THE POSSIBILITY OF THEORETICAL SYNTHETIC APRIORI KNOWLEDGE WITHIN A KANTIAN FRAME OF REFERENCE

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ABSTRACT

It is our concern in the present thesis to consider the problem whether theoretical synthetic apriori knowledge is possible. Since our position is in favor of the possibility in question, our consideration of the problem includes two main tasks: the first is to show that theoretical synthetic apriori knowledge is possible; and the second is to show that it has not been disintegrated though confronted with various challenges.

We begin our enquiry with a preliminary investigation which covers: (1) an elucidation of the significance of the problem involved - our point is that this problem is fundamental to Kant's epistemology which, in turn, constitutes the entire framework of his philosophy; (2) an introductory examination of the meaning of "apriori", which is an attempt to remove unnecessary misconceptions for the understanding of the theses we are going to discuss; and (3) an examination of the possibility of the analytic-synthetic distinction, which is planned for the provision of a necessary ground for our further discussion of theoretical synthetic apriori knowledge. In the chapter on the possibility of the analytic-synthetic distinction, we consider several approaches towards the problem. These

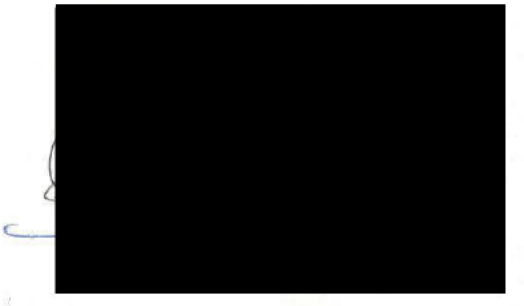
approaches are the metaphysical approach, the logical approach, the linguistic approach, and Kant's epistemological approach. Our consideration of these approaches leads to a conclusion which favors the analytic-synthetic distinction. This conclusion affirms that the analytic-synthetic distinction is in fact possible, even if the distinction has not been, in some sense, satisfactorily delineated.

We then pass over to the consideration of the possibility of theoretical synthetic apriori knowledge. This consideration includes a negative account as well as a positive account of the possibility involved. In the negative account of the problem, we try to show that there are apriori propositions which are not analytic; we thus assert the existence of theoretical synthetic apriori knowledge. In the positive account of the problem, we try to explain how such knowledge is possible within a Kantian frame of reference; we wish to establish that we can have apriori knowledge of space and time and nature, or a universal and apriori system of judgments and principles about space and time and nature. We try to show, in accordance with Kant's doctrine, that these judgments and principles, as conditions of the possibility of experience given by our cognitive faculty, are necessarily synthetic apriori.

Finally we try to show that it is possible to meet

the various objections which have been brought against Kant's position - those objections which imply that there could not be any synthetic apriori knowledge, or that synthetic apriori knowledge has already been dissolved by developments of mathematics and science. Our consideration of these objections shows that they are often grounded on misconceptions and are therefore unjustified. We conclude that theoretical synthetic apriori knowledge has not been so far disintegrated.

As a result of the above considerations, the general conclusion of this thesis may be stated thus: In all theoretical sciences of reason, it is possible that there are theoretical synthetic apriori judgments contained as principles. These principles are apriori because they are the presuppositions of both commonsense and scientific thought about nature; they are also synthetic because they express, in Kant's language, the conditions of possible experience. Theoretical synthetic apriori knowledge is therefore possible.



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CHAPTER I

INTRODUCTION

The Twofold Employment of Human Reason

Within the sphere of metaphysics the concepts of God, Freedom, and Immortality are always considered as having great significance. Although Kant in his "pre-critical" period had convinced himself that, from a Humean point of view, metaphysics is nonsense, he soon found that speculation about metaphysical concepts of this sort is in fact inevitable. He later concluded that to arrive at a satisfactory metaphysics was perhaps the most important task of the human intellect. But before we can frame a real metaphysics, Kant thought, it is necessary to take up an investigation of the human faculties of knowledge so as to determine what we can hope to know and what we can not know. It was this investigation which Kant first intended to pursue in the *Critique of Pure Reason.*

As a result of this investigation, Kant concluded that our objective knowledge can extend without limits in the world of Nature, but he also held that we can not have any objective knowledge about the existence of God, Freedom, and Immortality for they are beyond the world of experience. However, by this assertion Kant only means to restrict theoretical reason (which deals with the realms of the

conditioned = phenomena) to its empirical and legitimate employment, he does not mean that reason can never establish the existence of God, Freedom and Immortality. For, according to Kant, reason can be "related to its object in one or other of two ways, either as merely determining it and its concepts. . . or as also *making it actual*."[1] In the former function reason is *theoretical*, in the latter function it is *practical*, and it is through this latter function of reason that we can obtain practical knowledge of God, Freedom, and Immortality. So for Kant there is a twofold employment of human reason: when reason is employed theoretically he calls it *theoretical reason*, and when reason is employed practically he calls it *practical reason*.

With respect to theoretical reason, the employment of reason may be entitled legitimate in Kant's view if it is connected with experience directly or indirectly. In other words, a theoretical employment of reason is required to be empirical and to be used within the bounds of human experience. More precisely, it is forbidden to be used in a fashion which transcends all possible experience, for its remoteness from sense-perception is an ever-present source of danger which can result in an incorrect employment of reason. The temptation to misuse reason in this way springs

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[1] BX. All quotations from Immanual Kant's *Critique of Pure Reason* are taken from Norman Kemp Smith's translation (London: Macmillan, 1933); references to the *Critique* are given with the customary "A" and "B" for the first and second editions, as in that translation.

from deeply rooted human needs.[2] In the case of practical reason, the remoteness of employment from sense-perception is considered as natural and expected, for it concerns only matters in the moral world and not of a moral Nature. So while the theoretical employment of reason is legitimate only within the empirical world, Kant allows practical reason to be employed in a transcendental manner. In other words, whereas the former must be subject to the restrictions arising from the formal and the material conditions of possible experience, the latter is free from all these restrictions. This distinction is necessary and fundamental, and we must keep it in mind if we are to understand Kant's philosophy.

Two Attitudes Towards Theoretical Synthetic Apriori Knowledge

As we have mentioned, Kant's first *Critique* was originally devoted to an investigation of the human faculties of knowledge. However, Kant's actual endeavour may also be said to have two major aims. The one is to provide a philosophical basis for mathematics, natural sciences and all thought about matters of fact; the other is "to deny *knowledge*, in order to make room for *faith*."[3] The first one, as it is, involves one of the most fundamental problems in

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[2] Vide A797 = B825.

[3] BXXX.

in Kant's philosophy, viz., *how theoretical synthetic apriori judgments are possible in mathematical and scientific knowledge.* The second one, on the other hand, involves another fundamental problem which is of remarkable significance to Kant's ethical theory, viz., how practical synthetic apriori judgments are possible in metaphysics as a science; this is treated in detail chiefly in the *Critique of Practical Reason.*

In conformity with Kant's doctrine, synthetic apriori knowledge should be understood in two ways. By "practical synthetic apriori knowledge" Kant understands the creative principles as principles of actualizations of practical reason; by "theoretical synthetic apriori knowledge" he understands the regulative principles as well as the constitutive principles of theoretical reason, the consideration of which is the objective of the first *Critique* and the *Prolegomena to Any Future Metaphysics.* In these works Kant holds that *in all theoretical sciences of reason, theoretical synthetic apriori judgments are contained as principles.*[4] But his close references to (or apparent dependence on) Newtonian physics and Euclidean geometry has given rise to lots of objections, among which is the objection that Kant's synthetic apriori principles have already been disintegrated according to the modern development of mathematics and science.

The above objection is held, for example, by Hans

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[4] B14ff.

Reichenbach and other philosophers. On the one hand, the invention of non-Euclidean geometries 20 years after Kant's death has stimulated the axiomatization of mathematics, which is a development that runs right up to the Frege-Russell programme of reducing mathematics to logic.[5] On the other hand, Einstein's theory of relativity revises Newtonian physics, while at the same time quantum physics seems to repudiate the principle of causality.[6] In a word, the modern development of both mathematics and science certainly seems to suggest that theoretical synthetic apriori knowledge does not exist in these very sciences. Reichenbach therefore describes the development of mathematics and physics in the nineteenth and the twentieth century as "the process of the disintegration of synthetic apriori knowledge." His further claim is that in the realm of human knowledge, a proposition is either apriori and analytic, or synthetic and aposteriori (empirical). In other words, he claims that there are two and only two kinds of propositions: analytic apriori and synthetic aposteriori propositions. This anti-Kantian claim that excludes the possibility of synthetic apriori propositions, is in fact adopted by most of the philosophers within or clinging to the two major philosophical traditions in recent thought:

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[5] Vide Ch. VII.

[6] Ibid.

the Cambridge movement deriving from Moore and Russell, and the Logical Positivism of the Vienna Circle (Schlick, Carnap, *et al.*) together with the Scientific Empricism of the Berlin group (led by Reichenbach). This philosophical trend thus constitutes a powerful challenge to Kant's epistemology and metaphysics, both of which depend on his theory of synthetic apriori knowledge.

Summing up the above discussions, we find that there are two opposing attitudes towards the controversial problem concerning the possibility of theoretical synthetic apriori knowledge. Those following Kant would hold that theoretical synthetic apriori knowledge underlies all theoretical sciences of reason as their basis. But such philosophers as Reichenbach, Ayer, Sellars, Schlick and Nagel would argue, on the other hand, that there is no theoretical synthetic apriori knowledge at all, and that all propositions can be exhaustively classified into two categories, viz., analytic apriori and synthetic aposteriori (the category of synthetic apriori is excluded). This latter objection to Kant's view will become clearer when we come to the discussion in Chapter VII. There we shall try to show, objections of this sort are often based on some misinterpretations or misconceptions of Kant's doctrine, and are therefore mistaken and unjustified, as is the case for example of Reichenbach's objections.

Other Objections and Our Objectives

Although such philosophers as Reichenbach, Ayer, Schlick, and Carnap, deny the possibility of theoretical synthetic apriori knowledge, they nonetheless agree with Kant on the possibility of the analytic-synthetic distinction. In other words, they all accept Kant's view that every proposition is either analytic or synthetic exclusively. However, some philosophers go a step further to argue against this view and claim that no such distinction is possible insofar as, for them, all propositions are analytic (or all are syntethic, etc.); they assert that the distinction which the terms "analytic" and "synthetic" purport to mark simply does not exist. Chapters III and IV are concerned with these objections, and there we attempt to show that such philosophers' objections or criticisms of the distinction do not justify their rejection of it. Based on the analytic-synthetic distinction and assuming its soundness, the logical positivists and logical empiricists conclude that all necessary (apriori) propositions are analytic, and hence do not accept the possibility of necessary (apriori) propositions which are synthetic. However, in our further examination of the classification of propositions,[7] we find that there are certain necessary (apriori) propositions which can not in any clear way be

[7]This examination is presented in Chapter V.

counted as analytic, and if they are not analytic they must be synthetic if we accept this distinction. In order to clarify the matter, this negative account of the existence of synthetic apriori propositions is supplemented by another positive statement about their apriori syntheticity, which is actually a recapitulation of Kant's account of theoretical synthetic apriori judgments.

Finally, the present thesis may be said to have two main objectives:

(i) To show that theoretical synthetic apriori knowledge of the Kantian variety is possible. We do this by putting forward a negative account as well as a positive account of the possibility of such knowledge. In the positive account, we wish to establish the possibility that there can be apriori knowledge of space and time and Nature; viz., to establish that there can be a system of strictly universal and necessary principles about space and time and Nature which can never be derived from experience, but which are nonetheless applicable to it. Chapters V and VI are primarily devoted to this problem.

(ii) To show that theoretical synthetic apriori knowledge has not been disintegrated even though it has been confronted with challenges from the disciplines of philosophy, mathematics and science. This we do by showing how these challenges basically rest on

misconceptions rather than a reliable understanding of Kant's doctrine. Chapters III, IV, and VII are devoted to this purpose.

Having given a brief sketch of the whole problem with which we are concerned, let us now begin our enquiry in the following chapter with a preliminary examination of the meaning of "apriori."

CHAPTER II

THE MEANING OF "APRIORI"

The term "a priori" has been used very widely in all branches of knowledge with quite different meanings. Even in Philosophy it has been used in a variety of ways. So a historical examination of these different uses will surely help us to get a general conception of it.

The literal meaning of "apriori" is "from what is prior". "According to Aristotle, A is prior to B in nature if and only if B could not *exist* without A; A is prior to B in knowledge if and only if we cannot *know* B without knowing A."[1] For example, when we say "A is essentially prior to B" we mean that "B could not exist without the prior existence of A" and A is an instance of a substance. This is how "apriori" is sometimes used in metaphysics. Besides that, "apriori" is also used in epistemology. For example, when we say that "to know B through its cause or reason A" we mean that "the cause or reason A is prior to B (the effect) in knowledge." Finally, "apriori" is also used in logic. Aristotle believes that it is possible to demonstrate a causal relation through a syllogism which has

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[1] D. W. Hamlyn "*A priori & A posteriori*" in P. Edwards (ed.), *The Encyclopedia of Philosophy*, (N.Y.: Macmillan, 1967), [Abbreviation: *The Encyl. of Phil.*, Vol...].

the cause as its middle term. So when we say that "to know the conclusion of the premises" we mean by this "to know something in terms of a demonstrable causal relationship".

Medieval philosophers and theologians usually used this term in the same way as Aristotle. For example, St. Thomas' principles "everything has a cause" and "every movement has a mover" were thought to be apriori principles. The same was said of St. Anselm's principle "God is the absolutely perfect Being". Principles of this kind are for them ontological, and have but an obscure connection with experience.

It was not until Descartes and Leibniz that the concept of "apriori" was connected closely to the idea of "independence of experience." As for Descartes, self-evident principles such as "I think therefore I am" and knowledge about innate ideas such as "God" are apriori. They need not be derived "from any other source than certain germs of truth which exist naturally in our souls."[2] In other words, apriori knowledge of this sort is not derived from experience but only by logical deduction from a set of innate ideas inplanted by "nature", or precisely, by God.[3] Still, certain of Descartes' statements[4] suggest that innate

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[2] *Discourse on Method*, 6.

[3] Cf. Copleston, *The History of Philosophy* (London: Burns & Oates, 1946-64), Vol. IV, p. 83.

[4] Cf. Descartes' *Notes Against a Programme*, 12.

ideas are apriori forms of thought which in no way differ from the faculty of thinking, and that we, owing to this faculty's innate constitution, conceive things in certain ways. This, in some sense, anticipates Kant's theory of apriori intuitions and concepts.

As for Leibniz, there are two kinds of truths, i.e. truths of reason and truths of fact, while the former are themselves self-evident propositions or reducible thereto, the latter cannot be deduced from any of these self-evident truths. According to him, truths of fact are aposteriori, truths of reason are apriori. All truths of reason are based on the principle of contradiction or "what comes to the same thing, that of identity".[5] Their denials are logically impossible, i.e. one cannot deny them without self-contradiction, therefore they are analytic and necessary - to use Kant's terms. So from Leibniz's point of view, a proposition which is apriori is analytic and logically necessary, while one which is aposteriori is only contingent. As we shall see later, Leibniz' theory has had great influence upon recent Philosophy of mathematics. Another point is, for Leibniz, that all propositions are in one sense apriori, viz., apriori in the divine mind.[6] This,

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[5] C. I. Gerhardt's edition of *Die Philosophisches Schriften von G. W. Leibniz*, Vol. 4, p. 357. Copleston's translation in Ibid., p. 274.

[6] A more detailed description on this point is given in the following chapter.

indeed, indicates the fact that rationalists are always using "apriori" in a metaphysical manner than in a merely epistemological manner. But it is quite different in the case of Hume's empiricist philosophy, for Hume uses "apriori" only in an epistemological sense. And his use of "apriori", in turn, anticipates those theories of the apriori, which the characteristic empiricists of the twentieth century and the positivists have developed later.[7]

"All the objects of human reason or enquiry", Hume says, "may naturally be divided into two kinds, to wit, Relation of Ideas, and Matters of Fact".[8] Geometry, Algebra and Arithmetic are the sciences whose propositions are primarily of the first kind; they are either intuitively or demonstratively certain. For example, propositions such as "2 + 2 = 4" and "2 x 4 = 8" express only relations between numbers, and so their truth is discoverable by the mere operation of thought, without depending on what is existent anywhere in the universe.[9] In other words, we attain knowledge of the truth of these propositions by "reasonings apriori".[10] With respect to Matter of Fact statements Hume says, "... the knowledge of

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[7] Vide Copleston, ibid., Vol. V, p. 275.

[8] *An Enquiry Concerning Human Understanding* (La Salle: Open Court, 1958), Sect. IV, pt. 1 (p. 24).

[9] Ibid.

[10] Cf. ibid., p. 26.

this relation is not...attained by reasonings *a priori* but arises entirely from experience..."[11] It is evident here that "reasonings apriori" denotes those reasonings which depend only on the pure operation of thought and not on experience. So the meaning of the term "apriori" clearly denotes knowledge which arises from the mere operation of thought and not from experience.

Finally, let us turn to Kant's use of 'apriori'. In the *Critique of Pure Reason*, Kant said, "...we shall understand by *a priori* knowledge, not knowledge independent of this or that experience, but knowledge absolutely independent of all experience."[12] So an apriori proposition is a proposition which can be established without any reference to experience; an aposteriori proposition, in contrast, can not.

Knowledge independent of this or that experience may also, in one sense, be called apriori knowledge. For example, when we undermine the foundations of a house, we may say that we know apriori that the house will collapse, though it has not fallen as yet. But knowledge of this kind is only apriori in a relative sense, because we have, first of all, to learn from experience that bodies are heavy and will fall, when their supports are taken away.[13]

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[11] Ibid.

[12] B3.

[13] N. K. Smith, *A Commentary to Kant's 'Critique of Pure Reason'* (N.Y.: Humanities Press, 1923), p. 54.

In other words, we can know that something is apriori only after having some other experience, thus we call this relative apriori knowledge. For Kant, however, apriori knowledge is not apriori in this sense. Kant uses the term "apriori" in an absolute sense as is clear in the above quoted passage. For him "apriori knowledge" signifies knowledge which is absolutely independent of all experience, or, of all sense-impressions. Knowledge of this kind is called Absolute apriori knowledge.[14] One of the examples Kant cites is, "every change has its cause". This principle does not presuppose any empirical knowledge but is, on Kant's view, one of the foundations of experience. Kant says there are two kinds of absolute apriori knowledge, to wit, mixed absolute apriori knowledge and pure absolute apriori knowledge.[15] The above example, "every change has its cause," is an illustration of mixed absolute apriori knowledge, because "change" is an empirical concept. Absolute apriori knowledge which has no empirical concepts is pure absolute apriori knowledge. Examples of the latter would be, "substance is permanent" and "2 + 2 = 4".

To say that apriori knowledge is knowledge absolutely independent of all experience is merely to say that it is non-empirical. So far this is only a negative characterization. What, then, are the characteristics of apriori

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[14] Ibid., p. 55.

[15] Ibid.

knowledge? In order to answer this question, Kant puts forward criteria in terms of which we can infallibly distinguish pure (= apriori) knowledge from empirical knowledge.

> First, then, if we have a proposition which in being thought is thought as necessary, it is an apriori judgment; and if, besides, it is not derived from any proposition except one which also has the validity of a necessary judgment, it is an absolutely *apriori* judgment. Secondly, experience never confers on its judgments true or strict, but only assumed and comparative *universality*, through induction. We can properly only say, therefore, that, so far as we have hitherto observed, there is no exception to this and that rule. If then, a judgment is thought with strict universality, that is, in such manner that no exception is allowed as possible, it is not derived from experience, but is valid absolutely apriori.[16]

In other words, apriori knowledge is both necessary and strictly universal. But according to Pap, "a strictly universal proposition is one which has no conceivable exceptions, which is another way of saying that it is necessary."[17] So, we may say, Kant's two criteria are ultimately identical, that is, coalesce into one.[18] In the passage quoted above, he clearly states a distinction between "assumed and comparative universality" and "strict

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[16] B3-4.

[17] Arthur Pap, *Semantics & Necessary Truth* (New Haven: Yale Univ. Press, 1958), p. 23.

[18] Prichard, *Kant's Theory of Knowledge*, p. 4 n.3.

(or rational)[19] universality." In brief, Experience can only provide us with generalizations which have assumed and comparative universality, but cannot give us apriori knowledge with strict or rational universality as this can spring only from reason.

Summing up Kant's view, a judgement is apriori if it is logically independent[20] of all experience and sense-impressions, and exhibits necessity and strict universality.

The foregoing brief survey has shown that the notion of "apriori" has been construed in different ways in different systems. Still, we shall try to give a rough account of it: (S) A proposition or concept is apriori if it can be established or wholly justified without any appeal to sense-impression, or other empirical propositions, or ostensive definition, and is necessary in either a logical or an epistemological sense. We have not exhausted in this definition all the implications of this term, nor have we excluded all other possible definitions. For example, in a different way, phenomenologists such as Husserl and Scheler use the term "apriori" to denote "all those ideal meaning-wholes and propositions which, irrespective of any mode in which they are considered by the subject which thinks them

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[19] Paton, *Kant's Metaphysic of Experience* (London: Allen & Unwin, 1951), Vol. 1, p. 77, n.3.

[20] According to Körner, "two judgments are logically independent if neither of them entails either the other or the contradictory of it; and if the same is true also of their contradictories." In his *Kant* (Penguin Books, 1955), p. 19.

and of the nature of that subject, as well as irrespective of any mode in which an object is posited to which they might be applicable, are bodily given through the content of an immediate intuition."[21] Positivists, though agreeing with statement (S), go further to claim that "apriori" and "analytic" are co-extensional; in other words, all apriori propositions are analytic. Therefore statement (S) can at most suggest a general definition for "apriori", it will not exclude all other possible explanations of the term.

We shall close this discussion of "apriori" by noting also its opposing concept "aposteriori". Since a general definition of "apriori" has already been given, a corresponding general description of "aposteriori" may then be determined. Opposed to statement (S), statement (S') would run as follows: A proposition or concept is aposteriori if its establishment or justification must appeal to experience or sense-impressions, or other empirical propositions, or ostensive definitions, and it is not necessary in a logical or an epistemological sense. Aposteriori knowledge, as opposed to apriori knowledge, is therefore empirical, and its validation necessitates a procedure that does make reference to experience, sense-

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[21] Scheler's *Der Formalismus in der Ethik und die Materiale Wertethik*, 2nd ed., p. 43. M. Schlick's translation in "Is there a Factual *a Priori*?" in Feigl & Sellars (ed.), *Readings in Philosophical Analysis* (N.Y.: Appleton-Century-Crofts, 1949).

impressions and so on.[22] Propositions which express knowledge of this kind are, in Leibniz' words, truths of fact, or in Hume's words, matters of fact. All of them are contingent.

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[22]Cf. D. W. Hamlyn "Apriori and Aposteriori" in *The Encycl. of Phil.*, Vol. 1, p. 141.

CHAPTER III

THE POSSIBILITY OF THE ANALTYIC-SYNTHETIC DISTINCTION

The analytic-synthetic distinction has central importance in Kant's philosophy. For him, an analytic proposition is a proposition whose truth can be adequately known in accordance with the principle of contradiction alone; a synthetic proposition, on the other hand, is a proposition whose truth can not be thus recognized. This distinction, he further holds, is only a distinction within the world of experience, and is nothing at all if it transcends the field of experience. However, many philosophers have questioned the possibility of such a distinction.

In connection with the possibility of the analytic-synthetic distinction, we shall raise the following three fundamental questions. What do analytic and synthetic mean? Is it possible to classify all propositions clearly and distinctly into these two categories? If this classification is possible, what is the criterion for this classification? These questions have provoked endless controversy since Kant, and so far no consensus has been reached. As we know, many philosophers have criticised the strict distinction between analytic and synthetic propositions, and there are at least three objections to which we

should turn our attention. Some philosophers, for example Leibniz (who wrote before Kant), have held that all truths are innate and in a sense analytic.[1] After Kant, Hegel, Bradley and Bosanquet all held that every judgment is both analytic and synthetic. And recent philosophers such as Quine and White, on the one hand think that the criteria for the distinction are not satisfactory, and on the other hand put forward a gradualism or a continum to take the place of this distinction. These three major positions embody the main objections to Kant's view. We shall attempt to defend Kant's distinction against these objections, for if it is impossible to distinguish between synthetic and analytic propositions, then certainly we cannot divide apriori propositions into apriori *synthetic* propositions and apriori *analytic* propositions. In this chapter we shall only examine the first two views; the last view will be left to the following chapter which concerns problems of analyticity and criteria for the analytic-synthetic distinction.

Let us begin our discussion with Leibniz' account of propositions.

Leibniz divided true propositions into truths of reason and truths of facts. The former depend upon the principle of contradiction, and the latter depend upon the principle of sufficient reason. Again, the former are

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[1] Copleston, ibid., Vol. IV, p. 277.

either self-evident truths or capable of being reduced by us to self-evident truths, i.e. necessary and finitely analytic,[2] while the latter are contingent propositions incapable of being reduced to self-evident propositions, but can be known apriori by the divine mind through an infinite analysis. He said in his *De Scientia Universali seu Calculo Philosophico*,

> The difference between necessary and contingent truths is indeed the same as that between commensurable and incommensurable numbers. For the reduction of commensurable numbers to a common measure is analogous to the demonstration of necessary truths, or their reduction to identical truths. But, as in the case of surd ratios the reduction involves an infinite process and yet approaches a common measure so that a definite but unending series is obtained, thus also contingent truths require an infinite analysis, which God alone can accomplish.[3]

Perhaps we can call truths of fact analytic in this sense, i.e., infinitely analytic. But we should note that (1) truths of reason are not existential propositions, while truths of fact are; (2) truths of reason are necessary in all possible worlds, while truths of fact are only necessary in a particular possible world, and therefore are contingent in the strict sense, for they depend upon the

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[2]Copleston, ibid., Vol. IV, p. 277.

[3]Translation in Latta, *Leibniz: The Monadology and other Philosophical Writings* (London: Oxford Univ. Press, 1951), p. 61 n.

decree of God;[4] and (3) all truths are innate, the knowledge of which does not come to a Monad (or conscious Being) from outside itself, for there is no real causal interaction between substances, according to Leibniz.

