and Knowledge of Pedagogy (KP). These dimensions are embedded in an Educational Context (EC) that comprises the curriculum and standards; social, cultural, and economic context; teaching and learning conditions defined by the school district; and teaching conditions in the classroom (Figure 2).

Hence, the proposed PCK-EC model is framed as the teachers' Knowledge (KT, KLC, KSM, and KP) along with teachers' Attitudes integrated within an Educational Context to define the practice of teaching. The arrows around the circle indicate that the PCK-EC is not static, it changes and it is transformed by the teacher.

The dimensions of teachers' knowledge are described as follows:

- *Teachers' Attitudes* “The way teachers define their role is based on not only their conceptions of learning, but also their beliefs about teaching. These perspectives of learning and teaching underlie both the choice of tasks teachers make and the way in which they use the tasks” (Sullivan & Mousley, 2001, p. 148). Teachers' Attitudes may include their approaches, conceptions, opinions and perspectives in regards to teaching and learning. In practice, this affects what teachers consider fundamental for teaching a specific topic using TT.
- *The Knowledge of technology* is the knowledge a teacher might have for using a particular TT (pencil, chalkboard, origami, Cuisenaire rods, software, and digital devices, among others) in their teaching practice. The Knowledge of Technology requires an understanding of the affordances and constraints of the TT, what kind of content ideas could be taught, what strategies teachers could apply to facilitate students' understanding, and what kind of constraints of TT might limit teaching or student learning. The Knowledge of Technology also includes the “creativity and flexibility with available tools in order to repurpose them for specific pedagogical purposes” (Koehler & Mishra, 2008, p. 17). In practice, a teacher could choose different TT for teaching according to their understanding and attitudes with respect to the content subject matter and the use of that particular TT.
- *Knowledge of Learners' cognition* includes knowledge of how students gain “the knowledge of mathematics content being addressed, as well as understanding the processes the students will use and the difficulties and successes that are likely to occur” (Fennema and Franke, 1992, p. 162). In the practice of teaching, the knowledge of learners is represented by “the characteristics and preconceptions that students bring to a learning situation” (Angeli & Valanides, 2009, p. 158). Teachers need to identify students' difficulties and obstacles in understanding a mathematics topic. This allows teachers to include students' conceptions and misconceptions in their planning and to choose strategies that help to manage students' understanding of mathematical content.
- *The Knowledge of Subject matter* refers to teachers' “knowledge of the concepts, procedures, and problem-solving processes within the domain in which

they teach” (Fennema and Franke, 1992, p. 162). Thus, knowledge requires that teachers know “the procedures, but also to understand the concept underlying them. They need to know that something is so, and also why it is so” (Petrou & Goulding, 2011, p. 14). In practice, this knowledge is illustrated in the way teachers present subject matter concepts and procedures, give explanations, make representations of concepts, use examples, and clarify students’ questions in regards to mathematics.

- *The Knowledge of Pedagogy* refers to the knowledge needed by teachers in regards to “teaching procedures such as effective strategies for planning, classroom routines, behaviour management techniques, classroom organizational procedures, and motivational techniques” (Fennema and Franke, 1992, p. 162). In practice, the pedagogical knowledge is used to guide the development of student understanding of mathematics following one or more learning theories, to develop different ways in which teachers manage students’ situations and to present within a logical sequence the content and activities in the classroom. Teachers could use several pedagogical choices around the classroom activity including the materials and resources used for teaching.

The pedagogical content knowledge in an educational context (PCK-EC) for teaching a specific mathematics topic includes the way teachers integrate the Knowledge of Mathematics, Technology, Pedagogy, Learners’ cognition and their Attitudes as a unit during their teaching practices. Also, PCK-EC represents the way teachers use Knowledge and their Attitudes in the whole activity of teaching a particular mathematics topic situated in an EC. Teachers need to be aware that the PCK-EC is developed according to the way they unpack and make deeply meaningful the knowledge of mathematical content integrated with the other dimensions of teachers’ knowledge and their Attitudes. Teachers could transform and develop knowledge when they are teaching a specific topic. “A number of strategic research sites and key events are particularly illuminating for our understanding of how knowledge grows in teaching” (Shulman, 1986, p. 8). Then teachers use knowledge but also produce knowledge in their teaching. This feature implies that the PCK-EC is interactive, dynamic and it is transformed in

teaching during classroom interactions either as “reflection-in-action” and/or “reflection-on-action” (Schön, 1983, 1987 as cited in Park & Oliver, 2008). The reflection-in-action refers to the knowledge developed and enacted during teaching practices which requires integrating all components of PCK to provide reasons, explanations and justifications to students’ questions in situations that could be expected or unexpected (Park & Oliver, 2008). Reflection-on-action occurs after the teaching practice is completed and allows teachers to re-structure or modify the body of PCK for teaching a particular topic (Park & Oliver, 2008).

Previous studies of PCK and TPACK

Black (2007) evaluated teachers’ content knowledge, pedagogical content knowledge, and changes of instructional practices after professional development in mathematics secondary teachers. Her study highlights deficiencies in both content knowledge and pedagogical content knowledge, and also lack of connection between mathematical knowledge and instructional practice.

Cavin (2007) studied the development of technological pedagogical content knowledge (TPCK) in pre-service teachers during microteaching lesson study using two different technological instruments: a graphic calculator and an excel spreadsheet. This study followed the framework proposed by Mishra and Koehler (2006). Her study highlights three major aspects: the need for inquiring about the attitudes and beliefs of the pre-service teachers to the development of the TPCK, the need for using technology for teaching both procedural and conceptual knowledge, and also to establish a more quantitative measure of TPCK development.

Loughran, Mulhall and Berry (2004) examined science teachers’ pedagogical content knowledge (PCK) using two approaches: content-specific teaching procedures (Content Representation, CoRe) and teaching practice (Professional and Pedagogical experience repertoire, PaP-eR) across a range of science topics. They indicated that “the time and effort associated with developing cohorts of science teachers to work with, to detail their understandings of particular science content (CoRe) and associated pedagogical influences (PaP-eRs) is extensive” (Loughran et al. 2004, p. 381).

Stoilescu (2011) studied ways of using the Technological Pedagogical Content Knowledge (TPACK) framework in experienced teachers for integrating computer technology in mathematics education. He indicates that the TPACK framework allows analyzing and improving any activity in the classroom by being aware of the technology, mathematical content and pedagogy. He pointed out that three major challenges in using this framework are: a) it needs to be studied under different academic and socio-cultural settings; b) it needs to include “teachers’ attitudes, opinions, philosophy, and paradigms of teaching” (p. 198); and c) “lack of clarity in determining with accuracy the TPACK components of the teacher” (p. 198). Stoilescu (2015) also indicates “until this moment, there is no procedure available to determine with precision the level of knowledge in integrating Information and Communications Technology (ICT) that a teacher displays” (p. 542). Stoilescu (2015) describes the integration of technology in the classroom for three mathematics secondary teachers’ using the TPACK framework. Exploration of the TPACK for each teacher was based on naturalistic inquiry. The results were holistic and “presented intuitively” (weight of circles as a modified TPACK visuals).

The proposed PCK-EC model includes teachers’ Attitudes as another dimension integrated with teachers’ knowledge of Technology, Learners’ cognition, Subject matter, and Pedagogy. These dimensions are immersed in an Educational Context in order to illuminate concerns by the above researchers.

Methodology

This research was designed to explore teachers' pedagogical content knowledge in an educational context (PCK-EC, see Figure 2) when they are teaching a specific mathematics topic with the use of TT. A qualitative research approach was used because it allows for exploring and understanding "the contexts or settings in which participants in a study address a problem or issue" (Creswell, 2007, p. 40). In order to study teachers' PCK-EC, case studies were used to provide understanding of the dimensions of the proposed (PCK-EC) model in the practice of teaching and how it is affected when teachers are using different TT in their teaching. This chapter provides: a brief description of the case study methodology, an overview of the particular methods and research techniques used to gather evidence in this study, information on the procedures for data analysis as well as procedures to ensure validity of the data, and ethical considerations guiding the data collection.

Case study approach

A case study approach entails conducting an "empirical investigation of a contemporary phenomenon within its natural context using multiple sources of evidence" (Yin, 2003). Also, case study analyzes social interaction within a particular and historical context, and tries to identify and describe the social interaction instead of generalizing (Stark & Torrance, 2005). Thus, the case study approach involves the study of an issue explored through one (case study) or more cases (multiple case studies) within a bounded system (Creswell, 2007). According to Hancock and Algozzine (2006) there are three types of case studies: exploratory, explanatory and descriptive.

This is an exploratory multiple case studies designed to examine the potential utility of the proposed PCK-EC model (Figure 2). The cases of two mathematics teachers teaching in different settings were examined with this model to provide an initial pool of experience from which to begin refining it. In particular, this exploratory multiple case studies approach was used for addressing the following research questions:

- How effective is the proposed model PCK-EC (Figure 2) in supporting the examination of teacher practice and providing insights to the pedagogical content knowledge?
- How do different TT mediate or affect teachers' PCK-EC when mathematics teachers are teaching a particular topic?

In order to answer these questions, it was necessary to define the boundaries of each case study for this methodology. The boundaries were defined by the topic to be taught, specific tools that teachers were using for teaching, and the time frame for collecting data. This allowed for an in-depth understanding of the cases and a comparison of two cases through the methods used for collecting the data.

Methods

The advantages of the case study approach are that it allows researchers to explore an activity using several gathering methods for multiple perspectives (observer, participant), and it provides a rich description of a phenomenon (Stark & Torrance, 2005). In this particular research the phenomenon was defined as the teachers' PCK-EC teaching a specific mathematics topic with the use of different TT. Hence, in order to understand that phenomenon it was necessary to collect data through different methods that included: interviews (initial and final interview), videotapes of classroom teaching, teachers' reflection journals (after each teaching practice), direct observations during classes, and observations from video recorded classes, teachers' generated documentation (transcripts), and documents supporting educational context.

Recruitment of the participants. A call for participants was distributed through the BC Association of Mathematics Teachers email list (Appendix 1), with a request to pass along the invitation to any interested colleagues. Teachers who self-identified as potential participants were contacted to clarify details of the study, confirm willingness to participate, and to verify current use of TT in their teaching practice. Participant selection was based on the following features: (a) mathematics teachers who were teaching a specific mathematics topic of their choice (i.e., a unit) with substantive use of a recently introduced technological tool (i.e., iPad, tablet, Geogebra); (b) having experience using

the technological tool; and (c) the primary technological tool used in the teaching of the topic had to be different for each teacher. Two participants were identified, both of them located in the Greater Vancouver area. Each teacher had over ten years of experience teaching mathematics and at least two years using different TT.

In order to maximize the authenticity of the cases being observed (although there may have been some Hawthorne effect), the teachers were autonomous in planning and developing their activities with the TT, and there was no interference from the researcher in terms of planning, development of the topic or handling the TT. As a result, the time frame required for each participant was determined by the number of lessons teachers required to teach a specific mathematics topic, the time required to complete the reflective journal, and the initial and final interviews. The main participation of the teacher took place in the classroom, and they used different TT in their teaching. Letters of consent were issued to participants (mathematics teachers) and to the Principal of each school (Appendix 2 and 3, respectively). Letters of information and/or consent were sent to students' parents (Appendix 4 and 5, respectively).

Additionally, I was able to act like a participant observer because I was able to understand and interpret situations and actions in the classroom, while minimizing the interference of my observations, because of my own experience as a mathematics teacher.

Data collection methods. The proposed model PCK-EC (Figure 2) was used as the framework for collecting data. With the model we can describe a teaching situation in terms of the five dimensions: teachers' Attitudes, and teachers' Knowledge (KT, KLC, KSM and KP). Thus, by situating teachers' Attitudes and Knowledge in an EC, in which the cases were immersed, we can paint a picture to support communication and integration of teacher's PCK-EC in a dynamic environment that is guided by development of new digital technologies.

Examination of this model is important because it will allow us to gain conceptual and empirical understanding of the dimensions of PCK-EC for teaching a specific topic with the use of TT. This model may also constitute an instrument for guiding the examination of teachers' Attitudes and Knowledge. The arrows indicate the evolution

feature in the PCK-EC from the Educational Context perspective as well as from inside the teacher.

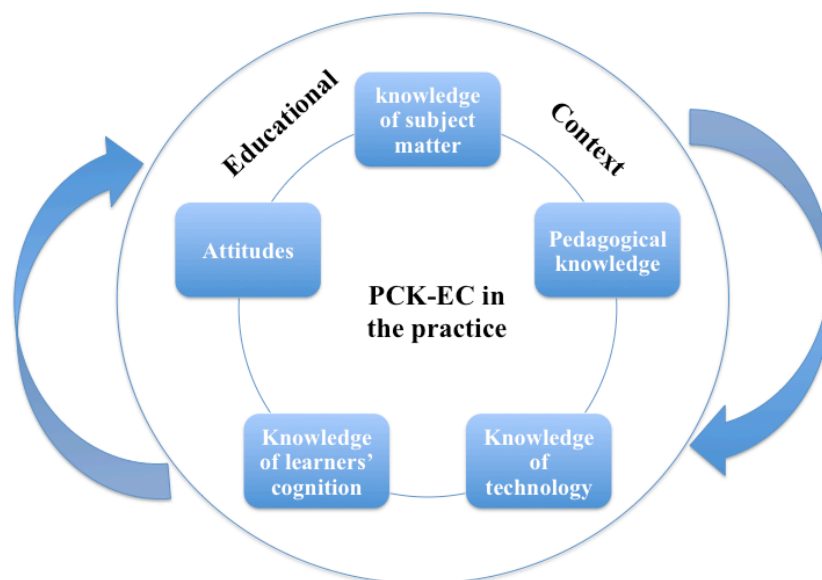


Figure 2. Theoretical Framework of Pedagogical content knowledge in Educational Context (PCK-EC) for teaching a specific mathematics topic.

In this research, data was collected having as a core study the dimension of KT and specifically to the knowledge of the main TT that teachers were using in their teaching. Thus, interview questions, direct observations during classes, observations from videotapes, and teachers' own reflections were framed for each dimension referred to the use of the main TT that teachers used in their teaching practices.

The data collection methods included:

- a) Semi-structured initial interviews that were performed before each teacher started to teach the mathematics topic;
- b) Semi-structured final interviews conducted after each teacher had finished their teaching of the mathematics topic;
- c) Teachers' journals of reflection (which teachers completed after teaching practices);
- d) Direct observations during classes;
- e) Observations from video recorded classes;
- f) Transcripts from initial and final interviews;

g) Other collected documents in regards to the educational context.

In order to maintain participant teachers' and schools' anonymity, the videographer technician and the reviewer of transcripts were asked to sign a confidentiality agreement (Appendix 6 and 7, respectively). The following table displays the information related to collecting of data.

Table 1

Data collection methods used in this study

Participants	Data	Techniques	Aims of data collection
Teachers	Initial interview	Use of transcripts from audiotapes and videotapes.	To provide evidence for identifying the
	Final interview	Use of the interpretive	dimensions of teachers' PCK-EC model. Also to
	Journal of reflections	repertoires for identifying themes in each dimension of the PCK-EC model.	determine how TT affect teachers' PCK-EC teaching a specific topic.
Researcher	Curriculum, Socio-economical information.	Gathering information from school and municipality.	To identify the Educational Context.
Researcher	Direct and indirect observations from lessons and videotapes.	Reflective notes on interactions of teachers, TT, and students (observational Protocol).	To obtain a more holistic and embedded sense of the ways teachers' PCK-EC is mediated through the use of TT.

Interviews. “The interview can be described in terms of individuals directing their attention towards each other with the purpose of opening up the possibilities of gaining insight into the experiences, concerns, interests, beliefs, values, knowledge and ways of

seeing, thinking and acting of the other” (Schostak, 2006, p. 10). In this study, the interviews were developed as semi-structured interviews with open-ended questions, following Hancock and Algozzine (2006) who point out: “semi-structured interviews invite interviewees to express themselves openly and freely and to define the world from their own perspective, not solely from the perspective of the researcher” (p. 40). Also, Creswell’s (2007) suggestion about design an interview protocol with open-ended questions was followed.

Two interview protocols were designed with open-ended questions. Questions for the interviews were elaborated for each one of the dimensions having as a focus the KT. Some of the questions were adapted from the paper entitled “Knowledge growth in teaching mathematics/science with spreadsheets: moving Pedagogical Content Knowledge (PCK) to Technology, Pedagogy and Content Knowledge (TPACK) through online professional development” by Niess, van Zee, and Gillow-Wiles (2010). The initial interview had 45-questions distributed in the five dimensions (Appendix 8), and the final interview was composed of 25-questions (Appendix 9) for the same five dimensions.

The first dimension in the interview protocol was teachers’ Attitudes, followed by KT, KLC, KSM, and KP (Appendix 8 and 9). As was mentioned above, the criteria to formulate interview questions had as a core study the KT, and focused on the main TT that teachers were using in their teaching practice. For example, in the teachers’ Attitude dimension, one of the questions was: “*Why do you think this TT (tablet, ipad) support your teaching?*” In the dimension KT “*Which are the constraints of this TT (tablet, ipad)?*” In the dimension KLC “*What is the previous knowledge that students need to know in regards to the use of this TT (tablet, ipad)?*” In the dimension KSM “*What mathematics topic do you teach with this TT (tablet, ipad)?*” and in the dimension KP “*How, why, and when do your strategies change for different types of learners?*”

Two personal interviews were conducted with each participant. In each case, one of the interviews was prior to the lessons observed, and the other one was after finishing each mathematics topic. The initial interview took approximately one hour. The final interview time frame varied from approximately half an hour to one. Both interviews were recorded with a digital voice recorder, and were fully transcribed.

Observations. “Observing in a setting is a special skill that requires addressing issues such as the potential deception of the people being interviewed, impression management, and the potential marginality of the researcher in a strange setting” (Hammersley & Atkinson, 1995 as cited in Creswell, 2007, p. 134). In this process, I followed Creswell’s (2007, p. 137) observational protocol to record information in a systematic way for analysis (descriptive and reflective notes). Also, Hancock and Algozzine (2006) highlight the fact that “case study researchers must actively attempt to identify and mitigate the effects of their biases and prejudices in order to ensure the impartiality of their conclusions” (p. 47). In this regard, observations were undertaken focusing on each teacher’s teaching of the specific mathematics topic in situ, with minimal intrusion, and passive observation.

Researcher observations were performed during teachers’ regular teaching lessons; notes for reflection were taken, following the practice of “bracketing or reduction” (van Manen, 2006), to distinguish between my objective observations and my thoughts derived from those observations. Video recording of the classes were conducted for further direct observations and reflections. The observational protocol (Appendix 10) followed the five dimensions of the proposed model PCK-EC (Figure 2), and consisted of several questions that were self formulated to guide the researcher in identifying the dimensions of the model during the teachers’ teaching practices. For example:

- *“Does the teacher use different mathematical representations of the concepts involved in the study topic? What are those representations? (KSM)*
- *Does the teacher present the content and activities in the classroom in a logical sequence? (KP)*
- *Do teachers assess or verify previous content mathematics knowledge required for understanding the topic? (KLC)*
- *Describe some constraints of the TT for the specific mathematics topic. Do teachers need to use others resources to clarify questions or to give deeper explanations? Which are the possible limitations on the use of the tool? (KT)*
- *Do teachers make explicit their Attitudes in regards to the ways the TT works better in class? (Teachers’ Attitudes)*

In total there were 14 classes that were video recorded with a total of 16.8 h duration for all the teachers' lessons, as it is presented in the following table:

Table 2

Duration of lessons and interviews (initial and final) recorded by participant

Participant	Teacher 1 – Mary	Teacher 2- Alex	Total
Number of lessons	<i>Grade 11: 3 lessons</i> <i>Grade 12: 5 lessons</i>	<i>Grade 10: 6 lessons</i>	14 lessons
<u>Duration (hours)</u>			
Initial interview	0.9 h	1.0 h	1.9 h
Final interview	0.6 h	0.9 h	1.5 h
Lessons	9.7 h	7.1 h	16.8 h
Total	11.2 h	9.0 h	20.2 h

Direct observations allowed following most of the interactions that were occurring during the teaching practice, while the video recording allowed seeing a particular interaction of what was happening during the class (i.e. the camera was directed towards the content being projected in the board, and as a consequence other interactions that were happening during the class were not followed by the camera; in other situations the camera was directed to the teacher while working with the TT. For that reason, reflective notes were taken during the lesson to be used as reminders in order to complete observations for each one of the dimensions of the model. Observations performed by watching the video recorded lessons were taken separately for each of the dimensions of the PCK-EC model. For example, the video recorded lessons were played to identify only the dimension of KSM obtaining quotes that exemplify this dimension. Same procedure was followed for each of the dimensions to complement the examination of the teachers' PCK-EC in the practice.

Documents collected in regards to Educational Context. Documents generated in this research included the teachers' journal of reflections, transcripts of initial and final interviews, videotape of teaching activities during lessons, and researcher's reflective

notes and observations. The journal of reflections aimed to explore teachers' thoughts in regards to teaching practices using the TT. Teachers were asked to complete their reflective journals after each teaching lesson was performed, and they were guided by a set of questions for each of the dimensions of the PCK-EC (Appendix 11).

The EC included: the curriculum and standards; social, cultural and economic context; teaching and learning conditions defined by the school district and teaching conditions in the classroom. Documents related to the EC were obtained from different websites. Curricular documents about the school districts, mathematics curriculum for middle and secondary school from the province of British Columbia, Statistics Canada, and school reports (Fraser Institute) were compiled to characterize the EC.

Data Analysis

Case study research requires summarizing and interpreting information from multiple sources focusing on the topic being studied. The process “involves examining and re-examining research questions and answers that evolve as information becomes available for analysis” (Hancock & Algozzine, 2006, p. 57). The analysis included the process of transcribing interviews; reviewing and categorizing answers from interviews and also from the reflective journals by using the interpretive repertoires method (discourse analysis) in order to identify collections of specific themes for each of the five dimensions of the proposed PCK-EC model. A protocol of observations (direct observations and video recordings) complemented the analysis of the text supporting the identification and validation of the themes (adding details to the description of the themes and providing some examples of these themes in teachers' teaching). Following this, themes were re-examined to discover patterns and identify correspondences between two or more dimensions. Through this multi-step analysis process a more detailed description of the teacher's PCK-EC emerged with respect to the topic s/he was teaching and TT s/he was using. Finally, the analysis of the cases and its comparison allowed providing some possible answers to the research questions related to the PCK-EC model.

Validity

The nature of validity arises because not all of the accounts of a situation or a phenomenon, or an activity are meaningful, credible or legitimate (Maxwell, 1992). Hence, the meaning that emerges from the collected data is exposed to different interpretations from individuals involved with the research. Validity may be defined as “how accurately the account represents participants’ realities of the social phenomena and is credible to them (Schwandt, 1997 as cited in Creswell & Miller, 2000, p. 124-125). In this study, a set of procedures was used to ensure the validity of the deep description of teachers’ PCK-EC as they taught a specific mathematics topic with the use of different TT. The procedures that were followed were suggested by Creswell & Miller (2010) and Creswell (2007), and included:

- Triangulation was used across data sources (two participants), the use of multiple data collection modalities (interviews, observations, journal of reflections, video-tapes, and related documentation), and theories that support evidence to corroborate the analysis of the cases.
- Prolonged engagement and persistent observation in the field was performed in classroom teaching for as long as the specific mathematic topic was taught. The engagement persisted by using the video recording of the lessons for extended periods of time during the analysis to corroborate data.
- An audit trail was made through committee members that were involved during the process of this research, particularly my advisor.

Ethical considerations

This exploratory multiple case study involved two different teachers that were teaching in different schools. As consequence, there was a set of guidelines that were completed in regards to ethical issues. The guidelines were defined in an Application for Ethics Approval for Human Participant Research presented to the Human Research Ethics Board from the University of Victoria (Appendix 12). The protocol was approved and registered with the number 13-044. Research was considered to be a minimal risk to the participants because they were not involved in any activities that differed from what they did in their everyday work.

Teachers' participation was voluntary, and formalized by providing a letter of consent in which teachers were informed of their right to withdraw at any time without any consequence (Appendix 2). Participants were provided with a written statement that describes the nature, purpose, and procedure of the research. Also, the statement explained what data would be collected and how it would be used, and a section for permission to take pictures and video recordings from the classes. Teachers were autonomous in planning and developing the activities that they do regularly in their work.

Pseudonyms were used to protect the privacy of participants and schools. A letter of consent was provided to each School's principal (Appendix 3). Letters of information were submitted to students and parents in one of the schools (Appendix 4), and a letter of consent was issued to the other school (Appendix 5). The videographer technician and the reviewer of transcripts signed a confidentiality agreement (Appendix 6 and 7, respectively). All data collected (interview transcripts, digital video-recording, notes, journal reflections, protocol of observations, email, etc.) are being kept on a secure computer, and will be maintained for approximately 5-years.

Results

This chapter presents the results obtained from exploring the teachers' knowledge needed for teaching specific mathematics topics with the use of TT using a new framework called "Pedagogical Content Knowledge in an Educational Context" (PCK-EC) (see Figure 2). The PCK-EC considers five dimensions: teachers' Attitudes, the Knowledge of Technology (KT), the Knowledge of Learners' Cognition (KLC), the Knowledge of Subject Matter (KSM), and the Knowledge of Pedagogy (KP). These dimensions are immersed in an Educational Context (EC).

The research used an exploratory multiple case studies approach. This chapter looks at the cases of two mathematics teachers, with the pseudonyms of Mary and Alex, who were teaching in schools located in the Greater Vancouver area. The boundaries of the case studies were defined by mathematics topic to be taught, specific technological tools that the teachers were using in their lessons, and the time frame available for collecting data (Table 3).

Table 3

Boundaries for the two case studies

Participant	Teacher 1	Teacher 2
	Mary	Alex
Type of School	Secondary Public School in BC, dual English and French immersion [teaching in English]	Technical Secondary Public School in BC [teaching in English]
Course	Pre-calculus and Calculus AP	Grade 10
Mathematics Topic	<i>Grade 11</i> : Linear and quadratic system <i>Grade 12</i> : Definitive integral and area under a curve	Introduction to trigonometry & Provincial exam review
Main technological tool	Tablet	iPad

Other technological tools used	Data projector	Data projector
	Graphing calculator	Scientific calculator
	Worksheets (paper and electronic)	Worksheets (paper and electronic)
	Chalkboard	Computer
	Textbook (paper and electronic)	Textbook (paper and electronic)
	Moodle class website	Class website
Number of classes observed	<i>Grade 11:</i> 3 lessons <i>Grade 12:</i> 5 lessons	6 lessons
<u>Duration</u>		
Initial interview	0.9 h	1.0 h
Final interview	0.6 h	0.9 h
Classes	9.7 h	7.1 h
Total	11.2 h	9.0 h
Reflective journals	17 pages	7 pages

Data was collected from a variety of sources: interview questions, direct observations during lessons, observations from video recordings of lessons, and teachers' journals. The analysis commenced by transcribing all of the audiotapes from interviews, as well as some verbal excerpts taken from the video recorded classes (used to complement and confirming observation notes) in order to have all the data in textual form for analysis. From the textual record of the teachers' statements, key comments were identified. A total of 355 and 252 comments and responses (quotes) from Mary and Alex, respectively, were extracted from the initial interview, final interview, and teachers' journal of reflections and analyzed in each one of the dimensions of the model (Table 4) using the method of interpretive repertoires (discourse analysis).

The analysis provided a deeper understanding of the teacher comments. By selecting, analyzing, and coding main ideas, a series of themes emerged in each one of the dimensions of the proposed model PCK-EC (teachers' Attitudes, KT, KLC, KSM, KP). These themes were expanded upon and polished after reflecting on cross-

dimensional comments and seeking corroboration from notes and quotes obtained from the observations (direct observations and video recorded classes). The descriptive theme names were chosen to summarize the pattern obtained through the analysis of the complete data set for each one of the dimensions in the proposed model (PCK-EC).

Table 4

Total number of quotes analyzed (initial and final interviews, and teacher's journal of reflections) to define the PCK-EC

	Dimensions				
	Attitudes	Technology	Learners' cognition	Subject Matter	Pedagogy
Mary's Data					
Initial interview	40	38	28	31	25
Final interview	27	21	12	9	10
Journal of reflections	21	21	13	12	14
Grade 11th					
Journal of reflections	8	7	6	5	7
Grade 12th					
Mary's quotes	96	87	59	57	56
Alex's Data					
Initial interview	30	23	8	16	23
Final interview	27	29	14	13	13
Journal of reflection	16	15	9	6	10
Alex's quotes	73	67	31	35	46
TOTAL QUOTES	169	154	90	92	102

Results of each case study are presented separately (Mary and Alex) using the PCK-EC model as the structure. First, the EC in which teachers structure their teaching practice is defined. Then, each of the dimensions of teacher knowledge proposed in the PCK-EC model (Figure 2) is presented in the following order: Attitudes, KT, KLC, KSM, and KP.

Educational Context

The educational context (EC) is the external structure that influences the manner by which a teacher's Attitudes and his/her KT, KLC, KSM, and KP are embedded in the practice of teaching a specific mathematics topic in a classroom. The EC in which Mary and Alex taught is situated in the Province of British Columbia (Canada), and includes the aspects interrelated directly with teaching practices: the curriculum and standards; social, cultural and economic context; teaching and learning conditions defined by the school district; and teaching conditions in the classroom.

Curriculum and standards. In the cases of Mary and Alex, the mathematics curriculum have been defined in the Common Curriculum Framework for Grades 10-12 Mathematics: Western and Northern Canadian Protocol (WNCP). The curriculum framework was developed by the six ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others (Western and Northern Canadian Protocol - WNCP, 2008). The Ministry of Education in the Province of British Columbia started in 2010 a curriculum redesign that was developed using the WNCP framework and this is what teachers are mandated to teach. The curriculum identifies five areas for development: personalized learning, quality teaching and learning, flexibility and choice, high standards, and learning empowered by technology (BC Ministry of Education, 2012a).

The province of BC has a total of 1604 public schools and 347 independent schools (2011-2012) distributed in 60 districts around the province, and they are governed by this curriculum framework (British Columbia Teachers' Federation - BCTF, 2012). During 2012/2013 the enrollment of students in the province of BC reached 638,841 students, 88.4% of students were in public schools and 11.6% were at independent schools. The average class size for the BC province during the 2012-2013 for grades 8-12 was 23.3 students (BC Ministry of Education, 2012b). The female/male proportion of students was 49.2% and 50.8%, respectively.

Social, cultural, and economic context. In regards to the social and cultural context, the schools in which this research was undertaken were located in the Greater Vancouver Regional District (GVRD); this region includes 21 municipalities, and 17 indian reserves (https://en.wikipedia.org/wiki/Greater_Vancouver). Mary's and Alex's cases were both located in public schools in one of the municipalities in this region.

Population. In 2011, Metro Vancouver had 2.3 million people. The population growth over a 25-year period from 1981 to 2006 was about 900,000 people, and between 2006 and 2011 increased by nearly 200,000 persons. Currently about 75%-80% of the region's population growth comes from immigration, and it is expected that this share will increase substantially in coming years (Metro Vancouver, 2009; Statistics Canada, 2011). The compound annual population growth rate between 1971 and 2001 for the community in which the schools are located was 3.2%, which is considerably higher than the GVRD which had a 2.1% growth rate for the same period (The Sheltair Group and Kelly & Associates, 2004).

Housing profile. The socio-economic profile in the GVRD indicates that approximately 61% are homeowners compared to 66% for BC in 2001, and 78% for the community in which the participant schools are located. Similarly, the percentage of single family home is 64% for the GVRD, 66% for BC and 75% for the community in which those schools are located (The Sheltair Group and Kelly & Associates, 2004).

Family structure. Household sizes are 2.59 for Metro Vancouver and 2.76 for the community in which the schools are located. The average number of children per family was 1.1 in the community (The Sheltair Group and Kelly & Associates, 2004). The family structure in BC was constituted by married couples in 72.7%, common-law couples 12.2%, and lone-parent families 15.1%. In the community where Mary's and Alex's schools were located, the family structure was constituted by married couples 73.7%, common-law couples 11.8%, and lone-parent families 14.5%, respectively (Statistics Canada, 2011).

Household income. The 2005 average household income for GVRD was \$73,158, and a projected average household income for 2008 equivalent to \$79,798 (Vancouver Economic Development, 2009). The reported household income during 2011-2012 for Mary's school was \$70,300; and the average family income for the same period for Alex's school was \$79,600 (Cowley and Easton, 2013).

Ethnic origin. The Canadian-born population in the GVRD was 73% compared to 83% in the community in which Mary's and Alex's schools were located. In the school area the top eight ethnic origins in 2001 were English, Canadian, Scottish, Irish, German, French, Dutch and Ukranian, while in the GVRD, the top eight ethnic origins were English, Canadian, Chinese, Scottish, Irish, German, East Indian, and Filipino. In 2001 17.7% of the GVRD's population is of Chinese origin, while in the community where the schools are located it was only 2.2% (The Sheltair Group and Kelly & Associates, 2004).

Teaching and learning conditions defined by the school district. Teaching and learning conditions of the schools in which Mary and Alex taught were defined by the same school district because they are located in the same municipality. The school district has maintained a leading role in developing "applications of technology to learning, promoting personalized learning programs and recognizing and celebrating diversity" (School District, 2013, 2012). The school district has explored the use of new technologies with the purpose of enhancing teaching and learning processes in the classroom. Some projects that have been implemented in secondary level, included: *1:1 iPads in Secondary Numeracy Grades* and *Tablets for Math Teachers*, among others (School District, 2013, 2012). Also, the school district has developed a plan for teacher professional development and one of the purposes is for improving pedagogy through the use of technology, and increasing the infrastructure needed to include technology within the classrooms.

The Secondary school in which Mary teaches is a dual English and French Immersion School, and it has a variety of programs including Advanced Placement (AP). The AP program allows highly motivated students to begin college or university studies in their grade 12 year. Total enrolment for the school during 2013 was 1,341 students.

Mary's school was participating in the program *Tablets for Math Teachers*, and also provides projectors and audio in the classroom.

The school in which Alex teaches is a technical secondary school, and offers an academic, as well as, trades programs (carpentry, metal fabrication, culinary arts, hairstyling and cosmetology). Enrolment during 2013 was 823 students. The school was participating in the program *1:1 iPads in Secondary Numeracy Grades*, and all the students in Alex's class were provided with an iPad for their use during the class.

Teaching conditions in the classroom.

Mary's case. Mary has been teaching mathematics in a public secondary school for seventeen years. She feels passionate about teaching and has taught mathematics at all levels for twenty-seven years in total. In this research, Mary taught a regular course in grade 11, and an AP course in grade 12, with the topics of Linear and quadratic systems and Definite integrals and area under a curve, respectively. The total number of lessons for the topic of Linear and quadratic systems were five (5) lessons, and for the topic Definite integral and area under a curve a total of three (3) lessons. Grade 11 was composed of 25 students, 8 males and 17 females. Grade 12 was composed of 14 students, 5 males and 9 females.

The tablet-computer was the main TT used during this case. Mary was also observed using other TT to support her practice. These included: OneNote, data projector, graphing calculator, worksheets (paper and electronic), a chalkboard, a textbook (paper and electronic) and a class website (Moodle).

Mary maintained a 'traditional' teaching environment. She took students' attendance either at the beginning or at the end of the lesson. The tablet was already set up for the lesson, and Mary was the only one who worked with the tablet. She typically sat at the middle-back of the classroom (beside the projector) to deliver the lesson. The presentation was projected onto a screen at the front of the classroom (Figure 8). Explanations and justification of the content of the lesson were discussed with students when Mary was working with the tablet. Sometimes when it was required she gave explanations at the front of the classroom, and occasionally she went directly to the student's desk.

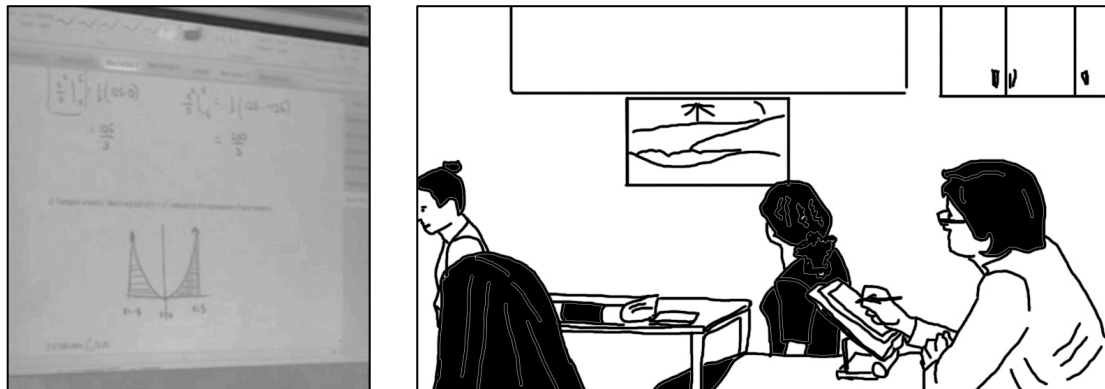


Figure 8. Mixed media sketch of Mary's primary teaching mode.

“The BCESIS website delayed the start of the lesson as it was slow to be called up. I usually take attendance at the start or the end of a lesson. Once started, I follow through as long as the process doesn't take longer than 5 minutes” (MJR, grade 12).

Mary showed good understanding of the topic, mastery of the tablet, and also good management of students' behaviour. Students remained in their assigned seats most of the time, but were free to communicate with each other. They were very quiet, and the environment of the lesson was very respectful in regards to questions, discussions and exchange of ideas.

Occasionally Mary used calculators, which were manipulated by her and by students. She provided calculators during the lessons and only a few students had their own (Figure 9).

