A HYBRID ANALOG COMPUTER
FOR BEAM OPTICS CALCULATIONS

by

ROBERT JOHN LOUIS
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ABSTRACT

Supervisor: R. M. Pearce

A hybrid analog computer is described which calculates particle trajectories in beam transport systems consisting of thin lenses, quadrupoles, bending magnets and drift spaces. Bending magnets can have constant or non-constant fields and normal or non-normal entry and exit angles. Calculations can be performed for systems containing up to sixteen magnets (i.e. sixteen thin lenses, quadrupoles, or bending magnets). The magnet strengths are set by potentiometers and the magnet lengths and positions are set by selecting pulses from a digital timing system consisting of a digital clock, AND gates, flip-flops and relays. Since all magnet parameters are set by potentiometers and switches, the problem set up time is small, changes in the beam transport system can easily be made and the effect on the beam is immediately apparent. The analog can run in a fast mode (with oscilloscope display) or a slow mode (with XY-plotter display). The problem can be stopped at any point to allow accurate measurement of the problem variables. The system has been found to be in error less than 0.07 cm in a system which has a maximum particle displacement of greater than 8.0 cm. The possibility of extending the system to the calculation of beam envelopes is discussed.

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I. INTRODUCTION

A particle accelerator emits a diverging beam of particles which can be contained and focussed using quadrupole magnets. The building which houses the accelerator and the layout of the experimental area often requires the beam to be bent; this is accomplished with bending magnets. The field free regions between these two types of magnets are called drift spaces. The magnets and drift spaces comprise a beam transport system.

The design of a beam transport system is difficult because a typical system will require many (perhaps 20 or 30) magnets. The effects of the magnets are not independent and each magnet affects the beam both in the horizontal and in the vertical planes, but in different ways.

The analog computer described here is an electrical analog of the problem described above. An electrical network solves differential equations which are similar to the equations of motion of the particles in the beam transport system. The theory of beam transport can be found in books by Steffen[1] and Banford[2]. Suitable scale factors relate the voltages on the analog computer to the displacements of the particles in the beam transport system. Distance down the beam axis corresponds to time on the analog computer, while displacement of the particle from the beam axis corresponds to a voltage.

This analog can calculate particle trajectories for systems containing up to sixteen magnets. The calculation takes about one second in the fast operating mode (with oscilloscope display for optimizing), and about ten seconds in the slow mode (with XY-plotter display for recording results). The quadrupole gradients and bending magnet fields are set by potentiometers and element positions are set by switches, so that the problem set up time is small and the positions and/or strengths of any element can easily be changed and the effect on the beam is immediately apparent.

Other beam transport analog computers have been built[3, 4, 5, 6, 7]. The analog computer described in this thesis is based on a commercial analog computer*. It incorporates the latest advances in analog computer techniques which make it easy to operate and very accurate. Also, it is small in size and can easily be switched between the fast and slow operating modes mentioned above.

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* The Systron-Donner 10/20

II. NOTATION AND EQUATIONS OF MOTION

We are concerned with systems consisting of drift spaces, thin lenses, quadrupoles and bending magnets. A bending magnet which has entry and exit faces not perpendicular to the beam axis can be treated[5] as a bending magnet with the entry and exit faces perpendicular to the beam direction and a certain thin lens (or focussing edge) at the entry and exit points.

The quantities used to describe these elements and the units used are:

1) drift spaces: length, L (cm.)

2) thin lenses: length, L (cm.)
focal length, f (cm.)

3) quadrupoles: length, L (cm.)
field gradient, G (kilogauss/cm. or kG./cm.)

4) bending magnets: length (measured along the beam axis), L (cm.)
bend angle, θ (radians)
magnetic field, H (kilogauss or kG.)
field index, n
radius of curvature of the beam axis through the magnet, r_o (cm.)

entrance angle, ϕ_1 (radians)

exit angle, ϕ_2 (radians)

The angles used in describing bending magnets are defined in Fig. 1. In Fig. 1, both ϕ_1 and ϕ_2 are positive (since positive angles are measured in the counterclockwise direction). Note the difference in the ways in which the entrance and exit angles are measured.

The field index n is a measure of the homogeneity of the field of a bending magnet and is given by

$$n = -\frac{r_o}{H}\frac{\partial H}{\partial r} \quad . \tag{1}$$

n is zero for a magnet which has a constant field.

The beam of particles can be described by the following quantities

energy, E (GeV.)

momentum, p (GeV./c)

magnetic rigidity, $(B\rho)$ (kilogauss cm. or kG.cm.)

momentum deviation, $\Delta p/p$ from some standard momentum.

The magnetic rigidity of a beam is a measure of how much force need be applied to bend the beam.

Some useful relations between these quantities are

$$r_o \text{ (cm.)} = \frac{10^4}{2.99793}\frac{p \text{ (GeV./c.)}}{H \text{ (kG.)}} \tag{2}$$

$$p \text{ (GeV./c.)} = 2.99793 \times 10^{-4} (B\rho) \text{ (kG.cm.)} \tag{3}$$

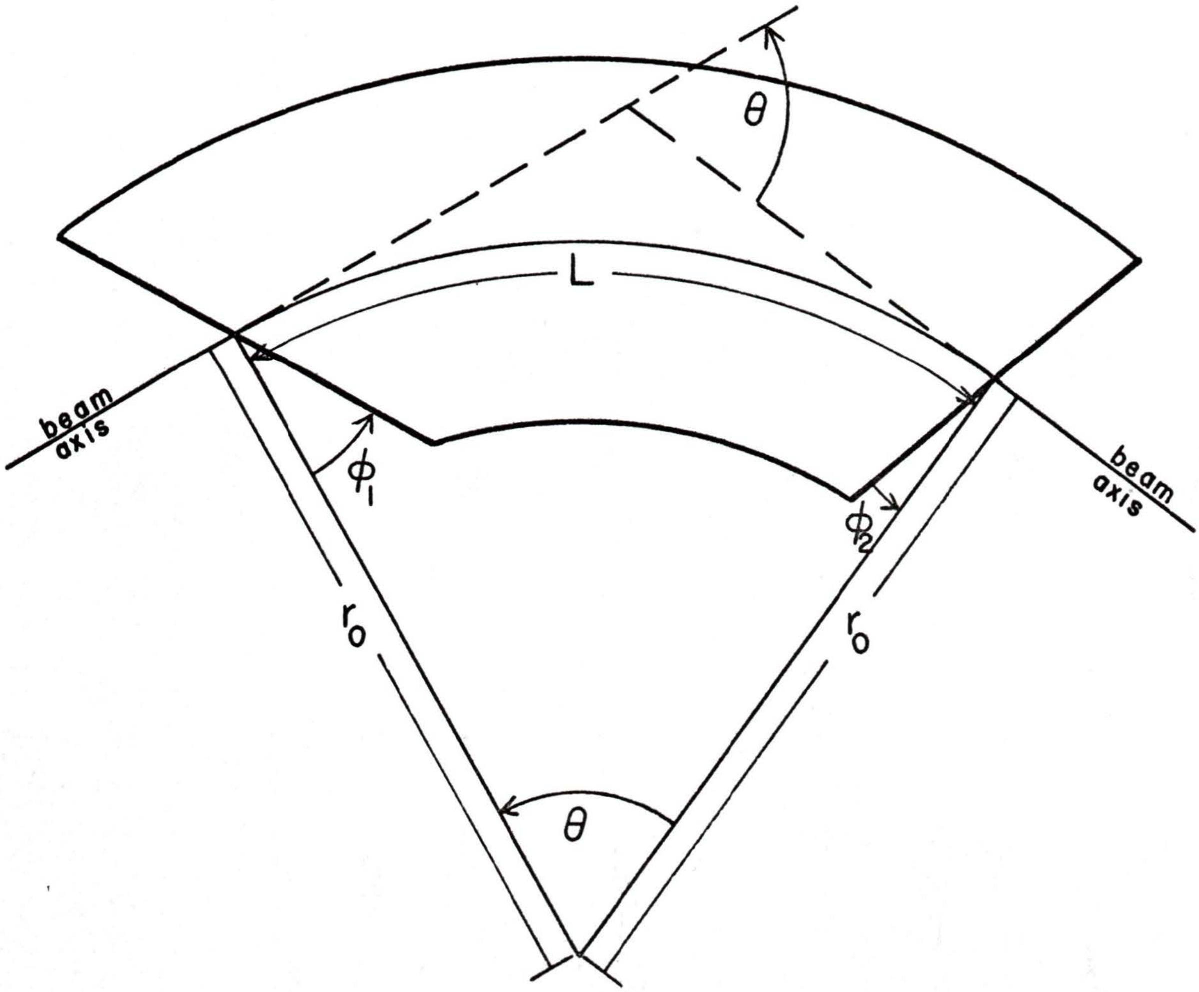

FIGURE 1

QUANTITIES USED IN DESCRIBING BENDING MAGNETS

$$r_o \text{ (cm.)} = \frac{(B\rho) \text{ (kG.cm.)}}{H \text{ (kG.)}} \tag{4}$$

$$\theta \text{ (radians)} = \frac{L}{r_o} \ . \tag{5}$$

The effect of the transport system on the beam can be described in two ways, either by calculating trajectories of individual particles through the system or by calculating the envelope of the whole beam of particles. Beam envelopes are discussed in Appendix II.

A particle trajectory is the displacement of the particle from the beam axis as a function of z, the distance down the beam axis. The beam axis is that trajectory which has zero initial displacement and slope. The displacement of the particle from the beam axis in the XZ-(horizontal) plane is denoted x, in the YZ-(vertical) plane, is denoted y. Since the effects of the magnetic elements are different in the horizontal and vertical planes, the horizontal and vertical trajectories will never be the same. A particle trajectory is completely defined by the description of the magnet system and the initial slope and displacement of the particle.

The equations of motion for particles in a beam transport system can be derived by substituting the magnetic fields produced by the various elements into the Lorentz equation.

In most high energy beam transport systems, the particle divergences are small so the velocity of the particles parallel to the beam axis remains constant and the following first order trajectory equations can be derived[8, 9].

$$\frac{d^2y}{dz^2} + g_y y = 0 \quad , \tag{6}$$

$$\frac{d^2x}{dz^2} + g_x x + d = 0 \tag{7}$$

where g_x , g_y and d are coefficients which describe the various magnetic elements. Expressions for these coefficients have been given by Hansford and Aspley[10] and are reproduced in Table I.

It should be noted that the expressions given for thin lenses and focussing edges* are derived with the assumption that the length of the element (thin lens or focussing edge) approaches zero. Since the analog computer cannot simulate elements which have zero length, these elements must be simulated by elements which have non-zero length. These lengths should be made as small as possible.

Equations (6) and (7) describe the trajectories of

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* A thin lens is an approximation to a thick lens (which is a quadrupole) which is useful for rough calculation when no access to computer is available. The approximation involved in going from thick to thin lenses is that the particle displacement does not change in the lens. Since thick lenses can easily be simulated on the analog, thin lenses will never be used in calculations on the analog computer. The thin lens coefficients are included in Table I for the sake of interest and because focussing edges (which are often used in beam transport calculations) are thin lenses with focal length $\frac{-r_0}{\tan \phi}$.

particles with respect to the beam axis. This axis bends through bending magnets and is defined at some fixed reference momentum. Since the radius of curvature is a function of momentum and since it is inconvenient to change the axis when a small change in momentum is to be studied, the term d in equation (7) has been added. This gives a first order correction to the trajectory for a small momentum change. That is, if a particle is tracked through the system at momentum p and a trajectory is desired for momentum $p + \Delta p$, including the $\frac{1}{r_o} \frac{\Delta p}{p}$ term (cf. Table I).gives the new trajectory (at momentum $p + \Delta p$) with respect to the old axis (at momentum p). Note that this correction term is non-zero only in bending magnets. The first order correction for small momentum change is zero in quadrupoles and in fact is zero in all elements which do not bend the beam axis[11].

As in most calculations on beam transport systems the analog computer performs calculations as if all magnetic elements are "hard-edged" i.e., as if the field rises abruptly from zero outside the element to some value which remains constant inside the element. In practice, the fields are not this shape but drop off to zero over some small distance. It has been shown[*,12] that if $\int G\, dz$ for the hard edge model is the same as $\int G\, dz$ for the actual magnet, the trajectories of the particles will be correct to the first order at least. Hence, the element lengths and

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[*] Blackstein, F. P. and A. J. Otter, Private Communication, January 1967.

fields used on the analog are often slightly different from the actual physical quantities.

Table I

Coefficients in Equations of Motion for Various Magnetic Elements

Element Type	Element Description	XZ-Plane			YZ-Plane	
		element length (L)	g_x	d	element length (L)	g_y
thin lens	focal length = f focussing horizontally	arbitrary but <<< than drift space between elements	$\frac{1}{Lf}$	---	arbitrary but <<< than drift space between elements	$-\frac{1}{Lf}$
	focal length = f defocussing horizontally	arbitrary but <<< than drift space between elements	$-\frac{1}{Lf}$	---	arbitrary but <<< than drift space between elements	$\frac{1}{Lf}$
quadrupole	field gradient = G effective length = L focussing horizontally	L	$\frac{G}{B\rho}$	---	L	$-\frac{G}{B\rho}$
	field gradient = G effective length = L defocussing horizontally	L	$-\frac{G}{B\rho}$	---	L	$\frac{G}{B\rho}$
bending magnet	angle of bend = θ radius of curvature of beam axis through the magnet = r_o entrance angle = 0^o exit angle = 0^o field index = n	$r_o\ \theta$	$\frac{1-n}{r_o^2}$	$\frac{1}{r_o}\ \frac{\Delta p}{p}$	$r_o\ \theta$	$\frac{n}{r_o^2}$

fields used on the analog are often slightly different from the actual physical quantities.

Table I (cont'd.)

Element Type	Element Description	XZ-Plane			YZ-Plane	
		element length (L)	g_x	d	element length (L)	g_y
focussing or defocussing edge*	at angle Ø to bending magnet with radius of curvature = r_o	arbitrary but <<< than drift space between elements	$\frac{\tan \emptyset}{L\ r_o}$	---	arbitrary but <<< than drift space between elements	$-\frac{\tan \emptyset}{L\ r_o}$
drift space	length = L	L	0	---	L	0

* The coefficients given here can be used to simulate non-zero entrance or exit angles. The edge will be focussing or defocussing depending on the sign of Ø.

III. DESCRIPTION OF THE COMPUTER

The analog computer calculates the particle trajectories by solving equations similar to equations (6) and (7) . Starting with given initial conditions, the equation appropriate to the first element is solved for the time which corresponds to the length of the first element. At the end of the first element, the values of displacement and slope become the initial conditions for the equation which describes the second element. Thus the particle trajectory is calculated by solving the equation for each element in sequence.

Since the equations of motion for all types of elements considered here are of the form of equations (6) and (7), the trajectories can be calculated by solving equations (6) and (7) and switching the coefficients g_x, g_y and d to the values appropriate for each magnet in sequence.

The computer consists of two main parts, an analog circuit which solves the differential equations (6) and (7) and a digital timing system which switches the values of g_x, g_y and d at the appropriate times.

A. <u>The Analog System</u>

The analog circuit is shown in standard notation in Fig. 2. The triangles represent summers, the triangles with rectangles on the end represent integrators. The input-output relationships for summers and integrators are given in Appendix I.

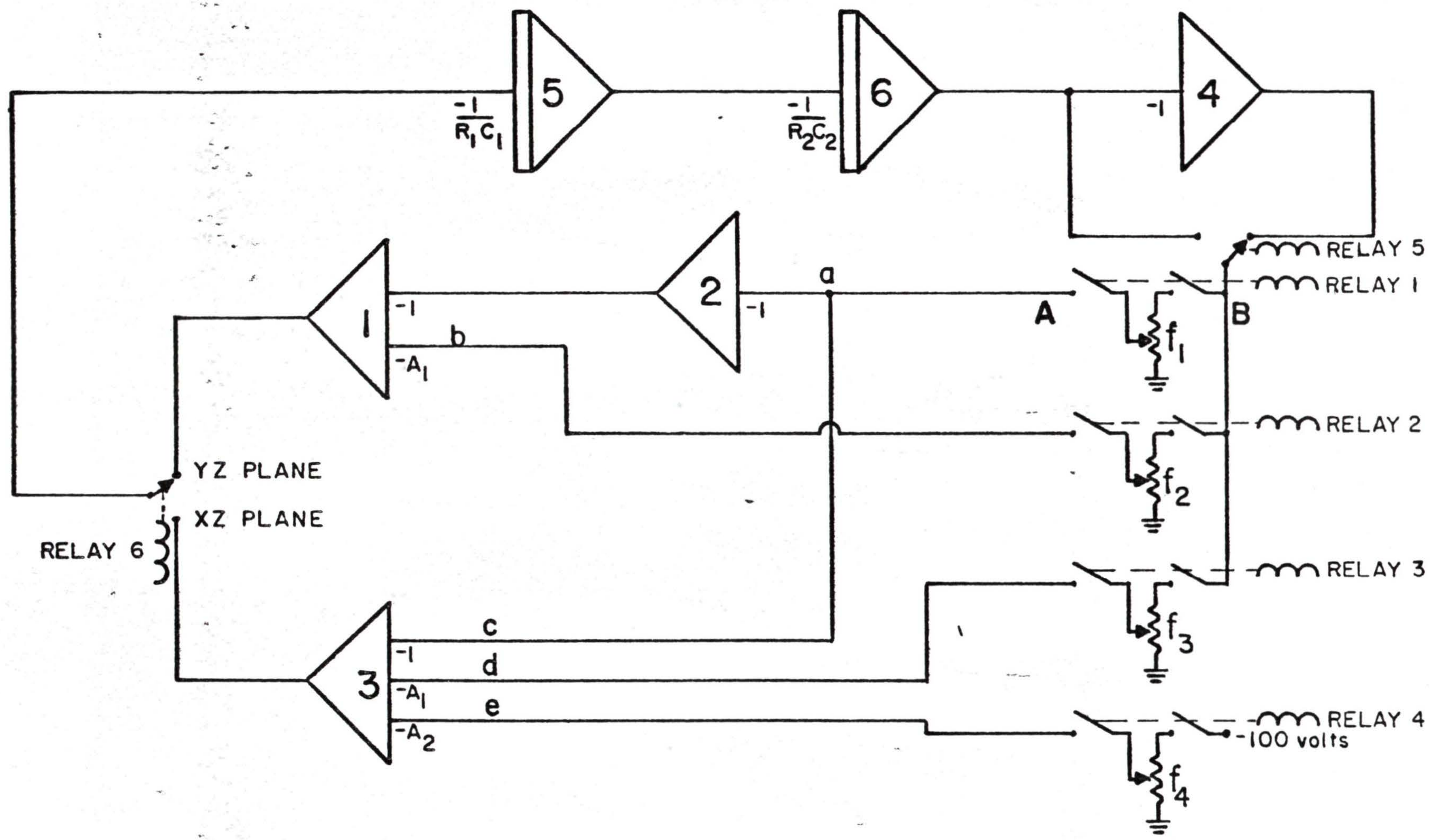

FIGURE 2 ANALOG CIRCUIT

In Fig. 2, the summer gains are noted at the input to each summer and the factor by which the integral is multiplied is noted at each integrator input.