This view of propositions has its real foundation in Leibniz' metaphysics. In his *Monadology*, we find out that monads are windowless,* thus one monad cannot be influenced by any other monads. As he observes, "Though each monad is a world apart, it changes in harmonious correspondence with the changes in all other monads according to a law or harmony pre-established by God."[5] In other words, God pre-established the universe "in the beginning of things, after which everything goes its own way in the phenomena of Nature, according to the laws of souls and bodies."[6] So we might say that every activity and future development of a monad has already been planned or pre-established by God, and nothing else can be added into it. Thus, although a proposition seems to be synthetic and empirical, it is *pg 105.

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[4]For more detail see Copleston, ibid., Vol. IV, p. 282: "Caesar's resolve to cross the Rubicon was indeed comprised in the notion of Caesar; but it does not follow that the possible world in which Caesar is a member is necessary. Granted that God selected this particular possible world, it was apriori certain that Caesar would resolve to cross the Rubicon; but it was not logically or metaphysically necessary for God to select this particular world."

[5]Copleston, ibid., Vol. IV, p. 308.

[6]Gerhardt (ed.) *Die philosophischen Schriften von G. W. Leibniz*, Vol. 3, p. 143 (a letter to Basnage), translation in ibid., p. 309.

actually analytic, as it has been pre-established by God that its predicate will be contained in its subject.

Leibniz' error involves, in Kant's view, the introduction of an ideal (i.e. the idea of God) and an idea of Pure Reason (i.e. the idea of infinity) into the world of experience in order to justify his claim that contingent truths are analytic. According to Kant, this is an illegitimate use of pure reason, and he would say that Leibniz' doctrine of the analyticity of contingent truths was the result of a misuse of pure reason.

Let us next examine Bernard Bosanquet's view. Let us start with an example. Consider this historical statement Q: "Caesar crossed the Rubicon". According to Bosanquet, this judgment does not assert that there was another Caesar besides the one who had before conquered Gaul or who was afterwards murdered on the Ides of March, but rather that there was one and the same Caesar who once crossed the Rubicon and who had before conquered Gaul and was afterwards murdered on the Ides of March. In asserting this, we in fact exhibit Caesar's identity in one of its differences through which it persisted. However, in the course of exhibiting such an identity in difference, there are two unavoidable processes. "On the one hand, I analyse the individual whole that is called Caesar by specifying one of the differences that may be considered as a part within it; on the other hand, I construct or make

synthesis of the individual whole in question, by exhibiting it as a whole that pervades, and absorbs in itself, each or all of its differences."[7] Accordingly these two processes prove that the judgment Q is both analytic and synthetic.

Another example we should consider here is an equation such as 7 + 5 = 12. Bosanquet points out that,

> It is obvious that if analytic and synthetic were reciprocally exclusive characters, the question, 'Is this equation an analytic or a synthetic judgment?' would be wholly unanswerable. If 12 were not the same number as 7 + 5, the judgement would not be true; if 7 + 5 gained nothing by being defined as 12, the judgment would cease to be a judgment at all.[8]

What he means here is that the judgment "7 + 5 = 12" is analytic because 12 is the same number as 7 + 5; but it is also synthetic, because 7 + 5 gained something by being *defined* as 12. And so the judgment "7 + 5 = 12" is both analytic and synthetic, because "by constructing 12 in this way, we *ipso facto* analyse it."[9]

Bosanquet's conclusion that "every judgment is both analytic and synthetic", is based on his assertion that judgment involves identity in difference, which in turn is

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[7] Bernard Bosanquet, "Analytic & Synthetic" in Nagel & Brandt (ed.) *Meaning & Knowledge* (N.Y., 1965), pp. 199-202. Fr. Bosenquet, *logic*, 2nd edition, Vol. 1, Chap. I.

[8] Ibid.

[9] Ibid.

founded on his metaphysical presupposition that in every judgment the *genuine* subject is reality. For example, the subject of statement Q seems to be "Caesar", but actually, according to Bosanquet, the whole statement Q is a predicate descriptive only of reality. This point can be seen more clearly in F. H. Bradley's writings. He said in *Appearance and Reality*, "In every judgment the genuine subject is reality, which goes beyond the predicate and of which the predicate is an adjective."[10] What is reality? Reality is the Absolute that is the "self-consistent"[11] and "comprehensive (non-relational) individual"[12] with existence as its form and sentient experience as its content. What then is Judgment? "Judgment proper is the act which refers an ideal content (recognized as such) to a reality beyond the act."[13] In other words judgment involves adding an adjective to reality, and this adjective is for Bradley an idea. An idea is necessary for thinking or judging, it is "an adjective divorced, a parasite out loose, a spirit without a body seeking rest in another, an abstraction from

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[10] F. H. Bradley, *Appearance & Reality* (2nd ed., London, 1897), p. 168.

[11] Vide, e.g., "Ultimate Reality is such that it does not contradict itself; here is an absolute Criterion" in ibid., p. 136.

[12] Vide, e.g. "... the real is individual. It is one in the sense that its positive character embraces all differences in an inclusive harmony." Ibid., p. 140.

[13] F. H. Bradley, *The Principles of Logic* (2nd ed., London, 1958), Vol. 1, p. 10.

the concrete, a mere possibility which by itself is nothing."[14] It "implies the separation of content from existence."[15] Therefore an idea is a mere feature of content which is used to qualify further the "that" of the subject. In so judging, we assert about nothing but reality, which is a "self-consistent and comprehensive individual." And judgment simply is "the re-union of two sides, 'what' and 'that', provisionally estranged."[16]

From a metaphysical point of view, the views of Bosanquet and Bradley are systematically consistent. Since reality is the Absolute, a judgment about it is analytic because the idea in the Judgment is merely a part of the absolute; the same judgment is also synthetic because it is essentially the "re-union of the idea and the 'that' (i.e. existence) of reality." With respect to this metaphysical theory of judgment we should consider at least the following two points. 1) According to Kant, we should not introduce the idea of reality into the world of experience as the "genuine subject" of all judgments, because the idea of reality is a *transcendental* idea of absolute totality, and this idea belongs only to the world of intelligibilis. 2) Even if we accept this metaphysical theory of judgment,

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[14] Ibid., p. 8.

[15] *Appearance & Reality*, p. 163.

[16] Ibid., p. 165.

we may still need another theory which applies solely to the realm of appearance without any self-contradiction.

On the basis of our brief examination of the views of Leibniz, Bosanquet and Bradley, we find that these views share one common point. They all involve the use of an idea (or ideas) of pure reason into the world of experience in order to justify their claim that it is impossible to classify all propositions clearly and distinctly into the two categories "analytic" and "synthetic". However, according to Kant's analysis in "Transcendental Dialectic" of the first *Critique*, such an introduction is illegitimate because an idea of pure reason is *transcendent* (that is, it transcends all possible experience), and its application to experience is unjustified and unwarranted. If Kant's analysis is correct, the views held by Leibniz, Bosanquet and Bradley would turn out to be merely the result of a misuse of pure reason and consequently would fail to repudiate Kant's account of the analytic-synthetic distinction.

So far we have only examined the views of Leibniz, Bosanquet and Bradley, who one and all take a metaphysical approach towards the problem concerning the possibility of the analytic-synthetic distinction. The consideration of other approaches towards the same problem is also necessary for a better understanding of the whole situation involved. We shall attempt to do this in the following chapter which begins with some discussions of analyticity.

CHAPTER IV

SOME DISCUSSIONS OF ANALYTICITY AND THE ANALYTIC-SYNTHETIC DISTINCTION

We shall now turn our attention to other discussions of the analytic-synthetic distinction. Generally speaking, the philosophers taking up these approaches are in fundamental agreement that the distinction is meaningful. However some contemporary philosophers, especially Quine and White, believe that such a distinction rests primarily on a dogma of empiricism and should be abandoned. Their rejection of the distinction, we shall argue, is unjustified. To begin the discussion, let us first consider problems surrounding the concept of analyticity.

The problems of analyticity can be discussed via (at least) three approaches, viz., (i) the logical approach, (ii) the linguistic approach, and (iii) the epistemological approach. We shall now consider these approaches.

1. The Logical Approach

Ludwig Wittgenstein's early researches into the nature of the propositions of logic provided a basis for interpreting and clarifying the notion of analyticity. For Wittgenstein, a proposition of logic is a truth-functional

tautology,[1] whose truth can be demonstrated by the truth-table method. For example, "(pV~p)" and "(p⊃q)⊃(~q⊃~p)" are propositions of this sort. If we set out to examine the nature of such propositions, we shall naturally find that: (1) a tautology is a compound proposition which essentially contains only logical constants and variables; (2) it is true regardless of the truth-values of its components, that is to say, it remains true no matter what is the case, it depends on no empirical facts; (3) it is devoid of factual content, it tells us nothing about the empirical world; (4) it is apriori and necessary, its denial is self-contradictory.[2] A proposition with these properties may be said to be logically true. In connection with this, Carnap holds that a sentence in a semantic system is logically true if it holds in every state-description.[3] As Quine has noticed, this account (tautology) is obviously an adaptation of Leibniz' conception of "truths of reason" as those truths that are "true in all possible worlds."[4] According to Quine's explanation, a state-description is "any exhaustive assignment of truth values to the atomic, or non-

omitted.

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[1] R. W. Ashby, "Linguistic Theory of the A Priori", in *The Encycl. of Phil.*, Vol. 4.

[2] Cf. Ashby, op. cit.

[3] See Carnap, *Meaning and Necessity: A Study in Semantics and Modal Logic* (Chicago 1947, 1st ed; 1956, 2nd enl. ed.), p. 10.

[4] Vide Quine, "Two Dogmas of Empiricism", in *From a Logical Point of View* (Cambridge, 1961), p. 23.

compound, statements of language."[5] For instance, in a language L, which contains only two individual constants "a" and "b" and two primitive predicates "P" and "Q", the expression that "Pa·Qa·Pb·~Qb" would be an example of a state-description.[6] Here, Quine also understands a logical truth as "a statement which is true and remains true under all reinterpretations of its components other than the logical particles."[7] In other words, a logical truth is necessarily true without any regard to the meanings that will ever be ascribed to its non-logical variables.

According to this view, all logical truths are analytic, and so an analytic statement may be defined thus: A statement is analytic if and only if it either (i) is a logical truth, or (ii) is transformable into a logical truth by the substitution of synonyms for synonyms.[8] Or, as Arthur Pap puts it: "A statement is analytic if it is a substitution instance of a logical principle or, in case defined terms occur in it, a definitionally abbreviated substitution instance of a logical principle."[9] Within these two possibilities of an analytic statement, the first

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[5] Ibid.

[6] A. Pap, "Are All Necessary Propositions Analytic?" in Olshewsky, *Readings in the Philosophy of Language* (N.Y., 1969), p. 382n.

[7] Quine, op. cit., p. 22. what does this mean?

[8] Ibid., p. 23.

[9] Pap, op. cit., p. 384.

alternative is typified by the example "All unmarried men are unmarried men", which is simply a substitution instance of a logical principle; the second alternative is typified by the example "All bachelors are unmarried", which is a definitionally abbreviated substitution instance of the logical principle "(x) [Px·QX⊃Px]", assuming the synonymous pair "bachelor" and "unmarried man".

An objection against this view has been widely debated. It is the thesis held by Quine and White that synonymity is no clearer a notion than analyticity. As has been shown, the proposition "all bachelors are unmarried" could be proved to be analytic only if "bachelor" and "unmarried man" turn out to be a real synonymous pair. It is the difficulties here involved in characterizing "a synonymous pair" that Quine points out. He writes with respect to this:

> We still lack a proper characterization of this ... class of analytic statements, and therewith of analyticity generally, inasmuch as we have had in the above description to lean on a notion of "synonym" which is no less in need of clarification than analyticity itself.[10]

In order to resolve this problem, we shall begin with some linguistic considerations and this is the topic of the next section.

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[10] Quine, op. cit. p. 23.

2. The Linguistic Approach

Carnap has attempted to meet the problem of synonymity in his article "Meaning and Synonymy in Natural Language."[11] According to Carnap, two predicates are synonymous if and only if they have the same intension, where "intension" is taken to mean the cognitive meaning which "may be roughly characterized as that meaning component which is relevant for the determination of truth."[12] The determination of such intensions is merely a matter of empirical operations, and depends wholly on a scientific procedure. For Carnap, the assignment of an intension is an empirical hypothesis which "can be tested by observations of language behavior"[13] in respect to those "responses concerning the application of a predicate to all the cases that can be found."[14] Thus, he defines the intension of a predicate as the range of the predicate which "comprehends those possible kinds of objects for which the predicate holds."[15] Then, by means of the same-

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[11] In Carnap, op. cit., pp. 233-247.

[12] Ibid.

[13] Ibid.

[14] By "all the cases..." we mean to include not only the actual cases but also possible cases. For this purpose, we are allowed to use modal expressions, or simply descriptions of such possible cases. Vide ibid.

[15] Ibid.

ness of intension, he defines the notion of "synonymity" and thereby also the notion of "analyticity"; he declares that a sentence is analytic if "it is true by virtue of the intensions of the expressions occurring in it."[16] Or more precisely, he writes: "A sentence is *analytic* in L for X at t if its intension (or range or truth-condition) in L for X at t comprehends all possible cases."[17]

Another response to the same problem is the suggestion that synonymity should be explicated without reference to the sameness of intension. According to this view, language is said to be extensional, and the sameness of meaning is considered equivalent to extensional equivalence.[18] That is to say, two expressions are regarded as synonymous if they are "interchangeable *salva veritate*," viz., "interchangeable in all contexts without any change of truth value."[19] But this account is by no means satisfactory. The extensional agreement of two expressions, say, "bachelor" and "unmarried man", need not imply any sameness of meaning; their interchangeability *salva veritate* may rest merely on accidental factors.[20] For example, in the case of "creature with a heart" and

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[16] Ibid.

[17] Ibid.

[18] Vide D. W. Hamlym, "Analytic and Synthetic Statements", in *The Encycl. of Phil.* (Vol. 1).

[19] Quine, op. cit., p. 27.

[20] Vide ibid., p. 31.

"creature with kidneys", the terms carry with them diverse meanings, but, as it happens, they always apply to the same things. This very fact shows that the interchangeability *salva veritate* does not afford a sufficient condition of synonymity.

Some, however, put aside all these and similar considerations, and set out to elucidate analytic propositions through the conception of "linguistic rules." They assert that the sentences that seem to express necessary propositions really express linguistic rules which only stipulate how certain symbols or words are to be used.[21] Wittgenstein, for instance, says in the *Tractatus Logico-philosophicus* that propositions such as "The colours green and blue cannot be in the same place simultaneously" express rules of logical grammar. This theory has, in fact, been held by M. Schlick, G. Ryle, Ayer, Strawson, and others. But Ayer later recognized that it is a mistake to say that such propositions are themselves linguistic rules. "For apart from the fact that they can properly be said to be true, which linguistic rules cannot", he writes, "they are distinguished also by being necessary, whereas linguistic rules are arbitrary."[22]

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[21] Ashby, op. cit. Cf also Ayer, "Truth by Convention".

[22] Ayer, *Language, logic and Truth* (2nd ed., N.Y., 1946), p. 17.

Consider the proposition (E): "If A precedes B, then B does not precede A". If we are tracing Quine's route, the proposition (E) may be said to be analytic if it is reducible to a logical truth. But after formalizing such a proposition, we obtain the formulae "(X)(Y)[XRy⊃~yRx], which is certainly no logical truth. For if we should try to reinterpret R as the relation of identity, we are bound to arrive at the false proposition (E'): "If A is identical to B, then B is not identical to A," which violates the criterion that "logical truths remain true under all reinterpretations." Thus we come to see that the necessary truth of propositions such as (E) rest not on their deducibility from logic, but perhaps rest, as Ayer, Strawson and N. Malcolm hold, on certain contingent, empirical "facts about verbal usage."[23] According to them, if we find that there exists a linguistic rule governing the usage of the verb "to precede", and which is formulated as "(x)(y)[xRy⊃~YRX], we are then entitled to claim (E) to be an analytic truth. If, on the other hand, we find another rule governing the usage of the verb "to identify", and which is formulated as "(x)(y)[xR'y⊃yR'x], we then conclude that (E') is false, simply because it violates such a rule. Following from the above consideration, we

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[23] Ibid., p. 16. Such philosophers as Malcolm, Ayer and Strawson are often called "verbalists" for holding that necessary propositions depend on certain contingent facts of verbal usage. Vide, e.g., A. Pap, op. cit., p. 393 ff.

come to this verbalist definition of "analytic": A proposition is analytic if it is "true by virtue of explicit or implicit rules of language."[24] According to this verbalist view, analytic propositions are necessary and do not describe any fact at all. They are necessary if certain linguistic rules are presupposed, and although they themselves are not rules, they nevertheless elucidite the proper use of words, signs, etc.[25] "[A]nalytic propositions", says Ayer, "are devoid of factual content, and consequently ... they say nothing ... although they give us no information about any empirical situation, they do enlighten us by illustrating the way in which we use certain symbols."[26]

Adherents of this *verbalist* view claim that in ordinary language, we consider propositions such as these - "If p implies q, and p is true, then q is true", "Every yard contains three feet", and "If A is earlier than B and B is earlier than C, A is earlier than C" - as necessary and analytic, since they follow from certain implicit linguistic rules which govern the usage of the constituent expressions in them.[27] Such implicit linguistic rules are

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[24]Pap, op. cit., p. 393.

[25]Vide Ayer, op. cit., pp. 16-17.

[26]Ibid., p. 79.

[27]Cf. Pap, op. cit., p. 393 ff; Ayer, op. cit., pp. 16-17.

simply linguistic habits or linguistic conventions, and their existence is a necessary and sufficient condition for the analyticity of the above-mentioned propositions and the like.[28] According to this verbalist interpretation, it is our linguistic habit to refuse to apply the word "yard" to anything less than three feet; and it is our linguistic habit to refuse to say that A is $\overset{\text{not}}{\wedge}$ earlier than C, if, e.g., A is earlier than B and B is ~~not~~ earlier than C; and so on.

In our view this verbalist account of analyticity involves several difficulties which may be considered as follows:

Firstly, we find that the implicit linguistic rules are merely arbitrary[29] conventions, and by this very fact, they are only contingent and must be subject to possible alterations. Just suppose the linguistic habits were to change in such a way that the word "yard" was taken to name a length of four feet, the proposition "Every yard contains three feet" would not remain necessary, while on the other hand the proposition "Every yard contains four feet" would become necessary instead. It would, however, be argued that the proposition which was formerly expressed by the sentence "Every yard contains three feet" would still be necessary; but if the necessity of the proposition

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[28] Ibid.

[29] Linguistic rules are arbitrary. Vide, e.g., Ayer, op. cit., p. 17.

were unaffected by such changes, it would be a mistake to claim that its analyticity *depends upon* linguistic rules.[30] *Secondly*, the propositions asserting those linguistic rules are themselves empirical, because they merely describe certain contingent facts of linguistic usage, or certain empirical observations of human linguistic behavior. And since whatever follows from an empirical proposition is itself also empirical, the *analytic* propositions that are claimed to follow from such empirical linguistic rules would turn out to be *empirical*, which is contradictory.[31] *Thirdly*, in a natural language, we find no firm ground for deciding whether certain constituent expressions of a proposition are used in accordance with linguistic rules or not, simply because we do not have something like a rule-book which enables us to distinguish certain true propositions from others that should be counted as linguistic rules. *Fourthly*, in case of more than one language, a proposition which is analytic in L_1, could become synthetic in L_2 if a diverse set of rules is adopted. *Finally*, since linguistic rules are simply empirical representations (as indicated in the second point), the verbalist thesis (that a proposition is analytic if it is true by virtue of linguistic rules) would seem to imply that

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[30] Pap, op. cit., pp. 393-4.

[31] Ibid., p. 395.

"a proposition is analytic if it is true by virtue of certain *empirical representations*." However, this seems to contradict the proposition which is held also by verbalists, that an analytic proposition is independent of any sense-impressions or any empirical representations.[32] For the reasons above, we may conclude that the verbalist thesis is indeed problematic.

Some philosophers have attempted to overcome some of these difficulties by constructing special languages. Carnap, for example, recognizes the vagueness of natural language, and he has attempted to construct a precise artificial language with explicit semantic rules. But no matter what the contents of these semantic rules may be, they come sooner or later to demarcate analyticity in the form that "A statement S is analytic for language L_o if and only if ...."[33] Quine has argued that such semantic rules are merely postulates,[34] the assignment of which is only a matter of choice. This has led M. G. White to observe that the analyticity involved here rests only on a convention, a rule explicitly stated in an artificial language, say, L_1. "But it is easy to see", he argues, "that we can construct

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[32] Ayer, for instance, holds that an analytic proposition is independent of experience, or facts of experience. Vide op. cit. pp. 78-9, etc.

[33] Quine, op. cit., pp. 33-34.

[34] Ibid., p. 35.

a language L_2 in which the reverse situation prevails and in which a linguistic shape which was analytic in L_1 becomes synthetic in L_2, etc."[35]

3. Some Remarks on the above discussions

(I) Given the above discussions we can now formulate at least four tentative criteria of analyticity. A true proposition or statement is *analytic*, if:

(C1) it is a logical truth, a statement which is true and remains true under all reinterpretations of its components;

or (C2) it is transformable into a logical truth by the substitution of synonyms for synonyms;

or (C3) it expresses a linguistic rule;

or (C4) it is true by virtue of explicit or implicit linguistic rules.

With these proposed criteria of analyticity in hand, we can construct the corresponding criteria of syntheticity, assuming that a proposition must be synthetic if and only if it is not analytic. For example, we may construct criteria for syntheticity as follows: $A_{\wedge}^{\text{true}}$ proposition or statement is *synthetic*, if:

(S1-2) it is neither a logical truth nor transformable into a logical truth by putting

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[35] White, "The Analytic and the Synthetic: An Untenable Dualism", in Linsky (ed.) *Semantics and the Philosophy of Language* (Urbana, 1952).

synonyms for synonyms;

or (S3) it does not express a linguistic rule;

or (S4) it is not true by virtue of any linguistic rules.

Among these supposed criteria for analyticity and syntheticity, we may say, the criteria (C3) and (S3) are the most untenable ones. For (C3) implies that true analytic propositions are linguistic rules which are arbitrary and contingent, and (S3) implies that propositions expressing contingent facts are not synthetic, and these implications are unacceptable. On the other hand, the criteria (C1), (C2) and (S1-2) would probably be the most plausible ones, for they involve only the problem of characterizing logical truths and synonymy.

However, Quine considers seriously the difficulties involved in characterizing synonymy. His consideration leads him finally to the claim that it is impossible to draw a distinction between analytic and synthetic propositions. "That there is such a distinction to be drawn at all," he tells us, "is an unempirical dogma of empiricists, a metaphysical article of faith."[36] He seems to claim the extreme thesis "not merely that the distinction is useless or inadequately clarified, but

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[36]Quine, op. cit., p. 37.

also that it is altogether illusory, that the belief in its existence is a philosophical mistake."[37] But Quine's criticisms[38] of the distinction do not justify his rejection of it. In an essay called "In Defence of a Dogma", Grice and Strawson correctly point out, where there is agreement on the *use* of the expressions involved with respect to an open class, there must be some kind of distinction present.[39] Now "analytic" and "synthetic" have a more or less established philosophical use, it would thus be absurd to claim that there is no such distinction. Furthermore, since Quine also extends his skepticism to other "intension concepts" such as synonym, meaning, and related ones, he is bound to be led to a paradoxical position. For as Grice and Strawson point out, if Quine is to be consistent in his adherence to this extreme thesis, then he must also maintain that "the distinction we suppose ourselves to be marking by the use of the expressions 'means the same as', 'does not mean the

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[37] H. P. Grice & P. F. Strawson, "In Defence of a Dogma" in Olshewsky, op. cit., p. 418.

[38] Quine says, for example, in "Two Dogmas of Empiricism": "Analyticity at first seemed most naturally definable by appeal to a realm of meanings. On refinement, the appeal to meanings gave way to an appeal to synonymy or definition. But definition turned out to be a will-o'-the-wisp, and synonymy turned out to be best understood only by dint of a prior appeal to analyticity itself." (Quine, op.cit., p. 32.)

[39] Vide Grice & Strawson, op. cit., p. 419.

same as' does not exist either."[40] But this simply involves saying that all talk of expression-synonym and sentence-synonym is meaningless and senseless.[41] This is extremely paradoxical.

As Grice and Strawson have made clear in their essay, there is a strong presumption in favor of the existence of the analytic-synthetic distinction. This presumption rests both on philosophical and on ordinary usage of language, and is not in the least shaken by the fact, if it is a fact, that the distinction in question has not been, in some sense, adequately clarified.[42] These considerations bring us to this point: although it may be the case that no adequate explication of analyticity has been given, the lack of such an explication does not imply that the explication is impossible. Moreover, on the basis the above described theoretical reasons, we agree with Grice and Strawson's view that the analytic-synthetic distinction does in fact exist, even if it is not very satisfactorily delineated.

Hilary Putnam, for example, agrees with this view, and argues in his essay "The Analytic and the Synthetic" that the analytic-synthetic distinction is

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[40] Ibid., p. 420.

[41] Vide ibid., p. 421.

[42] Vide ibid., pp. 421-422.

also necessary for physical sciences.[43] Even in the realm of mathematics, we find that the three most important modern theories about the nature of mathematics - logicism, formalism and intuitionism - all presuppose the exhaustive classification of propositions into analytic and synthetic.[44] This seems to us significant, and we are inclined to say it is possible, in principle at least, to make a sharp distinction between analytic and synthetic propositions.

We can see at this point that formulating the criteria for analyticity is a subject of some controversy. However, there are definite properties which can be ascribed to analytic propositions. Analytic propositions are:

(1) *necessary*, in the sense that they are "true in all possible worlds", that is true in every state-description, or true under all reinterpretations of components;

(2) *empty*, in the sense that they are void of factual content, they do not assert or deny the existence of any particular state of the world;

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[43] Vide, "The Analytic and the Synthetic", in *Minnesota Studies in the Philosophy of Science* (Minneapolis, 1962), Vol. III, pp. 358-397.

[44] See Körner, *The Philosophy of Mathematics* (N.Y., 1962), especially Chs. III and VI.

