

SOME MULTILINEAR GENERATING FUNCTIONS
FOR q -HERMITE POLYNOMIALS

by

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ABSTRACT

By applying the transformation theory of basic (or q -) hypergeometric series, a number of multilinear generating functions are developed for the q -Hermite polynomials studied by L.J. Rogers, G. Szegő, and others. Relevant connections of these general multilinear generating functions with various known results for the classical or basic Hermite polynomials are also indicated.

1. INTRODUCTION, NOTATIONS, AND THE MAIN RESULTS

For real or complex q , $|q| < 1$, let

$$(1.1) \quad (\lambda; q)_{\mu} = \prod_{j=0}^{\infty} \left[\frac{1-\lambda q^j}{1-\lambda q^{\mu+j}} \right]$$

for arbitrary λ and μ , so that

$$(1.2) \quad (\lambda; q)_n = \begin{cases} 1, & \text{if } n = 0, \\ (1-\lambda)(1-\lambda q)\dots(1-\lambda q^{n-1}), & \forall n \in \{1, 2, 3, \dots\}, \end{cases}$$

and

$$(1.3) \quad (\lambda; q)_{\infty} = \lim_{n \rightarrow \infty} (\lambda; q)_n = \prod_{j=0}^{\infty} (1-\lambda q^j).$$

Define, as usual, a generalized basic (or q -) hypergeometric function by (cf. [13, Chapter 3])

$$\begin{aligned}
 (1.4) \quad & r \phi_s \left[\begin{array}{c} \alpha_1, \dots, \alpha_r; \\ \beta_1, \dots, \beta_s; \end{array} \middle| q, z \right] \\
 &= \sum_{n=0}^{\infty} (-1)^{(1+s-r)n} q^{(1+s-r)n(n-1)/2} \frac{(\alpha_1, \dots, \alpha_r; q)_n}{(\beta_1, \dots, \beta_s; q)_n} \frac{z^n}{(q; q)_n},
 \end{aligned}$$

where, and throughout this paper, we find it to be convenient to write

$$(1.5) \quad (\lambda_1, \dots, \lambda_k; q)_\mu = (\lambda_1; q)_\mu \cdots (\lambda_k; q)_\mu,$$

and, for convergence of the infinite series in (1.4), $|q| < 1$ and $|z| < \infty$ when $r \leq s$, or $|q| < 1$ and $|z| < 1$ when $r = s + 1$, provided that no zeros appear in the denominator.

In his important memoirs on expansions of certain infinite products, L.J. Rogers ([8], [9], [10]) introduced the continuous q -Hermite polynomials $H_n(x|q)$ and the continuous q -ultraspherical polynomials $C_n(x; \nu|q)$, defined by

$$(1.6) \quad \sum_{n=0}^{\infty} H_n(x|q) \frac{t^n}{(q; q)_n} = \frac{1}{h(x, t)}$$

and

$$(1.7) \quad \sum_{n=0}^{\infty} C_n(x; \nu|q) t^n = \frac{h(x, \nu t)}{h(x, t)},$$

where [cf. Equation (1.5)]

$$(1.8) \quad h(\cos \theta, \tau) = (\tau e^{i\theta}, \tau e^{-i\theta}; q)_{\infty}.$$

As a matter of fact, Rogers used his results involving these polynomials in order to prove the celebrated Rogers–Ramanujan identities.

Another set of q -Hermite polynomials defined explicitly by (cf., e.g., [1], [2, p. 49], and [17])

$$(1.9) \quad h_n(x|q) = \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_x x^k, \quad \begin{bmatrix} n \\ k \end{bmatrix} = \frac{(q; q)_n}{(q; q)_{n-k} (q; q)_k} = \begin{bmatrix} n \\ n-k \end{bmatrix},$$

are closely related to the continuous q -Hermite polynomials via

$$(1.10) \quad H_n(\cos \theta | q) = e^{-in\theta} h_n(e^{2i\theta} | q),$$

so that any given result involving one set of q -Hermite polynomials would apply also to the other set.