Consider the circuit of Fig. 2. The potentiometer settings f_1, f_2, f_3 and f_4 will represent the coefficients in equations (6) and (7) which are given in Table I. Potentiometers like f_1 would be used for the $\frac{G}{B\rho}$, $\frac{1}{Lf}$ and $\frac{\tan \phi}{L\ r_o}$ terms for quadrupoles, thin lenses and focussing edges respectively. Potentiometers like f_2 would be used to produce the $\frac{n}{r_o^2}$ term, potentiometers like f_3 would be used to produce the $\frac{1 - n}{r_o^2}$ term. Potentiometers like f_4 would be used to produce the inhomogeneous $\frac{1}{r_o} \frac{\Delta p}{p}$ term (note that f_4 is fed by -100 volts rather than by $\pm V$ as the potentiometers which set coefficients of homogeneous terms are). Summer 2 gives the sign inversion necessary between XZ- and YZ-planes.

The quantities with which the analog computer computes are voltage V and time t. Time is measured from the time at which the computation starts.

If we choose to call the voltage at the input to Integrator 5 $R_1C_1R_2C_2 \frac{d^2V}{dt^2}$ then the output of this integrator will be $-R_2C_2 \frac{dV}{dt}$ and the output of Integrator 6 will be V. Summer 4 has a gain of -1 so its output will be -V hence after Relay 5, the voltage will be $\pm V$. This voltage goes to the top of many potentiometers of which only three (which have settings f_1, f_2 and f_3) are shown. If, for example, Relay 1 were closed, then the input to Summer 2 would be $\pm f_1 V$, the output would be $\pm f_1 V$, the output of Summer 1 would be $\pm f_1 V$.

This voltage goes to the input of Integrator 5 (when Relay 6 is in the position shown) which we have already called $R_1C_1R_2C_2 \frac{d^2V}{dt^2}$; hence it must be true that

$$R_1C_1R_2C_2 \frac{d^2V}{dt^2} = \pm f_1V$$

which is usually written

$$\frac{d^2V}{dt^2} \pm \frac{f_1}{R_1C_1R_2C_2} V = 0 \quad . \tag{8}$$

If Relay 6 is in the other position, arguments similar to the above give (with Relay 1 and Relay 4 closed)

$$\frac{d^2V}{dt^2} \pm \frac{f_1}{R_1C_1R_2C_2} V + \frac{f_4 A_2 100}{R_1C_1R_2C_2} = 0 \quad . \tag{9}$$

A_2 is the gain with which the f_4 term is introduced into Summer 3.

It can be seen that equations (8) and (9) are similar in form to equations (6) and (7). The relations connecting them are given in the next section.

For a drift space, no potentiometer would be connected into the circuit i.e. there would be an open circuit. There are twelve potentiometers available in the analog any of which can be wired for any of the four purposes described above. The relays are closed (by the timing system) in a sequence which gives the coefficients (g_x, g_y and d) necessary to simulate the transport system.

B. <u>The Timing System</u>

The timing system must connect one or more potentiometers into the circuit for each magnetic element. For a quadrupole, thin lens or focussing edge, one potentiometer is required (to produce the $\frac{G}{B\rho}$, $\frac{1}{Lf}$ or $\frac{\tan \emptyset}{r_o}$ coefficient respectively). For a non-uniform field bending magnet ($n \neq 0$, $\Delta p/p \neq 0$), three potentiometers are required; one each to produce the $\frac{1-n}{r_o^2}$, $\frac{n}{r_o^2}$ and $\frac{1}{r_o}\frac{\Delta p}{p}$ coefficients. For a drift space, no potentiometers are required since the circuit is open.

Each potentiometer must be connected in at a time which corresponds to the entrance position of the element and disconnected at the time which corresponds to the exit position. Thus, by switching in a series of potentiometers, a series of magnetic elements is simulated.

The timing system consists of a three-decade digital clock, 36 AND gates, 12 flip-flops and 12 relays. The circuit used to switch one potentiometer in and out is shown in Fig. 3. Points A and B on Fig. 3 correspond to points A and B on Fig. 2. The entrance and exit times for each element are selected by selecting the clock outputs which correspond to the desired times. The first set of pulses from the clock cause the AND gate to fire, turning the flip-flop on and closing the relay, hence connecting the potentiometer into the circuit. The second set of pulses from the clock causes another AND gate to fire, turning the flip-flop off and disconnecting the potentiometer . Each potentiometer is controlled by a circuit like the one in Fig. 3, i.e. the clock drives many AND gates in parallel.

The clock which sets the magnet entrance and exit times (and hence the magnet lengths) is a three decade digital clock, hence there are 10^3 possible entrance and exit times. This means that the magnet lengths are quantized, the size of one quantum being 10^{-3} of the total time available on the clock. In practice, this means that entrance and exit positions can be set only to a whole number of cm. Usually the magnet lengths are not exactly a whole number of cm. but if the g_x and g_y values are adjusted to make $\int G\,dz$ for the analog equal to $\int G\,dz$ for the real magnet, the trajectory is correct as discussed above for "hard edged" magnets.

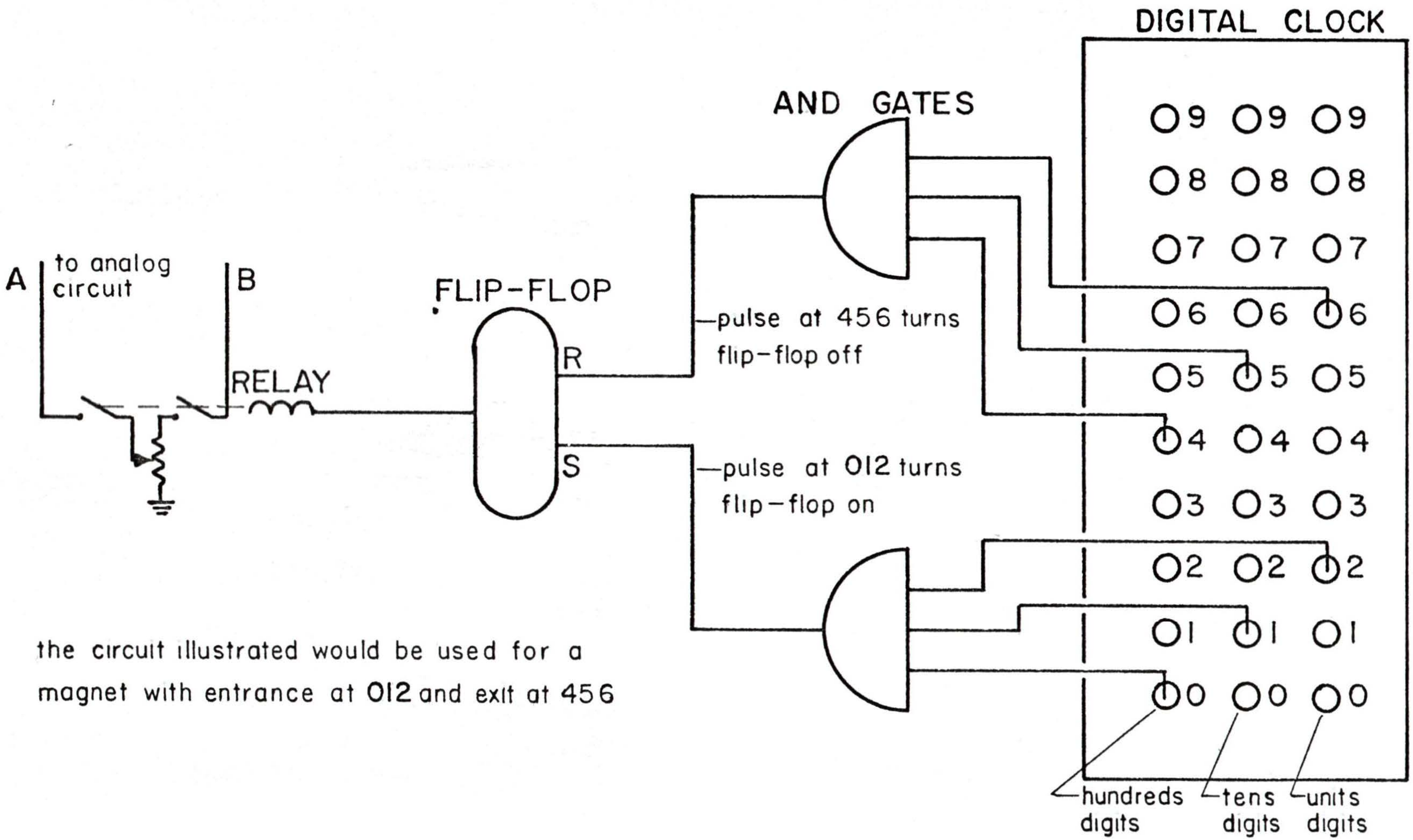

FIGURE 3 TIMING CIRCUIT

IV. SCALING PROCEDURES

In the analog computer, time t is the analog of distance down the beam axis z. These two quantities are connected by the scale factor α which is defined by

$$t = \alpha z \quad . \tag{10}$$

Voltage V is the analog of displacement of the particle from the beam axis. For trajectories in the XZ-plane, the displacement x is connected to the voltage V by the scale factor β where

$$V = \beta x \quad . \tag{11}$$

For trajectories in the YZ-plane, the displacement y is related to the voltage by

$$V = \beta y \quad . \tag{12}$$

If we substitute for t and V in equations (8) and (9), we obtain

$$\frac{d^2y}{dt^2} \pm \frac{\alpha^2 f_1 y}{R_1 C_1 R_2 C_2} = 0 \quad , \tag{13}$$

$$\frac{d^2x}{dt^2} \pm \frac{\alpha^2 f_1 x}{R_1 C_1 R_2 C_2} + \frac{\alpha^2}{\beta} \frac{A_2 f_4 \, 100}{R_1 C_1 R_2 C_2} = 0 \quad . \tag{14}$$

Comparing (13) and (14) to (6) and (7) we obtain the conditions which must be satisfied if the analog is to solve the desired equations (6) and (7)

$$g_y = \pm \frac{\alpha^2 f_1}{R_1C_1R_2C_2} \tag{15}$$

$$g_x = \pm \frac{\alpha^2 f_1}{R_1C_1R_2C_2} \tag{16}$$

$$d = \frac{\alpha^2}{\beta} \frac{A_2 f_4 \; 100}{R_1C_1R_2C_2} \tag{17}$$

Table I gives the values of g_x , g_y and d for various magnetic elements. If we put these values in equations (15), (16) and (17) and note that potentiometer settings like f_1 will be used for $\frac{G}{B\rho}$, $\frac{1}{Lf}$ or $\frac{\tan \emptyset}{Lr_o}$ terms, settings like f_2 will be used for $\frac{n}{{r_o}^2}$ terms, settings like f_3 will be used for $\frac{1 - n}{{r_o}^2}$ terms and settings like f_4 will be used for $\frac{1}{r_o} \frac{\Delta p}{p}$ terms, we find that the equations which must be satisfied are:

$$\frac{G}{B\rho} = \frac{\alpha^2 f_1}{R_1C_1R_2C_2} \quad \text{(if } f_1 \text{ represents a quadrupole)} \tag{18}$$

$$\frac{1}{Lf} = \frac{\alpha^2 f_1}{R_1C_1R_2C_2} \quad \text{(if } f_1 \text{ represents a thin lens)} \tag{19}$$

$$\frac{\tan \emptyset}{Lr_o} = \frac{\alpha^2 f_1}{R_1C_1R_2C_2} \quad \text{(if } f_1 \text{ represents a focussing edge)} \tag{20}$$

$$\frac{n}{{r_o}^2} = \frac{\alpha^2 A_1 f_2}{R_1C_1R_2C_2} \tag{21}$$

$$\frac{1 - n}{r_o^2} = \frac{\alpha^2 A_1 f_3}{R_1 C_1 R_2 C_2} \tag{22}$$

$$\frac{1}{r_o} \frac{\Delta p}{p} = \frac{\alpha^2}{\beta} \frac{A_2 f_4 \, 100}{R_1 C_1 R_2 C_2} \tag{23}$$

A_1 is the gain with which the f_2 and f_3 terms are introduced into Summer 3. The ranges encountered in the quantities which appear in equations (18) to (23) are:

0.1×10^{-3}	$\leqslant \frac{1}{B\rho} \leqslant 1 \times 10^{-3}$	(for 0.1 to 1 GeV protons)	$(\text{kG.cm.})^{-1}$
0.1	$\leqslant G \leqslant 1$		(kG./cm.)
0.01	$\leqslant \frac{1}{Lf} \leqslant 0.1$		$(\text{cm.})^{-2}$
0.1	$\leqslant \tan \emptyset \leqslant 10$		
0.1×10^3	$\leqslant r_o \leqslant 1 \times 10^3$		(cm.)
0	$\leqslant n \leqslant 1$		
1×10^{-3}	$\leqslant \frac{\Delta p}{p} \leqslant 1 \times 10^{-2}$		

Relay 5 can be operated by the same timing signal which controls any potentiometer so that values of G, $\tan \emptyset$, r_o, and $\Delta p/p$ of either sign can be generated.

In order to be able to operate the analog at two speeds (fast for oscilloscope display and slow for plotter display), we choose two values for α ,

$$\alpha_s = 10^{-2} \text{ sec./cm.} \quad \text{and,}$$

$$\alpha_f = 10^{-3} \text{ sec./cm.}$$

A convenient choice for β is

$$\beta = 10 \text{ volts/cm.}$$

f_1 to f_4 in equations (18) to (23) are potentiometer settings which can have any value between zero and one. For ease of setting the potentiometers, it is best to scale so that the values will be between 0.1 and 1.

If we set f_1 to $\frac{G}{B\rho} \times 10^3$ then from (18) we get

$$\text{for } \alpha_s\text{:} \quad R_1C_1R_2C_2 = 10^{-1} \text{ sec.}^2$$

$$\text{for } \alpha_f\text{:} \quad R_1C_1R_2C_2 = 10^{-3} \text{ sec.}^2$$

Convenient choices for the resistor and capacitor values are

for α_s: $R_1 = 0.1\ M\Omega$ $\quad R_2 = 1\ M\Omega$

$C_1 = 1\ \mu f$ $\quad C_2 = 1\ \mu f$

for α_f: $R_1 = 0.1\ M\Omega$ $\quad R_2 = 1\ M\Omega$

$C_1 = 0.1\ \mu f$ $\quad C_2 = 0.1\ \mu f$.

The computing resistors and capacitors in this computer are accurate to $\pm$ .01% . Thus when a resistor of $1\ M\Omega$ is stated, the actual value is $1.0000 \pm .0001\ M\Omega$. The values of $\frac{1}{Lf}$ and $\frac{\tan \phi}{Lr_o}$ are of the same order of magnitude as $\frac{G}{B\rho}$ so the above scale factors will also satisfy equations (19) and (20) (with the correct value of f_1).

Values for the amplifier gains A_1 and A_2 can now be deduced.

If we now consider equation (21) and set f_2 to $\frac{n}{{r_o}^2} \times 10^4$ and using the values of α , β , and $R_1C_1R_2C_2$ which have been derived above we get

$$A_1 = 0.1$$

Equation (22) will also give this result.

Considering equation (23) and setting f_4 to $\frac{1}{r_o}\frac{\Delta p}{p_o} \times 10^4$ and using the values of α , β , and $R_1C_1R_2C_2$ already derived we get

$$A_2 = 0.1$$

The scale factors derived here should cause the voltages for most common problems to be within the usable range of the computer (0.1 volt $\leqslant$ V $\leqslant$ 100 volts).

V. USES OF BEAM TRANSPORT ANALOG COMPUTER

Some of the problems to which the analog can be applied are

1) calculation of particle trajectories for magnet systems which are arbitrary
   a) at momentum p ,
   b) at momentum $p + \Delta p$ giving dispersion through bending magnets (the trajectory is calculated with respect to the beam axis for momentum p),
   c) at momentum p' (different from p). This gives a trajectory with respect to a new beam axis (for momentum p'). This shows the effects of changing the energy in both the quadrupoles and the bending magnets.
2) adjustment of a magnet system to give a desired beam, e.g. focus at a given point or parallel beam at a given point.
3) calculation of the transfer matrix of a system*.

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* Penner[11] has shown that the displacement and slope of a particle trajectory (x and x') at the end of a magnet system are related to the displacement and slope at the beginning of the system (x_o and x_o') by

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_o \\ x_o' \end{pmatrix}$$

where $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the transfer matrix of the magnet system.

If we use $x_o = 1.0$, $x_o' = 0.0$ then at any point in the system

$$x = a, \quad x' = c$$

and if we use $x_o = 0.0$, $x_o' = 1.0$ then

$$x = b, \quad x' = d .$$

So by stopping the calculation say at a point z, the transfer matrix up to this point can be found.

4) calculation of particle trajectories through periodic systems such as a muon channel (this can be accomplished by recycling the clock).

Sample problems which demonstrate some of these uses are given in Section VI.

The analog can also be used to solve any other problem which is described by differential equations of the same form as equations (6) and (7).

VI. SAMPLE PROBLEMS

Two sample problems are detailed here which illustrate the features of the analog.

The first is a 30° dispersionless bending system described by A. C. Paul[13]. It consists of five quadrupoles and two bending magnets. The bending magnet end faces are normal to the beam axis.

The second example is a 20° dispersionless bending system described by F. P. Blackstein and A. Otter*. It consists of three bending magnets. Two of the magnets have end faces at large angles to the beam axis. This demonstrates the "edge focussing".