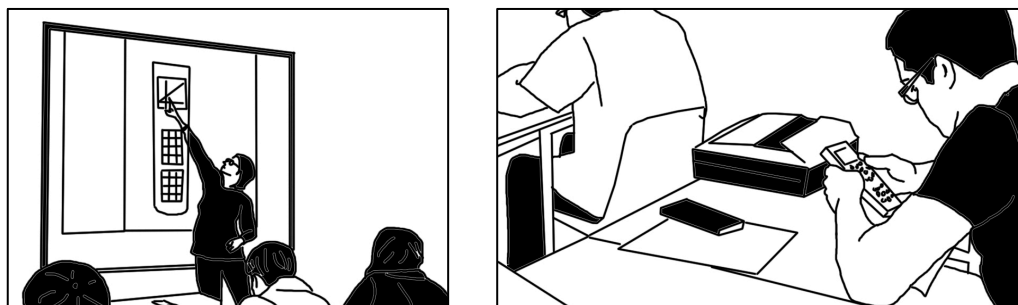


Figure 9. Sketch of Mary and students working with calculators

Alex's case. Alex has over twenty-eight years of teaching experience. He started teaching mathematics in 1985 at a public school in a Northern territory for two years, with two thirds of the population being Indigenous. This teaching experience helped him to learn about students' behaviour, and being flexible and creative in teaching mathematics because "there was not really much of a curriculum at the time". Then, he went to the Lower Mainland, where he had a "variety of experiences" teaching from grade eight up to grade twelve. Later, he went to administration (vice-principal), department head, and then teaching again Mathematics and Science.

Alex received a Bachelor of Education majoring in Secondary Mathematics, and is working towards a Master of Science degree. During the last three years he has been working part-time for the district, helping teachers with mathematics and pedagogy. He is returning as a full-time mathematics teacher in 2014.

During this research study, Alex taught a regular course in grade 10 as a visiting teacher, focusing on the topics of Introduction to trigonometry and the Provincial exam review. The total number of lessons was six (6). He developed both topics during lessons, dedicating more time towards the Introduction of trigonometry. His class was composed of 26 students, 15 males and 11 females.

The iPad was the main TT used during this case. It was also used by students who were participating in a *1-1 iPads in Secondary Numeracy Grades* district program. Alex was also using other TT to support his teaching. These included: data projector, scientific calculator, worksheets (paper and electronic), a computer, a textbook (paper and electronic) and class website.

Alex maintained a flexible teaching environment. Upon arrival to the class, students turned on their designated iPads and commenced chatting with each other while Alex set up his iPad. Alex and the students would then start discussing the lesson content in a more informal setting. The classroom was set up for a Science class, with a different distribution, which allowed students to be in closer proximity to each other (Figure 10).



Figure 10. Mixed media sketch of students' positioning in the classroom.

Alex was situated at the front in the middle of the classroom, and he expressed his discontent with the classroom physical environment with regard to lack of chalkboards and inadequate equipment to work with TT. He could not project his tablet and the iPad at the same time. He typically was standing at the middle-front of the classroom (beside the projector) to deliver the lesson. The presentation was projected onto a screen at the front of the classroom (Figure 11).



Figure 11. Mixed media sketch of Alex's primary teaching mode.

Alex introduced most of the lessons using the iPad, reviewing assignments, providing explanations and demonstrations of the activities for the lessons, responding to students' questions, or making announcements for lesson activities. Alex and students worked most of the time with the iPad during lessons, and occasionally Alex would turn off the iPad to monitor students' activities or to work with his computer while students

were working on their assignments. Sometimes, students went directly to Alex's desk to ask or clarify questions, otherwise students worked individually with the iPads and shared activity lessons (Figure 12). Alex designed an activity outside of the classroom in which students worked with iPads, and then came back to the classroom to continue working on the lesson.

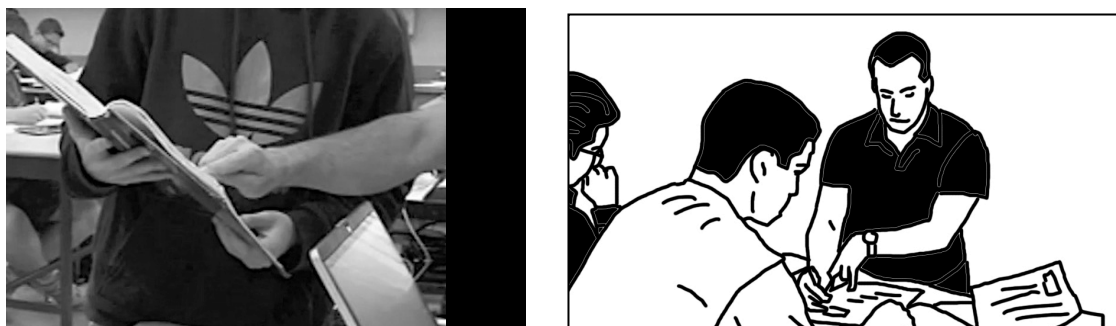


Figure 12. Mixed media sketch of Alex interacting with students during lessons.

Alex showed good understanding of the topic, mastery of the iPad apps, online resources and good management of students' behaviour. He designed the class website according to his needs and preferences. Students remained in their assigned seats most of the time, but were free to communicate with each other and move around the classroom. Students were attentive to Alex, and the environment of the lesson was very respectful in regards to questions, discussions and exchange of ideas.

Case 1 - MARY

This section uses the PCK-EC model to describe Mary's pedagogical content knowledge as she taught specific mathematics topics using a particular TT (tablet computer). The EC in which Mary taught was already presented. The dimensions of Mary's knowledge were identified following the proposed PCK-EC model, and they will be described in the following order: Attitudes, Knowledge of Technology (KT), Knowledge of Learners' Cognition (KLC), Knowledge of Subject Matter (KSM), and Knowledge Pedagogy (KP).

Dimension 1 – Mary’s Attitudes. Mary’s Attitudes were identified when she expressed an opinion, and provided her perspective of teaching/learning on the specific mathematics topics using TT. As was mentioned, Mary participated in this research by teaching two different topics in grade eleven and grade twelve. Then, Attitudes were summarized for both grades.

In response to questions, Mary expressed that in the process of teaching, TT are useful because they enhance and make a more interesting lesson, and provide a visual impact to the content of the lesson. On the other hand, she highlighted that students still need to be proficient with mathematical procedures using pencil and paper for understanding mathematics. Mary perceives herself as a learner, while she is integrating TT in her teaching.

She represented a total of six themes in this dimension and those themes were labeled: Useful, Visual enhancer, Convenience, Merging, Challenging, and Inspiring.

Useful. Mary represented this theme when summarizing the usefulness of TT in her teaching, not only for herself but also for students. The usefulness of the tablet in Mary’s teaching was expressed in regards to the ability to present the content of the class using notes and graphics simultaneously; sketching, highlighting, reviewing, accessing the content of previous lessons; and being able to direct the students’ attention to the knowledge that is being constructed.

Some of the quotes that supported this theme were:

I think, I was more able with the lessons being on the computer to go back to some things, and point to, and say... ‘Remember when we did this’ and it is right there, I don’t have to re-draw the whole thing, it is a little more efficient for me to get to (MII, May 1, 2013).

I think it is a cleaner lesson now. I am not standing in front of the notes. The notes are nice and clear to everybody wherever they are sitting. So, the convenience of that is extremely helpful. The diagrams the book provides, the founding instruction that I can use and build on, and I like that (MII, June 21, 2013).

Where as I'm restricted, if I'm just showing one person the calculator, that's not as good as having something that's simple as a screen and everybody can see it at the same time. It does make the flow of the lesson go more easily (MII, May 1, 2013).

“Yes, the tablet was again useful to [the] lesson. Students could easily write down notes. We were able to focus on what background knowledge was missing.” (MJR, May 23, 2013, Grade 11).

I also was able to call up a previous lesson to tie in previous material with what we were doing today. It also helped students that were absent from class as I was able to post lesson to the Moodle site that evening for students that were absent from class. One student was ill and the other was in Hawaii. I checked the participants on Moodle this morning and both had checked the notes (MJR, May 23, 2013, Grade 12).

Visual enhancer. This attitude involved the visual impact that Mary provides in teaching her students when she was interacting with the content of the lesson during and after lessons. Mary supported her attitude that the use of TT serves as a visual enhancer during lessons because they relay the content of the lesson in an easy and dynamic way. She presented mathematical objects with several colours; focusing on relevant parts of the content needed in teaching, making graphical representations, taking notes at any part of the lesson, highlighting important notes and emphasizing visual points of mathematical concepts and procedures. Students were able to visualize this content not only during the class, but were also able to access it after class.

“I look usually for visuals...that's where I've used it most... I highlight anything that they need to know, as far as notes, then we'll go through any visual” (MII, May 1, 2013).

“The visual has played a huge part, . . . the steps have always been there, and I have to explain them...I find a better visual is necessary for a lot of kids” (MII, May 1, 2013).

Very much try to make sure the visual - we do the algebra, what do we have to change about the formula - look at the diagram, and trying get them all to think of the two together, and to help them, and see what they are actually calculating (MFI, June 21, 2013).

“Visuals are extremely helpful, but you still need to give time to the students to absorb the information. It helps to be as obvious as possible” (MJR, May 2, 2013, Grade 12).

“I used the tablet and, to a smaller degree, the graphing calculator to support the lesson. The lesson, examples, and notes were all on the tablet. It enhanced the visual understanding needed for the lesson” (MJR, May 7, and May 9, 2013, Grade 12).

Convenience. Mary considered that TT used in her teaching were easy to manipulate and facilitated the communication of concepts in the lesson. She also indicated that the use of different TT helped her to use her time in a more efficient way by allowing more time for exploring students’ questions as well as connecting concepts that were already reviewed with new ones.

“The convenience to me was that I could make up a worksheet - test it out and then, polish it as every class went through. . . ” (MII, May 1, 2013).

The conversations that happen - usually are just at the first start of the class - where I ask for questions and then I can write it right in their space, as well as on the workbook. It is very nice to have that question right there, on the board, I don’t have to type it up, it’s there. It’s very time saving (MII, May 1, 2013).

“[I] Chose the tablet as it is convenient and it frees up some of my time in the lesson to spending more time on exploring questions” (MJR, May 14, 2013, grade 11).

“I believe that the tablet is a convenient tool to help explain the concept of splitting the areas by making the graphing so visual” (MJR, May 7 and May 9, 2013, grade 12).

“I choose the tablet for most of my lessons. I am familiar with its process and I like how I can download the worksheet onto the tablet and use it throughout the lesson. It’s very convenient” (MJR, May 2, 2013, grade 12).

“Convenience. I like how the notes are exactly what it is in the book. (I can add more examples if needed and I can change the entire lesson if I disagree with the lesson in the book)” (MJR, May 14, 2013, grade 11).

Merging. Mary combined both the use of traditional technology and the use of new technology in her teaching practice. Mary used traditional technology including the board, paper and pencil, and also, presents and develops the topic using the tablet, calculators, one notes program, and worksheets. She indicated the advantage of combining old and new technology during the teaching process in regards to pedagogical strategies: sharing some static information on the blackboard (for reference), having a ready copy of class notes available both in class (Tablet) and out (Moodle), and integrating TT with the use of some traditional procedures in the teaching process (e.g., manual sketching of graphs and comparing those results found on graphing calculators). Mary perceives herself as a learner, while she is integrating TT in her teaching.

“I still use the board now and then, especially when I’m teaching and I need some notes to refer back to constantly . . . [The] students need to see the steps up there” (MII, May 1, 2013).

If I can, I'll tell them how the calculator can help them. So, I would do one we would do with the lesson, we would do the long way. And then, I would plot the graphing calculators and say 'look what this can do for us (MII, May 1, 2013).

"I think that's what [the graphing calculators] do for me right now, this particular lesson, the students still need to do their own pencil and paper drawings" (MFI, June 21, 2013).

Used the graphing calculator to see the visual of points of intersection. Then went through the algebraic method to see how it agreed with the visual. Again, technology was there to enhance the concept. Prior knowledge of graphing calculators was an advantage (MFI, June 21, 2013).

I had them calculate the definite integral and then compare their answer to the graph of the curve given within the boundaries given. Then we altered our previous process, so that areas would not cancel each other out. Students learned as much about the technological tool as the mathematics. The table was simply there to enhance the lesson (MJR, May 2, 2013, grade 12).

Challenging. Mary expressed in this theme difficulties of managing her teaching when she was integrating new TT. The difficulties were assumed in regards to the proper use of TT (specifically, graphing calculators) and what might be the best way to introduce them in her lessons. On one hand, Mary recognized that graphing calculators supported her teaching with graphical representations needed; and on the other hand, she found the use of graphing calculators encumbers teaching in regards to development of manual basic skills for graphing. She indicated that using graphing calculators for guiding students' understanding of basic skills (graphing and interpretation of results) in her teaching was a challenge. According to her, graphing calculators display results or solutions to a mathematical problem, and students may not be prepared to understand what they are doing for them. Another issue that she raised was the awareness of being prepared for dealing with technical issues that may arise in her lessons that might limit

her teaching. These issues included students' skills for graphing with calculators, and also the availability of resources for students to manipulate and practice, during and after lessons, different versions of calculators and the class Moodle.

The expense of the graphing calculators makes it hard for me to insist that all students have one. If not all students can practice at home with the graphing calculator, and become fluent in their use, then you are constantly interrupting your lesson to help them (MFI, June 21, 2013).

“The kids still need basic skills, they still need paper and pencil as far as I am concerned for math” (MII, May 1, 2013)”.

I need the students to do the procedure. I don't want the computer to do the procedure. However, I know in real life, maybe you need to see all the steps in front of you. But when they [students] are learning it, I want them to come up with the steps (MII, May 1, 2013).

“Sketches were very helpful and quick with the use of the graphing calculators, but I do wonder if the students fully understand that the ordered pairs were making up the pattern of the curve or line” (MJR, May 14, 2013, Grade 11).

“The graphing calculator computed the points of intersection, but I am not sure that students knew what they are finding. I like the visual, but I prefer the algebraic method of finding the intersection points” (MJR, May 16, 2013, Grade 11).

It's a mixed bag. There are times when [graphing calculators] help, and especially, if they own the calculators themselves and can practice, and they are fluent when they are in classroom. Other times I do find them a little bit distracting as it takes up class time to show them over and over again how to use it (MFI, June 21, 2013).

Inspiring. Mary has a positive attitude for continuing using the tablet computer and for improving the integration of TT in her teaching practices. She considered that advantages of TT generally (Tablet computer, graphing calculators, Moodle class site) made the lesson more interesting and beneficial not only for students but for herself. Mary pointed out that she is still learning how to integrate technological tools in her teaching, and she devotes time to her own learning. She argued that students are living in a technological context, and that this familiarity encourages them to participate more in discussions about the topic. In particular, students started to inquire about the mathematical topic in her classes by handling the TT (graphing calculators).

“I always feel not as up to date as I could be. I look for my pro-D activities to help me out, and find out what is new out there for me. I am a learner at all times”. (MII, May 1, 2013).

It [using graphing calculators] does make the flow of the lesson go more easily and students when they are frustrated with the hard topic, that part of the lesson it’s now smoother. It helps with the overall tone as long as there’s no glitches (MII, May 1, 2013).

“The Moddle site - when it works, it’s wonderful. I think it is very helpful for kids to have that option” (MFI, June 21, 2013).

I always find that I’m learning something new. I would like more time. One of the things I enjoy about volunteering for this study is that it made me take a closer look at my technical use, it’s very easy to get busy as you saw this morning. I’m running and you take things for granted after a while (MII, May 1, 2013).

Dimension 2 – Mary’s Knowledge of Technology. Mary reveals her KT in her use of the TT and also in the ways in which she repurposes them in her teaching. She has been using a tablet computer for seven years (MII, May 1, 2013) and the tablet was the main TT used during her teaching practices (grade 11 and 12). Other TT included, a

computer (desktop or laptop), the OneNote program, a class Moodle, a data projector, graphing calculators, a traditional white board, textbook (copy and electronic), and worksheets (copy and electronic) (Figure 13). Mary showed a good understanding of the technical aspects involved in managing TT and this was further corroborated using direct observations in classes and from the video-recordings of classes.

She explained her technical use of TT as follows:

The computer and tablet are both in the classroom. That's how the lessons appear on the board with 'OneNotes' . . . Many worksheets that I need, I put on it and then write on top of that. Then, I post those to Moodle for the students . . . the workbook also comes with the PDF format, and I put them up on the board. I also used it for researching. I type up all my worksheets on it as well (MII, May 1, 2013).

Mary's claimed difficulties in working with the Tablet itself were not observed during her lessons. She wrote, highlighted, graphed, used color to distinguish elements, and added notes with a mastery of knowing the features of the tablet and the programs that she used (OneNote, e-book, graphing calculator, and adding worksheets).

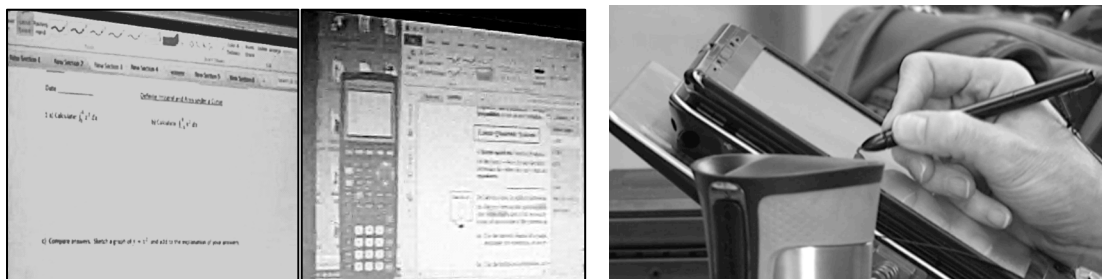


Figure 13. Technological tools used in Mary's teaching (MVO, May, 2013)

During Mary's teaching practices there were no major problems or technical issues except during one class in which she had a "minor glitch with the computer, but it worked itself out" (MJR, May 7 and 9, 2013). The equipment was set up in the classroom and it took a few minutes to turn on the tablet and projector. Most of the time Mary used

the tablet during her teaching practices, but sometimes she turned off the tablet when she considered it to be unnecessary.

Themes identified with regards to Mary's knowledge of technology were: Affordances, Selection, Constraints, Integration, and Interactions.

Affordances. Mary implicitly illustrated this theme when she mentioned the benefits of using TT in her teaching and in her use of the tablet in her teaching. Although the tablet computer was not specifically designed to support teaching, Mary discovered through her own efforts and through pro-D activities, the advantages of repurposing this TT to support teaching.

She started to explore the tablet and to acquire a deeper understanding of the features of this tool in order to communicate the content of the lesson. According to Mary, some of the benefits derived from using the tablet in her teaching included: the availability of having the textbook as e-copy allowed her to adjust the lesson at her convenience; the benefit of "being able to write on the tablet and put the lessons up there, it is invaluable . . ." (MFI, June 21, 2013; MII, May 1, 2013); the visual impact of the content of the topic "I think it is a cleaner lesson now. I am not standing in front of notes, the notes are nice and clear to everybody wherever they are sitting" (MFI, June 21, 2013); and the graphic representations: "the diagrams are far more accurate than what I can draw on the board, quickly, efficiently" (MII, May 1, 2013).

The use of different TT by Mary during her teaching increased the richness of her teaching practice. Thus, Mary's KT was complemented when she used the other TT in concert with the tablet (OneNote program, e-Textbook, paper and electronic worksheets, graphing calculators, Moodle website).

The OneNote program was used to insert notes onto lessons and worksheets that Mary made by herself on the tablet. It allowed her to organize and to plan the lesson at her convenience.

Mary recognized the advantage of having an eTextbook instead of having a hard copy of the textbook as she used previously in her traditional classes. The eTextbook is provided as a collection of PDF files, which she had already downloaded into the OneNote program allowing her to modify the content of the book. She had control of the

eTextbook, being able to make changes by making spaces to add notes, diagrams, graphs, colours, highlight words, shade graphs, create notes during the class, clarify questions with new examples, and to propose new exercises or assignments. During teaching practices it was evident that Mary did not have any technical issues working with the tablet and adjusting notes in the eTextbook.

Mary was able to save all the notes and comments made during the lessons, and made those resources available to students by posting the annotated lesson on the Moodle website (MJR, 2013, grade 11 and 12). The Moodle website was used to interact with students after class and to follow-up on students' work. Also, she interacted with students through e-mail when students needed to ask questions or to clarify assignments (MII, May 1, and MFI, June 21, 2013). The use of the Moodle website was not observed during Mary's teaching practices; it was identified from Mary's interviews (initial and final) and journal of reflections.

The main TT of study in Mary's case was the tablet working with the OneNote program, eTextbook and worksheets. According to her, other benefits derived from using those TT were: time savings (efficient presentation of prepared material), convenience of use (writing on tablet) and convenience of her location in the classroom for observing and listening to students during the lessons.

It's very nice that the workbook is a PDF, and I just have to call it up. I have to make sure I erase the notes from the other class, but at the same time, I've got to make sure that I don't erase them, before I put them on Moodle. I have to make sure my worksheets for the classes that aren't in a workbook...that I post them up, and give them to myself, so I have them up there . . . I've got to be prepared to be on the board, whenever it breaks down. I have to make sure my calculators are in the back and have backup batteries (MII, May 1, 2013).

I usually use the tablet to deliver the lesson. It is then easy to save the lesson to PDF format and save onto the class Moodle website, so students may check the notes. As well, the answer key with steps can be posted (MJR, May 7, 2013, grade 12).

“I downloaded the worksheet onto OneNote, and then photocopied it for students. I then projected it on the screen and used the OneNote program to enhance the lesson” (MJR, May 2, 2013, grade 12).

Finally, the graphing calculators were eventually used in the class to provide graphical representations of mathematical functions when Mary considered it was necessary in her teaching. She identified difficulties dealing with different versions of calculators that some students brought to the class, but no technical problems working with calculators were observed (MPO, May, June, 2013).

I didn't mention that calculators are used as well. They sometimes get in our way, so you have to really pick and choose when you are going to use them. They take more time than people realize to teach the different ones (MII, May 1, 2013).

[I] used the tablet to deliver the notes and the graphing calculator to sketch two functions and find their point of intersection. Yes, it is useful to have the tablet to teach this lesson. The tablet gives the students and myself the convenience of the notes being the same as in their workbooks and on the board. The tablet also allows me to highlight key notes in different colours and that helps students focus on the big ideas. I had the students use graphing calculators today to give us a visual on what we were finding (MJR, May 14, 2013, grade 11).

Selection. This theme illustrated the reasons for Mary choosing TT in her teaching. Mary taught the topics ‘Definitive integral and area under a curve’ (Grade 12) and ‘Linear and quadratic systems’ (Grade 11). The reasons for choosing TT included the visual impact as she presented the content of the lesson, the possibilities of revisiting the content of any previous lesson at any time, and also the opportunity to share graphical representations of functions during lessons. The Pro-D day activities and blogs helped Mary to reflect on, and make decisions about, what TT would be useful in her teaching.

I look usually for visuals, that's where I've used it most . . . I've explored what people mentioned, blogs, and things they have tried in the classroom, where they get really excited. So, I'll go read about it, and usually it's a little more tempered by the time I go [to a pro-D presentation] (MII, May 1, 2013).

[I] Chose the graphing calculator as it was a quick way to provide a visual for the lesson. [I] Chose the tablet as it is convenient and it frees up some of my time in the lesson to spend more time exploring questions (MJR, May 14, 2013, grade 11).

“Only with Math Twelve, do I really need [graphing calculators], when they're graphing the solutions to trigonometry, otherwise I use it to enhance the lesson . . . and I do like the graphing calculators for that reason. They make some things go a little bit quicker . . . we can see patterns” (MFI, June 21, 2013).

Constraints. This theme represented Mary's difficulties in using TT during her teaching. The main constraints that Mary mentioned were glitches or technical issues with the TT itself, and also, the time consumed in her teaching if students do not have the technical skills for working with TT provided. Specifically, she was concerned with graphing calculators because of the lack of students' previous knowledge with respect to manipulating these devices during her teaching and also about her technical knowledge working with different versions of graphing calculators.

Additional aspects that Mary indicated as limiting her teaching with regard to the use of TT (tablet, graphing calculators) included: administrative support from the school district (regarding the economic aspects of repairing or buying TT) and students' access to graphing calculators and/or Internet at home (for example, lack of knowledge of the TT, and restrictions with respect to their ability to follow-up on the Moodle website).

“The technical aspect of having computers in the classroom... it still hasn't been worked out to everybody's satisfaction. We will lose the wireless at any point for no reason” (MII, May 1, 2013).

Also, I use the graphing calculators in class. They have been harder to use as not all students own one and we have made policy where grade twelve (12s) now have to buy their own, or rent from us and it's not as strict as it could be there. And, again we have different versions of the calculator that goof us up every once in a while (MII, May 1, 2013).

The expense of the graphing calculators make it hard for me to insist that all students have one. If all students can't practice at home with the graphing calculator, and become fluent in their use, then you are constantly interrupting your lesson to help them. The lesson slows down. To speed things up, I try to get fellow students to help each other out. It has varying degrees of success (MJR, May 16, 2013, grade 11).

Mary made explicit an example of the difficulty of replacing her tablet when it broke. She had to use another tablet, which was very problematic, difficult to adjust to, and inefficient compared to the previous one.

"I find that tablets I've had over the years can break. Once, it took four (4) months to fix it and I was on a replacement tablet that didn't write in the middle. It was a horrible tablet" (MII, May 1, 2013).

The Moodle site caused problems. I would say four, five students continue (with problems), . . . other students suggest ideas as well, . . . we write them up on the board and they try them, . . . I don't know anything else to do. I give them over to the administrator to try and find out what the problem is with passwords. We seem to get it corrected and then it falls apart again. So, I am going to probably get another computer to . . . in here for students use, and I'm hoping at that time I can get them to connect and I can watch them, and see what is happening (MFI, June 21, 2013).

Integration. This theme is displayed by the way Mary combined new TT (tablet with OneNote program, eTextbook, worksheet (paper and electronic), class Moodle website and graphing calculators) in her teaching with the curricular content of the mathematical topic. The mathematical curriculum is integrated in a technological context through the eTextbook and worksheets. This integration was implicit in her planning when she organized lesson files, worksheets and the eTextbook at her convenience. During Mary's teaching, it was explicit that the integration of the mathematics content into a technological context took place because students (with the hard copy of the textbook, and the graphing calculator) and Mary (with the tablet, e-textbook and graphing calculator) were interacting at the same time while developing an understanding of the curricular content of the topic together (MPO, May and June, 2013). Because of the integration of those TT, students and teacher had the content of the lesson and notes available during classes.

In regards to graphing calculators, they were integrated occasionally when Mary considered it necessary for introducing lessons, graphing polynomial functions, and verifying sketching of functions (MFI, June 21, 2013 and MPO, May, June, 2013). The teacher taught the technical use of graphing calculators at the beginning of the year and it was reviewed during her lessons (MII, May 1, 2013).

We would actually expand the first couple of classes in the beginning of the year going over things that the calculator can do for you, and which buttons get you there, what the menus are, what the editing tools looks like. Some things that will be really useful to them (MII, May 1, 2013).

The worksheet that students were working on was the same one as on the screen. I was able to sketch the curves needed with the same conditions (space limits) on my drawing as the students. I can then focus attention with highlighters . . . the details central to the lesson (MJR, May 2, 2013, grade 12).

Interactions. This theme represents the form in which Mary and students interacted with TT. The teacher was the main agent who interacted explicitly with the

technological tool (tablet). She used the tablet all the time in the classroom. However, students and teacher interacted together to develop the content of the topic (students interacted by working in their textbook at the same time as Mary did with the eTextbook).

Graphing calculators (1:1) were manipulated by the students and Mary at the same time. She had available the graphing calculator within the tablet and she interacted directly on it. Students interacted with graphing calculators (their own or device provided by the teacher) and they were required to know how to use calculators. However, Mary guided students' interactions with graphing calculators and helped them to use those properly because not all students have their own calculator and thus cannot practice at home.

Also, interactions between two or more students who partnered to work together supported each other. This helped to solve the difficulties involved in knowing how to graph with calculators. Sometimes, interactions were produced between Mary and students when she asked for help solving technical issues created from students' working with different models of graphing calculators. "I know that different models of the calculators can trip us up sometimes as well, and then, I start looking for help within the classroom, often through another student, which can give me an idea where I can go next" (MFI, June 21, 2013).

The class Moodle was used by the teacher and students. Both of them interacted with the class Moodle to check lesson notes, to post assignments, and by students to review class notes that they had missed (students out of town, in other provinces and also in other countries). "Since I can put it on Moodle, it gives me the opportunity to interact with students all over the place right now. I have one boy in Hawaii in that class and I have a number coming back from a field trip to New York. It is so much simpler to have the convenience of putting it up on the Moodle" (MII, May 1, 2013).

Furthermore, students interacted with printed worksheets that Mary provided. The interactions with the worksheet occurred either individually or in groups, as well as with Mary as a teacher (developing the topic with an electronic worksheet on OneNote on the tablet). Interactions were observed during Mary's lesson and these observations were complemented with the video recordings of classes.

“I will have the worksheet up there and we will be talking about it. Then I will give them the actual answers to make sure they are on the right track . . .” (MII, May 1, 2013).

“I had one student in Hawaii. I can actually track if this student has signed on or not. So, I noticed that he was in Hawaii, and he was. He was getting the notes from the lesson which is wonderful . . .” (MFI, June 21, 2013).

Dimension 3 – Mary’s Knowledge of Learners’ cognition. The KLC was exemplified in Mary’s teaching in two ways. First, she assumed that students had basic knowledge in regards to the mathematics topic and the use of TT. Second, it was her own use of the TT during teaching practices that allowed her to understand the students’ learning process.

Mary assumed that students in grade eleven and twelve possessed basic skills concerning graphing functions using traditional technology (pencil and paper) as well as the ability to use graphing calculators. As a teacher of grade eleven ‘Linear and quadratic systems’ (pre-calculus), she expected that students would have previous knowledge to support new learning e.g., “[I] looked for signs that they were able to predict what the graph should look like and how quickly they accepted that they were finding an ordered pair and not just x” (MII, May 2, 2013). Similarly, in grade twelve where she taught “Definitive integral and area under a curve” (Calculus), Mary assumed that students knew the graphical representation or sketching of “how a polynomial looks like” (MII, May 2, 2013). In both cases, she assumed that students already possessed previous knowledge of how to sketch graphics and how to use graphing calculators.

During classes Mary identified students’ understandings and difficulties with the topic (grade eleven and twelve) by reviewing mathematical concepts and procedures of previous classes, and by questioning them on the mathematical content of each lesson. She highlighted that the use of TT (tablet using OneNote, eTextbook, worksheets) in her class allowed her to be in position to hear and participate in students’ conversations, and

that this was crucial for understanding students' difficulties in learning the mathematics topic being covered in a given lesson.

“[The tablet, OneNote, eTextbook, worksheet] allowed me time to stand back and listen to discussions between students . . . I am able to sit among the students to see and hear what they are discussing” (MJR, May 24, 2013, grade 11).

“Usually by listening to [the students] conversations, I find out their difficulties” (MFI, June 21, 2013).

“I don't have my back to the students while I'm doing that. Although they have their backs to me, I'm actually at the back of the room, so I can see what their heads are doing” (MII, May 1, 2013).

TT were also useful in guiding students' thinking about the topic and helping them with the visual aspect of the content, specifically, “the visual, the color, the highlighting, the ability to erase it and start over again” (MFI, June 21, 2013). Finally, the use of TT permitted both her and her students to review previous lessons, exercises, comments and notes. The use of TT supported more visual explanations and examples, and helped her listen to students' questions and comments.

The following themes summarized Marys' KLC: Review, Validation, Deficiency, and Assistance of TT.

Review. Mary illustrated this theme by reviewing the previous content of the lesson at the beginning of the lesson, by asking questions or solving problems that students had with the assignment, or during the lesson when students asked questions about the topic that required knowledge of previous examples or exercises. Previous knowledge of basic skills concerning manual graphing of functions and familiarity with graphing calculators was critical for Mary's teaching. The use of 'pencil and paper' to sketch mathematical functions was considered necessary to help students identify potential mistakes from graphing calculators and to analyze graphical functions required

for understanding the topic. Electronic worksheets facilitated Mary's reviewing previous content because assignments, examples, and notes were saved on the tablet and accessed at any time during the lesson.

"Notes are right before their homework, so it makes it easier to review as well"
(MFI, June 21, 2013).

I also was able to call up a previous lesson to tie in previous material with what we were doing today. It also helped students that were absent from class as I was able to post the lesson to the Moodle site that evening for students . . . (MJR, May 7 and 9, 2013, grade 12).

Although my assumption of graphing knowledge may have been optimistic, their lack of basic graphing skills was frustrating. I tried to trigger their memories of previous lessons, but it was weak. I may put in an introductory unit next September aimed at strengthening skills needed from previous grades (MJR, May 16, 2013, grade 11).

Validation. In this theme, Mary illustrated different ways in which she verified students' understanding or misunderstanding of the topic content. She stressed that in teaching calculus each lesson is building on the previous one (MII, May 1, 2013). She demonstrated the ability to access students' preconceptions and the understandings that they bring to a learning situation by questioning them.

Mary used an example where students had to find the area under the curve of a function. The students were asked to work with algebraic procedures before developing a graphical representation (sketch) and they found that the result of the area under the curve was zero (Figure 14).

$$3a) \text{ Calculate: } \int_{-2}^2 x^3 dx = \left. \frac{x^4}{4} \right|_{-2}^2 = \frac{1}{4} (16 - 16) = 0 !?!$$

Figure 14. Example of Mary's validating students' understanding (algebraic representation) (MVO, May, 2013, grade 12).

Then, she engaged the students in a discussion about this result: '*What is wrong with it*'. Students started to participate and to formulate different verbal explanations of the situation. Next, she proceeded to graphically represent the function (Figure 15).

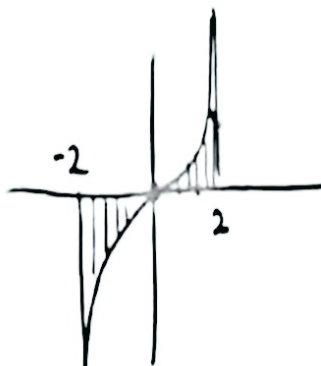


Figure 15. Example of Mary's validating students' understanding (graphical representation) (MVO, May, 2013, grade 12).

Once she finished the sketch, she started to ask questions again: '*What's happening?*' '*Why is the answer zero?*'. '*Something is wrong in the formula we are using*', '*How do we get the right answer?*' (MVO, May 2, 2013). Mary listened to students' responses and they started to analyze the graphic and to realize that the areas were positive and they were the same. This was very visible and explicit in the students' responses because of the help of observing the graphic representation of the function and teaching guidance. In this way, she continued asking questions to stimulate more elaborate responses from students that clarified the procedural result of the area. She used

expressions such as, *‘What else do we need?’ ‘Any other suggestions?’* (MVO, May 2, 2013). Mary listened carefully to students’ responses and exchanged ideas with them. She always encouraged them to continue the discussion leading to a conceptual understanding of the topic, and she used expressions such as *‘Sure that sounds good’, ‘But is there anything else that we need?’ ‘Ok, that makes sense’*. Thus, students built their own concept of symmetrical function. Finally, students and teacher answer the question *‘How can we change our calculations so that we get the actual area between the x-axis and the curve?’* They proceeded to write two approaches to finding the correct area:

- *Take the area from 0 to 2 and then double it and*
- *Multiply the negative area by -1 and then add it.*

In this sense, students made their own calculations and they found the correct procedure to find the area under symmetric curves.

She expressed her teaching goals directly to the class: *“...this is more a discussion class today”* (MVO, May 2, 2013), *“This is what is important here to do, the sketch”* (MVO, May 2, 2013), *“What else do we need to know to graph those?”* (MVO, May 7, 2013), and *“How do you approach the graphic?”* (MVO, May 7, 2013).

Mary stressed the need to identify the students’ understanding from previous lessons, because the knowledge is constructed consecutively, she points out:

Well, calculus is the only course that is so smooth. The last topic we did gives me the information about what I can build on. What I still need to give is more information or give more examples, or try to find out how we can fill up any gaps . . . (MII, May 1, 2013).

“I assume some things and when I put it up there, they don’t have that previous knowledge. That helps me see where they are and helps me adjust my lesson” (MFI, June 21, 2013).

“I want to access their prior knowledge, I want them to be thinking about what they already know, . . . and we are just building on to it” (MFI, June 21, 2013).

She recognized that students' understanding of previous topics is more visible in algebraic representations "If I say, linear graph, I get a lot of blank faces, but if I say $y = mx + b$, suddenly the lights come on" (MII, May 1, 2013). Mary is aware of the importance of knowing students' previous knowledge from previous grades before students can move forward in learning the topics. "I need to consider the background of the students and the previous knowledge that they have learned in their other classes. What has been enhanced and what is lacking?" (MJR, May 7 and 9, 2013, grade 12).

Deficiency. She identified students' difficulties primarily through students' questions, and also by the answers they provided to her questions. How students responded to questions supported in adjusting her teaching: "Again the quality of their questions helped me adjust my lesson" (MFI, June 21, 2013).

Mary mentioned explicitly some learning situations such as, listening to students' comments, answering questions and checking homework in her teaching practice. These learning situations demonstrated the way Mary identifies students' difficulties and her actions during her teaching practices.

Usually by listening to their conversations I find out where their difficulties are - as far as the lessons go, the conversations are what I'm trying to trigger, and that's where I find out what help do I need to give them . . . (MFI, June 21, 2013).

"I'm listening to their conversations to find out if they ask me questions, but they often ask each other. I'll be paying attention to that, trying to find out the difficulty level" (MFI, June 21, 2013).

Mary guided students' conversations during the lesson and explored important concepts of the topic in those conversations: "I listen to comments and answers made by students. My table where I sit when delivering the lesson is right in the middle of the students. I try to tweak/guide their conversations to consider the important concepts of the lesson" (MJR, May 1, 2013).

Assistance of technological tools. This theme represented the benefit of using the TT to gain access to students' understanding of the topics. According to Mary, the TT (tablet with the OneNote, worksheets, graphing calculators) supported her teaching by facilitating her ability to listen to the students' questions and providing more examples. Mary was "able to sit among the students to see and hear what they are discussing" (MJR, May 23, 2013, grade 11). The use of the tablet would be an enhancer because it was a proficient tool to present the content of the lessons when it was needed. Also, the use of graphing calculators supported Mary's teaching by providing a visual understanding needed for the lessons.

The technology can hopefully be an enhancer. I would have most of my drawings free hand, and I want them to relax about that. I need the shape and I need where it crosses the 'x' axis, the 'y' axis. I need the important part of the aspects. So, when I quickly give them this after they try the graph I can check to see that ... The sketch is not going to be on a grid, but I want the 'x' intercepts, the 'y' intercepts, and where the two graphs cross each other eventually. I need those pin points in. So the technology is a quick way to place things on the board and to check that we are on the same page (MII, May 1, 2013).