(3) *nonempirical*, in the sense that they are established independently of any sense-impressions or empirical facts, of any propositions or experience.

(II) The foregoing sections present a diversity of views about the notion of analyticity. Nevertheless, despite this diversity, all of these views seem to have one thing in common, namely, they all attempt to state a formal relationship between symbols in order to clarify, by means of this formal relation or characteristic, the notion of analyticity. This would supposedly in turn enable one to distinguish the boundary between analytic and synthetic propositions. But, perhaps if we push beyond a mere examination of the formal characteristics of symbols, to an examination of the conceptual content represented by these symbols, we will be able to determine the characteristics of analytic judgments in a quite different manner. This is in fact the approach that Kant adopted.

4. Kant's Account on Analyticity: The Epistemological Approach

Kant called his philosophy "critical" because it rests upon a critical enquiry of human knowledge - an enquiry into the sources, conditions, scope and limits of our knowledge, both apriori and empirical. This enquiry is

in fact one of the central topics of the *Critique of Pure Reason*. There Kant strongly emphasises the significance of synthetic apriori judgments for human knowledge, and he gives a full explanation of them. But before we can go into Kant's account of synthetic apriori judgments, let us first examine his theory of analytic judgments. This will enable us to complete the discussion under way in this Chapter.

Kant's account of judgments is based on the content of a judgment rather than its origin or its logical form. As he said in the *Prolegomena*,

> ... whatever be their origin or their logical form, there is a distinction in judgments, as to their content, according to which they are either merely explicative, adding nothing to the content of knowledge, or expansive, increasing the given knowledge. The former may be called analytical, the latter synthetical judgments.[45]

It is important to note that Kant's account is not merely about propositions, but about judgments. By "judgments" he means "activities of judging" or "propositions asserted by somebody."[46] In all judgments[47] there is a relation between subject and predicate, and this relation can be of

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[45] In Kant's *Prolegomena to Any Future Metaphysics*, translated by L. W. Beck (Indianapolis, 1950), §2 a. (Abbreviation: *Prolegomena*, §2 a).

[46] S. Körner, *Kant*, p. 18.

[47] Here Kant considers affirmative judgments only, the subsequent applications to negative judgment can be easily made.

two kinds. That is, "either the predicate B belongs to the subject A, as something which is (covertly) contained in this concept A; or B lies outside the concept A, although it does indeed stand in connection with it."[48] In the former case the judgment is, for Kant, *analytic*, and in such a judgment the connection of the predicate with the subject is thought through identity. Thus the predicate adds nothing to the concept of the subject, but simply unfolds what has all along been thought in it, and so the judgment is explicative. To construct a judgment of this kind, "I have merely to analyse the [subject] concept, that is, to become conscious to myself of the manifold which I always think in that concept,"[49] but I do not have to go outside the concept and appeal to the testimony of experience for its support. Since the predicate of an analytic judgment is already contained in the concept of the subject, we can not deny the judgment without contradiction. For instance, judgments such as "the whole is greater than its part" and "the whole is equal to itself" cannot be denied without a contradiction, i.e., their necessity is shown by the principle of contradiction, which Kant calls the supreme principle of all analytic judgments. It does not matter whether the terms or concepts in such a judgment are pure

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[48] B10.

[49] B11.

(non-empirical) or not: in the judgment "All bodies are extended", for example, despite the fact that "bodies" and "extended" are empirical concepts, we have framed an analytic judgment because we can not apprehend the concept of body without also apprehending the concept "through the characters of extension, impenetrability, figure, etc.,"[50] all of which are thought in the concept. We can make this point clearer by considering the construction of such a concept. Consider, e.g., the empirical concept "redness". First of all we must be given representations[51] of those things which are red, then we abstract from them the property of being red and bring it to the analytic unity of consciousness where the concept of redness is obtained. "[C]onsidered as made up of 'marks' every concept has an extension or, perhaps better, denotation."[52] The concept of body, therefore, can be considered as made up of marks (i.e., features) such as extension, impenetrability, figure, substance, etc., which are simply partial representations of the concept 'body'. So when we say "all bodies are extended", we merely "make explicit in the predicate what

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[50] B12.

[51] Representation is "an umbrella word corresponding to Locke's 'idea','whatsoever is present to the mind when it thinks'. Hence it refers indifferently to concepts, thoughts, images, and any kind of apprehension. Feelings are generally excluded." (T. D. Weldon's *Kant's Critique of Pure Reason*, 2nd ed. (Oxford, 1958), p. 124.)

[52] Paton, *Kant's Metaphysic of Experience*, Vol. I, p. 195. (Abbreviation: Paton, I, 195.)

is only implicit in the subject concept."[53] In doing this, we need nothing which goes beyond the concept "bodies" save those concepts or representations which are contained in or under it. On Kant's analysis, "all bodies are extended," then, is analytic, explicative and not empirically informative. Since it is analytic, it is logically necessary on the ground of the principle of analysis. But, in Kant's view, it is quite different in the case of the following synthetic judgment "all bodies have weight", for only bodies which are in interaction with other bodies, are found to develop the property of having weight. In other words, we claim that having weight is not part of the meaning of "body",[54] and that only in experience is weight bound to be "invariably connected with" those characters (such as extension, figure, etc.)[55] which are already conceived in the subject concept "body".

A further point, to which Paton calls our attention, needs to be made here: Although analytic judgments are made

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[53] In Vol. 9 of the Berlin edition of Kant's work, p. 111; English translation in Paton, I, 85.

[54] A. Pap, Semantics & Necessary Truth, p. 30. "And [Kant] would, had he been reared in the language of contemporary analytic philosophers, support this claim by appeal to the fact that the concept of weightless body (unlike that of a body devoid of inertia) is *not self-contradictory*; or that X is a body does not analytically entail (though it may factually imply, i.e. by virtue of an empirical law) X has weight.

[55] B12.

by the analysis of a concept (contrary to the analysis of a thing), they are not judgments about the concept, but judgments about the objects which are supposed to fall under the concept.[56]

In order to make the matter clearer, let us consider the following expression: $(x, y, z) \supset (x)$, where (x, y, z) is taken to be the subject-concept of a judgment, and x, y, and z are taken to be constituent concepts contained in (x, y, z). This expression, I think, reveals a feature of a topical analytic judgment (e.g. "all bodies are extended"). That is, although an analytic judgment is non-informative in character, it nonetheless exercises a function, namely, it contributes something to the clarification of a concept by representing clearly what was thought only obscurely in it. Analytic judgments have sometimes been referred to as "identical" judgments. However, Kant's account does not admit of this characterization. This is clear from what he says in the *Fortschritte*,

> Analytic judgments do, indeed, ground themselves upon identity and can be resolved into it; but they are not identical. For they demand analysis and serve for the explanation of the concept. In identical judgments, on the other hand, *idem* is defined *per idem*, and nothing at all is explained.[57]

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[56] Paton, I, 84, 85 n.1.

[57] In N. K. Smith, *A Commentary to Kant's 'Critique of Pure Reason'*, 2nd ed. (N.Y., 1923), p. 60. (Abbreviation: Smith, *Commentary*, p. 60.)

We are now in a position to draw this discussion together and to state summarily the properties of an analytic judgment. For Kant, they are as follows: 1) The predicate is implicitly contained in the concept of the subject, or in a less metaphorical expression, the predicate *only* represents clearly what was thought obscurely in the concept of the subject. This is considered a definition for analytic judgments. 2) "All analytic judgments depend wholly on the law of contradiction, and are in their nature apriori cognitions."[58] This expresses a necessary and sufficient condition, and a criterion, of analytic truths. 3) It gives no further information which lies outside the subject concept; it is independent of all experience. [This is another way of saying what is said under 1.] 4) It has the function of adjusting and explicating the meaning of a concept. 5) It is made by the analysis of a subject concept, but not of a thing.

Connected with this Kantian view of analytic judgments, there are at least two main criticisms: i) Kant's definition of an analytic judgment is unsatisfactory because it uses "a notion of containment which is left at a metaphorical level",[59] and therefore is too vague. ii) This definition limits itself to judgments of subject-predicate

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[58] *Prolegomena*, § 2 b.

[59] W. V. O. Quine, "Two dogmas of Empiricism", op. cit., p. 21.

form alone. These criticisms would seem to indicate that Kant's definition is not as perfect and as rigorous as it seems. However, we are not prepared to reject Kant's account, for besides the definition, he also provides us with a criterion for analyticity, which is quite free from the above criticisms. This criterion is simply the principle of contradiction which expresses the universal and sufficient condition of all analytic knowledge. "For," Kant says, " *if the judgment is analytic*, whether negative or affirmative, its truth can always be adequately known in accordance with the principle of contradiction."[60] And this has, indeed, been accepted by many contemporary logicians and linguists.

We shall now close the discussion of analyticity by giving a brief outline of Kant's view on synthetic judgments. To apprehend Kant's view on synthetic judgments, we must make reference to the definition and properties Kant has laid down for analytic judgments. But we should note that synthetic judgments, for Kant, are not wholly contrary to analytic judgments, for he does hold that there are synthetic judgments which are also apriori rather than aposteriori. According to Kant, a judgment is synthetic if its predicate lies outside its subject concept, viz., if we have to go beyond the particular subject concept in order

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[60] A151 = B190.

to find the complete extension of the concept.[61] Judgments of this kind may therefore be entitled ampliative, i.e. informative. "Judgments of experience, as such, are one and all synthetic."[62] "All bodies are heavy," for instance, is a judgment of experience because we are required to go beyond the concept 'body' and seek justification in experience. 'Having weight' is really not a feature which is contained in the concept 'body', but when we look back on the experience from which we have derived this concept of body, we find that weight is *invariably connected with* such properties as extension, impenetrability, etc.[63] As a result of this I can attach this concept of having weight to the concept of body synthetically. As Kant says in the *Critique*,

> The possibility of the synthesis of the predicate 'weight' with the concept of 'body' thus rests upon experience. While the one concept is not contained in the other, they yet belong to one another, though only contingently, as parts of a whole, namely, of an experience which is itself a synthetic combination of intuitions.[64]

It might be thought that Kant here shares the same view as

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[61] Vide A6-7 = B10-11.

[62] B11. However, we should notice that not all synthetic judgments are judgments of experience.

[63] A8 = B12.

[64] B12.

the positivists, and many other philosophers as well, that concrete experience is required in framing a synthetic judgment. But this is true for Kant only when the synthetic judgment is aposteriori. And Kant surely holds that we are in possession of synthetic judgments which are, on the one hand, independent of all experience, and on the other hand, are conditions of the possibility of experience, i.e., are presuppositions of all judgments of experience. We shall discuss this more fully in another chapter.

5. A Conclusion

After this examination of Kant's views, we see that his account actually anticipates the latter accounts on analyticity. For although his definition for analyticity is often considered as unsatisfactory, Kant's intent, as Quine has noticed, is "evident more from the use he makes of the notion of analyticity than from his definition of it."[65] It can be restated as follows, (Q): A proposition is analytic if it is true by virtue of meanings (in accordance with the principle of contradiction) and independently of fact. From a logical point of view, a proposition is analytic if it is or can be resolved into a tautology or a logical truth. From a Kantian point of view, all analytic judgments rest wholly on the law of contradiction. There is no fundamental incompatibility between these two views. For, on the one

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[65] Quine, op. cit., p. 21.

hand, although Kant does at times criticize identical judgments (e.g., tautologies with the form "p = p") as being empty and explaining nothing, he does not deny that tautologies are analytic; on the other hand, logicians such as Carnap and others do not, in principle, deny the analyticity of propositions having such forms as $(x, y, z) \supset (x)$, even though they might encounter difficulty in attempting to characterize these propositions.

However, some philosophers such as Quine and others have spoken against the notion of analyticity and therewith also the analytic-synthetic distinction. Basically, Quine himself does not question the analyticity of logical truths, and he also allows that the notion of analyticity could be formally explained in terms of the notion of synonymy. Yet he declares that the notion of synonymy is foggy, mysterious and unclarified, and he says the same things about the notion of analyticity. But as we have already indicated, even though the notion of analyticity has not been adequately clarified, the analytic-synthetic distinction is nevertheless real enough. The terms do, after all, have a use in philosophy.

Even though many difficulties remain, the endeavour of linguists to clarify the notions of synonymy, analyticity, intension, et al., have been fruitful and suggestive. Their contribution may be considered on the one hand as a suggestion to resolve the problems raised by

Quine; and, on the other hand, as an offer to provide a scientific procedure to complement Kant's account of the analytic-synthetic distinction. In a word, if the restatement of Kant's definition for analyticity (Q) is acceptable, what we further require is only a scientific procedure for the determination of meanings (intensions), synonyms, definitions, etc. - this scientific procedure might be possible and attainable, as Carnap has indicated. But as we have shown (in section 2), the attempts to elucidate analyticity directly through *linguistic rules*, would seem untenable. Nonetheless, we would say that while linguistic rules can not determine analyticity alone, they may be of some help in the determination of meanings, synonyms and so on.

Finally, as a result of our discussions in ~~both Chapters III and IV,~~ we conclude that the analytic-synthetic distinction is possible even if the distinction has not been so far very satisfactorily delineated. More precisely, we assert that all propositions are either *analytic* or *synthetic* exclusively, and can be in principle be classified exhaustively into these two categories. Such a conclusion, we should note, is acceptable not only to Kant and his followers, but also to most of the contemporary philosophers, including logical positivists, logical empiricists and others. Among these philosophers, there developed two different theories of the apriori: the one allows only that

analytic propositions are apriori; the other allows also some synthetic propositions, as well as analytic propositions, are apriori. That both of these two theories are correct would be rather paradoxical. We shall therefore attempt, in the following chapter, to find out whether all apriori proposition are analytic or whether there could be some apriori propositions which are synthetic.

CHAPTER V

THE CLASSIFICATION OF PROPOSITIONS

In this chapter, we shall examine the classification of propositions. Our aim will be to try to discover whether there are any propositions which could be synthetic apriori. In order to see whether there are possible synthetic apriori propositions, we have only to show that there are necessary apriori propositions which are not analytic. This is, indeed, a purely negative illustration of the existence of synthetic apriori propositions. In the course of this discussion, we shall give some arguments by which we hope to defeat claims that several necessary apriori propositions are actually analytic.

Let us first consider the following eight propositions:[1]

P1: Either it is raining out or it is not raining out.

P2: Flowers are flowers.

P3: Red flowers are red.

P4: All bachelors are unmarried men.

P5: Some cows are white.

P6: Nothing can be simultaneously red and blue all over.

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[1]These examples, which have been widely discussed, are taken from various texts or essays.

P7: $3 + 2 = 5$

P8: If A is earlier than B and B is earlier than C, A is earlier than C.

According to the previous relevant discussions, we may characterize the nature of each as follows:

P1 is a tautology of the form "pv~p", it is therefore analytic and apriori, for a tautology is true regardless of the truth-values of its components. (See Chapter IV, section 1 for details.) P2 is an identity of the form "p = p", assuming the denotation of the terms is consistent. In P3, the predicate-concept is contained in the subject-concept explicitly; the proposition P3 as such is of the logical form "(x)[Px·Qx⊃Px]". P4 is a tautology, assuming the terms "unmarried man" and "bachelor" are synonymous. All these propositions are analytic and apriori in Kant's sense of these terms; their denial in each case yields a self-contradiction. P5 is obviously a proposition of experience, and in Kant's analysis it is synthetic and aposteriori.

The above classification presupposes that, (1) logical truths are analytic, and (2) the synonym of a term can be determined. And given that our classification of the first five propositions is correct, the following additional propositions should be considered:

(i) All analytic propositions are apriori; and some apriori propositions are analytic;

(ii) All aposteriori propositions are synthetic; and some synthetic propositions are a posteriori;

(iii) A proposition cannot be both analytic and aposteriori.

Although these propositions show nothing contrary to our classification, they have provoked a great deal of controversy, for they leave open the possibility of synthetic apriori propositions. This possibility is in fact what most positivists and empiricists have objected to. As Moritz Schlick, the leader of the Vienna circle of logical positivism, writes, "The empiricism which I represent believes itself to be clear on the point that, as a matter of principle, all propositions are either synthetic a posteriori or tautologous; synthetic a priori propositions seem to me to be a logical impossibility."[2] Reichenbach, too, excludes such a possibility from a logical empiricist point of view: "The method of modern science can be completely accounted for in terms of an empiricism which recognizes only sense perception and the analytic principles of logic as sources of knowledge."[3] Philosophers in both of the above camps conclude that all apriori propositions are

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[2] "Is There a Factual a Priori?" in Feigl and Sellars (ed.), *Readings in Philosophical Analysis.*

[3] "The Philosophical Significance of The Theory of Relativity", in Feigl and Brodbeck (ed.), *Readings in The Philosophy of Science* (N.Y., 1953).

analytic - that is to say, a proposition is either necessary or contingent: if it is contingent, then it must be a-posteriori and synthetic; if it is necessary, then it must be a priori and analytic.

If we want to seriously challenge the above thesis, we have only to see that it is not the case that all necessary (or a priori) propositions are analytic. More precisely, if we can discover at least one necessary (apriori) proposition which is not analytic, we will have established that there can be a proposition that is both necessary and synthetic. Now let us go on to consider propositions P6, P7 and P8, which are commonly accepted as necessary.

P6 reads, "Nothing can be simultaneously red and blue all over." It might be argued that P6 is analytic because (i) it follows from another proposition A; or (ii) it is deducible from logic; or (iii) "nonblue" is part of the meaning of "red", that is, in some sense red excludes blue; or (iv) there is an implicit linguistic rule governing the use of the word "red", and P6 is true by virtue of that rule.

However, none of these arguments appear cogent. Let us consider each of them in turn.

In case (i), the argument runs: if we say "surface x is red at t" and also "surface y is blue at t", we have already implicitly asserted "$x \neq y$"; the formula, call it A,

corresponding to this proposition would be "(x)(y)(t) (Rxt·Byt⊃x≠y)", and from A, P6 follows.[4] While it is true that P6 follows from A, nonetheless since A involves the same situation as P6 it would not be analytic in any of the senses described above. And if A is not analytic, then P6 is not either.

In case (ii), the formula corresponding to P6 would be ~[(Ex)(Et)(Rxt·Bxt)]. This formula is certainly not logically necessary, P6 is not therefore deducible from principles of logic.

In case (iii), if "nonblue" is really part of the meaning of "red", then this means merely that "x is red at t" entails "x is not blue at t",[5] and our concern is just whether such an entailment may be regarded as analytic or not. Since it is generally held that color qualities are unanalyzable, "nonblue" can in no way be ascribed to the meaning of "red" through any so-called analysis of the *concepts* "red" and "blue". The proposition P6 therefore cannot, strictly speaking, be analytic: it tells us something more than what is contained in the concepts. Also, if non-blue is part of the meaning of "red", then so is non-yellow, non-green, etc. This just seems absurd.

In case (iv), the implicit rule in question may be formulated as "~[(Ex)(Et)(Rxt·Bxt)]" (in other words, we

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[4]Vide Pap, op. cit., p. 388.

[5]Ibid., p. 389.

should not apply the words "red" and "blue" to the same surface at the same time), and this is merely a linguistic convention. But if such a rule is merely a linguistic convention (linguistic habit), it must be subject to alterations owing to possible cultural changes, yet the truth expressed by P6 would seem unaffected by cultural changes, and this shows the above view is mistaken. The untenability of such a verbalist thesis has already been discussed in the preceding chapter, it is clear enough that P6 cannot by these means be shown to be analytic.

Let us now turn to proposition P7 in our list. Proposition P7 asserts "3 + 2 = 5", and it is a necessary proposition. If it is taken to be analytic, there are four possibilities:

(1) It follows from some premises or postulates;

(2) It is a tautology;

(3) "3 + 2" is contained in the meaning of "5";

(4) It is true by virtue of implicit or explicit rules for the use of symbols "3", "2", "+", "=" and "5".

Let us discuss each of these possibilities in turn.

(1) Let us now consider one of the most paradigmatic postulate systems from which the entire arithmetic of the natural numbers can be derived. This system,[6]

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[6]See C. G. Hempel, "On the Nature of Mathematical Truth", in Feigl and Sellars (ed.), *Readings in Philosophical Analysis*, pp. 226-7.

which was devised by the Italian mathematician G. Peano, contains three undefined primitives ("0", "Number" and "successor"), five postulates, and some definitions. The first two postulates, among others, are "0 is a number" and "The successor of any number is a number". According to Peano, 1 is defined as the successor of 0, or briefly as 0';[7] 2 as 1', 3 as 2' and so on. And his definition for addition is:

$$D_1 \begin{cases} \text{(a)} \quad n + o = n \\ \\ \text{(b)} \quad n + k' = (n + k)' \end{cases}$$

With these stipulations, we can determine the sum of 3 + 2 as follows:[8]

$$\begin{aligned} 3 + 2 &= 3 + 1' && \text{(the definition of 2)} \\ &= 3 + (0')' && \text{(the definition of 1)} \\ &= (3 + 0')' && (D_1 \text{ (b)}) \\ &= ((3 + 0)')' && (D_1 \text{ (b)}) \\ &= (3')' && (D_1 \text{ (a)}) \\ &= 4' && \text{(the definition of 4)} \\ &= 5 && \text{(the definition of 5)} \end{aligned}$$

The above proof seems to suggest that "3 + 2 = 5" is really analytic, because the latter follows from "the stipulations which have been laid down initially for

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[7] 0' is taken to mean the successor of 0; 1', the successor of 1; 2', the successor of 2; and so on. V. ibid.

[8] See ibid., p. 227.

for the arithmetical concepts."[9] But we think it is worth while, first of all, to examine carefully Peano's definition of addition (a), "n + o = n", where o and n are natural numbers. Here, we realize that "n + o = n" cannot be counted as tautological, since it is not equivalent to either "n + o = n + o" or "n = n", which are of the form "p = p". Moreover, we cannot argue that "n" is already contained in "n + o", because the relation of containment is irreversible, whereas the proposition "n + 0 = n" may be written as "n = n + o" without its truth being affected. Neither can "n + o = n" be said to be analytic in any of the other senses, and it is more plausible that it should be synthetic rather than analytic. In fact, "n + o = n" is for Kant a synthetic truth, and o (zero) expresses only the void of extensive magnitude in intuition. If we then admit that definition D_1(a) is nothing but a synthetic stipulation, the proof of "3 + 2 = 5" would turn out to be a synthetic procedure, for instance, we have to go synthetizing 3 and o into 3 in asserting "((3 +o)')' = (3')'", and so on. Hence the proposition "3 + 2 = 5" has not so far been shown to be an analytic truth, but it could be a synthetic necessary truth; its necessity is not bestowed upon it

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[9] Ibid., p. 227.

by stipulations, but is obtained through its intuitive certainty. This, at any rate, is Kant's view.

(2) "3 + 2 = 5" is not a tautology, because it is not equivalent to "3 + 2 = 3 + 2" or "5 = 5", which do have the form of the tautology "p = p".

(3) "3 + 2" is not contained in the meaning of "5". If someone does maintain that it is, I would certainly ask him whether he thinks "78 - 73", "$5\sqrt{3125}$", "7 x 3 - 16" and all other numerical functions to which 5 is equal are also contained in the meaning of "5", or conversely, whether "5" is contained in the meaning of "78 - 73", or "3 + 2", or "$5\sqrt{3125}$", and so on.[10] Surely, "3 + 2" is not contained in the meaning of "5", nor vice versa, for in framing the proposition "3 + 2 = 5" we need something more than the mere analysis of "3", "+", "2" and "5" - more precisely, in Kant's view, we must also appeal to the apriori construction of concepts in time.

(4) If "3 + 2 = 5" is said to be true by virtue of the rules for the use of symbols, it would involve an embarrassing situation similar to the one considered in case (iv) of P6. For the rules in question are merely conventions, and are subject to alterations. So

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[10] Vide A. C. Ewing, "The Linguistic Theory of A Priori Propositions", in H. D. Lewis (ed.), *Clarity Is Not Enough* (London, 1963), pp. 147-170.

it would be mistaken to hold that "3 + 2 = 5" is true by virtue of rules, insofar as the *truth* expressed by "3 + 2 = 5" would seem unaffected upon changes of these conventions. Other reasons given in the preceding chapter in connection with the criticisms on the verbalist thesis may also be adopted here.

The last proposition to be considered we stated earlier as this: "P8: If A is earlier than B and B is earlier than C, A is earlier than C."

Exactly the same example has been discussed in the preceding chapter, section 2, "The linguistic approach", so here we have only to give a brief sketch of the reasons for holding it is not analytic. These reasons are the following: (i) P8 is not a substitution instance of a logical principle (see Chapter 3, sections 1 and 2), it cannot be counted as logically true in any sense; (ii) The linguistic rule governing the use of "being earlier" cannot explain the supposed analyticity of P8; (iii) Finally, even if it can be deduced from some other premises, this is insufficient to prove its supposed analyticity.

As a result of the foregoing examinations of P6, P7 and P8, we find that they cannot in any obvious sense be counted as analytic, and if they are not analytic, they must be synthetic since this is the only possibility open. On the other hand, they are commonly accepted as necessary and apriori, in the sense that they are not deniable by any

empirical facts or dependent upon them. Thus we have proved there are necessary synthetic propositions, or in other words, there are synthetic apriori propositions. So we may conclude that there are three kinds of propositions possible using the various combinations of apriori, aposteriori, analytic, and synthetic, they are (1) analytic apriori, (2) synthetic a posteriori, and (3) synthetic apriori; the possibility of analytic a posteriori is excluded because all aposteriori propositions are synthetic.

However, the above discussions of the existence of necessary synthetic propositions or synthetic a priori propositions only represent only a negative statement, a positive statement as to their a priori syntheticity is now required. And this leads us to the next chapter, whose subject will be a detailed examination of the possibility of important theoretical synthetic apriori propositions.

CHAPTER VI

KANT'S DOCTRINE OF THEORETICAL SYNTHETIC APRIORI JUDGMENTS

Tracing the discussions of the preceding chapters, we arrive at the conclusion that there might actually exist a kind of knowledge which is both synthetic and apriori. In point of fact, the examination of this knowledge is one of the most important topics in Kant's philosophy. Now in order to elucidate the apriori syntheticity of this knowledge, we shall discuss the relevant portions of Kant's critical philosophy.