For the first example, a comparison of the trajectories calculated using the analog is given to trajectories calculated using a digital tracking program**.

I. 30° Dispersionless Bending System

The details of this system are given in Table II. The system is designed for 0.550 GeV protons (momentum 1.155 GeV/c and $B\rho$ 3.85×10^{3} kG.cm.). The details of this system as used on the analog computer are given in Table III. The

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* Blackstein, F. P. and A. J. Otter, Private Communication, January 1967.

** Tautz, M. F., TRIUMF Beam Tracking Program (1967).

magnet lengths have been rounded-off and the g_x and g_y values have been adjusted to compensate for this.

The $\frac{1}{r_o} \frac{\Delta p}{p}$ term is calculated for $\frac{\Delta p}{p} = 1\%$. The values of the g_x and $\frac{1}{r_o} \frac{\Delta p}{p}$ terms for bending magnets actually set on the computer are ten times larger than the values listed in Table III because these two terms are introduced into the circuit with a gain of 0.1 ($A_1 = A_2 = 0.1$).

Tables IV and V give a comparison of the trajectories calculated with the analog computer to those calculated with a digital computer. The values from the digital computer are labelled "correct". It can be seen that the largest error in displacement is 0.067 cm. This system has a maximum displacement of over 8 cm. The values given for displacements and slopes from the analog were obtained by stopping the problem at the desired point (say z = 100 cm.) using the "problem interrupt", then measuring the problem voltages with a digital voltmeter.

The trajectories used for the comparisons in Tables IV and V are shown in Figs. 4 and 5 respectively.

Fig. 6 shows trajectories in the horizontal plane for a range of initial slopes. It can be seen that this magnet system gives a focus in this plane. Fig. 7 shows trajectories in the vertical plane for a range of initial slopes.

Fig. 8 shows trajectories in the horizontal plane for a range of initial displacements. Fig. 9 shows trajectories in the vertical plane for a range of initial displacements.

Fig. 10 shows the effect of small variations in momentum. The values of $\Delta p/p$ used are larger than would be found in a normal beam transport system. This was done to make the $\Delta p/p$ effects more obvious. It can be seen that the system is dispersionless since the final displacements and slopes are the same for particles with $\Delta p/p = 0$, 1%, 2% and 5%.

The transfer matrices for the system were found to be

$$\begin{pmatrix} -1.709 & -1.508 \\ -.7333 & -1.236 \end{pmatrix}$$

for the XZ-plane and

$$\begin{pmatrix} -.9563 & 9.732 \\ -.0025 & -1.0072 \end{pmatrix} .$$

for the YZ-plane. The correct matrices are

$$\begin{pmatrix} -1.6742 & -1.4642 \\ -.7255 & -1.2318 \end{pmatrix} , \text{ and}$$

$$\begin{pmatrix} -.99603 & 9.6105 \\ -.00088 & -.99549 \end{pmatrix}$$

for the XZ- and YZ-planes respectively.

TABLE III

30° Bending System, Values used on Analog Computer

element	entrance position (cm.)	exit position (cm.)	$g_x \times 10^3$ $cm.^{-2}$	$g_y \times 10^3$ $cm.^{-2}$	$\frac{1}{r_o}\frac{\Delta p}{p} \times 10^3$ $cm.^{-1}$
bending magnet	0	64	0.01660	0.0	0.004075
quadrupole	94	137	-0.1429	+0.1429	---
quadrupole	168	211	0.1043	-0.1043	---
quadrupole	343	384	0.1086	-0.1086	---
quadrupole	516	559	0.1043	-0.1043	---
quadrupole	590	633	-0.1429	0.1429	---
bending magnet	663	727	0.01660	0.0	0.004075

TABLE IV

Comparison of Analog and Exact Values for 30° Dispersionless Bending System in the Horizontal Plane

z cm.	x_{analog} cm.	$x_{correct}$ cm.	error cm.	x'_{analog}	$x'_{correct}$	error
0	0.0006	0.0000	0.0006	0.7502	0.7500	0.0002
100	0.7537	0.7372	-0.0165	0.8124	0.7835	-0.0289
200	1.870	1.822	-0.048	0.7299	0.7096	-0.0203
300	2.374	2.330	-0.044	0.4909	0.4961	+0.0052
400	2.441	2.407	-0.034	-0.6691	-0.6476	+0.0215
500	1.773	1.760	-0.013	-0.6690	-0.6476	+0.0214
600	0.6919	0.7004	+0.0085	-1.221	-1.191	+0.030
700	-0.3196	-0.2832	-0.0364	-0.9592	-0.9422	+0.0170
780	-1.067	-1.025	-0.042	-0.9283	-0.9239	+0.0056

TABLE V

Comparison of Analog and Exact Values for 30° Dispersionless Bending System in the Vertical Plane

z cm.	y_{analog} cm.	$y_{correct}$ cm.	error cm.	y'_{analog}	$y'_{correct}$	error
0	0.0006	0.0000	-1.0006	0.7502	0.7500	-0.0002
100	0.7471	0.7483	+0.0012	0.6876	0.6902	0.0026
200	1.126	1.128	+0.0002	0.5901	0.5883	-0.0018
300	1.852	1.843	-0.0009	0.7355	0.7221	-0.0134
400	2.966	2.943	-0.0023	1.799	1.777	-0.022
500	4.772	4.721	-0.0051	1.798	1.777	-0.021
600	8.030	7.975	-0.055	3.184	3.108	-0.076
700	7.941	7.864	-0.077	-0.7568	-0.7466	+0.0102
780	7.334	7.267	-0.067	-0.7570	-0.7466	+0.0104

30° DISPERSIONLESS BENDING SYSTEM
HORIZONTAL PLANE

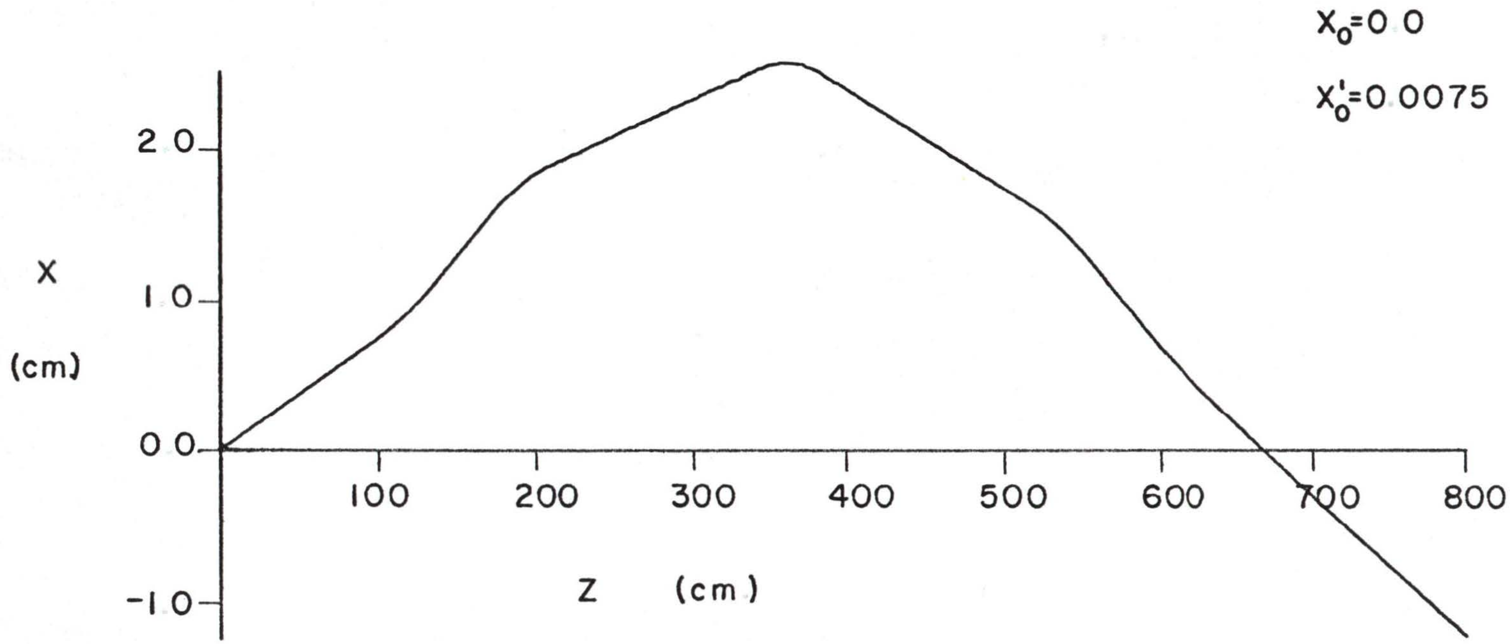


FIGURE 4

30° DISPERSIONLESS BENDING SYSTEM
VERTICAL PLANE

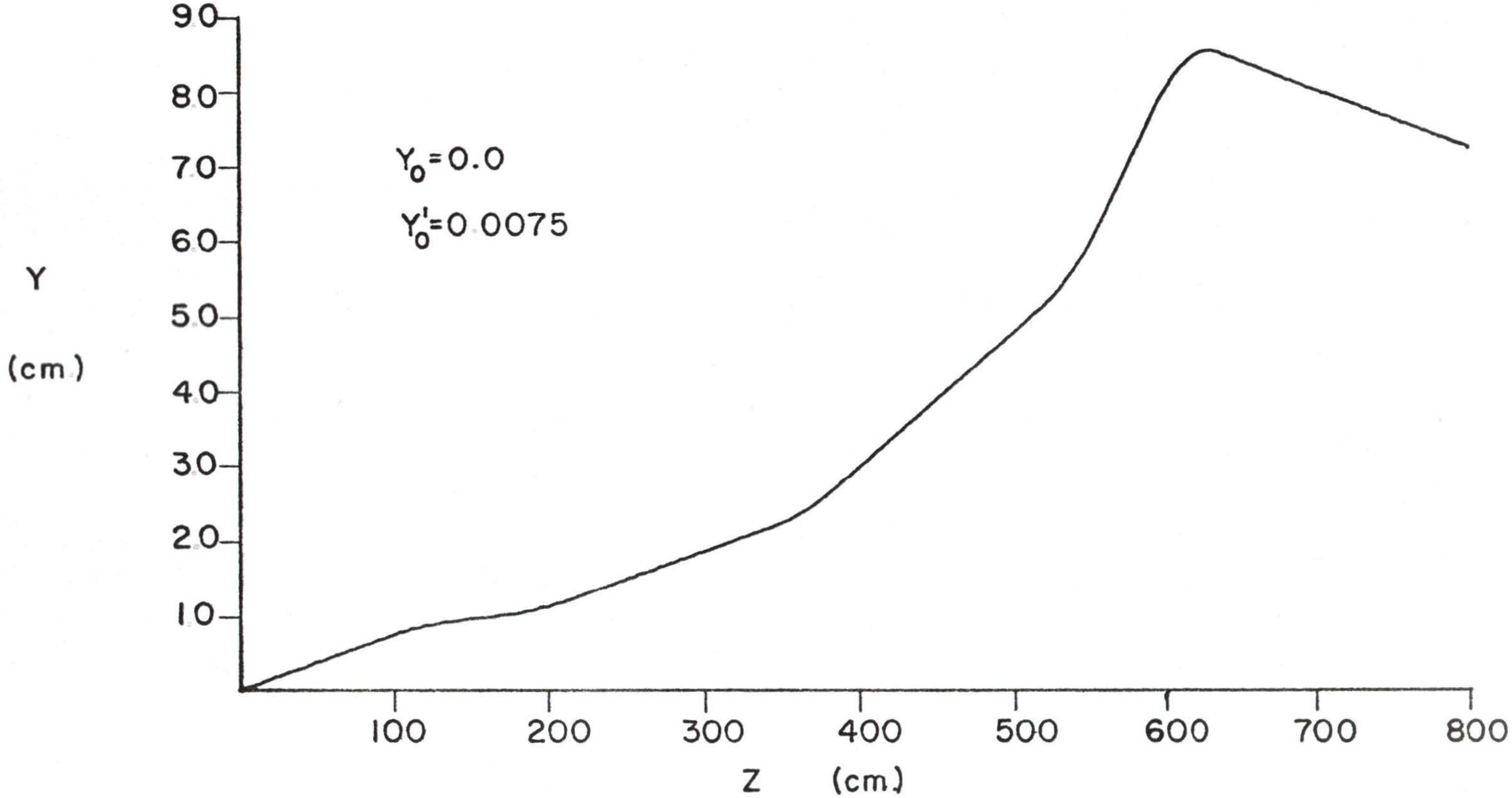


FIGURE 5

30° DISPERSIONLESS BENDING SYSTEM
HORIZONTAL PLANE

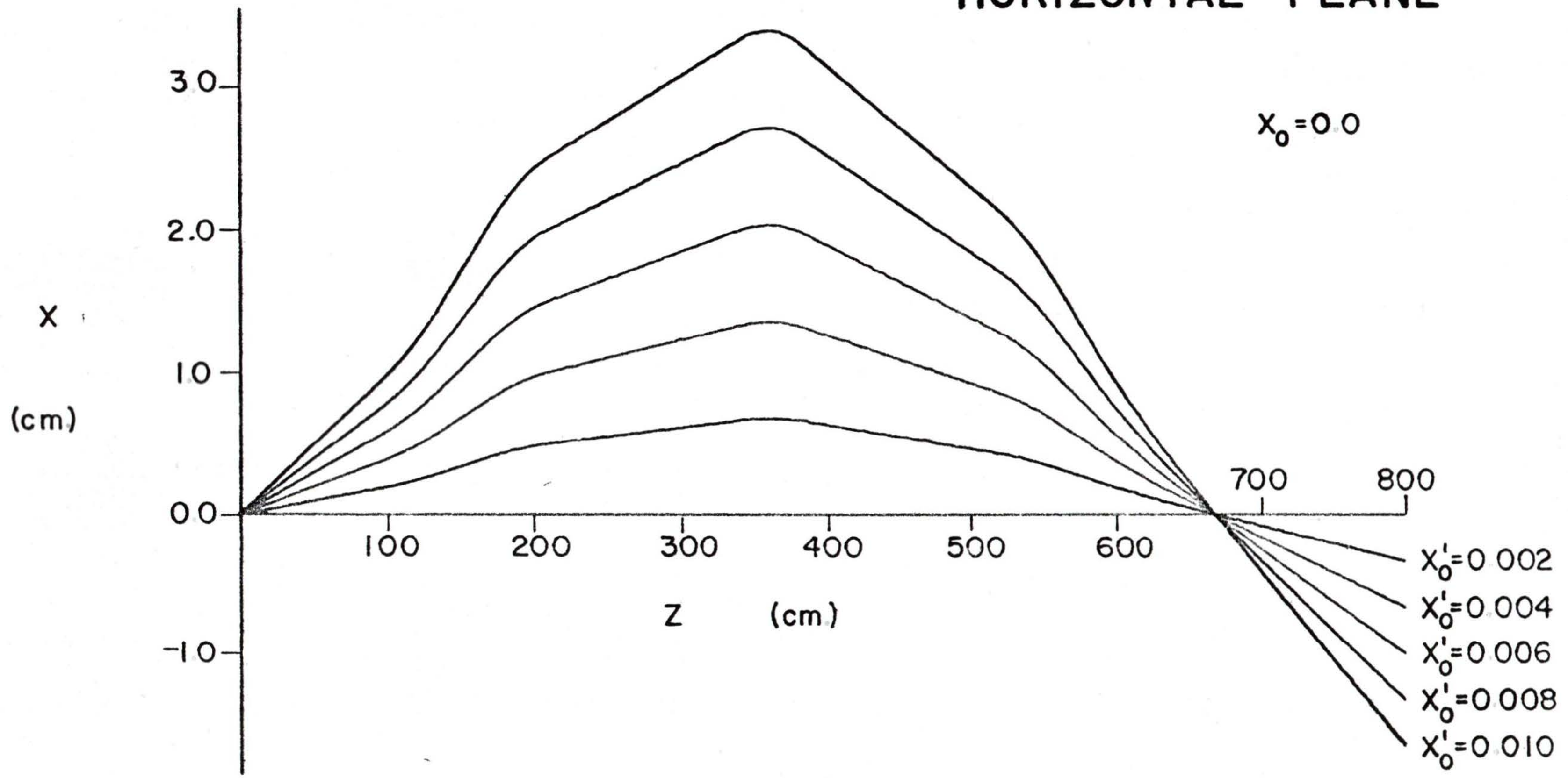
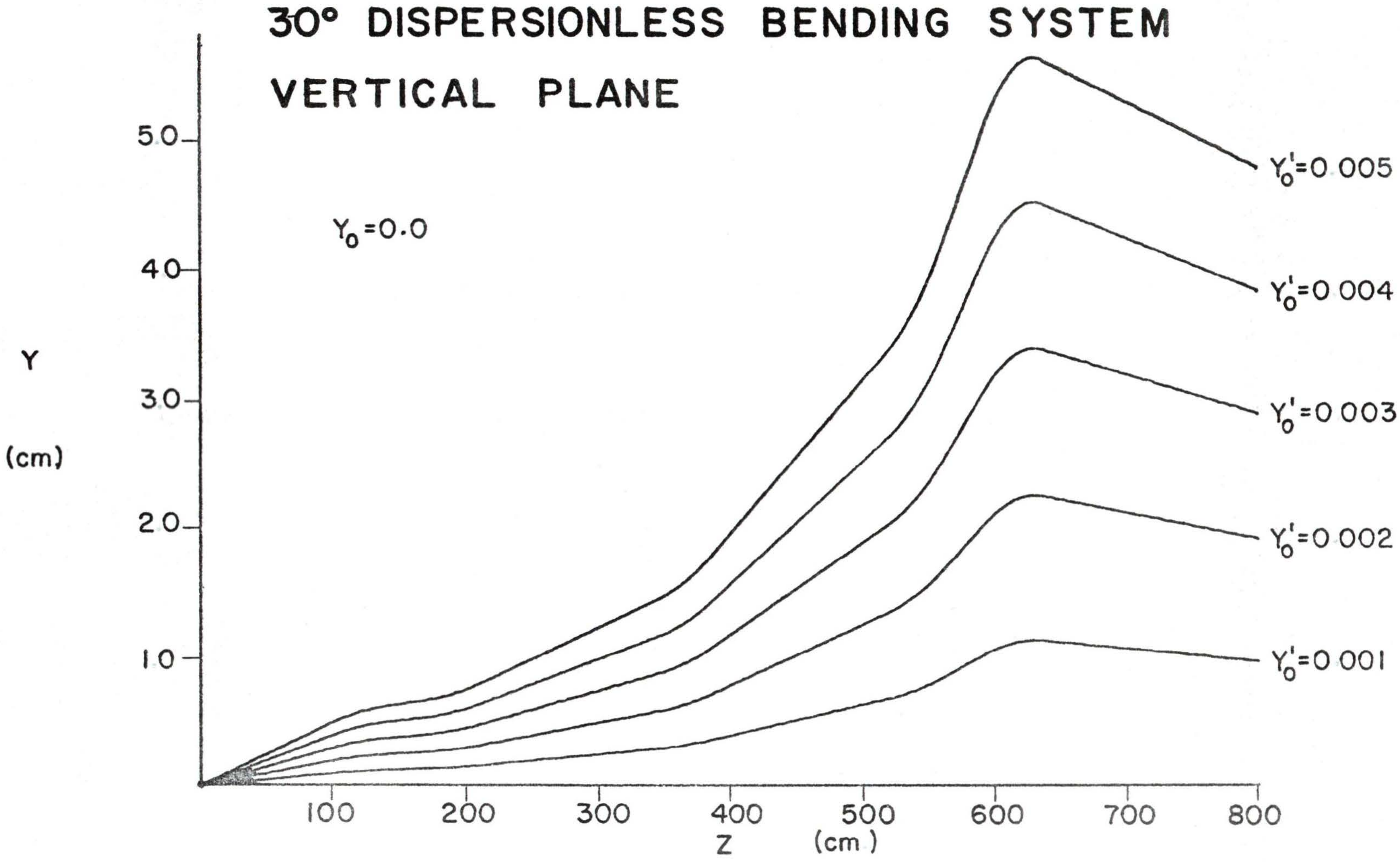