The use of TT helped Mary by allowing more time to pay attention to students.

"It allows me time to walk around the class to see how quickly students are doing their work. I also sit in the middle of class where I can hear quite a few student conversations" (MJR, May 2, 2013, grade 12).

"By making the concept more visual, I had more time to let the students work during class time. This gave me more of an opportunity to observe their work" (MJR, May 1, 2013, grade 11).

Dimension 4 – Mary's Knowledge of Subject matter. Mary represented her knowledge of the subject matter when she developed the mathematical content,

identifying connections between mathematical procedures and concepts related to the topics, and also when she provided explanations, examples and questions for students in order for them to gain mathematical understanding.

Mary used the tablet to present the mathematical content of ‘Linear and quadratic systems’ in grade eleven and ‘Definitive integral and areas under a curve’ in grade twelve. She indicated that she might teach any mathematical topic in grade eleven and twelve by using the tablet because it supports integrating the mathematics curriculum, and interacting with any class content at any time during the lessons. Mary used the same TT (tablet, OneNote, worksheets, graphing calculators, and Moodle website) for teaching in both grades. The use of TT supported her teaching in an efficient and easy way through presenting mathematical procedures, illustrating visual explanations, graphing more examples, and highlighting relevant details of graphing which were essential for understanding the mathematical concepts. In particular, Mary was concerned that students focused their attention to gain conceptual understanding of the topic and not only applying procedures without understanding.

Ahh... It is just making sure that the students know what we’re doing, that they are not just memorizing steps, that they really fully understand . . . that we are taking the area under the curve, and we are doing it by the height of infinite amount of rectangles times the width . . . and we are using definite interlocks to make that a shortcut (MII, May 1, 2013, grade 12).

“I do wonder if the students fully understand that the ordered pair was making up the pattern of the curve or line” (MJR, May 14, 2013, grade 11).

Themes identified in Mary’s KSM included: Visual connections, Disconnection, Technology aid, Mathematical thinking, and Representations.

Visual connections. This theme represented the way Mary linked the mathematical content using the TT to support students’ understanding. She presented the mathematical content of the lessons focusing the goals for learning within a technological

context. The use of TT supported visualization of concepts and procedures and facilitated the making of mathematical connections (Figure 16). Mary highlighted that having the opportunity to review previous notes, and graphs, during her lessons, supported her students in remembering the content of previous lessons and it helped Mary to situate the mathematical understanding needed for making connections with new mathematical content.

“To make more connections, so that they would remember it . . . the visual, the enhancement of it . . .” (MII, May 1, 2013).

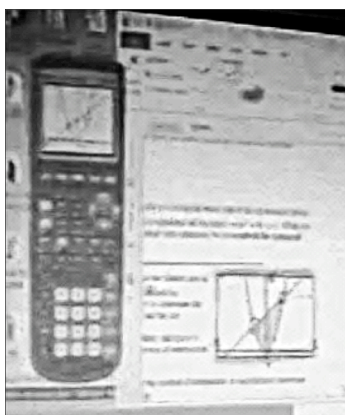


Figure 16. Mary simultaneously connecting two graphical representations, using the graphing calculator and the example of the eTextbook (MVO, May 2, grade 12).

She presented the procedural thinking (writing steps in OneNote) connecting with conceptual thinking (sketching functions and approaching intersections with the graphing calculator) (Figure 17). She used features of the tablet (e.g., colours, shading, sketches, annotations and a graphing calculator, etc.) to illustrate connections between the content of the lesson and the visual representation of it.

“So... they won’t just see steps, they also see a picture of what they are doing”.
(MII, May 1, 2013).

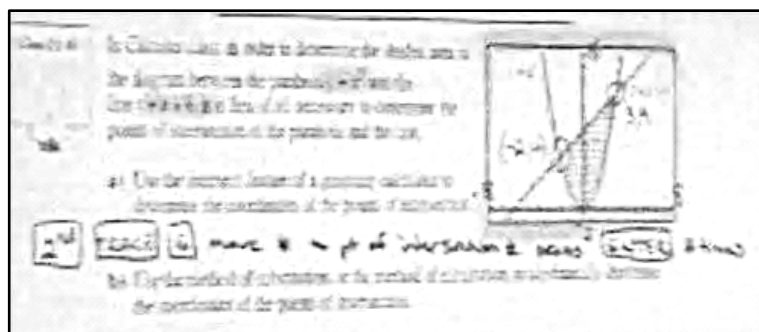


Figure 17. Mary connected graphical representations and mathematical procedures simultaneously working on the eTextbook (procedural and mathematical thinking) (MVO, May 14, grade 11).

In the following example: Mary compared the graphic presented in the eTextbook and the graphic that students and Mary drew with the use of the graphing calculator (Figure 18). Mary asked:

“Any reason why my calculator screen is different than the one on the book?”

“Could you tell me your point of intersection?”

“What happened with the graph?”

“Everybody agrees with that?”

(MVO, May 14, 2013, grade 11)

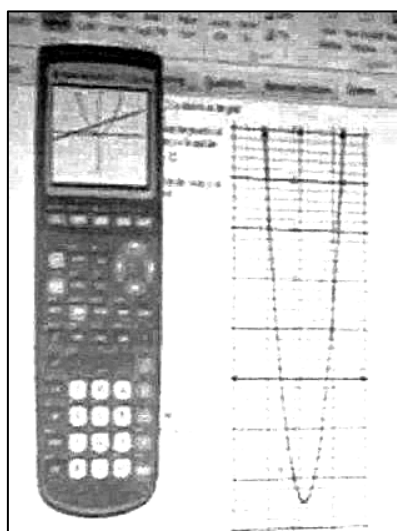


Figure 18. Mary compared and discussed simultaneously two graphical representations (graphing calculator and example from the eTextbook) from the same mathematical function (MVO, grade 11).

In this example, the eTextbook and the graphing calculator supported Mary's teaching to compare two graphical representations at the same time, and to identify the same graphic at a different scale (Figure 18). Mary guided understanding the results of the points of intersections with the graphing calculator because the scale values on the screen were not rounded to match the results presented in the eTextbook which were integers.

“Hopefully, the resources help install a connection between the visual and the algebra we do to find the points of intersections” (MFI, June 21, 2013, grade 11).

Used the graphing calculator to see the visual of points of intersection. Then went through the algebraic method to see how it agreed with the visual. Again, technology was there to enhance the concept. Prior knowledge of graphing calculators was an advantage (MFI, June 21, 2013, grade 11).

Mary expects that students might recognize different mathematical representations of the topics in the future because of the help of TT used (manual sketching for solving problems, and algebraic procedures) (figures 19 and 20).

And hopefully, that would help them next year at the university. They would have that visual to tie in with. The more connections I can make, even the singing with the songs, it is just ways of getting them to tap into more of what we were doing (MII, May 1, 2013).

I expected students to see the connection between the location of the boundary curves and a negative or positive area.

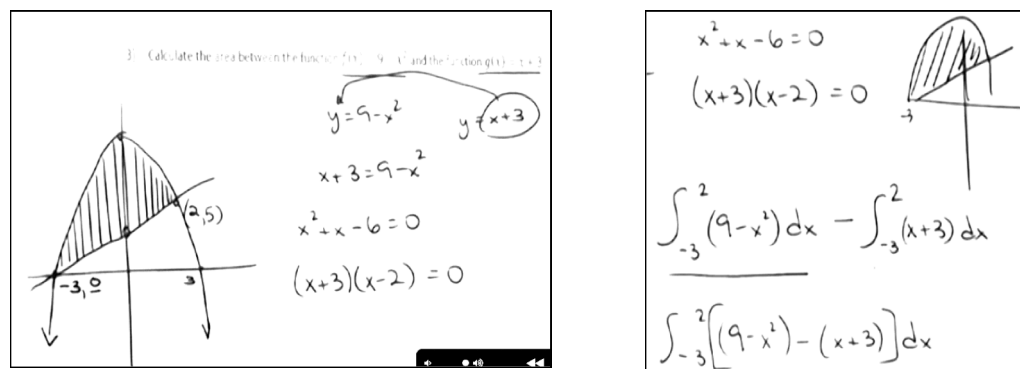


Figure 19. Mary solving a problem using visual connections through sketching, and connecting with mathematical procedures (MVO, May 7, grade 12).

I also wanted to see if they could not only see what we had to adjust in our method, but why. I was hoping they would see that our way of finding the height meant we had to consider which function was on top. I think they saw the negative value and the area below the x axis clearly enough. The ‘why’ may come with further lessons (MJR, May 2, 2013, grade 12).

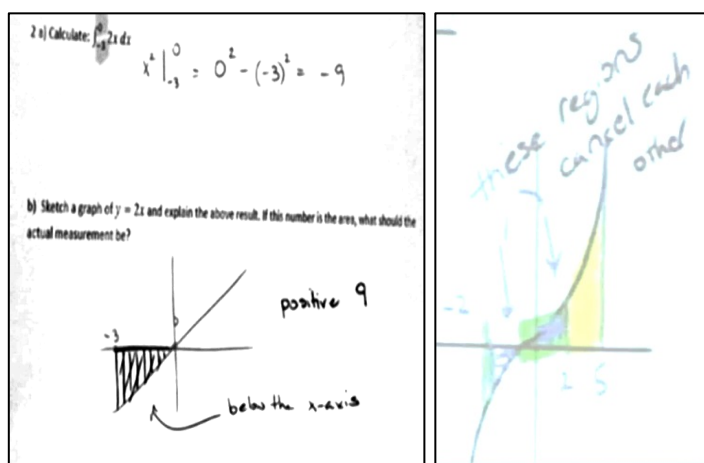


Figure 20. Solving problems of symmetrical functions using sketching (MVO, May 2, grade 12).

I had them calculate the definite integral and then compare their answer to the graph of the curve within the boundaries given. Then we altered our previous

process so that areas would not cancel each other out. Students learned as much about the technological tool as the mathematics. The tablet was simply there to enhance the lesson (MJR, May 2, 2013, grade 12).

Disconnection. Mary showed in this theme that the major difficulty in teaching the mathematics topics was that students struggled to make connections between conceptual and procedural understandings. She highlighted the difficulty of students identifying implicit concepts in graphical representations to link with procedural understandings. In both grades (eleven and twelve) she was explicit about the importance of students understanding what graphing calculators do before using them, and stressed the difficulty of students' building a conceptual understanding, recognizing patterns and connecting mathematical ideas to results presented on graphing calculators.

It is just making sure that the students know what we're doing ... that they are not just memorizing steps . . . Any time we take a shortcut, we sometimes lose what we are actually doing. I find with the graphing calculator that they don't see that and there is a disconnect. That happens, they turn it on, and they turn their brain off, sometimes - Just, little, little details, and then, math is details (MII, May 1, 2013).

"I make sure that they understand that the equations are the set of order pairs. There seems to be a disconnect" (MFI, June 21, 2013).

Another concern was the difficulty of students recognizing different forms of mathematical representations (graphical, symbolic, verbal, and algebraic) of both topics (Figure 21, 22). In particular connecting the graphic representation of a function with the algebraic representation in order to find areas under a curve (grade 12) and points of intersection (grade 11).

"To understand why the graph is important to finding the area and why we split up the integral" (MJR, May 7 and 9, 2013, grade 12).

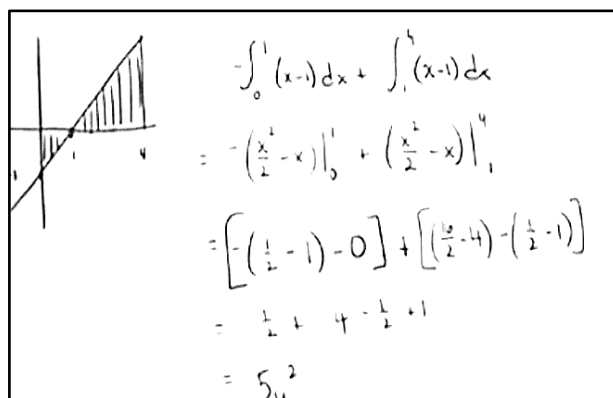


Figure 21. Mary's using manual sketching and algebraic representations to link conceptual and procedural thinking (MVO, May 2, grade 12).

“Some students did see clearly the connection between the graph and the answer we found algebraically. Others are still weak on this connection. Perhaps we could have found the answer algebraically and then found the points using the calculator” (MJR, May 14, 2013, grade 11).

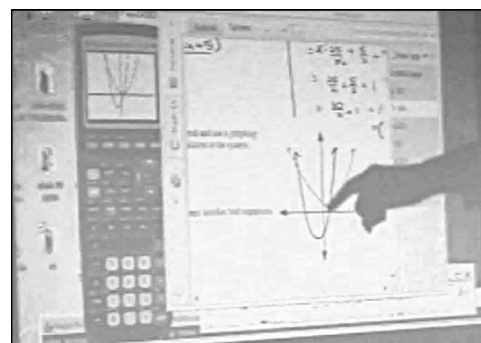
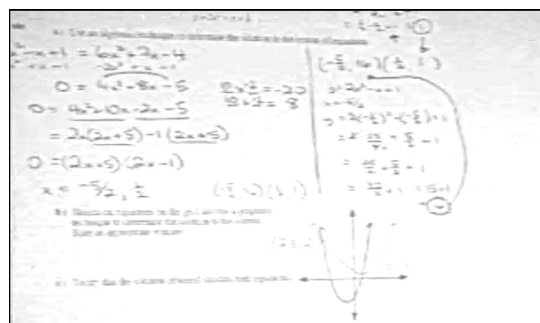


Figure 22. Mary comparing results obtained algebraically and graphically (using sketching and also graphing calculators) (MVO, May 16, 2013, grade 12).

Also, she was concerned about the weakness of basic mathematical skills, especially graphing manually and connecting that to the visualization presented on the graphing calculators.

“The reciprocal graph is always giving me problems, and I think is because when they use $y = mx + b$, they are using a short cut. They lose the fact that ordered pair is what they are looking at” (MII, May 1, 2013).

To be able to find points of intersections and to recognize that they were searching for ordered pairs. Some are fine, but there seems to be a disconnect for some students between the graph and ordered pairs. Graphing skills are weaker than I would like (MJR, May 14, 2013, grade 11).

My assumption of graphing knowledge may have been too optimistic. Their lack of basic graphing skills was frustrating. I tried to trigger their memories of previous lessons, but it was weak. I may put in an introductory unit next September aimed at strengthening skills needed from previous grades (MJR, grade 11, May 16, 2013).

Technology aid. The teacher highlighted that TT helped her to develop the mathematical understanding of the topic because of the visual impact of those TT when she is approaching the mathematical content of the topic. “The different colors, the highlighter, the drawings [the students] are able to tell me, if I am getting it right or wrong. It’s just much - much smoother when you are graphing” (MFI, June 21, 2013). “The visual provided by the graphic calculator does help the understanding of the points of intersection” (MJR, May 16, 2013, grade 11).

Mary illustrated that TT do not affect her mathematical thinking or the mathematical content, but rather they affect her approach to the content as well as the way she focuses mathematical thinking. “Actually, yes –not the thinking - not the content, just my approach. The content is still the same. It’s my approach that changes” (MII, May 1, 2013). “[the use of the tablet or graphing calculator] doesn’t change what I am going to teach, but does [affect] how I teach” (MII, May 1, 2013).

So, I think students get a mixed message there . . . that we don’t really want to learn the technology, the computer calculator part - and it is almost true. I want

them to know the concept more than I want to know how to push the buttons to get something, but I'm still trying to show how it supports the lesson (MFI, June 21, 2013).

Mary allowed students to use calculators when she considered it necessary in her teaching. Specifically, she used graphing calculators to introduce the lesson, to see patterns, to check previous knowledge, and to find points of intersection.

“The visual provided by the graphic calculator does help the understanding of the points of intersection” (MJR, May 16, 2013, grade 11).

“We also used the graphing calculator which helped me check out their previous knowledge of straight lines and parabolas” (MJR, May 16, 2013, grade 11).

“We used it [graphing calculators] to help us introduce the lesson, and to give us some patterns that we can see. I found it very helpful for that” (MFI, June 21, 2013).

She did not work all the time with the tablet, sometimes she turned off the computer and worked just with printed worksheets because she considered it useful to have students practice mathematical concepts and procedures in this way. Also, Mary shared some static information on the blackboard for student reference.

Sometimes, they don't flip back - they're the most helpless learners. They don't do a whole lot for themselves, the responsibility for learning is lower, and so, you put it up on the board, and you leave it up there. So that, when you are helping them now, because you are in the entire class you can get them to look back at the board. So I won't use the computer as often in that class, just because they need to constantly see the notes (MII, May 1, 2013).

Mathematical thinking. The mathematical thinking represented the way Mary approached the conceptual and procedural thinking in regards to teaching the content of the lesson. The conceptual thinking was illustrated in practical exercises and assignments to students during the lessons using a worksheet designed by Mary. She attempted to make this thinking explicit by questioning students during lessons about concepts related with the mathematical content. The procedural thinking was presented through examples where Mary explained the steps to solve a problem and the algebraic procedures that followed afterwards.

Mary developed the conceptual and procedural thinking in three ways:

- i. Mary made relevant the conceptual thinking first and then moved onto the procedural thinking (Figure 23).

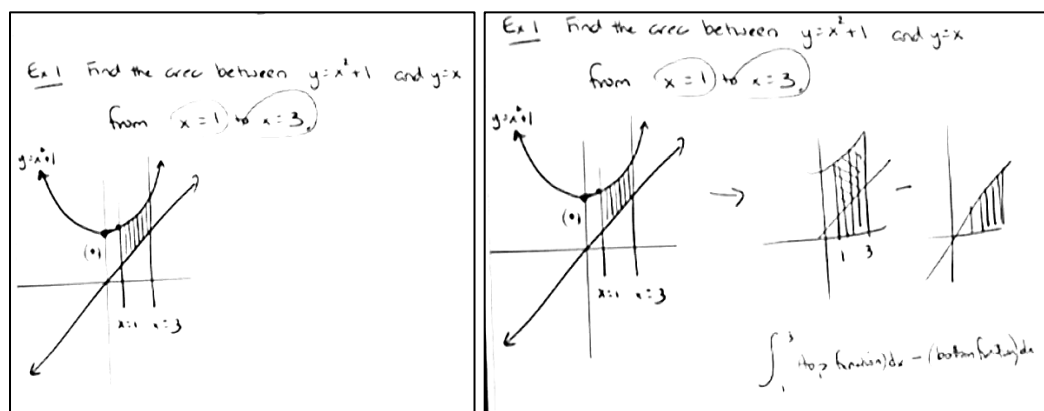


Figure 23. Example of graphical representations using sketching and focusing on conceptual thinking (MVO, May 9, 2013, grade 12).

She spent time questioning students about the graphic of the function(s) and looking for specific details into the graphic in order that students could better elaborate (articulate) their understanding of the conceptual part of mathematical content. It was expected that students gain knowledge of *why* it is important to visualize specific details into the graphic for connecting conceptual understanding (Figure 24).

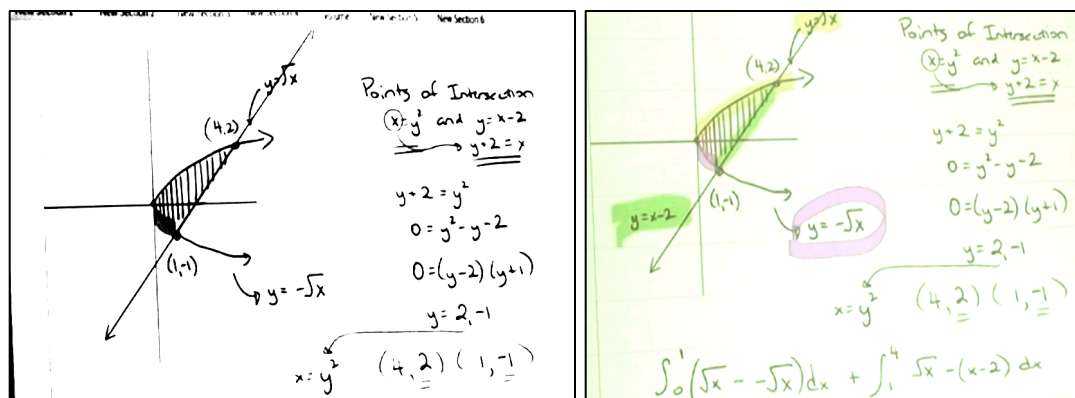


Figure 24. Example of graphical representations using sketching, focusing conceptual thinking and then procedural thinking (MVO, May 9, 2013, grade 12).

In grade twelve, Mary pointed out:

I expected students to see the connection between the location of the boundary curves and a negative or positive area. I also wanted to see if they could not only see what we had to adjust in our method, but why. I was hoping they would see that our way of finding the height meant we had to consider which function was on top. I think they saw the negative value and the area below the x axis clearly enough. The ‘why’ may come with further lessons (MJR, grade 12, May 2, 2013).

In grade eleven, Mary noted:

The goal was to learn how to find the points of intersection between a line and a parabola. I think that students need to be comfortable with the idea that these equations graph specific patterns. The procedure went well up to and including the steps of finding x . Some struggle with the y -value, because their connection to an ordered pair answer is weak. Some didn’t even recognize that they had to find a y (MJR, grade 11, May 16, 2013).

- ii. In this case, Mary emphasized first the procedural thinking first and then she allowed students to provide a sketch of the graphical representation of the function connecting mathematical ideas (conceptual). She posed different examples during the lesson and solved those –first making procedures and then

comparing graphical representation of the function (Figure 25). She then gave assignments that required following the same pattern.

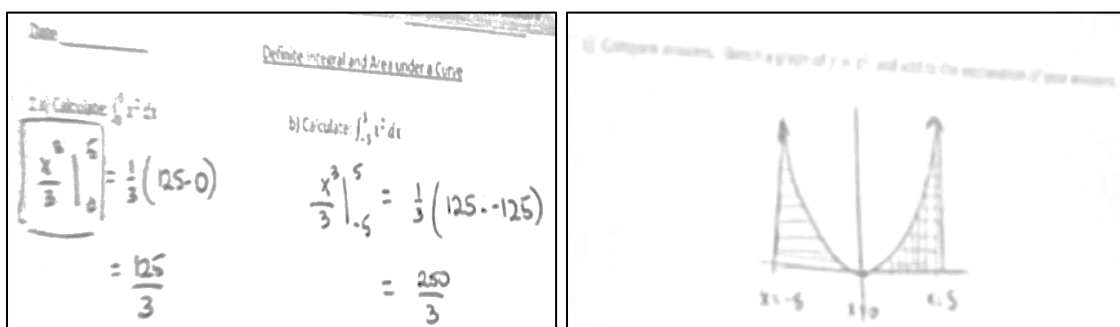


Figure 25. Example of focusing procedural thinking first, and then graphical representation of the function using sketches (MVO, May 2, 2013, grade 12).

- iii. Mary combined procedural and conceptual thinking simultaneously. She was switching between conceptual understanding—questioning and making graphical representations of different functions to find areas under a curve (grade twelve), and ordered pairs (grade eleven)—and applying algebraic procedures (Figure 26).

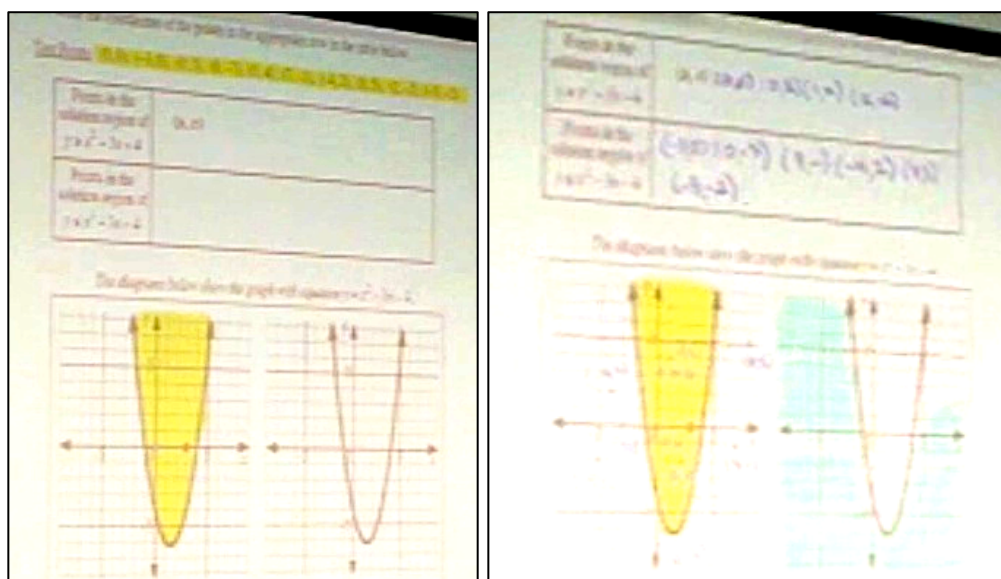


Figure 26. Example of the eTextbook focusing on conceptual and procedural thinking simultaneously (MVO, May 24, 2013, grade 11).

Representations. The mathematical representations are the different ways in which Mary represented a mathematical object during her teaching. Mary was working in both grades with functions and she made explicit verbal, symbolic, graphic and algebraic representations of those functions in her teaching. Mary focused on different mathematical representations of functions for reviewing basic skills of graphing and obtaining students' mathematical understanding.

Mary used mainly algebraic and graphical representations focusing on both the visual and symbolic representation of the functions. One example of those representations was a review in an electronic worksheet where students needed to understand polynomial functions in different representations. The review progressed through the linear function, the quadratic, reciprocal of a linear function, the radical, the absolute value, the exponential and the logarithm function. Mary and students first discussed verbal representations of those functions and then progressed to recalling graphical and algebraic representations.

The teacher asked: *'What do you remember in the last three years of math about graphing?'* (MPO, May 2, 2013, grade 12). The discussion started with the *'identity'* function. Some students talked about the graphic representation of this function remembering that it is a line, but they did not relate *'identity'* or *'linear function'* with the equation of the line or discussed their conceptual understanding of it. Once the teacher wrote the equation of the line, i.e. $y = mx + b$; the students immediately started to talk about the meaning of " m " and " b ". They then recognized that the identity function was a special case of the linear function. The name of slope and intercept was identified and the conceptual understanding was articulated for the students (verbal representation, graphical representation and symbolic representation) (Figure 27). Mary continued asking about the sketch of the graphic using the slope and intercept in the graphical representation.

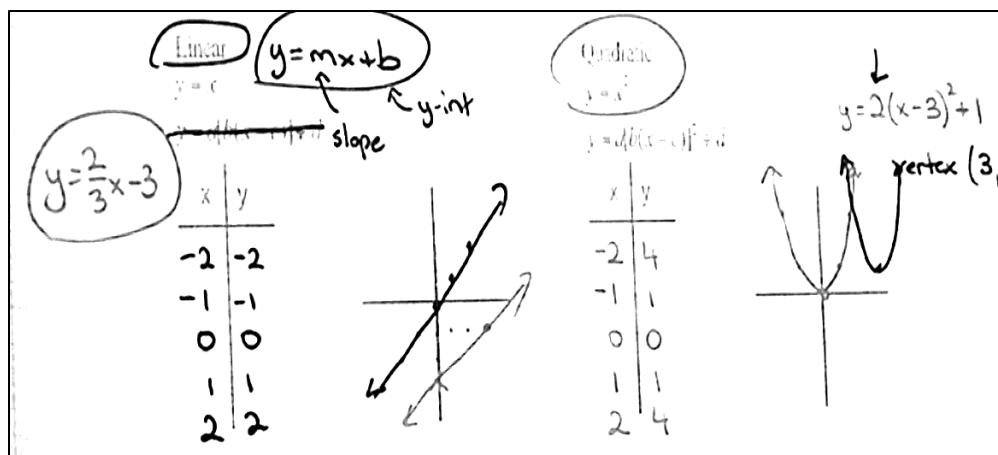


Figure 27. Mary reviewing basic skills of functions focusing on different mathematical representations (MVO, May 7, 2013, grade 12).

“I am going to assume that everybody knows these graphics, that they, [the graphics] are familiar to (you), and that you know how to start. That you have to make a table of values” (MPO, May 2, 2013).

Another example of making connections between representations was when the teacher asked students about the graphical representation of the circle and the half circle function sketching first and then using graphing calculators (Figure 28).

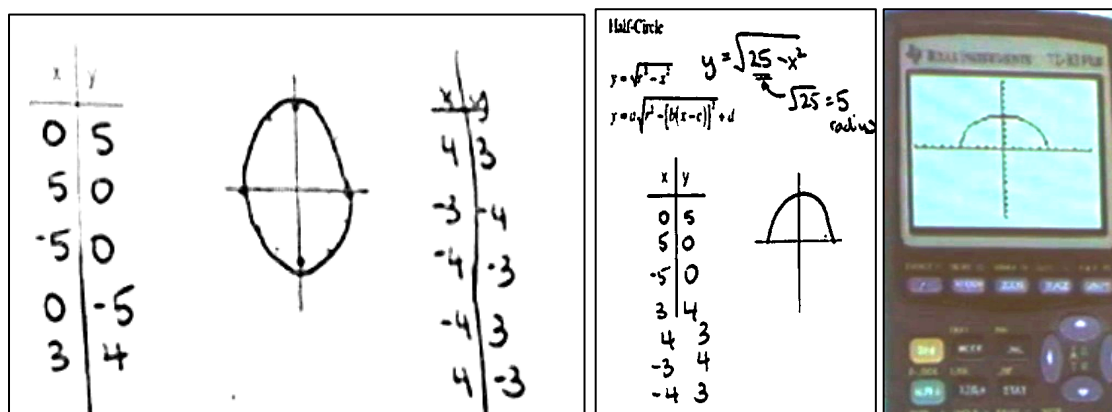


Figure 28. Representing the circle function and half circle function using a sketching and graphing calculator (MVO, May 7, 2013, grade 12).

When both, students and teacher obtained the graph, the teacher asked: “How can we get the other part of the circle?” The students answered correctly and everybody followed the procedure in their calculators. Thus, they obtained the graph of the circle (Figure 29).



Figure 29. The graphical representation of a circle function using graphing calculator (MVO, May 7, 2013, grade 12).

The approach of the graphing calculator did not provide the connection of both parts of the half circle and it was visible (on the screen of the calculator) that both parts did not touch the x-axis. The teacher pointed out: “. . . ok that’s better, I want to zoom [on the graphing calculator]”. Then, a student asked: ‘Why don’t they connect?’ and the teacher answered:

They don’t connect because of the technology. It won’t graph anything that is inaccurate, so it gets close. I don’t know. There seems to be a glitch, a couple of times where the graph doesn’t touch the x-axis as it should. So, It’s a good thing, we are still thinking (MVO, May 7, 2013, grade 12).

The representation of the whole circle manually and with the use of a graphing calculator became an important aspect of the lesson to emphasize the knowledge required for understanding points of intersections on the graphing calculator screen. Sometimes, the graphing calculator screen does not represent the points of intersection very well and

it is necessary to zoom in on the calculator screen to read those points. Thus, Mary pointed out to students to be aware of possible limitations of using graphing calculators and the need to understand how the function might be sketched before using graphing calculators. Students need to analyze what they see in the screen of the graphing calculator and avoid taking it for granted without mathematical thinking.

In each one of the theoretical characteristics of polynomial functions, the teacher implicitly stressed the importance of knowing multiple representations of functions such as verbal, numerical, visual and algebraic. She considered fundamental that students know how to describe in words a function (verbal representation), how to represent the function using a table of values (numerical representation), how to do a sketch or graph of the function (visual representation) and how to make an explicit formula (algebraic representation) before using the graphing calculator. According to Mary, it is not convenient to use graphing calculators without knowing the theoretical aspects of functions. It is necessary to examine different representations in order to avoid conceptual and procedural misunderstandings.

Also, Mary was explicit in her teaching of the importance of comparing graphical representations and reading points of intersections while sketching and using graphing calculators (Figure 30).

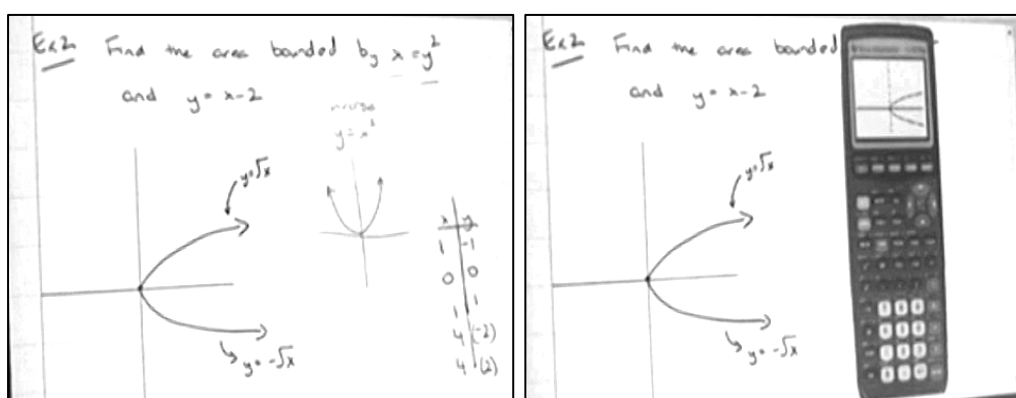


Figure 30. Examples of polynomial representations of functions using sketching and comparing results with graphing calculators (MVO, May 9, 2013, grade 12).

Mary compared a graphic representation from an example provided in the eTextbook and the same representation using graphing calculator (see Figure 18). The comparison helped her to identify that some students were confused reading points of intersection on the screen of the graphing calculators. Because the value of the points of intersection on the screen of the graphing calculator were not integers, students need to use rounding to corroborate the scales in order to recognize the points of intersection illustrated in the graphic of the eTextbook.

Today there were mixed results. Students became worried about the process in changing the form of the equation. The vertex numbers were not easy integers and that panicked a few students. In hindsight, I should have just gone with the x and y intersects and had them estimate where the vertex was (MJR, May 24, 2013, grade 11).

Dimension 5 – Mary’s Knowledge of Pedagogy. Mary’s pedagogical knowledge is defined by the strategies she followed in developing the mathematical content of the topics including: gaining students’ attention; reviewing previous content; presenting new content; planning for students’ practice activities; and having control of the class. Mary does not identify with any particular learning theory in her teaching. However, Mary might possess a blend between the constructivist perspective and the behaviourist learning theory. She used a constructivist perspective by providing resources (i.e. worksheets, Moodle website, calculators) to the students, to allow them to construct their own knowledge and then asked them to reflect on their own learning (MPO, May-June, 2013). Her approach could be described as behaviourist in her use of a stimulus-response technique when questioning during her teaching practices.

The main pedagogical strategy identified in Mary’s practice was guiding students through questioning and encouraging them to provide responses in order to build mathematical knowledge. Indeed, Mary used ‘discussion classes’, where she chose to be located “at the back, right in the middle of the students to listen to students’ conversations” in order to participate better in those discussions and to control students’ behaviour during lessons (MPO, May-June 2013, grade 11 and 12).

The mathematics topic was presented in a logical sequence. It was also neat, well organized and supported within a technological context. The curricular planning was available through the eTextbook and was adjusted and expanded by using worksheets and notes at Mary's convenience. She reviewed previous knowledge usually at the beginning of the lessons by asking about difficulties encountered in previous assignments, by questioning knowledge of previous topics, and situating the students in the context in which previous knowledge was learned (she used the tablet to recall images from previous lessons, procedures and specific examples). At the end of each class she summarized the content taught and gave assignments for practice and also provided advance organizers for further lessons.

The tablet was the main TT used by Mary to drive students' visual attention, and it allowed handling notes, graphics, and examples by shading, highlighting, graphing, and coloring specific details that she wanted to emphasize. Mary highlighted that she guides students' learning in her teaching, not the technological tools. She combined traditional technology (paper, pencil, keeping static information on the black board, and printed worksheets) with new technology (tablet, OneNote, eTextbook, eWorksheets, Moodle class website). TT assisted Mary in facilitating the communication of the content of the topic and focusing conceptual and procedural understanding.

Themes formed in this category included: Questioning, Teamwork, Guiding visualization, and Pedagogical tools.

Questioning. Mary's primary mode of teaching was through questioning (MPO, May, June, 2013). Mary indicated that most of her lessons were "discussion classes". She affirmed that the use of technological tools in her teaching allowed more time for exploring students' questions. In those teacher/student/technology interactions (questions-answers), Mary tried to identify students that required additional help: "That's when I have to find out more about them, some students grasp concepts easily, they don't even need the picture beside it, the other ones you need to find out from their questions, what parts need enhancing or showing up" (MII, May 1, 2013). Mary listened to students' ideas carefully and questioned them again and again seeking elaborated responses to develop understanding of the topic (MPO, May and June, 2013). The quality

of students' questions helped her to adjust the lesson: "it's again the quality of their questions, that's helped, me adjust my lesson" (MFI, June 21, 2013). During her lessons, she found, by listening to students' conversations, their level of understanding, particularly with graphical representations, and then augmented her lessons with additional topic related questions.

" . . . Encouraged students to talk about their own understanding and any areas of confusion" (MJR, May 9, 2013, grade 12).

"The tablet allows me to focus on questions and details of the work. Students can easily see the notes" (MJR, May 23, 2013, grade 11).

" . . . as far as the lessons go - the conversations are what I'm trying to trigger, and that's where I find out where and what help I need to give them" (MFI, June 21, 2013).

"I listen to comments and answers made by students. My table, where I sit when delivering the lesson is right in the middle of the students. I try to tweak/guide their conversations to consider the important concepts of the lesson" (MJR, May 2, 2013, grade 12).

Mary questioned previous knowledge of the topics at the beginning of the lessons. Usually, she started by asking students if they had any difficulties with the assignment and solved students' questions making notes on the tablet and sketching graphics when necessary (MPO, May, June, 2013). During classes, the use of the tablet helped Mary to make connections with previous lessons. She recalled previous lessons by providing visual explanations, justifications and responding to students' questions. Thus, by situating students into the context of the previous lessons, Mary encouraged questioning of mathematical concepts and procedures and supported such with the use of the tablet.

