Teaching numeracy using the literacy strategy of visualization for constructivist classrooms

by

Cindy Lancaster
B.A., University of British Columbia, 1995
B.Ed., University of Victoria, 2005

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Abstract

This project was conducted to investigate the potential application of the literacy strategy of visualization to the teaching of numeracy in middle schools. The project involved a comprehensive literature review of four related components: numeracy definitions and understandings, constructivist theory, visualization as a strategy, and professional development theories related to numeracy teachers. After completing the literature reviews in these areas, it was apparent that there is an abundance of research related to the use of visualization as a comprehension tool for developing abstract concepts. However, there was an absence of specific applications of visualization for students in a Grade 7 classroom. As a result, I created a handbook that can be used by teachers to apply this strategy in an explicit manner when teaching various numeracy concepts. The handbook is based on research, theory and government curriculum documents for British Columbia and describes practical applications of the theory in the form of five specific lessons. These five lessons in three numeracy strands (integers, decimals, and algebra) were developed because the application of visualization may not be obvious in these contexts. The handbook is also meant to be a springboard to further development of cross curricular strategies and an impetus for constructivist teaching in the classroom.

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Dedication

To my parents, who always filled our house with books and encouraged me to pursue my goals.

To my son, Roman, may you always love school and learning as much as you do right now.

CHAPTER ONE

Introduction

I believe in a holistic approach to teaching and learning that attempts to make connections across disciplines and topics, from theory to real world applications. It is these connections that make the learning of abstract concepts approachable and meaningful for students at any level. In particular, numeracy topics and concepts become more abstract and complex for Grade 7 students. Consequently, the repeated use of familiar strategies across curriculum disciplines allows students to build confidence in the tools and their applications in new contexts. For example, this year I explicitly taught the concept of morphemes in both language arts and science and reinforced the concept frequently. Students were challenged to use the strategy to determine the meaning of new words encountered in a variety of formats and subject areas. This simple lesson became a transferable skill that could be applied throughout the year and in future endeavours. It is this focus on integration with a constant eye on future applications of skills in academic and private contexts that informs my teaching practice. I perceive the classroom as an integral part of the students' lives and a place to learn, practice and take risks with new understandings before using these skills in more practical ways outside the school doors. The variety of strategies in my toolbox of instruction influences the integration of topics and allows me to see applications of literacy strategies in content areas that may not appear obvious. In particular, this includes the application of visualization to the teaching and learning of numeracy.

I am also drawn to and influenced by research and theory that can be applied directly in the classroom. The pragmatic nature of my personal character and style of instruction leads me to search out opportunities to try new ideas that have proven successful for other educators in similar circumstances. This quality has influenced the research structure of this project because I was constantly reflecting on a single question: What does this look like in the classroom? Theory and theoretical research are important for understanding the big ideas of education and learning but ultimately, I want to know how this understanding will benefit my students and make learning more engaging, relevant and long term. Consequently, I undertook a literature review of constructivist philosophy to provide a theoretical framework for pedagogy that is widely endorsed by research and current government curriculum documents in British Columbia. The investigation of constructivist pedagogy made clear connections to practical instructional strategies that could be applied to teaching literacy and numeracy. The popular strategies of flexible groupings, inquiry based activities, dialogue and collaboration and a relinquishing of some responsibilities for the learning by myself were already parts of my current practice but now they are solidly based in research and therefore, defensible. My investigation of visualization was important for understanding its popularity as a literacy strategy and its possible practical applications in numeracy. It was very encouraging to find several articles that described research that linked this strategy to numeracy topics and provided specific applications in the form of case studies. This result has encouraged me to relate these findings to the specific Grade 7 curriculum for British Columbia in a practical handbook that will be based in theory and research and yet meet the pragmatic needs of my personality and profession.

The Integrated Resource Package (IRP) for Mathematics in British Columbia prescribes the learning outcomes for students in numeracy and encourages the use of constructivist pedagogy for teaching. In the IRP (2007), making connections through visualization is specifically mentioned as part of the mathematical processes: “connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines,” and “develop visualization skills to assist in processing information, making connections, and solving problems” (p. 18).

However, the wording is vague and there are no direct examples for applications as to how these processes can be interpreted for the specific learning outcomes. It is this perceived need to connect process to learning outcomes that influenced my project and set the goal of a practical handbook for teachers to explicitly incorporate literacy strategies in the teaching of abstract numeracy concepts.

Visualization applications in numeracy: Connecting research and practice

The strategy of visualization is a very powerful tool for aiding comprehension and has been found effective with a broad cross section of students of differing ages, abilities and talents. In fact, Barry (2002) discovered that 84% of middle school and high school teachers surveyed were successfully using visual aids in content areas (p. 189). Most students are able to create some form of mental imagery when reading text and most of them have been exposed to this strategy repeatedly prior to Grade 7 (Douville, 2004; Lapp, Fisher & Johnson, 2010). It is this familiarity with the strategy and the ability of students to perceive an advantage to using the strategy that makes it so appealing to apply in cross curricular teaching. There is often no need to pre-teach the use of the strategy in a rudimentary sense prior to adapting its use for instruction in numeracy concepts and contexts because students are usually familiar with this strategy in literacy contexts. Visuals can be as complex as required, for example, detailed graphs and diagrams that relate numerous pieces of data, or as simple as a pictorial drawing of a number line.

Progression of visual representations: Basic to complex.

The different levels of visuals reflect a progression of visual representations from very basic pictorial images, “representations that depict the physical appearance of the elements described in the problem” (Zahner & Corter, 2010, p. 180), such as:


Figure 1: pictorial representation of an apple (www.hvnet.com)

to complex relational schematic images, “representations that depict relationships described in the problem” (Zahner & Corter, 2010, p. 180), such as:

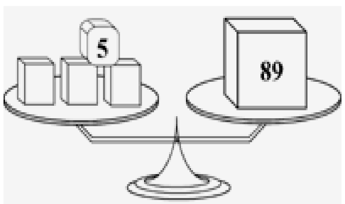

Figure 2: Schematic representation of scales (www.learnnc.org)

This progression of representations demonstrates conceptual understanding of abstract ideas when students can successfully interpret and create more complex representations. The fact that students can represent a variety of levels of understanding allows differentiation in the classroom and enables every student to have some success with solving a variety of mathematical problems. The complexity of the visuals that students are able to create and interpret can be an indication of internal concept images and schema as related to abstract concepts so they can be used for formative and summative assessment as well as instructional tools. It is clear from several research studies that students who use schematic images are much more successful in numeracy than those students using pictorial images. In fact, there is a negative correlation between pictorial images and success in mathematical problem solving (Steele, 2008; Styliano, 2011). Based on this research there appears to be a gap between current applications of visual representations and a true understanding of the differences between pictorial and schematic images in some numeracy classrooms. As van Garderen (2006) reminds us, “Instruction needs to go beyond getting students to try to ‘visualize’ the problem” (p. 505) and instead requires more complex representations of abstract concepts. Despite all these benefits of visuals, I did not seei a variety of visual representations in textbooks such as *Math Makes Sense* (Garneau et al., 2007) which focuses on the implementation of manipulatives. This absence may be partially attributed to confusion about the definitions of many related terms including: mental imagery, visualization and representation. As Stylianou (2010) stated, “It is noteworthy that researchers in mathematics education do not always agree on what representation means” (p. 326). Further research and investigation into the use of visuals in numeracy was needed to support my personal intuition that visual imagery is powerful for learning.

Constructivism: My natural choice of pedagogy

The investigation of constructivism as theory and pedagogy was influenced by the fact that I naturally apply these tenets in the classroom. In fact, it is the ability and willingness of middle school students to accept responsibility for their own learning and take risks in a collaborative, inquiry based activity (Perso, 2005) that makes my career so rewarding. It is these moments of knowledge creation by the group that are personally inspiring. This pedagogy is not only personally beneficial, but also is endorsed by the provincial government (British Columbia Ministry of Education, 2007) as the preferred approach to instructional strategies in the classroom and reinforced by professional development opportunities sponsored by my school district. An investigation into the theory of constructivism was needed to provide me with justification for its implementation in the middle years’ classroom and a better understanding of the placement of visualization within its tenets.

Link research to practice

There is a rich resource of research-based articles for various educational practices available to teachers. However, I do not see teachers reading these articles regularly in an attempt to stay current in their practice. Instead, they rely largely on workshops and professional development programs that are solidly based on educational theories and research and that offer practical strategies and lessons to move them forward in their practice. Stylianou (2010) noted, "Teachers grapple with *how* to integrate representation meaningfully in their instruction" (p. 340) which makes it understandable for teachers to want streamlined learning which is more efficient and less time consuming. The creation of a handbook (Appendix A) is my attempt to bring theory and research together into a practical format that can be easily read, accessed and used in the classroom.

Goal of the master's project: Numeracy handbook

After conducting the literature review into these various topics and an introductory investigation of the theories of professional teacher development, I created a handbook for teachers designed to link theory and research to practical applications of visualization strategies for the numeracy classroom. The handbook allows for connections to be made to educational theory when applying the presented lessons or specific graphic organizers. The handbook provides a practical framework of visualization strategies appropriate for numeracy instruction in a constructivist middle years' classroom.

Design considerations of the handbook.

Language accessibility.

The handbook needed to include some research and educational theory as a way to justify the lessons that are presented. However, the research and theory are not the focus of the handbook and academics are not the target audience. The audience for the handbook is teachers, both new and experienced, who may be looking for specific additional numeracy lessons to supplement their current repertoire. These teachers need language that is clear, thoughtful and easy to comprehend on first reading. These aspects are why there is limited academic vocabulary and terminology throughout the handbook.

Grade level.

There is a lack of specific, explicit visualization lessons in numeracy contexts for middle school students. Grade 7 is the focus of the handbook because this is the grade that I am currently teaching. This grade choice allows for some adaptations to be considered in either direction to facilitate the other two grade levels typically included in the middle years Grades 6-8. I am very familiar with the specific learning outcomes in mathematics and am able to provide personally tested instructional strategies for consideration.

Length.

The handbook is deliberately short and concise. There are many longer books and articles available about the application of visualization strategies in numeracy contexts for those teachers who want a more detailed explanation of theory or procedures. However, based on personal experience, the time constraints that teachers face daily do not allow for extended periods of time dedicated to reading professional resources. The short, accessible nature of this handbook means that it can be read during lunch and applied directly in the classroom later that day. After all, the goal is for the handbook to be a practical resource for daily teaching.

Lesson choices.

The three numeracy strands used for the five lessons in the handbook were chosen because they may not be obvious contexts for visualization strategies. Fractions and geometry use visualization strategies regularly so including these strands would have been a repetition of accepted practice. One of the goals of the handbook is for teachers to see and experience diverse applications of visualization in numeracy. The specific lessons are based on actual lessons that I have used successfully in the classroom. It is important to include tested applications to maintain credibility for the handbook.

Overall format.

The handbook has distinct sections that are designated by obvious titles in the center of the page. This simple consideration makes it easier for teachers to skim the handbook for applicable information and strategies. Background knowledge about research and theory is followed by visualization considerations, which is connected to curriculum documents and is finally applied to specific lessons. This organization makes the connections between the processes explicit and apparent. A glossary is provided at the end of the handbook to clarify some of the confusion about important vocabulary terms. Having the terms located in a single glossary allows the reader to quickly find a specific term when needed.

The lessons are structured as an outline in a numbered format so they can be easily read by the teacher. Learning goals are taken directly from the B.C. Ministry of Education curriculum documents so teachers do not need to consider where or whether or not this particular lesson fits into the mandated curriculum. The achievement indicators are included as a resource and demonstrate formative assessment tools for each learning outcome. The suggested assessment tools provide a starting point for teachers considering formative assessment for each of the lessons.

Finally, a discussion of the continuum of development of visual representations from basic pictorial images to complex, relational schematic images is included. This discussion attempts to make the differences between pictorial images and schematic images more explicit through the use of diagrams and examples. These diagrams can then be used to understand the level of conceptual understanding of students for formative or summative assessment. In this manner, the diagrams and examples become exemplars and goals that teachers can strive to achieve through the specific lessons provided or adapted in other contexts. Ultimately, it is the progression of complexity in representations from pictorial images towards schematic images that should inform instruction and practice because the ability for students to create complex representations show complex understanding. As a result, the handbook addresses the need to present practical, tested strategies based on sound research and theory about educational issues.

CHAPTER TWO

Literature Review

Teachers currently face unique challenges when attempting to prepare middle school students for the demands of a more global, technologically sophisticated society. An increase in the availability of digital technology, such as the Internet and social networking, has meant a shift towards a more mathematical and scientific world. It is with this understanding that mathematics teachers confidently say, "We live in a mathematical world" (National Council of Teachers of Mathematics [NCTM], 2000, Introduction, para. 1). It is a world comprised not only of numbers, but of patterns, shapes and problem-solving situations. Numeracy is evident throughout daily life; however, "numeracy is more than knowing about numbers and number operations" (British Columbia Ministry of Education, 2007, p. 11). As used throughout this literature review and described by Street, Baker and Tomlinson (2008):

> Numeracy is now generally understood as a competence in interpreting and using numbers in daily life, within the home, employment and society. Thus the meaning of numeracy must relate to the social context of its use and the social practices that are adoptedin that context. (p. ix)

This definition of numeracy is broad enough to include all strands of mathematics. It positions numeracy within mathematics as a distinct focus, with particular procedures, contextual considerations, and expectations.

The world is becoming more text based because "it is now no longer possible to understand language and its uses without understanding the effect of all modes of communication that are copresent in any text" (Kress, 2000, p. 337) and communication media more multimodal. Students will be required to use literacy in a variety of contexts to navigate their world including numerical literacy and the specialized discourse of mathematics. It is the “rich and varied discourse and visualization opportunities in mathematics [that] allow students to create links between their own language and ideas, and the formal language and symbols of mathematics” (Yore, Pimm & Tuan, 2007, p. 581). Shanahan and Shanahan (2008) discovered in their study with mathematicians, historians and scientists about specific literacy discourses in their disciplines that the language requirements for mathematics were unique and specific, noting that “texts serve to advance knowledge while at the same time serving to maintain a field’s hegemony. The end result [of this study] is that the literacy demands on students are unique, depending on the discipline they are studying” (p. 48). It is with this knowledge that numeracy teachers must attempt to integrate familiar literacy strategies within numeracy lessons to encourage conceptual understanding. Through the use of familiar literacy strategies in various contexts, students build success and deeper understanding of a variety of numeracy concepts.

This literature review begins with an overview of numeracy teaching and instruction. In this section the case for linking numeracy instruction to authentic tasks and real world experiences is presented, as are the positions represented by the existence of the so-called “math wars” (Draper, 2002; Marshall, 2005). Links between literacy and numeracy instructional strategies also are discussed. The following section focuses on the applications, for middle school teachers, of constructivist and social constructivist pedagogy in numeracy contexts. The section on visualization addresses some issues surrounding the definitions of key terms and presents research on the applications of this traditional literacy strategy in numeracy contexts. A section that focuses on professional development for numeracy teachers is included to provide research and background understanding for the creation of the Teacher’s Handbook, designed for this project. Finally, some gaps in the available research and literature are identified in the last section of this literature review.

Real World Connections

Teaching in the middle school requires teachers to understand the specific characteristics of our students in terms of emotional and cognitive development. Students are becoming increasingly reflective and able to think in more abstract ways (Perso, 2005) in many curricular areas, including numeracy. Students are also most engaged in academic tasks when the learning is connected to real world experiences (Perso, 2005). These two factors are important when developing instructional strategies for teaching numeracy and literacy for this age group. Consequently, students need to understand that mathematics is a way of viewing and interpreting the world rather than a compartmentalized discipline. This understanding allows students to perceive a value for their learning in the classroom.

Mathematics is everywhere.

"Mathematics is one way of trying to understand, interpret, and describe our world" (BCME, 2007, p. 13) and numeracy within mathematics cannot be disconnected from our life experiences. Street et al. (2008) found that making mathematics relevant to the child's life increased the success the child experienced in numeracy. They encouraged parents and families to actively and regularly discuss numeracy with children: "by talking about the maths you are using as you go about your day to day routine you can help your child understand what maths is used for…Maths is all around us not just in 'math books'" (p. 16). This holistic approach to learning encourages making connections, cross-disciplinary teaching, and a greater focus on conceptual understanding. When students learn new concepts in context they have an increased understanding and retention of the concept because "we know from research that comprehension skills are best learned in context and when applied to real text that is relevant and connected" (Jones & Thomas, 2006, p. 59). This knowledge about literacy can be easily transferred to the teaching of numeracy.

Authentic tasks.

To fully address the connection to real world applications students must be given authentic tasks in order to practice skills and explore concepts (Hyde, 2006; Jones & Thomas, 2006; Watson, 2004). "Our goal for mathematics teaching must be real conceptual understanding, and that means that at least some of the time, if not most of the time, students must work on complex, real-world problems, building mathematical models" (Hyde, 2006, p. 88) that reinforce concepts and skills taught in the classroom. For example, students will encounter a variety of mathematical concepts in daily life from the stories in newspapers (Crowe, 2010), which need to be analyzed, dissected and read with understanding so that the entire article can be comprehended. Thus, "numeracy is as essential to becoming an active and thoughtful citizen as literacy" (Crowe, 2010, p. 105). When the relevance of numeracy concepts is built explicitly into lessons students are encouraged to make connections between their learning and the real world. These connections are particularly important for middle years' students because as Perso (2005) stated, "contexts for mathematics learning should be relevant and meaningful for adolescents moving into a world of adults and trying to make sense of issues" (p. 27). This specific group of students, transitioning from childhood to adulthood, faces some unique developmental challenges. It is during this time that they are beginning to consider their long term goals and futures. Thus, it is critical that these students understand that "to be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (Watson, 2004, p. 34). Numeracy is not just a school subject that stays within the walls of academia.