FIGURE 6



30° DISPERSIONLESS BENDING SYSTEM
VERTICAL PLANE
$Y_0 = 0.0$
Y
(cm)
5.0
4.0
3.0
2.0
1.0
$Y'_0 = 0.005$
$Y'_0 = 0.004$
$Y'_0 = 0.003$
$Y'_0 = 0.002$
$Y'_0 = 0.001$
100
200
300
400
500
600
700
800
Z (cm)

FIGURE 7

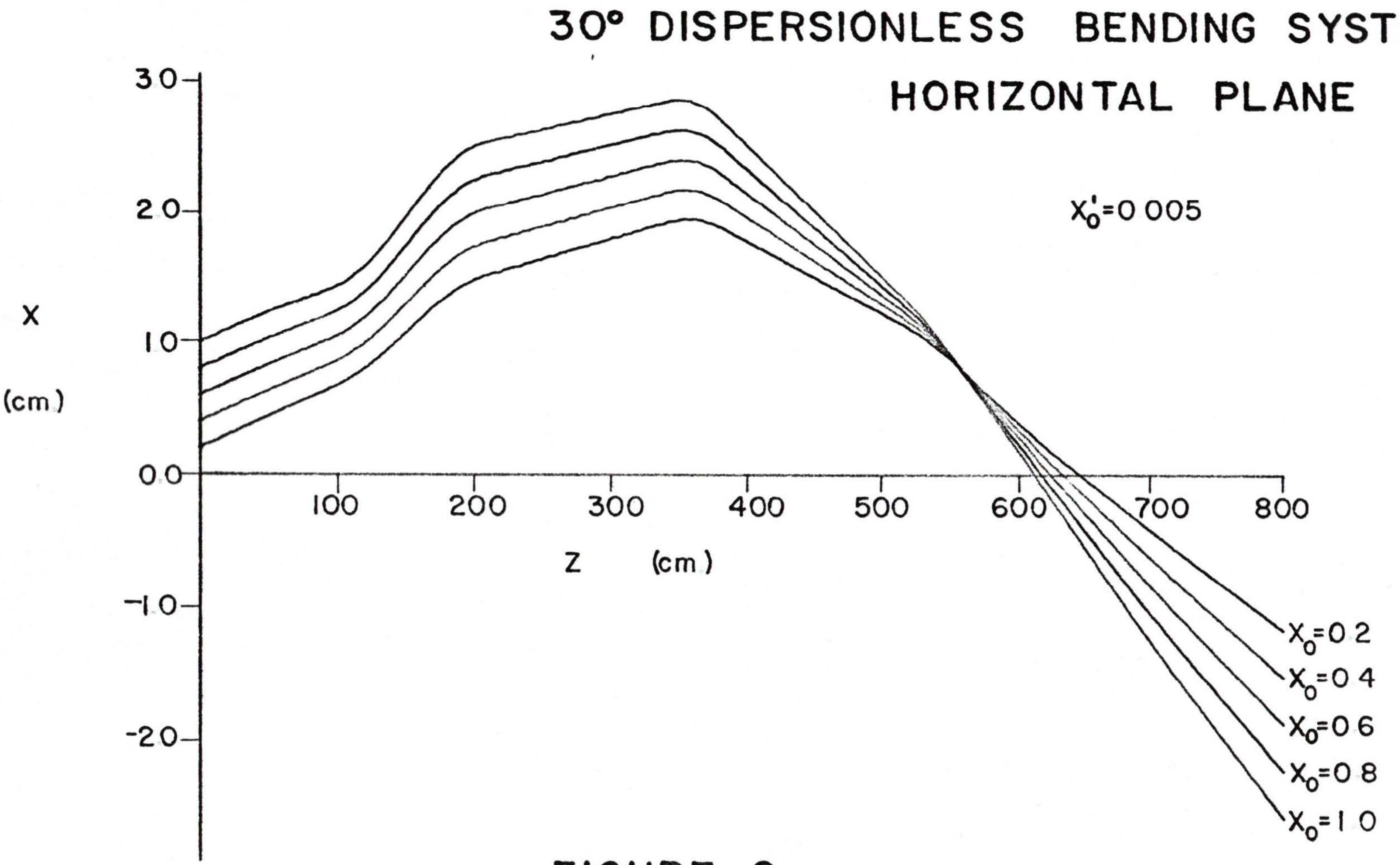

30° DISPERSIONLESS BENDING SYSTEM
HORIZONTAL PLANE
$X_0' = 0.005$
X
(cm)
3.0
2.0
1.0
0.0
-1.0
-2.0
100
200
300
400
500
600
700
800
Z (cm)
$X_0 = 0.2$
$X_0 = 0.4$
$X_0 = 0.6$
$X_0 = 0.8$
$X_0 = 1.0$

FIGURE 8

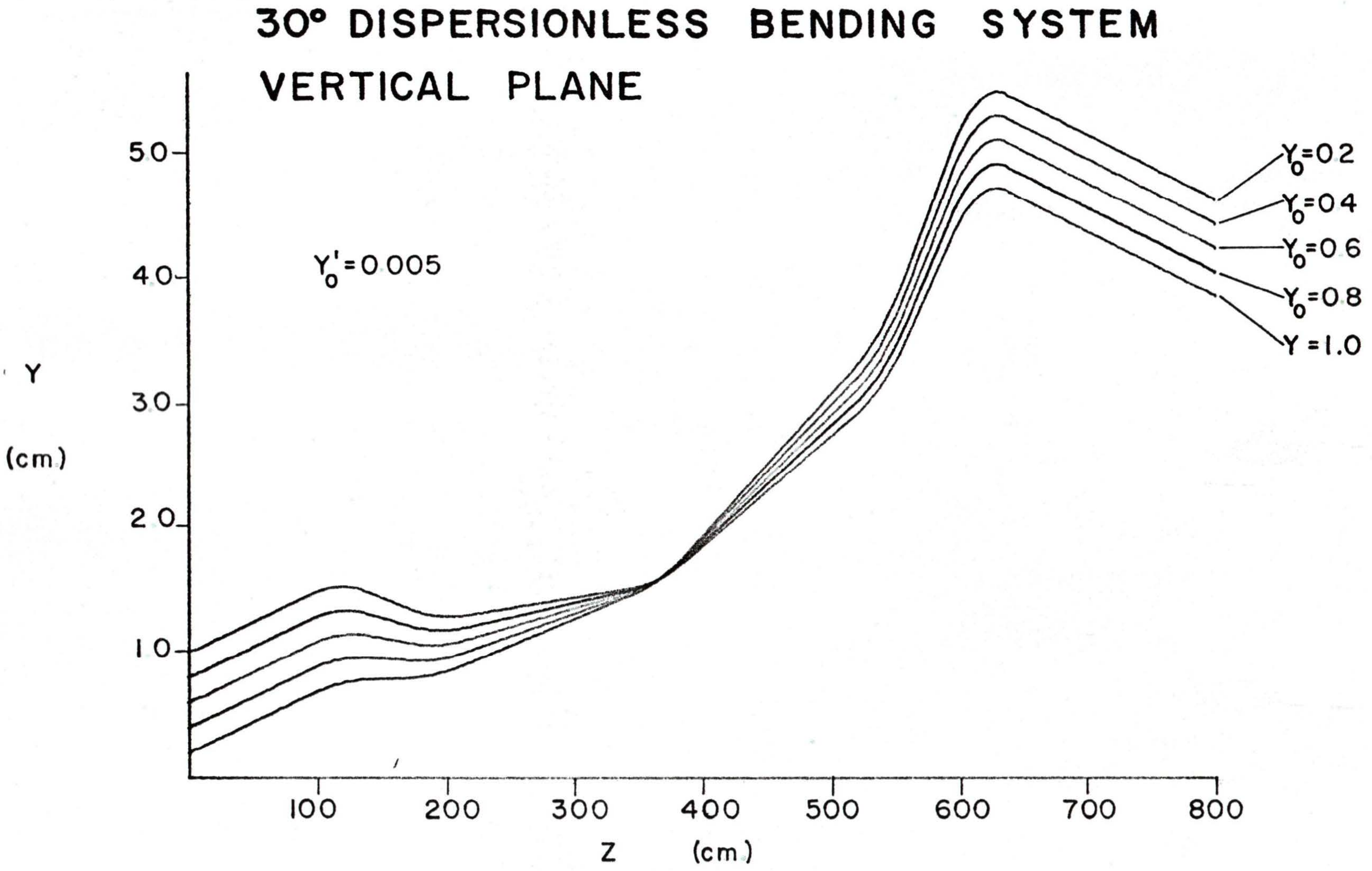

30° DISPERSIONLESS BENDING SYSTEM
VERTICAL PLANE
Y'o=0.005
Y
(cm)
5.0
4.0
3.0
2.0
1.0
100
200
300
400
500
600
700
800
Z (cm)
Yo=0.2
Yo=0.4
Yo=0.6
Yo=0.8
Y =1.0

FIGURE 9

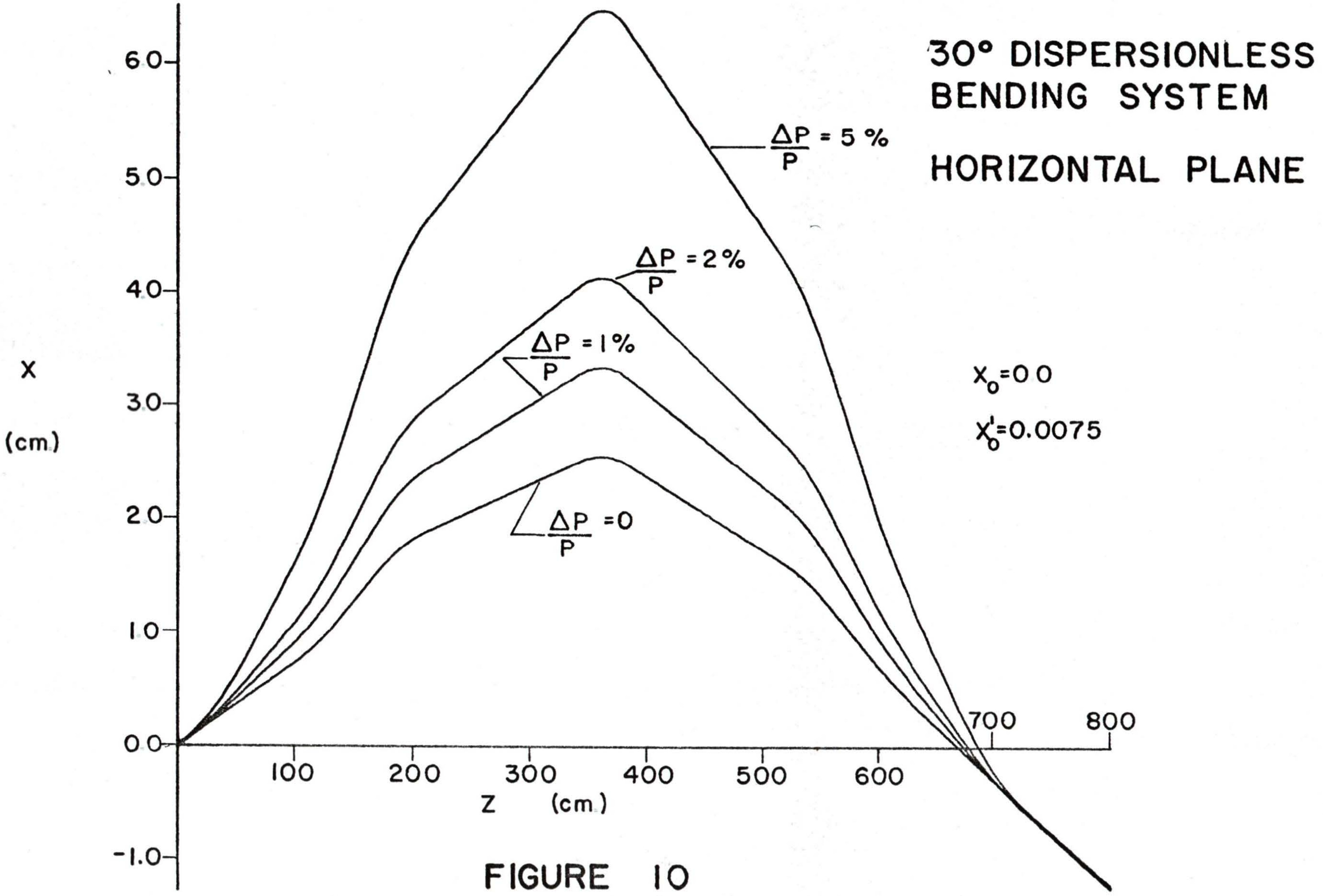

30° DISPERSIONLESS
BENDING SYSTEM
HORIZONTAL PLANE
ΔP/P = 5 %
ΔP/P = 2%
ΔP/P = 1%
ΔP/P = 0
X₀=0.0
X'₀=0.0075
X
(cm)
Z (cm)
6.0
5.0
4.0
3.0
2.0
1.0
0.0
-1.0
100
200
300
400
500
600
700
800

FIGURE 10

II. <u>20^o Dispersionless Bending System</u>

The details of this system are given in Table VI. The system is designed for 1.0 GeV protons (momentum = 1.696 GeV/c and $B\rho = 5.64 \times 10^3$ kG.cm.). The details of this system as used on the analog computer are given in Table VII. Again the magnet lengths have been rounded-off and the g_x and g_y values adjusted. The lengths of the focussing edges are arbitrary. 7 cm. has been chosen as a convenient length. The $\frac{1}{r_o}\frac{\Delta p}{p}$ term is calculated for $\frac{\Delta p}{p} = 1\%$. The values of the g_x and $\frac{1}{r_o}\frac{\Delta p}{p}$ terms for bending magnets set on the computer are ten times larger than those listed in Table VII because these two terms are introduced into the circuit with a gain of 0.1 ($A_1 = A_2 = 0.1$).

Fig. 11 shows a trajectory in the horizontal plane. The effect of the focussing edges can clearly be seen (the sharp bends in the trajectory).

Fig. 12 shows a trajectory in the vertical plane. Since all the bending magnets act as drift spaces in the vertical plane, the only magnetic elements which affect the beam are the focussing edges.

Fig. 13 shows trajectories in the horizontal plane for a number of values of $\frac{\Delta p}{p}$. The $\frac{\Delta p}{p}$ values used are much larger than would be encountered in a normal accelerator beam. This was done to make the effects of the $\frac{\Delta p}{p}$ terms more obvious. It can be seen that the system is dispersionless.