One of the major concerns of Kant's critical philosophy is to inquire into the nature and function of synthetic judgments. A fundamental question of the *Critique of Pure Reason* is: How are synthetic apriori judgments possible? With respect to theoretical synthetic apriori judgments, Kant discusses the various possibilities mainly in the sections of the *Critique* titled 'Transcendental Aesthetic' and 'Transcendental Logic'. In 'Transcendental Aesthetic', Kant shows that there are apriori forms of intuition under which all appearances must stand. He then shows how the apriori and synthetic character of mathematical judgments can be explained through these apriori forms.

In 'Transcendental Logic', he shows that there are non-mathematical apriori concepts which are applicable to experience, and how their proper application leads to those synthetic apriori judgments which are presupposed in common sense and scientific thought about nature.[1] In these expositions and deductions, Kant argues for the view that objects are made to conform to concepts instead of vice versa. He attempts to prove this view apodeictically.[2] This view was compared, by Kant, to Copernicus' primary hypothesis[3] in the sense that "Copernicus explained the *apparent* motions of the heavenly bodies as due to the motion of the observer on the earth [,and] Kant similarly explains the *apparent* characteristics of reality as due to the mind of the knower."[4] Objects of knowledge are made to conform to concepts, or in Kant's words, "the [apriori] representation alone must make the object possible."[5] This is a fundamental principle in Kant's epistemology. This representation, which is both synthetic and apriori, can be either of two kinds, namely, either of pure intuition or of pure concept. We shall now consider the former, and see in what

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[1] Vide Körner, *Kant*, p. 43.

[2] BXXII n.

[3] BXVI; BXXXII n.

[4] Paton, I, 75; vide also Smith, *Commentary*, pp. 22-25.

[5] B125.

sense, for Kant, this synthetic apriori representation makes an object possible.

1. Space and Time

Judging and perceiving are, according to Kant, irreducibly different. This is evident from his view that judging and perceiving are functions of two distinct faculties of mind, understanding and sensibility. These two faculties are the two sources of human knowledge; through the first objects are given to us, and through the second they are thought. Kant explains this as follows. "The capacity (receptivity) for receiving representations through the mode in which we are affected by objects, is entitled *sensibility*. Objects are *given* to us by means of sensibility, and it alone yields us intuitions; they are *thought* through the understanding and from the understanding arise *concepts*."[6] The undetermined object of an empirical intuition is called an appearance: the matter of which is sensation, which is the effect of an object upon the faculty of representation; the form of which is pure intuition, which so determines the manifold of appearance that it allows of being ordered in certain relations.[7] The *given*, for Kant, is an appearance, it is the result of the 'interaction' of sensibility and external things, and it must fall under certain

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[6] A19 = B32-33.

[7] Vide A20 = B34.

forms, namely the forms of space and time. These forms, which lie ready for sensations *apriori* in the mind, may also be termed the pure forms of sensibility, or pure intuitions.

In the "Transdendental Aesthetic" Kant says that there are two pure forms of sensible intuition which serve as principles of apriori knowledge, namely, space and time. The former is the pure form of outer sense, while the latter is the pure form of inner sense. They are the necessary forms through which an object can be given, that is to say, whatever can be given must be given in these pure forms of sensibility. The representation of an object, therefore, must have the formal characteristics of space and time. These formal characteristics, however, do not represent any property of things-in-themselves, nor do they represent them in their relation to one another.[8] They are nothing but the subjective conditions of sensibility, under which alone outer and inner intuitions are possible for us. In Kant's view, "space and time, as the *universal* and *necessary* system of relations which is the condition of the particular relations in which appearances are given, must be due to the nature of the mind."[9] The argument is explicitly stated in the "Transcendental Aesthetic". For example, with respect to time, he writes:

> Time is not something which exists of itself, or which inheres in things

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[8] A 26 = B42.

[9] Paton, I, 136.

> as an objective determination, and it does not, therefore, remain when abstraction is made of all subjective conditions of its intuition. Were it self-subsistent, it would be something which would be actual and yet not an actual object. Were it a determination or order inhering in things themselves, it could not precede the objects as their condition, and be known and intuited *a priori* by means of synthetic propositions. But this last is quite possible if time is nothing but the subjective condition under which alone intuition can take place in us. For that being so, this form of inner intuition can be represented prior to the objects, and therefore *a priori*.[10]

The argument with respect to space is formulated in the same way.[11] One more point should, however, be noted, and it is that, while space is only the apriori condition of outer appearances, time is the formal apriori condition of *all appearances whatsoever*. The reason for this is that all representations, either of outer sense or of inner sense, belong to our inner state as determinations of the mind, and must therefore conform to time - the formal condition of inner intuition. Time is only the mediate condition of outer appearances; the immediate condition of outer appearances is space. Since these forms are necessary if there is to be intuitions, and presumably appearances, they have empirical reality and "objective validity in respect of all objects which allow of ever being given to our senses."[12]

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[10] A32-33 = B49.

[11] A26 = B42.

[12] A35 = B52.

That is to say, no object can ever be given to us in experience which does not conform to the condition of time or space or both. Space and time are empirically real, but are also, for Kant, transcendentally ideal; i.e., they are nothing at all, if we cease to regard them as conditions of possible experience. They can not be ascribed to the objects in themselves (apart from their relation to our intuition) in the way either of subsistence or of inherence.[13] In other words, if we examine space and time carefully in the context of Kant's transcendental philosophy, we will find that spatial and temporal characteristics belong only to objects of experience, not to things-in-themselves.[14]

According to Kant, the above idea is fundamental to the understanding of pure mathematics. Kant made this point clearly in the *Prolegomena*. He writes,

> Without such a deduction [i.e., the transcendental deduction of the notions of space and time] and the assumption 'that everything which can be given to our senses (to the external senses in space, to the internal one in time) is intuited by us as it appears to us, not as it is in itself', the truth of pure mathematics may be granted, but its existence could by no means be understood.[15]

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[13] Ibid.

[14] Paton, I, 144.

[15] *Prolegomena*, § 12.

By means of a transcendental deduction or exposition, Kant exhibits the notions of Space and Time as principles from which the possibility of other synthetic apriori knowledge can be understood. The transcendental deduction shows, (1) that such knowledge does really flow from the given notions of space and time; (2) that this knowledge is possible only on the basis of a particular assumption about these notions.[16] Here the assumption is that space and time are apriori intuitions.

In the metaphysical exposition we are shown that space is at once a pure intuition and a necessary form of appearances (or form of intuition). Now provided that we may have an exact science of this pure form of intuition, that science must then not be obtained from experience, but can only be obtained apriori. Further, this science may be universally applied with validity to experience, and so thereby have a general and formal reference to, or description of it. It is therefore also synthetic. Geometry is a science of this kind. As Kant says, "Geometry is a science which determines the properties of space synthetically, and yet apriori."[17] For example, the geometrical judgment that "the straight line between two points is the shortest" is apriori, for it carries with it

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[16] A25 = B40.

[17] B40 = A25.

a necessity, which cannot be derived from experience,[18] and we do not have to measure the actual length between the two points in order to make such a judgment. It is also synthetic, for the concept of the shortest is wholly an addition, and cannot be derived, through any process of analysis, from the concept of a straight line - which contains nothing of quantity but only something of quality.[19] But we have to appeal to intuition, only with whose aid is the synthesis possible. Kant explains this point more fully in *Prolegomena*:

> For mathematics must first present all its concepts in intuition, and pure mathematics in pure intuition; that is, it must construct them. If it proceeded in any other way, it would be impossible to take a single step; for mathematics proceeds, not analytically by dissection of concepts, but synthetically, and if pure intuition be wanting there is nothing in which the matter for synthetic judgments *a priori* can be given.[20]

In other words, a priori construction is necessary in framing judgments of this kind. That is to say, they depend on the "construction of concepts", by which Kant means presenting apriori *in concreto* (i.e. in the single intuition) the intuitions which correspond to the concepts.[21] Besides

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[18] V. A10 = B14.

[19] B16.

[20] *Prolegomena*, § 10 (L.W.B. p. 30).

[21] B741; *Prolegomena*, §7 (L.W.B. p. 28).

these synthetic judgments in geometry, Kant admits that there are also in geometry analytic judgments such as "An equiangular triangle has three equal angles" and "An isosceles triangle has two equal sides", but these serve only as "links in the chain of method" and not as principles.[22] As principles, they are necessarily valid of space, and consequently of anything that might be encountered in space, because space is nothing other than the form of all outer appearances, and it is the form under which alone objects of sense can be given to us.[23] Outer appearances, therefore, can never contain anything, with respect to their form, other than what geometry prescribes for them.

According to Kant, time as the pure form of inner sense has, unlike space, no special science connected with it. Perhaps it is owing to the fact that time is the form of relations between inner states or representations that we can not work out an externalized determination of it. In the *Dissertation*, Kant expresses this basic view:

> Pure mathematics treats of space in geometry and of time in pure mechanics. To these has to be added a certain concept which is in itself intellectual, but which demands for its concrete actualisation the auxiliary notions of time and space (in the successive addition and in the juxtaposition

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[22] B17.

[23] *Prolegomena*, Part 1, Remark 1 (L.W.B. p. 34).

> of a plurality.) This is the concept of number which is dealt with in Arithmetic.[24]

Kant here says explicitly that since arithmetic achieves its concept of number through construction by the successive addition of units in time,[25] it must be based upon the pure intuition of time. Consider the arithmetic proposition 7 + 5 = 12, which is, for Kant, a synthetic apriori judgment. We cannot obtain the concept of 12 by a mere analysis of the concepts of 7, 5 and +, for the concept of the sum of 7 and 5 contains nothing except the union of the two numbers into one, and this suggests no particular single number which combines both. But we may call in the aid of dots or beads or fingers, with the help of which we can construct apriori the numbers in pure intuition, and, by adding one to the other, unit by unit, we arrive at the number 12 which is the sum.[26] In so doing, however, we have to go outside these concepts and exhibit apriori the intuitions which correspond to them. The judgment is therefore apriori and yet synthetic. This will be made clearer when we discuss the schematism and the axioms of intuition.

In addition to its being the basis of Arithmetic science, the pure form of time also makes possible the body of apriori synthetic knowledge which is presented in the

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[24] Translation in N. K. Smith, *Commentary*, p. 128.

[25] *Prolegomena*, §10 (L.W.B. p. 30).

[26] Vide B15-16 and Paton, I, 160.

general theory of motion, where motion, as alteration of place, is possible only through, and in, the representation of time.[27]

Finally, Kant concludes the "Transcendental Aesthetic" by answering the question with which he begins, "how are synthetic apriori judgments possible? His answer is as follows:

> When in *a priori* judgment we seek to go out beyond the given concept, we come in the *a priori* intuitions upon that which cannot be discovered in the concept but which is certainly found *a priori* in the intuition corresponding to the concept, and can be connected with it synthetically. Such judgments, however, thus based on intuition, can never extend beyond objects of the senses; they are valid only for objects of possible experience.[28]

However, since the act of judgment is not a function of sensibility, we cannot frame any judgment by merely having the manifold of pure intuition, for the conceptual element, i.e., the synthetic activity of the understanding, is essential and so far we do not have it.[29] The consideration

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[27] A32 = B48.

[28] B73.

[29] Vide B138: "To know anything in space (for instance, a line) I must draw it, and thus synthetically bring into being a determinate combination of the given manifold, so that the unity of this act is at the same time the unity of consciousness (as in the concept of a line); and it is through this unity of conscious that an object (a determinate space) is first known. The synthetic unity of conscious is ... an objective condition of all knowledge."

of this conceptual element and its principles is therefore also required. We thus come to the following section on pure concepts.

2. Pure Concepts of the Understanding

As we have said above, there is a fundamental principle in Kant's epistemology which states that all objects of knowledge must conform to the apriori representations of reason. This principle tells us that in constituting knowledge, the *formal elements* as well as the material elements are necessary. These formal elements include not only the apriori forms of intuition but also the *pure concepts* of the understanding. By means of sense, objects are given to us; by means of the understanding (from which concepts arise) objects are thought.[30] According to Kant, concepts are of two kinds, namely, aposteriori and apriori. There can be no doubt that the judgments in which we apply these concepts should ultimately refer to intuitions, either directly or indirectly.[31] However, the way in which they refer is different in each case. As aposteriori concepts they have been abstracted from intuitions and so can be used to refer to the objects of intuition. But the question arises, how and to what can apriori concepts refer? Kant provides a

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[30] B33.

[31] V. B33 and B747.

detailed answer to this question in the "Analytic of Concepts." If we are to understand this detailed answer, two points should be noted: (1) Apriori concepts differ from aposteriori concepts insofar as the former only describe intuitions which are given apriori; (2) the reference of apriori concepts to intuitions (while describing them) is a kind of ordering, whereas the reference of aposteriori concepts to intuition is but a mere describing.[32] But what is an apriori or pure concept of the understanding? Are apriori concepts necessary for objective judgments? In answering questions such as these, Kant, first of all, puts forward the following:

> The same function which gives unity to the various representations *in a judgment* also gives unity to the mere synthesis of various representations *in an intuition*; and this unity, in its most general expression, we entitle the pure concept of the understanding. The same understanding, through the same operation by which in concepts, by means of analytical unity, it produced the logical form of a judgment, also introduces a transcendental content into its representations by means of the synthetic unity of the manifold in intuition in general. On this account we are entitled to call these representations pure concepts of the understanding, and to regard them as applying *a priori* to objects...[33]

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[32] Vide Körner, *Kant*, p. 29.

[33] A79 = B104-105.

As Kant has made clear in the above passage, the pure concept of the understanding is a concept of the unity of the synthesis which is necessary to construct any object of experience, and this concept can receive a transcendental content (i.e., a schema) from the synthesis of time and space, with which it is then able to apply apriori to objects. The synthesis in question, we should note, are fundamental for framing knowledge of any kind. By synthesis, Kant understands the act of putting together different representations, and of grasping what is manifold in them in one act of knowledge.[34] Without such a synthesis, the manifold (be it given empirically or apriori) can not be known in any other way. Within this synthesis Kant distinguishes the following elements: (1) the manifold of intuition must first be given, and then, (2) brought into the synthesis of imagination, "a blind but indispensable function of the soul."[35] Then through (3) the application of concepts, a necessary synthetic unity or objective reference of this synthesis is given.[36] Such a synthesis is pure, if the manifold is not empirical and is given apriori. By unifying the manifold of pure intuition through the application of pure concepts, we confer objective reference upon it. These pure concepts, which

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[34] B103 = A77

[35] A78 = B103.

[36] A78-79 = B104.

Kant also calls categories, are twelve in number, and are classified into four groups. Here is Kant's list:

I. Categories of quantity:

(a) Unity;

(b) Plurality;

(c) Totality.

II. Categories of Quality:

(a) Reality;

(b) Negation;

(c) Limitation.

III. Categories of Relation:

(a) Inherence and Subsistence;

(b) Causality and Dependence;

(c) Community.

IV. Categories of Modality:

(a) Possibility-Impossibility;

(b) Existence-Non-existence;

(c) Necessity-Contingency.[37]

These categories, Kant claims, are apriori and objectively valid: although they can never be derived from experience, they are nonetheless necessarily applicable to it. However, in Kant's exposition it is not clear that the application of the Categories is a necessary condition for objective experience. This was quite explicit in the case

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[37] A80 = B108.

of space and time. In the case of the latter, it is only by means of such pure forms of sensibility that an object can appear to us, and so can be an object of empirical intuition; "space and time are pure intuitions which contain *apriori* the condition of the possibility of objects as appearances..."[38] In the case of the former, since there is no immediate relation between appearance and understanding, we may surely intuit something having it conform to the forms of the understanding. That is to say, we can have experience without passing objective empirical judgments, but we can not have objective experience without intuition. For this reason, Kant intended to solve the following problem in the "Transcendental Deduction", namely, how can subjective conditions of thought have objective validity? Or in other words, how can subjective conditions of thought furnish conditions for the possibility of all knowledge of objects?[39] In order to answer this question, Kant presents a directory principle for this whole inquiry which is this: "that [apriori concepts] must be recognized as *a priori* conditions of the possibility of [objective] experience whether of the intuitions which is to be met with in it or

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[38] A89 = B122.

[39] V. A89 = B122.

of the thought."[40] In the light of this principle, we shall proceed to an investigation of the relation between intuition and thinking and also of the way in which objective experiences are constituted.

What can be given through a sensible intuition is nothing but the manifold of representation received without any combination. It is, sans concepts, but "a stream of impressions," or, to use William James' expression, "a big blooming buzzing confusion."[41] This combination, since it is the spontaneous unifying work of the faculty of representation, can never come to us through the mere receptivity of sensibility. "[A]ll combination", says Kant, "- be we conscious of it or not, be it a combination of the manifold of intuition, empirical or non-empirical, or of various concepts - is an act of the understanding. To this act the general title 'synthesis' may be assigned, as indicating that we cannot represent to ourselves anything as combined in the object which we have not ourselves previously combined..."[42] "[T]he concept of combination includes, besides the concept of the manifold and of its synthesis, also the concept of the unity of the manifold. Combination is representation of the *synthetic*

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[40] A94 = B126.

[41] William James, *Some Problems of Philosophy* (New York, 1911), Ch. 4.

[42] B130.

unity of the manifold."[43] But such a synthetic unity is not to be identified with the category of unity, which however, like the other categories, must have its foundation grounded on the former. The synthetic unity of a manifold is therefore a necessary condition for applying categories. Having synthetic unity, the manifold must then be thinkable by the same subject, the 'I' in which it is found. Otherwise, thinking of the manifold would be rendered impossible.

> It must be possible for the 'I think' to accompany all my representations; for otherwise something would be represented in me which could not be thought at all, and that is equivalent to saying that the representation would be impossible, or at least would be nothing to me. That representation which can be given prior to all thought is entitled intuition. All the manifold of intuition has, therefore, a necessary relation to the 'I think' in the same subject in which this manifold is found.[44]

This necessary relation is precisely that the 'I think' must be capable of accompanying all of the manifold of intuition which is found in the 'I'. The representation of this

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[43] Ibid.

[44] B131-2.

relation Kant calls "pure apperception" or "original apperception". The apperception is pure because it is not empirical and contingent. It is also original "because it is that self-consciousness which, while generating the representation 'I think'..., cannot itself be accompanied by any further representation."[45] The unity of this apperception Kant calls the *transcendental unity of apperception* (or of self-consciousness), and it is only through this that apriori knowledge is possible. S. Körner puts Kant's view in more ordinary terms. He writes, "a manifold of [representation] may or may not be an *it* which can carry the burden of properties and relations. (In the latter case there is synthetic unity in the manifold.) There can be no *it* unless there is an *I* which could be aware of *it* and thereby of itself. (The possibility of this relation between *I* and *it* is the pure or original apperception.)"[46] In a word, if a manifold of representation is to be thinkable or to be able to become an object for me, the transcendental unity of apperception must be presupposed. The principle expressing this necessary condition for objective knowledge Kant considers the highest principle in the whole sphere of human knowledge. This principle Kant calls the principle of synthetic unity. It is stated like

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[45] B132.

[46] S. Körner, *Kant*, p. 62. I have put 'representation' for 'presentation' here.

this: "The supreme principle of the same possibility [of all intuition] in its relation to understanding, is that all the manifold of intuition should be subject to conditions of the original synthetic unity of apperception."[47] That is to say, in so far as the manifold of representations must allow of being combined in one consciousness, they are subject to this principle. The reason is simply that without such synthetic unity of apperception, nothing can be thought or known. This synthetic unity of apperception is an objective condition of all knowledge, since it alone constitutes the relation of the manifold (of representations of intuition) to an object or, in other words, that through which alone the former is united in a concept of the latter.[48] Nevertheless, "it is not merely a condition that I myself require in knowing an object, but is a condition under which every intuition must stand in order *to become an object for me.*"[49]

The above stated principle, is, indeed, an analytic proposition, for it asserts nothing but this: "all my representations in any given intuition must be subject to that condition under which alone I can ascribe them to the identical self as my representations, and so can comprehend them as synthetically combined in one apperception through

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[47] B136.

[48] Vide B137.

[49] B138.

the general expression, 'I think'."[50] Nevertheless, it shows the necessity of a synthesis of the manifold given in intuition, without which no human knowledge is possible.

So far, we have only shown that the synthetic unity of apperception is a necessary condition of the experience of an object. But here a question arises, how can we arrive at such a unity? In order to answer this question, Kant first examines judgments. According to him, a judgment is only "the manner in which given modes of knowledge are brought to the objective unity of apperception."[51] For instance, in the judgment 'Bodies are heavy', we only assert that the two representations 'bodies' and 'heavy' belong to one another *in virtue of the necessary unity of apperception* in the synthesis of intuitions, that is they are determined to be combined *in the object* according to principles which are derived from the fundamental principle of the transcendental unity of apperception.[52] Now that which can bring synthetic unity into the manifold of given representations must determine them by means of the logical functions of judgment, and thereby bring them under one pure apperception. And since the *categories* are just these functions of judgment, insofar as they are employed in

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[50] B138.

[51] B141.

[52] B142.

determination of the manifold of a given intuition, we come, at last, to the following important conclusion, "the manifold in a given intuition is necessarily subject to the categories."[53]

3. Schemata

In the preceding section, we have shown that according to Kant, the applicability of the categories to sensible intuitions is a necessary condition of objective experience. But if the categories can only determine the manifold insofar as it is given in an empirical intuition, we have as yet not shown it is possible to have any synthetic apriori knowledge of objects. Now fortunately, the categories are also able to apply to *sensible objects in general* through their transcendental determinations in time. Since time is the formal condition of the connection of all representations, the categories provide us with the necessary characteristics of any object of experience. Thus they enable us to anticipate these characteristics independent of experiences. But such a transcendental determination can not take place unless we call in the necessary unity of pure (productive) synthesis of imagination,[54] the principle of which is "the ground of the

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[53] B143.

[54] Imagination is "the faculty of representing in intuition an object that is *not itself present.*" (B151).

possibility of all knowledge."[55] Sensibility and understanding are two extremes that "must stand in necessary connection with each other through the mediation of this transcendental function of imagination,"[56] only by virtue of which the categories can be brought into relation to sensible intuition.[57] So by its means we can bring the pure intuition of time, through the functions of categories, into the original synthetic unity of apperception. In so doing, we determine apriori the pure intuition of time, and therefore also determine apriori the characteristics of objects of experience in general, for to this formal condition they must be subject. Or more precisely, through the transcendental synthesis of imagination, we determine in general the form (time) of the inner sense with respect to all representations,[58] and the determinations obtained are necessary characteristics of all objects so far as they are known to be combined by the same synthesis in one time.[59]

The notion of such a transcendental determination of time is, in fact, a central idea in Kant's Schematism, which treats of "the sensible condition under which alone

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[55] A118.

[56] A124.

[57] Ibid.

[58] A142 = B181.

[59] Paton, II, 39.

pure concepts of understanding can be employed."[60] By "applying pure concepts of understanding to objects", Kant means subsuming objects under them. Whenever we subsume an object under a concept there must be some homogeneity between the representation of the object and the concept.[61] But the categories, since they and sensible intuitions are heterogeneous, can never be derived from any intuition. So we must search for some third thing which is able to connect the categories and intuitions, and which will make the application of the former to the latter possible. This mediating representation must be pure, intellectual and sensible, and, of course, homogeneous both with the categories and with the appearances.[62] In Kant's view, such a representation is the transcendental determination of time in accordance with rules,[63] which, as the transcendental schema of the categories, first realizes them.[64] This transcendental schema is homogeneous, on the one hand, with the category (inasmuch as it is universal and rests on an apriori rule) and, on the other hand, with appearances in as much as every empirical intuition occurs in time and

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[60] A136 = B175.

[61] A137 = B176.

[62] A138 = B177.

[63] B184.

[64] B186 = A146.

lasts through time.[65] A schema, as a schema of a category, is quite different from the schema of either an aposteriori or a mathematical concept. (Of the latter Kant says, it is a "representation of a universal procedure of imagination in providing an image for a concept."[66]) The schema of a category is simply the rule of the pure synthesis,[67] or perhaps that "specific kind of *a priori combination* which is produced by the synthesis of imagination and is in conformity with the principle of synthesis (or rule of unity) conceived in the category."[68] This schema is a transcendental product of imagination, a product which concerns the determination of inner sense in general with respect to the conditions of its form (time), and with reference to all representations, so far as these must be connected apriori in one concept according to the unity of apperception.[69] This product results from the synthesis conceived in the schematized category, and therefore is a necessary characteristic of all temporal objects.[70] In other words, the schemata may be described as the universal temporal characteristics of sensible objects, namely, that

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[65] B178 = A139; Paton, II, 30.

[66] B180 = A140.

[67] A142 = B181 & Paton, II, 37, 38.

[68] Paton, II, 38.

[69] A142 = B181.

[70] Paton, II, 43.

without which objects would not be objects in time. And as such, they are also described as formal conditions of sensibility,[71] the conditions which the categories must contain apriori in themselves if they are to apply to any object.[72] Thus, while the transcendental schemata make possible the application of categories to sensible objects given under the form of time, they also restrict the employment of the categories.[73] That is, according to Kant, all employment of categories is merely *empirical* (i.e., within the bounds of possible experience) and relates solely to appearances; thus, the transcendental employment of the categories to things-in-themselves is considered illegitimate. If we omit such a restriction the categories will have a purely logical function signifying only the bare unity of representations, corresponding to which, however, there is no given object.[74] More precisely, the categories would, in such a case, be without meaning or content.[75] This will be clearer if we remember Kant's remark that "thoughts without content are empty, intuitions

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[71] V. B186 = A146 and B179 = A140. Cf. also B186, "The schema is, properly, only the phenomenon, or sensible concept of an object in agreement with the category."

[72] A140 = B179.

[73] V. A146 = B186 and A140 = B179.

[74] B186.

[75] B6.

without concepts are blind".[76] Thus if the categories are empty they will never yield knowledge of any objects,[77] unless it be merely the schemata as their "transcendental content."[78]

According to Kant, the schemata of the categories fall into four groups. Those four groups are as follows: quantity, quality, relation and modality. The schemata are apriori time-determinations in accordance with rules, and these relate to (in the above order) (1) the time-series, (2) the time-content, (3) the time-order, and (4) the scope of time in respect of all possible objects.[79] The schema of the category of quantity contains and makes possible the representation of the generation (synthesis) of time itself in the successive apprehension of an object.[80] The schema of the category of quality contains and makes possible the representation of the filling of time.[81] The schema of the category of relation contains and makes possible the representation of the connection of perceptions with one another at all times according to a rule of time-

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[76] B75.