She argued the importance of knowing the location of files containing previous lessons, notes and worksheets as part of her planning in order to provide prompt answers

to students' questions or to clarify concepts. At the end of the lessons Mary summarized concepts and explained assignments—confirming students' awareness of notes and worksheets through questions. The class Moodle assisted Mary pedagogically to refresh lessons, which allowed students to access up to date information and point out specific questions to enhance understanding of the lesson notes.

I couldn't find the lesson at first that I needed to have students revisit for the next class' lesson. However, once I did locate the notes it was easy to remind students of the past lesson. I also wanted them to take responsibility for relearning the material if they needed to (MJR, May 23, 2013, grade 11).

"I liked how this lesson reinforced the idea that Math builds on previous concepts. I would improve the lesson by making sure I knew where the needed notes were ahead of time" (MJR, May 23, 2013, grade 11).

"Yes, I go back over concepts to show them how is linked, so that is helpful as well. I just have to type on it and find it. It's good" (MFI, June 21, 2013).

I also was able to call up a previous lesson to tie previous material with what we were doing today. It also helped students that were absent from class as I was able to post lesson to the Moodle site that evening for students that were absent from class. One student was ill and the other was in Hawaii. I checked the participants on Moodle this morning and both had checked the notes (MJR, May 9, 2013, grade 11).

Teamwork. This theme is represented when Mary and students interacted together with the content of the topic (Mary using the eTextbook on the tablet and students working with the physical textbook), and also when students partner to develop knowledge of graphing skills (making sketches of functions and using graphing calculators). Mary highlighted the advantage of having students working as a team during her lessons. She indicated that students participated by teaching other classmates in the

manipulation of graphing calculators, finding graphical representations of functions, and clarifying questions. Thus, the advantage of teamwork was that it promoted students' conversations, and allowed for more discussions between students related to the manipulation of the TT and the mathematics topic.

They're not just reading steps, we're developing at the same time [the topic], and they copy it into the book. It's not like I just put the notes up on the board and they read. We are putting them down together and then it's efficiently in their book for them (MII, May 1, 2013).

When it comes to graphing calculator, having partners really does help me out. Sometimes I feel like I need roller skates to get to each one that is having a minor problem. The plot button is "on" for some reason and the student before them did something. I find that helping them help each other when they are using computers is extremely helpful (MII, May 1, 2013).

"To try to encourage them to learn how to use a graphing calculator, I often partner students up with someone who is knowledgeable. I try to make sure that both of them have a calculator" (MJR, May 14, 2013, grade 11).

"The lesson slows down. To speed things up I try to get fellow students to help each other out. It has varying degrees of success" (MJR, May 16, 2013, grade 11).

Mary recognized that using graphing calculators promoted students' mathematical thinking in regards to starting and supporting discussions during lessons. Specifically, she highlighted the manipulation of graphing calculators and the way interactions between students allowed them to flow between conversations between graphing calculator skills and the topic content. Many conversations produced between students were listened to by Mary and these helped her to identify difficulties, gave her opportunities to intervene in students' discussions, and to complement those discussions.

“I’m listening to their conversations to find out if they ask me questions, but they would often ask each other, and I’ll be paying attention to that, trying find out the difficulty level.” (MFI, June 21, 3013).

I love [discussions] when they start talking about that. The graphic calculator is a safe topic to start with. They don’t feel that they are in any way less intelligent because they don’t know how to use it. That’s something they are comfortable with—not knowing how to use it. So, asking questions to start with, breaks the ice for questions about the lesson, which they might be shy about. I can stand back and watch when a student explains a topic or something on the calculator. This can suddenly become something about the lesson as well, and that’s a wonderful moment. So I’m very happy with the discussions (MFI, June 21, 2013).

Guiding visualization. In this theme Mary identified the importance of the visual elements provided by the TT that supported her guiding of mathematical thinking. During her teaching practice, she used visualization to emphasize specific details that she considered were essential for learning the topic content. Mary guides students’ understanding of new concepts by showing previous examples, procedures, graphs or any previous visual content needed. She listened to students’ questions and then assisted their discussion by presenting an appropriate visual model. “I think the visual part is extremely helpful as strategy to start them, and to connect it to previous lessons” (MFI, June 21, 2013).

She guided visualization by combining traditional and new technology during lesson discussions. Mary used a traditional blackboard to visualize notes that she needed to refer back to constantly in her teaching (keeping information static for students’ reference), and also considered paper and pencil the best way for students to carry out mathematical procedures (she used printed worksheets). “I still use the board now and then especially when I’m teaching. Some notes that need to be referred back to constantly, and students need to see the steps up” (MII, May 1, 2013).

In regards to new TT, the tablet, OneNote program, graphing calculators, worksheets (hard and electronic copy), and eTextbook assisted Mary in enhancing the

visual impact of the lesson content by the use of colours, notes, diagrams and figures. She also used TT to access any part of the lesson content at any time to visualize specific points. “I go back over concepts, to show them, how it is linked. So, that is also helpful. I just have to ...it’s good” (MFI, June 21, 2013).

I believe that the tablet is a convenient tool to help explain the concept of splitting the areas by making the graphing so visual. The use of the colours makes it easy for me to stress the parts of the graph I am describing. It gives both the students and me a reference point to discuss any lingering questions. It’s also easy for me to call up points made in previous lessons to connect today’s lesson with concepts already explained (MJR, May 9, 2013, grade 12).

The students are engaged and their focus is on the answers and not on the minutia in the way the question is set up. I can also stress the big ideas easily with the tablet. It is easy for the mind to wander at any time, but the simplicity of the note taking for students lets them see clearly what is important in the lesson (MJR, May 24, 2013, grade 12).

Mary indicated that although some types of learners are not visual learners (students have difficulties showing understanding on visualizations), she uses the visual impact of TT for all types of learners especially for students in grade twelve. She makes comments that in higher grades (in this case, grade 11 and 12), students do not have so many learning differences like in the case of junior grades, and it is easier to follow the same strategy for all students. However, she has discovered that even though some students are not ‘visual learners’ in those grades (11 and 12), they required more time to adapt the strategy.

There are some [students] that are not visual learners, but you keep hoping they need more time. The ones that are in the academic programs often just need more time at it. They can’t get it in one day. I don’t see as many differences as I do in the junior grades (MII, May 1, 2013).

Mary highlighted the time required for obtaining conceptual and procedural understanding of the topic in regards to visualization provided through the TT (i.e., visualization provided by graphing calculators). She argued: “visuals are extremely helpful, but you still need to give time to the students to absorb the information. It helps to be as obvious as possible” (MJR, May 2, 2013, grade 12). Mary adjusted her lessons by allowing for more time to develop guiding visualizations when students were working with graphing calculators. She focused on the goal of visualizing points of intersections (grade 11) and finding areas under a curve (grade 12) instead of prioritizing the use of graphing calculators to obtain the graphical representation on the screen.

The visualization of the content of the topic with TT (projected on the screen with the tablet, OneNote, graphing calculator) allowed Mary “to stand back and listen to discussions between students” (MJR, May 16, 2013, grade 11), and “to give more examples” (MJR, May 23, 2013, grade 11). Mary considered her location in the classroom to be a good pedagogical strategy because she was “able to sit among the students, to see, and hear what they are discussing” (MJR, May 24, grade 11).

“So, for me it is just keep paying attention to what your students are saying, and trying to help with the visual” (MFI, June 21, 2013).

Pedagogical tool. This theme represented the way Mary used TT to support her teaching practices focusing on students’ understanding of the mathematical content. She was clearly affirming that the way she uses TT is what facilitated her teaching, not the tool itself. She argued “. . . Actually, not the content, just my approach, the content is still the same, it’s my approach that changes...how I teach it” (MII, May 1, 2013).

Mary incorporated the curricular planning into a technological context. The textbook was available on the tablet as a PDF file (through the OneNote program). She was able to make small or significant changes in the eTextbook as required. Mary recognized the value of having an electronic version of the textbook available in the tablet and considered the lessons to be a discussion process. She used the eTextbook as a pedagogical strategy to interact with her students in a dynamic way by co-developing the

content of the class. Both, teacher and students had the content of the class available at the same time, which allowed them to interact, discuss, ask questions, make diagrams, create notes, sketch functions, etc. In this sense, they were building mathematical procedures and concepts together. Also, the use of the class Moodle allowed her to maintain communication with students after lessons and students were able to discuss specific points of the lessons.

“It’s easier for students to understand their lesson and to reflect on it if needed. Notes are right before their homework, so it makes it easier to review as well” (MJR, May 16, 2013, grade 11).

“Students were able to follow the steps easily. Recognition of concepts was discussed” (MJR, May 23, 2013, grade 11).

The worksheets were created or enhanced using OneNote (and subsequently posted on the class Moodle). The integration of worksheets in her planning helped her to review previous knowledge, illustrate more examples related with the topic content, and allow students to understand better the mathematical topic by practicing exercises during the class and also at home. The combination of the tablet, worksheets and OneNote was used by Mary to organize the content of different lessons, to write notes at any time and to complement mathematical concepts and procedures with new examples. In this sense, she mentioned that TT allowed her to provide a very clear and organized visual lesson.

“It just makes it more colourful. It’s a tool, I don’t see it as leading the way, I see it as helping me teach the very same lesson” (MII, May 1, 2013).

“I will have the worksheet up there, we will be talking about it, and then I will be giving them the actual answers as well as making sure they are on the right track” (MII, May 1, 2013).

She does not need to read notes and to write on the board, or to carry a heavy book or to stay in the same place writing notes on the board. Mary indicated the convenience of the tablet in managing pedagogical aspects by linking previous lessons, notes, graphics, and worksheets with the development of the lesson content and also by introducing new mathematical concepts. The tablet was a convenient pedagogical tool that helped explain the topic by making the graphing visual. She pointed out that the use of colours in her teaching was important to focus parts of the content of the lesson including graphs that required special attention. Indeed, Mary likes that when she is doing manual graphing (sketching), or writing notes on the tablet, she is not an obstacle that limits students' view of the presentation.

“I am not standing in front of the notes. The notes are nice and clear to everybody wherever they are sitting” (MFI, June 21, 2013).

“The use of the colours makes it easy for me to stress the parts of the graph I am describing. It gives both the students and myself a reference point to discuss any lingering questions” (MJR, May, 2013).

The class Moodle was another technological resource that Mary used to integrate the notes of the lesson content and assisted her in maintaining communication with students after class. The benefit of the class Moodle was to make class lessons and notes available to all of her students including those that for any reason missed classes. She made relevant the pedagogical advantage for students interacting with the class Moodle when they came back to her with specific questions.

The ‘Moodle’ let me store lessons. The students are quite comfortable with getting any missing notes from there, and then, come back to me with specific questions rather than having to re-teach the whole topic. They still come at lunch hours sometimes and just ask for my notes (MII, May 1, 2013).

The use of graphing calculators was occasional and supported her teaching visually. The pedagogical strategy used by Mary when using calculators was that students focused on the identification of points of intersections in the graph showing mathematics understanding; contrary to focusing on skills related to using graphing calculators. She pointed out “give some time to [students] - making sure students know how to graph on a calculator, but if they still struggle, - emphasize that the importance of the graph is - it is visually showing you where the points of intersection are” (MJR, May 14, 2013). Then, she allowed students to use calculators when she believed that they had a good understanding of what calculators do for them. The teacher showed the importance of reviewing mathematical concepts and procedures by questioning students during graphing (making manual graphing or using graphing calculators). This factor helped her pedagogically to know students’ gaps in the process of learning the topics using graphing calculators.

“So the technology is a quick way to have things up on the board, and to just check that we are on the same page, and that they do have the background” (MII, May 1, 2013).

Case 2 – ALEX

The PCK-EC model is used to describe Alex’s pedagogical content knowledge as he taught specific mathematics topics using particular TT (iPad). The educational context in which Alex taught was already presented. The dimensions of Alex’s attitudes and knowledge (KT, KLC, KSM, and KP) are represented here.

Dimension 1 - Alex’s Attitudes. Alex expressed his Attitudes when he gave an opinion, thoughts, and his own perspective about the use of TT in his teaching, specifically in the topics ‘Introduction to Trigonometry’ and ‘Provincial Exam Review’. He expressed that TT have potential features to support his teaching with conceptual understanding, and provide assistance to students to individualize their learning. While teaching the specific topics, Alex highlighted the importance of using iPad apps, the class

website, and online resources to connect authentic and practical applications of trigonometry to formal mathematics. Practical samples of the provincial exam posted as online resources (class website) assisted Alex/students as they reviewed questions and solved problems. He indicated that the use of iPads allows students to work independently, visualize better the mathematical concepts, and experiment with mathematics. He also argued the need to be conscious that teaching with TT requires more time for planning, designing, and reviewing students' work because not all students adopt TT. Even though Alex perceived himself as technology literate, he highlighted that the main goal for him is to improve his skills for teaching with TT, not just his technical skills. Alex has a positive attitude working with TT and he thinks the use of TT is an opportunity to continue developing curricular projects to improve his teaching. The themes illustrated by Alex in this dimension were: Useful, Connections, Challenging, Consciousness, and Inspiring.

Useful. Alex established this theme when he mentioned the features of TT (iPad, tablet, class website) that assisted him in teaching practices, and the advantages with respect to helping students learn about the topic. The usefulness of the TT in Alex's teaching included: individualize learning; access to diverse materials and support an ample range of students; allow more discovery learning; facilitate authentic and practical applications of trigonometry for visualizing; and developing conceptual understanding.

Some of the quotes that supported this theme were:

[TT] allow access for more diversity, so students that need remedial assistance are now able to use technology to watch a video or watch my notes online. I didn't have that before, they would just put up their hands and they would have to listen to me or ask a peer. There are also activities that students can do if they finish early. So, handling diversity is a little more personal (AII, April 23, 2013).

I totally think [the class website] frees you up. I've done that for a few years now where I put my handouts on the website and then when one of the students says I'm away on Thursday, or I don't understand it, or I lost this puzzle and I say

‘Let’s go figure it out’. They all have access to the internet now because they can always come down here to the library and access it too; we allow them to print. I totally see that as crucial (AFI, June 19, 2013).

I believe that the tools helped – using photography to place students into the problem would not be possible without it. Their mathematical thinking was abstract – and then problem solving and patterns. They still need to work through the entire process to find missing parts of a right triangle (AJR, May 17, 2013).

“The iPads helped the students to learn how to download and practice provincial exam practice tests. They could also write on the screen over the question to show understanding” (AJR, May 27, 2013).

Connections. This theme was formed when Alex focused his interest on knowing how the use of iPad Apps (Camera, u-Clinometer, Explain Everything, and Dropbox), online resources and class website supported his teaching to make mathematics more authentic for students and also how those TT would be valuable for students connecting practical applications of trigonometry in more formal mathematics.

According to Alex, the use of iPad Apps allowed students to work independently by choosing, visualizing, discovering, and practicing the mathematics topic making connections with authentic and practical trigonometry applications (i.e., pictures from the school grounds, eTextbook, and online resources, were used to construct mathematical demonstrations using the Explain Everything app). The Apps “Explain Everything” and “Dropbox” helped Alex to verify if students transfer or connect iPad work done during lessons into their mathematical understanding.

With the iPad, I try and think about what applications they use and then... but then after they use it, they practice the mathematics, and there is an accountability where they have to show me that they transfer what they learn on the iPad or what they practice on the iPad to learning mathematics (AII, April 23, 2013).

I think because we are using the TT, we want them to understand some other issues, for example how technology can be helpful to make mathematics more real for them, also how maybe technology can be more helpful in terms of showing their understanding . . . to take a picture of something, and then have the student do the mathematics on top of the picture. That is a nice connection (AII, April 23, 2013).

I see a real potential in it. I do not know if I have obtained that potential yet, but my sense is: It does allow students to visualize on their own a little bit more what the applications are for trigonometry . . . (AFI, June 19, 2013).

Challenging. Alex expressed in this theme fears and concerns in regards to the use of iPads in his teaching practices. He felt fear using iPads because it could be inefficient and distracting for some students. Also, he would like to be sure that students are obtaining mathematical understanding of the topic and not just engagement with the iPad. Thus, the teacher was concerned that some “students are passive in the learning with the iPads, they are watching, reading a lesson, they are playing an app, but they are not thinking deeply with the iPad” (AII, May 2, 2013).

Right now in the beginning of this project, they weren’t using iPads in the class. I try to really embed in my practice as much as possible. But sometimes I have a fear that it’s inefficient, and I have a fear that it is distracting, and I have a fear that I am not able to get students to think as deeply as they could if I control the situation a little bit more, and they weren’t distracted by the iPad (AII, May 2, 2013).

The concern that I have with the iPads is that students will not transfer the procedures they have been practicing on to theoretical mathematics; also the iPads are actually distracting for some of the students and could inhibit learning. The concern too is that students are passive in their learning with the iPads, they are

watching a video, they are watching a lesson, they are playing an app but they are not thinking deeply (AII, April 23, 2013).

Other challenges expressed by Alex included that the process of incorporating TT in his teaching was time consuming in regards to planning (creating and posting notes on the class website) and providing feedback to students for work submitted after lessons (i.e., checking files saved in Dropbox or the class website). Additionally, some of the students that were working 1:1 with iPads do not like technology and it complicates the process of managing the learning in the classroom (i.e., some students did not submit work done during lesson).

You can submit, explain - you can explain a lot, you can put a different variety of files in Dropbox. But I was finding, I wasn't getting back to check those files. So, it is easier for me to walk around and just hold them accountable while they are sitting in a chair with me than going home and checking over their work (AFI, June 19, 2013).

Finally, Alex indicated that the arrangement of the classroom, and the desks distribution were not appropriate to teach with TT, and properly interact with students. This situation was uncomfortable for him and it was linked to the fact that he was teaching in a science classroom, and was a visiting teacher in that school.

I found teaching in a science class challenging. There was some board space around and I like to get kids writing stuff up on the boards. I found I liked to have boards as well as a projector. I like them to see a problem on [the board] and move to another place in the room... and that part I didn't like [wasn't convenient]. I didn't like the fact that I really couldn't arrange the desks and sometimes I like to have them working in groups of three (AFI, June 19, 2013).

Right, most classrooms have white boards all over the place, plus, I couldn't put up student work . . . I could have easily printed off a couple of those trick pictures

that the kids did and I could have put that up. I don't have any pull since those are not my classrooms. I'm a visiting teacher and I felt that part very challenging (AFI, June 19, 2013).

Consciousness. In this theme the teacher highlighted that TT and mathematics have been linked for a long time to the teaching-learning process. In this process, mathematics teachers might have been using TT more than teachers from other subjects. Alex indicated that it is necessary to be very conscious of the way TT had been used and also what you want to achieve with those TT “teachers have to be very thoughtful and deliberate about how they use technology in the classroom” (AII, April 23, 2013).

He is conscious that new TT may provide more learning, show students more real data, engage real life applications in mathematics, allow a more personalized approach (activities that students can now do at their own pace, at their own level), and help to handle the diversity in the classroom. On the other side, he is also conscious that TT require more time for planning, maintaining a website, reviewing students' work in files, researching class design for teaching with TT, and that not all students adopt TT during teaching practices.

It is really important for teachers to plan its use (TT) and not react to its use ... You need to do some serious planning, and I think you need to also stop quite a bit, and reflect and see if in fact you are changing the students' learning as intended (AII, April 23, 2013).

Another teaching strategy is to be very conscious about the culture in your classroom . . . of accepting diversity. Accepting it and not trying to isolate kids. So, you need a plan for that, just like you need a plan for the kids that get it right away (AII, April 23, 2013).

Finally he questioned the necessity of teachers to be conscious of improving their skills for teaching, not just improving their technological skills.

So, I think, I am improving my skills, but I don't think that I can ever get to the point where [I would ever be better than my students] because some of them are just so mentally gifted at it. I think I have to be cautious when improving my skills in technology (AII, April 23, 2013).

Inspiring. Alex has been working with TT since 1985, and this theme was represented when he indicated his commitment and willingness to continue working with TT in his teaching practices. He recognizes the power of the mobile devices and he is convinced that those are useful for teaching and learning. Alex not only identified several advantages of working with iPads in regards to student choices (working independently, accessing online material at their own pace, and finding real applications of trigonometry), but also he sees as an opportunity to do project and game based learning, as well as cross curricular projects and a potential for designing new software or apps that allow more teaching practices outside of the classroom.

He expressed his satisfaction of working with the iPad and he indicated the importance of combining the use of an iPad with a tablet, and suggested that finding additional equipment (pen for the iPad), other software (i.e., Geogebra) might help to develop the students' mathematical thinking of the topic, and lessons should be accessible online.

I would say, the other thing I like about it is perhaps, games or applications within the technology that the students can practice, could be logic puzzles, and that is really easy for the kids to motivate on, and if you can do the right research, and find the right applications for them, then maybe they're developing mathematical logic and ability that can be applied in more formal mathematics (AII, April 23, 2013).

I would like to see more project-based learning where maybe there are some cross-curricular projects where students are using technology such as videotaping. I would like to see it used for more work production. Right now they are using it for content understanding, or they are receiving the content through it, but they

are not creating work from it. I would love to see a tablet that has a pen, where you can actually scribe . . . because in mathematics right now the students are using their finger, and it is really poor writing... they can share their work with each other that way (AII, April 23, 2013).

It was useful definitely. I definitely would like to try it again. I see a real potential in it. I do not know if I have obtained that potential yet, but my sense is that it does allow students to visualize on their own . . . what the applications are for trigonometry (AFI, June 19, 2013).

I did like the idea of using it [iPad] as a meaning device. I did like the idea of using it as a device where they can write on it. I used this app—called Explain Everything—where the kids can write stuff on it, and again, I would ideally love to have some kind of digital portfolio to have with them (AFI, June 19, 2013).

“I think it was a positive experience for me as a teacher. I have evidence in terms of their grades that it was positive for their learning. I would say I would be interested in doing it again” (AFI, June 19, 2013).

Our lessons should be online, accessible 24 hours a day. I would love to see better practice than the Khan Academy has where the kids are getting feedback online... So the potential for organizing your lessons and even for doing a lot of things outside of class time is huge (AFI, June 19, 2013).

I would like to introduce videos as well – where we look at the construction or applications then work through it in class (I really like Dan Meyer’s 3 act lessons and this would fit in nicely with the technology I think) (AJR, May 27, 2013).

“I would like for students to . . . create their own videos – this might be more useful for auditory learners and may require more care and understanding from the students making the examples” (AJR, May 31, 2013).

Dimension 2 - Alex's Knowledge of Technology. Alex reveals his KT when he manipulated TT during lessons and also in the ways he used TT in his teaching. The iPad was the main TT used during his teaching. He was also observed using other TT to support his practice including: tablet, class website, a data projector, scientific calculator, and eTextbook. Alex has been using TT in his teaching since 1987. He began using graphing calculators, then he persuaded the use of Tablet computers for mathematics teachers in his district and he has used the Tablet for about five years. Recently he has been using the iPad for six months and is learning the integration of iPads in his teaching practices.

He showed mastery of working with iPad apps and using the features of the iPad to develop the design of his lessons combined with the Tablet computer. Alex designed the class website and the tablet computer supported him to post assignments, examples, and resources needed during lessons. He was concerned with adapting himself to the classroom because there was a lack of white boards, and the technological equipment did not allow him to use the tablet and the iPad at the same time (Alex ended up not using the tablet computer during lessons). During Alex's teaching practices, there were no major problems or technical issues that limited his teaching. He mentioned that occasionally they have unexpected data projector failures, problems with wireless speed, and some students' had difficulties posting assignments in Dropbox or the class website.

For most of the lesson time Alex and students interacted both with iPad apps, the class website, and online resources. He integrated the mathematical content of the topic into a technological context using the eTextbook, through iPad apps and the class website. He perceived himself as technologically literate and he is convinced that the power of new TT supports teaching and the learning process.

Themes identified in this dimension included: Affordances, Constraints, Integration, and Interactions.

Affordances. Alex characterized in this theme the exploration and use of the iPad Apps (Camera, Explain Everything, U-clinometer, Dropbox, Scientific calculator), online resources, eTextbook, tablet and his class website which benefitted his teaching practices. During this research Alex used the iPad as the main TT, and he mentioned that the main

benefit of using iPad is for “individualizing instruction, discovering learning, and searching more about mathematical applications”. He has learned and practiced the use of IT following his own motivation and interest for understanding the benefit of using such for teaching (AII, April 23, 2013).

The iPad supported Alex’s teaching by providing access to the class website, integrating different apps and allowing access to images (from the eTextbook and online images), supporting the development of content and providing resources.

Alex used the class website (Figure 31) to post a daily plan, class files, online practices, and other resources for teaching his lessons. He used a daily plan to introduce the lessons, to give instructions to students about the activities of the lessons and to communicate the mathematical content of the topic through eTextbook. The home page of the class website allowed students to submit assignments and to check grades.

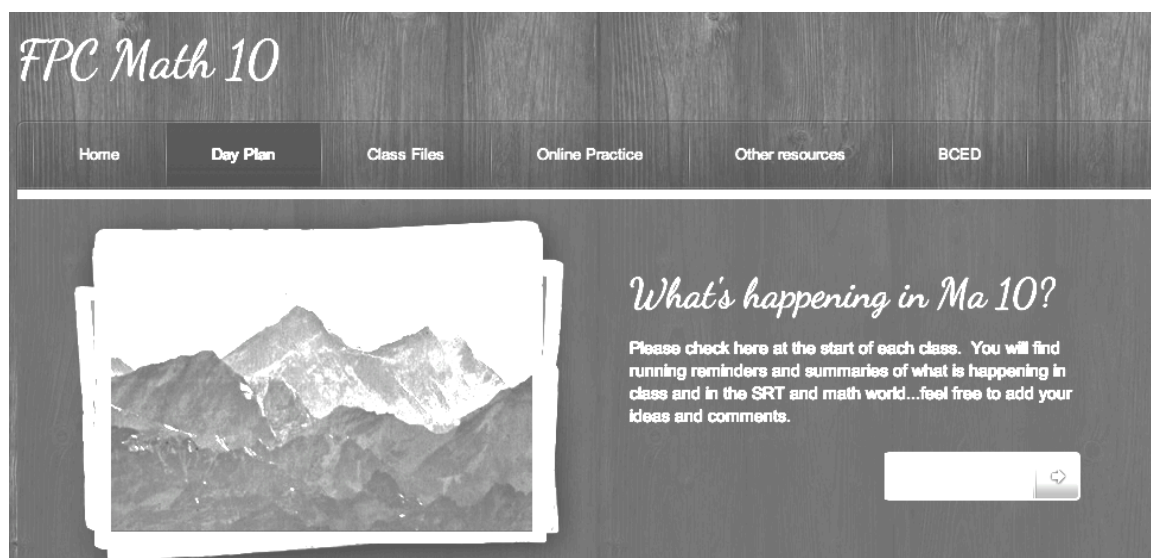


Figure 31. Class website used during lessons (AVO, May 2013)

During lessons, teacher and students were able to access the class website at their convenience.

They are now able to use technology to maybe watch a video or watch my notes online, I didn’t have that before... using technology I can give students a choice . . . there is just a lot more open choice with the iPad (AII, April 23, 2013).

The tablet was used to post notes, activities, examples, demonstrations, or other resources on the class website for reference by students. He did not use the tablet in his lessons.

The Camera app supported an activity outside of the classroom where students took photos of several objects on the school grounds. The U-clinometer app helped students approach measuring angles in the same activity.

The Explain Everything app facilitated students in importing photos and annotating them in order to create their own presentations (Figure 32).

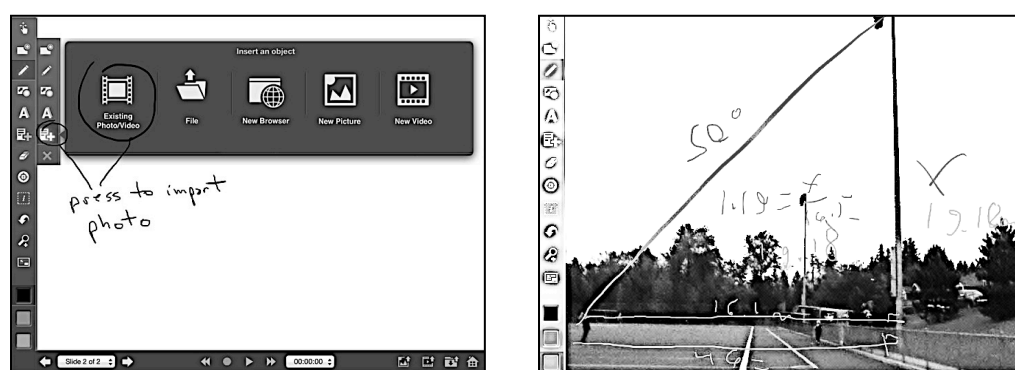


Figure 32. Demonstration of using Explain Everything app and example of students' presentation of work done in Explain Everything (AVO, May 2013)

Alex used the Explain Everything app during lessons for making demonstrations of practical applications of trigonometry, importing example pictures from the eTextbook, and also online pictures. This app also supported Alex's teaching by providing him with a board to write notes, share examples, write mathematical procedures, and to solve problems with students. Alex/students used the Explain Everything app to record work done during lessons and saved these files in Dropbox. Alex obtained evidence of students work during lessons through the Explain Everything app (during lesson and posted).

The use of iPads helped students to download samples of the provincial exams, and to practice several topic questions of the test based on their own interests. The Dropbox app allowed students to save and share files of work done during lessons. Alex

evidenced students' work done during lessons (solution to trigonometry problems, and solving question of the provincial test) through files saved on Dropbox and class website.

Constraints. Alex mentioned in this theme technical issues of the TT that limited his teaching, and also general concerns of working with TT.

Alex concerned about unexpected projector failures in regards to reliance on equipment (having trouble with his projector), wireless speed, and the time it takes to help students with the technology – including explain everything app (AJR, May 13, 2013). Also, he indicated that planning and posting on his website (the blog warm up activity, the lesson, and the wrap up activity) was time consuming for him. “Especially if you taught for a lot of years you might have a sentence in your daily plan. Like OK, angles, we are teaching angles today . . . where as now . . . I am typing every day” (AFI, June 19, 2013).

He mentioned limitations in his teaching in regards to disparity of interest and ability from students to post their own work either in Dropbox or in the class website (saving and posting files). He expressed that some students had difficulties using the Explain Everything app (importing photos, as well as, writing and doing the class work), and others were ‘technology phobic’, so not all of them completely adopted the use of the iPad.

“Some students posted a number of examples and some did not complete” (AJR, May 31, 2013).

“Many students had trouble with Dropbox, either posting their work or disconnecting their accounts” (AJR, May 22, 2013).

[Students indicated to Alex]. “I am not good at it for me to open up an app, and for me to save in Dropbox is too hard” (AII, April 23, 2013).

Alex expected that monitoring students' files posted on Dropbox was an easier task. However, he discovered that some students did not post their work, and in some

cases, he was not able to understand some of the students that posted their work (students used their fingers to write and the work was very disorganized). He was concerned that it is time consuming to provide individual feedback due to the amount of files to mark, either on Dropbox or the class website. Particularly, checking students' solutions to questions of the provincial exam.

One of my earlier goals was to try to monitor their learning [in Explain Everything app], and I found using Dropbox - where the students would save some of the work . . . to be really inconsistent. It is time for me to go back later on - and look at their Dropbox files . . . and they are not there. So, I do not know how meaningful that is. It is easy to glance [in Dropbox], that they've [students] placed something in there, but the quality of it for me . . . to actually mark it, and give them feedback [was difficult] . . . walking around the classroom, and watching, looking at them [students working on the iPad] . . . I think, I can get way more out of that, than using technology. So, I learned that (AFI, June 19, 2013).

Alex had trouble adapting himself to teaching in a science classroom because it lacked white boards, the distribution of the classroom was not appropriate for his teaching style, and he needed another projector. He indicated that a couple of classes were delayed because of problems setting the projector up.

I have the digital version [eTextbook] on my laptop and I can project that...also I can cut and paste a screen shot from my laptop, and then I can send it to my iPad because again I have trouble projecting with my laptop in that classroom (AFI, June 19, 2013).

Alex used the "Geometry Pad" app only to provide a brief explanation for measuring angles, and he identified that it was not appropriate for students' use. Also, he indicated that the U-clinometer app used to measure angles "was a little bit cumbersome" (AFI, June 19, 2013).

There was an app that I did not use very well, that I think it has a real potential, it's kind of live geometry or sketch pad. It is . . . I want to say to you . . . the 'Geogebra', but the one that is an actual app, that is really similar, we did not have the paid one . . . and I found it (Geometry Pad). It was not really good enough for the kids to use (AFI, June 19, 2013).

Integration. The theme integration was represented when Alex integrated TT (iPad apps, website, and tablet) in the context of the mathematics topic. This integration was focused on showing students “more real data, real life application in mathematics” (AII, April 23, 2013) in his teaching practices with the use of TT.

Alex's website is a compendium of an early TT integration where he used the tablet, computer, and Microsoft OneNotes. The tablet permitted to use the digital copy of the textbook (eTextbook) to capture images, examples, etc, and also to save files either on his website or Dropbox. Then he was able to take those files from there and use them on his iPad.

So, I would say for the past four to five years, I have used a tablet-computer, so like a Dell and HP, and I've used this regularly teaching mathematics, so I would write notes down using Microsoft OneNotes, and I would post my notes on my website. I can also do a few screen tests, it could be the lesson itself or it could be a tutorial (AII, April 23, 2013).

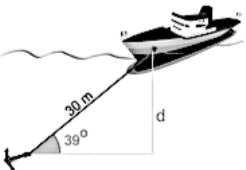
And then of course, on my tablet computer I was able to show. Actually, not so much in that class, if you recall. In that class my tablet does not sink up very well without a projector, so I was projecting using my Ipad a lot, but I do use my tablet a lot to record my screen casts and stuff (AFI, June 19, 2013).

Additionally, Alex integrated in his website the content of the topic mainly in the 'Day Plan'. He provided daily instructions of activities that students needed to follow during each lesson. Those instructions included a 'Warm Up' activity, the 'Lesson', and the 'Wrap Up' activity (Figure 33). The content of the topic was presented using several

examples of trigonometry from the textbook and also from pictures taken with the Camera app. Also, the provincial exam review was integrated with online resources posted on the class website.

Beyond Tangent

05/23/2013 0 Comments



Warm Up

Recall last day's lesson (or look below at some of the pictures). Next, open "explain everything" and write these four words in the corner of your screen: tangent, opposite, adjacent, angle.

Now on the middle of the page show how all of these words are related (use diagrams, explanations, examples, etc.). Save in your drop box when complete.


Lesson

Hold your finger on the image above (boat), then save this picture and open it in explain everything. We will use this as our fist example.

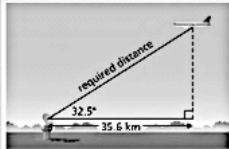
THINK ABOUT IT

Work with a partner.

The diagram shows measurements taken by surveyors. How could you determine the distance between the transect and the survey point?



Do the same with this image. Using one of the trig equations. Show all work in explain everything.



Now calculate the required distance in this image. Import, then show all your work on explain everything.

You can begin to practice your homework now - lesson #2 from your trig unit outline.

Wrap Up

Find at least one question on trigonometry from Sample Exam A. Complete, check and save it (in drop box). If you have time continue to work on sample exam A.

Figure 33. Structure of the ‘Day plan’ lesson on the class website (warm up, lesson and wrap up activities) (AVO, May 2013).

The home page of Alex’s website and the Dropbox app allowed him to gain access to students’ assignments. Alex integrated some assignments submitted in files and posted on either the website or the Dropbox as another source of examples presented in the content of the class, and also as evidence of students’ tasks (APO, May 2013) (Figure 34).

“That is how I used it mostly, so they could either send it to me through my website or they could it share it with me through Dropbox” (AFI, June 19, 2013).

“By circulating around the class I can see which questions the students select as well as notice any errors. Their solutions are also posted in dropbox” (AJR, May 13, 2013).

“The students wrote their work on their photo and then shared it in dropbox” (AJR, May 17, 2013).

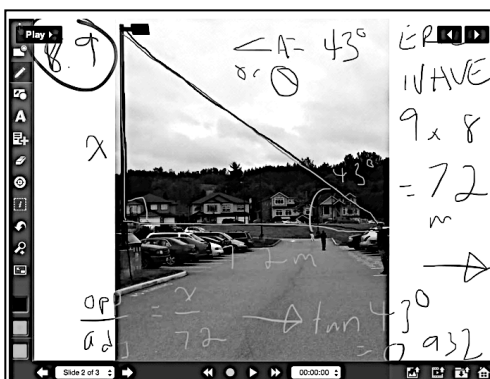


Figure 34. Examples of a student’s assignment in an activity outside of the classroom used by Alex as a resource for providing explanations during his teaching (AVO, May 2013)

The iPad was integrated with the development of the mathematical content through the apps (Camera, U-clinometer, Explain Everything, Dropbox, Calculator, and Geometry Pad). The Camera app was used for taking pictures of real objects that surround the students, rather than “drawing just an abstract triangle in the board” (AII, April 23, 2013). The U-Clinometer app was used for measuring angles of the objects in those pictures.

So by putting the camera back on, we will do an activity outside, and will take a picture of an object that is too tall to measure its height, and we will come back to the class, and then try to calculate its height with trigonometry (AII, April 23, 2013).

“This class was designed around the use of an iPad – for taking a photo and writing on the photo to find the height of an object on the school grounds” (AJR, May 17, 2013).

The “Explain Everything” app permitted integration of several features to allow demonstrations and examples for teaching the topics: imported pictures from the eTextbook, the school ground; allowed writing on top of the pictures; built a customized presentation; wrote demonstrations, solutions and explanations; and finally saved the edits as a file (Figure 35).

I did like the idea as using it [iPad] as a meaning device. I did like the idea of using it as a device where they can write on it. I use this app called “Explain Everything”- where the kids can write stuff on it (AFI, June 19, 2013).

So, I would have screen shots of problems and then I would put those on the screen, or I would put them on my app called “Explain Everything” and then I will do the problem in front of the kids on top of the problem (AFI, June 19, 2013).