TABLE VI

20° Bending System, Description of Elements

element	length (L) cm.	field gradient (G) kG./cm.	field (H) kG.
1) drift space	30.00	---	---
2) bending magnet constant field (n = 0) entrance angle = -62.97° exit angle = 0.0°	139.88	---	20.0
3) drift space	197.00	---	---
4) bending magnet constant field (n = 0) entrance angle = 0.0° exit angle = 0.0°	179.86	---	20.0
5) drift space	197.00	---	---
6) bending magnet constant field (n = 0) entrance angle = 0.0° exit angle = -62.97°	139.88	---	20.0
7) drift space	30.00	---	---

TABLE VII

20° Bending System, Values used on Analog Computer

element	entrance position (cm.)	exit position (cm.)	g_x	g_y	$\frac{1}{r_o} \frac{\Delta p}{p}$
focussing edge	23.	30.	- 0.9794	+ 0.9794	---
bending magnet	30.	170.	0.1220	0	0.03493
bending magnet	367.	547.	0.1220	0	0.03493
bending magnet	744.	884.	0.1220	0	0.03493
focussing edge	884.	891.	- 0.9794	+ 0.9794	---

20° DISPERSIONLESS BENDING SYSTEM
HORIZONTAL PLANE

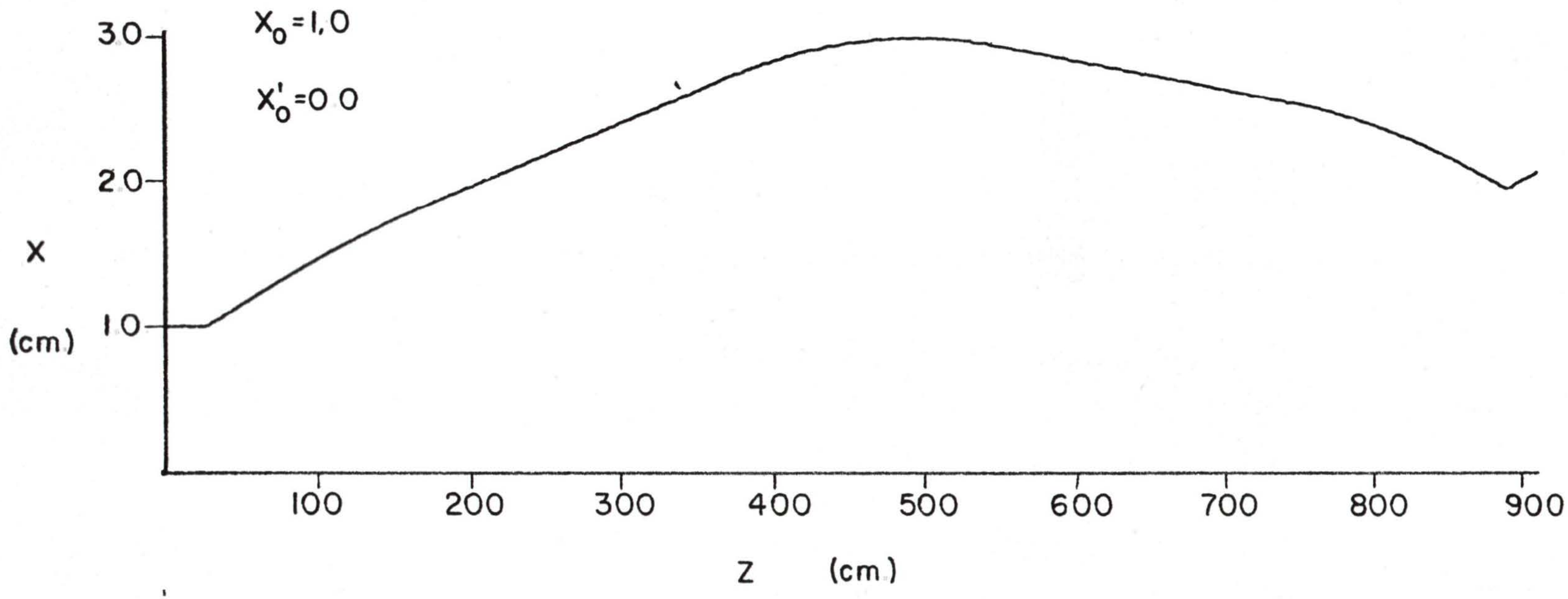


FIGURE 11

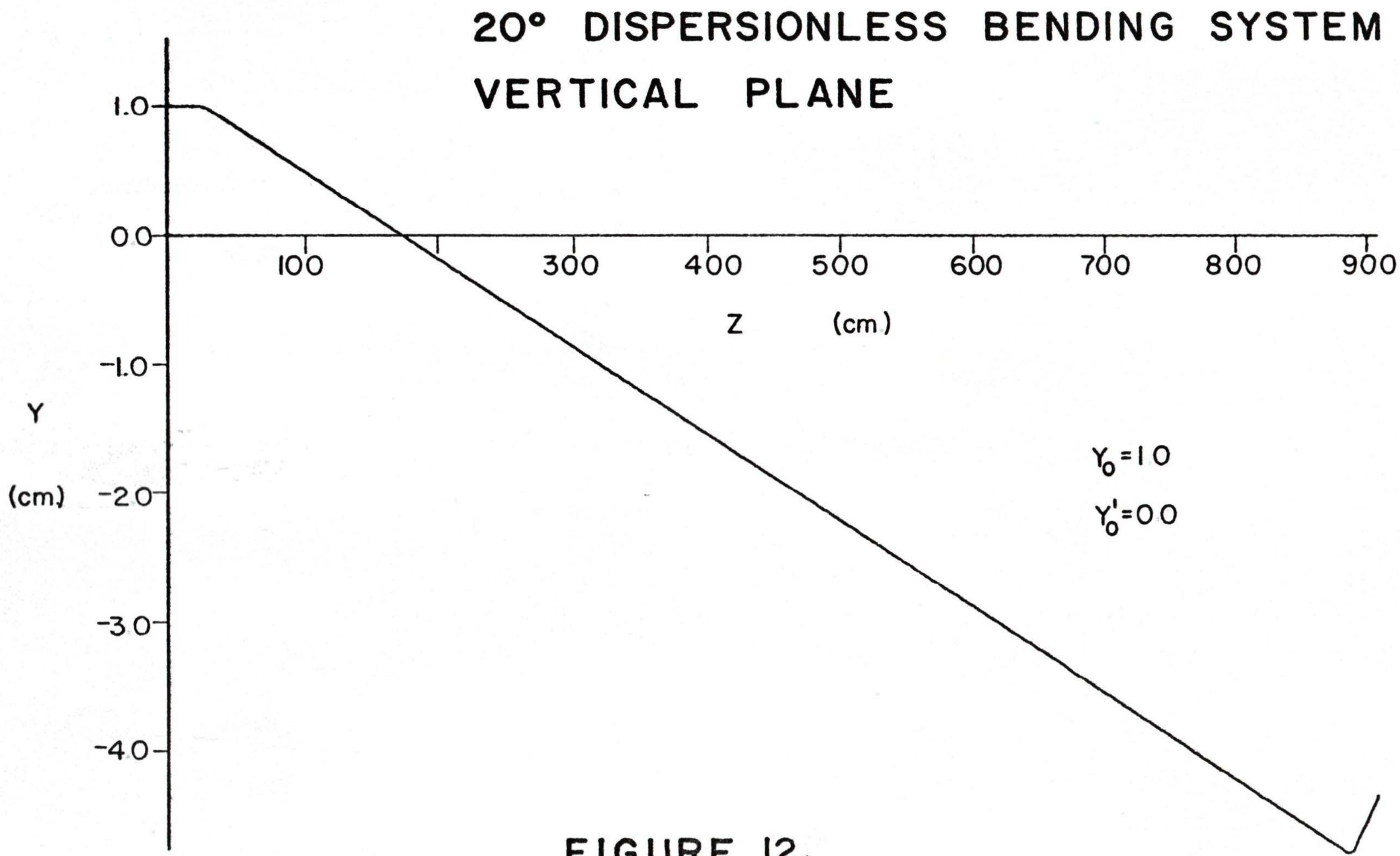

20° DISPERSIONLESS BENDING SYSTEM
VERTICAL PLANE
1.0
0.0
-1.0
-2.0
-3.0
-4.0
Y
(cm)
100
300
400
500
600
700
800
900
Z (cm)
Y0=1.0
Y0'=0.0

FIGURE 12.

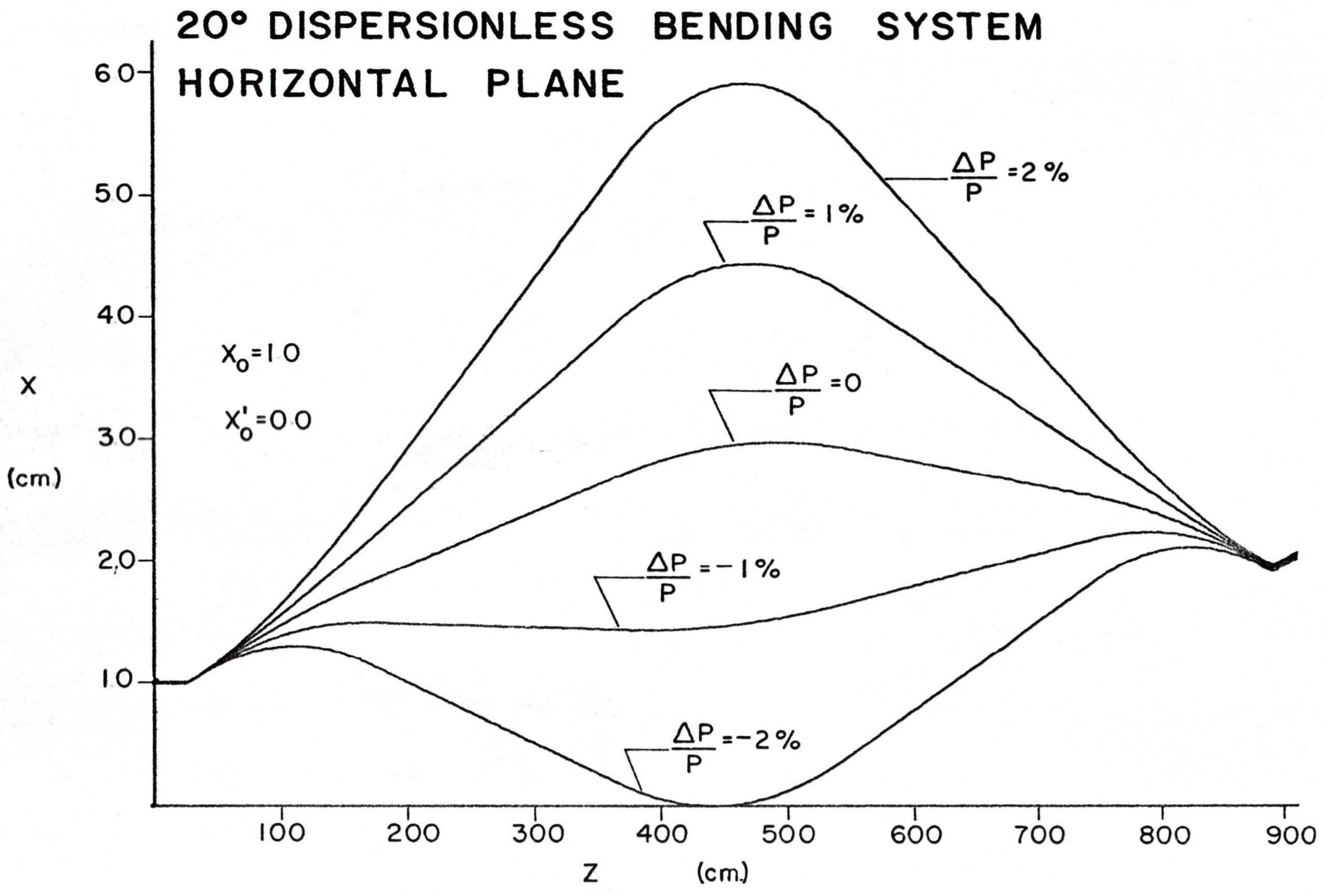

20° DISPERSIONLESS BENDING SYSTEM
HORIZONTAL PLANE
$X_0 = 1.0$
$X_0' = 0.0$
X
(cm)
$\frac{\Delta P}{P} = 2\%$
$\frac{\Delta P}{P} = 1\%$
$\frac{\Delta P}{P} = 0$
$\frac{\Delta P}{P} = -1\%$
$\frac{\Delta P}{P} = -2\%$
6.0
5.0
4.0
3.0
2.0
1.0
100
200
300
400
500
600
700
800
900
Z (cm)

FIGURE 13

VII. SUMMARY OF RESULTS

Sample problems showing the output of the analog are given in Section V.

In order to check the accuracy of the analog, Problem I in Section V was also run on a digital tracking program*. The particle trajectory displacements were found to be in error by less than 0.067 cm. in a system where the maximum particle displacement is greater than 8 cm.

The ability to stop the calculation at any point has been found to be very useful. It allows accurate measurement of the displacement and slope at any point in the transport system and it allows the transfer matrix of any part of the magnet system to be found.

The fast and slow modes of operation have also been found to be useful. Matching and optimization problems can be done quickly in the fast mode, then the slow mode can be used to make a permanent record of the results.

It is expected that the analog will become a part of the equipment in the control room of the TRIUMF accelerator. This would allow fast calculation and testing of beam lines, thus saving valuable accelerator time.

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* Tautz, M. F., TRIUMF Beam Tracking Program (1967).

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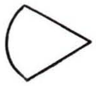
APPENDIX I. BASIC EQUATIONS FOR COMPUTING CIRCUITS

The equations derived here are the standard tools of analog computing and can be found in any book on the subject, for example[14, 15].

Figs. 14 and 15 show the circuits for summation and integration respectively. The symbol  denotes a high gain, direct coupled amplifier, commonly known as an operational amplifier.

Referring to Fig. 14, if the gain of the amplifier is $-k$ (the gain must always be negative or the circuit would be unstable because of the feedback) then the voltage e_o at the output must be related to the input voltage e by

$$e = \frac{-e_o}{k} . \tag{24}$$

The amplifier has a high input impedance so that the current which flows into the amplifier will be very small. If we take this current to be zero, then equate the currents flowing to point A to those leaving point A, we

$$\frac{e_1 - e}{R_1} + \frac{e_2 - e}{R_2} + \ldots + \frac{e_n - e}{R_n} + \frac{e_o - e}{R_f} = 0 .$$

Using (24), this can be written

$$e_o = \frac{-\left(e_1 \frac{Rf}{R_1} + e_2 \frac{Rf}{R_2} + \ldots + e_n \frac{Rf}{R_n}\right)}{1 - \frac{1}{k}\left(1 + \frac{Rf}{R_1} + \frac{Rf}{R_2} + \ldots + \frac{Rf}{R_n}\right)} . \tag{25}$$

k is usually very large (greater than 10^7 for the amplifiers used here) so the denominator is close to unity and (25) becomes

$$e_o = -(e_1 \frac{Rf}{R_1} + e_2 \frac{Rf}{R_2} + \ldots + e_n \frac{Rf}{R_n}) \quad . \qquad (26)$$

The derivation of the input-output relationship for the integrator is similar to that for the summer. Referring to Fig. 15 with both switches in the "compute" position, the voltage at point A at the input to the amplifier will be:

$$e = -\frac{e_o}{k} \quad . \qquad (24)$$

There will be essentially no current flowing into the amplifier so the current flowing through the input resistors must all flow through the capacitors. If the current flowing is i , then

$$e - e_o = +\frac{1}{C}\int i \, dt \quad .$$

The current which flows through the input resistors is

$$i = \frac{e_1 - e}{R_1} + \frac{e_2 - e}{R_2} + \ldots + \frac{e_n - e}{R_n}$$

hence, combining these two expressions for i :

$$e - e_o = -\frac{1}{C}\int (\frac{e_1 - e}{R_1} + \frac{e_2 - e}{R_2} + \ldots + \frac{e_n - e}{R_n}) \, dt \quad .$$

Using (24), we get

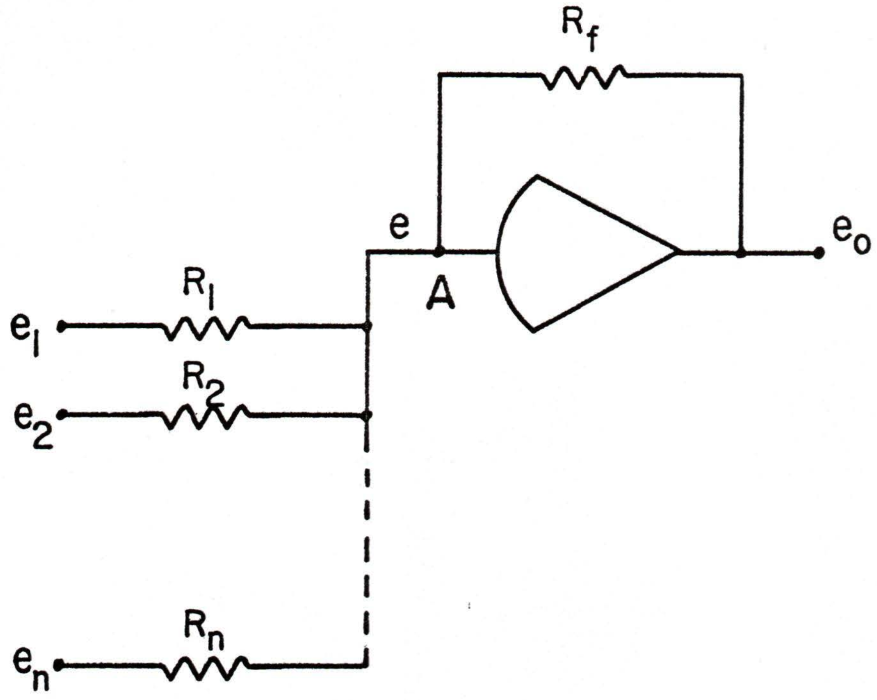


FIGURE 14 SUMMER CIRCUIT

$$e_o = - \frac{\int \left(\frac{e_1}{R_1C} + \frac{e_2}{R_2C} + \ . \ . \ . + \frac{e_n}{R_nC}\right) dt}{1 + \frac{1}{k} + \frac{1}{k} \left(\frac{1}{R_1} + \frac{1}{R_2} + \ . \ . \ . + \frac{1}{R_n}\right) dt} \ . \tag{27}$$

Again, for large k, we have

$$e_o = - \int \left(\frac{e_1}{R_1C} + \frac{e_2}{R_2C} + \ . \ . \ . + \frac{e_n}{R_nC}\right) dt \ . \tag{28}$$

If the upper switch in Fig. 15 is put in the "reset" position, the circuit is essentially a summer and the capacitor will be charged to the voltage applied to the I. C. terminal. This sets the initial conditions on the integrators.

If the input to the amplifier is removed by putting the bottom switch in Fig. 15 in the "hold" position, the amplifier output will remain at the value it had when the input was removed. Thus, by placing all integrators in the analog loop in the "hold" mode, the problem can be stopped. This is a convenient method for measuring problem voltages accurately using a digital voltmeter.

The analog computer used (Systron Donner Model 10/20) an internal logic system which actuates the RESET, COMPUTE and HOLD relays for all amplifiers simultaneously. This internal logic system can be controlled either manually from the console or from a REPETITIVE OPERATION function.