[77] B186.

[78] V. A79.

[79] A145 = B184.

[80] Vide Paton, II, 60 & n.3; A145 = B184, I partly adopt Paton's translation here.

[81] Ibid.

determination.[82] The schema of the category of modality contains and makes possible the representation of time itself as the correlate of the determination whether and how an object belongs to time.[83] All these schemata describe, in various aspects, the sensuous characteristics of objects of experience. The first two determine the features of the object; the third does not describe any features at all but only determines the object in its relation to other objects; the last one concerns nothing about the world of fact but only about its relation to our cognitive faculties.[84] Nevertheless, they all express one and the same function, that is, they mediate the subsumption of the appearance under the categories, and thus render possible the application of categories to appearances. I shall now set down the list[85] of transcendental schemata without any further description.

(1) The schema of the categories of quantity (unity, plurality, totality) is number[86], which Kant also called quantity as a phenomenon.

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[82] Ibid.

[83] Ibid.

[84] Cf. A233-5 = B286-7 and S. Körner, *Kant*, p. 89.

[85] Vide B182-4 and Paton, II, 44-60.

[86] B182: Number is "simply the unity of the synthesis of the manifold of a homogeneous intuition in general."

(2) The schema of the categories of quality (reality, negation, limitation) is degree.[87]

(3) The schemata of the categories of relation include the following:

a) The schema of substance is permanence in time.

b) The schema of causality is necessary succession, or succession of a manifold insofar as it is subject to a rule.[88]

c) The schema of community is the necessary co-existence of the accidents of one substance with those of another, or their coexistence according to a universal rule.

(4) The schemata of the categories of modality are these:

a) The schema of possibility is "the agreement of the synthesis of different representations with the conditions of time in general."[89]

b) The schema of actuality is "existence in some determinate time."[90]

c) The schema of necessity is "existence of an object at all times."[91]

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[87] B183.

[88] Ibid.

[89] B184.

[90] Ibid.

[91] Ibid.

4. The Principles of the Understanding

To the extent that we have discussed the application of a category, we have found that this is legitimate only insofar as it refers to the unity of a manifold combined synthetically in one apperception. Also, the category must contain in itself a sensible concept,[92] or rather, certain sensible conditions of the inner sense,[93] i.e., its schema, which entitles us to apply the category to objects in a synthetic apriori judgement.[94] Such a synthetic *apriori* judgment, which shows how the schematised category must apply to all objects of experience, is called a *principle* of the understanding. This principle is, for Kant, nothing but a rule for the objective use or empirical use of the schematised category.[95] In other words, it is a rule stating a restricting condition which governs its application to objects of experience. In the absence of this rule no objective experience would be possible. "[T]he conditions of the *possibility of experience* in general are likewise conditions of the *possibility of the objects of experience*,... for this reason they have objective validity in a synthetic

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[92] More precisely, a sensible concept "of an object in agreement with the category" (B186 = A146).

[93] B170 = A140.

[94] A148 = B187

[95] V. B200 = A161 and B223 = A180.

apriori judgment."[96] As a principle of the understanding, it must be apriori since it is a necessary condition of experience and so must be logically independent of all experience and all impressions of sense. Yet, it must also be synthetic, since it asserts something about experience, i.e. it asserts that every object of experience must be a substratum of the category, where the latter lies outside the concept of the former.

The synthetic apriori principles are dealt with by Kant in the "Analytic of Principles." They are found to be just those principles which render natural science possible, and this is no mere coincidence. Kant clearly says that "The principles of possible experience are ... at the same time universal laws of nature, which can be known apriori,"[97] where "nature" is taken to mean 'the complex of things as objects of possible experience'[98] or an 'aggregate of appearances.'[99] Nature, as the complex of objects of possible experience or as the aggregate of appearances - but *not* a thing in itself, must be subject to the principles of possible experience. Thus, these principles may be called the universal laws of nature. Since these principles,

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[96] A158 = B197.

[97] *Prolegomena* § 23.

[98] *Prolegomena* § 17.

[99] A114.

according to Kant, must also have their origin in our minds, it is not surprising that Kant says that "nature should direct itself according to our subjective ground of apperception, and should indeed depend upon it in respect of its conformity to law...,"[100] nor that he should say that the order and regularity of nature "we ourselves introduce,"[101] "we ourselves originally set them there."[102] Thus, according to Kant, these principles are universal laws[103] which the understanding *prescribes* to nature.[104] This will become clearer in the discussion that follows.

As we have noted, the principles of pure understanding are simply rules for the objective use of the categories; they relate the categories to possible experience. So on Kant's analysis, these principles fall naturally into four groups which correspond to the table of categories: (1) Axioms of Intuition, (2) Anticipations of perception, (3) Analogies of experience, and (4) Postulates of empirical thought in general. In the above list, the

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[100] A114.

[101] A125.

[102] Ibid.

[103] It should be noted that special laws, as opposed to universal laws, concern only those appearances which are empirically determined, and can not in their character be derived apriori from the understanding. See B165.

[104] *Prolegomena* § 36.

first two are concerned with only the formal aspect[105] of quantity and quality of objects of intuition, while the last two are concerned with the existence[106] of these objects in relation either to one another or to the understanding. The former are called the *Mathematical* principles,[107] which treat the synthesis of the homogeneous manifold where its consituents do not necessarily belong to one another.[108] The latter are called the *dynamical* principles,[109] which treat the synthesis of the manifold, where "its [heterogeneous] constituents *necessarily belong to one another*, as, for example, the accident to some substance, or the effect to the cause."[110] In other words, the mathematical principles deal with the apriori conditions of intuition - the absolutely necessary conditions of any possible experience; the dynamical principles deal only with the conditions of the existence of the objects of a possible empirical intuition.[111] Through these principles

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[105] A161 = B201.

[106] A160 = B199.

[107] A162 = B201.

[108] A162 = B201 n. The synthesis in question can be divided into the synthesis of extensive quantity and the synthesis of intensive quantity.

[109] A162 = B201.

[110] A162 = B201 n.

[111] A160 = B199.

of pure understanding, the special principles of mathematics and dynamics become possible. I shall attempt to discuss these principles in turn as follows.

I. Axioms of Intuition

There is one principle underlying the axioms of intuition. The principle is: "All intuitions are extensive magnitudes."[112] By extensive magnitude Kant means that in which the representation of the parts precedes and renders possible the representation of the whole.[113] Space and time are extensive magnitudes; their parts can only be known one after another.[114] The determination of space or time, or both, however, is that which is contained apriori in the appearance as their form. This is what Kant calls "an intuition in space and time,"[115] that is, the empirical form of the appearance as opposed to its matter given in sensation.

In his attempt to establish this principle he argues as follows: We cannot apprehend any appearance save through the successive synthesis of the homogeneous manifold of the determinate space or time which it

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[112] A162 = B202.

[113] B203 = A162.

[114] V. Paton, II, 113.

[115] A162 = B202.

occupies.[116] Thus, the perception of an object, as appearance, "is only possible through the same synthetic unity of manifold of the given sensible intuition."[117] But this synthetic unity of the homogeneous manifold is just what is thought in the concept of *magnitude*,[118] or more precisely, the category of extensive quantity.[119] And so all objects, as appearances, are necessarily subject to this category; in other words, they are all without exception extensive magnitudes.

Here Kant's primary purpose is to establish the applicability of the categories to objects of experience, or to put it in another way, he is trying to establish that every object of experience must stand under the categories. Yet, by means of the Principle of the Axioms, he also explains why pure mathematics is applicable to experience. In other words, he also explains how pure mathematics can have objective validity. "This transcendental principle of the mathematics of appearances greatly enlarges our *a priori* knowledge," Kant says, "for it alone can make pure

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[116] "The determinate space or time which it occupies" is supposed to be equivalent to the expression "the given sensible intuition" in the following sentence. V. Paton, II, 116 and n.2. V. B202 = A162 and B204 = A163.

[117] A162 = B203.

[118] A162 = B203.

[119] V. Paton, II, 116.

mathematics, in its complete precision, applicable to objects of experience."[120] As we have seen, the synthesis which is necessary for knowledge of objects of experience must involve a synthesis of the determinate space and time. Now the synthesis of spaces and times, as the synthesis of the essential form of all intuition, is that which renders possible at the same time the apprehension of appearance, and consequently every outer experience and all knowledge of physical objects.[121] Thus whatever pure mathematics prove of the synthesis of the form of apprehension is also necessarily valid of the objects apprehended.[122]

II. The Anticipations of Perception

The principle underlying the anticipations of perception is this: "In all appearances, the real that is an object of sensation has intensive magnitude, that is, a degree."[123] Needless to say, degree is degree (or intensive magnitude) of a quality. It is "a magnitude which is apprehended only as unity, and in which multiplicity can be represented only through

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[120] A165 = B206.

[121] A165-6 = B206.

[122] Ibid.

[123] A166 = B207.

approximation to negation = 0."[124] Here, I think, Kant distinguishes an intensive magnitude from an extensive magnitude which is a unity with its multiplicity represented by parts outside one another.[125] With respect to the form of appearances, the objects of experience must have extensive magnitudes; with respect to the matter of appearances, the objects of experience must have intensive magnitudes. Sensation is the sensum considered as a modification of the mind; the real, which corresponds to sensation, is the quality of the object revealed or given in the sensum.[126] In this principle, we are able in an apriori fashion to anticipate something, that is, to have apriori knowledge with respect to the *matter* of experience (which in itself is purely emprical). More simply, the Principle of Anticipations can tell us that all objects of perception, insofar as the perception contains sensation, must have a degree in them corresponding to the degree of our sensation.[127] But how is this possible? First of all, we should recognise that we are examining the nature of the sensation of an object,

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[124] A168 = B210.

[125] Paton, II, 136 n.1.

[126] Paton, II, 137.

[127] A166 = B208.

and also that the matter given in sensations is the real. Now we can represent, in abstraction from the synthesis of extensive magnitude, a synthesis of a sensation from its beginning in pure intuition = 0, up to any given empirical consciousness.[128] Sensation in itself[129] has therefore a magnitude which is *intensive* but not extensive, since there is no intuition of time or space to be met with in it.[130] In other words, it has a degree which is different from the extensive magnitude of an appearance. Correspondingly, we must also attribute an intensive magnitude or degree to all objects of perception, so far as the perception contains sensation.[131]

According to this principle, we can know apriori that (1) all sensations must have degree, and (2) all sensed qualities of objects, such as light, heat, sound, colour, taste, resistance and weight, which correspond to sensations, must also have degree. This explains the applicability of the categories of quality to objects of experience, and also establishes, according to Kant, an apriori basis for the application of pure mathematics to experience.

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[128] V. B208 and B218.

[129] That is, sensation *qua* sensation.

[130] A166 = B208.

[131] Ibid.

III. Analogies of Experience

The general principle that underlies the analogies is this: "Experience is possible only through the representation of a necessary connection of perceptions."[132] The principle is also formulated in the first edition as follows: "All appearances are, as regards their existence, subject *apriori* to rules determining their relation to one another in one time."[133] The "necessary connection" involved in the first formulation refers to the necessary unity of the transcendental synthesis of imagination, wherein the heterogeneous elements of the manifold are connected and related to one object and to each other.[134] As shown in the second formulation, this necessary connection is a necessary connection in time which is concerned with the *existence* of appearances in time. The principles governing such a necessary connection are expressed in the three Analogies. These Analogies are merely regulative,[135] Kant tells us, and are not constitutive principles, which means that they only "tell us what we must look for in experience, but they

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[132] A176 = B218.

[133] A177.

[134] Vide B201 n.

[135] B223 = A180.

do not enable us to construct it apriori."[136] To understand this point, we may examine a mathematical analogy, say, 4:5 = 2:x. This equation, on the one hand, tells us that 4, 5, and 2 stand in a certain quantitative relation to x, and on the other hand, it determines what x actually is. However, in the case of an Analogy of Experience all we have is a rule in our search for such an x,[137] which stands in a certain *qualitative* relation with something in experience. Such rules of necessary connection in time have certain marks, that is, duration, succession and coexistence, which are called the three modes of time.[138] By saying that duration, succession and coexistence are modes of time, Kant does not mean that they are modes of time itself, but rather that they are only relations in time,[139] and are the three fundamental temporal relations of all objects (so far as they are objects in time).[140] The Analogies, indeed, are just

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[136] Paton, II, 179.

[137] Since quality can not be constructed, we are unable to construct such an x, which can only be found in experience. V. Paton, II, 179.

[138] B219.

[139] V. A182 = B226 and A215 = B262.

[140] Vide, Paton, II, 166.

the rules for the synthetic unity of these time-relations of all perceptions or appearances.[141]

(a) The First Analogy, the principle of permanence of substance, is this: "In all change of appearances, substance is permanent; its quantum in nature is neither increased nor diminished."[142] Or expressed in another way, "All change (succession) of appearances is merely alteration (of substance)."[143] Time, Kant holds, is something permanent; the permanence of time is the condition of determining succession and co-existence. However, time itself cannot be perceived. Consequently the permanent, which is necessary for determining such time-relations, must be found in the appearances as their substance.[144] This substratum or substance is something abiding and permanent that represents or expresses the abiding unperceivable time. To say there is change or alteration is to say there is something that alters: that there is,

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[141] V. B220 = A177-8: "This synthetic unity in the time-relations of all perceptions, as thus determined apriori, is the law, that all empirical time-determination must stand under rules of universal time-determination. The analogies of experience...must be rules of this description."

[142] A182 = B224.

[143] B233.

[144] B224-225.

in other words, something which is altered but which *persists*, and is only changed in its states.[145] The existence of substance is therefore a necessary presupposition of all our experience of change and coexistence, whereas the latter are merely the ways in which the permanent exists.[146] Without such a permanent substratum, we cannot have any objective time-determinations of appearances, and so we cannot have any objective time-relations which could be distinguished from merely subjective or imaginary time-relations.[147] In other words, experience would lose its unity in the absence of the category of substance, and no distinction between objective change and subjective succession of representations would be possible.[148] The permanent substratum or substance is, therefore, the ultimate condition of determining any kind of objective time-relation, whether of succession or of coexistence; it is, as it were, "a necessary condition under which alone appearances are

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[145] B230 = A187.

[146] B226.

[147] Vide Paton, II, 198.

[148] Paton, II, 206.

determinable as things or objects in a possible experience."[149]

(b) The Second Analogy, the principle of succession in time in accordance with the law of causality is: "All alterations take place in conformity with the law of the connection of cause and effect."[150] The concept of substance allows existence in different states. Alteration is the way in which these various states follow one another.[151] Such a succession of states of existence, according to this principle, must occur in accordance with the law of causality. That is to say, everything that happens (or begins to be) presupposes something upon which it follows according to a rule.[152] Kant's point is that the distinction between the *objective* succession of events in the phenomenal world and the *subjective* succession of our representation is impossible unless we regard the objective succession as necessarily determined in agreement with the law of cause and effect.

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[149] B232 = A189.

[150] B232.

[151] B230 = A187.

[152] This is the formulation of the principle in the first edition (A189).

In looking at a house, for instance, our perceptions could begin with the apprehension of the top and finish with the bottom, or begin from below and end above. In the series of perceptions, the order is reversible and is subjectively but not objectively determined.[153] The reason for this is simply that there is nothing happening to the house which makes the perceptions follow upon one another in a necessary order according to a rule. But if, on the other hand, I see a ship moving down a stream, my perception of its lower position must follow upon my perception of its position higher up in the stream, but not vice versa.[154] The succession involved here is objectively necessary and irreversible according to a rule. With respect to this rule, Kant's words run as follows:

> This rule, by which we determine something according to succession of time, is, that the condition under which an event invariably and necessarily follows is to be found in what precedes the event. The principle of sufficient reason is thus the ground of possible experience, that is, of objective knowledge of appearances in respect of their relation in the order of time.[155]

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[153] V. B237 ff.

[154] B237 = A192.

[155] B246 = A200-1.

This principle of causality is certainly not an empirical principle which is derived by generalisation from experience; it is derived from the nature of time as such and the necessary unity of apperception.[156] Without such a principle, objective empirical judgements would not be possible nor would we have any coherent objective experiences.[157]

(c) The Third Analogy, the principle of coexistence in accordance with the law of reciprocity or community, is this: "All substances, insofar as they can be perceived to coexist in space, are in thorough going reciprocity."[158] In other words, all coexistent things stand in a reciprocal relationship. By coexistence of things, Kant means that they exist in one and the same time, and the perceptions of them in empirical intuition can follow upon one another reciprocally.[159] That is to say, in our apprehension of things, the order in the synthesis is a matter of indifference,

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[156] Paton, II, 274.

[157] Vide B247 = A202: "the relation of cause to effect...is the condition of the objective validity of our empirical judgements, in respect of the series of perception...it is the condition of experience."

[158] A211 = B256.

[159] B257 and B258.

and we can either start from A through B, C, D, to E, or in the reverse order from E to A.[160] Since such a succession is reversible, this simply means that there is not any causal relation among these elements or things. On the other hand, if we judge the reciprocal sequence of the perceptions of these things to be objective, then we have already presupposed that they must have reciprocal interaction,[161] or, more precisely, that they must act on one another and receive reciprocal influences in return.[162] For without interaction, each perception of an appearance in space would be isolated from every other, and "the chain of empirical representations, that is, experience, would have to begin entirely anew with each new object."[163] This thorough going community of mutual interaction is, therefore, a necessary condition of our knowledge of objective coexistence.[164]

The analogies of experience, as we have seen, are principles for the synthesis of the manifold with

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[160] B258.

[161] B258.

[162] A212 = B258.

[163] B260.

[164] Vide B260 and B257.

respect to time. They express the conditions of experience, and it is through these conditions that experience is possible. Furthermore, they "portray the unity of nature in the connection of all appearances under certain exponents..."[165] For nature is simply "the order and regularity in the appearances"[166] or, more precisely, "the connection of appearances as regards their existence according to necessary rules."[167] Therefore it must be subject to the analogies. The analogies are merely the rules governing the necessary connection of appearances.

IV. The Postulates of Empirical Thought in General

The three postulates are formulated as follows:[168]

(1) That which agrees with the formal conditions of experience, that is, with the conditions of intuition and of concepts, is *possible*.

(2) That which is bound up with the material conditions of experience, that is, with sensation, is *actual*.

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[165] A216 = B263.

[166] A125.

[167] A216 = B263.

[168] B265 = 266.

(3) That which in its connection with the actual is determined in accordance with universal conditions of experience, is (that is, exist as) *necessary*.

As the above statements make clear the postulates are concerned with the three categories of modality, namely, possibility, actuality, and necessity. But more precisely, they are concerned with, possible, actual, and necessary *existence*,[169] or with the possibility, actuality, and necessity of *things*, but not of thought.[170] These principles are the restricting conditions for all categories: they are nothing but "explanations of the concepts of possibility, actuality, and necessity, *in their empirical employment*; at the same time they restrict all categories to their merely empirical employment, and do not approve or allow their transcendental employment."[171]

The first postulate sets the conditions under which the existence of something can be considered as possible. That is, if things are to be possible, the

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[169] Vide the third postulate.

[170] Vide B267 = A219.

[171] A219 = B266 (emphasis added). As we have mentioned, an empirical employment is an employment within the bounds of possible experience, while on the other hand a transcendental employment is an employment which transcends all possible experience.

concept of these things should agree with the formal conditions of experience in general.[172] These formal conditions of experience in general, however, should be taken as *all* the formal conditions of experience, which includes not only the conditions of space and time, but also the conditions of the categories. Thus, a thing could be entitled possible only when the concept of it has objective reality, i.e., the synthesis of the concept belongs to experience, either as being derived from it, or as an apriori condition upon which experience in general in its formal aspect rests.[173] The possibility involved here is not a logical possibility, but rather a real or material possibility. For example, a Leibnizian monad (or a reality in Bradley's sense), is logically possible because the concept of it is free from self-contradiction. Yet, it is impossible in Kant's sense because the concept of it violates not only the formal conditions of intuition, but also violates the second and third analogies of experience - insofar as a Leibnizian monad is defined as a spiritual substance which, on the one hand, does not locate in space and time, and on the other hand, does not stand in any causal or reciprocal relationship with

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[172] A220 = B267.

[173] A220. In the former case of the two references to experience, the concept is empirical, in the latter case, pure.

other substances. It is possible, on Kant's analysis, only when the concept of it agrees with all the formal conditions of experience. "We can", says Kant, "indeed, prior to experience itself, know and characterise the possibility of things, merely by reference to the formal conditions under which in experience anything whatsoever is determined as object, and therefore can do so completely a priori. But, even so, *this is possible only in relation to experience and within its limits.*"[174]

The second postulate tells us that the mere concept of a thing does not establish that the concept has application to an actual thing.[175] The existence of its instance can only be found in perception, which supplies the content of the concept.[176] We therefore have to start from perception, if we are to assert something about the existence of a thing. But we do not mean that only that which is immediately sensed is actual, for we can say that a thing is actual if it is connected with some actual perception in accordance with the analogies of experience.[177] Hence we do not

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[174] B272 = A224. (Emphasis added.)

[175] Cf. A225 = B272.

[176] B273 = A225.

[177] A225 = B272.

restrict existence to the present, but also ascribe existence to the cause, just as we make such ascription to the effect. "Thus from the perception of the attracted iron filings we know of the existence of a magnetic matter pervading all bodies," Kant tells us, "although the constitution of our organs cuts us off from all immediate perception of this medium."[178] And surely, we infer the existence of fire, in the like manner, from the perception of a house which has burned down. However, existence or actuality should not, in any case, be ascribed to things-in-themselves except as things in the world of appearances, since "our knowledge of the existence of things reaches...only so far as perception and its advance according to empirical laws can extend",[179] that is to say, our knowledge cannot reach anything outside the advance of our experiences.

The third postulate is concerned with the necessity of the existence of objects. The necessity involved here is not a logical necessity; it is, as it

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[178] A226 = B273.

[179] A226 = B273, vide also A493 = B521: "The objects of experience, then, are *never* given *in themselves*, but only in experience and have no existence outside it... For everything is real which stands in connection with a perception in accordance with the law of empirical advance."

were, a material or causal necessity.[180] This necessity (of existence) can never be known merely from concepts. The knowledge of it rests wholly on the connection with the actual, a connection which must be in accordance with the universal conditions of experience.[181] These universal conditions of experience, however, include not only the apriori conditions of experience but also the laws of causality, i.e., the empirical laws of nature.[182] Thus something that occurs according to the empirical laws (those apriori conditions are, of course, presupposed) may be entitled necessary. For instance, if we see a stream of water flowing downwards, this is only a sign of something actual. But if, further, we can establish that the stream of water is flowing downwards as a consequence of the very existence of the gravity of the earth, then we have established that the stream of down-flowing water is something necessary. It is necessary because it is determined as the effect of a certain cause in accordance with empirical laws of causality. In a word, it is necessary *qua* effect of something else.[183] The necessity in question is

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[180] B279 = A226.

[181] B279 = A227.

[182] V. A227 = B279-80.

[183] V. ibid.

therefore also conditioned and hypothetical.[184] Thus Kant says, "Necessity concerns only the relations of appearances in conformity with the dynamical laws of causality...";[185] we can understand it only through the concepts of effect and causality. The criteria of necessity, therefore, lies solely in "the law that everything which happens is determined *a priori* through its cause in the [field of] appearance"[186] (the Second Analogy of experience). Hence, the criterion applies only to the world of appearances since it cannot extend any further than the field of possible experience.

5. Conclusion

As a result of our examination of the Principles we can now see that they relate the categories to the possibility of objective experience, and moreover that they are simply rules for the legitimate, empirical, and objective employment of categories. By means of the application of categories, appearances are brought under one pure apperception which, in turn, constitutes the objective experience of appearances. Needless to say, the application of a category is legitimate and objective only if it connects with the synthetic unity of a manifold of intuition, and provided that the category in question is not

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[184] V. A228 = B280.

[185] Ibid.

[186] B280 = A227.

a bare one, but necessarily a schematized one. That is to say, it must also contain in itself a certain sensible temporal condition (viz., schema), without which no object can be perceived and thought at the same time.[187] Such schemata, together with the principles, restrict all categories to their merely empirical employment, and allow no transcendental employment of them; hence the principles render possible their employment only in relation to experience and within its limits, but do not allow them to be ascribed to any other thing save those which occur in the world of appearances.

A thing may be entitled possible, if it agrees with the formal conditions of experience, that is, with the conditions of intuition and of concepts.[188] With respect to the possibility of intuition, Kant writes:

> The supreme principle of the possibility of all intuition in its relation to sensibility is, according to the Transcendental Aesthetic, that all the manifold of intuition should be subject to the formal conditions of space and time. The supreme principle of the same possibility, in its relation to understanding, is that all the manifold of intuition should be subject to conditions of the original synthetic unity of apperception. Insofar as the manifold representations of intuition are *given* to us, they are subject to the former of these two principles; insofar as they must allow of being

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[187] S. Körner, *Kant*, p.76.

[188] Vide B265.

> *combined* in one consciousness, they are subject to the latter.[189]

These two principles are significant, because "all our knowledge must relate, finally, to possible intuitions, for it is through them alone that an object can be given."[190] Accordingly, if we do transcend such limitations and go out beyond the field of possible experience, no knowledge with objective reality could be obtained.

In the case of synthetic apriori knowledge, indeed, we find that for Kant the apriori concepts involved also refer to intuitions; these concepts are of two kinds, that is, "either already includes in itself a pure intuition... or...includes nothing but the synthesis of possible intuitions which are not given apriori."[191] It is by virtue of these apriori concepts that synthetic apriori knowledge be constituted either intuitively through the construction of concept, or discursively in accordance with concepts.[192] Whichever is the case, the synthetic unity of consciousness is necessary, since it is an objective condition of all knowledge[193] and the necessary ground for judgment.[194]

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[189] B136.

[190] A719 = B747.

[191] A719 = B747.

[192] Vide ibid.

[193] Vide B137-8 and B197 = A158.

[194] Vide, e.g., B143.