Math Wars

There is a debate about shifting conceptions of the instruction of mathematics and numeracy that is often described by researchers and educators as the "math wars" (Draper, 2002; Marshall, 2005). On the one side are proponents of instruction focused on the learning of algorithms and the memorization procedures. On the other, are those advocating real world applications, creativity and problem-solving strategies. While both of these descriptions are simplifications, the teaching of mathematics traditionally has been based on a transmission model of knowledge acquisition. However, in the 1990's researchers and educators were starting to question practices that compartmentalized learning and forced students to simply memorize processes and algorithms. As Draper (2002) notes:

> Mathematics reform has worked to move instruction away from the tradition in which knowledge is viewed as discrete, hierarchical, sequential, and fixed and toward a classroom in which knowledge is viewed as an individual construction created by the learner as he or she interacts with people and things in the environment. (p. 521)

Reading research has shown that collaboration, communication and teacher understanding of the personal schema of students have greatly affected comprehension of texts. It was this pedagogical shift in literacy that forced other disciplines, including numeracy, to reevaluate approaches to teaching. Yore and his colleagues (2007) point out that "these reforms had common learning goals focused on contemporary literacies for all students and common pedagogical intentions focused on constructivist approaches and authentic assessment" (p. 567). Of course, a dramatic shift in ideology and pedagogy cannot occur uniformly and is rarely universally embraced; immediately, there is tension, discussion, debate and sometimes, resistance to research. This tension and debate is what is referred to as math wars. Despite ongoing controversy about educational practices involving numeracy and literacy, there is one clear outcome: “Today’s mathematics classrooms look quite different from classrooms twenty years ago” (NCTM, 2000, para. 4). In the end, the transmission model of instruction has been abandoned as the dominant pedagogical model in favor of constructivism and social constructivism at least as described by key and influential curriculum documents (British Columbia Ministry of Education, 2007; NCTM, 2000).

Linking Literacy and Numeracy

Changes in educational philosophies in mathematics classrooms have led to instructional practices that are diverse, challenging and individual. There no longer exists the pervasive practice of streaming: that is, the practice of grouping students with similar abilities within schools. Therefore, teachers must be more flexible in their use of instructional strategies to ensure students are engaged and learning at their various levels of readiness. “Teachers need to understand the ‘affordances’ that children arrive with at school” (Baker & Street, 2004, p. 20) and various literacy theories have attempted to address this notion of individuality in the classroom. Baker and Street (2004) articulated this concern within their discussion of mathematics as social when they stated that “numeracy practices are not only the events in which numerical activity is involved, but are the broader cultural conceptions that give meaning to the event, including the models that participants bring to it” (p. 19). Students are no longer viewed as empty vessels waiting for teacher knowledge by most educators. The ability to see connections between new and old information can be taught by providing multiple examples in explicit teaching or through an integrated curriculum approach as espoused in many middle years curriculum documents (Whitehead, 2005). Integration of curriculum, when thoughtfully approached by educators, leads to deeper, more complex schema in a variety of disciplines. Additionally, students must have opportunities to discuss the mathematical solutions to a variety of problems as "an emphasis on having students actively compare, reflect on, and discuss multiple solution methods is also identified as a key feature of expert mathematics instruction" (Jitendra et al., 2009, p. 252). An understanding of literacy practices, such as the need for explicit teaching of reading comprehension strategies including schema and visualization, has led to this shift in recommended teaching practices across the curriculum, including numeracy.

This literature review describes the currently advocated change in mathematics instruction and its relationship to a specific literacy strategy. The influence of research in reading comprehension by Allington, Pearson and Vygotsky on the development of specific pedagogy will become apparent as this review unfolds. My reading of the literature and research has made it evident that "mathematics is a 'language' all its own" (Phillips, Bardsley, Bach & Gibb-Brown, 2009, p. 468), but it is a language nonetheless. Consequently, adaptation and incorporation of literacy practices grounded in theory and research should help middle school students better relate, learn and understand mathematics.

Constructivism and Social Constructivism: Theory and Practice

The shift in educational pedagogy resulting from the math wars includes a movement towards social constructivism and student-centered learning (Draper, 2002). Students are encouraged to explore ideas in an inquiry based structure, to communicate and collaborate with peers in a variety of groupings and deepen understanding by using a number of different representations of learning. The basic tenets of social constructivism generally include: "active learning, positive social interactions, metacognitive awareness, and various ways for exploration" (Yore et al., 2007, p. 567) and are important to the new teaching of numeracy in our classrooms. These tenets have been used successfully in literacy classrooms to increase students' reading comprehension of various texts (Duke & Pearson, 2002) and can be transferred into various numeracy contexts for contemporary classrooms.

History.

Constructivism as an educational theory has a long, rich history. Although there are many definitions of this teaching philosophy, I use the one provided by Gordon (2009): "constructivism is based on the assumption that learners actively create, interpret, and reorganize knowledge in individual ways" (p. 39). Constructivist pedagogy is concerned with how knowledge is created both within the individual and community rather than with specific instructional strategies, even though instructional strategies drive pedagogy within the classroom. Glimpses of constructivism are found in the educational writings of 18th century French philosopher, Jean Jacques Rousseau, who stated that, "Instruction should involve dialogue between teacher and learner and should be centered around objects more than books" (Null, 2004, p. 184). Here the foundations of modern educational constructivism are characterized as dialogue, active learning and an increased partnership between student and teacher in learning. Over the years constructivism has borrowed ideas and attempted to transfer theories from other disciplines including: sociology, psychology, anthropology and neurology. Some aspects of these theories are very useful in creating pedagogy, but theories developed in other disciplines cannot be unproblematically transplanted into the field of education (Davis & Sumara, 2002; Gordon, 2009). This has created fracture and an overgeneralization of the philosophy within the educational system to the point that Phillips (1995) famously wrote, "constructivism, … whatever else it may be, [is] a 'powerful folktale' about the origins of human knowledge" (p. 5).

Despite these concerns and cautions, constructivism has an educational history based on research and promoted by the well-known and respected theorists such as Jean Piaget, Lev Vygotsky, and John Dewey.

Piaget.

In the realm of constructivism, Piaget is viewed as focusing on the construction of individual knowledge rather than collective knowledge. His work focused on young children but the general principles can transfer to any age group when a new concept is being introduced and learned because "new knowledge could be constructed only when the learner is confronted with objects that could not be assimilated into prior knowledge" (Harlow, Cummings & Aberasturi, 2006, p. 45). Piaget stressed that "children in particular- construct knowledge out of their actions with the environment" (Harlow et al., 2006, p. 45). Learning, whether about science topics, mathematics concepts or simply exploration of the learner's universe universe must be an active process and an interaction with the environment. It is this interaction that leads to the specific process of knowledge creation that occurs:

> if the exploration of the object or idea does not match current schema, the child experiences cognitive disequilibrium and is motivated to mentally accommodate the new experience. Through the process of accommodation, a new schema is constructed into which information can be assimilated and equilibrium can be temporarily reestablished. (Harlow et al., 2006, p. 45)

It is this tension between what the individual already knows and assumes, and new ideas that leads to new knowledge creation. If the child can simply reinforce current schema with the new ideas or concepts, then genuine new learning has not occurred. Piaget viewed learning and knowledge construction as a specific process with distinguishable steps: assimilation, accommodation, equilibration, construction, and internalization of action schema (Harlow et al., 2006; Phillips, 1995). Educators have been able to take his theories and create instructional strategies for many content areas. The use of manipulatives and inquiry based models are the practical result of Piaget's theories. For numeracy in particular, these strategies are encouraged in the NCTM Standards (2000) and the B.C. IRP for Mathematics (2007) as a means to engage diverse learners with higher and more complex numeracy concepts in the middle years' grades while meeting the needs of struggling learners in the same classroom.

Vygotsky.

Vygotsky is considered the leading theorist on social constructivism and he focused on how people especially children learn new knowledge within social constructs and situations. For Vygotsky, "learning is thus considered to be a largely situation-specific and context-bound activity" (Liu & Matthews, 2005, p. 388) because he believes that it is the social interactions that happen between individuals within the environment that influence the construction of knowledge. "Knowledge is not mechanically acquired but actively constructed within the constraints and offerings of the learning environment" (Liu & Matthews, 2005, p. 387) and this environment includes the semiotic systems used and the cultural influences placed on the participants. It is from these basic beliefs that Vygotsky defined the Zone of Proximal-Development. This zone is the area where students can achieve the understanding of new knowledge with the help of other experts, peers or teachers, but cannot be successful when left to rely solely on individually constructed knowledge (Cole & Walsh, 2004, p. 4). This process requires the collaboration of individuals and communication within a social context. Unlike Piaget, Vygotsky insisted that teaching focus on the potential not the actual learning ability of the student to ensure that learning and knowledge growth remain a progression (Gordon, 2009, p.

52). The process of socially constructed knowledge improves and challenges ideas in a manner that allows students to exceed individual capabilities. It is this belief that "provided means for educational researchers to access and incorporate [into constructivist pedagogy] several strands of critical inquiry that had not found a niche in the field" (Davis & Sumara, 2002, p. 416). Anderson (1996) noted that the focus on collaboration and on the influence of the world on the individual contributed to the philosophy that teaching is "an interactive process during which teachers and learners worked together to create new ideas in their mutual attempt to connect previous understandings to new knowledge" (as cited in Null, 2004, p. 182). In the teaching of numeracy, the influence of Vygotsky can be seen in the focus on small group interactions and the emphasis on collaboratively communicating thinking strategies among classmates.

Dewey.

Dewey wrote about the need to create knowledge by being an active participant in the environment emphasizing that "if we see that knowing is not the act of an outsider spectator but of a participator inside the natural and social scene, then the true object of knowledge resides in the consequences of direct action" (as cited in Phillips, 1995, p. 6). Dewey reflects the beliefs of Piaget that the student or child needs to explore and experience the environment in order to learn. The concept of active learning is used in numeracy by encouraging students to use manipulatives and apply knowledge to real world concepts and exploration. Despite the emphasis Dewey placed on active learning, in 1956 he cautioned that "in education, extremes are dangerous and that teachers should avoid approaches that either marginalize the needs, experiences, and interests of children or focus entirely on these factors" (as cited in Gordon, 2009, p. 48). The consequences of such extremism were experienced by the teacher in the case study by Gordon (2009) who tried to let students immerse themselves in constructivist and inquiry based learning in a Grade 9 mathematics classroom. The activities, such as independent study of mathematical concepts, large group brainstorming and written explanations for various problems were engaging and thought-provoking but in the end students commented that they were often confused by concepts when they had no guidance by the teacher. The teacher realized that constructivism could not be the only instructional approach utilized in the classroom and that there needed to be more integration of a variety of teaching philosophies to reach all students. "A good constructivist classroom is one in which there is a balance between teacher- and student-directed learning, and one that requires teachers to take an active role in the learning process, including formal teaching" (Gordon, 2009, p. 47), so the answer is not a wholesale application of constructivist activities and instructional practices in the numeracy classroom but a greater understanding of its role in successful learning across the curriculum.

Creation of knowledge.

Constructivist philosophy and pedagogy rely on one fundamental belief: knowledge is constructed and not just transmitted: "humans are born with some cognitive or epistemological equipment or potentialities… but by and large human knowledge, and the criteria and methods we use in our inquiries, are all constructed" (Phillips, 1995, p. 5). It is true that some things can be learned by transmission, such as the names of objects, multiplication tables and other factual items. However, in this case new knowledge is not created or constructed by the learner. It is the creation of new knowledge and understandings within students that is the focus of constructivist teachers and researchers. Sheat Harkness (2009) characterizes "knowledge as a dynamic process of inquiry, characterized by uncertainty and conflict which lead to a continuous search for a more refined understanding of the world" (p. 245). Learning should be the search for greater understanding and not just an accumulation of facts.

The teaching of numeracy has a natural affiliation with constructivist pedagogy, despite a lack of universal implementation of this partnership. Originally,

> the structures of mathematics were thought to be attained by the capacities for reason, logic, or conceptual processing. In this, mathematical structures were regarded as having a mind-independent existence, and the function of rationality was to come to know these fundamental structures. (Steffe & Kieren, 1994, p. 711)

Creativity and originality were not meant to be taught in mathematics because of the belief that knowledge already existed in the discipline, and so it just had to be discovered and transmitted effectively. However, it is now generally acknowledged that the development of knowledge in numeracy requires conjecture and construction; that is, "optimizing knowledge creation calls for norms that encourage creative problem solving and treating all knowledge as potentially improvable" (Bereiter & Scardamalia, 2010, p. 10).

Some mathematics teachers became educational researchers to better address the issues and problems experienced when implementing teaching research philosophies in classroom situations. The results of this research-teaching relationship led to the development of more constructivist strategies in the mathematics curriculum as described by the NCTM Standards (2000) and the British Columbia IRP for Mathematics (2007). The focus of these documents on strong conceptual understanding reflects the research in mathematics that reminded us that, "to become productive, knowledge must be lived by the learners. It must be worked with and used in various contexts, explored and questioned, connected not only with other explicit ideas but also with institutions and habits" (Bereiter & Scardamalia, 2010, p. 5). Despite all that is known in research and promoted in government documents, the pressure to achieve assessment standards has meant that these core beliefs have too often been ignored. As Bereiter and Scardamalia

(2010) made clear: “Knowledge Building is incompatible with the mile-wide, inch-deep curriculum, with its demands for rapid coverage of a multitude of topics” (p. 12). In this case Knowledge Building is defined as “the creation and improvement of knowledge of value to one’s community” (Scardamalia & Bereiter, 2010, p. 8). Thus, time constraints have meant that true knowledge building is not always possible in the middle years’ classroom. The pressures on teachers to meet curriculum assessment standards and guide students in developing deep conceptual understanding has led to conflict within the development and implementation of constructivist pedagogy.

Middle years’ studies.

Several research studies addressed the issue of knowledge creation by explicitly teaching students schema-based instructional strategies for numeracy (Jitendra, Hoff & Beck, 1999; Jitendra et al., 2009). The seventh-grade students in the study by Jitendra and his colleagues (2009) followed a specific sequence of activities to build on prior knowledge of numeracy concepts related to ratios. The concept of ratios and ratio proportions was first introduced with exemplars so that students could start to recognize generalities in the problem structures. Students were then expected to recognize the needed schematic diagram required to successfully solve the problem. The modeled, scripted lessons were followed by collaborative opportunities with peers where knowledge was constructed as a community. A pre-test was used to provide a baseline of conceptual knowledge and a post-test was given after the instruction to measure the influence of the schema-based instruction (SBI). Jitendra et al. (2009) were able to confirm their initial hypothesis that there was “a statistically significant difference in students’ problem solving skills favoring the SBI condition, suggesting that SBI represents one promising approach to teaching ration and proportion word problems” (p. 260). This study provides not only a successful example of knowledge creation in a middle years' classroom, but also, a template for diverse conceptual applications.

Activation of schema and knowledge creation based on prior knowledge was an important component of the study done by Pape (2004) when middle school students were observed solving a variety of word problems. Participants were videotaped while solving twelve word problems and the results were coded according to grounded theory procedures. From the videos it became apparent that students used either a Direct Translation Approach (DTA), "a predominant focus on numerals and direct translation of the problem into arithmetic operations" (p. 192), or Meaning Approach (MA), "focus on relational terms and the context of the problem" (p. 193), with some variations. The participants in this study confirmed previous research by Halford (1993) that "an accurate model is constructed through active transformation of the text base, activation of problem-type schemas, and integration of the problem elements within these schemas" (Pape, 2004, p. 189). The students who were most successful with the solutions were able to activate personal schemata and place the problems within a context (MA). "The more successful students provided evidence that they translated and organized the given information by rewriting it on paper, and they used the context to support their solutions" (p. 208). Students were able to accurately transfer skills, processes and concepts to new situations of learning. The success of these students emphasized the importance of knowledge creation in the constructivist classroom using the strategies of personal responsibility for learning and providing a variety of representations of learning.

Other studies.

In a study by Sheats Harkness (2009) a group of pre-service teachers were immersed in a constructivist mathematics course that focused on problem solving. This particular case study was part of a series of research investigations and a much bigger project. Although this study does not specifically involve middle years' students, the strategies used, such as collaboration and open-ended discussions are applicable to this age group (Perso, 2005). Students were required to work in small groups on two or three word problems during the class. Data were collected as videotapes, transcripts, reflections and interviews. The object of the study was to have pre-service teachers experience the structures and approaches of a constructivist classroom and the emphasis on developing new knowledge rather than simple memorization of numeracy facts. "In mathematics classrooms, students co-construct their knowledge through collaboration and meaningful tasks. When they do so, they make connections to previous mathematical understanding and refine their thinking; they are not empty vessels waiting for information deposits and accumulation" (Sheats Harkness, 2009, p. 248). By constantly challenging assumptions made by the students and encouraging mathematical thinking, the students were able to move beyond the "how" of problem solving to the "why". This shift demonstrated the construction of new knowledge by the individuals through collaboration and dialogue as encouraged in the IRP for Mathematics for Grade 7 (2007). The focus on the construction of knowledge rather than on the "right answer" requires a shift in philosophy towards progressive problem solving that many teachers do not often attempt (Scardamalia & Bereiter, 2010). Progressive problem solving requires teachers to continuously monitor student progress and challenge students to progress in their learning because

> people who become experts and who continue to advance in expertise are people who practice progressive problem solving; as parts of their work become automatic, demanding fewer mental resources, they reinvest those resources into dealing with tasks at a higher level, taking more complexity into account. (Scardamalia & Bereiter, 2010, p. 6)

It is this higher order thinking that most educators want for their students in order for them to become successful, productive citizens.