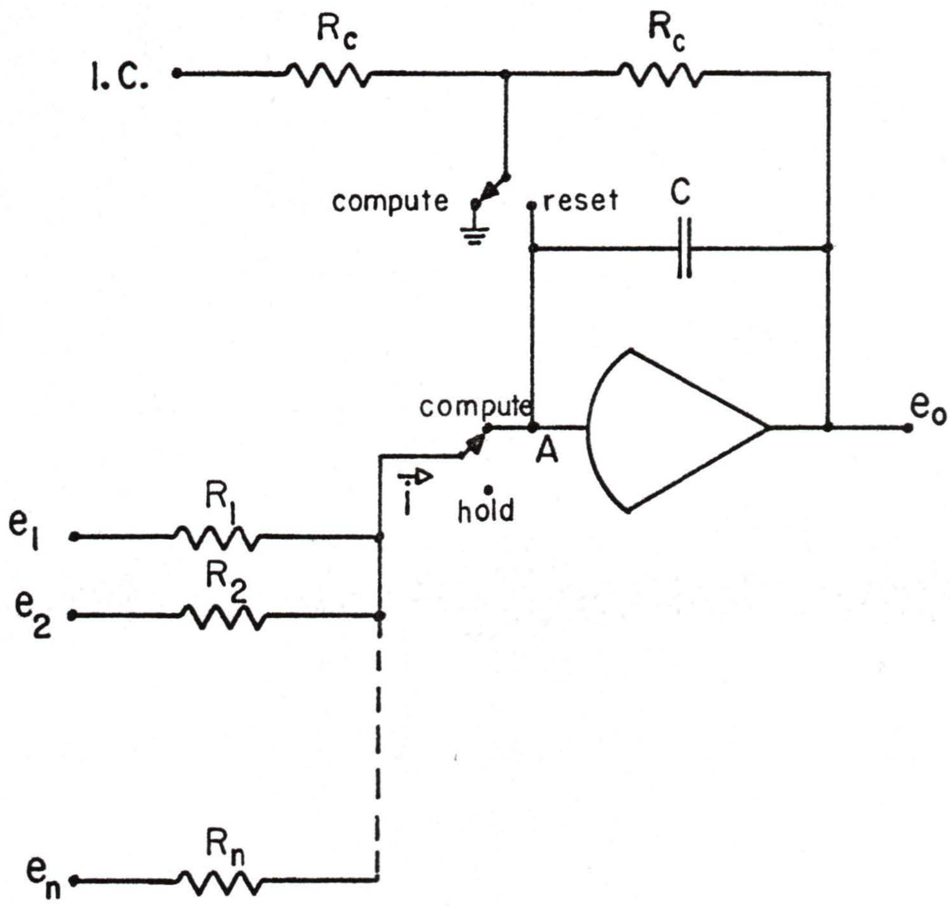


FIGURE 15 INTEGRATOR CIRCUIT

APPENDIX II. POSSIBILITIES FOR ENVELOPE TRACKING

It is often helpful to calculate beam envelopes rather than particle trajectories.

The beam envelope is a curve within which lie the trajectories of all the particles in the beam. E_x denotes the curve which encloses all the trajectories in the XZ-plane and E_y denotes the curve which encloses all the trajectories in the YZ-plane.

When discussing the beam as a whole (rather than individual particles), the beam can most conveniently be described by stating a phase space volume. Phase space for a beam of particles is a six dimensional space in which the coordinates are the displacements and momenta in three mutually perpendicular directions. Each particle in the beam can be represented by a point in this phase space and the beam is said to occupy a volume in phase space. This volume is a volume which encloses most (usually a large fraction, say 99%) of the phase space points. Liouville's Theorem, of which Steffen[16] gives a proof for the case of particles in a beam transport system, states that the phase space volume of the beam remains constant. Since in a beam transport system there is not coupling (at least to first order) between motions in the X-, Y- and Z-directions, this invariance extends to the individual phase areas. If the phase areas are assumed to be enclosed by upright ellipses, these phase space areas are given by

$$A_x = \pi x_{max}\, p_x = \pi x_{max}\, p \sin \theta_x \approx x_{max}\, p\, \theta_x$$

for the XZ-plane and

$$A_y = \pi y_{max}\, p_y = \pi y_{max}\, p \sin \theta_y \approx y_{max}\, p\, \theta_y$$

for the YZ-plane. A_x and A_y are the phase areas for the XZ- and YZ-planes respectively, x_{max} and y_{max} are the maximum displacements in the X- and Y-directions, p_x and p_y are the momenta in the X- and Y-directions and θ_x and θ_y are the maximum particle divergences in the X- and Y-directions. p is the momentum in the Z-direction.

The quantities usually used to describe a beam are the beam emittances which are closely related to A_x and A_y. The emittances are denoted by ε_x and ε_y for the XZ- and YZ-planes respectively and are defined by

$$\varepsilon_x = \frac{A_x}{\pi p} = x_{max}\, \theta_x$$

$$\varepsilon_y = \frac{A_y}{\pi p} = y_{max}\, \theta_y \quad .$$

For a given momentum, ε_x and ε_y are constants of the motion as A_x and A_y are.

A more complete description of the ideas of phase space applied to beam optics is given by Steffen[17].

A typical range of ε_x and ε_y is $0.1 \times 10^{-3} \leqslant \varepsilon \leqslant 1 \times 10^{-3}$ cm. rad[18,19].

There are two distinct methods of calculating beam envelopes with an analog computer. Only one method has been attempted but both will be described for completeness.

A. <u>Envelopes in Terms of Conjugated Trajectories</u>

Steffen[20] shows that, if two particle trajectories y_1 and y_2 in the YZ-plane are calculated, starting with the initial conditions

$$y_1\,(0) = E_y\,(0) \qquad \frac{dy_1}{dz}\,(0) = \frac{dE_y}{dz}\,(0)$$

$$y_2\,(0) = 0.0 \qquad \frac{dy_2}{dz}\,(0) = \frac{\varepsilon}{E_y}\,(0) \tag{29}$$

then the beam envelope in the YZ-plane is given by

$$E_y = \sqrt{{y_1}^2 + {y_2}^2}\ . \tag{30}$$

The zeroes in the brackets refer to values at $z = 0$. Similarly, for two trajectories x_1 and x_2 started with similar initial conditions in the XZ-plane, the envelope in the XZ-plane will be

$$E_x = \sqrt{{x_1}^2 + {x_2}^2}\ , \tag{31}$$

Thus, if we had two analog loops, one calculating y_1 and one calculating y_2, two multipliers and some device to take the square root would give us the envelope E_y. This approach was not used for the following reasons:

1) the computing equipment necessary for the second analog loop was not available.

2) the two computing loops would require two sets of potentiometers to set the quadrupole gradients. It would be necessary either to set two sets of potentiometers or to have two potentiometers mounted on one shaft. The first alternative would be time consuming when setting the problem up, the second would be inaccurate.

3) multiplication of two voltages varying in time and the taking of square roots are operations which involve considerable error.

For these reasons, it was decided to use the second approach given below.

B. <u>The Envelope Equation</u>

Steffen[21, 22] derives the differential equations for beam envelopes. These equations in the notation used here are

$$\frac{d^2E_y}{dz^2} + g_y\, E_y - \frac{\varepsilon_y{}^2}{E_y{}^3} = 0 \tag{32}$$

$$\frac{d^2E_z}{dz^2} + g_x\, E_x - \frac{\varepsilon_x{}^2}{E_x{}^3} = 0 \tag{33}$$

where E_x and E_y are the beam envelopes in the XZ- and YZ-planes respectively and ε_x and ε_y are the beam emittances in XZ- and YZ-planes respectively. Except for the ε^2/E^3 terms, equations (32) and (33)* are the same as equations (6) and (7) and can be solved by adding another loop to the analog circuit of Fig. 2. This loop produces the ε^2/E^3 term and adds it to the other terms at the first integrator. The new circuit is shown in Fig. 16. To calculate trajectories as before, the output of the upper loop is not included by leaving the switch in the "trajectories" position.

The function generator produces the function $V_{out} = s\ (V_{in})^{-3}$ where s is some scale factor. Arguments similar to those in Section IV show that the equations solved by the analog are, for the switch in the upper loop in the "envelopes" position:

$$\frac{d^2V}{dt^2} \pm \frac{V\ f_1}{R_1C_1R_2C_2} + \frac{f_5 sV^{-3}}{R_3C_1R_2C_2} = 0 \tag{34}$$

$$\frac{d^2V}{dt^2} \mp \frac{V\ f_1}{R_1C_1R_2C_2} + \frac{f_5 sV^{-3}}{R_3C_1R_2C_2} = 0 \tag{35}$$

We will use the scaling equations

$$t = \alpha z \tag{10}$$

$$V = \beta E_x \tag{36}$$

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* Equations (32) and (33) are valid only for $\frac{\Delta p}{p} = 0$.

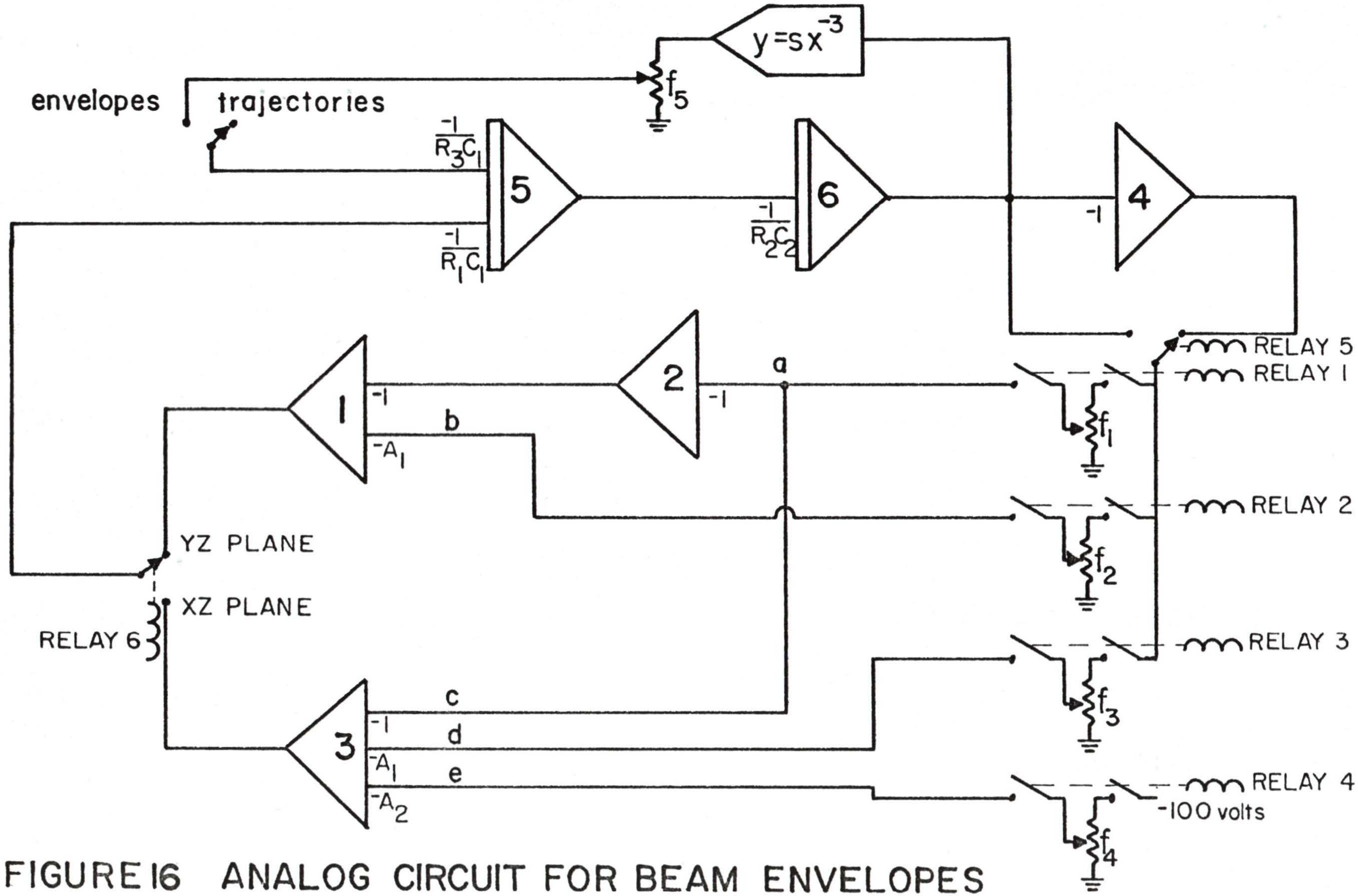


FIGURE 16 ANALOG CIRCUIT FOR BEAM ENVELOPES

for envelopes in the XZ-plane and

$$t = \alpha z \tag{10}$$

$$V = \beta E_y \tag{37}$$

for envelopes in the YZ-plane.

Substituting for t and V in equations (34) and (35) we obtain

$$\frac{d^2E_y}{dt^2} + \frac{\alpha^2 f_1 E_y}{R_1C_1R_2C_2} + \frac{\alpha^2}{\beta^4}\frac{f_5 s E_y^{-3}}{R_3C_1R_2C_2} = 0 \tag{38}$$

$$\frac{d^2E_x}{dt^2} + \frac{\alpha^2 f_1 E_x}{R_1C_1R_2C_2} + \frac{\alpha^2}{\beta^4}\frac{f_5 s E_x^{-3}}{R_3C_1R_2C_2} = 0 \tag{39}$$

Comparing (38) and (39) to (32) and (33) we find that equations (18), (19), (20), (21), and (22) must be satisfied and in addition

$$\varepsilon^2 = \frac{\alpha^2}{\beta^4}\frac{sf_5}{R_3C_1R_2C_2} \quad . \tag{40}$$

If we take $\varepsilon^2 \leqslant 1 \times 10^{-7}$ cm. radian as a reasonable range then setting f_5 to $\varepsilon^2 \times 10^7$ and using, as before

$$\alpha = 10^{-2} \text{ sec./cm.}$$

$$\beta = 10 \text{ volts/cm.}$$

we get

$$\frac{s}{R_3C_1R_2C_2} = 10\,\frac{\text{volts}^4}{\text{sec.}^2} \quad .$$

In Section IV, we have already set

$$R_2C_2 = 1 \text{ sec.} \qquad C_1 = 1\ \mu f$$

Values available for R_3 are

$$0.1,\ 1.0,\ 10.0\ M\Omega$$

hence the choices available for s are

$$s = 0.1,\ 1.0,\ 10.0\ .$$

However, the following problems were encountered:

1) because of limitations in the maximum slope which the function generator can produce, it was not possible to generate $y = s\,x^{-3}$ for values of s less than 10^5.

2) in a beam waist, the value of E will be about 0.1 cm. which corresponds to a voltage of 1 volt on the analog. At $V = 1$ volt, the output of the function generator should be $y = 10^5(1)^{-3} = 10^5$ volts but the maximum voltage allowable in the analog computer is 100 volts.

The only solution to this dilemma seems to be to increase the amplitude scale factor β . Various values of β were tried but in all cases it was found that one or the other of the problems above occurred or, for very large β, it was found that the dynamic range of E (range of values over which the analog would operate) was so small that no useful problems could be solved.

For these reasons, the analog cannot at present calculate beam envelopes.

The solution to the problem is either to

1) obtain a function generator which can produce

$y = 10x^{-3}$, or

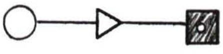
2) use the method of calculating envelopes given in A.

APPENDIX III. CIRCUIT DETAILS

There are some details of the timing circuits which were omitted for clarity in Section IV; there is a control system which controls the various modes of operation of the analog circuit.

A. Timing System

It was found that the clock could not supply the current required to drive many AND gates (as would be needed if a particular digit happened to occur in many magnet entrance and exit positions). To correct this, the circuit shown in Fig. 17 was devised. Each clock output goes to the input of a circuit like the one shown in Fig. 17 and the outputs of these circuits go to the switches which select the input pulses to the AND gates.

The flip-flops could not supply the current required to drive the relays. Relay driver circuits are provided in each AND gate and flip-flop module in the computer. The symbol for these circuits on the patch-board is

. One of these driver circuits is required between each flip-flop and relay.

Relay 5 which changes the sign of the term in g is always operated in parallel with a relay which controls a potentiometer so it is necessary to drive Relay 5 through a diode. If this is not done, since Relay 5 is operated in parallel with many other relays, each time one relay should

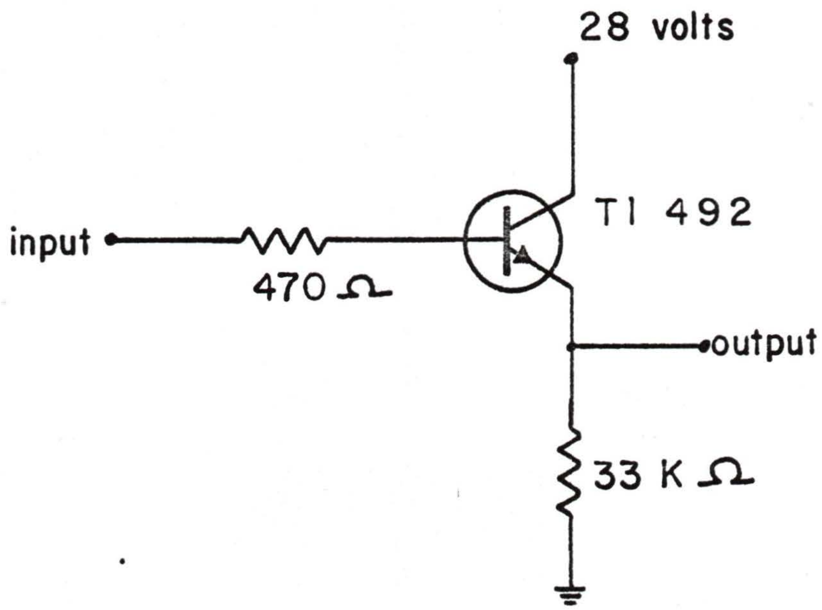


FIGURE 17 AND GATE DRIVER CIRCUIT

close, they all will close because all the relay supplies will be connected together at Relay 5.

The wiring from AND gate to flip-flop is as follows. The signal from the AND gate goes to the "S" terminal of the flip-flop if it is desired that the signal turn the flip-flop on and to the "R" terminal if it is desired that the signal turn the flip-flop off. If it is desired to use the signal to make the flip-flop change state each time the signal occurs, the signal should be connected to the "T" input of the flip-flop.

B. Analog Control Circuit

There are five operating modes for the Systron-Donner 10/20 analog. These are described in the computer instruction manual* and will be reproduced here.

1) Hold: opens the inputs to all integrators, thus holding the computer voltages (cf. Fig. 5).

2) Compute: the inputs of the integrators are connected into the circuit, the reset relays are opened (i.e. the initial conditions are removed) (cf. Fig. 5). This starts the computation.

3) Reset: opens all hold relays and applies initial conditions to all integrators (cf. Fig. 5).

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* Systron-Donner Model 10/20 Instruction Manual p. 13.

4) Rep-Op: automatically switches the computer between compute and reset modes so that the problem is solved repetitively.