For Kant, the possibility of synthetic apriori knowledge rests wholly on the conception that *the judgments and principles, as conditions of the possibility of experience given by our cognitive faculty, are synthetic apriori.* In order to explain this conception, Kant exhibits analysis and criticisms of theoretical reason. The outcome of such analyses and criticisms tells us that the attainment of objective knowledge must follow the procedure stated below: first of all, the manifold of intuition must be given, then, such a manifold of intuition is brought into a synthesis through the function of imagination, and finally, it is brought into the state of objective unity through the application of categories which is the function of understanding.[195] Kant reminds us that it is through this last function (i.e., the function of bringing the synthesis of imagination to concepts) of the understanding that we "first obtain knowledge properly so called."[196]

Within the process of the constitution of knowledge, the apriori elements provided by the cognitive faculty are fundamentally the formal conditions of space and time, and the conditions of the original synthetic unity of apperception or consciousness, viz., the pure concepts of the understanding, or, in other words, the categories. Besides these, the apriori elements include also the trans-

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[195] Vide, e.g., A77-79 = B102-104.

[196] A78 = B103.

cendental schemata of imagination (that is, the transcendental determinations of time that render possible the application of categories to sensible objects in general) and the apriori principles of understanding. All of these elements taken together are necessary conditions for the objective employment of categories. Since these apriori elements are just the conditions of the possibility of objective experience, they are then said to provide us with a system of apriori determinations of objects in general through the determination of possible experience. These apriori determinations of objects in general are in fact necessary conditions of objects as objects of our experience. In other words, if such and such a thing does not agree with such apriori determinations, it can never become an object of our experience. Geometry, for instance, is a science of the apriori determinations of space, and since space is the apriori form in which alone all outer objects of experience can be given, geometry therefore contains in itself a system of apriori determinations of spatial relations that expresses the necessary conditions of outer appearances. So we may say, it is only because we have the apriori determinations of spatial relations, e.g. of triangle, that we can intuit triangular bodies in space. Thus, geometrical judgments are known to have objective validity, not merely from the character of the concepts involved, but because they express the formal conditions of

experience. The same applies with respect to all other theoretical synthetic apriori judgments.

In short, theoretical synthetic apriori judgments are nothing but statements about conditions we ourselves require in knowing an object, while at the same time conditions under which every intuition must stand in order to become an object for us.[197] They are certainly apriori, because they are necessary and strictly universal, and also logically independent of all experience and sense-impressions. But in what sense are they synthetic?

So far as we are concerned, we find that there are several senses in which these judgments may be entitled synthetic. For instance, we may call a synthetic apriori judgment synthetic in the sense that its predicate lies outside its subject concept, and requires that we go beyond that particular subject concept in order to find extension as bound up with it. We may also call a synthetic apriori judgment synthetic in the sense that it is universally and necessarily applicable to experience since it expresses nothing but a formal condition under which appearances must stand. Perhaps it may also be called synthetic in the sense that it is necessarily connected with possible experience, or that it is simply a condition of possible experience. In any case, however, its syntheticity is

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[197] Cf. B138.

explained through its reference to experience, and this very fact again explains its having merely empirical reality instead of any transcendental reality.

These conclusions follow from Kant's enquiry into the sources, conditions, scope, and limits of our knowledge. And as we should notice, the whole *Critique of Pure Reason* is in fact an analysis of our experience into its formal and material elements.[198] Kant's analysis is remarkable because it shows that, in constituting knowledge, not only the material elements, but also the formal elements, are necessary. That is to say, mere sensations or perceptions constitute no knowledge, but can do so only when they are subsumed under the formal conditions of sensibility, of imagination, and of understanding. Here, it must also be noted, it is just those synthetic apriori judgments which express and govern all these formal conditions of objective cognitions, and which render possible all knowledge within the world of human experience. Kant also puts stress on "experience" when he begins his introduction to the *Critique*. He says, "all our knowledge begins with experience."[199] By this, however, he does not mean that all our knowledge arises out of experience, but only means that without such a starting point in experience, we could never gain any empirical knowledge of objects, and consequently no

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[198] Vide Paton, I, 138.

[199] B1.

apriori knowledge would be possible either, since they are merely restricting principles of the former. Indeed, this idea has echoes throughout the *Critique*, and we can find many similar descriptions such as: "all our knowledge relates finally to possible intuitions;"[200] "intuition is that through which knowledge is in immediate relation to objects, and to which all thought is directed;"[201] "that an object be given, means simply that the representation through which the object is thought relates to actual or possible experience;"[202] *et al.*

The aim of this chapter has been to recapitulate Kant's account of theoretical synthetic apriori knowledge represented in his *Critique* and *Prolegomena*. In point of fact, we are in fundamental agreement with these Kantian theses, and shall try, in the following chapter, to show that it is possible to meet objections which have been brought against them. However, our survey of such objections will be restricted to those which seem to be typical and significant.

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[200] Vide A719 = B147.

[201] Vide A19 = B33.

[202] Vide A156 = B195.

CHAPTER VII

CONTROVERSIES CONCERNING THE POSSIBILITY OF THEORETICAL SYNTHETIC APRIORI KNOWLEDGE

In the last chapter we have discussed Kant's elaborate justification of theoretical synthetic apriori judgments. This is a dominant theme of the *Critique of Pure Reason.* But doubts concerning the possibility of synthetic apriori judgments (and knowledge) have not been laid to rest. These doubts have provoked endless controversies since Kant's death. Kant's doctrine is now confronted by various challenges from philosophy, physics, and mathematics. In the realm of mathematics, the development of non-Euclidean geometries and Russell's analysis of mathematics, seem to suggest that mathematics and geometry are only analytic but not synthetic. On the other hand, in the realm of physics, the discovery of principles such as the principle of the conservation of mass-energy and the principle of indeterminacy seem to suggest that Kant's principles are only synthetic but not apriori. Furthermore, philosophers in such camps as those of Logical Positivism and Empiricism have claimed that in the realm of human knowledge, no synthetic apriori cognition is possible at all. These challenges to Kant's doctrine, we shall argue,

are unjustified. In the first section we shall examine Reichenbach's criticisms together with other objections from physics; in the second section we shall consider objections from geometry; and in the final section we shall confine ourselves to mathematics in its narrower sense, and consider whether it is in fact analytic or not.

1. Scientific Aspect (natural science)

Among those who speak against the possibility of synthetic apriori knowledge, Hans Reichenbach, a scientific empiricist, is one of the most significant. He describes the development of mathematics and physics, in the nineteenth and the twentieth centuries, as the disintegration of synthetic apriori knowledge. He tries to prove, by virtue of the outcome of modern physics and of non-Euclidean geometries, that there is no synthetic a priori knowledge, and that all human knowledge is either synthetic a posteriori or analytic a priori. We shall first confine our discussion to the objections he connects with natural science.

Reichenbach's basic views about traditional philosophy are expressed in his essay "Rationalism and Empiricism: An inquiry into the roots of philosophical error".[1] There he points out that the errors of traditional philosophy involve a misconception of the

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[1] In *The Philosophical Review*, July (1948).

nature of human knowledge. This misconception is a result of regarding mathematical knowledge as the prototype of all knowledge. There are two origins of this misconception: The first is the result of dependence on the development of science, the second is the extra-logical motives of intending to establish certainty for human knowledge. After advancing these claims Reichenbach suggests the following means for ridding ourselves of this misconception: "Since philosophy is dependent on science, we should make this dependence the conscious condition of our work; we should know that the nature of knowledge can be studied only through analysis of science. The idea of a philosophical theory of knowledge that derives the general outlines of knowledge from the structure of the mind, or from an insight into the nature of being, should forever be abandoned."[2] This statement, indeed, represents a typical positivist's view concerning the relation between philosophy and science. The goal of modern science, he insists, is only inductive reliability and not certainty.[3] The search for certainty is one of the most dangerous sources of error so far as it is usually connected with the claim to have a superior knowledge.[4]

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[2]Loc. cit.

[3]Hans Reichenbach, *The Rise of Scientific Philosophy* (Berkeley, 1951), p. 30.

[4]Ibid., p. 37.

With respect to Kant's synthetic a priori, Reichenbach writes,

> Kant mobilizes the science of his day for the proof that certainty is attainable; and he claims that the philosopher's dream of certainty is borne out by the results of science.. .. He regarded the physics of Newton as the ultimate stage of knowledge of nature and idealized it into a philosophical system.[5]

He continues,

> Had Kant lived to see the physics and mathematics of our day he might very well have abandoned the philosophy of the synthetic a priori. So let us regard his books as documents of their time, as the attempts to appease his hunger for certainty by his belief in the physics of Newton. In fact, Kant's philosophical system must be conceived as an ideological superstructure erected on the foundation of a physics modelled for an absolute space, an absolute time, and an absolute determinism of nature.[6]

As we know, it is true that Kant does take the results of scientific investigations seriously, however, the account of Kant's views expressed above cannot be correct. Surely Kant never regarded the physics of Newton as the paradigm of certainty, nor the ultimate stage of knowledge of nature. As Kant himself said in the Antinomy of Pure Reason,

> In natural science, on the other hand, there is endless conjecture,

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[5]Ibid., p. 42.

[6]Ibid., p. 44.

> and certainty is not to be counted upon. For the natural appearances are objects which are given to us independently of our concepts, and the key to them lies not in us and our pure thinking, but outside us; and therefore in many cases, since the key is not to be found, an assured solution is not to be expected.[7]

This passage seems to show that Reichenbach's account of Kant is not the whole story, and might even be mistaken. Again, similar views are found in Reichenbach's account of Kant's transcendental deduction.

> [Kant] argues that mere observation does not supply experience, that observations must be ordered and organized before they can become knowledge. The organization of knowledge, according to him, is dependent on the use of certain principles, such as the axioms of geometry and the principles of causality and the conservation of mass, which are innate in the human mind and which we employ as regulative principles in the construction of science. They are, so he concludes, necessarily valid because without them science would be impossible. He calls this proof the transcendental deduction of the synthetic a priori.[8]

In the *first* place, Reichenbach employs the term "innate" to describe synthetic a priori principles, which is certainly not appropriate.[9] In the *second* place, he only

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[7] A480 = B508.

[8] Reichenbach, ibid, p. 45.

[9] Please vide Ch. 2 "The Meaning of 'Apriori'." We are quite sure that Reichenbach is using the word in Descartes' sense.

mentions principles but never categories. However, the transcendental deduction, according to Kant, is the demonstration of the objective and necessary applicability of the *categories* to objects.[10] In the *third* place, the demonstration of the validity of axioms of geometry is dealt with only in the transcendental aesthetic, not in the deduction. And the principle of conservation of mass is not a principle of the understanding. In the *fourth* place, axioms of geometry are not regulative principles, and even the principle underlying the axioms of intuition (the principle that asserts all intuitions are extensive magnitudes) is not a regulative, but a constitutive principle. From the above, we see that Reichenbach's interpretation of Kant is, at least, seriously inaccurate if not totally mistaken. Consider further Reichenbach's attempt to explain the synthetic a priori using the analogy of the blue spectacles.[11] That is to say, employing the synthetic apriori principles is taken to be analogous to wearing blue glasses. We are born with these glasses on, and only through them do we see the world and acquire experience.[12] Of this analogy Reichenbach says,

> But how does he know that experience will always be possible? Kant had no

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[10] Vide, e.g., A85 = B117.

[11] Reichenbach, ibid, p. 45.

[12] This analogy will be of some help if we bear in mind that all human beings have one and the same kind of glasses, and which can not be changed.

> proof that we would never arrive at a totality of observations which could not be ordered in the frame of his *a priori* principles and which would make experience impossible, at least experience in the Kantian sense. In the language of our illustration this case would occur if the physical world contained no light rays of the wave length corresponding to blue; the man with the blue glasses would then see nothing. If the corresponding case were to occur in science, if experience of the Kantian kind should become impossible, Kant's principles would be shown to be invalid for the physical world. And because of the possibility of such a disproof, the principles cannot be called *a priori*. The postulate that experience in the frame of the *a priori* principles must always be possible is the unwarranted assumption of Kant's system, is the undemonstrable premise on which his system hinges.[13]

In the above passage, we find Reichenbach arguing from the possibility of experience of a kind which contradicts the apriori principles, to a disproof of the validity of these principles. But the argument he presents is not sound, since the possibility of an all-embracing experience incomprehensible to the human mind is not really contrary to Kant's doctrine, and so one cannot then pass logically from it to the conclusion advanced. On the other hand, Reichenbach's belief, that Kant had no proof that we would never arrive at a totality of observations which contradicts the apriori principles, is proved to be

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[13]Reichenbach, ibid, pp. 47-8.

erroneous, if we check it in the light of the outcome of the transcendental deduction. For the transcendental deduction only establishes the following thesis:[14] the supreme principle of the possibility of all intuition, in its relation to sensibility, is that all the manifold of intuition should be subject to the formal conditions of space and time; the supreme principle of the same possibility, in its relation to understanding, is that all the manifold of intuition should be subject to conditions of the original synthetic unity of apperception, and consequently to the categories. In a word, all intuition is possible only through the conditions of sensibility and understanding, which of course includes those apriori principles since they are the universal rules of unity in the synthesis.[15] Since this is the case, there would not be any empirical observation that can exist apart from the synthetic unity of appearances, the synthesis according to the synthetic apriori principles of understanding. And as a result, we could never arrive at a totality of observations which contradict the apriori principles. Here, we are merely taking the totality to be that of the world of appearances (we think that Reichenbach means only this), and if it is to be taken as an *absolute* totality, the concept would also be regarded as empirically impossible,

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[14] V. B136 and B143.

[15] Vide B196 = A157.

according to Kant's Antinomy of Pure Reason.[16] There is no need to discuss this point any further.

Even granted that there is such a totality of observations that could not be fitted into the frame of the apriori principles that would render experience in the Kantian sense impossible, we cannot, on this basis, deny these principles. For experience, in the Kantian sense, involves an empirical determination of an object through perception;[17] that is to say, it involves two factors, namely, matter and form. The matter is given by sensibility, and the form is imposed by both pure intuition and pure understanding; without all of these elements there is nothing that could be called human experience. "Even, therefore, with the aid of [pure] intuition," says Kant, "the categories do not afford us any knowledge of things; they do so only through their possible application to *empirical intuition.* In other words, they serve only for the possibility of *empirical knowledge;* and such knowledge is what we entitle experience."[18] Experience depends therefore upon apriori principles for its form, and there is no experience apart from these principles. Such a totality of observations lies, therefore, outside the world of experience if it could not be fitted into this frame and

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[16]Vide A408 = B435 ff.

[17]Vide B218 = A176.

[18]B147.

thereby obtain its form. Hence, we cannot argue from such a possibility to the denial of these principles.

Reichenbach next argues the corresponding case in science. He says, the man with the blue glasses would see nothing if the physical world contained no light rays of the wave-length of blue. This is quite true, for what this amounts to is that we cannot have any experience if there is no object given through perception. But if we thereupon conclude that the apriori principles would then be shown to be invalid for the physical world, we have indeed misunderstood Kant's position. For Kant repeatedly emphasizes that these synthetic apriori principles have only conditioned but not absolute validity.[19] There is an explicit relation of interdependence between experience and their objective validity. To use Kant's own words:

> Their objective reality, as necessary conditions of experience, and indeed of its very possibility, can always be shown in experience. Apart from this relation synthetic *a priori* principles are completely impossible. For they have then no third something, that is, no object, in which the synthetic unity can exhibit the objective reality of its concepts.[20]

As this passage makes clear, the principles cannot be regarded as holding universally, holding even when there is

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[19] That is to say, they are restricted to their merely empirical employment, and in which only they show their objective reality.

[20] A187 = B196.

no object given through perception. They nonetheless hold conditionally, that is they hold within the field of appearances.[21] So what Reichenbach thinks to have proved is only based on a misinterpretation of Kant, and this makes his argument against Kant's position untenable.

Reichenbach's question of how Kant knows that experience will always be possible has a presupposition: it presupposes that Kant does assert that experience will always be possible. But as a matter of fact, Kant has never expressed beliefs of this kind, and his first *Critique* is not aimed at trying to decide whether experience will always be possible or not, but is instead an examination of the subjective conditions of the very possibility of experience. As has already been indicated, human experience depends upon empirical intuition for its matter, and depends upon pure intuition and pure understanding for its form. With respect to the *occurrence* of the sensibly given, that is sensation, we can do nothing - we cannot obtain it from ourselves, since our cognitive mind is not a creative faculty. And with respect to objects of experience, the principles of understanding are only

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[21] Vide A239 = B298: "Therefore all concepts, and with them all principles, even such as are possible apriori, relate to empirical intuitions, that is, to the data for a possible experience. Apart from this relation they have no objective validity, and in respect of their representations are a mere play of imagination or of understanding."

universal rules which bring them into the synthetic unity of apperception.[22] No experience is therefore possible unless there is a matter given to sense, and so the impossibility of experience still remains. Here, if we should introduce the postulate of things-in-themselves to act as the causes corresponding to the occurrences of appearances or sensations, it might seem that the occurrence of experiences would depend on these as causes. But from Kant's point of view, such introduction would be erroneous because the category of cause and effect can have legitimate use only within the field of appearance.[23] In fact, Kant does not take things-in-themselves to be the cause corresponding to occurrences of sensations; things-in-themselves have for him a negative function, which expresses the limit or domain of human knowledge.[24] In this way, understanding cannot decide whether experience will always be possible or not, nor can it decide whether there is another series of appearances, nor whether there is another kind of cognitive mind different from ours. Nevertheless, these possibilities do

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[22]Vide B196 = A157.

[23]Vide the Second Analogy of Experience, B233 ff. The Second Analogy is the principle of determination of the existence of *appearances* in time, according to the relation in time as a successive series. See B262 = A215. Vide also B305 = A248: "The pure categories, apart from formal conditions of sensibility, have only transcendental meaning; nevertheless they may not be employed transcendentally, such employment being in itself impossible....they cannot, when separated from all sensibility, be employed in any manner whatsoever...."

[24]Vide, e.g., B310 and B311.

no harm to the objective validity of principles, which are employed solely in an empirical manner and never in a transcendental manner. To terminate the above discussion of Reichenbach's argument, we shall quote Kant's own words:

> Other forms of intuition than space and time, other forms of understanding than the discursive forms of thought, or of knowledge through concepts, even if they should be possible, we cannot render in any way conceivable and comprehensible to ourselves; and even assuming that we could do so, they still would not belong to experience - the only kind of knowledge in which objects are given to us. Whether other perceptions than those belonging to our whole possible experience, and therefore a quite different field of matter, may exist, the understanding is not in a position to decide. It can deal only with the synthesis of that which is given. . . That yet another series of appearances in thoroughgoing connection with that which is given in perception, and consequently that more than one all-embracing experience is possible, cannot be inferred from what is given; and still less can any such inference be drawn independently of anything being given - since without material nothing whatsoever can be thought.[25]

The short-comings of Reichenbach's criticism, we may say, are essentially based on two facts, the first of which is his misinterpretation of Kant's doctrine, as has been illustrated above; the second of which is his misunderstanding of the relation between Newton's physics and Kant's

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[25] A230-2 = B283-4.

philosophy. In an essay called "The Philosophical Significance of the Theory of Relativity,"[26] Reichenbach writes:

> The development of science, on the other hand, has led away from Kantian metaphysics. The principles which Kant regarded as *synthetic a priori* were recognized as being of a questionable truth; principles contradictory to them were developed and employed for the construction of knowledge. These new principles were not advanced with a claim to absolute truth but in the form of attempts to find a description of nature fitting the observational material. Among the plurality of possible systems, the one corresponding to physical reality could be singled out only by observation and experiment. In other words, the synthetic principles of knowledge which Kant had regarded as *a priori* were recognized as *a posteriori*, as verifiable through experience only and as valid in the restricted sense of empirical hypotheses... The process of the dissolution of the *synthetic a priori* is one of the significant features of the philosophy of our time."[27]

In this passage Reichenbach is telling us that we can now obtain knowledge from principles that differ from those of Kant. His contention is this: the occurrences of these principles deny the validity of Newtonian physics, and thereby also deny the validity of those principles in Kant's philosophical system. In other words, the claim is that Kant's doctrine of the synthetic apriori will be denied

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[26] Reprinted in H. Feigl and M. Brodbeck (ed.) *Readings in the Philosophy of Science.*

[27] Op. cit., pp. 208-209.

whenever Newtonian physics is denied. However, we have argued that Kant's epistemology is not dependent on Newtonian physics.

Among the four basic problems with which Kant's first critique is concerned, there is one formulated as this question: "How is pure science of nature possible?"[28] To this question, Kant's answer is simply that our cognitive faculty possesses subjective laws which hold of things as objects of possible experience, and under which alone an empirical knowledge of things is possible.[29] These subjective laws are just those synthetic apriori principles of our understanding. Here we should also note that the term 'principles', which Reichenbach sometimes uses to refer to the laws of natural science, is equivalent only to 'particular empirical laws'[30] as opposed to 'laws'[31] or 'principles' in Kant's sense.[32] In Kant's view, the subjective laws, that is the synthetic apriori, are always necessary and strictly universal, whereas particular empirical laws cannot have either of these characteristics.

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[28]Vide B20 and *Prolegomena* § 14.

[29]Vide *Prolegomena* § 17.

[30]Particular empirical laws are sometimes called simply empirical laws or special laws.

[31]At times, however, Kant uses the word 'laws' to cover both 'empirical laws' and 'apriori laws'. Vide A126.

[32]Vide A126-8, B165 and B263.

The former, being issued apriori from the understanding itself, are not borrowed from experience, but are principles which make experience possible.[33] The latter, namely, the empirical laws of natural science, are only special determinations of the former,[34] and it is from them that they obtain the necessary formal element which is distributed by the understanding alone. Moreover, these empirical laws can never be derived from the pure understanding.[35] To search for empirical laws, we must resort to experience aided by the instruction of these apriori laws.[36] That is to say, "empirical laws can exist and be discovered only through experience, and indeed in consequence of those original laws through which experience itself first becomes possible."[37] Thus, natural science, as a system of empirical laws and concepts through which we can comprehend nature, depends on the intellectual form[38] of all our knowledge of objects, that which is found in the synthesis of apperception and in the categories. Nevertheless, as we have shown, this does not imply that the empirical laws can be deduced completely from the principles of understanding; on the contrary, the employment of these

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[33] A126.

[34] Vide A126 and A128.

[35] Vide A127 and B165.

[36] B165.

[37] A216 = B263.

[38] Vide A129-130.

principles depends upon the given of sense-date so as to construct empirical knowledge of objects. Hence the principles of understanding only enable us to determine the formal aspect of scientific concepts and laws in general, but nothing as regard their contents - which concern appearances that are empirically determined.[39] In this way, if the understanding is to construct empirical knowledge of objects, it has to be subject to the restrictions of sense-data, and make corresponding special determinations of the apriori principles of different types of sense-data. That is to say, these special determinations are possible only through observation based upon experience, and are therefore not entitled to claim any certainty, since some particular experience may occur and thus make special determinations, other than the one previously adopted, possible. Even corresponding to one and the same observational material, the understanding may make more than one special determination by means of the apriori principles, and these special determintions will have the same validity in relation to the empirical data. Consequently, based on the same foundation of understanding, it is possible to construct a Newtonian physics, but it is also possible to construct a non-Newtonian physics. Thus Kant's position does not force him to consider any particular system of science as the ultimate or the only possible one. The

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[39] Vide B165.

reason why his theory is so closely connected with Newtonian physics is not because he regards Newton's principles as necessary and synthetic truths, since this is not the case, but rather because he regards Newton's system as a model of the appropriate empirical employment of the understanding. This system enables him to show what type of effort is involved.

Despite the differences in terms, we find that there is *no real disagreement* between Kant's theory on the one hand, and those of contemporary philosophers such as Dewey, Hempel, Feigl and Reichenbach on the other, with respect to their views of natural science.[40] Again, we find Einstein expressing a similar view in his essays. "*Science*," he says, "*is the attempt* to make the chaotic diversity of our sense-experience correspond to a logically uniform system of thought ... The sense-experiences are the given subject-matter. But the theory that shall interpret them is man-made."[41] He also maintains that a concept or theory is "suggested" by empirical data, but not deduced from it;[42]

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[40] Vide Milmed, *Kant and Current Philosophical Issues* (New York, 1961), pp. 10-12.

[41] A. Einstein, "The Fundaments of Theoretical Physics", reprinted in Feigl and Brodbeck (ed.), op. cit., p. 171. Compare Kant's view that unity and uniformity is known apriori, but the particular ways in which that uniformity is realized can be known only through experience.

[42] A. Einstein, "On the Method of Theoretical Physics," in *Idea and Opinion* (New York, 1954), p. 274.

the theory itself is rather a "free invention", so constructed as to provide a logical scheme into which empirical data will fit.[43] Accordingly, Kant writes, "Reason has insight only into that which it produces after a plan of its own ... it must not allow itself to be kept, as it were, in nature's leading-strings, but must itself show the way with principles of judgment based upon fixed laws, constraining nature to give answer to questions of reason's own determining."[44]

So far we have examined Reichenbach's criticisms of Kant's doctrine of synthetic apriori knowledge. Within these criticisms Reichenbach shows a strong belief that Kant's principles have been displaced or disproved by modern theories of physics. But this belief can be shown to be unjustified by turning our discussion to two special problems. The first problem concerns the principle of the conservation of substance in conjunction with the principle of the conservation of mass-energy; the second problem concerns the principle of causality in conjunction with the principle of indeterminacy developed in quantum physics. Let us now discuss these problems in turn.

(1) It has been argued that Kant identified substance with matter in his statement of the principle of the

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[43] Ibid., p. 272.

[44] Bxiii.

conservation of substance, and so, it is said, Kant's principle no longer holds since matter has been displaced by the principle of the conservation of mass-energy. This shows, it is claimed, that the former principle is only empirical and not synthetic apriori.[45] However, this criticism will not bear scrutiny, and, in fact, as we will argue, Kant's principle of the conservation of substance has not been displaced in the manner suggested.

Kant's First Analogy of Experience,[46] as it stands, only asserts *that* there must be a permanent substance, and not *what* it is; and what the analogy really demands is that "a physical theory should contain some conservation law be it of matter, energy, matter-or-energy or what you will."[47] It is in the *Metaphysical Foundations of Natural Science* that Kant narrows down this most general notion of substance and conceives it as matter. There he says that substance is determined by the possible spatial perception of matter, and therewith the concept obtains a sensible content. Matter is described thus: (1) Matter is the movable in space;[48]

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[45]Vide Reichenbach, "The Philosophical Significance of the Theory of Relativity," op. cit.