Lesh, Doerr, Carmona and Hjalmarson (2003) question the basic premise that all new knowledge needs to be constructed claiming that: "constructing is far too narrow to describe the many ways and nuances of ways that significant conceptual systems are learned" (p. 215). Instead they suggest students should use a variety of models and a modeling perspective to understand numeracy concepts. This theory has foundations in Piagetian research because these researchers believe that learning ultimately happens when there is disequilibrium between new information and the existing models or schema. The models and the modeling perspective are based on the following tenets:

> a) people interpret their experiences using models; b) these models consist of conceptual systems that are expressed using a variety of interactive media (concrete materials, written symbols, spoken language) for constructing, describing, explaining, manipulating, predicting or controlling systems that occur in the world; and c) models developed in and for the world are constantly interpreted and reinterpreted. (p. 213)

New ideas and concepts must be judged based on the individual's existing models of understanding in numeracy. These models are a more limited and specialized construction of the student's general schema compared to the Piagetian explanation of knowledge creation. Eventually, students are able to build new understanding and learning through resolving cognitive dissonance. It is important that teachers observe and evaluate the students during this process to prevent frustration and provide adequate scaffolding within each student's zone of proximal development as described by Vygotsky. The construction of new knowledge is not an instantaneous process, as "early understandings usually are characterized by fuzzy, fragmented, poorly coordinated, confused, and partly overlapping constructs that only gradually become sorted out in such a way that similarities and differences become clear" (p. 218). However, it is the ability to integrate new knowledge within current schemata that is a reflection of conceptual learning and transferability of skills.

Communication.

Communication is essential in the process of collaboration and knowledge construction within the constructivist paradigm. Without the sharing of ideas through a variety of semiotic systems, knowledge cannot be developed or created. "Learning is a generative process of meaning making, enhanced by social interactions" (Sheats Harkness, 2009, p. 246) and collective and collaboratively developed knowledge has a greater potential to be deeper and more resilient because it is based on several individual schemas. Vygotsky articulated the idea that: "for those who adopt the sociocultural approach, acting and thinking with others drives learning and at the heart of the process is dialogue and interaction" (as cited in Stephen, 2010, p. 21). The NCTM standards as devised in 2000 placed a key emphasis on communication which promoted a dramatic shift in numeracy practices and pedagogy.

This shift was reflected in the 2007 B.C. IRP, which emphasized the point that "students need to be encouraged to use a variety of forms of communication while learning mathematics" (p. 18). In order "for students to understand mathematical concepts they use language" (as cited in Hyde, 2006, p. 7; NCTM, 2000) which facilitates communication amongst peers, within the classroom and within the wider world. This heightened focus on communicating understanding rather than regurgitating answers has meant that numeracy classrooms must be more dynamic, interactive and relevant. Educators have adapted various literacy strategies to address these learning goals.

A research study.

Moss and Beatty (2010) worked with Grade 4 classes to study the value of collaboration in developing pre-algebra knowledge through on-line interactions. This research project attempted to address the reasoning process standards, a major component of the NCTM Standards which include: "making conjectures, abstracting mathematics properties, explaining reasoning, validating their assertions, and engaging in discussions and questions regarding their own thinking and the thinking of others" (p. 19). Despite the study participants being younger than middle years' students, the NCTM standards addressed and the numeracy concept are applicable to middle grades in British Columbia. The study was based on the premise that learning mathematics and numeracy is much more complex than memorizing algorithms. For eight weeks, students participated through asynchronous discussion threads and posts to "develop collective knowledge in progressive discourse" (p. 14). The results were dramatic when the final posts were examined. Students were able to create knowledge and demonstrated a deeper understanding of mathematical concepts than was usually experienced with grade 4 students. This success can be attributed to the dynamics of the dialogue situations that were created but not facilitated by the teachers. Students were strictly in charge of the discussion, and their "commitment to finding solutions, negotiating multiple solutions, and articulating justifications for conjectures meant that these students were able to work at a higher level of mathematics than has been previously shown" (p. 19). The use of technology in this study allowed students to take more responsibility for their learning than standard pen and paper activities often allow. The need in such discussions to articulate and justify thinking when solving mathematical problems supports the belief that a sense of understanding is further developed by verbalizing thoughts (Ridlon, 2009).

Challenges to collaboration.

Collaboration within the constructivist framework presents some challenges in the classroom. Collaboration relies on the ability of the teacher to create small groups that are homogeneous in academic abilities, talents or interests and that are supportive. Scardamalia and Bereiter (2010) make the point that “a supportive environment and teacher effort and artistry are involved in creating and maintaining a community devoted to ideas and their improvement” (p. 8). In numeracy classrooms there often exists a “hierarchy of students’ mathematical achievement and status” (Moss & Beatty, 2010, p. 7), but working in small homogeneous groups means that students are less affected by the structure and can experience a “democratization of knowledge”. As with any instructional strategy there are some concerns that occur because of the need to use a single strategy for a diversity of learners and individuals. Some students in a study by Ridlon (2009) expressed the opinion that they “enjoyed mathematics because they felt empowered and could make sense of mathematics themselves, [but] group work was slower, and not all members participated” (p. 213). In any situation where a group of individuals are expected to work together to solve a problem, there are always those who feel constrained by the group and those who are “freeriders” (p. 217). Active and responsive teaching during this classroom time would potentially prevent any serious issues. Teachers must offer guidance about the expectations for group work including the need for articulating points of difference and negotiating solutions.

Responsive teaching.

Constructivist teaching needs an active, responsive educator to lead the discourse and set the conditions for learning. This is because “observing and listening to the mathematical activities of students is a powerful source and guide for teaching, for curriculum, and for ways in which growth in student understanding could be evaluated” (Steffe & Kieren, 1994, p. 723). This paradigm cannot be successful in an environment where students are expected to work independently without guidance, scaffolding or explicit teaching: most students will simply flounder. Although the goal of constructivism is to give the learner more responsibility for acquiring and building knowledge, “the teacher’s role is to create the conditions, including the tasks and the tools, that support diverse ways of interpreting problem situations” (Lesh et al., 2003, p. 228). The environment that students participate in and focus their learning around must be carefully conceptualized and structured by the teacher to allow for inquiry within boundaries and limits (Peters, 2010).

Middle years’ studies.

In a case study by Peters (2010) a science teacher created a dynamic learning situation for Grade 7 students by integrating the topic of genetics and practice in oral language in the form of a mock trial. Despite the fact that this study takes place in a science context, the same principles of constructivist pedagogy can be applied to the numeracy classroom. Peters made a point of noting that: “In this study, the environment includes the physical set up of the room, the roles of the teacher and students in teaching and learning, and the assignment given to the class” (p. 342), and all of these factors contribute to the success of the constructivist method. The results of Peter’s study supported the general research findings about inquiry based learning that: “Inquiry is linked with many positive student outcomes, such as growth in conceptual understanding, increased nature of science knowledge, building relationships between the student and teacher, reducing errant learning, and development of research skills” (p. 330).

There are startling data in the research on numeracy skills in middle school and the decline in students’ mathematical self-concept (Hackenberg, 2010; Reid & Roberts, 2006; Ridlon, 2009), especially in constructivist classrooms that support the claim that students simply are not learning or engaging in mathematics in later years: “By about age twelve, students who feel threatened by mathematics start to avoid courses, do poorly in the few math classes they take, and earn low scores on math achievement tests” (Ridlon, 2009, p. 188). Hackenberg (2010) supported the finding that girls in particular are vulnerable to attacks on their mathematical self concept which is the “collective perceptions of one’s ability to do and know mathematics, and such perceptions are formed in relation to others” (p.61). Essentially this suggests that those who can do computations well absorb their mathematical ability into an enhanced sense of their identity. The development of mathematical self concept became problematic in Hackenberg’s study when the researcher realized that she was not being a responsive teacher and was failing to accommodate the student’s need for further explanation. The teacher had created a constructivist environment which focused on learner centered approaches but the teacher’s role was limited. In the end, the student was unsuccessful and distrust had formed between teacher and this particular student.

Ridlon’s (2009) study was an ambitious project based on constructivist approaches that lasted two years during which time she worked with two groups of grade six students. In this case, “educational reform comes from the mobilization and coherence of forces both within the school (administrators, teachers and students) and outside the school (parents and community)” (p. 189), as there was a perceived need to address numeracy concerns with middle school students. The research and reform that occurred was school initiated rather than implemented by a government directive. Selected students were placed in a class that adopted the Problem Centered Approach, students focused on learning numeracy concepts through word problem strategies and explicit connections to authentic tasks. In this classroom, the teacher no longer assumed the role of the authority in mathematical knowledge and the traditional transmission model of instruction was abandoned. Students were encouraged to work in small groups to solve a variety of problems and then share the solution and methods with peers. The teacher was able to support student thinking and initiate classroom dialogue but the key to allowing students to explore ideas was that the "teacher remains nonjudgmental because the viability of solution methods is determined by the class, not the teacher" (p. 196). This simple procedural difference meant that the students were empowered by their peers and themselves to create new understandings. This empowerment led to greater risk taking in learning and a deeper comprehension of numeracy concepts through the understanding that "teachers must develop each student's mathematical power by respecting and valuing their ideas, ways of thinking, and mathematical dispositions" (p. 199). Knowing the students and their abilities meant that the teacher was able to offer problems that were engaging, challenging and that addressed learning outcomes. If the teacher had not been responsive to student needs in this situation, students would have become increasingly disengaged and groups ineffective. It is the complex need to know when and how to scaffold students so they are successful in their zones of proximal development that creates some problems with constructivist classrooms. "Discussions about how and when to scaffold, and what kinds of adult actions and interactions move children to new understandings and competences with the tools of their society are less common" (Peters, 2010, p. 24), and their rarity indicates a need for such a focus in ongoing professional development available to teachers. This ability to recognize a need is not easily taught and learned but can be mastered with repeated practice and attention. In the end, students who participated in the test group "had a significantly higher gain achievement than those in the traditional explain-practice group" (Ridlon, 2009, p. 221) and their attitude towards mathematics also improved. These encouraging results highlight the need to make numeracy tasks relevant to middle years' students in a manner that is responsive to their needs, abilities and interests.

Concerns related to responsive teaching in constructivist classrooms.

Despite "research from a variety of theoretical perspectives [which] suggest[s] that a defining feature of a supportive environment is a responsible and responsive adult" (Peters, 2010, p. 21), this characteristic of constructivism is not always apparent in some classrooms. Student-led learning, inquiry based activities, small group learning and a focus away from the teacher - all tenets of constructivist philosophy- have meant that some educators remove themselves from the classroom all together. Sriprakash (2010) found this to be the case when observing several students in India following the government imposed programs related to child-centered learning. In some classrooms the teachers tried to follow the philosophies despite a lack of resources and time. However, in other classrooms the situation was much different, "to a point, the notions of children's independence and responsibility were used to justify non-active teaching and even absence, despite the high demands made by the child-centered pedagogy of teachers" (p. 303). These teachers took these opportunities to completely disengage from the classroom community and in some cases physically leave. This is a danger of constructivist pedagogy when not carefully considered or fully understood by teachers and utilized in an inappropriate manner.

A second concern for constructivism related to responsive teaching is the need to teach explicit skills for student centered learning. This concern was raised by several students in the studies (Gordon, 2009; Ortiz-Robinson & Ellington, 2009; Peters, 2010; Ridlon, 2009; Wohlfarth et al., 2008) when they participated in interviews and surveys completed at the end of their courses. Students felt they needed explicit teaching of skills related to discussion within small groups, research methods, organizational tools and ways to determine the importance of information discussed within a group. Any hesitation felt by students towards the constructivist classroom was often related to apprehension about their abilities to cope with greater responsibility for their own learning (Ortiz-Robinson & Ellington, 2009). Students who had the most difficulty were students who were unaccustomed to the constructivist teaching approach.

Meeting diverse learning needs.

Constructivist instruction can meet the needs of a diverse group of students with explicit instruction of activity expectations and goals because it does not rely on the notion that one set of structured activities will be a learning panacea for everyone. The premise of schema theory that knowledge is created and influenced by the background experiences of the individual supports this understanding. In fact, learning and knowledge construction are most successful when teachers apply “curriculum in harmony with the child’s real interests, needs, and learning patterns” (Chung & Walsh, 2000, p. 215). It is not strictly the material and concepts that are paramount in the classroom but rather the individual students who are the key component of the learning dynamic.

When teaching numeracy, it is important to understand that students come with different home experiences that influence their conceptions of abstract mathematical ideas (Street et al., 2008). Mathematics and arithmetic have traditionally been taught via an authoritarian or transmission model, but changes as a result of the “math wars” have led to a better understanding of the role of schema in learning. We now better appreciate the extent to which “ teachers should consider prior student knowledge when they plan lessons, as well as the notion that teachers should make learning as natural as possible” (Null, 2004, p. 181). A constructivist approach to numeracy instruction means that teachers understand that “children with different developmental backgrounds may be able to get the same answers on an arithmetical task, but the ways in which they do so might differ significantly" (Steffe & Kieren, 1994, p. 719). Not only is this the reality in a classroom of diverse learners, but these differences should be celebrated and shared within the community to support the creation of collective knowledge. Following the research of Vygotsky, students become the expert peers through collaboration and collective knowledge creation who can help each other move into their zones of proximal development. In the end, the learning community benefits from the constructed knowledge of individuals.

Inquiry method of instruction.

The inquiry method of instruction that coexists within the constructivist framework allows students "the choice of the topic, methods, processes, and resources" (Peters, 2010, p. 345). The tasks are open ended questions or word problems, in the case of numeracy, that students can interpret in a variety of ways within a constructed framework to ensure targeted learning outcomes are the goal. It is this flexibility in resources, methods and products that allows students to meet individual needs within the classroom. Students are encouraged to explore questions through a variety of methods that suit their learning styles, and share the outcomes with peers. This exploration is not only necessary for active learning, but is also a reflection of learning outside the classroom because "children learn to acquire the tools for thinking and acting through observation and participation in authentic tasks that are a part of everyday life" (Stephen, 2010, p. 24). This focus on experiential learning is supported by the research and writing of Vygotsky on childhood development and learning. Active learning is essential for building knowledge because new information cannot be discovered or acquired without interacting with the world and others. It is this interaction that creates the personal experience and the resulting diversity of the learning because "individuals construct their own reality through actions and reflections" (Steffe & Kieren, 1994, p. 721).

Middle years' studies.

Street et al. (2008) conducted an ethnographic study of numeracy practices that involved observations of three schools and included case studies of three students over three years. These students were considered at-risk: minimally achieving grade level in numeracy. The question researched was how school practices and home practices related to numeracy affected student achievement. Results showed that school numeracy practices were "top-down: they [teachers] were concerned with the activity (concept) skill that was centrally decided [government curriculum] on by the teacher" (p. 32). In these situations, students were assessed using traditional pen and paper tasks. In the classrooms that relied on transmission models of instruction students struggled to comprehend numeracy concepts and apply their learning to various situations. Conversely, home practices were solution driven and exploratory in nature. For example, one student played card games with his brothers to learn his numbers while another student helped with the grocery shopping and paying for the items. Students showed an understanding of numeracy when encouraged to explore a variety of contexts, practices and processes to arrive at a solution. This finding supports the need to allow students to utilize the tenets of constructivism within the numeracy classroom.

One research project with middle school students that seemed to successfully increase numeracy comprehension was conducted by Reid and Roberts (2006). The GO GIRL project was designed to address the decline in academic success of middle school girls in mathematics. As the authors report "the middle school years are a pivotal point for many girls and often the beginning of a downward spiral of competence and confidence" (p. 300). However, in this age group, boys do not seem to experience the same decline in mathematical self-concept (Lowrie, 2011). The pilot project was an intervention program that created a mentor relationship between middle school students and university graduate students. Through interviews, surveys, observations and journal entries, the researchers determined that the implemented framework of support that was based on the principles of social constructivism had enhanced the understanding of the girls and created a foundation of success. There was a "significant increase in attitude and ability" (Reid & Roberts, 2006, p. 296) as a result of the program. Small group interactions, exploration of concepts through journal entries and the personal collaboration with mentors were all cited as important contributors to the girls' success. Thus, it is important that "constructivist teachers reject the transmission model of teaching or the pedagogy of control or telling" (Draper, 2002, p. 522) as the sole means of instruction, so middle school girls can be more likely to experience success within their classrooms.

Obstacles to implementing constructivism.