For these q -Hermite polynomials, Ismail and Stanton [6] have derived several interesting multilinear generating functions with the help of certain integrals which they studied systematically. Combinatorial proofs of some of Ismail–Stanton results were given by Ismail, Stanton, and Viennot [7]. The object of the present paper is to apply the transformation theory of the q -hypergeometric series (1.4) with a view to deriving the following general multilinear generating function for the q -Hermite polynomials:

$$\begin{aligned}
(1.11) \quad & \sum_{n_1, \dots, n_k=0}^{\infty} h_{n_1+\dots+n_k+s}(x|q) h_{m_1+n_1}(x_1|q) \dots h_{m_k+n_k}(x_k|q) \frac{t_1^{n_1}}{(q; q)_{n_1}} \dots \frac{t_k^{n_k}}{(q; q)_{n_k}} \\
& = \sum_{r_1=0}^{m_1} \dots \sum_{r_k=0}^{m_k} q^{m_1 r_1 + \dots + m_k r_k} \prod_{j=1}^k \left\{ \frac{(q^{-m_j}; q)_{r_j}}{(q; q)_{r_j}} (x_j t_j)^{r_j} h_{m_j-r_j}(x_j|q) \right\} \\
& \cdot \left\{ \frac{1}{(x, t_1, x_1 t_1, \dots, t_k, x_k t_k; q)_{\infty}} 2k^{\Phi} 2^{k-1} \left[\begin{matrix} t_1, x_1 t_1, \dots, t_k, x_k t_k; \\ q, q^{1+s+R} \end{matrix} \right] \right. \\
& \left. + \frac{x^s}{(1/x, x t_1, x x_1 t_1, \dots, x t_k, x x_k t_k; q)_{\infty}} 2k^{\Phi} 2^{k-1} \left[\begin{matrix} x t_1, x x_1 t_1, \dots, x t_k, x x_k t_k; \\ q x, 0, \dots, 0; \end{matrix} \right] \right\},
\end{aligned}$$

where, for convenience, $R = r_1 + \dots + r_k$.

For $m_1 = \dots = m_k = 0$, the multilinear generating function (1.11) simplifies considerably, and we thus obtain

$$\begin{aligned}
(1.12) \quad & \sum_{n_1, \dots, n_k=0}^{\infty} h_{n_1+\dots+n_k+s}(x|q) h_{n_1}(x_1|q) \dots h_{n_k}(x_k|q) \frac{t_1^{n_1}}{(q; q)_{n_1}} \dots \frac{t_k^{n_k}}{(q; q)_{n_k}} \\
& = \frac{1}{(x, t_1, x_1 t_1, \dots, t_k, x_k t_k; q)_{\infty}} 2k^{\Phi} 2^{k-1} \left[\begin{matrix} t_1, x_1 t_1, \dots, t_k, x_k t_k; \\ q, q^{1+s} \end{matrix} \right] \\
& + \frac{x^s}{(1/x, x t_1, x x_1 t_1, \dots, x t_k, x x_k t_k; q)_{\infty}} 2k^{\Phi} 2^{k-1} \left[\begin{matrix} x t_1, x x_1 t_1, \dots, x t_k, x x_k t_k; \\ q x, 0, \dots, 0; \end{matrix} \right],
\end{aligned}$$

which, for $s = 0$, corresponds to a result due to Ismail and Stanton [6, p. 28, Equation (5.14)].

Formula (1.12) may be looked upon as a q -analogue of a multilinear generating function for the classical Hermite polynomials, which was given by Srivastava and Singhal (cf. [15, p. 140, Equation (4.6) with $s = 0$]; see also [16, p. 1239, Equation (4)] and [14, p. 496, Problems 11 and 12]).

We develop our proof of the general result (1.11) in many stages. Indeed, in this process, we derive several interesting generating functions for the q -Hermite polynomials.