[46]B224 ff.

[47]S. Körner, *Kant*, p. 85. Vide also Paton, II, 209.

[48]Kant, *Metaphysical Foundations of Natural Science*, translated by J. Ellington, Indianpolis, 1970, p. 18.

(2) matter is the movable insofar as it fills a space;[49] (3) matter is the movable insofar as it is something having a moving force;[50] (4) matter is the movable insofar as it can as such be an object of experience.[51] Thus R. P. Wolff, for instance, calls this a dynamical theory of matter, in contrast to both the theories of the atomists and the Cartesians.[52]

Kant's conception of matter, we should note, is in fact quite different from that of Newton's. This will become clearer as our discussion develops. In Kant's view, matter is the movable insofar as it fills a space. However, it does so not by its mere existence, but by a special moving force,[53] which is a fundamental characteristic of all physical reality. Such fundamental moving forces are those of repulsion and attraction,[54] where the former is commonly called impenetrability[55] and the latter, gravity.[56] The

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[49]Ibid., p. 40.

[50]Ibid., p. 95.

[51]Ibid., p. 118.

[52]R. P. Wolff, *Kant's Theory of Mental Activity*, (Cambridge, 1963), p. 232.

[53]J. Ellington, op. cit., p. 41.

[54]Ibid., p. 43 ff.

[55]Ibid., p. 47 ff.

[56]Ibid., p. 70.

quantity of matter, again according to Kant, is the number of its movable parts in a determinate space.[57] And so far as all its parts are considered as simultaneously active (moving), it is called "mass".[58] Hence, "matter" in Kant's sense is not to be identified with "matter" in a Newtonian sense. According to Newton, matter, as a physical reality, is quite different from force or moving force. Force is not an element constituting matter, but a function that changes the motional state of it. To Newton, the proposition "a body (matter with a special shape) has a moving force" is only a synthetic proposition; but for Kant it is certainly an analytic proposition, since matter *must* possess moving force.[59] Matter, for Kant, can be thus comprehended as involving a field of force.[60] This view is different from the views of Newton and others; however, it is, in fact, a view similar to Einstein's. Einstein suggests that there are special kinds of fields which have modes of motion such that there are pulse-like concentrations of fields, and fields represent energy, and energy

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[57]Ibid., p. 96.

[58]Ibid.

[59]Ibid., p. 95 ff.

[60]Vide Wolff, op. cit. p. 232; also ibid., p. XXIX.

represents matter.[61]

Tracing the route of thinking described, we find that Einstein's view does essentially coincide with Kant's theory of matter, and consequently we say this: Newton's principle of the conservation of mass and Kant's principle of the conservation of matter are essentially different, since the notion of matter in these two principles has a different content; on the other hand, there is no fundamental difference between Kant's principle and Einstein's principle of the conservation of mass-energy, since their notions of matter belong to the same mode. Since this is the case, the principle of the conservation of substance, which Kant regards as synthetic apriori, has not hitherto been replaced nor disproved by any theory in modern physics, but has instead acquired powerful support from it. Nonetheless this still seems insufficient to establish that the First Analogy has obtained the strict universality it should claim. We are inclined to think it is important to consider the views in Kant's *Metaphysical Foundations of Natural Science* distinct from those of *The Critique*. The principle of the conservation of substance should be left in its most general form. In this form it will then apply to any physical reality which physical theory may

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[61]Vide Ellington, op. cit., p. XXIX.

discover to be permanent in space.[62]

(2) Like the principle of the conservation of substance, Kant's Second Analogy (the principle of causality) has also been attacked as untenable. The Second Analogy of Experience is stated thus: "All alteration takes place in conformity with the law of the connection of cause and effect."[63] The law of causality in question is, Kant holds, "an indispensable law of *empirical representation* of the time-series that the appearances of past time determine all existences in the succeeding time, and that these latter, as events, can take place only in so far as the appearances of past time determine their existence in time, that is, determine them according to a rule."[64] However, since the recent development of the principle of indeterminacy in quantum physics, some have argued that the law of causality should be abandoned, or, at the very least, since its universality and necessity no longer hold for subatomic phenomena, it should be seen as only synthetic and not apriori.[65] What the principle of indeterminacy tells us is that the most complete

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[62]Cf. Paton, II, p. 209.

[63]B232.

[64]B244.

[65]Vide Reichenbach, op. cit.

knowledge we can have of a certain event is not a sufficient condition to allow us to predict the occurrence of another event. This knowledge only allows us to compute the probability of such an occurrence, and thus there is the possibility that the one event could occur without or before the other.[66] Is this indeterminacy principle fatal to Kant's theory of causation? We shall now examine this in detail.

(a) First of all we should note that this well-known indeterminist theory developed by Bohr and Heisenberg is not an ultimate one; it is only one among those more deterministic ones suggested by Einstein, Louis de Broglie, David Bohm, etc.[67] Many hold that the indeterminacy principle is insufficient. They think it is conceivable that some day a physical theory will be constructed which is based on the principle of causality. Such a theory will, they think, be more powerful than the purely statistical contemporary theories.[68] Secondly, as Louis de Broglie has recognized, the

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[66] L. W. Beck, "The Second Analogy and the Principle of Indeterminacy", *Kant-Studien* (1966), reprinted in T. Penelbum and J. J. MacIntosh, *The First Critique* (Belmont, 1969), p. 91.

[67] Vide Ellington, op. cit., p. XXXI.

[68] Compare S. Körner, "On the Kantian Foundation of Science and Mathematics", *Kant-Studien* (1966), reprinted in *The First Critique*, p. 106.

indeterminacy in predictions of subatomic events "is completely masked by the errors introduced in the course of experience, and everything happens therefore as though it did not exist at all..."[69] On the basis of this observation, it is therefore likely that one day a more effective experimental procedure and apparatus will be devised and made available that could diminish such errors, and also exposes the underlying deterministic characters of such subatomic phenomena.

(b) The following argument is even more telling than the above. We find that the Second Analogy is required in arguments designed to establish the indeterminacy principle itself and that their relation depends on an epistemological basis;[70] what is confirmed or refuted by experience is certainly not the regulative principle according to which physical theories, either causal or statistical, are constructed, but merely the *theories themselves.*[71]

Fundamentally, modern physicists agree that events can be distinguished from continuing

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[69]Louis de Broglie, *Matter and Light*, p. 230.

[70]The argument is presented by L. W. Beck in op. cit., pp. 89-96, and we shall follow here.

[71]Compare S. Körner, op. cit., p. 106.

states of affairs revealed in regular order,[72] and surely, it would otherwise be completely insignificant to say such and such things are not causally related since it might well be the case that they are not events at all.[73] Just here, as Beck points out, indeterminists have to call upon the Second Analogy so as to make such a point, and it is only on this basis that they are able to assert that there are *events* not causally related. Consider a simplified experimental situation which backs up the principle of indeterminacy. We have a clock to tell times t1, t2, etc., and we call the corresponding successive positions of the hands C1, C2, etc. the clock-series. In the experiment, on Beck's analysis, flash Fx which is observed at tx is interpreted as evidence (or representation) for any subatomic event Ex, but we will deny for any particular events that the one (E1) is the cause of the other (E2) because the latter is not perfectly correlated with the former.[74]

As Beck has noticed, to arrive at such a conclusion, we are obliged to assume the Second Analogy in order to fix the sequence of the C-

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[72] L. W. Beck, op. cit., p. 92.

[73] Ibid., p. 93.

[74] Ibid., pp. 92-93.

events and thereby also the sequence of F-events,[75] and such a *prior decision* [in a middle-sized object, here, a clock] is absolutely essential. Still another principle is required for us to pass from the sequence of F-events to that of E-events. In the present case, the principle is only a postulate, which might be formulated thus: The temporal relation between the clock events Cx and the subatomic events Ex gives rise to a report Fx at Cx.[76] Indeterminists argue on the basis of this, that since experience shows that the F-series is not correlated with the C-series invariably, then the E-events are not causally related.[77]

This much is clear: the causal relations among microscopic objects are denied only after the temporal ordering of C-events and also the synchronization of E-events and F-events, and, as a result, the Second Analogy and the postulate stated have already been presupposed.[78] Therefore from an epistemological point of view, we

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[75]Ibid., p. 94.

[76]Ibid., p. 94.

[77]Ibid., p. 93, p. 94.

[78]Ibid., p. 95.

> assert that our knowledge of indeterminacy could not be established without grounding it on our apriori knowledge of causal determinacy in general.[79]

To end these discussions, we would like to point out again that the principles of understanding are synthetic apriori principles which underlie all branches of knowledge as their basis; if knowledge is to be possible at all, these principles must be presupposed. Thus, no experience or empirical knowledge which must presuppose these principles could ever disprove them. In order to disprove them a transcendental argument, rather than an empirical argument, is required. And as we have seen, instead of being disproved, they in fact acquire confirmations from scientific studies, and even the indeterminacy principle, which appears so incompatible with the Second Analogy, turns out at last to be grounded on that principle. Hence we may conclude as follows: What Kant does establish is "*something* about a universal, apriori system of law and order in the world; but it is something quite different from what we should today call 'physics'."[80] This 'physical system' is therefore not to be identified with the Newtonian physics or any other physical theories, since it does not represent

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[79] Ibid.

[80] Milmed, *Kant & Current Philosophical Issues*, p. 55.

physics itself; but it is advisable to understand it as "a logic of physics",[81] or as H. J. Paton calls it, "a metaphysics of experience." So even if Newtonian physics or some other current scientific theories are proven to be false, this shows nothing but that physical theories which were accepted previously are now denied by some modern physical theories on good empirical grounds; nevertheless, all these physical theories must be grounded on the same apriori foundations - the synthetic apriori principles of human understanding.

2. The Geometrical Aspect

Undoubtedly the best known axiom system for geometry is Euclid's. Indeed, admiration for its rigour and thoroughness has been expressed frequently. Also, its deductive method has been widely employed, for instance, by Spinoza in the *Ethics*, by Newton in mechanics, by Lagrange in analytic mechanics and by Clausius in thermodynamics.[82]

There are two types of propositions in Euclidean geometry, namely, axioms and theorems. Axioms are known by direct intuition with certainty; theorems are proved by reasoning, and are said to have a double certainty that arises from both demonstration and the sensual evidence originated in spatial intuition. The supposed certainty of

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[81] Ibid.

[82] Hao Wang, *Logic, Computers and Sets* (New York, 1970), p. 2.

geometry thus suggested the idea of a demonstrative system of absolutely certain philosophical principles,[83] and this ideal has often been adopted by metaphysicians in their search for certainty. However, during the first third of nineteenth century, the authority of Euclidean geometry was facing severe challenges because different consistent geometries were constructed independently by N. I. Lobachevski, J. Bolyai and K. F. Gauss. These new geometries share one common characteristic: they all abandon the parallel axiom and adopt the assumption that there exists more than one parallel through a given point. In 1854 B. Riemann brought out the conept of a finite but unbounded space and developed geometries with the assumption that there exist no parallel lines at all. In contrast to Euclidean geometry, all of these geometries are called non-Euclidean.[84]

The development of non-Euclidean geometries led to a fundamental change of view in mathematics. This change had the following consequences:[85] (1) geometrical propositions were no longer regarded as necessary synthetic truths; (2) geometries were regarded as formal deductive systems, and this led to the development of formalization or axio-

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[83]Vide, C. S. Peirce, "The Architecture of Theories," *The Monist* (1891).

[84]Vide Reichenbach, *The Rise of Scientific Philosophy*, Ch. 8.

[85]Vide ibid. and Henri Poincare, "Non-Euclidean Geometries and the Non-Euclidean World", in Feigl & Brodbeck (ed.), op. cit.

matization of mathematics; (3) it was thus claimed that abstract mathematics should be separated from spatial intuition; and, (4) the application of geometries and mathematics to nature, was therefore regarded as problematic. As regards Kant's doctrine, the discovery of non-Euclidean geometries was said to be further evidence of the disintegration of the synthetic apriori.[86]

As a result of the separation of mathematics from spatial intuition, mathematics should no longer rely on diagrams. Euclidean geometry was regarded only as an axiom system which had not been fully formalized. It was claimed that strict axiomatization should involve total abstraction from the meaning of the concepts. Adhering to this viewpoint, axiomatics separated the logical form from the factual and intuitive content, and treated the logical-formal as the only subject-matter of mathematics.[87]

In general, an axiom system must contain a set of primitives. These primitives are undefined, and therefore without any explicit meaning. And from combinations of these primitives, we obtain a number of elementary propositions - that is axioms or postulates which are stated to be 'true' unconditionally and which involve

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[86] Vide Reichenbach, op. cit.; Reichenbach, "Rationalism and Empiricism," loc. cit.; "The philosophical significance of the theory of relativity", loc. cit.

[87] Vide A. Einstein, "Geometry and Experience," reprinted in Feigl and Brodbeck (ed.), op. cit., pp. 189-194.

neither empirical nor linguistic meanings. Then through the process of definition and deductions according to rules, we arrive at a series of derived propositions, i.e. theorems. Their truth follows from primitives, axioms and definitions. That is to say, they are deductive, logical, and analytical truths, but they are no longer necessarily true in the physical world. In a word, all symbols and propositions which occur in an axiom system are entirely formal and void of all content of experience. In systems of this kind, a basic logic of inference, for instance, the theory of quantification or the predicate calculus, is usually taken for granted. And it deals with only a few logical constants such as "if-then", "not", "all", "some", "or", "and", "if and only if". If we adjoin one such standard axiom system to an axiom system for geometry, we get a highly formalized system of geometry. According to this description, Euclidean geometry is certainly not a thoroughly formalized axiom system. The most famous axiom system for geometry was proposed by D. Hilbert in 1899. Others have been put forward by Peano, Veblen, Huntington, etc.[88]

Since the separation of geometry from spatial

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[88]Vide, e.g., Hao Wang, op. cit.

intuition have been so made,[89] another problem manifests itself. The problem is this: how can geometry, as an axiom system which by itself contains no assertions about experience, apply to empirical or perceptual objects? The answer given by A. Einstein is simply this: "To be able to make [assertions about physical objects], geometry must be stripped of its merely logical-formal character by the co-ordination of real objects of experience with the empty conceptual framework of axiomatic geometry."[90] For example, if we add the statement that solid bodies behave according to the theorems of three-dimensional Euclidean geometry, then the propositions of Euclid contain assertions concerning the relations of physical bodies. The geometry as thus completed is called practical geometry.[91] But another problem arises: If there exist two or more geometries which are self-consistent but contradict one another, how can we decide which of them (if either) is true of physical reality? This problem has led most mathematicians, scientists and philosophers to take up one and the same

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[89] Vide Einstein, op. cit., "The progress achieved by axiomatics consists in its having neatly separated the logical-formal from its objective or intuitive content; according to axiomatics the logical-formal alone forms the subject-matter of mathematics, which is not concerned with the intuitive or other content associated with the logical-formal." The same quotation in Frank's book (op. cit.) takes "axiomatic geometry" for "axiomatics".

[90] Einstein, op. cit.

[91] Vide ibid.

attitude: they consider that in axiomatic geometry, the terms "point", "straight line" etc. represent only empty conceptual schemata,[92] and axioms are only arbitrary conventions governing the employment of terms, but not laws or hypotheses about physical reality. Such a conception is suggested by the French mathematician Henri Poincare, and it is called conventionalism. "*The axioms of geometry*," he says, "...*are neither synthetic apriori judgments nor experimental facts.* They are *conventions;* our choice among all possible conventions is *guided* by experimental facts; but it remains *free* and is limited only by the necessity of avoiding all contradiction."[93] In other words, it is meaningless to ask questions such as the one formulated above, for we may choose any geometry arbitrarily bound only by our specific physical hypotheses concerning measurement of length and time, etc.[94] What experience tells us is only which one is the most *convenient* geometry, but not which one is the most accurate.[95] Our choice remains free.

Einstein agrees with this. "*Sub specie aeterni* [under

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[92] Op. cit.

[93] H. Poincare, "Non-Euclidean Geometries and the Non-Euclidean World," in Feigl and Brodbeck, ibid.

[94] Einstein points out that all these hypotheses are conventions, and "all that is necessary to avoid contradictions is to choose the remainder of (P) so that (G) and the whole of (P) are together in accord with experience," where (G) stands for geometry and (P) stands for the purport of physical laws. See Einstein, op. cit.

[95] Poincare, op. cit.

the aspect of eternity] Poincare," he says, "in my opinion, is right."[96] Geometry in itself asserts nothing about the relations of real things. (It can make such assertions only if it is combined with physical laws.)[97] Thus, not even one geometry among others is to be regarded as necessary to physical space; the question as to which geometrical system physical space "actually" belongs is merely a matter of convention and convenience.

On the basis of the above accounts, we come to this claim: Geometry is either apriori, or synthetic, but not both. For instance, we have Einstein's statement as follows: "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality."[98] Since geometry has been regarded as merely a formalized axiom system, the whole system is but an empty conceptual frame-work. Axioms in it are conventions which are stated to be unconditionally true. All other propositions are logical inferences from the axioms, and are therefore analytic. In this sense, geometry is certain and apriori. But after an interpretation with co-ordinative definitions and operational definitions etc. have been added to it, the system becomes

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[96] Einstein, op. cit.

[97] Ibid.

[98] Ibid.

descriptive of the physical world. In that sense, geometry is synthetic, empirical and uncertain. So Reichenbach writes again, "There is no synthetic apriori of geometry: either geometry is apriori, and then it is mathematical geometry and analytic - or geometry is synthetic, and then it is physical geometry and empirical."[99]

Now let us turn to examine Kant's view of geometry more carefully.

Apart from a small number of analytic propositions[100] which serve as links, the propositions of geometry, Kant holds, are synthetic without exception.[101] Yet Kant also notes that all these propositions, "strictly so called, are always judgments apriori."[102] That is to say, they are necessary, and are made independently of experience; yet they are not analytic, but synthetic judgments. Thus, they cannot be obtained by dissecting concepts analytically;[103] or more precisely, although they are necessarily subject to the principle of contradiction,[104] they cannot be demon-

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[99] Reichenbach, *The Rise of Scientific Philosophy*, p. 140.

[100] For instance, the whole is equal to itself (a=a), the whole is greater than its parts (a+b>a) etc. See B17.

[101] Vide B14 and B16.

[102] B14.

[103] *Prolegomena*, §10.

[104] The principle of contradiction is "the universal, though merely negative, condition of all our judgments in general" (B189). Vide B14: "All mathematical inferences proceed in accordance with the principle of contradiction."

strated merely by means of this principle. In order to establish their truth, a third something is required so as to make the synthesis of the subject-concept and the predicate-concept possible,[105] since the latter is not to be extracted from the former in a synthetic judgment. This third something, however, must be found in intuition for otherwise the concept would be empty; further, this intuition must be pure, otherwise the judgment would not be apriori.[106] In other words, all concepts in geometry must be first presented in pure intuition. This is another way of saying that an apriori construction of concepts is necessary.[107]

Pure intuition is apriori, for it removes the empirical element of sense, and contains only the necessary and universal relations in which appearances stand.[108] Thus, in Kant's view, pure intuition contains in itself an apriori manifold[109] which is nothing but a system of relations, a sensible given arising out of the nature of our sensibility. Space is certainly a pure intuition, it is necessarily represented apriori and underlies all outer

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[105] A155 = B194.

[106] Vide B73 and *Prolegomena*, § 10.

[107] Ibid.

[108] Vide B34-5.

[109] Vide A77 and A25 = B39.

intuitions.[110] All kinds and determinations of space, that is parts of space, can and must be represented apriori: they are also pure intuitions.[111] And only by means of pure intuition are geometrical judgments possible: "if pure intuition be wanting", says Kant, "there is nothing in which the matter for synthetic judgments apriori can be given."[112] According to him, all geometrical knowledge depends on the "apriori construction of concepts", which means to exhibit apriori the intuition which corresponds to the concept.[113] In other words, we have to represent directly and apriori by imagination[114] the configurations to which the concepts refer, where configurations are simply determinations of space[115] or perhaps relations among positions in the pure intuition of space. Through construction of concepts, the latter acquire as their content the pure manifold of intuition, without which they cannot have any meaning or significance. It should be noted, however, that if space were not to be regarded as a formal apriori condition of

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[110] A245.

[111] A29.

[112] *Prolegomena*, §10.

[113] A713 = B741.

[114] B204: 'The mathematics of space (geometry) is based upon this successive synthesis of the productive imagination in the generation of figures.'

[115] Cf.A25,A29, and B606: "All figures are only possible as so many different modes of limiting infinite space."

outer experience, these concepts, and therewith the whole of geometrical knowledge, would be impossible - they would be "nothing but a playing with a mere figment of the brain."[116]

For example, consider this geometrical proposition, "Given three straight lines, a figure is possible." We find the fact is: no matter how much we labour, we cannot derive the above proposition by merely analysing or dissecting the concept of straight lines and of the number three. Nonetheless, we can do so if we have recourse to intuition.[117] In other words, we have to construct apriori three straight lines in pure intuition and then think about them - soon we will find that a figure can be easily constructed, for instance, by joining the ends of the straight lines one to another in accordance with a certain procedure. In this way, we can make geometrical judgments synthetically, and yet, in an apriori fashion.

With the construction of concepts, we see that geometrical knowledge is possible apriori, and indeed, through, and only through, the pure intuition of space. Yet it is possible not in a merely logical sense, but in the sense that it must have objective reality. It is so only insofar as it expresses the formal conditions of experience as found in space; space is nothing but the form of all

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[116] A157 = B196.

[117] Vide B65-66.

outer intuition.[118] Thus Kant writes, "Empirical intuition is possible only by means of the pure intuition of space ... What geometry asserts of pure intuition is therefore undeniably valid of empirical intuition."[119] Geometry is for Kant *a science which determines apriori the properties of space synthetically.*[120] It must then be grounded on the pure intuition of space, which in turn must be regarded as the essential form of all outer intuition. For that reason, all geometrical judgments are synthetic apriori, and must also hold of objects of outer sense, that is to say, they are necessarily applicable to whatever objects we observe about us.

We shall now try to get to closer grips with Kant's difficult account by considering some of the criticisms that have been made of it. These criticisms may be summarized as follows:[121]

(1) Kant assumes that there is only one geometry (Euclidean geometry), but now a number of other geometries have been developed.

(2) Kant assumes that there is only one space (Euclidean space), but now different geometries

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[118] Vide A26 = B42.

[119] A165 = B206.

[120] B40.

[121] Vide D. P. Dryer, *Kant's solution for verification in Metaphysics* (London, 1966), p. 160.

describe different possible spaces.

(3) Kant holds that geometrical judgments can only be made by recourse to intuition, or, more precisely, we must resort to the construction of configurations, either in pure or in empirical intuition. But now modern geometries have devised purely deductive proofs in which none of these is required.[122]

(4) Kant holds that geometrical judgments are synthetic apriori, but now they have been shown to be either analytic apriori or synthetic a posteriori, exclusively.[123]

We shall now discuss each of these criticisms in the order in which we have stated them.

(1) As we have already seen, non-Euclidean geometries are not sciences which are concerned with the nature of space or with the properties of spatial configurations; they are merely logical systems or axiom systems. In these axiom systems no attempt is made to verify the axioms, the truth of them is only presupposed. The truth of theorems in such systems is therefore hypothetical, that is to say, they are asserted in this manner: if such and such and such, then such and such and such.[124] For instance, we do not assert that

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[122] Vide C. G. Hempel, "Geometry and Empirical Science', *American Mathematical Monthly*, 52 (1945).

[123] Vide Einstein, op. cit; Reichenbach, op. cit., p. 140; Hempel, op. cit.

[124] Vide Dryer, op. cit., p. 161.

through two points only one straight line can pass, but we only assert that if such and such conditions are admitted, then through two points only one straight line can pass. Hence, non-Euclidean geometry, as a geometry, is taken in a sense quite different from Kant's. A non-Euclidean geometry is only a deductive system, and it says nothing about the determination of the properties of space. With respect to these deductive systems, therefore, we are not concerned with anything that relates to the empirical world. We are concerned only with the contextual significance of propositions - i.e. their validity within the system.

(2) Strictly speaking, Kant does not assume that there is only one Euclidean space. However he does hold that there is only one space. This space, as pure intuition, contains nothing but the necessary and universal relations in which outer appearances stand. And this space can be described by Euclidean geometry. Spaces, which are said to be described by geometries, have a quite different significance; they mean only so many sets of relationships which are in conformity with the axioms and theorems of so many geometries.[125] For instance, a geometry is said to describe a curved space if it describes only

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[125]Vide Dryer, op. cit., pp.164-5.

relations on the surface of a sphere; on the other hand, it describes only an elliptic space, if it describes only relations on the surface of an ellipsoid. Thus, it is clear enough that this conception of spaces is by no means a denial of Kant's view that there is only one space. Furthermore, according to Kant, these many sets of relations may be said to be only parts[126] of the one whole system of relations that expresses the necessary conditions of outer appearances. In fact, Henri Poincare finds that all non-Euclidean geometries are translatable into branches of the whole system of Euclidean geometry.[127] In order to do this, we have to construct a dictionary, which would enable us by a simple translation to transform, e.g., Lobachevski's theorems into theorems of Euclidean geometry.[128] Therefore, it is advisable to consider it in this way: if non-Euclidean geometries are to be rendered intelligible they may be interpreted and transformed into Euclidean geometry, then they are only branches of one geometry and describe only one and the same space;

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[126] Or more precisely, limitations or determinations.

[127] Vide H. Poincare, op. cit.

[128] Poincare, op. cit. For example, according to a certain dictionary, the theorem that the sum of the angles of a triangle is less than two right angles may be translated thus: "If a curvilinear triangle has for sides circle-arcs which prolonged would cut orthogonally a certain plane, the sum of the angles of this curvilinear triangle will be less than two right angles."

on the other hand, if they are to be regarded as pure axiom systems as such, they do not describe the physical world, and the concept of spaces is only the result of the play of imagination.

(3) As indicated in (1), non-Euclidean geometries are only axiom systems, and therefore like other axiom systems, they could do without any use of diagrams or any recourse to intuition. But geometry in Kant's sense is a system of synthetic knowledge about space, and must be thus based on intuition. [See also the descriptions above in (2).]