There are numerous benefits to implementing constructivism in the classroom that are documented by research. So, why then are more teachers not embracing the pedagogy and using its tenets to teach numeracy for middle years' students? The answer may in fact be quite simple: "the standards themselves fall short in the guidance they offer to teachers who lack the experience and confidence to teach in a way they were not taught themselves" (Marshall, 2003, p. 193). It is never enough to mandate reforms and new curriculum without offering practical support to teachers for implementation. Professional development opportunities need to be created and offered to address the issues of teachers' mathematical self efficacy (Ly & Brew, 2010) which are beliefs in personal abilities related to mathematics and their pragmatic concerns related to using a constructivist model in classrooms with a diverse range of students. Only a multi-faceted approach will succeed in providing support for teachers implementing the standards in the spirit intended. As Stephen (2010) points out:

> … policy, curriculum and practice guidance, the experience of initial and continuing professional education, personal beliefs and value systems and the community of practice in which they work can all be expected to make a difference to what practitioners do and, therefore, to the experiences of children.(p. 19)

This professional development needs to be based in research, but must not rely on theory as the only foundation; positive, practical experiences provided by expert teachers are essential.

Constructivist classrooms require teachers to provide guidance and structure while giving students substantial responsibility for their own learning. Gordon (2009) points out that "research shows that even experienced educators have a hard time putting this type of instruction into practice because of the multiple challenges it poses for teachers, such as managing classroom interaction, understanding content, and assessing student knowledge" (p. 43). The instruction strategies paradoxically must be structured and yet flexible enough for active learning and subjective knowledge construction to occur. Consequently, "well-mastered routines are necessary in order to free conscious attention as much as possible so that it can focus on aspects of a task which are novel and problematic" (Marshall, 2003, p. 200). This idea applies to learning a specific numeracy concept, understanding the appropriate ways to collaborate as a group, and the need to develop structure and routine in a learning community in general. Teachers must always be monitoring student learning for misconceptions or frustrations and through responsive teaching provide appropriate scaffolds for learning specific concepts. These qualities and abilities cannot always be acquired from reading research or attending workshops, but often are the result of numerous attempts to take risks with teaching strategies and try new frameworks with different groups of students. As a result, "it is they (first-rate young-at-heart personalities) who must be the solution, and they must be nurtured" (Marshall, 2003, p. 200), for these are the teachers willing to explore options for pedagogy frameworks and most likely those who have already been encouraged to implement constructivism in their own educational experiences.

Ly and Brew (2010) studied a cohort of Vietnamese pre-service teachers and a cohort of Australian pre-service teachers to determine the effects of philosophical beliefs on implementing constructivist tenets in the mathematics classroom. The impetus for the study was the changes in curriculum in both nations that had been driven by educational reforms in many disciplines. Ly and Brew comment that “pressure for curriculum change has not only come from global pressures and government initiatives, but also from demands of other teachers, pupils and parents who are aware of the outdated nature of the curriculum” (p. 69). Despite reforms being mandated by governments, change has been slow in Vietnam due to a lack of knowledge, training and skills in implementation. Teachers are still relying on traditional means to teach numeracy and authoritarian structures within the classroom. New teachers and pre-service teachers responded that they “believe mathematics is a creative endeavor” (p. 76): however, there was a lack of connection between beliefs and instruction as observed in lesson plans. This problem can be attributed to the concept of teacher self efficacy; that is, “the extent to which the teacher believes he or she has the capacity to affect student performance” (p. 71). Teachers who did not feel empowered were less willing to implement constructivist frameworks and beliefs in the classroom, because according to Ly and Brew, “teachers who feel less self efficacious are likely to want to create a greater sense of control and authority in the classroom among their students” (p. 82). Many are afraid to be challenged on their own level of understanding by students so they avoid situations where this will occur. It is only through time and experience that teachers will be able to improve their perceptions of their competency in numeracy with resultant positive impact on self-efficacy. Supportive learning communities within schools and thoughtful professional development will encourage this change and growth (Carrington, Deppeler & Moss, 2010).

A final consideration for the implementation of constructivism in the classroom is the understanding that constructivism is a pedagogy. Alexander (2004) defined pedagogy as “the act and discourse of teaching,” (as cited in Stephen, 2010, p. 17) and the application of professional judgments or “any conscious activity by one person designed to enhance learning in another” (p. 17). It is not as simple as providing opportunities for students to collaborate and have dialogue in small groups or provide a variety of resources, methods and assessment choices. Constructivist teaching is a mindset that must dictate all aspects of how instruction is delivered, measured and assessed. It relies on trust and a belief that students want to learn and are capable of advocating for their learning needs in an effective manner within the classroom. This idea can be very difficult for some educators, because “although all educators do not have the same mindset, many see teaching in terms of controlling: what students learn, how they learn, and how the learning is measured” (Reynolds, 2006, p. 55). Relinquishing control to students in an attempt to engage, motivate and develop a sense of personal responsibility can be frightening for some educators, particularly in the climate of accountability that is the consequence of the current emphasis on standardized testing. The benefits of adapting existing and familiar literacy strategies, such as visualization, in the numeracy classroom addresses some of the concerns related to time constraints and standardized testing.

Visualization and mathematics

Visualization has been a popular strategy used for teaching reading comprehension for many years. Throughout this literature review, visualization is defined as “the construction and formation of internal images (e.g. mental images) and/or external images (e.g. with the aid of pencil and paper) and then using those images effectively for mathematical discovery and understanding” (Zimmermann & Cunningham, 1991 as cited in van Garderen, 2003, p. 246). It is widely accepted that the ability to think in images requires a different activation of schema than does simply working within words and printed text, and we now recognize that “one stark difference between individuals is the degree to which they appear to think with images, finding diagrams and pictures more useful than explanations in words” (Woolner, 2004, p.18). The New London Group (1996) in their seminal work about multiliteracies articulated a need to reach all students in instruction by using a variety of methods. They stated that: “the mission of education…is to ensure that all students benefit from learning in ways that allow them to participate fully in public, community, and economic life” (p. 60). As a result teachers need to include alternative methods of communication and a variety of literacies in their classrooms. This does not imply that teachers abandon traditional instructional strategies to commit fully to images and pictures in teaching. Not all ideas can be taught in images neither can all ideas be represented in words.

The strategy of visualization has been used successfully to teach reading comprehension in literacy classrooms: “it is not surprising that decades of research have proven that, indeed, getting students to create visual images before, during, or after reading is a viable way of enhancing comprehension” (Wood & Endres, 2004, p. 346). Wood and Endres (2004) conducted a study to investigate the creation and application of a specific strategy based on visualization. The imagine, elaborate, predict, confirm (IEPC) strategy was tested with Grade 4 students using both fiction and nonfiction texts. The results confirmed that students were able to demonstrate increased comprehension and engagement with the text. Further research has been done in a variety of classrooms at different age levels including middle school, and the results have been consistent. Overall, the strategy of visualization has been used to increase connections across symbol systems and text mediums in literacy. Through modeling and explicit teaching of the strategy for aiding reading comprehension, students have demonstrated a deeper understanding of concepts and text (Hibbing & Rankin-Erickson, 2003; Lapp, Fisher & Johnson, 2010). It is the successful application of visualization in literacy that makes the strategy appealing in the contexts of numeracy classrooms.

Consequently, Hyde pursued research in various classrooms by applying literacy strategies to numeracy instruction. Hyde is a strong supporter of visualization techniques in the teaching of mathematics, noting: “there are two ways students use visualization in mathematics that should come as no surprise: creating mental images as they read and creating representations of their mental images” (2006, p. 67). This statement does not appear to be revolutionary and most literacy teachers would agree that this process is happening when students are attempting to comprehend text. In fact, this strategy can be explicitly taught, scaffolded, guided and encouraged by literacy teachers. However, can numeracy teachers say the same? It requires a shift in understanding and pedagogy to realize that

> for every symbol that students write there must be a concrete referent in their heads of what that symbol refers back to. They must be able to conjure up a mental picture, an image of some sort, that this symbol is a symbol of something specific, or we might say that this symbol represents something specific. (p. 86)

Checking understanding verbally with students by asking them to explain the significance of the symbol and through multiple experiences allows students to create a deeper understanding of numerical concepts. This process reflects the beliefs some researchers have about how students learn in general: that they “engage prior knowledge, organize knowledge, monitor and reflect on [their] learning" (Hyde, 2004, p. 11). Visual images allow students to follow this process with concrete examples and representations to guide their thinking and learning. In 2007 the British Columbia government included visualization explicitly within the new curriculum package for mathematics, stating that "visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers" (p. 19). The IRP stressed that "visualization is fostered through the use of concrete materials, technology and a variety of visual representations" (p. 20). Once these ideas appeared in the mandated curriculum documents it was expected that educators would begin including the related strategies and pedagogy in their current practice. This does not always happen and teachers often require more direction and guidance before they are able to implement the concepts of visualization to meet the needs of a diverse and specific classroom of students.

However, there are several benefits to adapting the literacy strategy of visualization in the numeracy classroom. Visual imagery described by Hegarty and Kozhevnikov (1999) as "a presentation of the visual appearance of an object, such as its shape, color, or brightness" (p. 685), can be used to introduce students to problems and new concepts. Kotsopoulos and Cordy (2008) used this specific sequence of visualization to integrate mathematics and science in a Grade 7 classroom. Students responded to the use of the visual imagery with increased engagement and a deeper comprehension of the abstract concept being investigated. In this particular case, the visual imagery was used as a hook to gain the attention of the students. Visualization can also be applied throughout the lesson sequence in numeracy contexts that reflect the positioning of before, during and after reading text in literacy contexts. In this case, the representations are flexible because "a representation, be it complex or simple, is [a] vital part of explanations teachers provide of new concepts, illustrations of problem-solving processes, and creating connections among concepts" (Stylianou, 2010, p. 329). Here, the representation, "a configuration that stands for something else" (Stylianou, 2010, p. 266), becomes a way for teachers to observe the internal schema of a student. Thus, the strategy of visualization is an important means for teachers to check students' understanding of abstract concepts (McVee, Dunsmore & Gavelek, 2005) and to create responsive lessons. Finally, visualization supports an integrated approach for teaching the curricular disciplines in middle school. In the study by Begoray (2001), teachers were encouraged to incorporate viewing and representing strategies into their current practice. Over the two-year period of the study, Begoray observed or teachers reported more than 70 different approaches of visual imagery in a variety of contexts (p. 206) used in this middle school. The study confirmed that students experienced increased engagement and deeper comprehension of ideas and "because middle-years students are still moving from concrete to abstract thinking, their use of visual representations seemed to enhance their learning" (p. 210).

A muddle of visualization terms.

There are many different terms specific to the strategy of visualization, including imagery, representation and visual imagery. Each of these terms has been used by teachers and researchers to describe a process or a product related to a visualization activity, but there is some inconsistency in their use. Stylianou (2010) recently noted that, "researchers in mathematics education do not always agree on what representation means" (p. 362). This statement highlights a small piece of the fuzzy definitions puzzle: academics cannot agree on important definitions related to their research. As a result, teachers in the classrooms are inconsistent in their understanding and use of the various terms. For example, "teachers' conception of representation as a process and a mathematical practice appears to be less developed, and, as a result, representations have a peripheral role in their instruction" (Stylianou, 2010, p. 325). Consequently, specific definitions for the important terms are provided throughout the literature review and the glossary located at the end of Chapter 2.

Purpose of visualization.

The purpose of incorporating visualization techniques in mathematics is not strictly to act as a motivational tool for lower achieving students, although there is some evidence that this may be a factor. "Illustrations frequently serve an affective or motivational function for students" (Hibbing & Rankin-Erickson, 2003, p.762), especially when encountering text that is dense or difficult to comprehend for a variety of reasons. For this literature review, illustration is defined as a drawing or picture that reflects the text and the mental imagery associated with the text (Hibbing & Rankin-Erickson, 2003). Not only can visualization engage students in an activity but it also has the specific purpose of helping organize information and contribute to comprehension of the task. In fact, "students who lack the ability to create visual images when reading often experience comprehension difficulties" (Hibbing & Rankin-Erickson, 2003, p. 758). This applies to whether students are reading a story, textbook or mathematical problem. If students cannot comprehend the meaning and information that is communicated within the text, they will not be successful in providing accurate solutions for a variety of problems. Thus, "visual imagery has a role in establishing the meaning of a problem, channeling problem-solving approaches, and influencing cognitive constructions" (van Garderen, 2006, p. 496). Visualization then becomes a tool and a means to organize ideas related to mathematical questions. Visualization should not be dedicated only to the domain of word problems but can be effectively used in any number of situations if taught explicitly and reinforced regularly as a comprehension strategy. Hyde (2006) discussed the importance of visualization and the need for educators to move beyond the understanding of imagery as strictly pictorial representations. For word problems, he defined pictorial representations as those “that depict the physical appearance of the elements described in the problem” (Zahner & Corter, 2010, p. 180). Visual imagery which is the representation of an object (Hegarty & Kozhevnikov, 1999), and visualization, the act of bringing an image into mind (Kotsopoulos & Cordy, 2009) then move beyond a literacy comprehension strategy to become a tool for communication in mathematics.

Visualization and visual imagery are multiliteracy practices in numeracy instruction. Thus, it is through this integration of various literacies that teachers demonstrate an awareness of the importance of meaning making in any situation within the classroom, because “in a profound sense, all meaning-making is multimodal. All written text is visually designed” (NLG, 1996, p. 81). Today, students are not limited to communicating in the single literacy format of traditional print text, but also use visual images in a variety of situations to share ideas, construct understanding and develop schemata throughout their daily lives. For example, students utilize social media such as Facebook and text messaging which integrate icons and images with written text. As teachers of numeracy and literacy we can use these so-called ‘hidden literacies’ as a springboard to academic literacies (Perry, 2006) and extend our understanding of how our students make meaning based on their experiences, interests and needs (Cervetti et al., 2004; Sriraman, 2004). Thus, it is imperative that teachers consciously plan learning opportunities about mathematical concepts with attention to students’ background knowledge and schema activation as a component. After all, these personal experiences influence the types of visual images and representations students can create and accurately interpret in numeracy contexts.

Middle years' studies.

In Pyke's (2003) study 174 eighth grade students completed eight questions to determine whether students were able to use representation, referential, and transformative strategies in their visual representations. Representation strategies meant students reproduced the information in the same text type; referential strategies meant students translated information from written text to imagery or vice versa; and transformative strategies meant students transferred information from visual or written text to mathematical number systems. The results showed that students were more successful with representation (text-text/image-image) and referential (text-image/image-text) than transformational (numbers/formulas) (p. 420). Pyke (2003) discovered that "many students tend to view diagrams as pictures rather than resources that might reveal important information about a problem's structure" (p. 425). Students in his study had not been explicitly taught the value of creating and using dynamic diagrams to test ideas and solution strategies so the diagrams became models that were used only at the end of solutions for justification rather than as tools for solving the problem. This finding stressed the need for instructors to understand the processes involved in creating visual imagery in order to support a more relevant use of its forms. As Pyke points out, "representations produced by students do not emerge simply, automatically, and naturally from an external presentation into a single mental image in the mind" (p. 428). There are a number of different schema connections that must be made including: identifying the objects presented, placing the objects within a known context, identifying the problem structure, recognizing previous solution processes and finally applying this knowledge in a cohesive manner to communicate understanding. It is the understanding of the role of schematic structures and activation of prior knowledge that allows teachers to effectively and efficiently provide instruction to meet students' needs in a numeracy classroom.

In 2001 Lowrie investigated the problem solving strategies used by sixth grade students. The focus of the study was “to identify how visual and nonvisual (verbal) abilities effect problem solving in mathematics” (p. 355). Students were given a set of 20 questions to solve and a variety of other measures were used to examine related factors in mathematical success including: preference efficiency measure, numeracy measure and item difficulty questionnaire. The results showed that “boys were more inclined to represent [with numerals and formulas] and solve problems” (p. 359). This result reflected a tendency for the boys in the study to use mathematical formulas rather than the pictorial representations used by the girls. However, a conclusion was made that “students who predominantly used visual methods outperformed the students who predominantly employed nonvisual methods” (p. 360). Apparently, using visualization in the classroom in middle school is an effective way to help students solve mathematical problems: “teachers should provide students with opportunities to develop powerful visual representations when engaged in problem solving- particularly when the tasks are novel or complex” (p.360). This study confirms that visualization has a place in the instructional strategies of numeracy as well as literacy.