2. DERIVATION OF BILINEAR FORMULAS

We begin by proving the following bilinear generating function for q -Hermite polynomials:

$$(2.1) \quad \sum_{n=0}^{\infty} h_{n+s}(x|q) h_n(y|q) \frac{t^n}{(q; q)_n} = \frac{(xyt^2; q)_{\infty}}{(t, xt, yt, xyt; q)_{\infty}}$$

$$\cdot \frac{(xt; q)_s}{(xyt^2; q)_s} {}_2\phi_1 \left[\begin{matrix} yt, q^{-s}; \\ q^{1-s}/xt; \end{matrix} \middle| q, q/t \right]$$

or, equivalently,

$$(2.2) \quad \sum_{n=0}^{\infty} h_{n+s}(x|q)h_n(y|q) \frac{t^n}{(q; q)_n}$$

$$= \frac{1}{(x, t, yt; q)_{\infty}} {}_2\phi_1 \left[\begin{matrix} t, yt; \\ q/x; \end{matrix} \middle| q, q^{1+s} \right] + \frac{x^s}{(1/x, xt, xyt; q)_{\infty}} {}_2\phi_1 \left[\begin{matrix} xt, xyt; \\ qx; \end{matrix} \middle| q, q^{1+s} \right],$$

where s is a nonnegative integer.

Formula (2.1) is a q -analogue of a bilinear generating function due to Carlitz [3, p. 127, Equation (6.5)]. In our proof of (2.1) we shall make use of each of the following known results:

$$(2.3) \quad \sum_{n=0}^{\infty} h_{n+s}(x|q) \frac{t^n}{(q; q)_n} = \frac{1}{(t, xt; q)_{\infty}} \Phi_k^{(t)}(x, q),$$

where $\Phi_n^{(\alpha)}(x, q)$ denotes a set of orthogonal q -polynomials studied by Hahn ([4], [5]), and by Al-Salam and Carlitz [1]; in fact, we have

$$(2.4) \quad \Phi_n^{(\alpha)}(x, q) = \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix} (\alpha; q)_k x^k,$$

so that, obviously,

$$(2.5) \quad \Phi_n^{(0)}(x) \equiv h_n(x|q);$$

$$(2.6) \quad \sum_{n=0}^{\infty} h_n(x|q) \Phi_n^{(\alpha)}(y, q) \frac{t^n}{(q; q)_n} \\ = \frac{(\alpha xy t; q)_{\infty}}{(t, xt, xyt; q)_{\infty}} 2^{\Phi} 1 \begin{bmatrix} \alpha, xt; \\ q, yt \\ \alpha xy t; \end{bmatrix},$$

which, for $\alpha = 0$, reduces immediately to the q -Mehler formula [2, p. 50, Example 9]:

$$(2.7) \quad \sum_{n=0}^{\infty} h_n(x|q) h_n(y|q) \frac{t^n}{(q; q)_n} = \frac{(xyt^2; q)_{\infty}}{(t, xt, yt, xyt; q)_{\infty}}.$$

The generating function (2.3), which essentially is equivalent to (2.7), was given by Ismail and Stanton [6, p. 29, Equation (5.17)]. Formula (2.6), on the other hand, corresponds to the special case $a = 0$ of a result given by Al-Salam and Carlitz [1, p. 49, Equation (1.17)].