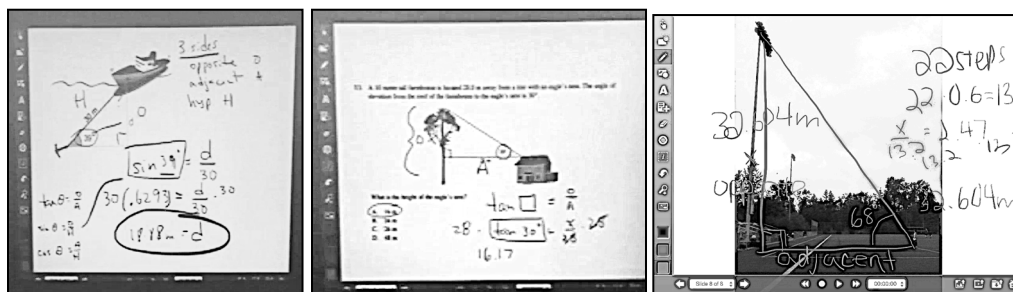


Figure 35. Alex/students using Explain Everything app integrating a photo imported from the eTextbook for solving a problem – an online picture (sample of the provincial exam review), and student’s own presentation from an activity outside of the classroom (AVO, May 2013).

The Calculator app was used for finding the measure of angles when Alex/students were solving trigonometry problems during a lesson, as well as for solving examples of the Provincial exam review. The Geometry Pad app was integrated by Alex just to show another way of finding the measure of an angle that could be used to solve trigonometry problems, but this app was used briefly during the lesson (Figure 36).

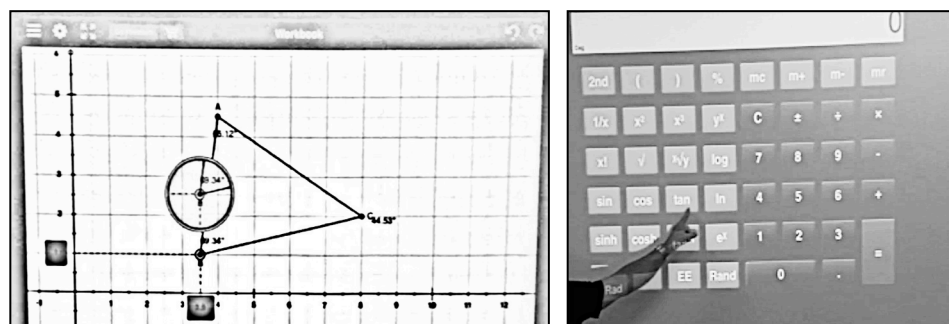


Figure 36. Alex showing how to find the measure of an angle using scientific calculator and Geometry Pad (AVO, May, 2013).

Interactions. The theme interactions constituted the manner in which Alex and students interacted with TT. Interactions in the classroom were produced between Alex-TT, student-TT, and Alex-students-TT.

Alex's class was part of the pilot project to provide an iPad for each student (1:1), and for that reason the interaction of student-TT started from the beginning of the lessons, when each student pick up the assigned iPad. Then, teacher and students interacted most of the time with an iPad to develop different activities during teaching lessons.

Alex interacted individually with the tablet, Microsoft OneNotes to post lessons, exercises, and notes in his website. He introduced the lessons interacting with the website through the iPad and the projector. Also, Alex interacted with the iPad apps (Calculator, Explain Everything, Dropbox, U-Clinometer, Geometry Pad) to develop activities of the lessons and at the same time explaining the use of the TT. Alex interacted mostly with the Explain Everything app to provide explanations, examples, justifications, notes, with the aim of either posting on his website or to address the topics during his teaching practices. The interaction with the Geometry Pad was brief and only to demonstrate

another way of measuring angles. Alex interacted with students during class, by checking students' work on the iPad, and solving individual or group questions. Sometimes, he had to take the iPad from a student to solve a question.

So, the students would not really interact with the technology itself except through the products that it produces. The students interact directly with the iPad, so they each are going to be given one, and they are going to use it to do a variety of activities, and hopefully those activities will make learning mathematics more effective (AII, April 23, 2013).

Sometimes the activity includes the use of the iPad first, sometimes not, and I will continue with that structure. The reason I show that is because I really feel that this is an opportunity, and I want to try to make the most of this opportunity (AFI, June 19, 2013)

Students interacted explicitly with the iPad apps (Camera, U-Clinometer, Explain Everything, Calculator, Dropbox), hard copy of the textbook, printed worksheets, and online resources provided by the teacher to obtain samples of the Provincial Exam. (Figure 37).

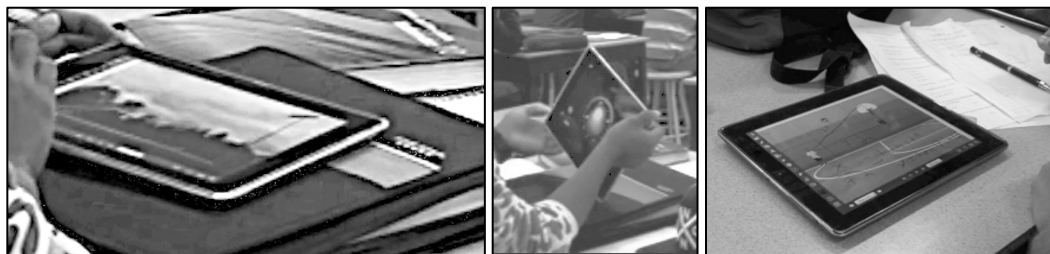


Figure 37 - Students interacting with the iPad during lesson activities (AVO, May, 2013)

“The tech tool’s main use was working on some examples from the text, on their device, and posting to Dropbox. Their practice was the textbook practice from their trig unit outline” (AJR, May 22, 2013).

“Practice again came in the form of textbook work – although they did need to work out some questions with me first. Their assignment/practice will be from the text from their unit outline” (AJR, May 27, 2013).

The students-teacher-TT interaction occurred at the beginning of the class, when the teacher and students used the iPad to check instructions of the lessons on the website. On several occasions during the development of the lessons, interactions were observed between students-teacher-TT including demonstrations of the use of the iPad apps, and solving students’ questions.

Right, so I kind of set it up now, so that my structure is the open assigned device, each of them and they are accountable for that device . . . and so when they come in they pick it up, they check what is going on in the class on my website, and then we embark on our activity of the day (AFI, June 19, 2013).

“Yes, the goal was for them to solve all types of trig problems and they did that using the iPads. They also used the calculator in the iPad and posted their examples online” (AJR, May 22, 2013).

Dimension 3 - Alex’s Knowledge of Learners’ Cognition. Alex focused KLC guiding students to discover basic concepts of trigonometry through practical demonstrations of trigonometry using TT, discussing with students examples of questions of the provincial exam review, and guiding students to apply basic concepts to solve problems. Alex assumed that students had basic knowledge for solving equations and students did not require previous knowledge of the use of TT (iPad apps and class website). Alex made demonstrations of the use of iPad apps during lessons and also posted instructions on the class website.

During lessons, Alex identified students’ understanding and difficulties of the topics by reviewing mathematical concepts and procedures of previous lessons, and by questioning the mathematical content of each lesson. The use of the Explain Everything app supported Alex’s teaching to obtain evidence of students’ understanding and

difficulties of the topic. Likewise, monitoring students' activities while they were working with TT, questioning students during lessons and circulating around the classroom for checking students' work was also conducted.

The use of TT in Alex's teaching allowed students to have more choices for visualizing practical applications of trigonometry and experiment with the mathematical content using different apps. The use of TT facilitates students to work independently during lesson activities and share work done during lessons on files posted on Dropbox or the class website.

Themes illustrated in this dimension included: Review, Validation, Deficiency and Assistance of TT.

Review. Alex illustrated in this theme the way he reviewed previous knowledge needed for learning the topic. He reviewed previous content knowledge using the iPad at the beginning of lessons and also when students were developing activities. He introduced the topic mostly through the 'class website' with the 'warm up' activity posted in the 'Day Plan'. The 'warm up' activity facilitated Alex to review the topic content by providing an explanation of what was done in previous lessons, by reviewing previous assignments and also by clarifying writing instructions in regards to the use of iPad apps needed for activities during the class.

Files posted on the class website and the assistance of the Explain Everything app allowed Alex to show to students how to access information (i.e., samples of the provincial exam review), how to review notes of previous lessons, or how to further review new topic content if desired.

I could continue to do, what I have done in the past with both, the iPad and the tablet, and that is through my website. There is going to be information for them to access that they can push themselves with, or that they can get from remedial resources (AII, May 2, 2013).

Alex spent time encouraging students to review samples of the provincial exam and clarify main topic concepts. The instructions posted on the class website and

explanations for accessing online resources with the iPad, assisted Alex in guiding students to make the review of the topics independently.

We talked about what my goals were for them, and I really try to tell them...that my goal for them is to be a little bit more independent. So, I spend a lot of time showing them that, if they need to access some information or have questions, they can do that through the iPad or through the internet. So that was one of my big goals (AFI, June 19, 2013).

I encouraged, but did not force, the use of online resources to practice (prep for their quiz as well) – many find Khan Academy interesting to begin with, but they [students] give up when challenged. I was hoping they would personalize their review and practice (AJR, May 29, 2013).

Validation. Alex validated students' understanding by focusing practical applications of the topic first and then formulating theoretical questions. "Wanted to see how the students reacted to an application before learning the concepts" (AJR, May 17, 2013). He validated conceptual understanding of the topic 'Introduction to trigonometry' identifying first "the concept of what ratio is, and what it can be used for" (AII, May 2, 2013) and applying the concept of trigonometry ratios for solving problems. In the topic of provincial exam review, he suggested to limit each unit to three main ideas for students selecting and practicing questions of the provincial exam review.

Alex verified students' understanding of the topic during the lesson reviewing previous concepts, discussing examples of applications of trigonometry, circulating continually around the class, questioning about the topic, and checking students' working with the iPad. He discussed some samples of students' work done during lessons in order to validate students' understanding.

In the topic 'Provincial Exam Review, he wants students to be independent and discover difficulties on their own. "My goal is to develop independent and self-aware students. By self-selecting the questions to review they must take some responsibility to identify what they are weak in" (AJR, May 13, 2013). He validated students'

understanding by encouraging students to share lesson activities with their classmates, ask questions, check answers, and post problem solving on Dropbox.

The Explain Everything app supported Alex's teaching to obtain evidence of student work done during lessons, and he called it 'accountability': "that whole piece of evidence at the end of the hour of what they've done and what they've learned" (AFI). He focused students' accountability to verify understanding of the topic. This accountability was placed on files when students submitted or saved their assignments and lesson tasks either on Dropbox or the class website. "There is an accountability where they have to show me that they transfer, what they learn on the iPad or what they practice on the iPad to learning mathematics" (AII, April 23, 2015).

Alex identified that students tend to validate their understanding of the topic when they compare notes and discuss with classmates using iPads, more than when they use the hard copy of the textbook. However, he found it easier, to validate students' understanding by circulating around the class, and watching students as "they worked through their work" than using accountability with iPads, in particular accountability of the provincial test review.

"I find students have a tendency to share when they are working with the iPads, kind of compare each other's notes and they talk a little bit, a little bit more than when they are working out of a textbook" (AII, April 23, 2015).

Deficiency. Alex identified students' difficulties by reviewing previous assignments, circulating around the class checking students' work, questioning students (individually and collectively) during lessons, and checking accountability of students' work on files. "By circulating around the class, I can see which questions the students select as well as notice any errors. Their solutions are also posted in Dropbox" (AJR, May 13, 2013).

He highlighted explicitly the following situations addressing topic content difficulties:

“Getting around the classroom as much as possible when they are doing their work” (AFI, June 19, 2013).

“Trying as much as possible to have stuff on my website that points out how to correct those issues” (AFI, June 19, 2013).

“Repeating, so every lesson there would be some kind of picture on my website about the day before and just repeating over and over again and not only because the vocabulary is new to them” (AFI, June 19, 2013).

“Identifying what they know or don’t know, repeating on the fundamentals and the basics, and a lot of kids going in for extra help” (AFI, June 19, 2013).

Alex observed that some difficulties in regards to teaching the topic content by using TT was the willingness of some students to adopt TT. “Some students are technology phobic, believe it or not, still at this age, where they don’t want to use an Ipad” (AII, April 23, 2013). Even though some students did not post accountability of lesson activities, Alex persuaded students to develop their lesson activities at their own level and pace allowing them to adopt more the use of TT. Many students discover difficulties for themselves using TT and they came to the teacher with specific questions. This helped Alex to discover deficiencies.

Alex identified students’ difficulties of conceptual understanding of the topic when he was checking accountability of some students’ work posted in files using the Explain Everything app. He pointed out: “some of the students wrote down the distance from the base of the pole on the line that was actually the hypotenuse and not the base of the triangle. So, I could see that was a misconception right away. That was helpful” (AFI, June 19, 2013).

Alex promoted independence and self-awareness to allow students to identify their own difficulties (reviewing the provincial exam and introduction to trigonometry). “My goal is to develop independent and self-aware students. By self-selecting the

questions to review they must take some responsibility to identify what they are weak in” (AJR, May 13, 2013).

Assistance of TT. Alex expressed in this theme benefits of using TT in his teaching practices in regards to students’ learning. He likes to use TT daily during teaching practices and his main goal “is to use it for discovering the principles, understanding conceptual, and looking how the mathematics is applied” (AII, April 23, 2013). Using the iPad apps in his teaching practices allowed more choices for students to visualize and develop understanding of the topic content in regards to the concept of ratios, “where they come from, what they represent” (AII, April 23, 2013); and learning about authentic applications of trigonometry, “The best way would be to use [the iPad] to learn about real applications of trigonometry” (AII, April 23, 2013). Also the use of an iPad promoted individualized instruction because students had the opportunity to select and practice by themselves several samples of the provincial exam review, and also construct their own examples of practical trigonometry applications: “TT allow a little bit more personalized approach, so there are activities that students can now do at their own pace, at their own level” (AII, April 23, 2013).

Alex indicated that it is crucial to develop students’ understanding of “how technology can help their learning, how technology is an asset for them, either to find remedial sources or to apply their learning” (AII, April 23, 2013). He indicated that the use of TT assisted his teaching in enhancing the visualization of trigonometry applications:

[Students] could visualize the applications of [trigonometry] more. Again, to me trigonometry is about pictures and diagrams and not equations, and so, I hoped that by embedding the diagrams in the textbook, on my projector, and in my lessons – that aspect of being a little bit more visual...helped (AII, April 23, 2013).

The use of the tablet helped the teacher to post several resources on the class website for “reference for students that have been away, or students that have difficulties and need to go back, and look on their own” (AII, April 23, 2013). However, Alex

mentioned in his reflections that he did not observe that students took advantage of this resource.

The use of Explain Everything app and the eTextbook assisted Alex to guide the review of the provincial exam by showing students how to access questions from samples of tests. Alex/students used examples of pictures from the eTextbook and from online samples of the test, and they were able to write on top of those pictures for solving trigonometry problems of the provincial exam using the Explain Everything app.

“To me geometric type problems should be diagram based and it makes more sense if the students have the actual diagram and can write on it. They then, saved it in Dropbox” (AJR, May 22, 2013).

“The iPads helped the students to learn how to download and practice the provincial practice test. They could also write on the screen over the question to show understanding” (AJR, May 27, 2013).

In the topic provincial exam review, using iPad apps permitted students to have more choices for selecting and practicing questions by themselves, sharing questions, and solving questions together with Alex during lessons (students worked using the Explain Everything app and posted files on Dropbox or the class website).

Dimension 4 - Alex’s Knowledge of Subject matter. Alex illustrated his KSM when he developed the mathematical content of the topics providing explanations, examples, making demonstrations and questioning students to obtain mathematical understanding (conceptual and procedural). “My goal is for them to have a good understanding of how it [trigonometry] is used and the skills to use trigo [trigonometry] to solve problems” (AFI, June 19, 2013). He used the iPad to present mainly the mathematical content of Introduction to trigonometry and also to discuss samples of the provincial exam review. He mentioned that he might teach any mathematical content using iPads, but that the use of the tablet is more convenient for the integration of the

mathematics curriculum because the possibility for saving files and the option to interact with files to develop lesson content.

Alex used TT (iPad apps, tablet, class website, and online resources) to teach both topics. This TT supported his teaching in an efficient way by first visualizing practical applications of trigonometry to obtain conceptual understanding of the topic. Then, he focused the use of basic concepts (Introduction to trigonometry and Provincial exam review) to solve problems.

The use of TT supported Alex mainly by connections between familiar examples of students' environment (i.e., pictures from school ground, online pictures and the eTextbook), and visualizing and linking conceptual understanding with procedural thinking. However, some students had difficulties transferring the knowledge obtained working with iPads to mathematical understanding. He indicated that the use of TT supported his teaching with conceptual thinking more than with procedural thinking. The use of TT assisted him in illustrating more pictures and practical demonstrations of the topics, and students could write on top of pictures showing their mathematical understanding. Also, the use of TT supported mathematical representations linking the mathematical content when teacher/students shared conceptual and procedural understanding through the eTextbook, online pictures, and their own trigonometry representations.

The themes represented in this dimension were: Visual connections, Disconnections, Technology aid, Mathematical thinking and Representations.

Visual connections. Alex made mathematical connections to the first topic 'Introduction to trigonometry' by focusing first on practical applications of trigonometry. He showed several applications of trigonometry to connect the concept of ratio and right triangles (angles and sides) to real objects (pictures of buildings, boats, etc), instead of first presenting basic trigonometry concepts: "[I] wanted to see how the student reacted to an application before learning the concepts" (AJR, May 17, 2013); "I've tried to stress two things and one of them was the concept that trigonometry is ratios, ratios in a triangle, so from that, I tried to get them to visualize it as much as possible" (AFI, April

23, 2013). Then, he focused on the importance of knowing the concept of ratio to solve trigonometry problems.

Alex linked trigonometry with the students' environment by having them take a picture of an object on the school grounds using the camera app on the iPad. Students imported the photo to the "Explain Everything" app, and they constructed their own trigonometry demonstration to find the height of the object they photographed. He made connections to trigonometry using examples from the eTextbook and other imported online pictures, highlighting triangles, and adding measured or found angles and side lengths. The most important aspect for the teacher was that students were able to connect or transfer their work done with iPads during classes to trigonometry concepts and use them to solve problems. Alex expressed, "having them draw triangles, [helps the students] draw relationships and see the power of knowing a simple pattern like a ratio and a triangle and how that simple pattern let you solve some big problems" (AFI, June 19, 2013).

Alex pointed out:

I hope that technology - by letting them record their understanding, by taking pictures that they see - that they are connecting, real type applications on the school ground. And then instead of just showing a picture on a piece of paper, you actually have the picture there, and they actually transfer the mathematics to real life, hopefully (AII, May 2, 2013) (Figure 38).

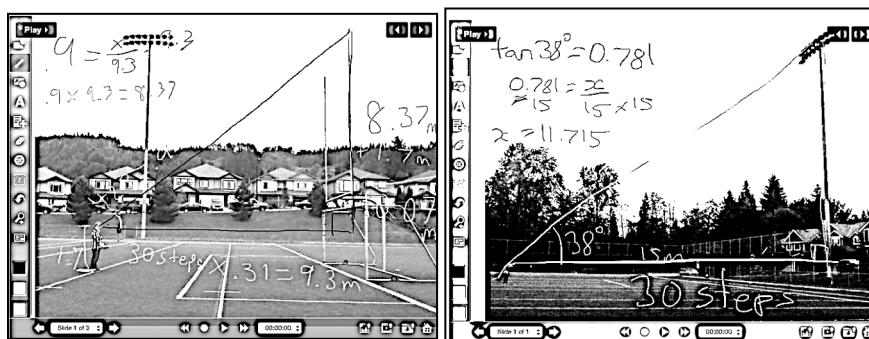


Figure 38: Examples of students creating their own trigonometry application (AVO, May, 2013)

Another aspect of making mathematical connections to the topic was using examples of modern construction to convey trigonometry applications. Alex highlighted the benefit of using technology for measuring angles, and he argued that students could apply that knowledge to modern building construction and to observe “the usefulness of technology for construction purposes and processes, etc.” (AII, May 2, 2013).

In the topic “Provincial exam review”, Alex used online samples to make connections between relevant concepts in the units under review (including trigonometry) (Figure 39). “I almost try to limit each unit to three big ideas and try to almost assess those separately, and to also stress what they are to them, and so, I try to organize those three ideas in the online practice” (AFI, June 19, 2013).

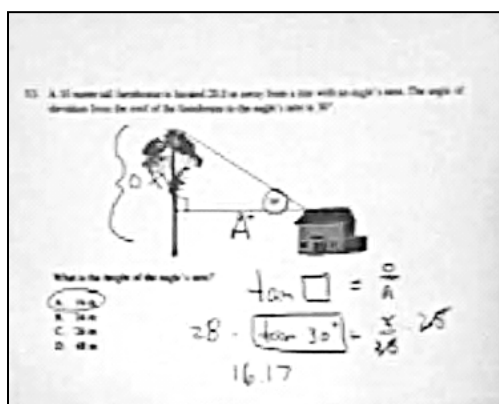


Figure 39 – Example solving a question of the provincial exam about trigonometry and using the Explain Everything app (AVO, May, 2013).

Disconnections. Alex found that some students had difficulties with the language of trigonometry because the vocabulary was new. “They have a hard time writing and pronouncing the new words (i.e., sine, cosine), struggle with the symbols (i.e., Greek letters for angles), and “mixing side versus angle, or the ratios” (AII, April 23, 2013). Also, some students had difficulties solving equations: “They struggle, even beyond the solving equations that is for sure . . . fractions [especially]” (AII, April 23, 2013). Many students required extra help and review/support. Alex needed to provide several examples to help students’ recall/understand the concepts associated with the new vocabulary and

symbols. He would provide visual applications of trigonometry for solving equations and explanations.

They have seen the word hypotenuse before, [although] they [may] have forgotten it, but the word ‘adjacent’ used in mathematics is new to them . . . tangent is a word that they have heard before but not applied to trigonometry . . . a lot of them were saying ‘sin’ instead of sine and . . . lot of them wouldn’t be able to write out cosine. [The review process requires] identifying what they know or don’t know, repeating . . . the fundamentals . . . and . . . [providing a lot of] extra help (AFI, June 19, 2013).

Alex identified difficulties teaching the mathematics content using the iPads. He indicated some students were passive when working with an iPad, others get distracted, while still others do not like iPads. He was concerned that they are not thinking deeply about the mathematical topics or building understanding. Some students had difficulties “transferring the procedures they have been practicing to theoretical mathematics” (AII, April 23, 2013).

On the topic ‘Provincial exam review’, the teacher had difficulties monitoring student work submitted through Dropbox or the class website because of the amount of time required to check files and provide students with individual feedback. Although he wasn’t able to completely review student work, he was able to share several samples of students’ work during lessons to support explanations and clarifications.

The teacher identified other underlying problems that some students struggle with, separate from issues related with the use of TT. Issues included misunderstanding of previous mathematics concepts identified during questioning and reviewing. An additional challenge of working with technology is overcoming the students’ frustrations and disappointment in previous classes. “I find a lot of students that struggle . . . Their confidence is so low, often my first step is just to build their confidence” (AII, April 23, 2013).

Technology Aid. The use of iPad apps changed Alex’s former pattern of introducing concepts first, followed by mathematical demonstrations. Now, with the use

of iPads in his teaching, he first introduces demonstrations of trigonometry and follows with the basic concepts. He thinks that this new approach helps the students better understand the nature of trigonometry by using iPads to demonstrate the application of trigonometry in authentic contexts and examine basic trigonometric concepts using visual representations.

Class examples were found in the school environment, in the textbook, and in online resources and were used by the students to construct visual trigonometry demonstrations. These in turn were used as resources to support the development of students' mathematical understanding through discussions of trigonometric concepts and procedures.

I think because we are using the technology tools, we want them to understand some other issues, for example how technology can be helpful in making mathematics more real for them, and also how maybe technology can be more helpful in terms of showing their understanding (AII, April 23, 2013).

I hope the fact that they've actually seen the triangles around in the school and in the environment . . . and they've now copied it onto their page, and they [have] done measurements, that they have [a] stronger conceptual understanding. Instead of me drawing just an abstract triangle on the board and saying this is a sine ratio. So, I am hoping for a better conceptual understanding (AII, April 23, 2015).

The combination of the iPad apps (Camera, Explain Everything, U-clinometer, eTextbook and online resources) helped students to construct and visualize basic trigonometric concepts (ratio, right triangles, sides and angles). The "Explain Everything app" allowed the teacher and the students to represent their mathematical understanding and procedures directly on an image from an authentic/familiar context. In turn, this helped to improve the students' conceptual understanding of the topic.

Students can take pictures, they could import these pictures and look for triangles, real triangles that are around us, . . . and by measuring those triangles and looking

at the ratios of those triangles they can . . . [build] conceptual understanding . . . and then we can formulate from that the [trigonometric] ratios (AII, April 23, 2013).

I believe that the tools helped – using photography to place students into the problem would not be possible without it. Their mathematical thinking was abstract – and then problem solving and patterns. They still need to work through the entire process to find missing parts of a right triangle (AJR, May 17, 2013).

In the topic ‘Provincial exam review’, the use of iPads helped students obtain online samples of the provincial exam questions, to personalize their selection of the review questions, to share their solutions of selected sample questions, and to self-identify gaps in their knowledge.

I believe the resources helped some . . . The mathematical thinking was different depending on the unit or questions that they picked. Overall my goal was to have more students share their work so they can see examples of success as well as examples of challenges – in other words sharing experiences with others may make them feel more comfortable (AJR, May 31, 2013).

“I believe that technology can be used to have students post and share their own work – it gives them accountability and it gives some students the choice of seeing [other] student[s] work out problems . . . ” (AJR, May 31, 2013).

The use of Explain Everything assisted Alex and the students by importing examples of problems with pictures from the eTextbook and samples of the exam review. They had the opportunity to draw and write on top of those pictures for solving the problems, and showing their mathematical understanding.

“To me geometric type problems should be diagram based and it makes more sense if the students have the actual diagram and can write on it” (AJR, May 22, 2013).

“The iPads helped the students to learn how to download and practice provincial exam practice tests. They could also write on the screen over the question to show understanding” (AJR, May 27, 2013).

Mathematical thinking. After demonstrating the application of trigonometry, the teacher focused the mathematical thinking (concepts and procedures) of the topic ‘Introduction to Trigonometry’ first on conceptual understanding of and then approached procedural thinking. He presented real life applications of trigonometry and from there focused on key visual elements needed for discovering basic trigonometry concepts: “I think for me it was the idea of a little bit of the applications of it. This unit, I try to focus on the applications of it, and for them to visualize, especially visualize a right triangle” (AFI, June 19, 2013).

The use of the iPad was very helpful to introduce the topic with more examples and diagrams of applications of trigonometry. Teacher and students were able to discuss basic concepts of trigonometry using those examples and were supported in their understanding by writing on top of the diagrams.

I tried also to get them to visualize a little bit more, how important it is to use, you know, how to write a right angle triangle on a diagram right from the start. Often what we’ll do is start with the concept of a ratio, and then look at the applications later. I kind of flipped it around because of the technology this time (AFI, June 19, 2013).

Alex gave students time to explore and think about conceptual understanding of trigonometry, helping students to visualize right triangles, and to discover patterns between ratios by themselves. “I was trying to get them a bit more of chance to discover for themselves the applications of trig, instead of me, pointing out where they are once

they learn how to do trig” (AFI, June 19, 2013). He supported the students’ developing conceptual understanding by letting students discover the concept of ratios, guiding them in how to find sides, how to find angles and then working with more complex questions using more than one triangle. He mentioned explicitly the importance of students obtaining mathematical understanding of the topic using the iPad: “I just have to make sure that their mathematical understanding is improving, not just their engagement in the class. So, I am still thinking about it”.

Alex used the “Explain Everything” app to annotate pictures and diagrams, which facilitated students’ mathematical thinking. It was reflected when students pointed out their conceptual understanding on the diagrams (ratios, sides, angles) and also when they developed mathematical procedures for solving trigonometry problems (writing on the screen over the question or diagram):

When they are solving problems . . . and again hopefully by them actually drawing a triangle on a diagram on an iPad. I hadn’t [done] that before, because we don’t allow the kids [to] draw in their textbooks . . . hopefully that would help them in their mathematical thinking later (AFI, June 19, 2013).

He also mentioned that the use of iPad apps allowed students to manipulate diagrams for solving problems and this helped to enhance conceptual understanding: “I think the concepts were enhanced, by again seeing the diagrams and seeing the applications in the diagrams and drawing on the diagrams” (AFI, June 19, 2013). However, he did not think that TT supported procedural thinking: “I think I could have written just the steps on the board for them to do. So, I don’t know if the technology really helped that very much” (AFI, June 19, 2013).

Procedural understanding of the inverse operations was relevant for solving trigonometry problems. “I try to show my students patterns in mathematics – such as showing them inverse operations to solve for y , then getting them to predict how to solve for y if you are looking for an angle in a trig. equation”(AJR, May 27, 2013). It was also important to practice exercises for obtaining procedural understanding. The teacher used the textbook and printed worksheets for practicing the topic: “Practice again came in the

form of textbook work – although they did need to work out some questions with me first. Their assignment/practice will be from the text from their unit outline” (AJR, May 27, 2013). Students were able to choose whether they used iPads or paper for practicing mathematical procedures:

If they are actually solving an equation or they’re doing their homework or just practicing for an assignment, they were readily encouraged to use paper. They don’t need to use the iPad for that. I don’t think I’d force them to do that again unless I want them to write on top of a picture which would be easier on iPad (AFI, June 19, 2013).

In the topic ‘Provincial exam review’ the teacher focused on conceptual understanding within the various units delimiting them to three big ideas and highlighting the bigger concepts for each unit separately. The teacher guided mathematical thinking showing several examples of the exam and also by solving some questions with students. The teacher also encouraged students to share examples of problem solving by inviting them to post their work in Dropbox or on the class website. In this experience, the teacher approaches conceptual and procedural thinking while at the same time discussing those examples during the lessons and providing general feedback to students. Students had the opportunity to develop mathematical thinking for the provincial exam by selecting, exploring and solving several questions by themselves and also sharing experiences with their classmates.

Representations. Alex introduced the topic ‘Introduction to Trigonometry by displaying a series of ‘Google Images’, and asking students to visualize common features of the images that illustrated trigonometry. By discussing implicit information on those images, Alex and the students developed general concepts about the unit (Figure 40).

Alex and students made representations of the topic using the Explain Everything app. He used images imported from online resources and their eTextbook to share examples, explanations and demonstrations of the topic. Having students draw on top of the images was very important to help visualize main concepts related with the topic and

for solving trigonometry problems. Also, they used Explain Everything to share mathematical procedures for solving problems (Figure 41).

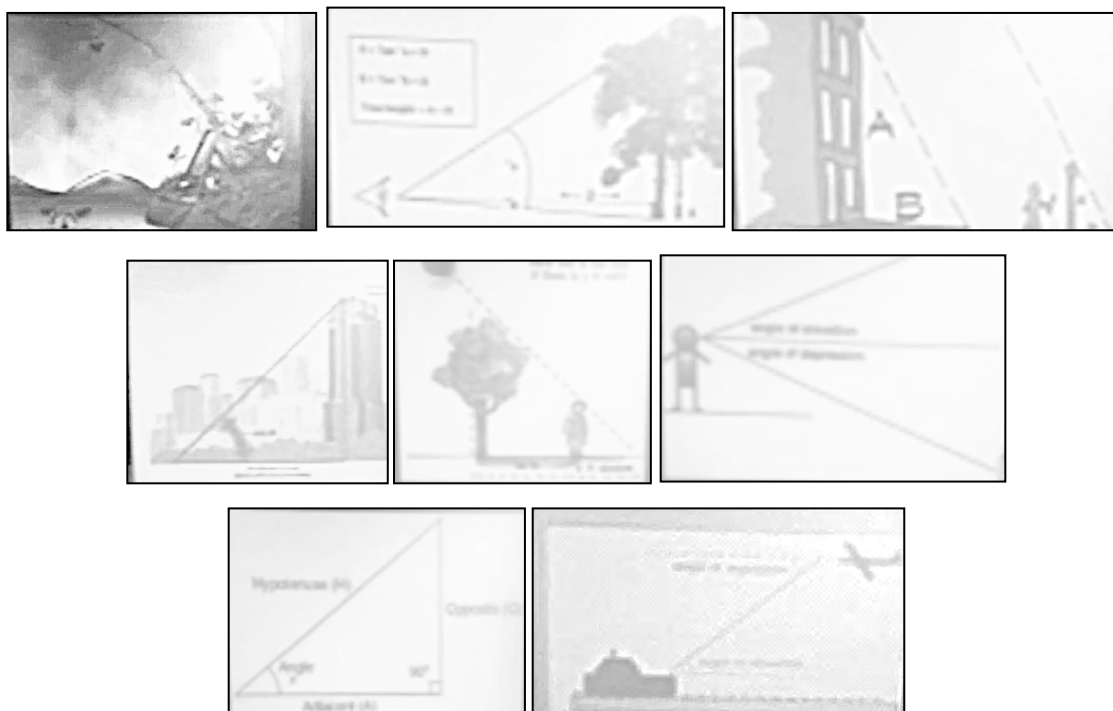


Figure 40 - Examples of online images used during a lesson to introduce the trigonometry topic (AVO, May, 2013).

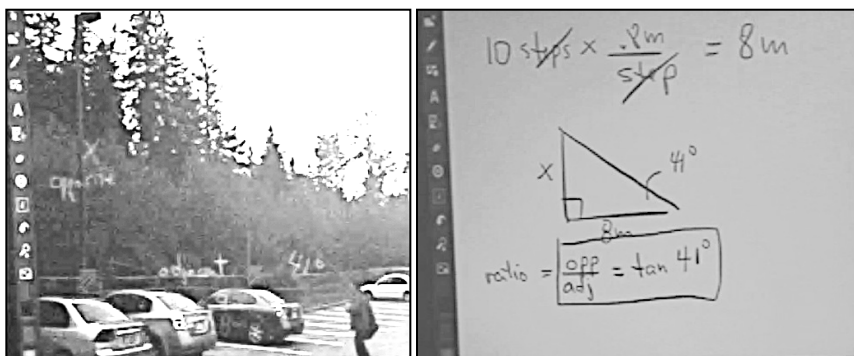


Figure 41. Example of students (creating their own trigonometry application) and Alex (writing procedures solving a problem) using Explain Everything app (AVO, May, 2013).

Students created their own trigonometry representations by finding the height of objects found on the school grounds. The teacher used some of those examples to provide representations of trigonometry applications (Figure 42).

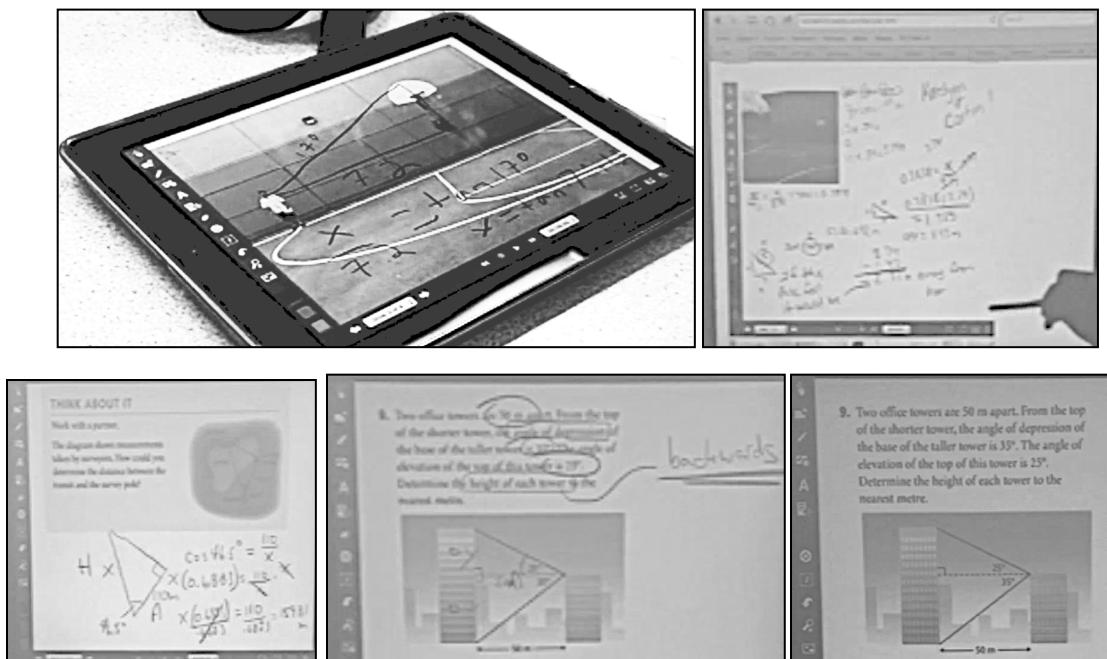


Figure 42: Representations of trigonometry applications (AVO, May, 2013).

Dimension 5 – Alex’s Knowledge of Pedagogy. Alex’s KP was represented when he expressed his teaching perspective, the strategies for developing the mathematical content of the topics, the way he obtained students’ attention while reviewing, explaining instructions on the class website, developing lessons activities and having control of students’ behaviour during lessons. Alex influenced his teaching mainly through a constructionist perspective. He guided students with the goal that they would discover and construct their own understanding of the mathematical topic with the assistance of TT (iPad apps, class website, eTextbook and online resources) during lessons. “As a mathematics teacher - I strongly believe in the constructionist theory. On that, if it is going to have meaning and be long lasting, they have to construct their own understanding” (AII, May 1, 2013). Also, he agrees with a sociological perspective in regards to the importance of the classroom culture established and interactions produced (Alex – TT – students) in a classroom that could be crucial for learning a topic. In particular he indicated:

At the same time . . . I also believe there is sociological -where they are going to be getting their understanding. Also by how they interact, so your classroom culture is really crucial, so if you set up a culture where errors are not celebrated but learned from and there is no negative punitive feeling from it - that it's, just part of the learning process, then hopefully that will help them (AII, May 1, 2013).

The main pedagogical strategies identified in Alex's teaching practices were that students experimented independently with the mathematical content supported by the use of TT. He highlighted the strategies of engaging students with open questions, monitoring their learning, circulating around the classroom, and providing students' feedback while students were working with iPads.