5) Balance/Pot-Set: converts all amplifiers to a gain of 2500 so that the junction offset can be monitored and adjusted to zero; grounds the summing junctions of all amplifiers so that the potentiometers can be set under loaded conditions; connects 100 volts to the top of each potentiometer so that they can easily be set.

There are various circuits on the patch board which control these modes and the XY-plotter semi-automatically to make the analog easier to use for the specific problem at hand. These circuits will be described as they apply to each mode.

AND gates 35 and 36 define the compute and reset times when the analog is in the Rep-Op mode. Gate 35 turns Flip-flop 12 on, Gate 36 turns it off. Flip-flop 12, through a relay driver, controls the reset and compute of the computer (as described below) and controls the pen of the XY-recorder.

Flip-flop 11 sets in which plane (XZ- or YZ-) the analog is calculating. The input to this flip-flop is selected through Function Switch 2. If Function Switch 2 is in the lower position, Flip-flop 11 will change state at the beginning of each reset cycle, i.e. the computer will alternate between XZ-and YZ-planes. If Function Switch 2 is in the middle position, there will be no input to Flip-flop 11 and the flip-flop will not change state. Putting Function Switch 2 in the upper position causes the flip-flop to change state so that the computer can be set to the XZ- or YZ-plane manually. When the light on Flip-flop 11 is on, the computer is operating in the XZ-plane. Flip-flop 11, through a relay driver, controls the relay in the analog circuit which selects the plane (Relay 6 in Fig. 2) and controls a relay which selects the appropriate initial conditions.

The various computer modes are controlled by Flip-flop 12 and other circuits as described below:

1) <u>Hold</u>: The computer can be put in the hold mode by pushing the hold button on the control panel or applying 25 to 100 volts to the hold trunkline (labelled H on the patch board). The hold trunkline is used for the "problem interrupt". AND gate 34 is connected to Flip-flop 10 in such a way that each pulse from the gate causes the flip-flop to change state. The flip-flop is connected to the hold trunkline through a relay driver and Function Switch 1 so that when the flip-flop is on and Function Switch 1 is closed, the problem is in the hold mode. Thus, be selecting the input to Gate 34, the problem can be put into

the hold mode at any desired time. The next pulse from the gate (which comes after the clock has made one complete cycle) causes the gate to turn off, the problem reverts to the compute mode, and the solution is continued. If it is desired not to use the "problem interrupt", Function Switch 1 is left open.

2) Compute: This mode is never used, rep-op is used instead because it is necessary to have the plotter synchronized with the compute cycle.

3) Reset: This mode is never used because reset is part of the rep-op cycle which is always used for the reason given above.

4) Rep-Op: This is the mode which is used for all beam transport calculations. The compute time switch is set to the "external" position, then signals to the compute and reset trunklines determine the compute and reset times. Flip-flop 12 is on when it is desired to have the analog in the compute mode and off when it is desired to have the analog in the reset mode. Using the normal and inverted output of Flip-flop 12 and two relay drivers, +28 volts is applied to the compute trunkline and 0 volts is applied to the reset trunkline when Flip-flop 12 is on; 0 volts is applied to the compute trunkline and +28 volts is applied to the reset trunkline when the flip-flop is off. Thus,

in the rep-op mode, the compute and reset times are set by the clock and can bet set to whatever values are appropriate for the problem being solved.

5) Balance/Pot-Set: This mode is used to zero the amplifiers and set the potentiometers as described above. If the potentiometers are to be set accurately, it is necessary to set each one with the load connected, i.e. when the relay which controls it is closed. This is accomplished in the following manner: Function Switch 3 turns the clock on and off so, by watching the lights on the flip-flops, the system can be stopped by Function Switch 3 with one flip-flop on, i.e. with one relay closed. The potentiometer controlled by this relay can now be adjusted under load conditions. Now by turning the clock on again and waiting until the next relay is on, the next potentiometer can be adjusted. This can be continued until all potentiometers have been set. The same procedure should be followed to read values from potentiometers.

The analog can operate at two speeds, fast for oscilloscope display and slow for XY-plotter display. To change the speed of operation, two changes must be made; the capacitors on the integrators must be changed and the speed of the clock must be changed.

There is provision for patching two sets of capacitors on each integrator. Which one is used is selected by a relay. This relay is activated, i.e. the time scale is changed, by pushing the "time-scale" button on the control panel.

To change the speed of the clock, an extra decade counter must be added (if it is desired to have the clock run more slowly) or removed (if it is desired to have the clock run faster). For example, if the clock was set up with the 1 mHz. oscillator output fed into the decade 1 input, the decade 1 output into the decade 4 input, the decade 4 output into the decade 5 input and the decade 5 output into the decade 6 input, the clock can be speeded up by a factor of 10 by putting the 1 mHz. oscillator output into the decade 4 input and leaving the other connections as they were, i.e. decade 1 has been left out. These connections are made at the top of the clock module.

The connections described above are the ones used for the two clock speeds given in Section IV.

When operating the analog in the fast mode (with oscilloscope display), there is a pulse provided at the beginning of each compute cycle. This pulse is provided at the "sync" terminal on the side of the analog computer and can be used to trigger the oscilloscope.

APPENDIX IV. PROBLEM SET UP AND OPERATING INSTRUCTIONS

The instructions given below are intended as a guide to assist persons who are somewhat familiar with the operation of the analog.

In running a typical problem, the following steps should be followed:

1) Prepare a table of element lengths, g_x-, g_y-, and d-values done for the example problems in Section V.
2) Set the element position switches to the entrance and exit positions for each active element, i.e. set the first set of switches to the entrance position of the first active element, the second set of switches to the exit position of the first active element, the third set of switches to the entrance position of the second active element and so on for every active element in the system.
3) Patch the AND gates to the flip-flops and the flip-flops to the relays as described in Appendix III. The patching and potentiometer setting for each element type is:
   a) Quadrupole: one potentiometer (set at $\frac{G}{B\rho}$) connected to points a and c in Fig. 2. Relay 5 should also be activated (through a diode) if the element is horizontally focussing, i.e. if $G \geqslant 0$.
   b) Thin lens: as for a quadrupole but the potentiometer should be set at $\frac{1}{Lf}$.

c) <u>Focussing edge</u>: as for a quadrupole but the potentiometer should be set at $\frac{\tan \phi}{B\rho}$.

d) <u>Bending magnet</u>: three potentiometers, all driven from the same relay driver. One (set at $\frac{1 - n}{r_o^2}$) connected to point d on Fig. 2, one (set at $\frac{1}{r_o} \frac{\Delta p}{p}$) connected to point e on Fig. 2, and one (set at $\frac{n}{r_o^2}$) connected to point b on Fig. 2. The potentiometer set at $\frac{1}{r_o} \frac{\Delta p}{p}$ is fed by -100 volts rather than by $\pm V$ as all other potentiometers are.

4) After the element position switches have been set and the AND gates and flip-flops have been connected, the lights on the flip-flops should flash in the same sequence as the magnet system which is being simulated.

5) To set the potentiometers under load conditions as described in Appendix III the following steps should be followed:

a) put the computer in the balance/pot-set mode.

b) turn the clock on (Function Switch 3 to lower position) and watch the light on the first flip-flop; when it goes on, turn the clock off. Now the timing sequence has been stopped with the potentiometer which represents the first magnet connected into the circuit.

c) set the first potentiometer in the usual manner.

d) turn the clock on again until the second flip-flop is on etc.

6) Set the switches which control Flip-flop 12 to the range of compute and reset time which is desired (000 to 800 in example I in Section V).

7) With the computer in the reset made, set the desired initial conditions. When setting the initial conditions it should be remembered that the voltage at the output of Integrator 5 is

$$-R_2C_2 \frac{dV}{dt} = -R_2C_2 \frac{B}{\alpha} \frac{dx}{dz}$$

and the initial voltage must be set accordingly. Similarly, the voltage at the output of Integrator 6 is

$$V = \beta x \quad .$$

8) Now, putting the computer in the rep-op mode will cause the problem to be solved repetitively. If Function Switch 2 is in the lower position, the problem will be solved in XZ- and YZ-planes alternately.

9) To stop the solution of the problem, the computer should be put in either the reset or the hold mode.

The "problem interrupt" is used by setting the inputs to AND gate 3 and to the desired interrupt position and closing Function Switch 1. (This connects the hold signal to the hold trunkline.)

APPENDIX V.
CONVERSION TABLES OF ENERGY TO MOMENTUM AND MAGNETIC RIGIDITY FOR PROTONS, PI MESONS AND MU MESONS

In the following table, momentum p and magnetic rigidity ($B\rho$) are the quantities described in Section II. The other quantities listed are

$$\text{BETA} = v/c$$

$$\text{GAMMA} = \frac{m}{m_o}$$

where v is the velocity of the particle, c is the velocity of light, M the relativistic mass of the particle and m_o is the rest mass of the particle.

The expression E + 06 in the table means 10^6 thus, $.45814 \times 10^6$ would be printed in the table as .45814 E + 06.

DATA FOR PROTONS MASS = 938.256 MEV

ENERGY	MOMENTUM	BETA	GAMMA	BETA*GAMMA	MAGNETIC RIGIDITY	1/(MAGNETIC RIGIDITY)
MEV	MEV/C				GAUSS CM	1/(GAUSS CM)
10.	137.34	.14484	1.01065	.14638	.45814E+06	.21826E-05
20.	194.75	.20324	1.02131	.20757	.64963E+06	.15393E-05
30.	239.15	.24699	1.03197	.25489	.79773E+06	.12535E-05
40.	276.87	.28303	1.04263	.29509	.92355E+06	.10827E-05
50.	310.36	.31405	1.05329	.33078	.10352E+07	.96594E-06
60.	340.86	.34146	1.06394	.36329	.11370E+07	.87950E-06
70.	369.12	.36610	1.07460	.39341	.12312E+07	.81216E-06
80.	395.62	.38853	1.08526	.42166	.13196E+07	.75776E-06
90.	420.69	.40913	1.09592	.44838	.14032E+07	.71261E-06
100.	444.57	.42819	1.10658	.47383	.14829E+07	.67432E-06
110.	467.45	.44593	1.11723	.49821	.15592E+07	.64132E-06
120.	489.47	.46252	1.12789	.52168	.16326E+07	.61248E-06
130.	510.73	.47809	1.13855	.54434	.17036E+07	.58698E-06
140.	531.33	.49276	1.14921	.56629	.17723E+07	.56423E-06
150.	551.34	.50662	1.15987	.58762	.18390E+07	.54375E-06
160.	570.82	.51975	1.17052	.60838	.19040E+07	.52519E-06
170.	589.83	.53222	1.18118	.62865	.19674E+07	.50826E-06
180.	608.41	.54407	1.19184	.64845	.20294E+07	.49274E-06
190.	626.60	.55537	1.20250	.66784	.20901E+07	.47843E-06
200.	644.43	.56616	1.21316	.68684	.21496E+07	.46519E-06
210.	661.94	.57647	1.22381	.70550	.22079E+07	.45289E-06
220.	679.14	.58634	1.23447	.72383	.22653E+07	.44142E-06
230.	696.05	.59581	1.24513	.74186	.23217E+07	.43070E-06
240.	712.71	.60488	1.25579	.75961	.23773E+07	.42063E-06
250.	729.12	.61361	1.26645	.77710	.24321E+07	.41116E-06
260.	745.31	.62199	1.27710	.79436	.24860E+07	.40223E-06
270.	761.28	.63007	1.28776	.81138	.25393E+07	.39379E-06
280.	777.06	.63784	1.29842	.82819	.25919E+07	.38580E-06
290.	792.64	.64534	1.30908	.84480	.26439E+07	.37821E-06
300.	808.05	.65257	1.31974	.86123	.26953E+07	.37100E-06
310.	823.29	.65955	1.33040	.87747	.27462E+07	.36413E-06
320.	838.38	.66630	1.34105	.89355	.27965E+07	.35758E-06
330.	853.31	.67282	1.35171	.90947	.28463E+07	.35132E-06
340.	868.10	.67913	1.36237	.92523	.28956E+07	.34534E-06
350.	882.76	.68524	1.37303	.94086	.29445E+07	.33960E-06
360.	897.29	.69115	1.38369	.95634	.29930E+07	.33410E-06
370.	911.70	.69688	1.39434	.97170	.30411E+07	.32882E-06
380.	925.99	.70244	1.40500	.98693	.30887E+07	.32375E-06
390.	940.18	.70783	1.41566	1.00205	.31360E+07	.31886E-06
400.	954.25	.71305	1.42632	1.01705	.31830E+07	.31416E-06
410.	968.23	.71813	1.43698	1.03194	.32296E+07	.30962E-06
420.	982.10	.72306	1.44763	1.04673	.32759E+07	.30525E-06
430.	995.89	.72785	1.45829	1.06142	.33219E+07	.30102E-06
440.	1009.58	.73251	1.46895	1.07602	.33676E+07	.29694E-06
450.	1023.19	.73703	1.47961	1.09052	.34130E+07	.29299E-06
460.	1036.72	.74144	1.49027	1.10494	.34581E+07	.28917E-06
470.	1050.17	.74572	1.50092	1.11928	.35029E+07	.28547E-06
480.	1063.54	.74989	1.51158	1.13353	.35475E+07	.28188E-06
490.	1076.84	.75395	1.52224	1.14770	.35919E+07	.27839E-06
500.	1090.07	.75791	1.53290	1.16180	.36360E+07	.27502E-06

DATA FOR PROTONS MASS = 938.256 MEV

ENERGY	MOMENTUM	BETA	GAMMA	BETA*GAMMA	MAGNETIC RIGIDITY	1/(MAGNETIC RIGIDITY)
MEV	MEV/C				GAUSS CM	1/(GAUSS CM)
510.	1103.23	.76176	1.54356	1.17583	.36799E+07	.27174E-06
520.	1116.32	.76552	1.55421	1.18978	.37236E+07	.26855E-06
530.	1129.35	.76918	1.56487	1.20367	.37671E+07	.26545E-06
540.	1142.32	.77275	1.57553	1.21750	.38103E+07	.26244E-06
550.	1155.24	.77623	1.58619	1.23126	.38534E+07	.25950E-06
560.	1168.09	.77963	1.59685	1.24496	.38963E+07	.25665E-06
570.	1180.89	.78295	1.60751	1.25860	.39390E+07	.25386E-06
580.	1193.63	.78619	1.61816	1.27219	.39815E+07	.25115E-06
590.	1206.33	.78935	1.62882	1.28571	.40238E+07	.24851E-06
600.	1218.97	.79244	1.63948	1.29919	.40660E+07	.24593E-06
610.	1231.57	.79545	1.65014	1.31261	.41080E+07	.24342E-06
620.	1244.12	.79840	1.66080	1.32599	.41499E+07	.24096E-06
630.	1256.62	.80128	1.67145	1.33931	.41916E+07	.23857E-06
640.	1269.08	.80410	1.68211	1.35259	.42331E+07	.23622E-06
650.	1281.49	.80685	1.69277	1.36582	.42746E+07	.23393E-06
660.	1293.86	.80955	1.70343	1.37901	.43158E+07	.23170E-06
670.	1306.20	.81218	1.71409	1.39215	.43570E+07	.22951E-06
680.	1318.49	.81476	1.72474	1.40526	.43980E+07	.22737E-06
690.	1330.74	.81728	1.73540	1.41832	.44388E+07	.22528E-06
700.	1342.96	.81975	1.74606	1.43134	.44796E+07	.22323E-06
710.	1355.14	.82217	1.75672	1.44432	.45202E+07	.22122E-06
720.	1367.29	.82453	1.76738	1.45726	.45607E+07	.21926E-06
730.	1379.40	.82685	1.77803	1.47017	.46011E+07	.21733E-06
740.	1391.48	.82912	1.78869	1.48305	.46414E+07	.21544E-06
750.	1403.52	.83134	1.79935	1.49588	.46816E+07	.21359E-06
760.	1415.53	.83352	1.81001	1.50869	.47217E+07	.21178E-06
770.	1427.52	.83565	1.82067	1.52146	.47616E+07	.21000E-06
780.	1439.47	.83775	1.83132	1.53419	.48015E+07	.20826E-06
790.	1451.39	.83980	1.84198	1.54690	.48413E+07	.20655E-06
800.	1463.28	.84181	1.85264	1.55958	.48809E+07	.20487E-06
810.	1475.15	.84378	1.86330	1.57222	.49205E+07	.20322E-06
820.	1486.99	.84571	1.87396	1.58484	.49600E+07	.20161E-06
830.	1498.80	.84761	1.88461	1.59743	.49994E+07	.20002E-06
840.	1510.58	.84947	1.89527	1.60999	.50387E+07	.19846E-06
850.	1522.34	.85130	1.90593	1.62252	.50779E+07	.19692E-06
860.	1534.07	.85309	1.91659	1.63503	.51171E+07	.19542E-06
870.	1545.78	.85485	1.92725	1.64751	.51561E+07	.19394E-06
880.	1557.47	.85657	1.93791	1.65996	.51951E+07	.19248E-06
890.	1569.13	.85827	1.94856	1.67239	.52340E+07	.19105E-06
900.	1580.77	.85993	1.95922	1.68480	.52729E+07	.18964E-06
910.	1592.39	.86156	1.96988	1.69718	.53116E+07	.18826E-06
920.	1603.99	.86317	1.98054	1.70954	.53503E+07	.18690E-06
930.	1615.56	.86474	1.99120	1.72188	.53889E+07	.18556E-06
940.	1627.12	.86629	2.00185	1.73419	.54274E+07	.18424E-06
950.	1638.65	.86781	2.01251	1.74648	.54659E+07	.18295E-06
960.	1650.16	.86930	2.02317	1.75875	.55043E+07	.18167E-06
970.	1661.66	.87077	2.03383	1.77100	.55426E+07	.18041E-06
980.	1673.13	.87221	2.04449	1.78323	.55809E+07	.17918E-06
990.	1684.59	.87363	2.05514	1.79544	.56191E+07	.17796E-06
1000.	1696.02	.87502	2.06580	1.80763	.56573E+07	.17676E-06