Now we turn to the proof of the bilinear generating function (2.1). Denoting the left-hand side of (2.1) by $A_S(x, y, t)$, and making use of (2.3) and (2.6), we find from (2.1) that

$$(2.8) \quad \sum_{s=0}^{\infty} A_S(x, y, t) \frac{u^s}{(q; q)_s} = \sum_{n=0}^{\infty} h_n(y|q) \frac{t^n}{(q; q)_n} \sum_{s=0}^{\infty} h_{n+s}(x|q) \frac{u^s}{(q; q)_s} \\ = \frac{1}{(u, xu; q)_{\infty}} \sum_{n=0}^{\infty} h_n(y|q) \Phi_n^{(u)}(x, q) \frac{t^n}{(q; q)_n}$$

$$\begin{aligned}
&= \frac{(uxyt; q)_{\infty}}{(t, yt, xyt, u, xu; q)_{\infty}} {}_2\phi_1 \left[\begin{matrix} yt, u; \\ q, xt \\ uxyt; \end{matrix} \right] \\
&= \frac{(xyt^2; q)_{\infty}}{(t, xt, yt, xyt, xu; q)_{\infty}} {}_2\phi_1 \left[\begin{matrix} xt, xyt; \\ q, u \\ xyt^2; \end{matrix} \right],
\end{aligned}$$

where we have employed the familiar Heine's transformation [2, p. 19, Corollary 2.3]:

$$(2.9) \quad {}_2\phi_1 \left[\begin{matrix} a, b; \\ q, z \\ c; \end{matrix} \right] = \frac{(b, az; q)_{\infty}}{(c, z; q)_{\infty}} {}_2\phi_1 \left[\begin{matrix} z, c/b; \\ q, b \\ az; \end{matrix} \right].$$

Equating the coefficients of u^s in the first and last members of (2.8), we obtain

$$\begin{aligned}
(2.10) \quad \Delta_s(x, y, t) &= \frac{(xyt^2; q)_{\infty}}{(t, xt, yt, xyt; q)_{\infty}} \frac{(xt, xyt; q)_s}{(xyt^2; q)_s} \\
&\quad \cdot {}_3\phi_2 \left[\begin{matrix} q^{-s}, q^{1-s}/(xyt^2), 0; \\ q^{1-s}/(xt), q^{1-s}/(xyt); \\ q, q \end{matrix} \right].
\end{aligned}$$

Finally, we transform this ${}_3\phi_2$ series by applying a particular case ($a = 0$ and $c = 0$) of Sears's formula [11, p. 167, Equation (8.3)]:

$$(2.11) \quad {}_4\phi_3 \left[\begin{matrix} a, b, c, q^{-n}; \\ e, g, h; \end{matrix} \middle| q, q \right] = \frac{(g/c, eg/ab; q)_n}{(g, eg/abc; q)_n} \cdot {}_4\phi_3 \left[\begin{matrix} e/a, e/b, c, q^{-n}; \\ e, cq^{1-n}/g, cq^{1-n}/h; \end{matrix} \middle| q, q \right],$$

$$egh = abcq^{1-n} \quad (n = 0, 1, 2, \dots),$$

and (2.10) leads us at once to the bilinear generating function (2.1).

In order to obtain the equivalent form (2.2), we use a known transformation [18, p. 370, Equation (3)]

$$(2.12) \quad {}_2\phi_1 \left[\begin{matrix} a, b; \\ e; \end{matrix} \middle| q, \frac{ec}{ab} \right] = \frac{(e/a, e/b; q)_\infty}{(e, e/ab; q)_\infty} {}_3\phi_2 \left[\begin{matrix} a, b, c; \\ abq/e, 0; \end{matrix} \middle| q, q \right],$$

$$a \text{ or } b \text{ or } c = q^{-n} \quad (n = 0, 1, 2, \dots),$$

in (2.1) twice, so that

$$(2.13) \quad \Delta_s(x, y, t) = (yt)^{-s} \frac{(xyt^2, q/(xyt^2), yq; q)_\infty}{(t, xt, yt, xyt, q/t, q/xt; q)_\infty}$$

$$\cdot {}_2\phi_1 \left[\begin{matrix} yt, xyt; \\ yq; \end{matrix} \middle| q, \frac{q^{1-s}}{xyt^2} \right],$$

which yields (2.2) in view of another result due to Sears [12, p. 183, Equation (4.1)]:

$$\begin{aligned}
 (2.14) \quad & (q/b, aq/c, c; q)_\infty {}_2\phi_1 \left[\begin{matrix} a, b; \\ q, z \\ c; \end{matrix} \right] \\
 &= \frac{(a, c/b, q^2/c, bz, q/bz; q)_\infty}{(c/bz, bzq/c; q)_\infty} {}_2\phi_1 \left[\begin{matrix} aq/c, bq/c; \\ q^2/c; \\ q, z \end{matrix} \right] \\
 &+ \frac{(c, q/c, aq/b, abz/c, cq/(abz); q)_\infty}{(bz/c, cq/bz; q)_\infty} {}_2\phi_1 \left[\begin{matrix} a, aq/c; \\ q, \frac{cq}{abz} \\ aq/b; \end{matrix} \right].
 \end{aligned}$$

3. MULTILINEAR EXTENSIONS

If, in the bilinear generating function (2.2), we replace s by $s + m + r$, y by z , and t by v , multiply both sides by

$$\frac{y^r}{(q; q)_r} \frac{u^{m+r}}{(q; q)_m},$$

and sum the resulting expression with respect to m and r from 0 to ∞ , we shall obtain the trilinear formula:

$$\begin{aligned}
(3.1) \quad & \sum_{m,n=0}^{\infty} h_{m+n+s}(x|q)h_m(y|q)h_n(z|q) \frac{u^m}{(q;q)_m} \frac{v^n}{(q;q)_n} \\
&= \frac{1}{(x,u,v,yu,zv;q)_{\infty}} 4^{\Phi_3} \left[\begin{matrix} u,v,yu,zv; \\ q,q^{1+s} \\ q/x,0,0; \end{matrix} \right] \\
&+ \frac{x^s}{(1/x,xu,xv,xyu,zxv;q)_{\infty}} 4^{\Phi_3} \left[\begin{matrix} xu,xv,xyu,zxv; \\ q,q^{1+s} \\ qx,0,0; \end{matrix} \right],
\end{aligned}$$

which, for $s = 0$, corresponds to a result due to Ismail and Stanton [6, p. 27, Equation (5.10)].

Now replace y and z in (3.1) by yt and $z\tau$, respectively, multiply both sides by

$$t^{\alpha-1} \tau^{\beta-1} (tq;q)_{\lambda-\alpha-1} (\tau q;q)_{\mu-\beta-1},$$

and q -integrate with respect to t and τ using the familiar result:

$$(3.2) \quad \frac{\Gamma_q(\alpha)\Gamma_q(\beta)}{\Gamma_q(\alpha+\beta)} = \int_0^1 t^{\alpha-1} (tq;q)_{\beta-1} d(q,t),$$

where the q -Gamma function $\Gamma_q(z)$ is defined by

$$(3.3) \quad \Gamma_q(z) = \frac{(q;q)_{\infty}}{(q^z;q)_{\infty}} (1-q)^{1-z}.$$

Upon letting $\lambda, \mu \rightarrow \infty$, and replacing q^{α} and q^{β} by α and β , respectively, we thus find from (3.1) and (2.4) that

$$\begin{aligned}
(3.4) \quad & \sum_{m,n=0}^{\infty} h_{m+n+s}(x|q)_{\Phi_m^{(\alpha)}}(y,q)_{\Phi_n^{(\beta)}}(z,q) \frac{u^m}{(q;q)_m} \frac{v^n}{(q;q)_n} \\
&= \frac{(\alpha y u, \beta z v; q)_{\infty}}{(x, u, v, y u, z v; q)_{\infty}} 4^{\Phi_3} \left[\begin{array}{c} u, v, y u, z v; \\ q, q^{l+s} \end{array} \right] \\
&+ \frac{(\alpha x y u, \beta z x v; q)_{\infty}}{(1/x, x u, x v, x y u, z x v; q)_{\infty}} x^s 4^{\Phi_3} \left[\begin{array}{c} x u, x v, x y u, z x v; \\ q x, \alpha x y u, \beta z x v; \\ q, q^{l+s} \end{array} \right].
\end{aligned}$$