Based on Lowrie’s (2001) study results and the belief that visualization should be used to teach numeracy, Steele (2008) created a small research study with grade seven students. The goal of the study was to investigate the methods that middle years’ students use to generalize patterns in growth and change problems. The research question focused on the ways “students use representations to make generalizations within and across pictorial growth and change problems related in pattern structure” (p. 100). In this case, representation is defined by Stylianou (2010) as “a configuration that stands for something else” (p. 266). Students were given eight problems to solve without any teacher or researcher modeling or guidance. For example:

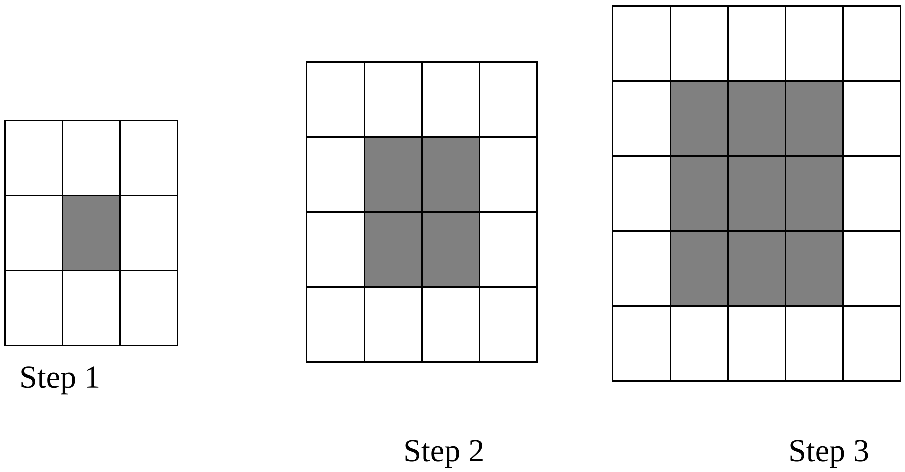

Figure 3. *Borders and Blues* pictorial growth and change problem (Steele, 2008)

Students shared their solutions in small groups and the discussions were audio taped for further analysis. In the example above, students were instructed to provide the pattern for the shapes in an algebraic equation that could be generalized to any number of squares. The results showed that "all eight students demonstrated flexibility in linking different external representations-pictorial, verbal, tables (lists), and symbolic notation" (p. 104). Even though the study was conducted with a very small number of students (only eight), the results confirm that visualization as a strategy can play a useful role in a numeracy classroom. The implications described in the study can be applied to other middle years' grades and a broader range of students in terms of ability and learning needs. For example, students used a diagram, "a visual representation that presents information in a spatial layout" (Diezmann & English, 2001 as cited in Pantziara, Gagatsis, & Elia, 2009, p. 40), as a means to picture the change in the representations and identify repeating patterns. In this manner, visualization was an invaluable tool for the students to organize and construct knowledge.

Other studies.

Tanisli and Ozdas (2009) studied the abilities of fifth grade students to detect patterns in a variety of situations based on prior experience or existing schema. Although this study focused on a younger age group, the results are relevant to middle years' students because similar concerns are apparent in grade six. Tanisli and Ozdas based the study on the premise that the identification and application of patterns "is a key concept for understanding of mathematical knowledge and concepts" (p. 1486). Being able to recognize when to activate certain problem solving schema makes success possible for students. The results of the study showed that "in the visual approach, students focused on the structure of the shape and in this context, they used the strategies in recursive [manners]" (p.1489). Many students were able to use schemata specific to the problem and the context in an appropriate manner, in this case, visual imagery was a dominant tool. Low achieving students focused on the visual imagery as a problem solving strategy, which indicates the activation of a specific schema, but high achieving students used numerical and visual strategies to complete the problems (p. 1492). These more able students were able to access multiple schemata and utilize visualization in an appropriate numeracy context.

Forms of visual imagery.

There are many different forms of visual imagery that are relevant to the understanding of mathematics, but all forms can be placed within three categories: internal images, external images, and manipulatives. Students must use visual imagery and representations to understand the abstract concepts that are central to mathematics learning (Pape & Tchoshanov, 2001). The definitions for the three main categories are provided by Pape and Tchoshanov (2001) and are general enough to apply to many studies addressed in this literature review. Internal imagery is the “abstractions of mathematical ideas or cognitive schemata that are developed by a learner through experience” while external imagery is the “manifestations of mathematical concepts that ‘act as stimuli in the senses’ and help us understand these concepts” (p. 119). The usefulness of manipulatives is that they are concrete objects that can be manipulated to represent mathematical concepts. These terms are fully described in the following sections.

Internal imagery.

Internal visual imagery is a reflection of the learner’s current schemata and cannot be objectively observed by a teacher. This is because “mental imagery is generally agreed to be the process of forming internal pictures of objects or events not present to the eye that can affect later recall and comprehension” (Douville, 2004, p. 36). However, mental images are an intrinsic part of the learner’s schema and allow appropriate connections to happen between old and new knowledge. These mental images are differentiated according to the individual, the context and the environment. Douville (2004) made a distinction between basic metal imagery and complex imagery when she noted: “it is important to note that multisensory images are not only more consistent with those generated by effective imagers, but are also those that are more elaborated and therefore more memorable for students” (p. 36). Teachers must move beyond the simple instructional strategy of basic pictorial visualization to a more complex schema activation that is multisensory to assure retention of the skill and concept. It is not enough to have students activate schema and create mental images if we are not attempting to build and make connections through that schema for further learning. Allowing students to build individual mental images as a reflection of personalized schema can be a very powerful tool when constructing external diagrams as representations, and “a number of studies have shown that self-constructed diagrams are powerful heuristic strategies in problem solving” (Pantziara et al., 2009, p. 42). After all, it is the internal mental images that facilitate cognitive function in a meaningful manner and lead to vivid, detailed external imagery or representations. Ultimately, external representations and imagery provide a window to a student's level of understanding because teachers "can make inferences about a student's internal representations on the basis of his or her external representations" (Steele, 2008, p. 98). It is the relationship between the internal and the external imagery that teachers must recognize and value for guiding instruction in a constructivist numeracy classroom.

External imagery.

External imagery is the "manifestations of mathematical concepts that 'act as stimuli in the senses' and help us understand these concepts" (Pape & Tchoshanov, 2001, p. 119). External imagery includes a variety of visual imagery including graphic organizers, manipulatives, symbol systems, numbers, formulas, graphs, diagrams, schematic representations and pictorial imagery. External visual imagery is meant to be an accurate representation of the mental or internal imagery, thus it can be very simple or very complex.

Graphic organizers.

Graphic organizers are "visual representations that help students identify, organize, and remember important information" (Boardman et al., 2008, p. 23). This form of external imagery has been used successfully in literacy contexts through a variety of standard formats including Venn diagrams, flow charts and mind maps. In fact, Duke and Pearson (2002) explicitly discuss visual representations as a meaningful strategy for improving reading comprehension because "a visual representation is … [a] *re*-presentation; literally, they allow us to present information *again*" (p. 219). It is this reconstruction of information and transformation into another medium that allows the student to demonstrate a deeper understanding of abstract concepts. Similarly, complex schemata of numeracy concepts need to be nurtured, developed and explicitly taught: "there is growing evidence regarding the benefits of explicit schema training using visual representations on students' learning of mathematics" (Jitendra et al., 2009, p. 252). These visual representations referred to can be several different graphic organizers that help organize information and essentially become an external representation of the internal schema of the student. In numeracy contexts, students typically use graphic organizers to organize information based on word problems.

Middle years' studies.

Zollman (2009) had students use a four square and a diamond organizer in nine middle school classrooms. Students were given a pretest of word problems to solve without the help of the organizer and then a posttest after instruction was provided. The students were explicitly shown how to use the organizer and then participated in guided practice. This particular strategy allowed "the student, and the teacher, to identify missing information or absent connections in one's strategic thinking" (p. 5) so that follow-up instruction could happen for clarification. The organizer became a symbolic representation of the student's schema as, "it allows a student to quickly organize, analyze, and synthesize one's knowledge, concepts, relationships, strategy, and communication" (p. 5) so that errors in schema construction can be addressed effectively. Teachers can determine where students are confused about certain aspects of the mathematical problem by quickly observing the completion of the boxes in the graphic organizer.

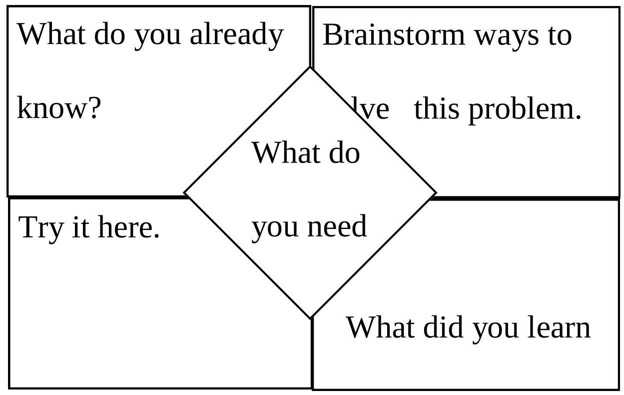

Figure 4. Four square and diamond organizer (Zollman, 2009)

The results indicated that students of all abilities were able to improve their success with word problems after explicit instruction and use of the graphic organizer.

Peled and Segalis (2005) conducted a study in Israel in an attempt to address concerns about poor performance on standardized international tests. From their investigations with two grade six classes, it became clear that students were expected to memorize procedures with limited understanding of the numeracy concepts behind the processes. This situation meant that students were not using a variety of external imagery strategies to help organize information and contribute to understanding. Teachers need to “facilitate connections by identifying the related structures for students, by guiding them to abstract a general schema, and by using several number domains” (p. 209) before a conceptual understanding can be realized. The intervention employed by the researchers was to explicitly teach the process of making connections and to map solution steps to various subtraction problems. Further, “in the process of generalization, the child was encouraged to make connections and map steps in the second procedure to steps in the first procedure” (p. 214). In this case, students drew and wrote about the various steps required to solve subtraction questions. The reinforcement of a sequenced, structured approach to the steps created a flexible graphic organizer for the solutions. This process demonstrated that those students able to make connections and use the map process effectively were more successful in solving the mathematical problems.

Manipulatives.

Manipulatives are an intrinsic part of mathematics instruction in many classrooms. Students are introduced to the concept of addition by physically grouping objects, or in some cases, specially designed base-ten blocks, so the process of the concept can be seen clearly by the learner. Visualization is the first level in the model constructed for geometric thinking by van Hiele as described by Jacobbe (2008). In this process students must be able to physically see and manipulate objects to learn geometric properties of shapes and measurement. However, students cannot be expected to apply the inquiry method of learning with manipulatives and accurately create a concept image for that mathematical idea. A specific process was described by Hall (1998) in Pape and Tchoshanov (2001) which includes three steps in the procedural analogy theory: 1) unguided exploration of the manipulative; 2) guided modeling by the teacher with the manipulative to make connections to mathematical concept; and 3) symbolic and written representation of the concept (p. 123). “This process is successful only to the degree that the concrete material procedures are analogous to procedures with symbols and the degree to which this connection is made explicit for the learner” (p. 123). Consequently, the use of manipulatives cannot be haphazard in the numeracy classroom where deep understanding of concepts is the goal. Students need to understand the connection between the physical representation and the abstract concept fully in order to appreciate the power of the manipulative. For example, students must learn and understand the correspondence between the base ten unit blocks and the place value system of numbers to use the manipulatives effectively for counting and operations. Giardino (2010) reiterated this concern about the use of concrete objects when teaching mathematical concepts, stressing that “manipulatives must be used with caution, don’t use them when the objects don’t respond to the abstracts” (p.36), as this will only lead to inaccurate concept images or frustration for the learner. Therefore, many forms of visual imagery, such as graphic organizers, schematic imagery and manipulatives, are available for students to implement when attempting to comprehend abstract mathematical concepts and these should be applied in multiple representations throughout the curriculum to ensure comprehensive understanding.

Pictorial and schematic imagery.

“Schematic representations are those that depict relationships described in the problem, while iconic (pictorial) representations are those that depict the physical appearance of the elements described in the problem” (Zahner & Corter, 2010, p. 180). It is schematic representations that should be the goal of good teaching. Unfortunately, many educators never move beyond having students draw pictures to help understand word problems, yet “instruction needs to go beyond getting students to try to ‘visualize’ the problem” (van Garderen, 2006, p. 505). If the goal is to help students construct complex schema and concept images that facilitate deep mathematical understanding, then schematic representations should be the instructional goal for problem solutions. For example, a pictorial representation of a problem that asked students to solve the jam jar sizes of a specific number of jars in relation to a balanced scale was drawn like this by a sixth grade student.

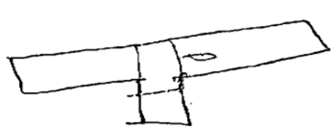

Figure 5. Pictorial representation of a scale (van Garderen & Montague, 2003, p.249)

The schematic representation of the same problem includes numerical labels to distinguish between jam jar sizes, the number of jars is drawn in relation to the scale and a total weight is provided as part of the references.

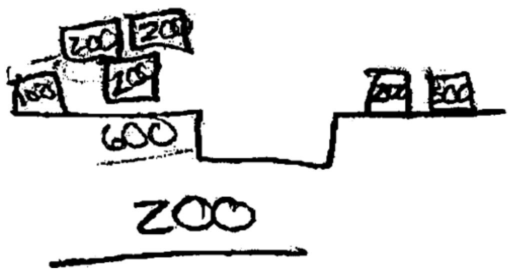

Figure 6. Schematic representation of a scale (van Garderen & Montague, 2003, p. 249)

It is the relationships and connections proposed and observed between diverse data or situations that demonstrated understanding of mathematical principles. The use of schematic images was positively related to success in mathematical problem solving, whereas the use of pictorial images was negatively related to success in mathematical problem solving. This result was repeated in several studies including those by Pape and Tchoshanov (2001), van Garderen (2006), and Zahner and Corter (2010). Construction of relevant schemata in a constructivist classroom should be the goal of all teachers and awareness of the distinctive types of visual imagery will assist this process.

Middle years' studies.

In a study by van Garderen and Montague (2003), 66 sixth grade students of varying abilities were given 13 mathematical word problems to solve. Each student was interviewed about the solution strategy and whether visual imagery was used. Additionally, each visual representation was rated as either a pictorial image or schematic image. The results showed that all gifted students used schematic representations and that no learning disabled student used schematic images more than 5 times. Not surprisingly, "schematic representation was positively and significantly correlated with mathematical problem solving. Pictorial representation was negatively and significantly correlated with mathematical problem solving" (p. 251). Most students attempted to use visualization for eight of the thirteen questions, although the level of complexity in the representations varied. The findings of this study confirmed other work (Hegarty & Kozhhevnikov, 1999) that found schematic representations to be more successful. However, one limitation discussed was the need for "further research, in particular an intervention study involving the use of schematic instruction" (p. 252). It was unclear whether the relationship between schematic representation and accuracy was purely causal or simply correlative.

Other studies.

Zahner and Corter (2010) completed a study with graduate students that focused on the implementation of visual representations for problem solving. Although this study focused on an older participant group, the findings are consistent with other studies done in middle years' classrooms (Stylianou, 2011; van Garderen & Montague, 2003). They found correlations between the types of imagery used and the rate of success which reinforced research conducted by previous studies. Essentially, schematic imagery had a higher rate of success than simple pictorial imagery. One interesting result was the prevalence of a spatial reorganization of a given information strategy used by nearly all the participants to solve the problems. Zahner and Corter speculate that "reorganization may aid in the abstraction of a problem schema from the text of a word problem in part by selecting out the critical problem information from the mass of superficial story detail" (p. 190). This strategy appeared to be used spontaneously by the participants that could demonstrate its power and usefulness within the classroom, as it "may make it easier for novice problem solvers to check for needed or missing information, to break down the problem into subparts, or to make visual associations to relevant formulas" (p. 187). The flexibility of the strategy means it can be applied by various students across a variety of numeracy concepts.

Progression of development of visual imagery.

Two studies (van Garderen, 2006; Zahner & Corter, 2010) discussed the notion that visual representations are created by students with varying degrees of sophistication. Pictorial (iconic) imagery, "representations that depict the physical appearance of the elements described in the problem" (Zahner & Corter, 2010, p. 180), reflect only a basic understanding of the mathematical word problem and the abstract numeracy concept. For example, in a question that asks students to determine the number of trees planted in a given distance, a pictorial representation includes several trees but nothing else.

Figure 7. Pictorial representation (van Garderen & Montague, 2003, p. 249)

On the other hand, the schematic representation of the same problem includes numbers to show the distance relationship between trees and a mathematical unit for the solution.

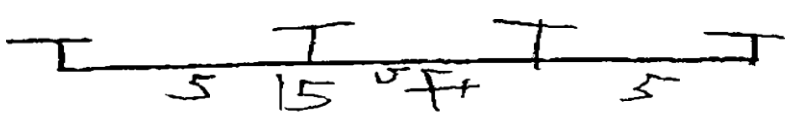

Figure 8 Schematic representation (van Garderen & Montague, 2003, p. 249)

These examples show the different details that students can include in visual imagery to demonstrate their level of understanding about the question and the numeracy concept. The pictorial representation allows the teacher to see that the student has a very vague understanding that trees are present in the problem, but no relationship between the trees or distance is shown. However, the schematic representation shows that the student understands the relationship between the trees (they are in a row) and the distance (trees are evenly spaced) and the units used to solve the problem (ft.). All three components are demonstrated by the spatial positions of the objects relative to each other and the specific numerical and word labels are provided. Complex connections between various numeracy schemata are demonstrated by the level of details. Thus, visual representation can be described as a developmental continuum where each step of progression is shown by additional levels of relational details and connections within the image. The flow chart below provides a graphic display of this conceptual understanding. At one end of the continuum are iconic and pictorial images, the most basic visual imagery: "representations that depict the physical appearance of the elements described in the problem" (Zahner & Corter, 2010, p. 180). At the other end of the continuum are schematic representations, "representations that depict relationships described in the problem" (Zahner & Corter, 2010, p. 180) with sophisticated labels (numerical and word) and a variety of symbols to show relationships within the image. Teachers should be focusing on developing schematic representations with numeracy concepts because these representations show deep, connected conceptual learning.