The mathematics topic was presented in a logical sequence. The curricular planning was very well organized and it was illustrated in the class website at his convenience. Alex introduced lessons discussing first with students the general aspect of the topics and then proceeded with lesson activities. He focused his teaching on the topic of 'Introduction to Trigonometry' in most of his lessons. He used the iPad at the beginning of lessons to review students' previous knowledge by asking whether difficulties were encountered with assignments, clarifying students' questions and discussing the activities of the new lesson posted on the class website including demonstrations of how to use iPad apps or online resources.

Sometimes, he started the lessons teaching first the topic 'provincial exam review' and other times it was teaching at the end of the lessons. He showed students samples of the exam, illustrated guidelines for selecting exam questions, and solved particular trigonometry problems. Students had to work independently to develop the topic 'provincial exam review' during lessons.

On the topic 'introduction to trigonometry', Alex focused his teaching to experiment with the mathematical content assisted with the use of TT, mainly by the Explain Everything app, and the Camera app for importing pictures from the eTexbook, school ground and online resources to make practical demonstrations of trigonometry. He

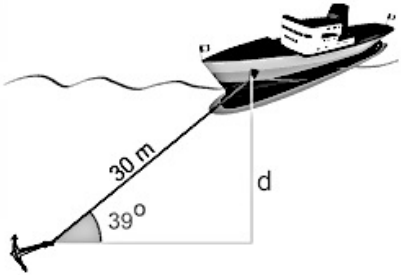
supported students with verbal and writing instructions (class website) to guide them to develop the activities and to work independently during the lessons. The use of the Explain Everything app assisted Alex's teaching for solving problems focused on visualization of mathematical representation of trigonometry for identifying concepts and applying those for solving problems. At the end of classes, Alex discussed new assignments for practicing the content that was taught, provided online resources for further lessons, and highlighted the conceptual understanding for solving problem.

Themes represented in the dimension KP included: Questioning, Experimental learning, Individualization, and Pedagogical tools.

Questioning. Alex focused his teaching on 'Introduction to trigonometry' and "Provincial exam review", by first discussing with students the goals for the lesson, taking instructions posted on the class website (warm-up activity, lesson, and wrap-up activity) and doing demonstrations. He clarified activities during the lesson. In this way, Alex developed the lessons by questioning and interacting with students (posted in class website).

In the warm-up activity (class website) Alex questioned students mainly about previous assignments and reviewed previous lessons (Figure 43). He brought up student samples and clarified students' misunderstandings (i.e., amended some mistakes of students' work posted in Dropbox or class website), answered questions related to students' difficulties with the mathematical content, and discussed technical aspects of students working with TT (i.e., difficulties related to saving and posting work done on classes in Dropbox and class website).

05/23/2013 0 Comments



Warm Up

Recall last day's lesson (or look below at some of the pictures). Next, open "explain everything" and write these four words in the corner of your screen: tangent, opposite, adjacent, angle.

Now on the middle of the page show how all of these words are related (use diagrams, explanations, examples, etc.). Save in your drop box when complete.

Figure 43. Example of the ‘warm up’ activity posted in the class website (AVO, May, 2013).

According to the lesson plan, the teacher discussed ‘the lesson’ activity (also posted on the class website) questioning students in order to obtain students’ understanding of the instructions. During the development of each lesson, the teacher and students interchanged ideas related with the activities and also with the mathematical content of the topics. Alex solved individual and collective questions using the iPad or textbook. He considered monitoring a crucial strategy in his teaching and demonstrated this by circulating around the class, posing open questions, and providing students with feedback (Figure 44).



Figure 44. Alex and students interacting with the mathematical content and also with the use of TT – Mixed media sketch of Alex and students (AVO, May, 2013).

“I also believe that getting around the class and giving students feedback is key, so giving them a variety of questions to work on - allows me to circulate more” (AJR, May 22, 2013).

On solving problems, the teacher thinks that it was a useful strategy to ask open-ended questions. He encourages students to think more on their own, to employ different options for solving problems, and to even learn even making mistakes.

“As much as possible having the students think more and not just watching you think is essential . . . posting problems for the kids, letting them try different things, letting them make mistakes and then talking about that afterwards is essential” (AFI, June 19, 2013).

Alex considered it a good strategy to 1) examine students’ work done with iPads during the class and 2) use questioning to clarify misunderstandings. He guided students to correct mistakes by individually/collectively questioning them, and posting resources and examples on the class website to clarify misunderstandings.

I think first of all it is knowing that they are having difficulties and that is just getting around the classroom as much as possible when they are doing the work. And then addressing those difficulties when they appear and also trying as much as possible to have stuff on my website that points out how to correct those issues (AFI, June 19, 2013).

Experimental learning. Alex approached the topic ‘Introduction to trigonometry’ by making mathematics more experimental for students. He introduced the topic by showing pictures and diagrams, and augmented them with mathematical notations and representations to guide students in visualizing and discovering trigonometry applications:

In terms of my teaching style, I was trying to get them a bit more [of] a chance to discover for themselves the applications of trig instead of me pointing out where they are, once they learn how to do trig (AFI, June 19, 2013).

“[Using TT in my classroom] does allow students to visualize on their own a little bit more what the applications are for trigonometry” (AFI, June 19, 2013).

He proposed several activities where students had to produce mathematical work from trigonometry demonstrations using the Explain Everything app. This app assisted the teacher in making mathematics more real for students by importing images within their own environment, online resources, and eTextbook examples: “I think, one of my big thoughts for [students] is try to show them how mathematics has a real life application. And, I think I could do it easier with an iPad” (AII, April 23, 2013). A specific activity done in class that illustrated making mathematics more experimental occurred when students explored and visualized the applications of trigonometry by taking pictures of objects on the school grounds. They analyzed the pictures, using the ‘Explain Everything’ app to demonstrate how they would calculate the height of the object.

Under pedagogy as mentioned above my goal was for the students to see the patterns and appreciate the applications of trig right at the beginning of the unit (and not wait to apply it until the end). This requires a bit more patience on both my part and the students – but I did like some of the intelligent questions that came out of today’s lesson – I will definitely try this again (AJR, May 17, 2013).

Alex illustrated the utility of mathematical representations for solving trigonometry problems and he guided students to experiment with graphics, pictures or diagrams to understand the problem and also to do mathematics on top of the diagram for solving the problem. The teacher gave example explanations and demonstrations and then he asked students to practice by themselves using the iPad.

So, I would have screen shots of problems and then I would put those on the screen, or I would put them on my app called “Explain Everything” and then I will do the problem in front of the kids on top of the problem and these are questions out of graphics that I read out of the book, and again my goal is to also

show them that when you are problem solving you want to use a diagram right above the diagram and not just look at the diagram and be frustrated by it being in the textbook, so I used the digital copy of the textbook to embed some of the examples and project those examples on the screen with my students (AFI, June 19, 2013).

Students experimented with mathematics by working on practical applications of trigonometry while visualizing the topic, discovering, and applying basic concepts of trigonometry for solving problems. The teacher focused the use of the iPad for students on “discovering the principles, understanding conceptual, and [seeing] how the mathematics is applied” (AII, April 23, 2013). Alex’s approach is to guide students to develop an understanding of “how technology can help their learning, how technology is an asset for them, either to find remedial sources or to apply their learning” (AII, April 23, 2013). However, he found that some students were not motivated to fully engage during lessons, even though he tried to approach the topic in a different way using TT. “I still notice that motivated students do the work (at any time) versus unmotivated students – who won’t do the work during a flipped class much” (AJR, May 29, 2013).

In regard to “Provincial exam review” the teacher provided specific instructions (posted on his class website) for students to experiment with the selection of specific questions in each unit in order to review the exam by themselves: “Under pedagogy my goal was for the students to do some self-reflection and decide on their own which topics/questions to review” (AJR, May 13, 2013), “Wanted to reinforce that they can download and practice provincial exams on their own. I like how they can quickly personalize which questions to work on” (AJR, May 27, 2015).

Alex mentioned the advantage of teamwork for students experimenting with different levels of questions. Students selected and solved sample exam questions on the ‘Explain Everything’ app. He asked students to share their work by posting files on Dropbox or the class website. The work they had done on the Explain Everything app might help other students obtain a reference point for solving a problem or to discover their weaknesses.

“Overall my goal was to have more students share their work, so they can see examples of success as well as examples of challenges – in other words sharing experiences with others may make them feel comfortable” (AJR, May 31, 2013).

“Hopefully they are interacting more directly with the questions and not simply going right to the abstract, where they have to write out the equation and solve it” (AJR, May 27, 2013).

Individualization. Alex expressed that the use of iPads helped him to individualize and diversify learning for his students. He guided students to be more independent by developing activities during class and also by finding technological resources (iPad apps, class website) to support their learning.

I really try to tell them that my goal for them is to be a little bit more independent. So, I spend a lot of time showing them that if they need to access some information or have questions they can do that through the iPad, or through the Internet. So, that was one of my big goals. I did a lot of talking and demonstrating about that (AII, April 23, 2013).

Alex encouraged individualized learning in his teaching using a class website where students have access to lessons, notes, and online resources at any time (Figure 46). Both the students and teacher interacted individually with this website during lesson development and sometimes afterward for assignments. Students developed activities during lessons at their own pace, and at their own level (i.e., selecting and solving questions/problems of the provincial exam and also developing lesson activities posted on the class website). “Under pedagogy my goal was for the students to do some self-reflection and decide on their own which topics/questions to review” (AJR, May 13, 2013). “... by simply investigating on the Internet some images of what “trig” is – I was hoping to again develop some independence and also to get them thinking on their own what this unit will be about” (AJR, May 13, 2013).

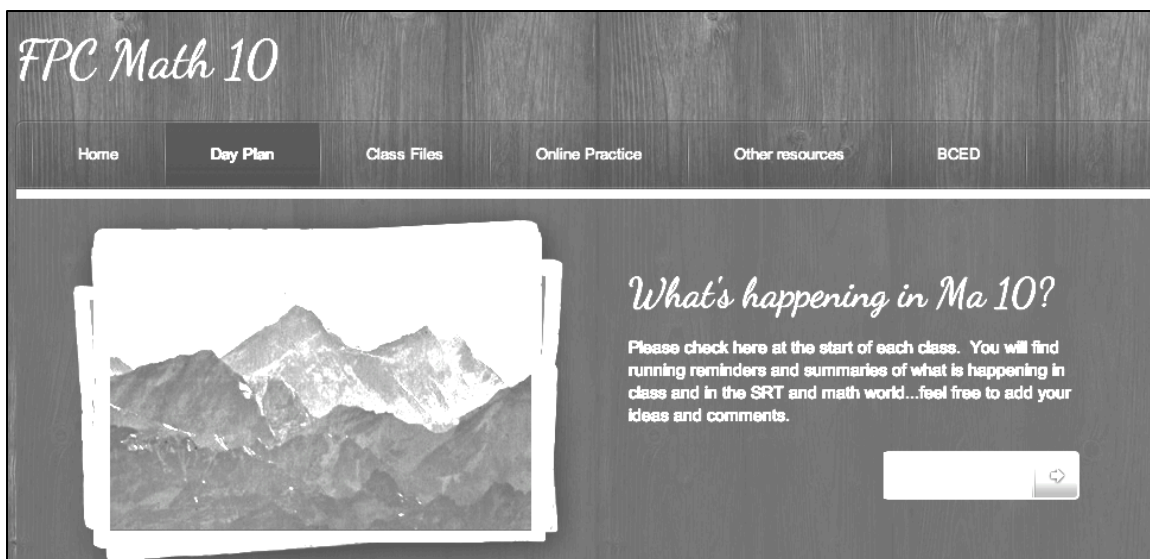


Figure 45. Class website (AVO, May, 2013).

Students also had available resources posted on the class website that could assist or to go further in their development of other topics.

That is really one of my big goals. So, if a student is stuck, or if a student wants to go ahead, they can just access my website. They can go back and look at the notes from the previous lesson. They can look at the previous topics - all that kind of stuff. Or the students that want to work ahead, they know that I have under my online practice - they can actually go and watch a video on Kahn Academy and see what the next unit [is] . . . (AFI, June 19, 2013).

Alex used TT in his teaching for diversifying learning and pointed out “My classroom is now designed for more diversity in the classroom” (AFI, June 19, 2013). The class was structured around the use of an iPad considered by the teacher to be an ‘open assigned device’ for each student. Students were responsible for the device and the class website was designed to support a range of students’ needs:

Through my website, there is going to be information for them [students] to access. They can push themselves . . . or they can get [help] from remedial

resources. So, I think for me personally I am also trying to use the technology to handle the diversity in the classroom (AII, April 23, 2013).

He focused the diversity in the classroom by providing more students' choices and experiences of doing practical demonstrations of trigonometry. Students had the choice of watching lessons at any time, reviewing notes made during classes and also using remedial resources posted on the class website.

So students that need remedial assistance, they are now able to use technology . . . maybe . . . watch a video . . . or watch my notes online - I didn't have that before. They just put up their hand and they would have to listen to me or [they could] ask a peer. There are also activities that students can do . . . if they finish early. So, handling diversity is a little more personal (AII, April 23, 2013).

Alex mentioned the importance of being conscious of the culture in a classroom for accepting diversity when using TT and the need to be prepared with a plan to address this need.

Accepting it [diversity] and not trying to isolate kids, . . . some students are not going to understand it right away. So, you need a plan for that, just like you need a plan for the kids that get it right away (AII, April, 23, 2013).

The teacher identified that some students do not like technology and he considered a good strategy to try to expose them to the use of TT and provide first the opportunity that students adopted TT in his teaching.

Some of the students do not like technology believe it or not, but I still want to expose them to it, because of the fact that if they can use it and adapt it to their own learning styles. It could assist them . . . (AII, April 23, 2013).

Pedagogical tools. Alex illustrated the use of iPad apps (Camera, Explain Everything, Dropbox, Geometry Pad, U-Clinometer) and the class website, as tools that helped him manage pedagogical issues including: planning, reviewing previous knowledge, visualizing concepts needed for understanding the topic, constructing authentic trigonometric demonstrations, showing more examples during their lessons, sharing students' work done in class, representing concepts on pictures and solving trigonometric problems.

“I believe that the tools helped – using photography to place students into the problem would not be possible without it” (AJR, May 17, 2013).

“Yes, thought of ways that I could use the iPads for activating prior knowledge. (AJR, May 13, 2013).

Alex designed his own activities for using TT (iPad apps, tablet and class website) in his teaching practices. He guided the technical use of iPad apps during lessons concurrently with the development of mathematical content. Alex used a bootstrap process to develop his skills with the iPad and create useful classroom activities.

I was kind of unsure how to use iPads – when you are teaching you don't have a lot of time to sit down and read other blogs and copy other things that people are doing . . . A lot of it was fairly spontaneous (AFI, June 19, 2013).

With iPads Alex was able to easily create an activity that took students outside of the classroom. As an example, the Camera app was used to obtain pictures of the school grounds and the U-Clinometer app to measure the angle of inclination. Interestingly, he discovered that the U-Clinometer was not precise enough to measure the angle of inclination and he planned to use also a protractor to be more accurate in the future.

“ . . . We will do an activity outside, and will take a picture of an object that is too tall to measure its height, and we come back to the class, and then try to calculate its height with trigonometry” (AII, April 23, 2013).

“I think, I would add estimating the angle next time, and perhaps add measuring with a protractor (as that is a learning outcome on it’s own)” (AJR, May 27, 2013).

The class website was used as a pedagogical tool to support the introduction and presentation of lesson activities, to review trigonometric concepts and to access online resources. Alex recognized one benefit of using his class website to support lessons was that students could access the content of the lesson at any time

“I see our actual mathematics, our lessons should be online, accessible 24 hours a day . . . So the potential for organizing your lessons and even for doing a lot of things outside of class time is huge . . . and I don’t see . . . going back” (AFI, June 19, 2013).

The class website was a channel of communication between teacher and students to support discussions of daily plans, submission of student assignments, and access to grades.

The Explain Everything app was a very important tool both in supporting Alex’s preparation, construction of activities, and in supporting class activities. During the lessons it helped by allowing students to capture pictures from the school grounds or the diagrams from the eTextbook, and then provided them with a tool to record their work as they completed examples from the eTextbook, generated trigonometric demonstrations, solved trigonometric problems, and solved questions from the provincial exam review samples. Also, this app supported Alex’s teaching as a virtual whiteboard for sharing conceptual ideas, examples, justifications, and procedures for solving problems.

He pointed out the need for students to better organize the work done in Explain Everything and to improve their writing. “Students find it challenging to write on the

iPads . . . poor and very large hand writing” (AJR, May 22, 2013). “Students are fairly comfortable doing mathematics on the iPad but again they are not organizing their work in their dropbox” (AJR, May 27, 2013).

The Dropbox app was used as a pedagogical tool to capture the evidence of students’ work done during lessons. The accountability of this work was crucial for Alex to identify during lessons students’ understanding the level of difficulty of the topics. “Accountability is one I’m still working on. You know, where, that whole piece of evidence at the end of the hour of what they’ve done and what they’ve learned” (AFI, June 19, 2013).

He used some files posted on Dropbox to provide feedback to students and to review assignments during lessons.

“I also believe there has to be accountability, so I will be looking at students’ dropbox folders to see who actually completed the activity” (AJR, May 22, 2013).

“ I believe that technology can be used to have students post and share their own work – it gives them accountability and it gives some students the choice of seeing a student work out problems that they may share” (AJR, May 31, 2013).

Alex used the eTextbook as a source of examples – capturing and posting trigonometric problems on the class website. Students preferred to use the textbook for practicing exercises instead of the electronic copy. “. . . Students can have access to the digital version... they can get it through their iPad, or they can get it at home. It requires a license, and we have an agreement for a license, but I just find the students are not interested in it, they would rather have a hard copy of the textbook” (AFI, June 19, 2013). He was autonomous in lesson planning and he approached the topic in a different way than suggested by the textbook.

Alex mentioned other pedagogical factors related to the classroom. It was a science classroom with large tables and one screen, meaning that he was not able to share the screen from the iPad and tablet at the same time. Also, the lack of whiteboards in this

classroom limited his ability to supply students with continuous access to required supporting information (e.g., relationship between ratios).

“ . . . Most classrooms have white boards all over the place . . . I couldn't put up student work, . . . I could have easily printed off a couple of those trick pictures that the kids did and I could have put that up” (AII, May 1, 2013).

Discussion and Interpretation of Findings

In this section, the utility of the proposed PCK-EC model for understanding Mary's and Alex's PCK while teaching specific mathematics topics with the use of TT will be discussed. This discussion will include the Educational Context, and the dimensions identified within Mary's and Alex's PCK-EC. Next, possible answers to the two research questions proposed in this study will be presented.

1. How effective is the proposed model PCK-EC (Figure 2) in supporting the examination of teacher practice and providing insights to the pedagogical content knowledge?
2. How do different TT mediate or affect teachers' PCK-EC when mathematics teachers are teaching a particular topic?

Finally, in order to enhance understanding of Mary's and Alex's PCK-EC, the inter-relationships between themes across dimensions will be discussed.

Understanding Mary's and Alex's PCK-EC

Understanding a teacher's knowledge for teaching a specific topic with the use of technological tools (TT) requires us to link the teacher's theoretical knowledge and teaching practice, but this has been difficult because "for many teachers, their practice and the knowledge/ideas/ theories that tend to influence that practice are often tacit" (Schön, 1983; Loughran et al. 2004). To facilitate this exploration it is important to develop a theoretical framework and a collection of prompts to support teachers as they describe their knowledge and attitudes with respect to the use of TT in their teaching practice. Our first goal in examining the proposed PCK-EC model was to identify the Educational Context (EC) and to describe each one of the dimensions of the PCK-EC by identifying useful themes within each dimension.

Educational Context. The PCK-EC model is proposed as a framework to support teachers, coaches and researchers in the examination of teachers' Attitudes and Knowledge within an Educational Context (EC), and using a particular focus. Considering that any component of teacher knowledge needed for teaching is situated in

an educational context, the EC was defined as the external structure that influences the manner by which a teacher's Attitudes and his/her KT, KLC, KSM and KP are embedded in the practice of teaching a specific mathematics topic in a classroom. The inclusion of the Context as external factor that influences teachers' knowledge has been advocated by different authors (Stoilescu, 2015; Loughran et al. 2004; Anderman & Anderman, 2000; Barnett, 1999). However, there has not been a systematic inclusion of the context in TPACK framework publications and the nature of context has had different meanings in those publications (Porrás-Hernández & Salinas-Amescua, 2013; Rosenberg & Koehler, 2015; Kelly, 2010). In this research, it is proposed that the EC includes: the curriculum and education standards; social, cultural and economic context; teaching and learning conditions defined by the school district; and teaching conditions in the classroom.

According to Doering et al. (2009) it is necessary to include teacher's knowledge in an EC. The knowledge that a teacher uses "depends on varied factors including the specific classroom culture, students characteristics, school and district policy, and numerous other factors that can neither be predicted nor accounted for a priori" (p. 336). The EC for Mary and Alex was situated in the same municipality and school district within the Province of British Columbia. Both teachers are governed by the same curriculum, and share the same social, cultural and economic context. The mathematics curriculum was defined within the Common Curriculum Framework for Grades 10-12 Mathematics: WNCP, and the topics taught were in grade 10 for Alex and grades 11 and 12 for Mary. Population growth in the community where the schools were located exhibited a higher growth rate compared to the region (3.2% vs. 2.1%), with a high socio-economic profile represented by 78% of residents being homeowners in that particular community, and a reported average family income of \$70,300 for Mary's school, and \$79,600 for Alex's school (Cowley and Easton, 2013). Several researchers indicate that the economic context is important because of its influences on academic achievement, motivation and a variety of social and emotional outcomes from students (Murdock, 2000; Murdock, 1999; McLoyd, 1998; Duncan & Brooks-Gunn, 1997), which in turn might influence teachers' teaching and the culture of the classroom.

In regard to teaching and learning conditions, the EC for Mary and Alex was different and defined by the type of school, grade level, mathematics topic, the use of

specific TT for teaching, students' characteristics, and other situations related with classroom culture, and the teaching practice. Mary taught in a dual English and French Immersion School, and Alex taught in a Technical Secondary School. Both, Mary and Alex are experienced teachers with over 27 and 28-years of teaching experience respectively. In 2013, the total enrollment of students was 1,341 in Mary's school and 823 in Alex's school. The number of students in Mary's case was 25 students in grade 11 (68% female), and 14 students in grade 12 (64% female). In Alex's case the number of students was 26 (42% female).

The schools in which each teacher worked were each involved in a special technology integration programs. Mary's school was participating in the program *Tablets for Math Teachers*, and Alex's school was participating in the program *1:1 iPads in Secondary Numeracy Grades*. In both cases technology integration was part of the school district's plan to support the teaching and learning process with the use of TT. The use of TT has been stated in the Technology Principle by the NCTM as "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000a, p. 24). In 2010, the Common Core State Standards in Mathematics (CCSSM), and the Common Core Standards for Mathematical Practice (CCSMP) were released, which state in the principle 5, *Use Appropriate Tools Strategically*. More recently in 2014, the NCTM updated the Technology Principle as the Tools and Technology Guiding Principle to promote the use of tools and technology (Lassak, 2015).

Due to their individual schools involvement in a technology integration program, both teachers had access to two different TT: tablet and iPad. Mary used mostly the tablet during class instruction, and she was the only one who interacted with the tablet. The iPad was the main TT used by Alex, and students were also provided with iPads for their use during the lessons. Mary taught the topic 'Linear and quadratic systems' in grade 11, and the topics 'Definite integrals and area under a curve' as part of an AP course in grade 12. Alex taught two topics in the same classroom. The dominant topic was 'Introduction to trigonometry', and the other topic was the 'Provincial Exam Review'. There were some other issues related with teaching practice (use of TT, position of the teacher using the TT, classroom distribution, classroom environment, lesson activities) that might

influence teachers' Attitudes and Knowledge (KT, KLC, KSM, KP). Doering et al. (2009) pointed out "context influences both teacher knowledge and practice. In turn, teacher knowledge influences practice and practice influences which types of knowledge are used more in the classroom" (p. 336).

PCK-EC model dimensions for Mary and Alex. The PCK-EC theoretical model was explored empirically within Alex's and Mary's case studies. Interpretive repertoires method was used to examine Mary's and Alex's answers to interview questions and their reflective journals. This method supported the process of identifying the collection of themes in each one of the dimensions of the model. A form of discourse analysis, the interpretive repertoire can be defined as "the building blocks [teachers] use for constructing versions of actions or cognitive processes" and are "constituted out of a restricted range of terms used in specific stylistic and grammatical fashion" (Whetherell and Potter 1988, p.172). Mary and Alex drew several interpretive repertoires through their responses and reflections. These interpretive repertoires constituted collections of themes, which described Mary's and Alex's dimensions of the model PCK-EC (Figures 46 and 47, respectively).

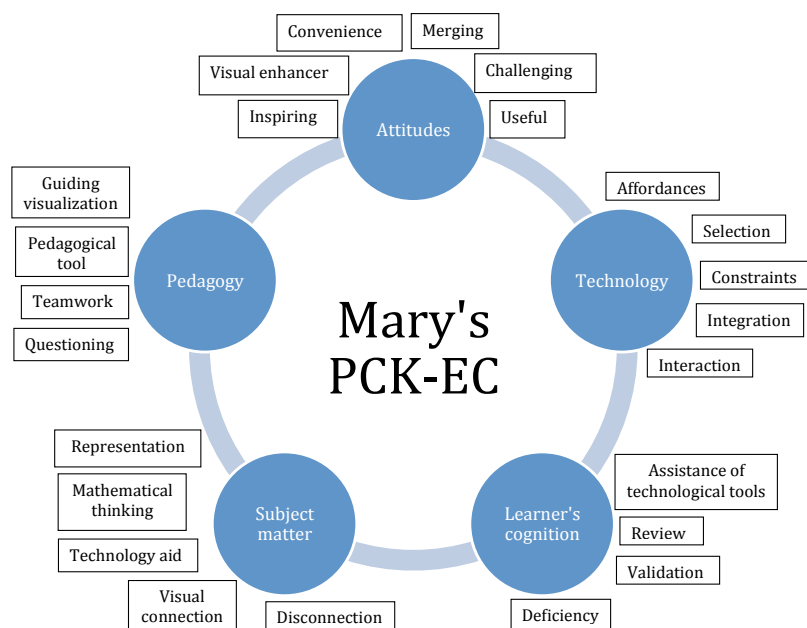


Figure 46. Mary's collections of themes identified in the five dimensions of the model PCK-EC

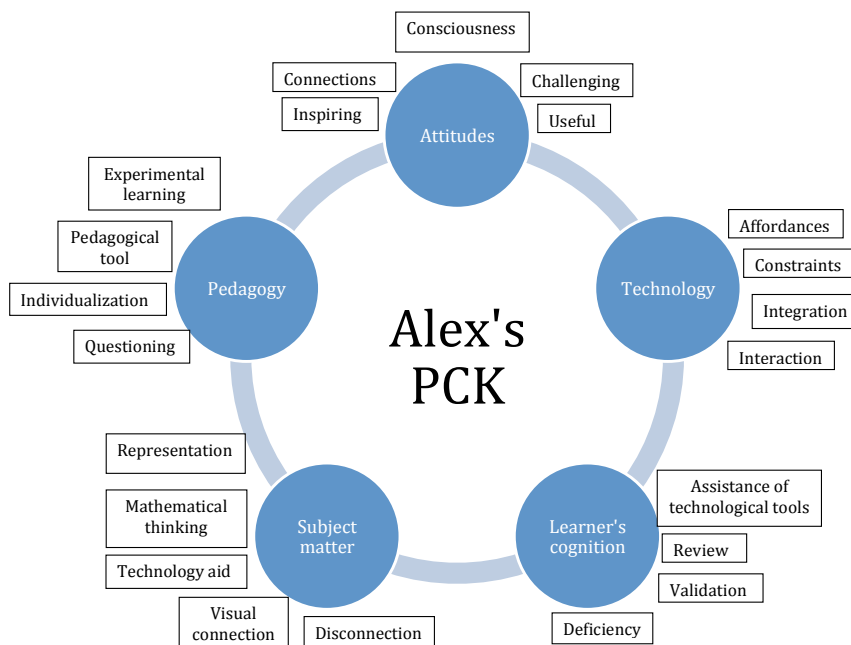


Figure 47. Alex's collections of themes identified in the five dimensions of the model PCK-EC

Dimension 1 - Attitudes. Teachers' Attitudes may include their opinions, perspectives, values, or preferences in regards to teaching and learning processes. Teachers' attitudes and perceptions toward technology play a significant role for understanding integration of technology into classroom practices (An, Alon & Fuentes, 2015). Several researchers have found that beliefs and attitudes of in-service and pre-service teachers predict and influence the use of TT in the classroom (Abbitt, 2011; Anderson & Maninger 2007; Albion & Ertmer, 2002; Pajares, 1992).

In this study, Mary's and Alex's Attitudes were identified through different themes that represented their perceptions, approaches, and preferences for teaching the specific mathematics topic with the particular TT. Mary illustrated in her Attitudes the themes Useful, Challenging, Merging, Convenience, Visual enhancer and Inspiring. The dominant theme represented in her responses and reflections was the theme Visual enhancer. Mary highlighted in her Attitudes that the tablet and graphing calculators were used to enhance the visualization needed for focusing mathematical understanding. In this

research, Mary taught two topics: one in pre-calculus and the other in calculus. One of the main goals for teaching pre-calculus and calculus is focusing on conceptual understanding, and researchers have identified that the use of computers facilitate that understanding by the use of visuals (Muzangwa, 2013). Visualization in teaching contributes to the development of both mathematical ideas and the relationships between mathematical concepts. Zimmermann and Cunningham (1991) define mathematical visualization as “the process of forming images (mentally, or with pencil and paper, or with the aid of technology) and using such images effectively for mathematical discovery and understanding” (p. 3).

Alex illustrated his Attitudes through the themes Useful, Challenging, Consciousness, Connections and Inspiring. Alex taught two topics ‘Introduction to Trigonometry’ and ‘Provincial Exam Review’. The dominant theme represented in his responses and reflections was Connections. For him, the use of TT (iPad apps, tablet computer, class website, online resources) assisted his teaching practice in making Connections to mathematical concepts through practical applications of trigonometry. NCTM (2000b) pointed out, “An emphasis on mathematical connections helps students recognize how ideas in different areas are related” (p. 13), and “Students should connect mathematical concepts of their daily lives, as well as to situations from science, the social sciences, medicine, and commerce” (p. 14).

Mary and Alex illustrated three common themes in their Attitudes: Useful, Challenging and Inspiring (see Figure 48). However, the description of these themes was a bit different for each teacher. Both of them identified TT are valuable for use in their mathematics teaching (Useful). Even though the exploration and incorporation of TT represented a difficult task (Challenging), they have a positive attitude for continuing using TT (Inspiring). Alon, An and Fuentes (2015) indicated “The teacher’s beliefs about what mathematics means to them and about how students make sense of mathematics will affect how they approach instruction” (p. 129).

Mary and Alex also illustrated some different themes in their Attitudes. Mary also represented the themes Visual enhancer, Convenience, and Merging; and Alex, the themes Connections, and Consciousness (see Figure 48). The theme Visual enhancer and Connections were the strongest Attitudes represented by Mary and Alex, respectively,

and it might be related to what teachers considered the most critical aspect for supporting understanding of the topics. The attitudes that a teacher might have toward a TT in regard to learning goals influence whether they use it in their teaching practice (Ertmer & Ottenbreit-Leftwich, 2010). The theme Convenience could be related to Mary's preferences for comparing her traditional teaching to the advantages of using new TT. The theme Merging could represent the advantages of approaching old and new technologies in regard to conceptual understanding and pedagogical strategies. In Alex's case, the theme Consciousness might be related to internal and external factors that teachers need to be aware of when teaching with TT and how these factors might facilitate or limit teaching (i.e. TT allow a more personalized approach, usually require more time for designing lessons, promote diversity in a classroom, students display different levels adopting TT, unexpected glitches, etc.).

Dimension 2 - KT. The KT requires an understanding of the affordances and constraints of the TT, what kind of content ideas could be taught, what strategies teachers could apply to facilitate students' understanding, and what kind of constraints of TT might limit teaching or student learning. The Knowledge of Technology also includes "creativity and flexibility with available tools in order to repurpose them for specific pedagogical purposes" (Koehler & Mishra, 2008, p. 17).

Mary and Alex represented their KT through the same themes: Affordances, Constraints, Integration and Interactions. Additionally, Mary illustrated the theme Selection (Figure 48). They represented their KT by integrating the mathematics curriculum with the use of the eTextbook (Mary) and the class website (Alex). The main TT used in Mary's teaching was the tablet and she was the only one who interacted with it. Mary and her students both interacted with graphing calculators when it was needed. Mary expressed that some of the benefits of using the tablet in her teaching included: having an eTextbook to adjust the lessons at her convenience, being able to present the content of the topic at any time, and having the possibility to compare graphical representations.

Alex used the iPad as the main TT and he expressed that the main benefit for using the iPad was for individualizing instruction, discovering learning, and searching for

additionally information about mathematical (trigonometric) applications. Some of the benefit of using an iPads during his teaching included: checking the class website, working with different apps on activities, obtaining pictures (from the eTextbook and online) that supported mathematical representations of the topics, and using online resources for developing the content of the class (i.e., samples of the provincial exam). Difficulties of using TT during teaching were not particularly relevant for Alex and Mary and both indicated those difficulties did not limit teaching the topics.

Understanding both Mary's and Alex's KT lies in the analysis of their descriptive names themes: Affordances, Selection, Constraints, Integration and Interactions. The theme Affordances might represent the features of the TT that benefit teaching practices. Reichert and Mouza (2015) highlighted this in their work: "the success of tablets and mobile apps largely depends on teachers' abilities to create learning experiences that capitalize on the affordances of these tools" (p. 125). Meletiou-Mavrotheris, Stylianou, Mavrou, Mavromoustakos and Christou (2015) indicated that the affordances of the iPad/tablets allow teachers "to promote communication, collaboration, differentiation of instruction, and other promising practices in mathematics education (inquiry-base learning, discovery learning, problem solving, etc.)" (p. 186). Also, Reichert and Mouza (2015) argued that one of the affordances of the mobile devices is "to support interactions with peers in a variety of settings" (p.122).

The theme Constraints might be related to teachers' difficulties and challenges when using TT during teaching practice. Carruthers, Martinovic and Pearce (2015) affirmed that "integrating new technology into a classroom, school or district can be an overwhelming task" (p. 288). Several authors indicated that challenges integrating TT into the classroom includes amount others the size of the required investment, lack of institutional support, teacher behaviour, commitment of teachers (An et al. 2015; Carruthers et al. 2015, Keengwe, Onchwari, and Wachira, 2008). Integration of technology in a classroom might face some obstacles that according to Ertmer (1999) could be either extrinsic or intrinsic to the teacher. Extrinsic barriers might be related directly to the school structure, inadequate access to technology support, gaps between technology and curriculum, to name a few. These constraints might be represented in the EC, while intrinsic barriers are related to the teachers' Attitudes and knowledge. Kumi-

Yeboah and Campbell (2015) point out the limitation of using tablets/iPads is that some of the operating systems are not compatible. This issue was expressed by Alex as part of the limitation of saving files on the iPad.

The theme Integration might be represented when teachers combine the TT (old or new) into their teaching practices with the curricular content. “As pre-service teachers learn more about how technology relates to teaching and learning, they perceive that knowledge about technology is more important to their ability to successfully integrate technology in the classroom” (Abbitt, 2011, p. 141). The use of the tablet/iPad in literacy assists teachers to design and implement curricular integration because they are able to link concepts of the content they learn in the classroom (Kumi-Yeboah & Campbell, 2015). In Mary’s and Alex’s cases, they were able to design and implement curricular integration following their own planning, and also they were able to adjust and expand it at their convenience.

Interactions of TT could represent the form in which teachers and students interact with TT during teaching practices. Kilpatrick, Hoyles and Skovsmose (2005) point out that interactions occur between teachers, students, text, software and other artifacts as part of the communication during the teaching and learning of mathematics. In Mary’s and Alex’s cases, interactions were produced between teacher and students; teacher, students, and TT (tablet, iPad apps, class website, graphing calculator, online resources); students and TT; teacher and TT; and student – TT – student. Many of these interactions are very complex to analyze, and also difficult to be captured only by researcher’s observations during teaching practices. The complexity of identifying these interactions is even bigger when students are able to interact with TT (i.e., using the class Moodle and class website for Mary’s and Alex’s cases, respectively). “Alongside the human participants, artifacts may play a pivotal role in structuring the interactions in teaching/learning situations – by the way they are designed and the tools made available” (Kilpatrick et al. 2005, p. 131). Theories of instrumental genesis and semiotic mediation have been studied to understand the communication between artifacts, teachers and students (Artigue, 2013; Mariotti, 2002, Noss & Hoyles, 1996). Mariotti (2002) highlighted “the relationship between the pupil and the teacher may be transformed by

the introduction of the computer, making communication between them more efficient and reciprocal” (p. 705).

Dimension 3 - KLC. The KLC includes knowledge of how students gain “the knowledge of mathematics content being addressed, as well as understanding of the processes the students will use, and the difficulties and successes that are likely to occur” (Fennema and Franke, 1992, p. 162). In the practice of teaching, the knowledge of learners is represented by “the characteristics and preconceptions that students bring to a learning situation” (Angeli & Valanides, 2009, p. 158). Teachers need to identify students’ difficulties and obstacles in understanding a mathematical topic. This allows teachers to include students’ conceptions and misconceptions in their planning and to choose strategies that help to manage students’ understanding of mathematical content.

Mary and Alex represented their KLC through the same descriptive theme names: Review, Validation, Deficiency, and Assistance of TT. Mary illustrated most frequently her KLC through the themes Review and Deficiency. The tablet supported Mary’s teaching to Review previous concepts, clarify assignments, add notes, and to compare different graphical representations at the same time. Mary identified students’ understanding (Validation) and misconceptions (Deficiency) with respect to the mathematical content by asking questions to students. The Assistance of TT (tablet) in Mary’s teaching was highlighted for reviewing the mathematical content rather than for identifying students’ understanding and difficulties as in Alex’s case.