Formula (3.4) would obviously reduce to (3.1) when $\alpha = \beta = 0$. It may be of interest to remark in passing that, by applying the aforementioned process of q -integration for augmenting parameters in the multilinear generating function (1.12), we can easily deduce the following interesting result:

$$\begin{aligned}
(3.5) \quad & \sum_{n_1, \dots, n_k=0}^{\infty} h_{n_1 + \dots + n_k + s}(x|q)_{\Phi_{n_1}^{(\alpha_1)}}(x_1, q) \dots \Phi_{n_k}^{(\alpha_1)}(x_k, q) \frac{t_1^{n_1}}{(q;q)_{n_1}} \dots \frac{t_k^{n_k}}{(q;q)_{n_k}} \\
&= \frac{(\alpha_1 x_1 t_1, \dots, \alpha_k x_k t_k; q)_{\infty}}{(x, t_1, x_1 t_1, \dots, t_k, x_k t_k; q)_{\infty}} 2k^{\Phi_{2k-1}} \left[\begin{array}{c} t_1, x_1 t_1, \dots, t_k, x_k t_k; \\ q, q^{l+s} \end{array} \right] \\
&+ \frac{(\alpha_1 x x_1 t_1, \dots, \alpha_k x x_k t_k; q)_{\infty}}{(1/x, x t_1, x x_1 t_1, \dots, x t_k, x x_k t_k; q)_{\infty}} x^s 2k^{\Phi_{2k-1}} \left[\begin{array}{c} x t_1, x x_1 t_1, \dots, x t_k, x x_k t_k; \\ q x, \alpha_1 x x_1 t_1, \dots, \alpha_k x x_k t_k, 0, \dots, 0; \\ q, q^{l+s} \end{array} \right]
\end{aligned}$$

which, for $\alpha_1 = \dots = \alpha_k = 0$, yields (1.12) by virtue of (2.5).

The multilinear generating function (1.12) can be proven directly by suitably iterating our method of derivation of the trilinear formula (3.1). We now outline our derivation of the general result (1.11) for $k = 2$.

In view of (2.3), the generating function (3.4) may be rewritten in the form:

$$(3.6) \quad \sum_{i,j=0}^{\infty} \frac{\alpha^i}{(q;q)_i} \frac{\beta^j}{(q;q)_j} \sum_{m,n=0}^{\infty} h_{m+n+s}(x|q) h_{m+i}(y|q) h_{n+j}(z|q) \frac{u^m}{(q;q)_m} \frac{v^n}{(q;q)_n} \\ = S_1 + S_2,$$

where, for convenience,

$$(3.7) \quad S_1 = \frac{(\alpha y u, \beta z v; q)_{\infty}}{(x, u, v, y u, z v, \alpha, \beta, \alpha y, \beta z; q)_{\infty}} 4^{\Phi} 3 \left[\begin{array}{c} u, v, y u, z v; \\ q, q^{1+s} \\ q/x, \alpha y u, \beta z v; \end{array} \right]$$

and

$$(3.8) \quad S_2 = \frac{(\alpha x y u, \beta z x v; q)_{\infty}}{(1/x, x u, x v, x y u, z x v, \alpha, \beta, \alpha y, \beta z; q)_{\infty}} x^s 4^{\Phi} 3 \left[\begin{array}{c} x u, x v, x y u, z x v; \\ q, q^{1+s} \\ q x, \alpha x y u, \beta z x v; \end{array} \right].$$

Expanding S_1 and S_2 in double series of powers of α and β , and equating the coefficients of $\alpha^i \beta^j$ from both sides of (3.6), we finally obtain the trilinear case ($k=2$) of (1.11). The multilinear case can indeed be proven similarly, and we omit the details involved.

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