DATA FOR PI MESONS MASS = 139.580 MEV

ENERGY	MOMENTUM	BETA	GAMMA	BETA*GAMMA	MAGNETIC RIGIDITY	1/(MAGNETIC RIGIDITY)
MEV	MEV/C				GAUSS CM	1/(GAUSS CM)
10.	53.77	.35949	1.07164	.38525	.17936E+06	.55750E-05
20.	77.35	.48471	1.14328	.55417	.25801E+06	.38757E-05
30.	96.30	.56790	1.21493	.68996	.32124E+06	.31129E-05
40.	112.98	.62918	1.28657	.80948	.37688E+06	.26533E-05
50.	128.28	.67669	1.35821	.91910	.42792E+06	.23368E-05
60.	142.65	.71476	1.42986	1.02200	.47583E+06	.21015E-05
70.	156.33	.74595	1.50150	1.12005	.52148E+06	.19176E-05
80.	169.50	.77196	1.57314	1.21441	.56541E+06	.17686E-05
90.	182.27	.79395	1.64479	1.30588	.60800E+06	.16447E-05
100.	194.72	.81275	1.71643	1.39504	.64951E+06	.15396E-05
110.	206.89	.82899	1.78807	1.48230	.69014E+06	.14489E-05
120.	218.85	.84312	1.85972	1.56798	.73003E+06	.13698E-05
130.	230.63	.85552	1.93136	1.65232	.76930E+06	.12998E-05
140.	242.24	.86645	2.00300	1.73552	.80803E+06	.12375E-05
150.	253.72	.87616	2.07465	1.81774	.84631E+06	.11815E-05
160.	265.07	.88482	2.14629	1.89910	.88419E+06	.11309E-05
170.	276.32	.89259	2.21793	1.97971	.92172E+06	.10849E-05
180.	287.48	.89957	2.28958	2.05965	.95895E+06	.10428E-05
190.	298.56	.90589	2.36122	2.13901	.99590E+06	.10041E-05
200.	309.56	.91161	2.43287	2.21784	.10326E+07	.96842E-06
210.	320.50	.91682	2.50451	2.29621	.10690E+07	.93537E-06
220.	331.38	.92158	2.57615	2.37414	.11053E+07	.90466E-06
230.	342.20	.92593	2.64780	2.45170	.11414E+07	.87605E-06
240.	352.98	.92993	2.71944	2.52890	.11774E+07	.84930E-06
250.	363.71	.93361	2.79108	2.60579	.12132E+07	.82424E-06
260.	374.40	.93700	2.86273	2.68239	.12488E+07	.80071E-06
270.	385.06	.94014	2.93437	2.75872	.12844E+07	.77855E-06
280.	395.68	.94304	3.00601	2.83480	.13198E+07	.75765E-06
290.	406.27	.94574	3.07766	2.91066	.13551E+07	.73791E-06
300.	416.83	.94824	3.14930	2.98632	.13903E+07	.71921E-06
310.	427.36	.95058	3.22094	3.06178	.14255E+07	.70149E-06
320.	437.87	.95276	3.29259	3.13706	.14605E+07	.68466E-06
330.	448.35	.95480	3.36423	3.21217	.14955E+07	.66864E-06
340.	458.81	.95670	3.43587	3.28713	.15304E+07	.65340E-06
350.	469.26	.95849	3.50752	3.36195	.15652E+07	.63886E-06
360.	479.68	.96017	3.57916	3.43663	.16000E+07	.62497E-06
370.	490.09	.96175	3.65080	3.51118	.16347E+07	.61170E-06
380.	500.48	.96324	3.72245	3.58561	.16694E+07	.59901E-06
390.	510.85	.96464	3.79409	3.65994	.17040E+07	.58684E-06
400.	521.21	.96596	3.86574	3.73415	.17385E+07	.57518E-06
410.	531.55	.96721	3.93738	3.80827	.17730E+07	.56398E-06
420.	541.89	.96839	4.00902	3.88230	.18075E+07	.55323E-06
430.	552.21	.96950	4.08067	3.95624	.18419E+07	.54289E-06
440.	562.52	.97056	4.15231	4.03010	.18763E+07	.53294E-06
450.	572.81	.97157	4.22395	4.10387	.19107E+07	.52336E-06
460.	583.10	.97252	4.29560	4.17758	.19450E+07	.51413E-06
470.	593.38	.97343	4.36724	4.25121	.19793E+07	.50522E-06
480.	603.65	.97429	4.43888	4.32478	.20135E+07	.49663E-06
490.	613.91	.97511	4.51053	4.39828	.20477E+07	.48833E-06
500.	624.16	.97589	4.58217	4.47172	.20819E+07	.48031E-06

DATA FOR PI MESONS MASS = 139.580 MEV

ENERGY	MOMENTUM	BETA	GAMMA	BETA*GAMMA	MAGNETIC RIGIDITY	1/(MAGNETIC RIGIDITY)
MEV	MEV/C				GAUSS CM	1/(GAUSS CM)
510.	634.40	.97664	4.65381	4.54511	.21161E+07	.47255E-06
520.	644.64	.97735	4.72546	4.61844	.21502E+07	.46505E-06
530.	654.87	.97803	4.79710	4.69171	.21844E+07	.45778E-06
540.	665.09	.97867	4.86874	4.76494	.22185E+07	.45075E-06
550.	675.30	.97930	4.94039	4.83812	.22525E+07	.44393E-06
560.	685.51	.97989	5.01203	4.91126	.22866E+07	.43732E-06
570.	695.71	.98046	5.08367	4.98435	.23206E+07	.43091E-06
580.	705.91	.98100	5.15532	5.05740	.23546E+07	.42468E-06
590.	716.10	.98152	5.22696	5.13041	.23886E+07	.41864E-06
600.	726.28	.98202	5.29861	5.20338	.24226E+07	.41277E-06
610.	736.46	.98250	5.37025	5.27632	.24565E+07	.40706E-06
620.	746.64	.98297	5.44189	5.34922	.24905E+07	.40151E-06
630.	756.81	.98341	5.51354	5.42209	.25244E+07	.39612E-06
640.	766.98	.98384	5.58518	5.49493	.25583E+07	.39087E-06
650.	777.14	.98425	5.65682	5.56773	.25922E+07	.38576E-06
660.	787.30	.98464	5.72847	5.64051	.26261E+07	.38078E-06
670.	797.45	.98502	5.80011	5.71325	.26600E+07	.37593E-06
680.	807.60	.98539	5.87175	5.78597	.26938E+07	.37121E-06
690.	817.75	.98574	5.94340	5.85867	.27277E+07	.36660E-06
700.	827.89	.98608	6.01504	5.93133	.27615E+07	.36211E-06
710.	838.03	.98641	6.08668	6.00398	.27953E+07	.35773E-06
720.	848.17	.98672	6.15833	6.07659	.28291E+07	.35345E-06
730.	858.30	.98703	6.22997	6.14919	.28629E+07	.34928E-06
740.	868.43	.98732	6.30161	6.22176	.28967E+07	.34521E-06
750.	878.56	.98761	6.37326	6.29432	.29305E+07	.34123E-06
760.	888.68	.98788	6.44490	6.36685	.29643E+07	.33734E-06
770.	898.80	.98815	6.51654	6.43936	.29980E+07	.33354E-06
780.	908.92	.98841	6.58819	6.51185	.30318E+07	.32983E-06
790.	919.04	.98866	6.65983	6.58433	.30655E+07	.32620E-06
800.	929.15	.98890	6.73148	6.65678	.30993E+07	.32265E-06
810.	939.26	.98913	6.80312	6.72922	.31330E+07	.31917E-06
820.	949.37	.98936	6.87476	6.80164	.31667E+07	.31577E-06
830.	959.48	.98958	6.94641	6.87405	.32004E+07	.31245E-06
840.	969.58	.98979	7.01805	6.94644	.32341E+07	.30919E-06
850.	979.68	.99000	7.08969	7.01881	.32678E+07	.30600E-06
860.	989.78	.99020	7.16134	7.09117	.33015E+07	.30288E-06
870.	999.88	.99039	7.23298	7.16352	.33352E+07	.29982E-06
880.	1009.98	.99058	7.30462	7.23585	.33689E+07	.29683E-06
890.	1020.07	.99076	7.37627	7.30817	.34025E+07	.29389E-06
900.	1030.16	.99094	7.44791	7.38047	.34362E+07	.29101E-06
910.	1040.25	.99111	7.51955	7.45276	.34699E+07	.28819E-06
920.	1050.34	.99128	7.59120	7.52504	.35035E+07	.28542E-06
930.	1060.43	.99144	7.66284	7.59731	.35372E+07	.28270E-06
940.	1070.51	.99160	7.73448	7.66957	.35708E+07	.28004E-06
950.	1080.60	.99176	7.80613	7.74181	.36044E+07	.27743E-06
960.	1090.68	.99191	7.87777	7.81404	.36381E+07	.27486E-06
970.	1100.76	.99205	7.94941	7.88627	.36717E+07	.27234E-06
980.	1110.84	.99219	8.02106	7.95848	.37053E+07	.26987E-06
990.	1120.92	.99233	8.09270	8.03068	.37389E+07	.26745E-06
1000.	1130.99	.99247	8.16435	8.10287	.37726E+07	.26506E-06

DATA FOR MU MESONS   MASS = 105.659 MEV

ENERGY	MOMENTUM	BETA	GAMMA	BETA*GAMMA	MAGNETIC RIGIDITY	1/(MAGNETIC RIGIDITY)
MEV	MEV/C				GAUSS CM	1/(GAUSS CM)
10.	47.04	.40675	1.09464	.44524	.15692E+06	.63725E-05
20.	68.01	.54128	1.18928	.64374	.22688E+06	.44075E-05
30.	85.08	.62720	1.28393	.80528	.28381E+06	.35234E-05
40.	100.26	.68834	1.37857	.94893	.33444E+06	.29900E-05
50.	114.30	.73433	1.47322	1.08184	.38128E+06	.26227E-05
60.	127.58	.77019	1.56786	1.20755	.42559E+06	.23496E-05
70.	140.32	.79887	1.66250	1.32813	.46808E+06	.21363E-05
80.	152.66	.82226	1.75715	1.44484	.50922E+06	.19637E-05
90.	164.67	.84165	1.85179	1.55857	.54930E+06	.18204E-05
100.	176.44	.85793	1.94644	1.66991	.58854E+06	.16991E-05
110.	188.00	.87175	2.04108	1.77933	.62710E+06	.15946E-05
120.	199.39	.88360	2.13572	1.88715	.66510E+06	.15035E-05
130.	210.64	.89385	2.23037	1.99363	.70263E+06	.14232E-05
140.	221.77	.90277	2.32501	2.09897	.73976E+06	.13517E-05
150.	232.80	.91060	2.41966	2.20335	.77654E+06	.12877E-05
160.	243.74	.91750	2.51430	2.30688	.81303E+06	.12299E-05
170.	254.60	.92362	2.60894	2.40969	.84927E+06	.11774E-05
180.	265.40	.92908	2.70359	2.51185	.88527E+06	.11295E-05
190.	276.13	.93396	2.79823	2.61345	.92108E+06	.10856E-05
200.	286.81	.93835	2.89288	2.71454	.95671E+06	.10452E-05
210.	297.45	.94231	2.98752	2.81519	.99218E+06	.10078E-05
220.	308.04	.94590	3.08216	2.91543	.10275E+07	.97322E-06
230.	318.59	.94916	3.17681	3.01531	.10627E+07	.94098E-06
240.	329.11	.95213	3.27145	3.11487	.10978E+07	.91090E-06
250.	339.60	.95485	3.36610	3.21413	.11327E+07	.88277E-06
260.	350.06	.95734	3.46074	3.31311	.11676E+07	.85640E-06
270.	360.49	.95963	3.55539	3.41186	.12024E+07	.83161E-06
280.	370.90	.96173	3.65003	3.51037	.12371E+07	.80827E-06
290.	381.29	.96368	3.74467	3.60868	.12718E+07	.78625E-06
300.	391.65	.96548	3.83932	3.70680	.13064E+07	.76544E-06
310.	402.00	.96715	3.93396	3.80474	.13409E+07	.74574E-06
320.	412.33	.96870	4.02861	3.90252	.13754E+07	.72705E-06
330.	422.65	.97014	4.12325	4.00015	.14098E+07	.70931E-06
340.	432.95	.97148	4.21789	4.09764	.14441E+07	.69243E-06
350.	443.23	.97274	4.31254	4.19500	.14784E+07	.67636E-06
360.	453.51	.97391	4.40718	4.29223	.15127E+07	.66104E-06
370.	463.77	.97501	4.50183	4.38936	.15469E+07	.64641E-06
380.	474.02	.97604	4.59647	4.48637	.15811E+07	.63243E-06
390.	484.26	.97701	4.69111	4.58329	.16153E+07	.61906E-06
400.	494.49	.97792	4.78576	4.68012	.16494E+07	.60625E-06
410.	504.71	.97878	4.88040	4.77685	.16835E+07	.59398E-06
420.	514.93	.97959	4.97505	4.87351	.17176E+07	.58220E-06
430.	525.13	.98035	5.06969	4.97009	.17516E+07	.57088E-06
440.	535.33	.98107	5.16433	5.06659	.17856E+07	.56001E-06
450.	545.52	.98175	5.25898	5.16303	.18196E+07	.54955E-06
460.	555.70	.98239	5.35362	5.25940	.18536E+07	.53948E-06
470.	565.87	.98301	5.44827	5.35571	.18875E+07	.52978E-06
480.	576.04	.98359	5.54291	5.45196	.19214E+07	.52042E-06
490.	586.21	.98414	5.63756	5.54816	.19553E+07	.51140E-06
500.	596.37	.98466	5.73220	5.64430	.19892E+07	.50269E-06

DATA FOR MU MESONS MASS = 105.659 MEV

ENERGY	MOMENTUM	BETA	GAMMA	BETA*GAMMA	MAGNETIC RIGIDITY	1/(MAGNETIC RIGIDITY)
MEV	MEV/C				GAUSS CM	1/(GAUSS CM)
510.	606.52	.98516	5.82684	5.74039	.20231E+07	.49427E-06
520.	616.67	.98563	5.92149	5.83644	.20569E+07	.48614E-06
530.	626.81	.98608	6.01613	5.93244	.20908E+07	.47827E-06
540.	636.95	.98651	6.11078	6.02840	.21246E+07	.47066E-06
550.	647.08	.98693	6.20542	6.12432	.21584E+07	.46329E-06
560.	657.21	.98732	6.30006	6.22019	.21922E+07	.45615E-06
570.	667.34	.98769	6.39471	6.31603	.22260E+07	.44923E-06
580.	677.46	.98805	6.48935	6.41184	.22597E+07	.44251E-06
590.	687.58	.98839	6.58400	6.50761	.22935E+07	.43600E-06
600.	697.70	.98872	6.67864	6.60335	.23272E+07	.42968E-06
610.	707.81	.98904	6.77328	6.69906	.23610E+07	.42354E-06
620.	717.92	.98934	6.86793	6.79474	.23947E+07	.41758E-06
630.	728.03	.98963	6.96257	6.89039	.24284E+07	.41178E-06
640.	738.13	.98990	7.05722	6.98601	.24621E+07	.40614E-06
650.	748.23	.99017	7.15186	7.08160	.24958E+07	.40066E-06
660.	758.33	.99043	7.24650	7.17717	.25295E+07	.39533E-06
670.	768.42	.99067	7.34115	7.27272	.25631E+07	.39013E-06
680.	778.52	.99091	7.43579	7.36824	.25968E+07	.38507E-06
690.	788.61	.99114	7.53044	7.46374	.26305E+07	.38015E-06
700.	798.70	.99136	7.62508	7.55922	.26641E+07	.37535E-06
710.	808.78	.99157	7.71973	7.65468	.26978E+07	.37066E-06
720.	818.87	.99177	7.81437	7.75012	.27314E+07	.36610E-06
730.	828.95	.99197	7.90901	7.84554	.27650E+07	.36165E-06
740.	839.03	.99216	8.00366	7.94094	.27987E+07	.35730E-06
750.	849.11	.99234	8.09830	8.03632	.28323E+07	.35306E-06
760.	859.18	.99252	8.19295	8.13169	.28659E+07	.34892E-06
770.	869.26	.99269	8.28759	8.22704	.28995E+07	.34488E-06
780.	879.33	.99285	8.38223	8.32237	.29331E+07	.34093E-06
790.	889.40	.99301	8.47688	8.41769	.29667E+07	.33707E-06
800.	899.47	.99317	8.57152	8.51299	.30003E+07	.33329E-06
810.	909.54	.99332	8.66617	8.60828	.30339E+07	.32960E-06
820.	919.60	.99346	8.76081	8.70355	.30674E+07	.32600E-06
830.	929.67	.99360	8.85545	8.79881	.31010E+07	.32247E-06
840.	939.73	.99373	8.95010	8.89406	.31346E+07	.31901E-06
850.	949.80	.99386	9.04474	8.98929	.31681E+07	.31563E-06
860.	959.86	.99399	9.13939	9.08451	.32017E+07	.31232E-06
870.	969.92	.99411	9.23403	9.17972	.32353E+07	.30909E-06
880.	979.97	.99423	9.32867	9.27492	.32688E+07	.30591E-06
890.	990.03	.99435	9.42332	9.37011	.33024E+07	.30280E-06
900.	1000.09	.99446	9.51796	9.46528	.33359E+07	.29976E-06
910.	1010.14	.99457	9.61261	9.56045	.33694E+07	.29678E-06
920.	1020.20	.99467	9.70725	9.65561	.34030E+07	.29385E-06
930.	1030.25	.99478	9.80190	9.75075	.34365E+07	.29098E-06
940.	1040.30	.99488	9.89654	9.84589	.34700E+07	.28817E-06
950.	1050.35	.99497	9.99118	9.94101	.35036E+07	.28541E-06
960.	1060.40	.99507	10.08583	10.03613	.35371E+07	.28271E-06
970.	1070.45	.99516	10.18047	10.13124	.35706E+07	.28006E-06
980.	1080.50	.99525	10.27512	10.22634	.36041E+07	.27745E-06
990.	1090.55	.99533	10.36976	10.32143	.36376E+07	.27490E-06
1000.	1100.59	.99542	10.46440	10.41651	.36711E+07	.27239E-06

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UNIVERSITY OF VICTORIA 1962 to 1966

Degrees, Diplomas, Etc., Awarded, with Dates and Names of Institutions:

B.Sc. (Honours) 1966 University of Victoria, Victoria

Honours and Awards:

University of Victoria Fellowship, 1966/67

Publications:

R. J. Louis and R. M. Pearce, A Hybrid Analog Computer for Beam Optics Calculations, presented at the Western Regional Nuclear Conference, February 1967.