Figure 9. Progression of development in visual imagery

In fact, “pupils’ texts can be viewed as ‘one kind of evidence’ of the cognitive processes that they have engaged in” (Jewitt, Kress, Ogburn & Tsatsarelis, 2001, p. 7) and a reflection of understanding more abstract concepts. The ability to interpret information as imagery from written text is called transduction which is defined as “semiotic material [that] is moved across modes” (Bezemer & Kress, 2008, p. 169). Bezemer and Kress (2008) describe the different affordances or resources that are associated with imagery which can be observed to determine complexity. These affordances include: “position of elements in framed space, size, colour, shape, icons, and spatial relations” (p. 171). Although there is some specificity lost in the visualization process, there are gains made in the generality of the information represented (Bezemer & Kress, 2008). As a student visualizes an object or scenario small, minute details more become less important and are replaced by a general understanding of the abstract concept. In the previous tree example, it is not important what type of trees are planted rather the relationships between the trees. This ability to filter information and still represent detailed images is a reflection of schematic imagery. Consequently, teachers should be applying this continuum to inform instructional practices, lesson sequences and assessment opportunities.

Visual images as a tool in mathematics.

Numerous studies have shown that creating and using visual images are powerful tools for problem solving when employed effectively by students (Pantziara et al., 2009; Stylianou & Silver, 2004; Zahner & Corter, 2010). These studies were conducted in a variety of countries, with students of different ages and abilities, and completed around numerous mathematical topics. However, the outcome was the same in all studies. The students who were most successful in using visual imagery used the strategy as a learning tool rather than a product of the solution. These representations of visual images were not simply pictorial in nature and included a variety of representational forms. Tversky (2001), explains that

> external inscriptions may be used for a variety of purposes, including summarizing problem information, recording and reasoning about situations/story elements, offloading memory storage, coordinating the results of intermediate calculations, representing numerical or functional relationships via graphs and making abstract relationships concrete.(as cited in Zahner & Corter, 2010, p. 180)

In fact, the most successful students used a variety of representations throughout the many stages of problem solving which demonstrated the different purposes for each type of image. These images included lists of information reorganized spatially, pictorial representations, illustrations, diagrams and schematic representations. The value of students simply using symbols and external inscriptions of a multitude of variations cannot be overestimated in the middle years classroom, because "symbols and symbol systems support the cognitive activity by reducing the cognitive load (i.e., by reducing all that the individual must think about to accomplish a task), clarifying the problem space, and revealing immediate implications" (Pape & Tchoshanov, 2001, p. 120). This is why teachers should not settle for the standard student response from some middle years' students, "I can do it all in my head." This perception can lead to inaccurate solutions, misconceptions about mathematical concepts, and frustrations for most students who cannot organize all the needed ideas without some concrete representation.

Middle years' studies.

Montague and Applegate (2000) investigated the role of visualization and visual imagery in the perceptions and persistence of middle years' students in mathematical problem solving. Students of three levels of ability (learning disabled, average-ability and gifted) from grade seven and grade eight were given six word problems to solve. There was no significant difference in strategies amongst the groups in relation to the 1-step word problems. However, there were significant differences in the 2-step and 3-step word problems (p. 223). Results showed that "good problem solvers used more strategies overall" (p. 224) and the LD students struggled the most with strategies that facilitate problem representation. LD students struggled with not only creating complex visual representations, but with deciding on an appropriate initial visual image (p. 225). These results highlight the need for teachers to explicitly demonstrate a variety of visualization strategies throughout the numeracy curriculum. It is a reality of current classrooms that many levels of abilities exist and students must receive appropriate differentiated instruction to be successful.

Stylianou (2011) utilized her findings from an earlier study (Stylianou & Silver, 2004) to investigate the various representations created by middle school students. Stylianou (2011) determined that the experts (university mathematics professors) from the previous study had used visual imagery as: "a means to understand information; a recording tool; tools that facilitate exploration; and monitoring and evaluating devices" (p. 271). These categories were then applied to the representations created by the middle years' students in the study. Students used visual imagery as a tool in a variety of formats and means which reflected the same categories as used by experts. In fact, students used the representations in social contexts within the constructivist classroom to explore numeracy concepts. Stylianou (2011) notes that,

> Throughout our work, students used representations as *presentation tools* for their work, both informally to their peers in their groups, and more formally to present their solutions to their class. Students also used their representations as *tools to negotiate* and *co-construct meaning* and strategy with their peers. (p. 276)

In this manner, the representations were dynamic tools for exploring learning, demonstrating understanding and creating knowledge collaboratively in this constructivist classroom.

Two studies done in middle years' schools found that there were some gender differences regarding the use of visual imagery to solve mathematical problems. Lowrie and Diezmann (2011) investigated the performance of 317 Australian students in relation to the Graphical Languages in Mathematics, a way to classify graphics used in mathematics. Bertin (1967/1983) describes "graphics in terms of information within the graphic, the properties of the system, and the underlying components that govern and combine these properties" (as cited in Lowrie & Diezmann, 2011, p. 110). Based on this definition, graphics are organized into six "graphical languages": axis, apposed-position, map, connection, retinal-list, and miscellaneous. Students in the study completed a test composed of six questions from each "language". The results showed that over the three year period, boys consistently outperformed girls across all categories. This study demonstrates that a gender difference may exist in the ability to interpret specific graphic representations. However, further studies need to be done to determine a relationship between the creation of schematic representations and gender rather than an indication of previous experience.

Casey, Nuttall and Pezaris (2001) conducted a study with eighth grade students to investigate gender differences in visualization related to numeracy and the influence of self-efficacy. Students were given two different subtests based on conventional beliefs of gender abilities. The Male and Female subtest varied in the number of questions dedicated to each numeracy topic and in the level of difficulty for the questions. For example, the Male subtest had 40% problem solving questions, 20% simple procedures; but the Female test had 27% problem solving and 47% simple procedures (p. 37). The study showed that boys outperformed girls on the Male test due to two indirect factors “a) better spatial-mechanical skills on average and b) the increased self-confidence they have” (p. 45). The results of this study are consistent with earlier studies investigating gender differences in mathematics. Although boys performed better in relation to spatial skills, girls did better on verbal skills. This finding articulates the need for classroom teachers to provide a variety of instructional strategies through explicit modeling and student practice that will reach all students in the classroom. Overall, “teachers should understand that when boys are doing well on particular mathematics problems, they may be solving these mathematical problems by drawing on their visualization skills, referring to and manipulating mental images” (p. 50). Consequently, teachers should utilize visualization in the classroom and guide the development of existent visual images towards complex, schematic representations.

Other studies.

A pilot study was conducted by Pape and Tchoshanov (2001) as a single experiment in a high school mathematics department. Despite the fact that the participants were older students, the study is relevant to middle years because it stresses the importance of explicit instruction of visualization in the numeracy classroom. Three classes were involved in the study. One class was taught trigonometry lessons using strictly analytical tools (formulas); a second class was taught the same content using only pictorial images; and the third class received instruction with a combination of strategies. The results were that students who received only one form of instruction were very rigid in their application of knowledge and had difficulty adapting to non-routine problems. The students in the combination class were much more successful and demonstrated a flexibility of problem solving strategies because of their exposure to multiple representations. The authors concluded that students need to be exposed to a variety of visual representations and given adequate opportunities for practice so that “representations must be thought of as tools for thinking, explaining, and justifying" (p. 126). There must be caution when encouraging students to use visual imagery in mathematics because not all tools are the same and the schemata of students are diverse. Consequently, teachers must provide specific instruction and tools for specific situations. This is because

> students can be aided in developing their comprehension of a text as teachers model for them how to apply and regulate these strategies, which also include constructing mental images from text cues and creating graphic representations of the text structure as supports to getting the big picture of what the text is about. (Lapp, Fisher & Johnson, 2010, p. 423)

Once again guidance within the framework of constructivist inquiry is the key for successful application of strategies in mathematics.

Stylianou and Silver (2004) completed a study that focused on the implementation of visual representations for problem solving with undergraduates and university professors. This study had participants of a much higher age and ability levels in mathematics, but it is included here as a seminal piece of research investigating visual representations in numeracy. The overall research question investigated was: did visual representations aid in the successful completion of mathematical word problems? The study concluded that visual imagery helped the participants organize information and derive solutions, although there were varying degrees of success. Stylianou and Silver (2004) found that both mathematical experts and novices used a variety of visual imagery but the success rate was different. Experts used diagrams that were more detailed and often reflected the deep structure of their understanding of the mathematical concepts, with deep structure "defined as the principles of the discipline or archetypical methods of solution, or information on problem-solving strategies" (p. 362). This result suggests that the schemata of experts are more complex, intricate and dynamic as a result of many more experiences in the designated field of study than those of novices. There was one significant difference between the two groups in the application of diagrams as tools during the solving process. The novices tended to use the diagrams to organize the information that was presented during the comprehension stage of the process and then abandoned the diagrams in favor of formulas and numerical representations. However, the experts used diagrams during the exploration stage and later continued to use the same diagrams in a more dynamic manner (p. 379). The diagrams became a means to test possible outcomes prior to arriving at a solution for the problem. This dynamic use of the diagrams seemed to solidify understanding of the concept in the problem and allowed for deeper access to schema structures. Although the experts' diagrams were detailed and very precise for the task, there is evidence that simpler diagrams may be more appropriate for teaching mathematical concepts. The authors suggest that "simple (but correct) diagrams may assist the user in further exploration. Furthermore, simple diagrams are probably more likely to be viewed as general representations of a concept rather than illustrations of a one instance of the general case" (p. 368). This finding implies that that the quality and type of visual representations impacts the construction of mathematical concepts for students.

Cautions for using visual imagery in mathematics.

Despite all the benefits associated with visual imagery in mathematics there must be some cautions for teachers to consider when applying the strategy within the numeracy classroom. Visual imagery and representations can contain errors and be interpreted in a number of ways by students. Visual imagery can contain pre-visual or post-visual errors. The pre-visual errors are representations that are made based on a misunderstanding of the problem presented and do not accurately depict the information. Post-visual errors occur when learners have difficulty accurately interpreting the visual imagery used, so assumptions made about the diagram are false. Either one of the scenarios stress the need for students to be cognizant of the mental images created when comprehending text and the accuracy of the external representations. A visual representation is a form of text that must be interpreted by the reader to make meaning of the ideas or concepts being communicated, and “visuals can be interpreted in a variety of ways depending on what is being said or asked” (Brating & Pejlare, 2008, p. 354). It is the context and the background knowledge of the student that determines the information detected in the visual imagery in any given situation, and “the individual reacts with a mathematical visualization in a way which is better or worse depending on previous knowledge and the context” (p. 354). The multiple interpretations of a single diagram and the usefulness of the diagram are directly related to schema and context. Although visual imagery is a powerful tool for students to develop an understanding of abstract mathematical concepts, there must be caveats for its effective use in the classroom. Teachers must be aware that students may inappropriately use this strategy and create understandings that are inaccurate and difficult to alter. Brating and Pejlare (2008) note that “a reason why a visualization can be problematic to an individual observer is [that it is] a simplified and limited view of what the picture actually represents. This can be due to limited knowledge or experience” (p. 356). It is the responsibility of the active teacher to recognize these instances of confusion and provide adequate scaffolding for the learner.

The use of visual imagery can be assumed to be a predominant strategy for understanding various texts and connecting effectively to current schemas, but teachers are cautioned to “not assume students can use visual imagery” (Douville, 2004, p. 39). There are multiple ways of constructing knowledge and interacting with the world: using imagery is only one method. This is why students must be explicitly taught when to use visual imagery, how to use visual imagery and the different characteristics that determine a successful application of the strategy. Students do not inherently understand the characteristics of diagrams and do not possess an innate ability to use diagrams in an appropriate manner with a variety of problem schemata. This ability to work with diagrams and understand their multiple applications is described as diagrammatic literacy, which is “a skill not inherent in all students and needs to be developed” (Pantziara et al., 2009, p. 55). Students need to be exposed to multiple representations, including diagrams, in a variety of situations with explicit teaching of their applications. The ability to choose the appropriate visual imagery to match the current problem was related to the success of the problem solving (Zahner & Corter, 2010) and proved to be the most difficult component of visual imagery. The caution in this case is that educators provide the multiple exposures and opportunities to practice representations in an organized manner so that problem schemata can be built based on good quality source problems and clear repetition.

Finally, there was no study that had a highly significant result where visual imagery could be credited with directly causing higher success rates for all students. In many studies the results were mixed because the visual imagery was not used in a comprehensive or cohesive manner across the sample of participants. The studies by Stylianou and Silver (2004), Tanisli and Ozdas (2009) and van Garderen (2006), all demonstrated that the participants who were most successful in problem solving were able to use a variety of visual imagery including numerical representations and schematic representations. The other participants attempted to use visual imagery in most cases, but the images were simplistic and lacked the connections required to demonstrate clear understanding of the underlying mathematical concepts. In other words, this strategy works for some students some of the time and once again proves there is no holy grail of instructional strategies (Woolner, 2004). This understanding reinforces the belief that teachers need to provide students with a variety of comprehension strategies for many different situations and be flexible and responsive in their teaching so concepts can be addressed in a multitude of ways.

Professional development in numeracy education

There have been many dramatic changes in the instructional delivery of numeracy concepts in classrooms as a result of the “math wars” and government curriculum documents. In fact, Stylianou (2010) states that

> While the recommendations for increased and fluid use of representations in mathematics classrooms place significant demands on teachers, there is little evidence that teachers have received the necessary support to implement them; there is little evidence that either professional development programs or teacher preparation programs have been preparing teachers and prospective teachers to meet these demands and integrate them in instruction effectively. (p. 326)

The focus of the research presented in this section of the literature review is to understand the context of professional development in numeracy, characteristics of professional development and current challenges facing teachers. The teacher’s handbook (Appendix A) I created for this project is based on my review of research and educational theory related to professional development and numeracy.

The role of pedagogy.

A discussion of teacher education must begin with the role and development of pedagogy. Pedagogy is understood “as a practice or a craft representing the teacher’s accumulated wisdom with respect to their teaching practice acquired over time” (Carrington, Deppeler & Moss, 2010, p. 2) and it is influenced by various forms of knowledge, beliefs and values. Essentially, pedagogical beliefs are the framework and foundation of all instructional practices in the classroom including the pursuit of professional development. Teachers have beliefs that guide their practice about a multitude of issues: "teachers do hold implicit theories about students, the subjects they teach and their teaching responsibilities, and that these implicit theories influence teachers' reactions to teacher education and to their teaching practice" (Carrington et al., 2010, p. 2). For example, for teachers to implement constructivist pedagogy effectively in the classroom they must have a foundational belief that they are not the sole authority and gatekeeper of knowledge; that there is significant value in having students explore activities to create knowledge and that discussion and dialogue are a pathway to further learning. These tenets then inform the types of instructional strategies regularly used in the classroom, the forms of assessment employed and the nature of the interactions between student and teacher. The importance of pedagogy as a framework for teaching is recognized in Australia where the focus on professional development is related to changes in pedagogical structures (Chamberlin, 2009). In Chamberlin's study, teachers attended a two week summer institute to improve their knowledge of numeracy concepts. This program followed the standard philosophy that "in professional development that is classified as content-based, the entry point is to involve teachers in doing significant mathematics" (p.22). The goal was to have teachers experience numeracy learning in situations that simulated the classroom environment as much as possible and to address pedagogical concerns as well, so that "content is supplemented with pedagogical topics such as investigations of students' mathematical thinking or examining constructivism as a theory of learning" (p. 23). In the end, teachers learned that the most effective strategies for numeracy instruction were collaboration, visuals, test and revise, and group schema building (p. 28). This realization highlighted the need to create a constructivist classroom to employ the most effective strategies for numeracy learning.

Pedagogical change is never easy or quick because the shift from the transmission model to inquiry-based learning requires *fundamental change;* that is, transformation of self and practice (Goos & Geiger, 2010). Changing teaching philosophy and pedagogy requires changing basic personal beliefs and values. The most effective way to orchestrate such a monumental event is with conscious effort and design. Hedges (2010) makes the point that “research on teacher effectiveness demonstrates that working with teacher beliefs, helping teachers link theory, research, philosophy and practice, and providing opportunities to engage in discussion about new research findings are recognized as integral to ongoing success” (p. 300). The focus of pedagogical change and professional development should be incremental progressive change towards a clear goal which will reflect the best interests of both student and teacher.

The role of government curriculum and educational policy documents.

The public education system has several characteristics inherent in structural and power relationships that can limit professional development including an emphasis on top-down management and requirements for accountability. Carrington et al. (2010) described the situation in Australia eloquently:

> the way that teachers are trained, the way that schools are organized, the way that the educational hierarchy operates, and the way that education is treated by the political decision makers, results in system more likely to retain the *status quo* than to change. (p. 1)

In fact many of the problems related to professional development are a result of systemic interactions between the various policies and stakeholders in education. In Australia, for example, Hardy (2009) claims that the government is attempting to influence and in some cases dictate the direction of educational philosophy and professional development. He points out that

> the provision of teacher professional development, or “PD” as it is typically described in

> Australian education, is currently constructed as a crucial element of school reform policies, where schooling and its quality are seen as central to the economic competitiveness of the nation. (p. 73)

Immediately, issues of accountability, measurability and the focus of cost-effective measures in education delivery surface. While government documents do not endorse specific professional development programs, there are general recommendations and a focus which support the curriculum standards. Teachers in Australia are encouraged to follow school dictated initiatives for professional development rather than individual plans, which may be perceived as a negative application of personal development but can also generate learning communities of support in the immediate environment. There is also a heavy emphasis on constructivist principles related to specific disciplines and a link to funding professional development in these particular areas (Carrington et al., 2010; Hardy, 2009). The role of government in influencing professional development for teachers is not inherently negative when considering the possible explicit links to curriculum development. However, this top-down approach to teacher learning is reminiscent of the transmission model of classroom teaching which has been proven to be ineffective for long-term conceptual learning.