Mary persuaded students to use ‘pencil-and-paper’ to sketch mathematical functions in order that students may identify potential mistakes in graphs illustrated on the screen of the graphing calculator. Also, she stressed that students need to apply their knowledge of basic graphing skills for analyzing graphs with graphing calculators. Hitt (2003) indicated the importance of promoting skills for using different mathematical representations including paper, pen and technology, and to identify students’ misconceptions interpreting graphing when supported by TT (i.e., graphing calculators, graphing mathematics software), by doing a reflective use of technology for teaching mathematics.

Alex illustrated his KLC primarily through the themes Assistance of TT, Validation, Deficiency and it was less represented through the theme Review. The use of

iPad apps assisted Alex to identify students' understanding and difficulties learning the topic. Files posted by students in Dropbox and class website allowed him to maintain accountability of students' performance. Also, he circulated around the classroom monitoring students' understanding when they were working with iPads.

The class website assisted Alex in reviewing assignments, and providing clarifying instructions for using iPad apps. Files posted in the class website assisted students to review or access materials and additional resources (i.e., samples of the provincial exam review). The Explain Everything app supported Alex's teaching—showing trigonometry demonstrations to students, presenting examples, and also providing explanations or justifications of the mathematics topics. The Explain Everything app supported students in creating their own trigonometry demonstrations and also supported posting them either on the Dropbox or class website. Carruthers et al. (2015) indicated some of these advantages including: “using a whiteboard app such as Explain Everything...they can then explain their thinking while writing out their solutions to mathematical exercises” (p. 287) and saving “...all created content to a cloud-based storage option (originally Dropbox, then Google Drive), which allowed them to share folders with their classmates, their teacher, or their parents, thus ensuring that all stakeholders had easy access to each student's work” (p. 287).

The theme Deficiency might be related specifically to the way teachers identify students' difficulties in understanding with respect to the topic content. Park and Oliver (2008) indicated “Teachers' understanding of students' misconceptions impacted their decisions made throughout the entire teaching process from planning to assessment, which ultimately improved their PCK. As teachers develop better understanding of students' misconceptions, their PCK became more sophisticated” (p. 275, 276). Mary expressed explicitly students' misunderstandings using graphing calculators due to students' lack of conceptual and graphic skills. In Alex's case, he was concerned about students' difficulties transferring the knowledge generated when using the Explain Everything app into mathematical understanding.

Dimension 4 - KSM. The KSM refers to teachers' “knowledge of the concepts, procedures, and problem-solving processes within the domain in which they teach”

(Fennema and Franke, 1992, p. 162). Thus, KSM requires that teachers understand “the procedures, but also to understand the concept underlying them. They need to know that something is so, and also why it is so” (Petrou & Goulding, 2011, p. 14). In practice, this knowledge is illustrated in the way teachers present subject matter concepts and procedures, give explanations, make representations of concepts, use examples, and clarify students’ questions.

Mary and Alex represented the same collection of themes in the dimension of KSM (Visual connections, Disconnections, Technology aid, Mathematical thinking and Representations). Mary represented strongly her KSM in her comments, reflections, and teaching practices observations. She illustrated most frequently her KSM through the theme Visual connections. Mary focused her KSM connecting conceptual and procedural thinking through visualization of the mathematical content and linking graphical representations. Gorev and Gurevich-Leibman (2015) argued that visualization and representation of mathematical objects have become important in the last two decades due to the integration of technology into mathematics classrooms.

Alex, also represented his KSM strongly. He also illustrated most frequently his KSM through the theme Visual connections. Alex approached his KSM guiding students to experiment with mathematics by transferring work done with iPads and linking with conceptual understanding. The use of TT (iPad apps, Tablet, class website, and online resources) supported his teaching first through visualizing practical applications of trigonometry for obtaining conceptual understanding of the topic. Then, he focused on the use of basic concepts within the topics (Introduction to trigonometry and Provincial exam review) for solving problems.

Mary and Alex illustrated strongly the theme Visual connections for supporting mathematical understanding. Mary did not express explicitly in her comments and reflections the aid of TT for supporting Mathematical thinking and Representations. However, she illustrated clearly during teaching practices her KSM supported by the integration of TT. In particular, when she was approaching conceptual and procedural understanding of graphical representations of functions, the tablet assisted her to link different mathematical representations (i.e., graphical examples from the eTextbook,

manual sketching and the use of graphing calculators). Zimmermann and Cunningham (1991) indicated:

If mathematics is the science of patterns, it is natural to try to find the most effective way to visualize these patterns and to learn to use visualization creatively as a tool for understanding. This is the essence of mathematical visualization (p. 3).

Visual connections and Representations have been considered as skills that foster Mathematical thinking. Some teachers that use TT in their teaching practices relate the affordances of TT for supporting conceptual understanding. Artigue (2013) pointed out “One acknowledged affordance of digital technologies is the new forms of representations of mathematical objects that they give access to, together with new means for acting on these representations and for connecting them in a dynamic way” (p. 12).

Alex expressed explicitly in his comments and reflections the aid of TT for supporting Mathematical thinking, and Representations (i.e., the use of Explain Everything was used to create mathematics demonstrations, and students could write on top of pictures showing their mathematical understanding). Carruthers et al. (2015) argued: “having students create their own artefacts by using screencast or by sharing their work with others cultivates a feeling of confidence as well as the desire necessary for applied students to progress in often procedurally based courses like mathematics” (p. 289). Also, Bell, Juersivich, Hammond and Bell (2012) indicated, “visual representations, such as illustrations, photographs, graphs, maps, analogies, physical manipulatives, and three-dimensional models, are often used by effective teachers to promote conceptual understanding” (p. 104).

Disconnections might be related to major difficulties in teaching the mathematics topic. Some of these difficulties in Mary’s and Alex’s cases could be related to students’ difficulties visualizing mathematical concepts; time-consuming review of previous knowledge (i.e., students’ lack of previous concepts or lack of basic mathematic skills for graphing); gaps in linking conceptual and procedural thinking; and students’ difficulties transferring their knowledge produced in an activity using TT into formal mathematics.

The theme Technology aid represented the way Mary and Alex used TT for guiding mathematical understanding of the topics. Bell et al. (2012) indicate, “Teachers who have a deep knowledge of their content and of the concepts students have most difficulty grasping can apply their understanding of digital technologies to select the best form of dynamic representation when appropriate” (p. 105).

The theme Mathematical thinking might represent teachers’ approaches of conceptual and procedural thinking during their teaching practices. “Dynamic representations generated by technology, such as graphing calculators, geometry tools, calculator-based laboratories (CBLs), and applets, can foster students’ attainment and development of concepts skills, and problem solving (Duke & Graham, 2006 as cited in Bell et al. 2012).

The theme Representations could illustrate the different way in which teachers may represent a mathematical object during their teaching practices. NCTM (2000b) stated the standard Representation as: “create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; and use representations to model and interpret physical, social, and mathematical phenomena” (p. 14).

The KSM was less visible in Alex’s teaching practices compared with Mary’s teaching, in that he guided students to produce mathematical work on iPads individually. Because these students’ work was performed electronically on their iPad and digitally submitted from their iPads to the class Dropbox, the researcher was not able to fully observe those interactions. Alex and Mary agreed that the use of TT supported conceptual thinking.

Dimension 5 - KP. The KP refers to the knowledge needed by teachers in regard to “teaching procedures such as effective strategies for planning, classroom routines, behaviour management techniques, classroom organizational procedures, and motivational techniques” (Fennema and Franke, 1992, p. 162). In practice, the pedagogical knowledge is used to guide the development of student understanding following one or more learning theories.

Mary and Alex represented two commonalities themes in the dimension of KP: Questioning and Pedagogical tool. They each represented two additional themes: Guiding visualization and Teamwork for Mary; and Experimental learning and Individualization for Alex.

Mary represented strongly her KP in her comments, reflections, and observations of her instructional practices. She most frequently illustrated her KP through the theme Pedagogical tool. She highlighted that the way she uses TT is what supported her teaching, not the tool itself. In particular, Mary indicated that she is the one who guides the visualization of the topic content combining traditional and new technology during lessons. She expressed the need of not taking for granted students' understanding of the content of the topic, and to be certain that students have the appropriate time for obtaining conceptual and procedural thinking.

The main strategy identified in Mary's teaching was guiding students through Questioning and encouraging them to discuss the topics for building mathematical understanding. The Common Core State Standards for Mathematics (CCSSM, 2012) in the standards for mathematical practice include the principle: Use appropriate tools strategically. This states:

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. . . They are able to use technological tools to explore and deepen their understanding of concepts (Principle 5, p. 7).

Alex illustrated most frequently his KP through the themes of Pedagogical tool and Experimental learning supported by the use of TT. Alex used iPad apps, class website, eTextbook and online resources as tools that helped him to manage pedagogical issues, to individualize and diversify learning. Kumi-Yeboah and Campbell (2015) indicated that with the emergence of tablets/iPad in the classroom "students have become self-directed and independent learners" (p. 58).

Alex highlighted the strategies of engaging students with open questions, monitoring students by circulating around the classroom and providing students' feedback while students were working with iPads was more efficient for him than to check students' files on Dropbox. Contrarily, Carruthers et al. (2015) highlighted that by using the Dropbox app for checking students' files the "teacher feedback was almost instantaneously making easy for the teacher to provide assistance as required" (p. 287).

Mary and Alex represented strongly their KP during their teaching practices and both agreed that the use of TT supported conceptual understanding. Mary expressed that using a tablet in her teaching practices is an effective Pedagogical tool, as long as she does not have glitches or technical problems with the TT during lessons. She pointed out the need to enhance students' basics skills for graphing before using graphing calculators. Mary also mentioned that students needed time to analyze graphs and find strategies for deciding whether to use manual sketching or graphing calculators according to their understanding.

Alex indicated that he is discovering and learning how to teach with TT, just as the students are. He mentioned that it is challenging to design lessons using TT, but those TT support his teaching and assisted him to create an activity outside of the classroom. Bledsoe and Pilgrim (2015) point out, "In order for students to get hands-on experience with real world learning they must engage in more informal class activities as well as learning opportunities outside the classroom" (242).

Alex identified the need for having whiteboards in the classroom to share static notes as reference for students, and he mentioned that the use of a tablet combined with iPad could support better teaching. Carruthers et al. (2015) suggested that for avoiding disruption in the classroom by using the iPad a gradual transition is needed "to take the resources that are currently provided effectively via chalkboard, overhead, or handout and to save the content in a portable document for (PDF) file form for simple annotation on the iPad" (p. 288).

Mary and Alex mentioned that some students had difficulties manipulating TT during their lessons, either because some students lack technical skills for using TT, or they simply do not like technology, and this situation has to be accommodated for in the lesson planning.

The theme Pedagogical tool might refer to the way teachers use TT to support teaching practices focusing students' understanding of the content of the topic. Gorev and Gurevich-Leibman (2015) highlighted:

The effectiveness of technology integration into the mathematical classroom depends on the teacher, on his/her experience using the technology and on his/her proper selection of mathematical tasks according to his/her content knowledge of the discipline that allow him/her to take advantage of the technology's features (p.737).

The theme Questioning might be related to the pedagogical strategy for considering the teaching of a topic as a discussion class. Teacher and students interact with lesson activities by a dialectic question-answer relation. The teacher as a key agent in the teaching process guides students to elaborate better answers to questions in order to support understanding of the content of the lesson. Students could also help the teacher to adjust the lesson content according to the quality of questions they ask. Mary expressed explicitly, "It's, again, the quality of their [students'] questions that help me adjust my lesson" (MFI, June 21, 2013). In regard to the impact of students in teachers' PCK, Park and Oliver (2008) found the following three major aspects: First "when students posed challenging questions to teachers, these questions frequently facilitated both deepening and broadening of teachers' subject matter knowledge" (p. 271). Second, "students' responses such as enjoyment, evidence of learning, and nonverbal reactions to instructional strategies affected teachers' decisions to replace, modify, or validate the strategies employed" (p. 272). Third, "students' creative and critical ideas stimulated teachers' creation of innovative instructional ideas for future classes" (p. 274).

The theme Teamwork might be related to social interactions produced between teacher-students; students-students; teacher-TT-students. These interactions support understanding of the content of the lessons. Then, teamwork includes the benefits of students discussing between them the use of the TT or the content of the lesson during teaching practices.

The theme Guiding visualization could refer to the visual assistance provided by the use of TT (old, new or both) that needs to be guided by the teacher in order to support students' mathematical thinking.

The theme Experimental learning might refer to teachers approaching their topic by making the subject matter content presented more experimental for students. Lassak (2015) indicates that technology is a tool that supports representations and "representations are essential to effective mathematics experiences and allow students to best make sense of the mathematics they are studying" (p. 183).

The theme Individualization might be related to the use of TT in guiding students to be more independent learners. "Teachers who frequently integrate technology into instruction are likely to hold student-centered, constructivist pedagogical beliefs (Becker, 2000; Hermans, Tondeur, van Braak, & Valcke, 2008; Hernandez-Ramos, 2005; Martin & Schulman, 2006; Rakes, Flowers, Casey, & Santana, 1999)" as cited in Anderson, Groulx, and Maninger, 2011, p. 324)

Answering the first research question

The first research question is: How effective is the proposed model PCK-EC (Figure 2) in supporting the examination of teacher practice and providing insights to the pedagogical content knowledge? The discussion of each one of the dimensions (and their constituent themes) represented by Mary and Alex suggest that the proposed PCK-EC model was useful in identifying Mary's and Alex's Attitudes and Knowledge (KT, KLC, KSM and KP) as they taught specific topics with the use of TT.

By combining different sources of data (observations, interviews, journals) it was possible to describe collections of themes to support the examination of teacher practice. Baxter and Lederman (1999) indicated that teacher's PCK must be articulated not only by observations, but also by asking the teacher questions directly. Because Attitudes must be expressed to be captured objectively, only interviews and reflective journals supported the identification of the collection of themes that described the Attitude dimension. For the description of the collections of themes in the other dimensions (KT, KLC, KSM and KP), the interview and reflective journals were augmented with observations. For example: in Mary's case, the description of collection of themes in the dimension of

KSM was very well supported and augmented with observations (direct observations and video-recordings). These observations complemented the description of Mary's KSM providing textual examples from her lessons and showing different graphical representations that she used to support mathematical thinking.

Descriptive names of themes emerged when codifying common patterns in the examination of Mary's and Alex's responses and reflections in each one of the dimensions. Analyzing the descriptive theme name may provide insights for providing a guideline to help teachers to describe and reflect in their own PCK-EC.

The unique set of descriptive theme names in each of the five dimensions could provide a guiding collection of themes to support teachers in expressing and describing their knowledge in the model (see Figure 48). The framework could also support coaches to guide teachers in the process of understanding their own PCK-EC.

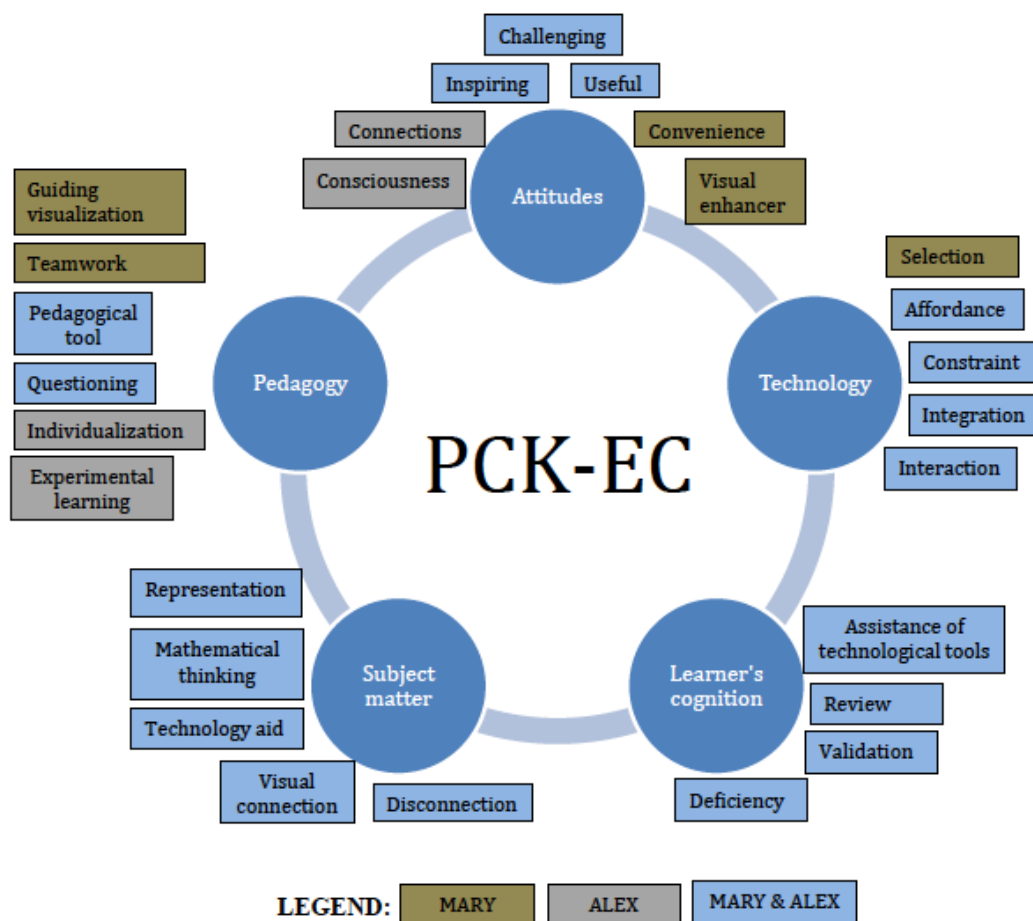


Figure 48. Alex's and Mary's collections of themes represented in each dimension of the model PCK-EC

Answering the second research question

The second research question is: How do different TT mediate or affect teachers' PCK-EC when mathematics teachers are teaching a particular topic? A description of each one of the dimensions (and constituent themes) of the PCK-EC model illustrated several factors that showed how different TT affect Alex's and Mary's PCK-EC when they were teaching a specific topic. Some of these factors included the following: The use of different TT affected the way Mary and Alex presented the planning of the lessons; the way they reviewed previous knowledge; and their approach for developing lesson activities.

One affordance of the TT, (within the theme of affordances) allowed Alex to do an activity outside of the classroom, and assisted Mary as she adjusted lessons during class or planned lessons outside of class. The communication features inherent in different TT (within the theme of interactions) affected the way teachers were able to monitor students i.e., Mary monitored students mainly by questioning them and checking paper worksheets; Alex usually monitored students by checking their work on iPad during lessons and also by checking files posted on Dropbox and class website.

The following three examples are provided to illuminate how the use of different TT affected Mary's and Alex's PCK-EC.

Example 1. Mary highlights that one characteristic of teaching pre-calculus and calculus is that each lesson is built on the previous one. This encouraged Mary in her use of a tablet to enhance the visualization needed for understanding the content of the topic. In particular, she retrieved visual representations from previous lessons/notes when she needed to review previous knowledge.

Similarly, Alex believes that trigonometry is based on "pictures, diagrams, and ... patterns" (AFI, June 19, 2013). This need influenced Alex in his use of iPads. Focusing first on practical and authentic applications of trigonometry to support the development of conceptual understanding instead of his traditional approach of presenting abstract concepts first and then the applications. Thus, Mary's and Alex's Attitudes influenced the way they used TT in their teaching practices. However, the use of TT also affected Mary's and Alex's Knowledge (KT, KLC, KSM, and KP).

Example 2. Results suggested that the use of different TT (Tablet, iPad) affected the way Mary and Alex designed and presented their lessons. Mary used the eTextbook to present the content of the lesson through the tablet, while Alex used the class website to present a daily plan of the content of the lesson. Mary's complete collection of resources (worksheets and eTextbook) and annotated lesson notes (from previous lessons) were available at any time during lessons. Also, Mary adjusted and expanded her planning during the lessons by using worksheets and notes according to her needs.

Alex posted the lesson plans in the class website which was accessible at any time, but he was unable to make ad hoc changes to the documents during teaching because of a technical limitation (iPad only during lesson, Tablet during prep). Thus, Mary's and Alex's KT and KP were mediated by the use of the Tablet and iPad.

Example 3. Mary's position in the classroom was mediated by the use of the tablet. She explained that the use of TT in her class allowed her to be in a position to hear and participate in students' conversations. She was located at the middle back of the classroom (beside the projector) to support listening to students' conversations (KLC) and managing students' behavior (KP). Carruthers et al. (2015) pointed out that one of the benefits of using technology is the opportunity for observing students having "... enriched conversations, sometimes even arguments, about procedures and concepts all related to understanding mathematics" (p. 289). Mary provided specific explanations of the topic at the middle front of the classroom when it was needed. Alex, however, was positioned at the center front of the classroom (beside the projector). He expressed its importance to students seeing him at the front of the classroom in order to manage student behaviours and have them focus on the lessons.

Interestingly, Mary and Alex expressed different Attitudes in regards to their physical location in the classroom. Carruthers et al. (2015) highlighted "teachers can deliver content through their own iPads while being anywhere in the room, given them the flexibility to work *with* the students instead of *in front of* them" (p. 286). Thus, the use of different TT affected Alex's and Mary's Attitudes in regards to their location with in the classroom. This was inter-related with their KT, KLC and KP.

From the previous examples it could be stated that the use of different TT affects teachers' PCK-EC when mathematics teachers are teaching *a particular topic*. Another way of illustrating how different TT affect teachers' PCK-EC is by identifying how Mary's and Alex's Attitudes, and Knowledge (KT, KLC, KSM and KP) are interrelated in their teaching practice (teachers' PCK-EC). This is very challenging and difficult to approach, but it is presented as a first attempt to illuminate a graphical representation of Mary's and Alex's PCK-EC.

Mary's and Alex's PCK-EC

Finding inter-relationships between the collections of themes across dimensions. According to Fennema and Franke (1992):

Little research is available that explains the relationship between the components of knowledge as new knowledge develops in teaching, nor is information available regarding the parameters of knowledge being transformed through teacher implementation . . . The challenge is to develop methodology and systematic studies that will provide information to enlighten our thinking in this area. The future lies in understanding the dynamic interaction between components of teacher knowledge and beliefs, the role they play, and how the roles differ as teachers differ in the knowledge and beliefs they possess (p. 163).

The method of interpretative repertoires supported the identification of the collection of themes in each one of the dimensions. Also, this method helped to identify the nature of teachers' inter-relating their Attitudes and Knowledge (KT, KLC, KSM, and KP) when they were asked to articulate an Attitude or Knowledge. For this reason, an individual quote might support more than one theme within or across dimensions (identifying an inter-relationship between themes). Thus, teachers' Attitudes and their knowledge (KT, KLC, KSM, and KP) are not articulated separately, they are inter-related across dimensions.

Example 1. In Mary's journal of reflection (grade 12), one of the questions she was asked to answer focused on the dimension of Attitudes: Do you think that the use of resources and materials including the technological tool(s) were useful in teaching this topic? Please, could you provide some details about how those resources support your teaching? Mary answered:

(1) Yes, the tablet enables me to highlight the visual part of the lesson. (2) The worksheet that students were working on, was the same one as on the screen. (3) I was able to sketch the curves needed with the same conditions (space limits) on my drawings as the students. (4) I can then focus attention with highlighters the details central to the lesson, i.e, where the area was symmetrical, the area was doubled, where the area was below the x-axis, the area was negative. (5) We needed to change our procedures to accommodate these aspects. At the moment, we are getting negative values. (MJR, May 2, 2013, grade 12).

Analysis of Mary's answer. Mary's answer was taken as only one quote to be analyzed as follows:

From (1) she pointed out that the tablet is useful because it enables her to highlight the visual part of the lesson. Thus, the quote supported the theme Useful in the dimension of Mary's Attitudes.

From (2) and (3) Mary talked about some affordances of the TT. The OneNote program on the tablet allowed Mary to bring worksheets into the lesson and to allow both students and her to work simultaneously on the content during lessons (Mary with the tablet projected on the screen and students with the printed worksheet). Then, this quote supported the theme of Affordances in the dimension of KT.

From (3), Mary argued the possibility to sketch the curve needed with the same conditions (space limits on drawing) as the students. In this case, the quote supported the theme Pedagogical tool in the dimension KP.

Mary identified that the ability to focus attention by highlighting details central to the lesson was helpful and gave an example relating her KSM. From (4) and (5) the quote

supported the theme Visual connection in Mary's KSM and the theme Guiding visualization in her KP.

As a result, the quote obtained in the dimension of Attitudes not only supported a theme in Mary's Attitudes, but also supported other themes across dimensions as follows:

- In the dimension of Attitudes, the quote supported the theme Useful.
- In the dimension KT, the quote supported the theme Affordances.
- In the dimension of KSM, the quote supported the theme Visual connections
- In the dimension KP, the quote supported the theme Pedagogical tool and Guiding visualizations.

Hence, it was inferred from the analysis of the quote that Mary not only represented her Attitudes in an isolated fashion, she inter-related her response (quote) with her KSM, KT, and KP at the same time.

Example 2. Another question in the dimension of Attitudes: Do you think that your beliefs about the materials that you used in your class today are reflected in your teaching? Please, explain some examples or details if it is possible. One quote taken from Mary's answer was:

(1) I believe that the tablet is a convenient tool to help explain the concept of splitting the areas by making the graphing so visual. (2) The use of the colours makes it easy for me to stress the parts of the graph I am describing. (3) It gives both the students and myself a reference point to discuss any lingering questions. (4) It's also easy for me to call up points made in previous lessons to connect today's lesson with concepts already explained (MJR, May, 7 & 9, grade 12, 2013).

Analysis of Mary's answer. Mary's answer was taken as only one quote to be analyzed as follows:

From (1), the quote supported the theme Visual enhancer in Attitudes.

From (1) and (2), the quote supported the theme Affordances in the dimension KT.

From (1) and (2) the quote supported the theme Visual connection in the dimension of KSM.

From (3) and (4) the quote supported the themes Validation and Review in the dimension of KLC.

From (1), (2), (3), and (4) the quote supported the themes Guiding visualization, Pedagogical tool and Questioning in the dimension KP.

In this case, Mary's response (the quote) in the dimension of Attitudes, inter-related not only her Attitudes, she also inter-related her KT, KLC, KSM and KP.

Example 3. In Alex's final interview a question focused on the dimension of Technology: Did you integrate TT to the mathematics curriculum? How it was possible.

One of the quotes in Alex's answer was:

(1) So, I would have screen shots of problems and then, I would put those on the screen, or I would put them on my app called 'Explain Everything', (2) and then I will do the problem in front of the kids on top of the problem – (3) and these are questions out of graphics, that I read out of the book – (4) and again my goal is to also show them that when you are problem solving, you want to use a diagram write above the diagram and not just look at the diagram and be frustrated by it being in textbook. (5) So, I used the digital copy of the textbook to embed some of the examples and project those examples on the screen with my students (AFI, June 19, 2013).

Analysis of Alex's answer. Alex's answer was taken as only one quote to be analyzed as follows:

From (1), the quote supported the theme Connections in the dimension of Attitudes

From (1), (4) and (5), the quote supported the themes Affordances and Integration in the dimension KT.

From (2), (3), (4) and (5), the quote supported the themes Representation, Mathematical thinking, Visual connection and Technology aid in the dimension of KSM.

From (2), (3), (4) and (5), the quote supported the themes Experimental learning, Questioning, and Pedagogical tool in the dimension of KP

In Alex's response, he supported the description of two themes (Affordances and Integration) in the same dimension (KT), and also in this quote he inter-related other themes across dimensions that included his Attitudes, KLC, KSM and KP.

These examples illustrate how Mary's and Alex's Attitudes and Knowledge (KT, KLC, KSM, and KP) dimensions were interrelated. Inter-relationships could be described as both outgoing and incoming. An outgoing link from a specific theme in a dimension refers to a connection that Mary or Alex made when they were responding to questions on a specific dimension, and made a connection to a theme in another dimension. An incoming link refers to when a connection to the theme was made in response to a question on another dimension.

Graphical representation of Mary's and Alex's PCK-EC. Examining the outgoing and incoming connections between themes (in all dimensions) helps us to further define Mary's and Alex's PCK-EC. The average value for the number of outgoing and incoming inter-relationships was used to assign weights to Mary's and Alex's PCK-EC (see Figure 49, 50).

Mary's PCK-EC weight. These values suggest that Mary relied more heavily on the Subject matter and Pedagogy dimensions to illustrate and support her choices, explanations, and reflections. Subject matter received the largest number of connections (average 21 outgoing and incoming links, representing 28% of the total links). Pedagogy was the next most frequently connected dimension (average 19.5 outgoing and incoming links, representing 26.5% of total links), with the other dimensions Attitudes, Learners' cognition and Technology receiving fewer links (17%, 14.5%, and 14%, respectively).

Mary's PCK-EC is not fixed or static. It is dependent upon the educational context (topic, class, etc.), and it evolves through her experiences and as she reflects on her practice. Using this PCK-EC model to examine Mary's reflections and our observation of her practice, we can see that Mary's choices to pragmatically adopt TT have enhanced her ability to communicate concepts/procedures through improved

dynamic/interactive visual representations. Importantly, she indicated the advantages of sharing some static information on the blackboard (for reference), having a ready copy of class notes available both in class (Tablet) and out (Moodle), and integrating TT with the use of some traditional procedures in the teaching process (e.g., manual sketching of graphs and comparing those to results found on graphing calculators). Mary pointed out “I still use the board now and then, especially when I’m teaching and I need some notes that need to be referred back to constantly, and students need to see the steps up there” (MII, May 1, 2013). Mary perceives herself as a learner, while she is integrating TT in her teaching.

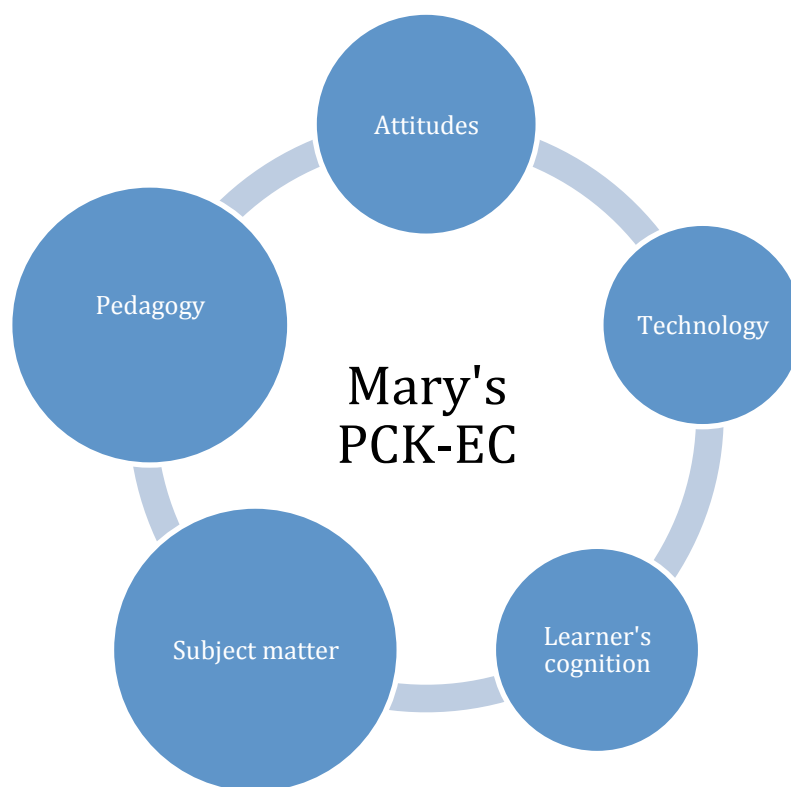


Figure 49. Mary’s PCK-EC with dimensions weighted according to the average percentage of outgoing and incoming comments (weights in proportion to area).

Alex’s PCK-EC weight. In Alex’s case, the average value for the outgoing and incoming connections suggests that Alex relied more heavily on his KSM, KP and KLC to represent and support his responses and reflections for teaching the specific topics.

Subject matter and Pedagogy received the largest number of connections (Average 39 outgoing and incoming links in each dimension, representing 22% of total links per dimension). KLC was the next most frequently connected dimension (Average 36.5 outgoing and incoming links, representing 21% of total links), with the other dimensions KT and Attitudes receiving fewer links (19.5% and 15.5% respectively).

Alex's PCK-EC is also dynamic. It is dependent upon the EC (topic, TT used, class, etc), and it evolves through his experiences and as he reflects on his practice. Alex recognized the improvement of his technological skills for teaching the topics including the development of the class website, typing, screen casting a lesson, demonstrating an app, and supporting the diversity of students using TT in his teaching practice. The use of iPad apps, online resources and class website (TT) facilitated a change to his previous style of teaching, in the first topic 'introduction to trigonometry' they were used to visualize and make connections to authentic applications of trigonometry for developing conceptual understanding. He recognized the potential of TT for doing more experimental mathematics and for encouraging students to work independently. Alex was well motivated to continue working with TT and he highlighted in his reflections the need to promote more discovery learning using a dynamic software like GeoGebra. He also indicated to produce more work during the lessons making mathematics more experimental for students.

In this study, Mary's and Alex's cases have been described and analyzed for understanding the way they unpack and articulate their Attitudes and Knowledge for teaching a specific topic with the use of particular TT. Loughran et al. (2004) indicated that "teachers' professional knowledge is difficult to categorize and therefore exceptionally difficult to articulate and document" (p. 370). The use of different sources of data and the method of interpretive repertoires allowed the identification of themes in each dimension of the PCK-EC. These themes supported a deep description of Alex's and Mary's cases. Additionally, it was possible to recognize the inter-relationships (outgoing and incoming) between themes (within and across dimensions) which helped to enhance understanding of Mary's and Alex's PCK-EC. These inter-relationships show us the complexity of describing teachers' PCK-EC during teaching practices as a whole. It is suggested that the frequency of the outgoing and incoming inter-relationships might give

us an average weight for each dimension of the PCK-EC to obtain a graphical representation.

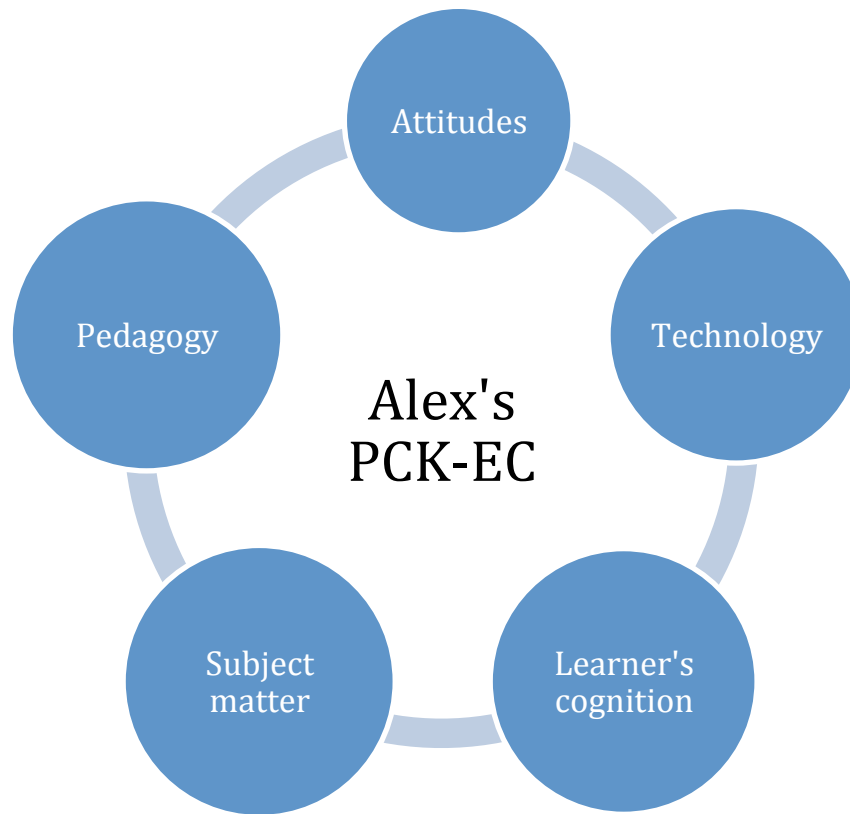


Figure 50. Alex’s PCK-EC with dimensions weighted according to the average percentage of outgoing and incoming comments (weights in proportion to area).

The TPACK framework has been used to represent graphically the knowledge needed for teaching using different size circles to identify a content-dominated knowledge base, but this approach has been intuitive (Doering et al. 2009; Stoilescu, 2015). Some challenges associated to the use of the TPACK framework include:

- “when a teacher is in the classroom, actively working to engage students and draw them into a lesson, the knowledge the s/he *possesses* is less important than the knowledge s/he *uses*” (Doering et al. 2009, p. 335).

- knowing that “depending of the context of a situation and the various levels of knowledge a teacher has, certain domains of knowledge may be used more than other” (Doering et al. 2009, p. 336).
- “shows lack of clarity in determining with accuracy each teacher’s components (Stoilescu 2015, p. 542).
- “the TPACK framework does not offer the possibility to rigorously take into account teachers’ attitudes, opinions, philosophy, and paradigms of teaching” (Stoilescu 2015, p. 542).

The proposed PCK-EC model might give insights to discuss potential ways to overcome limitations described above. The PCK-EC framework supported a deep description of Mary’s and Alex’s case studies identifying Attitudes and Knowledge used in their teaching practices. This analysis might help teachers to reflect on their Attitudes and Knowledge that they consider important for teaching the specific topic and for the integration of the use of TT into their practices.

Conclusions

The proposed PCK-EC model is a framework of dimensions that can be used to identify themes to describe teachers' Attitudes and Knowledge (KT, KLC, KSM, KP) within an Educational Context by combining different sources of data.

The PCK-EC model is context dependent, and it was particularly effective and useful as a tool in supporting the identification of collections of themes within each dimension by examining a teacher's responses and reflections using the method of interpretive repertoires.

The collection of themes identified in this study provides an initial map of the dimensions in the PCK-EC model. This collection may assist teachers (and coaches) in identifying and understanding their own PCK, to reach consensus about their central ideas teaching a specific mathematics topic with the use of TT, and to assist researchers, teacher educators, coaches and supervisors as they reflect on the development of new teacher knowledge produced during teaching.

Additional case studies across different educational context may be used to refine and augment the collections of common themes in each of the dimensions of the PCK-EC model. These collections of themes might be useful for understanding teachers' PCK-EC within a specific school community according to the way teachers and coaches adjust this framework at local levels in a common consensus.