(4) Kant holds that all geometrical judgments are synthetic apriori, except for a few which are analytic and which only serve as a method of concatenation,[129] but not as principles. The truth of these analytic judgments, such as "the whole is equal to itself" (a=a) and "the whole is greater than its parts" (a+b>a), is unaffected by the geometrical concepts employed. In the case of other judgments, such as "a cube must have square faces" and "an isosceles triangle has two equal sides", we must also consider the meaning of the geometrical concepts involved in order to verify them. Analytic judgments alone would not constitute a science of geometry, since the latter contains in itself a body of

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[129] *Prolegomena*, § 2c. Vide also B17.

synthetic apriori principles. These synthetic apriori principles carry with them necessity which cannot be obtained from experience.[130] Judgments such as "through two points can pass only one straight line", and "through a point can pass only three straight lines which are perpendicular to each other" are just instances of these synthetic apriori principles. And since they all describe the properties of space, they are one and all synthetic. However, we could, if we liked, construct a variety of axiom systems for such a science of space. Euclid's geometry is one such example. Usually, axiom systems of this kind must contain on the one hand the same set of axioms and theorems as those contained in that geometry, and on the other hand some additional axioms, so as to make the system purely deductive.[131] Thus, some argue that theorems of Euclidean geometry are analytic, because they are deducible in an axiom system. But we should also note the following points. In the *first* place, to repeat again, axioms and theorems in an axiom system are purely formal and carry no content of experience, and are therefore to be taken in a sense quite different from that of Kant's - as a result, their analyticity in the corresponding axiom system gives rise to no contra-

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[130] Vide *Prolegomena*, § 2c; B14-5.

[131] Vide Dryer, op. cit. p. 161, also Hao Wong, op. cit. pp.2-3.

diction to the syntheticity of axioms and theorems in Euclidean geometry. In the *second* place, even granted that the axiom system is given an interpretation in conformity with Euclidean geometry, we find that the axioms are still presuppositions, from which theorems are just logical inferences. This does not mean that the axioms and what follows from them is not synthetic. The axioms and theorems *themselves*[132] are synthetic. Yet a synthetic proposition "can indeed be established by the law of contradition, but only by presupposing another synthetic proposition from which it follows."[133] Others argue that if they are to be synthetic, then they must be synthetic in the sense that they are only empirical and aposteriori, and rest essentially on induction from experience.[134] Compare Reichenbach's words on space: he says, "space is not a form of order by means of which the human observer constructs his world - it is a system formulating the relations of order holding between transported solid bodies and light rays and thus expressing a very general feature of the physical world..."[135] The conception of space

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[132] They can be verified directly in intuition.

[133] *Prolegomena*, § 2c.

[134] Vide Einstein, op. cit., Reichenbach, op. cit. pp. 138-140.

[135] Reichenbach, op. cit., p. 139.

in question is only a physical one, and therefore should not be put to the same level as that of Kant's. The science of such a space is certainly empirical, and might be in fact regarded as a branch of physics.[136] But in this sense, it confirms only that physics is empirical, and Kant's contention is that the science of space is a pure science. It is apriori because the propositions in it can be verified wholly apart from experience by construction of concepts in pure intuition; nevertheless, it must be applicable to experience because space is just the essential form of outer intuition or experience. Space in Kant's doctrine is the formal condition of outer observation, without which no natural science would be possible; thus even physical or practical geometry, as a natural science, if possible at all, must presuppose space.

Some, however, question the necessary applicability of geometry to experience. They do this by reference to physical observations. They argue, for example, that light rays (on which visual observations depend) describe only curves, and also that real solid bodies are not in nature rigid. The conclusion they want to draw from this is that the immediate relation between geometry and physical reality appears

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[136] Vide Einstein, op. cit.

destroyed,[137] and that geometry, if applicable at all, must be subject to correction on the basis of these observations. And geometry is for that reason only empirical. But we think that the conclusion of the foregoing argument goes beyond its premises, for at most it shows that the criteria for applying the concepts of "straight line", "triangle", "congruence", etc. are defective.[138] The meaning of any geometrical concept, such as "straight line" or "triangle", should not be thus tied strictly to any one type of experimental outcome, or any one physical phenomenon or procedure;[139] and when scientists assert that the path of a light ray is not straight, they are just using the same concept of straightness as occurs in Euclidean geometry.[140] We may well search for some better criteria for applying geometrical concepts, but this by no means disproves the aprior-ity of geometrical judgments and their necessary applicability to objects

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[137] Ibid. Also Reichenbach, op. cit. pp. 130-133.

[138] Compare Dryer, op. cit. p. 169.

[139] E. G., "straight line" is interpreted as meaning the path of a light ray, or as meaning the path along which a measuring rod would have to be laid down the fewest times, etc. Vide Stephen F. Barker, *The Philosophy of Mathematics* (Englewood Cliffs, 1964), pp. 51-2.

[140] Vide Dryer, op. cit., p. 169.

of experience.[141]

On the basis of the foregoing discussion, we come to the following conclusions: Geometry is both apriori and synthetic. It is different from an axiomatic geometry on the one hand, and a physical geometry on the other. Geometry in Kant's sense is only an apriori science of space, where space is the essential form of outer appearances. Kant's theory of geometry is not to be identified with Euclidean geometry; it only supplies us with an apriori foundation for the possibility of a science of the pure form of space. Such a science may be Euclidean geometry, but it may not be. However, it must be a science of the pure form of space. Now Euclidean geometry is a science of this sort, so Kant's geometrical theory may be regarded as the apriori foundation of its possibility; but non-Euclidean geometries are not sciences of this sort, and consequently Kant's theory is not in a position to account for their possibilities. As a result, the discovery of non-Euclidean geometries does not, by itself, disprove Kant's apriori theory of geometry.

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[141] Compare Gottlob Frege's words in his *The Foundations of Arithmetic*, p. 20: "...the truths of geometry govern all that is spatially intuitable, whether actual or product of our fancy. The wildest visions of delirium, the boldest inventions of legend and poetry, where animals speak and stars stand still, where men are turned to stone and trees turn into men, where the drowning haul themselves up out of swamps by their topknots - all these remain, so long as they remain intuitable, still subject to the axioms of geometry."

3. Mathematical Aspect

Since we have already discussed many of these problems in connection with geometry, we shall now confine ourselves to mathematics in its narrower sense. Kant's view here is basically the same as that which is expressed in his theory of geometry. Despite the fact that mathematical propositions follow from axioms according to principles of logic, he holds that the axioms and propositions are not themselves principles of logic or an application of them;[142] rather, they are descriptive of the pure intuition of time and of constructions in it, where the pure form of time, and also mathematical judgments, hold true only of things as the mind intuits them, not of things as they are in themselves.[143] Reichenbach claims that the disintegration of synthetic apriori knowledge began with developments within mathematics. He writes, "The logical development which pushed mathematics from its throne... reaches its final stage in Russell's analysis of arithmetic: mathematical truth is analytic, mathematics is not descriptive of physical reality."[144] Our major concern here is whether mathematics is deducible from, or can be reduced to, logic; in short, we would like to ask whether it is really analytic or not.

In the realm of modern mathematics, there developed

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[142] B14; Körner, *The Philosophy of Mathematics*, p. 72.

[143] Vide B46-B53.

[144] Reichenbach, "Rationalism and Empiricism", op. cit.

three significant views, namely,

(1) the logicism represented by Frege and Russell;

(2) the formalism represented by Hilbert; and

(3) the intuitionism represented by Brouwer.

We shall examine these in order.

Russell's view is clear. What he holds is simply that "all pure mathematics deals exclusively with concepts definable in terms of very small numbers of fundamental logical concepts, and that all its propositions are deducible from a very small number of logical principles."[145] But this presupposes a logic which is to be understood broadly: i.e., one which includes the logic of truth-functions, the extensional logic of classes, the logic of quantification, and also the postulates and rules of inference drawn from these three branches of logic.[146] (Many modern logicians do in fact accept Russell's view that the notions of mathematics can be defined in purely logical terms. Quine, for instance, holds that arithmetical notions are all definable in terms of three notional devices,[147] but he does not claim to have reduced arithmetic to logic.)[148]

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[145] Russell, *Principles of Mathematics*, preface.

[146] Körner, op. cit., p. 39, p. 50.

[147] They are membership, joint denial and qualification with its variables. See Quine, *Mathematical Logic*, p. 126.

[148] Körner, op. cit., p. 50.

According to Frege and Russell, the number '1' is defined as a class, more precisely the class of all those classes each of which contains only one element. We then assert that 'x' is a member of the class of classes '1' under two conditions, namely, (1) there exists an entity, say 'u', such that ($u \varepsilon x$), and (2) for any two entities 'v' and 'w', if ($v \varepsilon x$) and ($w \varepsilon x$) then $v=w$.[149] In a similar way, the number '2' is defined and explained. Then in terms of these definitions, the expression "1 + 1 = 2" can be defined through the logic of truth-function, then the logic of quantification, and the logic of class. It is defined thus: "1 + 1 = 2" $=_{df}$ "$(x)(y)(((x \varepsilon 1) \& (y \varepsilon 1)) \equiv ((x \cup y) \varepsilon 2))$" - that is to say, here we have assumed that x and y are non-empty classes with no common member, then for any classes x and y, if x is an element of 1 and y is an element of 1, then their logical sum is an element of 2.[150] In this way, arithmetic is said to be reducible to logic. But the question arises, is such a logical interpretation sufficient to prove the analyticity of mathematics?

Firstly, we notice that logicism assumes that there is a common general characteristic "L" of logical propositions. Further, logicism asserts that certain propositions of mathematics also possess L. Finally, it assumes all other propositions that can be formally deduced

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[149] Ibid., p. 52.

[150] Ibid., p. 53.

from these mathematical propositions also possess this characteristic (L).[151] But such a characteristic is lacking in Russell's logical system, and even also lacking in other more recent systems, including Quine's logico-mathematical system.[152] If we are given a number of propositions, can we distinguish those which are logical propositions? We find no characteristic which enables us to do so. For Quine, and some others, the postulates are merely enumerated, and no general characteristic is supplemented.[153] The absence of such a common general characteristic in logical systems is admitted by most contemporary logicians.[154] Russell's account of logic is therefore very obscure, and his reduction of mathematics to logic may turn out to be just a translation. Furthermore, as S. Körner points out, "the logicist account of logic is philosophically inadequate beyond its mere obscurity."[155]

Secondly, we shall consider problems concerning the Frege-Russell definition of natural numbers. According to them, a certain number is defined in terms of the similarity of classes. For instance, the number 2 is defined as a class of classes of doublet, the number 3 is

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[151] Ibid., pp. 55-6.

[152] Ibid., p. 56.

[153] Ibid.

[154] Vide Carnap, *Introduction to Semantics* (Harvard Univ. Press, 1942), Chapter C.

[155] Körner, op. cit., p. 56.

defined as a class of classes of triplet, etc. But this analysis is circular, for in distinguishing a class of doublet, or a class of triplet, from others, we must have already applied the concept of 2 or 3.[156] Furthermore, Russell defines every natural number n as having a unique successor n + 1, and he also assumes the axiom of infinity so as to ensure that n is not equal to n + 1.[157] However, this axiom is a non-logical hypothesis, and if the concept of number is indeed a purely logical concept, it should not be defined by reference to a non-logical hypothesis.[158] Thus the attempted reduction of mathematics to logic turns out at last to be merely a reduction of logic *plus* non-logical hypotheses.[159] Besides, the axiom of infinity implies the existence of infinite classes; that is to say, it implies the possibility of actual infinity, and this involves some other difficulties - the difficulties generated by the notion of actual infinity.[160] The reason is simply that it describes no concrete objects.[161]

What we have tried to show above is this: the Frege-Russell programme of reducing mathematics to logic is

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[156] Vide ibid., p. 58.

[157] Ibid., p. 59.

[158] Ibid.

[159] Ibid.

[160] Ibid., p. 73.

[161] Ibid.

open to serious question, and the claim that all mathematical truths are analytic has not been so far justified. In the discussion that follows, we shall see that Hilbert's formalism and Brouwer's intuitionism do not differ fundamentally from the philosophical position of Kant.

Hilbert does in fact adopt Kant's principal idea expressed in the Transcendental Aesthetic. As Hilbert wrote in *Die Gundlagen der Mathematik:*

> ...something which is presupposed in the making of logical inferences and in the carrying out of logical operations, is already given in representation: i.e., certain extra-logical concrete objects which are intuitively present as immediate experience, and underlie all thought. If logical thinking is to be secure, these objects must be capable of being exhaustively surveyed, in their parts; and the exhibition, the distinction, the succession of their parts, and their arrangement beside each other, must be given, with the objects themselves, as something that cannot be reduced to anything else or indeed be in any need of such reduction.[162]

Although Hilbert is not a strict Kantian, his views of mathematics agrees with Kant's in the claims that (1) mathematics includes descriptions of concrete objects and constructions, and that (2) actual infinities, among others, are introduced only as ideal elements to amplify the field of mathematics, and this requires a proof of the

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[162]Hilbert, *Die Grundlagen der Mathematik*, Vol. 6, p. 65, translation in Körner, op. cit., p. 73.

consistency of the amplified system. In other words, we have to prove such an 'introduction' produces no contradiction within the system.[163]

Hilbert's conception of construction is closely connected with the construction of perceptual objects. To him, the formulae constructed are perceptual objects "which can be distinguished and classified by means of perceptual characteristics which are possessed either by the formulae themselves or by the process of their production..."[164] These objects are stroke-expressions, and it is the propositions about these stroke-expression and their production that constitute the elementary arithmetics.[165] For instance, the proposition "1 + 2 = 3" is analysed into "1 + 11 = 111", and the commutative law is analysed into "11 + 111 = 111 + 11", wherein the signs "1", "11" and "111" represent one stroke, two strokes and three strokes.[166] Their structure or situation as presented is so simple and transparent that we can read off the synthetic relation between the concepts.[167] This seems to show that Hilbert agrees with Kant as to the syntheticity of mathematical propositions.

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[163] Körner, op. cit., pp. 73-4.

[164] Ibid., p. 84.

[165] Ibid.

[166] Ibid., pp. 77-8.

[167] Ibid., pp. 101-2.

Intuitionism also follows Kant's line of thought. Basically, both formalists and intuitionists share the following conceptions: (1) mathematical theories are synthetic and cannot be deduced from logic;[168] (2) no doctrine of the actual infinite is admitted, but only the doctrine of the potential infinite, or of indefinite totalities.[169] But intuitionism is even nearer to Kant than formalism is, since Brouwer holds, in accord with Kant, that mathematical truths are synthetic apriori and are grounded upon the pure intuition of time. They are, more precisely, grounded upon the intuited non-perceptual objects and constructions in time.[170]

In his essay "Historical Background, Principles and Mathods of Intuitionism" Brouwer expresses his intuitionism as follows:

> The *first act of intuitionism* completely separates mathematics from mathematical language, in particular from the phenomena of language which are described by theoretical logic, and recognizes that intuitionist mathematics is an essentially languageless activity of the mind having its origin in the perception of *a move of time*, i.e., of the falling apart of a life moment into two distinct things, one of which gives way to the other, but is retained by memory. If the two-ity thus born is divested of all quality, there remains

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[168] Ibid., p. 85, p. 119.

[169] Ibid., p. 120; S. F. Barker, *Philosophy of Mathematics*, p. 74.

[170] Körner, op. cit., p. 120.

> the *empty form of the common substratum of two-ities*. It is this common substratum, this empty form, which is the basic intuition of mathematics. [171]

This is substantially what Kant maintains in his *Critique*; mathematics presupposes an intuition which is the invariant form of experience, and which differs from the apprehension of logical connections between concepts or statements.[172] And the separation of mathematics from mathematical language is in fact a very important idea, since the former is essentially a mental activity and the latter is but a description of its externalization, systematization and formalization. As professor Mou points out, the apriority and syntheticity of mathematics rest not upon those written mathematical systems, but upon the transcendental foundation which is presupposed by them.[173] In the same manner, he holds, we should also distinguish "logic itself" from "written logical systems", while the latter might well be analytic or deductive, the former, which is the apriori foundation of all logical systems, and which concerns the principles of the synthesis of understanding, must be synthetic and apriori.[174] Thus, the

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[171] Brouwer, "Historical Background, Principles and Methods of Intuitionism", in *South African Journal of Science*, Oct.-Nov. (1952).

[172] Körner, op. cit., p. 122.

[173] T. S. Mou, *The Critique of Cognitive Mind* (Chinese edition, Taipei, 1950), pp. 150-250.

[174] Ibid.

written systems of mathematics and logic should only be regarded as ways of exhibiting the externalization of pure reason.

To the intuitionist, mathematics is the construction of entities in pure intuition,[175] where the construction denotes only self-evident experience, but not external perception.[176] And since mathematical statements are merely reports on self-evident constructions in time,[177] their truth is therefore dependent on such constructions; that is to say, every true mathematical statement can be justified by a corresponding construction.[178] Or more precisely, if we are to assert the truth of a mathematical statement, we must possess a constructive proof of it; similarly, if we are to assert the falsity of a mathematical statement, we must possess a constructive disproof of it.[179] Take for example the proposition "A square circle cannot exist". This is a theorem, according to Brouwer and Heyting, since in constructing such a square circle, we have to construct a square which is at the same time a circle, and in so doing we arrive in intuition at a contradiction.[180]

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[175] Körner, op. cit., p. 125.

[176] Ibid., p. 136.

[177] Ibid., p. 138.

[178] Ibid., p. 136.

[179] Barker, op. cit., p. 75.

[180] Körner, op. cit., p. 138.

On the basis of the above observations, we may conclude as follows. As we have seen, Russell's logicism is insufficient to prove the deducibility of mathematics from logic, it is thus so far insufficient to prove that all mathematical knowledge is analytic in the sense that it is logical or deducible from logical truths. Hence, Reichenbach may be mistaken when he says that the development of mathematics has shown the dissolution of synthetic apriori knowledge. The same applies to his claims that the development of mathematics, which dissolves synthetic apriori knowledge, reaches its final stage in Russell's analysis of mathematics, and that this development has led away from Kant's doctrine of the synthetic apriori. For we have already seen that the recent development of mathematics actually leads closer to Kant's line of thought; this development embodies two significant views which explicitly deny the possibility of deducing mathematics from logic, but affirm the syntheticity of mathematical knowledge. This very evidence has proven Reichenbach's claim (that there is no mathematical synthetic apriori knowledge) untenable. Consequently we see that theoretical synthetic apriori knowledge is in effect possible, and it has not been so far disintegrated by the development of mathematics or the philosophy of mathematics.

CHAPTER VIII

GENERAL CONCLUSION

With respect to the possibility of theoretical synthetic apriori knowledge, the aim of the previous chapter has been to defend Kant's position against various objections. These objections, as we have attempted to show, have often been based on a misinterpretation of Kant's doctrine and so, as it were, hang in the air. Reichenbach's criticisms, for instance, are mistaken in just this uncritical way. We think it is worth while to stress several important points that we have made in the last chapter as follows.

One of the most important tasks that Kant accomplished was his analysis of experience. Human experience, as Kant recognized, involves the use of concept. Among these concepts are those which cannot be derived from experience, but are nevertheless applicable to it. Concepts of this sort Kant calls the categories. According to Kant, these categories, as concepts which condition experience, can have legitimate and objective application only within the field of experience. The propositions that express the rules for such legitimate and objective application of the categories Kant calls the principles of the understanding.

These principles can be established apriori, and are presupposed in all our thought about, and experience of, the world. Consequently, we can never arrive at a totality of observations which contradict these apriori principles, for it is just these principles that set the restrictions and limits for all our experience and knowledge. And since these principles necessarily relate to the empirical world, they are therefore synthetic as well as apriori.

Another point to be noted is that, for Kant, the synthetic apriori principles of the above description are not special laws of natural sciences. The principles, as principles of the understanding, are necessarily apriori, and they actually constitute a universal and apriori system of law and order about nature as we experience it. But laws of natural sciences, as modes of empirical knowledge about nature, must be empirical and dependent on the intellectual form which the apriori principles prescribe for them. For Kant, the system of synthetic apriori principles is therefore not a special determinate theory of science, but rather, a "logic of science," or a "metaphysics of experience."

In the field of mathematics, Kant's view should also be noted. Geometry is for Kant an apriori science which determines the properties of space synthetically, and space is the essential form of all outer intuition. Geometry in this sense is to be distinguished from an axiomatic geometry on the one hand, and a physical geometry on the other. An

axiomatic geometry is apriori and analytic; a physical geometry is synthetic and empirical. But a geometry in Kant's sense is quite different: it is apriori, but it is not analytic; it is synthetic, but it is not empirical. It is apriori because it is strictly universal and unaffected by any empirical fact or observation about us; it is synthetic because it is necessarily applicable to the empirical world. A geometry in this sense is synthetic apriori, for it includes in itself a system of synthetic apriori judgments which are principles governing all outer observations. Kant's theory of geometry is therefore not a special geometrical system of any kind, but rather, an apriori theory for any geometry that can ever be counted as an apriori science of space. Kant's view of arithmetic is similar. In a word, arithmetic, like geometry, can be verified wholly apart from experience by apriori construction of concepts in intuition. Mathematics thus understood includes reference to intuition: it is synthetic and cannot be deduced from logic. This Kantian thesis has in fact been adopted by most formalist and intuitionist mathematicians. Moreover, it is important to note that the apriority and syntheticity of mathematics rest not upon those written mathematical systems, but upon the transcendental foundation which is presupposed by them. Thus, for Kant, merely logical or analytic propositions constitute no mathematics; the possibility of mathematics lies solely in its reference

to intuition. Mathematical judgments, therefore, like those apriori principles of nature, must hold true of things - they hold true of things as the mind intuits them, not of things as they are in themselves. Mathematics, as a real mathematics, is a mathematics of appearances, as Kant calls it.

As a result of the discussions given in the previous chapter we see that, contrary to some claims, theoretical synthetic apriori knowledge has not been dissolved by the development of science and mathematics; on the contrary, it has in effect obtained possible support from this development.

The above restatement should be at the least helpful to an understanding of Kant's theses which constitutes the fundamental focus of the present thesis. Now in order to state clearly our line of thinking in the thesis, we shall give a brief note about the interrelation of the discussion in the preceding chapters.

It has been our concern in this thesis to consider the problem whether theoretical synthetic apriori knowledge is possible in the realm of human knowledge. We begin our enquiry by outlining in chapter one the significance of this problem, from a Kantian point of view, and the complicated situation which the problem has caused, owing to the controversies among philosophers both for and against the

claim that theoretical synthetic apriori knowledge is possible. But since the claim that admits the possibility of synthetic apriori knowledge presupposes the prior possibility of the analytic-synthetic distinction, our consideration of the latter possibility in chapters three and four has been a preliminary investigation that necessarily precedes our later consideration of the former possibility in chapters five and six. Our conclusion of the fourth chapter affirms the possibility of the analytic-synthetic distinction, and this sets up a necessary basis that enables us to pass over to the discussion which is planned to establish the possibility of theoretical synthetic apriori knowledge. This discussion is given in chapters five and six: in chapter five we have tried to show that there are synthetic apriori propositions; in chapter six we attempted to explain how they are possible within a Kantian frame of reference. Finally, the chapter on controversies (chapter seven) was designed to meet the objections arising from philosophical, mathematical, and physical enquiries - the objections which have been brought against the Kantian theses presented in chapter six. Our discussion attempted to show that these objections are untenable.

Now we shall close our whole enquiry by putting forward the following Concluding Remarks:

[1] So far as we are concerned, we find that there are several senses in which a proposition or a mode of knowledge may be entitled apriori. But few among these senses are commonly accepted. However, here is an eligible one that could be stated thus: A proposition or a mode of knowledge is apriori if it is necessary in either a logical or an epistemological sense, and can be established or wholly justified without any appeal to sense-impressions, or ostensive definitions, or empirical propositions or knowledge. In accordance with the discussion exhibited in chapter six, an epistemologically apriori proposition may be further described as follows:

(a) It can be established by methods that depend on considerations that are not purely logical;

(b) It is presupposed in all intelligible discourse; it can be derived from the nature of human reason;

(c) It states certain fixed limits for human knowledge.

An epistemological apriori proposition of this sort would also be synthetic in Kant's view. But in what sense may a proposition or a mode of knowledge be entitled synthetic? Kant's answer is simply this: A proposition or a mode of knowledge is synthetic if it refers to experience directly or indirectly. A synthetic proposition may or may not be aposteriori. If it is not aposteriori, then clearly it is synthetic

apriori. A synthetic apriori proposition is synthetic because it is connected with experience, in the sense that:

(a) It necessarily applies to our experience of objects in space or time (or both);

(b) It is a condition of possible experience - it expresses a formal condition under which appearances must stand;

(c) It states certain fixed limits for our experience.

An epistemologically apriori proposition in Kant's analysis is, of course, a synthetic apriori proposition of the above description.

[2] Now we think it is appropriate at this point to quote one of Kant's most significant remarks as follows: *"Thoughts without content are empty, intuitions without concepts are blind."*[1] This statement is absolutely essential to the possibility of knowledge of any kind. In all empirical knowledge, the apriori element is just as necessary as the perceptual or intuitional elements; such knowledge can result only if the one is applied to the other. In all synthetic apriori knowledge the perceptual or intuitional element is also necessary; such knowledge would be nothing if it has not been related to possible experience. The quoted passage

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[1] A51 = B75. Emphasis added.

therefore indicates the boundary of human knowledge: our knowledge cannot extend any further *beyond* the world of experience, but it can extend without limits within such world; all our knowledge relates finally to possible intuitions.

[3] Finally, as a result of the preceding chapters, we see that there are apriori principles of natural science and mathematics which are synthetic apriori, and which have not been so far disintegrated, even though they have been confronted by various objections. They are instances of an epistemologically apriori propostion or a synthetic apriori proposition described in [1]. To put it briefly, these principles are synthetic apriori because they are conditions of the possibility of experience given by our cognitive faculty. Another way of putting this would be to say that these principles state the conditions necessary for the conduct of rational enquiry about nature. We therefore come to this statement: *Theoretical synthetic apriori knowledge is possible within the realm of human knowledge.*

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THE POSSIBILITY OF THEORETICAL SYNTHETIC APRIORI KNOWLEDGE WITHIN A KANTIAN FRAME OF REFERENCE

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May, 74
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