Traits of effective professional development.

Mathematics teacher education has three themes currently: 1) the role of reflection to inform on-going learning; 2) growing awareness of *social* dimension of professional development; and 3) *organizational context* of teacher's work and the extent to which this context provides resources to support change (Goos & Geiger, 2010). Although these themes have been present in many educational areas, the focus is relatively new for mathematics. "Reflective practice has an established history in teacher education and teacher practice as a source of self-improvement"

(Hedges, 2010, p. 300), and can be implemented formally or informally in the classroom. Some teachers and pre-service teachers are encouraged to maintain reflection logs to document ideas and musings about philosophy, pedagogy and beliefs (Carrington et al., 2010). These logs can then provide documentation and impetus for direction in professional development by highlighting questions or concerns in their teaching practice. As Carrington et al. (2010) noted: "supporting teachers to critically reflect and learn from each other builds professional knowledge and identity and nurtures pedagogical development and improvement" (p. 8). The act of reflection can be an individual pursuit done on a consistent basis or it can be facilitated externally as part of a larger professional development enterprise. However, either method can be used "to enhance the chances that teachers will internalize and consider incorporating similar changes in their instruction, teachers should reflect on their own personal learning experiences within the professional development" (Chamberlin, 2009, p. 32). It is the reflection and examination of personal learning that will contribute to the building of teaching schema and possible changes to pedagogy.

Collaboration is an intrinsic part of constructivist pedagogy and an important component of effective professional development. These instances of collaboration can be highly structured as in the study by Swan and Swain (2010) who worked with 24 teachers of numeracy in a variety of alternative programs or educational settings. The goal of the study was to move teachers from the traditional and standard approach of individual instruction based on practice worksheets and transmission model strategies to more constructivist approaches to learning. The participants (teachers) "were encouraged to try out new classroom activities using prepared classroom resources. They were offered a mentor and a network of support as they did this" (p. 168). This particular instance of collaboration with integrated systemic support led to many successful outcomes for the participants and their students. Other forms of collaboration are embedded strategies of larger professional development programs and are often conducted over multiple meetings and incorporate the principles of reflection, to capitalize on the understanding that "facilitated and scaffolded social dialogue combined with self-reflection can assist teachers to develop their professional, situated and individual identity" (Carrington et al., 2010, p. 11). There are opportunities within these communities of learning and support to discuss various strategies that were tried in classrooms and the experience of others can help inform professional growth in individuals. A goal of such collaborative meetings is to reinforce some current practices of teachers engaging in professional development, and help teachers "become more aware of viable alternatives rather than proceeding on impulse and intuition based on outdated ideals and practice" (Carrington et al., 2010, p. 11). Sharing of experiences and strategies is an important focus of collaboration but there also needs to be collaboration in creating professional developmental opportunities. "Teacher involvement in determining professional activities is important if practice is to be transformed" (Hedges, 2010, p. 304), as professional development that is perceived to address a specific need either at a school or district level has greater "buy-in" from teachers. The impact of these opportunities can be seen immediately and personally by those participants undertaking the learning experience.

The final characteristic of effective professional development noted by Hedges (2010) is the ability of teachers to see the application of the process or strategy in their current classroom context. As Hedges emphasizes "it is vital that educational research has applied validity otherwise findings are unlikely to be taken up by teachers in their professional learning and implemented in practice" (p. 299). Although teaching and education are based on theoretical knowledge, it is the pragmatic application of this academic pursuit that has the greatest impact on students and learning. It is the observable success of the theoretical frameworks that has influenced teachers to change practice, beliefs and pedagogy. Swan and Swain (2010) found this premise to be supported within their research study:

> It might be assumed that in order to change a teacher's practice, one has to first change through persuasion his or her beliefs about teaching. However, we would suggest that changes in beliefs are more likely to follow changes in practice, after the implementation of well-engineered, innovative methods, as processes and outcomes are discussed and reflected upon. (p. 175)

In this case, it was the application of new strategies and the success of these strategies that influenced the changes in pedagogy. Personal success in the classroom is very powerful for altering paradigms but teachers also recognize the relevance of documented case studies for developing their understanding of applications in the classroom. In fact, Carliner et al. (2009) surveyed trainers and educators about the dissemination of research for the teaching of various disciplines and found that "70% mentioned the importance of being able to use the [research] content" (p. 14) directly in their own practices. There is overwhelming evidence that teachers only incorporate research and theory in their classrooms when applications of the pedagogy are clear, pragmatic and beneficial for their students and themselves. As Hedges (2010) stresses "[e]xternal opinion is insufficient. Teachers need to see actual evidence of problems, or the benefits of changes to their practices through enhanced learning for children before making lasting changes to practices or the underlying beliefs that inform them" (p. 309). This simple understanding of professional development can help to explain the reluctance of some teachers to implement specific numeracy strategies as dictated by the curriculum documents, in particular, those related to schema activation and visualization.

Challenges for professional development.

In addition to some of the challenges already addressed in terms of changing pedagogical structures, governmental influence on professional development topics, and the need to incorporate reflection, collaboration and application in PD programs, there also is the influence of accountability. Some teachers see “curriculum and assessment requirements…as constraints that prevent them from changing their practice” (Goos & Geiger, 2010, p. 502). Inherent in this belief is the assumption that changes to pedagogy will require more time for instruction, assessment and planning with limited resources or support, because “if it were that uncomplicated for teachers to improve practice, they would likely do so. Instead, teachers are often resistant to changing established practices” (Hedges, 2010, p. 300). Essentially, many teachers focus on a multitude of reasons why pedagogy should not and could not change, rather than attempting to make adaptations to move towards a pedagogical goal. There are ways to incorporate curriculum and assessment requirements in a meaningful manner but these strategies must be investigated, explored and searched for by willing teachers. Just as there is no panacea of successful instructional strategies for our students there is not one perfect method of professional development for teachers, and it is a fair claim that “the history of professional development for teachers is a landscape littered with failed approaches” (Hedges, 2010, p. 300).

After reviewing the traits of effective professional development for numeracy instruction, I applied the principles to the creation of my teachers’ handbook. The provided lessons are based on successful lessons that I have used in my classroom. This characteristic addresses the need for teachers to see a practical application based on specific case studies. The format includes a detailed procedure linked to a specific learning outcome which addresses the need to have practical applications to the curriculum drawn from educational theories and research.

Finally, the tenet of collaboration exists by creating lessons that can be adapted and adopted for particular classrooms. The considerations, cautions and adaptation suggestions allow teachers to build upon the lessons. In fact, this form of collaboration is similar to asynchronous discussion and can be very effective. As a result, the handbook addresses possible barriers to professional development through a practical resource.

Gaps in Literature

After completing a literature review of constructivism, visualization, and professional development as related to middle years' numeracy teaching, there exist some apparent gaps in the available research and literature. There is a noticeable lack of Canadian research studies published since 2000. In the review of constructivism only the articles by Bereiter and Scandamalia (2010) described Canadian studies. In contrast, work by Australian researchers and writers were abundant in all areas of the literature review. This result may be an indication of an increased emphasis and effort by the Australian government since 2000 to support curricular changes towards constructivism described in a variety of government documents: or it may be an indication of the difficulty in finding Canadian articles through current databases. It is important that provincial governments in Canada realize that teachers need Canadian research and specifically research based pedagogy to inform changes in teaching practice. The provincial standards are specific to our culture, needs and goals. We should not rely on the transferability of other international research. Thus, this is a call to the government and universities to facilitate more research within our schools and support teachers in publishing current, applicable and constructivist research in a productive, accessible manner.

Visualization and visual imagery are important instructional tools for teaching abstract concepts in the numeracy classroom. Steele and Johanning (2004) explicitly stated that diagrams were powerful tools to build problem solving schemas. In fact,

> Some students used diagrams as a means of recognizing the similarities among problems and identifying the types of problems they had stored in their schemas. Students who based their reasoning on the physical sense- the diagrams they drew- rather than only reasoning about numeric patterns were able to interpret the relationship between quantities in the problem and represent their thinking with symbolic algebraic generalizations. (p.87)

It is the use of visual imagery as a tool for solving mathematical problems that can be very powerful in the classroom. Despite the general agreement about the importance of visualization amongst researchers, teachers, and curriculum documents, there is no consensus about the nature of the strategy. As previously noted, van Garderen (2006) stated that "instruction needs to go beyond getting students to try to 'visualize' the problem" (p. 505). More current research done in middle years' classrooms that explored the differences between pictorial and schematic imagery is needed. Specifically, there is need for further research into the possibilities of schematic representation instruction as an intervention strategy for struggling students.

The confusion that exists within the community of researchers, teachers and curriculum documents around relevant vocabulary related to visualization needs to be explored and articulated in academic articles. It is very difficult for teachers to consistently reinforce visualization as a comprehension strategy across curriculum when basic vocabulary is confusing. Investigation and research into the implications and existence of this confusion are needed before appropriate programs for professional development can be created.

It is with these beliefs and assumptions that I created a handbook for middle years' teachers that addresses the Grade 7 numeracy curriculum as dictated by the British Columbia Ministry of Education IRP for Mathematics (2007). This document encourages numeracy instruction within a constructivist model that emphasizes flexible small groupings, communication, connections and visualization as instructional strategies. After working with a variety of textbooks and investigating different instructional strategies, I have found that students are more successful when explicitly taught using visual representations and connecting new knowledge to existing knowledge to build conceptual schema. It is my belief based on research and my review of the literature that more teachers will employ the tenets of visual imagery, and constructivism when provided with specific, pragmatic applications (Carrington et al., 2010; Chamberlin, 2009) for their classrooms.

Glossary

This glossary contains the relevant terms for the literacy strategy of visualization. These terms are included here because there is often confusion expressed by teachers and researchers with respect to the general definitions due to the variety of terms in use. In fact, Stylianou (2010) suggests that "it is noteworthy that researchers in mathematics education do not always agree on what representation means" (p. 326) or on many of the other important terms related to the topic. The following definitions are the ones used and cited throughout this document:

diagram: "a visual representation that presents information in a spatial layout" (Diezmann & English, 2001 as cited in Pantziara et al., 2009, p. 40)

external imagery: "manifestations of mathematical concepts that act as stimuli on the senses and help us understand these concepts" (Janvier, Giraradon & Morand, 1993 as cited in Pape &

Tchoshanov, 2001, p. 119). Examples include: numerals, algebraic equations, graphs, tables, diagrams, or charts

graphic organizer: "visual representations that help students identify, organize, and remember important ideas from what they read" (Boardman et al., 2008, p. 23). Examples include: Venn diagrams, KWL charts, concept maps, schematic diagrams or framed outlines

illustration: a drawing or picture that reflects the text and the mental imagery associated with the text (Hibbing & Rankin-Erickson, 2003)

imagery: "the ability to form mental representations of the appearance of objects and to manipulate these representations in the mind" (Hegarty & Kozhevnikov, 1999, p. 684)

internal imagery: "abstractions of mathematical ideas or cognitive schema that are developed by a learner through experience" (Pape & Tchoshanov, 2001, p. 119)

manipulatives: concrete representations and tools used to teach abstract concepts. Examples include: base-ten blocks, fraction circles or integer tiles

mental imagery: "the process of forming internal pictures of objects or events not present to the eye that can affect later recall and comprehension" (Douville, 2004, p. 36)

pictorial (iconic) imagery: "representations that depict the physical appearance of the elements described in the problem" (Zahner & Corter, 2010, p. 180)

representation: "a configuration that stands for something else" (Stylianou, 2010, p. 266). Examples include: symbolic expressions, drawings, written words, graphs, diagrams or numerals. <u>** Caution</u>: "representation became imbued with various meanings and connotations in the changing paradigms of the last two decades" (Pressing, 2006, p. 206 as cited in Stylianou, 2010)

schematic imagery: "representations that depict relationships described in the problem" (Zahner & Corter, 2010, p. 180)

spatial imagery: "a representation of the spatial relationships between parts of an object and the location of objects in space or their movement" (Hegarty & Kozhevnikov, 1999, p. 685)

visualization: "the construction and formation of internal images (e.g. mental images) and/or external images (e.g. with the aid of pencil and paper) and then using those images effectively for mathematical discovery and understanding" (Zimmermann & Cunningham, 1991 as cited in van Garderen, 2003, p. 246), or "the act of bringing a visual onto the mind's screen" (Mazur, 2003 as cited in Kotsopoulos & Cordy, 2009, p. 261)

visual imagery: "a presentation of the visual appearance of an object, such as its shape, color, or brightness" (Hegarty & Kozhevnikov, 1999, p. 685).

CHAPTER THREE

Reflections on Content and Process

Triumphs

Throughout this challenging endeavour to expand my personal learning of educational theory, research and philosophy there have been many triumphs. The process led to an affirmation that many of the instructional strategies and tools that I use regularly in the classroom are valid choices for my students. I consciously provide students with opportunities to collaborate with peers on a variety of inquiry based activities across the curriculum. My initial motivation for creating these opportunities was because the activities were engaging for most students and addressed multiple learning needs in my classroom. Now, I can base these activities firmly in the research and theory of constructivism regarding the generation of learning and knowledge. This understanding allows me to focus on providing these activities within the broader context of constructivist pedagogy. Understanding the foundations of pedagogy as the personal values, beliefs and knowledge of the individual teacher provides me with an increased awareness of the effect of my personal qualities on the environment of my classroom. While I already had a general sense of my role in creating a valuable learning experience for all my students, now my more explicit understanding of the nature and the extent of this role forces me to reevaluate all instructional choices against the articulation of my personal teaching beliefs and philosophies. The awareness of my influence in the learning community that I developed from completing this project, will lead to more informed decisions about instructional practices in the future.

Secondly, by completing copious amounts of reading based in research and theory, I gained many useful new strategies to use in the classroom. My professional commitment as an educator to provide engaging, appropriate and relevant activities was enhanced by completing this project. I am excited to try many of the successful research based activities in my classroom which will have benefits for my students as lifelong learners. I already incorporated the graphic organizer created by Hyde (2006) and some of his strategies into my problem solving lessons. The success of these lessons in my own classroom for all my students, regardless of ability and interest level, has encouraged me to expand the applications next year. The exposure to new instructional strategies and knowing that each strategy is based on sound research has increased my potential to be a "master teacher" in the classroom and the school.

Surprises

My personal experiences in the classroom and school community had led to many assumptions about visualization strategies in the numeracy classroom. Textbooks that are endorsed by the Greater Victoria School District emphasize multiliteracy and constructivist pedagogy. Despite this apparent focus on multiple representations, there was little guidance in the IRP to create lessons that employed visual imagery. Instead, visual imagery was limited to number line representations and pictorial images. My intuition and experiences in the classroom led to my belief that a greater importance needed to be placed on the role of imagery in developing numeracy concepts. I began the research with the assumption that there existed limited research in the area of this cross curricular approach, but I was wrong. There actually exists an abundance of theoretically based research articles exploring the importance of visual imagery as a tool when teaching abstract numeracy concepts. The depth of these studies and the variety of situations explored were surprising and very encouraging. I now have a theoretical and research based justification for having students use multiple visual representations in the classroom that exists beyond the use of standard manipulatives.

The research I found about the different forms of visual imagery was particularly enlightening. I was guilty of having students just "visualize" in my classroom in many different contexts. The result was some students could not create a basic mental image and others had mental images that were inaccurate. I have since learned that more explicit instruction, modeling and guidance are needed to help students create accurate images. The discussion of pictorial representations versus schematic representations confirmed my intuition that students need to move towards more complex representations to demonstrate abstract conceptual knowledge in numeracy (Stylianou, 2011; van Garderen & Montague, 2003; Zahner & Corter, 2010). The articles I found have provided concrete examples to use when developing assessment tools and lessons.

Finally, it was surprising that there exists so much confusion about basic vocabulary related to visualization. Basic characteristics of visualization cannot be clearly articulated by researchers and teachers when the language is imprecise. This reality may contribute to the ineffective and inconsistent use of visualization in numeracy and other contexts.

Influential Researchers

There are three researchers that I discovered while completing the literature review who are particularly influential in my current teaching practice. Pape completed two research projects that investigated students' abilities to understand numeracy concepts. The first project was completed with Tchoshanov in 2001 in a high school context. Three different classes were taught the same trigonometry concepts using three different instructional strategies: strictly symbolic formulas or visuals or both visuals and formulas. The results showed that students in the mixed methods class were more successful overall when solving numeracy problems. As a result, in my classroom I provide many different methods of instruction and representation to reach all my diverse learners. Pape's second study (2004) with middle years' students emphasized the need for students to have opportunities to complete think alouds and use a variety of methods to represent knowledge. This study also articulated the need for students to understand context when attempting to solve word problems. Visualization of the scenario can help some students understand the context of problems and experience greater success.

Stylianou completed three studies (2004, 2010, 2011) that I read for the literature review. Her overall focus was to determine the role of representation both as a process and a product in numeracy instruction. Her study with Silver (2004) was conducted with experts (university mathematics professors) and undergraduates. This study showed that experts use representations in dynamic ways that allow for them to be a process or tool and not just a product to show understanding. In my classroom, I am attempting to explicitly teach this skill as a means for students to deepen abstract conceptual understanding.