Observation of teachers' practices can be useful to corroborate and complement the examination of teachers' responses and reflections. However, observations (direct and video recording) of teachers' practices are limited in their potential with respect to independently understanding teachers' PCK-EC. Fully understanding a teachers' PCK-EC requires that the teachers articulate their Attitudes and their Knowledge (KT, KLC, KSM, and KP) directly through interviews and journals.

The description of the collections of themes illustrated by teachers indicates their approach to the use of TT during their teaching practices. This approach affects the way teachers interrelate their Attitudes and Knowledge (KT, KLC, KSM and KP) in each of the dimension of the proposed PCK-EC model. For example: In Mary's teaching, students' misunderstanding using graphing calculators affected the way she approached

graphing calculator use in her teaching. In Alex's teaching, he had difficulties using the Dropbox app for reviewing students' work done during the lesson, and this affected his Attitude to providing students' feedback during lessons.

By examining a teacher's responses and reflections using the method of interpretive repertoires, it is possible to identify a useful collection of themes in each dimension, and illuminate the inter-relationships (outgoing and incoming) between themes, within and across dimensions.

It is difficult to examine the dimensions, themes and interrelationships in a teacher's PCK-EC and the difficulty increases when more than one TT is in use by either the teacher or the students. In Alex's case, the presence of several TT used by students and the teacher during teaching practices highlighted a higher frequency of strong-inter-relationships compared to Mary's case.

The utility of identifying inter-relationships between themes across dimensions could enhance the understanding of how teachers' PCK-EC is affected by the use of different TT in each one of the dimensions of the model. In particular, it is suggested that the frequency of the outgoing and incoming inter-relationships might assist researchers in finding an average weight of teachers' Attitudes and Knowledge expressed when they are using TT in their teaching practices.

The protocol applied in identifying the PCK-EC model can be used to support teachers in reflecting on their own practice. With the PCK-EC framework, teachers can unpack and integrate their knowledge of KSM with the other dimensions in the model (Attitudes, KT, KLC, KP) to transform and develop their knowledge when they are teaching a specific topic, and using particular TT. This implies that the PCK-EC is transformed in teaching during classroom interactions either as "reflection-in-action" and/or "reflection-on-action" (Schön, 1983). The reflection-in-action refers to the knowledge developed and enacted during teaching practices which requires integrating all components of PCK to provide reasons, explanations and justifications to students' questions in situations that could be expected or unexpected (Park & Oliver, 2008). Reflection-on-action occurs after the teaching practice is completed and allows teachers to re-structure or modify the body of PCK for teaching a particular topic (Park & Oliver, 2008).

The PCK-EC model maintains the main components of teacher's knowledge needed for teaching identified by Shulman (1986, 1987), including Knowledge of Subject matter or content, Knowledge of Pedagogy, and Knowledge of Learners' cognition. The PCK-EC includes the dynamic and interactive nature of teacher's knowledge (Teacher's knowledge is transformed during teaching practices) and teachers' beliefs as external factor that influence teachers' knowledge proposed by Fennema and Franke (1992). However, the PCK-EC framework includes teachers' attitudes as another dimension that is linked to teachers' knowledge instead of viewing teacher's beliefs as an external factor that influence teachers' knowledge. Finally, the PCK-EC includes technology as one of several important interconnected dimensions of teachers' knowledge instead of pulling it out as a defining dimension as in TPACK.

The PCK-EC model might give insights to address some challenges from previous PCK models, and TPACK, in regard to:

- The inclusion of teachers' Attitudes given that they are a major agent in the process of teaching;
- The need of situates teacher's Attitudes, KT, KLC, KSM and KP within an EC;
- The identification of EC including four elements: curriculum and standards; social, cultural and economic context; teaching and learning conditions defined by the school district; teaching conditions in the classroom;
- The examination of PCK-EC model showing one way of identifying each dimension of the model by using a collection of common themes (adjusted at a local level) to support teachers in unpacking their understanding of their PCK-EC;
- And finally the implication of generating a graphical representation of teachers' Attitudes and Knowledge (KT, KLC, KSM and KP) within their PCK-EC.

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Appendix 1 Email Invitation

I am inviting you to participate in my research study entitled '*Evaluating Pedagogical Content Knowledge in an Educational Context*'. This study is being conducted by me, Mercy Lili Pena-Morales a Ph.D. candidate in the Department of Curriculum and Instruction at the University of Victoria, in partial fulfillment of the requirements of my doctoral degree in Education. You may contact me by email at melipemo@uvic.ca if you have any questions about the study. My graduate supervisor at UVic is Dr. Tim Pelton, who may be contacted at tpelton@uvic.ca.

The purpose of this study is to explore the utility of a new framework called Pedagogical Content Knowledge in an Educational Context (PCK-EC). Specifically it aims to explore the teachers' knowledge needed for teaching a specific mathematics topic with the teacher's knowledge of technology as a focus.

In particular the research is designed as a multiple case study and the research questions are:

1. How do different technological tools mediate or affect teachers' pedagogical content knowledge in an educational context when mathematics teachers are teaching a particular topic.
2. How effective is the PCK-EC model for examining teacher practice and providing insights to the pedagogical content knowledge (in an educational context) for teaching a specific mathematics topic with the use of technological tools.

If you are mathematics teacher with at least one year of experience using a technological tool (tablets, graphic calculators, document camera, gps devices, Geogebra, Cabri, smart board, etc.) in your teaching practice, you could be a potential participant in this project.

Participation in this project will require choosing a mathematics topic in which the technological tool will be used in your teaching practices. your participation will include an initial and final interview, video recording of all classes required to develop the mathematics topic, and developing a journal of reflections during the period of observations – all classes involved in the developing of the mathematics topic. the general time commitment could be 4 to 8 hours of classes depending on the topic, interviews may take up to 2 hours each, one hour for reviewing transcripts of the interviews and the time required to complete the reflective journal might be 15 to 30 minutes per day of instruction. once you decide to join the study i will seek permission from the school's principal or institution's director. If you are interested in being part of this study, please contact me for further information.

I really appreciate any help you may offer to support this research project.

Best regards,

Appendix 2 Participant Consent Form – Teachers

[Department letterhead]

[Date]

[Teacher name]

[School name]

[Address]

[Community] BC [Postal Code]

EVALUATING PEDAGOGICAL CONTENT KNOWLEDGE IN AN EDUCATIONAL CONTEXT

Dear [name]

I am inviting you to participate in my research study entitled ‘*Evaluating Pedagogical Content Knowledge in an Educational Context*’. This study is being conducted by me, Mercy Lili Pena-Morales a Ph.D. candidate in the Department of Curriculum and Instruction at the University of Victoria, in partial fulfillment of the requirements of my doctoral degree in Education. You may contact me by email at melipemo@uvic.ca if you have any questions about the study. My graduate supervisor at UVic is Dr. Tim Pelton, who may be contacted at tpelton@uvic.ca. In addition, you may verify the ethical approval of this study, or raise any concerns you might have, by contacting the Human Research Ethics Office at the University of Victoria at ethics@uvic.ca.

Purpose and Objectives

The purpose of this study is to explore the utility of a new framework called Pedagogical Content Knowledge in an Educational Context (PCK-EC). Specifically it aims to explore the teachers’ knowledge needed for teaching a specific mathematics topic with the teacher’s knowledge of technology as a focus.

In particular the research is designed as a multiple case study and the research questions are:

1. How do different technological tools mediate or affect teachers’ pedagogical content knowledge in an educational context when mathematics teachers are teaching a particular topic.
2. How effective is the PCK-EC model for examining teacher practice and providing insights to the pedagogical content knowledge (in an educational context) for teaching a specific mathematics topic with the use of technological tools.

Importance of this Research

To provide a model that could give insights for understanding the challenge of identifying theoretical elements of teachers’ knowledge in the practice of teaching.

To develop a framework for studying teachers' knowledge for teaching that takes into account teachers' attitudes, opinions, philosophy and paradigms for teaching.

To contribute to the literature of how technology could be integrated in mathematics teaching.

The framework could be useful for guiding teacher education programs and also for professional development to guide teachers' teaching

Participants Selection

You are being asked to participate in this study because you have experience using a technological tool in your current practices as a mathematics teacher.

If you consent to voluntarily participate in this research project, your participation will include an initial and final interview to understand how you use technological tools in your current mathematics teaching practices. The interview will be conducted in person at a location and time convenient to you. The interviews will be recorded (video/audio) to allow for later transcription. Once your interview has been transcribed, I will send you a copy of the transcription via email to allow you to verify the information that you provided during the interview, and to allow you to provide any further thoughts that you may wish to add.

The research also involves video recording of all classes required to develop a certain mathematic topic, which we will choose in common agreement. The video recording may require the presence of a technician, as well as me for keeping and recording observations regarding to the interactions in the classroom while you are teaching. While observing the class I will sit in the back of the class.

Finally, I am asking that you develop a journal of reflections during the period of observations – all classes involved in the developing of the mathematics topic. This journal of reflections is intended to explore your thoughts in specific questions regarding your teaching practice using the technological tool.

Risks and Inconvenience

Participation in this study does not present any known or anticipated risks to you. Your identity and the identity of your school will be protected through the use of pseudonyms. However, participation in this research may cause some minor inconvenience to you, due to the time required for participation, but I hope that you will not find it to be an onerous commitment.

Benefits

The potential benefits of your participation in this research include personal growth and professional development as you engage in the process of examining your practice. In addition your participation will be contributing to the literature with respect to validating a theoretical model and expanding our empirical understanding of teacher's knowledge needed for teaching a specific mathematics topic with the use of technological tools.

Finally, your participation will provide insight with respect to how technology might be integrated in mathematics teaching.

The validation of the proposed model '*Pedagogical Content Knowledge in an Educational Context*' could be useful for guiding teacher education programs and also for professional development to guide teachers' teaching, which might help to improve the quality of teaching.

Voluntary Participation

Your participation in this research must be completely voluntary. If you do decide to participate, you may withdraw at any time without any consequences or any explanation. If you do withdraw from the study your data will be used only if you give permission to do so. If you do not want to give permission for the use of the collected data, then, data will be removed and destroyed.

Anonymity

In order to protect your anonymity and privacy, pseudonyms will be used with any example quotes or images (separate permission required below) that may be drawn from the observations or interviews.

Use and Disposal of Data, and Confidentiality

Data collected during this research may be analyzed in subsequent research activities by the researcher in future publications other than this research project. Data and notes from this research will be stored in a safe location in the researcher's house on a password protected computer to prevent unauthorized access and protect your privacy by maintaining the confidentiality of your data. Furthermore, the assistant in transcribing the interview and also the person who will make the video recording will be required to sign a confidentiality agreement. Electronic data, paper documents and all data collected during this research will be erased / removed from researcher's computer, and paper documents will be shredded 5 years after completion of the researcher's dissertation.

Dissemination of Results

It is anticipated that the results of this study will be shared with others in the following ways: in my dissertation, as published journal article(s), book chapter(s), or conference presentations. Also, I may use some pieces of the interview transcripts or journal reflections to exemplify results in academic work. When it is complete my dissertation will also become a publically available document that will be available through UVic's D-space. I will be happy to provide you a link to it if you wish when it is complete.

Contacts

Individuals that may be contacted regarding this research include Dr. Tim Pelton, who may be contacted by email at tpelton@uvic.ca. In addition, you may verify the ethical approval of this study, or raise any concerns you might have, by contacting the Human Research Ethics Office at the University of Victoria at ethics@uvic.ca.

Appendix 3
Consent Form - School Principal / Institution Director

[Department letterhead]

[Date]

[School Principal/Institution Director name]

[School name]

[Address]

[Community] BC [Postal Code]

**EVALUATING PEDAGOGICAL CONTENT KNOWLEDGE IN AN
 EDUCATIONAL CONTEXT**

Dear _____,

I am a Ph. D candidate in the Department of Curriculum and Instruction at the University of Victoria, working under the supervision of Dr. Tim Pelton. In my doctoral dissertation called '*Evaluating Pedagogical Content Knowledge in an Educational Context*', I am investigating the elements of teachers' knowledge needed for teaching a specific mathematics topic having as a core study the knowledge of technology. This research will validate a new framework for identifying those elements called Pedagogical Content Knowledge in an Educational Context (PCK-EC). The Human Research Ethics Office at the University of Victoria has approved this study

The purpose of this study is to explore the utility of a new framework called Pedagogical Content Knowledge in an Educational Context (PCK-EC). Specifically it aims to explore the teachers' knowledge needed for teaching a specific mathematics topic with the teacher's knowledge of technology as a focus.

In particular the research is designed as a multiple case study and the research questions are:

1. How do different technological tools mediate or affect teachers' pedagogical content knowledge in an educational context when mathematics teachers are teaching a particular topic.
2. How effective is the PCK-EC model for examining teacher practice and providing insights to the pedagogical content knowledge (in an educational context) for teaching a specific mathematics topic with the use of technological tools.

Research of this type is important because it might provide a model that could give insights for understanding the challenge of identifying theoretical elements of teachers' knowledge in the practice of teaching. It also may contribute to the literature of how

technology could be integrated in mathematics teaching. Finally, validation of the framework could be useful for guiding teacher education programs and also for professional development.

I would like to ask your permission for [participant name] to participate in this project. Participation will include interviews, reflective journaling and the observation and video recording (may require a support technician) of a unit of study in one of the participant's classes. The number of hours that I might spend in the teacher's classroom depends on the mathematics topic to be studied, but it could be from 4 to 8 hours of classes.

Participation in this study is completely voluntary, and does not present any known or anticipated risks to you, your teacher or the school. Aside from a few exemplar quotes and images (in which only the teacher will be identifiable – with permission), I will not publish any name or information that might identify the school, the students or the teachers in the written work, oral presentations, or publications. The information remains confidential. Your teacher is free to withdraw at any time, even after she/he has consented to participate. She/he may decline to answer at any specific questions. Data collected will be retained in a secure location, used only by the researcher, and appropriately disposed of once research is complete (within 5 years of my dissertation completion).

The potential benefits of participation for your teacher include contribution to the literature of developing a theoretical and empirical understanding of teacher's knowledge needed for teaching a specific mathematics topic with the use of technological tools. Also, to provide insight to the literature of how technology could be integrated in mathematics teaching. The validation of the proposed model '*Pedagogical Content Knowledge in an Educational Context*' could be useful for guiding teacher education programs and also for professional development to guide teachers' teaching, which might help to improve the quality of teaching.

It is anticipated that the results of this study will be shared with others in the following ways: in my dissertation, as published journal article(s), book chapter(s), or conference presentations. Also, I may use some pieces of the interview transcripts or journal reflections to exemplify results in academic work. When it is complete my dissertation will also become a publically available document that will be available through UVic's D-space. I will be happy to provide you a link to it if you wish when it is complete.

Individuals that may be contacted regarding this research include Dr. Tim Pelton, who may be contacted by email at tpelton@uvic.ca, and me at melipemo@uvic.ca. In addition, you may verify the ethical approval of this study, or raise any concerns you might have, by contacting the Human Research Ethics Office at the University of Victoria at ethics@uvic.ca.

Please sign the attached form, if you agree with this research. The second copy is for your records. Thank you very much for your help.

Yours sincerely,

Mercy Lili Pena-Morales

<i>Name of School's Principal/Director</i>	<i>Signature</i>	<i>Date</i>
--	------------------	-------------

A copy of this consent will be left with you, and a copy will be taken by the researcher.

Appendix 4 Information Letter – Parents

[Department letterhead]

[Date]

[School name]

[Address]

[Community] BC [Postal Code]

EVALUATING PEDAGOGICAL CONTENT KNOWLEDGE IN AN EDUCATIONAL CONTEXT

Dear Parent/Guardian,

I am a Ph. D candidate in the Department of Curriculum and Instruction at the University of Victoria, working under the supervision of Dr. Tim Pelton. I am investigating the elements of teachers' knowledge needed for teaching a specific mathematics topic having as a core study the knowledge of technology. Research of this type could be useful for guiding teacher education programs and also for professional development to guide teachers' teaching. Also, it might help to understand theoretical elements of teachers' knowledge in the practice of teaching, and may contribute to the literature of how technology could be integrated in mathematics teaching. The Human Research Ethics Office at the University of Victoria has approved this study.

During my research I will interview the mathematics teacher only and will observe them in classrooms. I will not interview any students and I will not interfere with classroom activities. However, I will be videotaping some classes in which your child may be present. While, it is possible that the video recordings of the class may also capture your child's image, I will not use any images of your child nor what he/she says. The focus of this research is on understanding the teacher. In addition, all video files will be held confidentially and disposed of appropriately when my research is complete.

The video recording implies the presence of a technician that will be doing the video recording, as well as me for keeping and recording observations regarding to the interactions in the classroom. I and my video technician both have valid police background check documentation.

Please contact me if you have any questions - by email at melipemo@uvic.ca. If you need additional information or have concerns regarding this research, you may also contact my supervisor, Dr. Tim Pelton, who may be contacted by email at tpelton@uvic.ca. In

addition, you may verify the ethical approval of this study, or raise any concerns you might have, by contacting the Human Research Ethics Office at the University of Victoria at ethics@uvic.ca.

Thank you very much for your help.

Yours sincerely,

Mercy Lili Pena-Morales

Appendix 5 Consent Letter – Parents

[Department letterhead]

[Date]

[School name]

[Address]

[Community] BC [Postal Code]

EVALUATING PEDAGOGICAL CONTENT KNOWLEDGE IN AN EDUCATIONAL CONTEXT

Dear Parent/Guardian,

I am a Ph. D candidate in the Department of Curriculum and Instruction at the University of Victoria, working under the supervision of Dr. Tim Pelton. I am investigating the elements of teachers' knowledge needed for teaching a specific mathematics topic having as a core study the knowledge of technology. Research of this type could be useful for guiding teacher education programs and also for professional development to guide teachers' teaching. Also, it might help to understand theoretical elements of teachers' knowledge in the practice of teaching, and may contribute to the literature of how technology could be integrated in mathematics teaching. The Human Research Ethics Office at the University of Victoria has approved this study.

During my research I will interview the mathematics teacher only and will observe them in classrooms. I will not interview any students and I will not interfere with classroom activities. However, I will be videotaping some classes in which your child may be present. While, it is possible that the video recordings of the class may also capture your child's image, I will not use any images of your child nor what he/she says. **The focus of this research is on understanding the teacher.** In addition, all video files will be held confidentially and disposed of appropriately when my research is complete. The video recording implies the presence of a technician that will be doing the video recording, as well as me for keeping and recording observations regarding to the interactions in the classroom. The video technician and I, both have valid police background check documentation.

Your signature below indicates that you are informed of the conditions of this research project, and that you consent that your child will be present in the classroom while the video recording is occurring. You may opt out and in that case your child will be moved to another section of the class that will not be video taped.

Please contact me if you have any questions - by email at melipemo@uvic.ca. If you need additional information or have concerns regarding this research, you may also contact my supervisor, Dr. Tim Pelton at tpelton@uvic.ca. In addition, you may verify the ethical approval of this study, or raise any concerns you might have, by contacting the Human Research Ethics Office at the University of Victoria at ethics@uvic.ca.

Thank you very much for your help.

Yours sincerely,

Mercy Lili Pena-Morales

Name of Parent/Guadian

Signature

Date

Provide initials *if you consent*:

- Presence of your child while videotaping: _____
- Opt out – child will be moved to another section while video taping is occurring _____

Appendix 6 Confidentiality Agreement – Videographer

1. Confidential Information

Mercy Lili Pena Morales [the researcher] is conducting a research study entitled ‘Evaluating Pedagogical Content Knowledge in an Educational Context’ [the study] and hereby confirms that she will be employing _____ as a videographer [the videographer], and that the videographer will be observing and recording classroom activities for the study.

The classroom observations and recordings made by the videographer are considered confidential information [confidential information].

2. Obligations of Videographer

A. The videographer hereby agrees that the confidential information related to the study is to be used solely for the purposes of said study and should only be discussed with the researcher.

The videographer hereby agrees not to disclose, publish or otherwise reveal any of the confidential information observed or captured as part of the study.

B. Materials (media) containing confidential information must be stored in a safe location so as to prevent access to said materials by third persons. Confidential Information shall not be duplicated by the videographer.

3. Completion of the Work

Upon the completion of the work and/or at the request of the researcher, the videographer shall transfer all recorded confidential information captured or received (and the media containing it) to the researcher within ten (10) days of such request.

I _____ (the videographer) hereby agree to maintain the confidentiality of all observations related to this study and return all recorded confidential information on request.

Signature and Date

Appendix 7 Confidentiality Agreement – Transcriptionist

1. Confidential Information

Mercy Lili Pena Morales [the researcher] is conducting a research study entitled ‘Evaluating Pedagogical Content Knowledge in an Educational Context’ [the study] and hereby confirms that she will disclose certain confidential information to _____ who will be acting as an interview transcriptionist [the transcriptionist].

Confidential information shall include all observations, data, materials, audio or video recordings transmitted orally, in writing, or by any other media, to the transcriptionist by the researcher.

2. Obligations of Transcriptionist

A. The transcriptionist hereby agrees that the confidential information for the study is to be used solely for the purposes of said study and should only be discussed with the researcher.

The transcriptionist hereby agrees not to disclose, publish or otherwise reveal any of the confidential information received from the researcher

B. Materials containing confidential information must be stored in a safe location so as to prevent access to said materials by third persons. Confidential Information shall not be duplicated by the transcriptionist except in support of the transcription process.

3. Completion of the Work

Upon the completion of the work and/or at the request of the researcher, the transcriptionist shall return all recorded confidential Information received in written or tangible form, including copies, or reproductions or other media containing such confidential Information, within ten (10) days of such request.

I _____ (the transcriptionist) hereby agree to maintain the confidentiality of all confidential information related to this study and return all recorded confidential information on request.

Signature and Date

Appendix 8 Initial Interview

The questions that are in red color were adapted from the paper entitled “Knowledge Growth in teaching Mathematics/Science with Spreadsheets: Moving PCK to TPACK through Online Professional Development” by Niess et al. (2010).

<i>Attitudes</i>	<i>KT</i>	<i>KLC</i>	<i>KSM</i>	<i>KP</i>
<p><i>1. When you think about teaching and learning mathematics, what is your conception of how technological tools are useful?</i></p> <p><i>2. Why do you think this technological tool(s) support mathematics teaching?</i></p> <p><i>3. What is your current view and understanding about integrating this technological tool in mathematics?</i></p> <p><i>4. When and how should this</i></p>	<p><i>1. With which technological tools are you a consistent user? How do you use these technologies? (Personal uses as well as professional uses).</i></p> <p><i>2. Do you consider yourself as technology literate with these technologies? (Do they have novice or strong content and process working knowledge of these technologies – for their use personally?)</i></p>	<p><i>1. What is the previous knowledge that students need to know in order to introduce this mathematical topic in regards to the subject matter?</i></p> <p><i>2. What is the previous knowledge that students need to know in order to introduce this topic in regards to the use of this technological tool?</i></p> <p><i>3. How do you identify students’ understanding about this mathematics</i></p>	<p><i>1. What mathematics topics do you teach with this technological tool?</i></p> <p><i>2. What other technological tools would be useful in teaching this unit/topic?</i></p> <p><i>3. Describe the goals for learning when this technological tool is integrated in the mathematics curriculum. (Do they focus on the subject matter or the subject matter within a technology context or</i></p>	<p><i>1. What learning theories do you consider in your curricular activities (planning, classroom routines, management and motivational techniques) when you use this technological tool?</i></p> <p><i>2. How do you integrate this technological tool for teaching this unit?</i></p> <p><i>3. When the technological tool is a new one to the</i></p>

<p><i>technological tool be used in teaching a mathematics topic?</i></p> <p>5. <i>Why do you think that this technological tool is useful for learning mathematics?</i></p> <p>6. <i>What changes do you see in students' mathematical thinking in regards to mathematical concepts when they use this technological tool in learning this unit as opposed to when they do not use this technological tool?</i></p> <p>7. <i>What changes do you see in students' mathematical thinking in regards to mathematical procedures when they use this</i></p>	<p>3. <i>How long have you been working with this particular technological tool for teaching mathematics?</i></p> <p>4. <i>How do you integrate this technological tool into the mathematics curriculum?</i></p> <p>5. <i>Describe how are the interactions in the classroom with the use of this technological tool (If the teacher is the only one that interacts with the tool, or student-tool, student-teacher-tool)</i></p> <p>6. <i>Which are the affordances of this technological tool?</i></p> <p>7. <i>Which are the</i></p>	<p><i>topic?</i></p> <p>4. <i>How do you identify students' difficulties about this mathematics topic?</i></p> <p>5. <i>How would you assess students' understandings in this unit/topic when you use the technological tool? (Thinking about incorporating the technology in assessing students' understanding and thinking ON this unit/topic).</i></p> <p>6. <i>How is the <u>specific technological tool</u> useful in guiding students' thinking about this topic?</i></p> <p>7. <i>How has your knowledge about students'</i></p>	<p><i>something else?).</i></p> <p>4. <i>Do you have difficulties in teaching the mathematical content of this unit? What is the major difficulty that you have found about the content in this unit?</i></p> <p>5. <i>Have the materials, resources or technological tools that you have used to teach this unit helped you to develop the mathematical content? Have those tools changed the mathematical content.</i></p> <p>6. <i>Describe how the use of this tool allows you to develop the mathematical thinking (conceptual and procedural) in this unit?</i></p>	<p><i>students, what strategies do you use to guide them in learning mathematics using this technology? (To identify if they work in learning about the technology as they are learning the specific topic, or if they focus on learning about the technology and later as the mathematic context)</i></p> <p>4. <i>How, why, and when do your strategies change for different types of learners?</i></p> <p>5. <i>How, why, and when are your strategies the same regardless of learner differences?</i></p> <p>6. <i>Describe useful strategies when</i></p>
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<p><i>technological tool in learning this unit as opposed to when they do not use this technological tool?</i></p> <p>8. <i>What do you think is the best way in which this technological tool could be integrated in teaching this unit?</i></p> <p>9. <i>What do you think are the possible effects of integrating this technological tool in mathematics teaching with respect to:</i></p> <ul style="list-style-type: none"> <i>a. Curriculum</i> <i>b. Learning skills</i> <i>c. Technology</i> <i>d. Social and cultural issues</i> <p>10. <i>How has your conception of incorporating technological tools</i></p>	<p><i>constraints of this technological tool?</i></p> <p>8. <i>According to those affordances and constraints how do you use this technological tool for educative purposes?</i></p> <p>9. <i>What technological tools do you still want to learn more about? Why and for what purposes?</i></p> <p>10. <i>How have your knowledge and skills with these and other technologies changed through your work?</i></p>	<p><i>understandings, thinking, and learning in subject matter topics with specific technological tools changed through your career?</i></p>	<p>7. <i>Which are the differences of teaching the mathematical content of this unit with the use of this specific tool?</i></p> <p>8. <i>Explain how the concepts/processes are enhanced through the use of the technological tool.</i></p> <p>9. <i>How would you structure the integration of the technological tool in this unit? Explain why you would do it this way?</i></p> <p>10. <i>How has your knowledge about integrating technologies in mathematics curricular areas changed through your career?</i></p>	<p><i>students are interacting with this technological tool for learning about a mathematics topic</i></p> <p>7. <i>Discuss useful strategies that assure equal access to the technology for all students.</i></p> <p>8. <i>How has your knowledge about instructional strategies changed through your career when integrating technological tools?</i></p>
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<i>in teaching specific topics in mathematics changed through your career?</i>				
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Appendix 9
Final Interview

<i>Attitudes</i>	<i>KT</i>	<i>KLC</i>	<i>KSM</i>	<i>KP</i>
<ol style="list-style-type: none"> 1. Do you think that the way you taught this unit was different in regards to past experiences? Could you explain that differences. Does it caused changes in your teaching style? 2. Do you think that the technological tools used in class were useful in teaching this unit? 3. What changes did you see in students' mathematical thinking in 	<ol style="list-style-type: none"> 1. Did you integrate this technological tool to the mathematics curriculum? How it was possible? 2. Describe how were the interactions in the classroom between teacher, technological tool and students. How do you evaluate those interactions? 3. The way you or your students interacted with the technological tool allowed more students' discussions and participation in the class. How 	<ol style="list-style-type: none"> 1. How did you manage students' difficulties about this mathematics topic? 2. How the use of the technological tool(s) helped you to assess students' understanding in this unit? 3. How was the specific technological tool (s) useful in guiding students' thinking about this topic? 4. Any relevant aspect that you need to take in 	<ol style="list-style-type: none"> 1. How did you manage the conceptual and procedural mathematical thinking in this unit in the presence of technological tools? 2. Which were the differences of teaching the mathematical content of this unit with the use of this specific tool? 3. Explain how the concepts/processes were enhanced through the use of technological tool(s)? 4. Do you have any comment to add to 	<ol style="list-style-type: none"> 1. Did your planning fit with the development of the class in teaching this unit? Did you find something unexpected? How did you manage it? 2. Could you describe what strategies were relevant for teaching this topic? What would you recommend for future teachers? 3. Describe useful strategies when students were interacting with this technological tool (s) for

<p><i>regards to the use of technological tool(s).</i></p> <p>4. <i>Do you think that the use of technological tools in your classes helped students to obtain the learning outcomes?</i></p> <p>5. <i>What did you learn about teaching this topic using the technological tool(s)?</i></p>	<p><i>about those discussions?</i></p> <p>4. <i>In which ways the technological tool (s) used in your classes improved or limited your teaching?</i></p> <p>5. <i>Any fact relevant to improve your teaching in regards to the use of technological tools for the specific topic?</i></p>	<p><i>account for future classes to teach this topic in regard to students' learning.</i></p>	<p><i>the presence of the technological tool in your teaching in regards to the mathematical content in this unit?</i></p>	<p><i>learning about this mathematical topic?</i></p> <p>4. <i>What would you change in your teaching if you would teach this topic again?</i></p>
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Appendix 10

Protocol of Observations

Attitudes

- Does the teacher talk about the importance of using the tool for teaching this unit with the students?
- Do teachers make explicit beliefs in regards to the ways the technological tool works better in class?
- Are there interactions that indicate teacher's belief in regards to the technological tool use in support teaching? Which interactions are they?
- Does any teacher's beliefs limit teaching? What are the teacher's approaches in regards to the tool?
- To analyze a discourse between teacher-students' interactions or teacher-tool interaction or teacher-tool-students to identify: what are the teacher's conceptions of the tool? What is the teacher's meaning of the tool for teaching?
- Why the teacher considers it important to use the tool to teach a specific topic?
- Why does the teacher prefer to use the tool?
- Why does the teacher consider it fundamental to use that tool for teaching a specific topic?

Knowledge of Technology (KT)

- Does this particular technological tool affect the mathematical content?
- To describe some affordances of the tool for the specific mathematics topic?
- To describe some constraints of the tool for the specific mathematics topic? (Do teachers need to use other resources to clarify questions or to give deeper explanations? Which are the possible limitations on the use of the tool?)
- What kind of constraints in the use of the tool may limit teaching?
- What are the strategies that teachers use to facilitate students' understanding of the mathematical topic with this specific tool?
- How does the teacher repurpose the tool for specific pedagogical purposes? (To analyze creativity and flexibility of the teacher teaching with available tools).
- What is the teacher's understanding of the tool?
- What is the teacher understanding of mathematics with the tool?
- How does the teacher use the tool for educative purposes?

Knowledge of Learners' Cognition (KLC)

- Does teacher assess or verify previous content mathematics knowledge required for understanding the topic.
- Does the teacher demonstrate understanding about characteristics and preconceptions that students bring to a learning situation? Do they focus in the content of the topic or in the technological tool, or both?

- Do teachers use the technological tool to access students' thinking about the topic? What is the way teachers show this access? Give examples.
- How teachers figure out students' difficulties and obstacles in understanding the mathematics topic during their teaching practices? Does the teacher make explicit those misunderstandings?
- What are the ways teachers try to solve students' misunderstanding in regards to the content of the classes or in regards to the use of the technological tool in their classes? Do they use the technological tool and focus on the curriculum? Write examples.

Knowledge of Subject Matter (KSM)

- How teachers manage the goals for learning when they are integrating the technological tool in class (Do they focus on the subject matter? or the subject matter within a technological context? or something else?).
- How the mathematical content is presented by the teacher in regards to the specific topic? (Does the teacher introduce first the conceptual mathematical thinking and then the procedural thinking? both at the same time? just the conceptual thinking? just the procedural thinking? how does it work in class?).
- What are the examples, metaphors, justifications, explanations or other strategies that teachers use in class to explain concepts and/or procedures.
- Does the teacher use different mathematical representations of the concepts involved in the studied topic? Are those representations explicit? What are those representations?
- How teachers answer students' questions in regards to mathematical content knowledge.
- How teachers interact with students and the technological tool in regards to the content knowledge

Knowledge of Pedagogy (KP)

- What is the learning theory approach used by the teacher. To write details to analyze teachers' approaches.
- Do students interact with the tool?
- Does the teacher interact alone with the tool?
- Do teachers and students interact both with the technological tool? To write details about those interactions.
- Do teachers follow a textbook or pre-existing curriculum or do they develop the activities by themselves following a curriculum?
- How teachers manage students' situations in the classroom. Exemplify student situations and give examples of teacher actions.
- Does the teacher present in a logical sequence the content and activities in the classroom.

- How the teacher guides students in learning mathematics with the use of the technological tool (The students work in learning about technology as they are learning the specific topic or if they focus on learning about the technology and later as the mathematical context).
- How the teacher integrates the technological tool into the curriculum.
- Describe useful strategies that teachers use for teaching in the classroom (Survey of protocol Beardsley, 2002, same questions) (Engaged students as members of a learning community; permitted students to drive the focus and directions of lesson; respected students' prior knowledge and their preconceptions inherent therein; permitted student exploration before formal representations; encouraged students to seek and value alternative modes of investigating or problem solving)

Appendix 11

Journal of reflections

I am asking your reflections after your teaching practice was completed for each one of the classes involve in the developing of the topic. The purpose of this journal of reflections is intended to explore your thoughts in specific questions regarding your teaching practice using any material, resource including the technological tool(s).

Attitudes

1. What did you learn about teaching this topic?
2. Do you think that the use of resources and materials including the technological tool (s) were useful in teaching this topic? Please, could you provide some details about how those resources support your teaching?
3. Do you think that your beliefs about the materials that you used in your class today are reflected in your teaching? Please, explain with some examples or details if it is possible.

Technology

1. Did you integrate the technological tool(s) used in your class with the content of the mathematics topic today? How it was possible?
2. In which ways were the affordances and constraints of the technological tools facilitating or limited your teaching. How did you manage these issues today?
3. How the use of resources in your teaching today reflects teaching and learning aspects? What changes would you suggest to improve teaching and learning this topic?

Learners' cognition

1. How did you identify students' difficulties in regards to mathematics content in this class?
2. How did the use of the technological tool help you to assess students' understanding in this class?
3. What did you expect from students in your class today? How was in the final?

Subject matter

1. What was the major difficulty that you found to teach the mathematical content in this class?
2. Have the materials, resources or technological tools that you used to teach this class helped you to develop the mathematical content? Explain, how was the development of the mathematical thinking in the presence of those tools?

3. What mathematics goals were achieved? What mathematical goals still need more work in order to achieve them?

Pedagogy

1. Did your curricular planning fit with the development of your class today? Any frustration unexpected in this class? How did you handle this one?
2. What strategies did you use to guide students in learning the topic today? Did you identify if they work in learning about the technology as they are learning the specific topic, or if they focus on learning about the technology and later as the mathematical context.
3. Describe useful strategies when students were interacting with this technological tool(s) for learning about a mathematics topic?
4. What would you change in your teaching if you would teach this topic again?

Appendix 12 Ethical Approval



Office of Research Services | Human Research Ethics Board
Administrative Services Building | Rm B202 | PO Box 1700 STN CSC, Victoria BC V8W 2Y2 Canada
T 250-672-4545 | F 250-721-8960 | uvic.ca/research | ethics@uvic.ca

Certificate of Renewed Approval

PRINCIPAL INVESTIGATOR: Mercy L. Pena Morales UVic STATUS: Ph.D. Student UVic DEPARTMENT: EDCI SUPERVISOR: Dr. Tim Pelton	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px;">ETHICS PROTOCOL NUMBER</td> <td style="padding: 2px;">13-044</td> </tr> <tr> <td colspan="2" style="padding: 2px;"><i>Minimal Risk - Delegated</i></td> </tr> <tr> <td style="padding: 2px;">ORIGINAL APPROVAL DATE:</td> <td style="padding: 2px;">28-Feb-13</td> </tr> <tr> <td style="padding: 2px;">RENEWED ON:</td> <td style="padding: 2px;">15-Jan-16</td> </tr> <tr> <td style="padding: 2px;">APPROVAL EXPIRY DATE:</td> <td style="padding: 2px;">27-Feb-17</td> </tr> </table>	ETHICS PROTOCOL NUMBER	13-044	<i>Minimal Risk - Delegated</i>		ORIGINAL APPROVAL DATE:	28-Feb-13	RENEWED ON:	15-Jan-16	APPROVAL EXPIRY DATE:	27-Feb-17
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PROJECT TITLE: Pedagogical content knowledge in an education context RESEARCH TEAM MEMBER: None DECLARED PROJECT FUNDING: None											
CONDITIONS OF APPROVAL											
<p>This Certificate of Approval is valid for the above term provided there is no change in the protocol.</p> <p>Modifications To make any changes to the approved research procedures in your study, please submit a "Request for Modification" form. You must receive ethics approval before proceeding with your modified protocol.</p> <p>Renewals Your ethics approval must be current for the period during which you are recruiting participants or collecting data. To renew your protocol, please submit a "Request for Renewal" form before the expiry date on your certificate. You will be sent an emailed reminder prompting you to renew your protocol about six weeks before your expiry date.</p> <p>Project Closures When you have completed all data collection activities and will have no further contact with participants, please notify the Human Research Ethics Board by submitting a "Notice of Project Completion" form.</p>											
Certification											
<p>This certifies that the UVic Human Research Ethics Board has examined this research protocol and concluded that, in all respects, the proposed research meets the appropriate standards of ethics as outlined by the University of Victoria Research Regulations Involving Human Participants.</p> <hr style="width: 20%; margin: auto;"/> <p style="text-align: center;">Dr. Rachael Scarth Associate Vice-President Research Operations</p>											

Certificate Issued On: 15-Jan-16

13-044 Pena Morales, Mercy L.