Finally, Street has written extensively about the importance of social literacies and numeracies (2008). His contention that all students come to school with a variety of abilities and experiences related to numeracy has influenced the movement towards constructivism in my classroom. In particular, social constructivism in the numeracy classroom is more powerful when students can recognize and build on each other's experiences and expertise. The students then actively become the teacher and scaffolding expert for their peers' learning. In my classroom, I incorporate peer tutoring opportunities and time for discussion amongst students to highlight the diversity that exists.

Recommended Articles

There are three articles that I read for the literature review that I would recommend to colleagues. Each article describes practical applications for the middle years' classroom and shows some surprising insights in the research. Gordon (2009) describes case studies to highlight the application of constructivist pedagogy in the classroom. The discussion of the history of the educational theory in the beginning of the article provides some insight into the general tenets of the pedagogy. The resulting discussion of pragmatist constructivism attempts to describe the benefits and shortfalls of the pedagogy when applied in real life situations. This article provides some specific frameworks and considerations for teachers when applying constructivism in a variety of contexts.

Zahner and Corter (2010) completed a study with a group of undergraduate students who applied specific diagrams when solving Grade 7 probability problems. This article discusses the difference between pictorial and schematic imagery and provides clear examples for the reader's consideration. The most powerful conclusion of the project was the spontaneous reorganization of information into lists and charts by most of the participants. This simple procedure proved to be helpful for the participants to solve the questions with increased accuracy. As a result, this strategy should be explicitly taught in numeracy classrooms as a dynamic form of representation.

Finally, Stylianou's (2011) article reports on an extension and application of her previous study with experts (2004). She used the results from the earlier study with Silver (2004) to examine the use of representations by students in Grade 7. Stylianou found that middle years' students used representations in many of the same ways as the experts when solving word problems. The most surprising aspect of her findings was the importance of discussion for students after creating their representations. This application of social constructivism integrated with visualization is a refelction of the developmental needs of adolescents and a successful means to engage these learners (Perso, 2005).

Challenges

As with any undertaking for personal growth and development there were challenges along my journey including the fact that, it was difficult to focus my topic to a manageable discussion in the beginning because all aspects seemed new and exciting. There are so many applications for literacy strategies in the teaching of numeracy including, to list just a few: oral literacy related to partner and group work, focusing on communication and justification of word problem solving strategies, having students write about numeracy concept development in the form of math journals, and so on. All these strategies are complex and have proven successful in my personal classroom. In the end, a decision had to be made and these other topics were put aside for future investigations. Finally, the amount of time required to pursue and complete a master's project cannot be underestimated. It is time consuming to locate appropriate academic research articles, read the articles, summarize and articulate an argument and then complete a relevant project to display learning, insightfulness and creativity. There is a reason why not every teacher chooses to pursue a graduate degree and time constraints are definitely a big issue. However, the professional learning that I have gained and the satisfaction in knowing that I completed a challenging task has made the endeavour a rewarding experience.

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Appendix A

Teaching numeracy using the literacy strategy of visualization: A teacher's handbook

This handbook is a result of completing research in the form of a literature review about integrating literacy strategies, specifically visualization, in numeracy instruction. The handbook is deliberately concise and is based on Grade 7 curriculum outcomes in numeracy for British Columbia. The 5 lessons outlined in three numeracy strands (integers, decimals, algebra) are meant to be general suggestions that teachers can adapt to individual groups of students and their own contexts. The hope is that these lessons will act as a springboard to various applications of other literacy strategies in numeracy. It is my belief that all students are capable of complex conceptual understanding in numeracy when given appropriate tools through explicit teaching and practice.

The handbook includes: a summary of key points raised in current literature; discussion of the myriad of terms associated with visualization; a depiction of the continuum of complexity with respect to types of representations; and 5 lessons that depict visualization beyond images and diagrams in an unexpected numeracy context.

Theory and research foundations

In 1983 the National Commission on Excellence in Education published *A Nation at Risk* which provided an impetus for a revision of the numeracy curriculum standards in the United States. The new *Principles and Standards* (National Council of Teachers of Mathematics, 2000) advocated a new approach to numeracy in the educational system and started the mathematics reform described as follows by Draper (2002):

> Mathematics reform has worked to move instruction away from the tradition in which knowledge is viewed as discrete, hierarchical, sequential, and fixed and toward a classroom in which knowledge is viewed as an individual construction created by the learner as he or she interacts with people and things in the environment. (p. 521)

This shift towards constructivist pedagogy followed a spate of earlier literacy reforms that emphasized the importance of a variety of comprehension strategies in learning new concepts. A shifting of pedagogy is not always a swift or comprehensive process because pedagogy is shaped by the values, beliefs and philosophies that inform teaching practice and these are not easily changed. Understanding and embracing the philosophy of constructivism are important if the advocated changes to numeracy instruction are to occur. Constructivism encourages student centered learning based in inquiry activities that utilize collaboration with peers and teachers. This framework is based on the research and writings of Piaget, Vygotsky and Dewey who all described the different ways that students generate knowledge and learning. The overarching principle proposed by these theorists would be that new learning must be generated by interaction with peers and the environment and it cannot simply be transferred from one individual to another through ‘telling’. It is the underlying premise that knowledge and learning must be dynamic, experiential, and evolving that informs the type of instructional strategies now being recommended. As Sheats Harkness (2009) expresses it, “knowledge [is] a dynamic process of inquiry, characterized by uncertainty and conflict which leads to a continuous search for a more refined understanding of the world” (p. 245). Thus, teachers must continue to plan lessons in numeracy that challenge students’ thinking and perceptions of abstract concepts with the understanding that knowledge construction is slow and incremental, and that “early understandings usually are characterized by fuzzy, fragmented, poorly coordinated, confused and partly overlapping constructs that only gradually become sorted out in such a way that similarities and differences become clear" (Lesh, Doerr, Carmona & Hjalmarson, 2003, p. 218). Multiple exposures to a numeracy concept in a variety of representations which allow for discovery of connections will lead to greater learning and understanding in the classroom.

Visualization, "the construction and formation of internal images (e.g. mental images) and/or external images (e.g. with the aid of pencil and paper) and then using those images effectively for mathematical discovery and understanding" (Zimmermann & Cunningham, 1991 as cited in van Garderen, 2003, p. 246), can be comprehensively applied across mathematical domains, including fractions, integers and algebra, when taught explicitly as a learning tool. This is because "visual imagery has a role in establishing the meaning of a problem, channeling problem-solving approaches, and influencing cognitive structures" (van Garderen, 2006, p. 496). It is this ability to transcend specific context and concepts that makes visual imagery and visualization a powerful tool for numeracy instruction and learning. For example, students can use visualization to understand the specific setting of a story in literacy or the particular context of a word problem in numeracy. Visualization includes mental imagery, defined as "the process of forming internal pictures of objects or events not present to the eye that can affect later recall and comprehension" (Douville, 2004, p. 36); external imagery, "manifestations of mathematical concepts that act as stimuli on the senses and help us understand these concepts" (Janvier, Giraradon & Morand, 1993 as cited in Pape & Tchoshanov, 2001, p. 119); and manipulatives. Manipulatives, concrete representations and tools used to teach abstract concepts, are a popular and well-used resource for introducing young learners to the characteristics of abstract numeracy concepts. Visual imagery can be used in the same manner in later grades to emphasize relationships and processes related to mathematical concepts. However, the true power of visual imagery exists in the multitude of forms and functions it can provide for the numeracy student. For example, external inscriptions (imagery) can be used to:

- summarize information,
- record story elements of word problems,
- reduce cognitive load on working memory storage,
- coordinate intermediate calculations of complex problems, and
- make the abstract appear concrete (Zahner & Corter, 2010).

With all the possible positive applications of visual imagery being highlighted, it is imperative that these uses and strategies be explicitly taught in the classroom.

Vocabulary Confusion

Despite the popularity of visualization as a comprehension strategy, there is confusion related to the accepted definitions of important vocabulary terms. As Stylianou (2010) stated, “It is noteworthy that researchers in mathematics education do not always agree on what representation means” (p. 326). This confusion may be a contributing factor to the reluctance for many teachers to incorporate visualization into various numeracy contexts. As a result, I provide definitions throughout the text of this handbook for important vocabulary terms. I have also provided a glossary at the back of the handbook.

Developmental Continuum of Imagery

Based on research about visualization in numeracy classrooms, it is clear that the different types of representations fall along a continuum of development or complexity. The continuum begins with the most basic pictorial representations, “representations that depict the physical appearance of the elements described in the problem” (Zahner & Corter, 2010, p. 180). For example, if the word problem asked the student to determine how many trees can be planted in a given distance the pictorial representation would be:

Figure 1. Pictorial representation (van Garderen & Montague, 2003, p. 249)

There are no numbers or labels attached to the representation. In fact, the number of trees drawn may not even be an accurate reflection of the answer. On the other hand, a schematic representation, "representations that depict relationships described in the problem" (Zahner & Corter, 2010, p. 180), would include numbers, mathematical units, and indicate the spatial relations between the trees. Observe the example below,

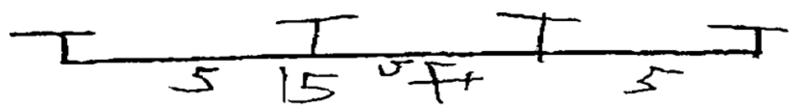


Figure 2 Schematic representation (van Garderen & Montague, 2003, p. 249)

The differences between the two representations are the presence of labels, numbers and relationships within the image. As each layer of details is added to the image, another connection is made to conceptual understanding. In fact, "pupils' texts can be viewed as 'one kind of evidence' of the cognitive processes that they have engaged in" (Jewitt, Kress, Ogburn & Tsatsarelis, 2001, p. 7) and a reflection of understanding more abstract concepts. The ability to interpret information as imagery from written text is called transduction which is defined as "semiotic material [that] is moved across modes" (Bezemer & Kress, 2008, p. 169). Bezemer and Kress (2008) describe the different affordances or resources that are associated with imagery which can be observed to determine complexity. These affordances include: "position of elements in framed space, size, colour, shape, icons, and spatial relations” (p. 171). Although there is some specificity lost in the visualization process, there are gains made in the generality of the information represented (Bezemer & Kress, 2008). As a student visualizes an object or scenario small, minute details more become less important and are replaced by a general understanding of the abstract concept. In the previous tree example, it is not important what type of trees are planted rather the relationships between the trees. This ability to filter information and still represent detailed images is a reflection of schematic imagery. The graphic below provides an example for instructional reference of the representational continuum,

Figure 3 Developmental continuum for imagery

Consequently, instruction should be informed by the assumption that students need to demonstrate deep understanding through schematic representations. Explicit teaching, modeling and guidance will help move students along the continuum when using visualization.

Government Curriculum Documents

Government prescribed curriculum documents are a reality of teaching in the public education system in many countries. These documents attempt to provide guidance and a progressive structure for teaching all disciplines, and teachers are legally bound to follow them.

As well as the pragmatic intention of defining specific learning outcomes, usually based on research, the documents attempt to provide a unified vision of educational philosophy and pedagogy to be practiced in the province, state, or country. In the case of the *Practice Standards* (NCTM, 2000), the influence on numeracy instruction was internationally felt. The opening statements of the document set the criteria for the changes:

> We live in a mathematical world….All students deserve an opportunity to understand the power and beauty of mathematics. Students need to learn a new set of mathematics basics that enable them to compute fluently and to solve problems creatively and resourcefully. (Introduction, para. 1)

The document provides a clear theoretical basis for the subsequent mathematical processes described and used to organize the more specific learning outcomes. These processes are reflected in the British Columbia IRP (2007) document for numeracy appearing under the headings of: communication, connections, mental mathematics and estimation, problem solving, reasoning, technology, and visualization (p. 18). A quick glance at these topics reveals the influence of constructivist and literacy pedagogy on the reforms to numeracy instruction. The document stresses that these topics should be:

- incorporated into every strand of the numeracy curriculum,
- taught explicitly for transference, and
- reinforced regularly in a variety of activities.

Also emphasized is the focus on schema (connections) and the importance that schema building has in the numeracy classroom. As the IRP states, "when mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated" (p.18). Visualization and visual imagery in mathematics should be applied in multiple representations with multiple exposures because “the use of visualization in the study of mathematics provides students with the opportunity to understand mathematical concepts and make connections between them” (p.19). The IRP makes clear that when visualization is integrated into instruction it offers students a powerful learning strategy. The curriculum documents for British Columbia not only outline the specific learning outcomes for each grade, but also describe specific assessment strategies under the title “Suggested Achievement Indicators”. These suggestions are very useful when planning instruction for learning and assessment because they are directly related to specific learning outcomes and are created for application within a constructivist pedagogical orientation. As a result, I use some of these suggestions when applying the examples of visualization and schema building lessons in the next section.

Application of Visualization in Numeracy - What does it look like?

Integrating visualization into numeracy instruction requires a basic understanding of multiliteracies, a broadened and updated conceptualization of literacy that acknowledges the technologically complex print and media environment in which our students are immersed, and a recognition that students learn and communicate knowledge in a variety of media. This understanding of multiliteracies encourages students to become proficient when explaining numeracy concepts through using personal schematic understanding and a choice of representations for this knowledge. This application of such a broadened conception of literacy encourages alternative methods of learning and expression, but does not negate the importance of eventually learning standard symbolic mathematical formulas and representations. These formulas and numerical representations are essential for communication within the discipline of mathematics and are building blocks for future learning. Visualization, schema building and the use of graphic organizers should be intermediate steps in the process of building accurate, comprehensive abstract conceptual understanding. Graphic organizers are used as a way to organize information in a familiar, structured manner. It is repetition that makes graphic organizers a part of schema building and an example of visual imagery that can be easily transferable across numeracy domains and various topics addressed in the academic curriculum.

The following sections describe three specific applications of visualization strategies to teach learning outcomes in five lesson examples. Although lesson structure is outlined, the structure is intended to be flexible and fluid to allow for adaptations needed to respond to differences in ability, grade level or Individual Education Program (IEP) requirements. Each application will be based on a separate numeracy topic as described by the IRP (2007) and a specific outcome related to the Grade 7 curriculum. All are successful strategies I have used with my own students. However, I have deliberately avoided focusing on applications of visualization for teaching geometry because this strand already offers obvious connections to the use of visual imagery. A short discussion of cautions and adaptations is included with each description as a means to connect the lessons to theory and research.

Numeracy strand: Number. Topic: Integers.

Visual strategy: Schematic imagery

Lesson Example One

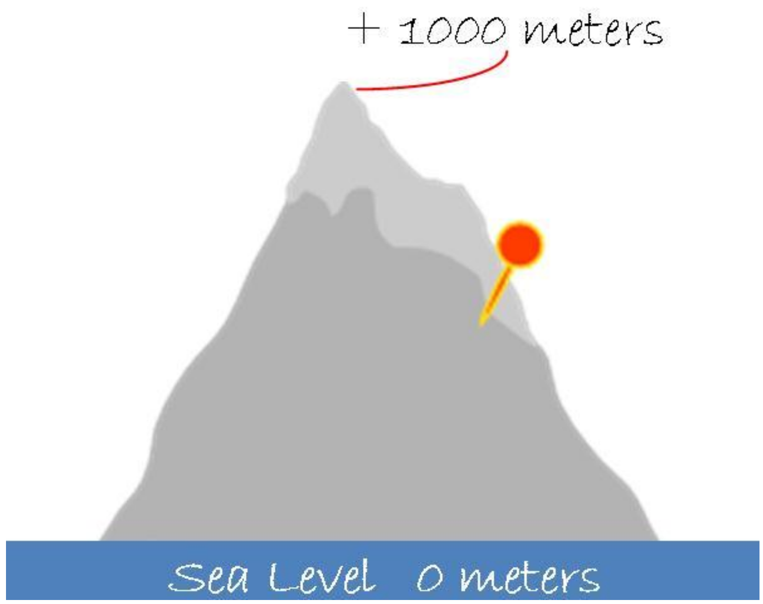
Learning Outcome: Students will demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically (British Columbia Ministry of Education, 2007, p.40).

Suggested Achievement Indicator: "add two given integers using concrete materials or pictorial representations and record the process symbolically" (BCME, 2007, 52). (In this case, students will use pictorial representations to add integers).

Teacher Considerations: Most students have a basic understanding of integers as positive and negative numbers based on the Grade 6 curriculum and real-world experiences. However, simply writing the vocabulary term integers on the board will not be enough to access the complex schemata students already possess because specific numeracy vocabulary is difficult to retrieve from memory. Students need some prompting and context to place the vocabulary term.

Lesson Procedure:

1. Begin by asking students what they already know about integers and record the results on the board. This provides a record of collective knowledge for the class.
2. Tell students that they will be drawing a scene that will have many different integers represented and relationships between the integers will become clear (in this first case they will copy the picture being drawn from the board before creating their own original).
    - It is important to reinforce that although the drawing should be accurate, it does not have to be artistically magnificent. This reassurance will address some of the concerns of students who perceive themselves as unable to draw images. Teachers should model the type of drawing required.
3. Draw a line across the middle of the board horizontally and label the line 0 (sea level). This drawing can be as detailed as you want depending on time and the amount of formative assessment needed to show understanding.
4. Draw a mountain on the land and label it an arbitrary height (1000m). Ask the class if that is +1000m or - 1000m.



5. Draw a shark swimming in the ocean and label him an arbitrary depth (50m). Ask the class if that is +50m or -50m. Label the depth as - 50m.
    - You can then add an airplane in the sky, an oil well and any other objects at various heights and depths until students demonstrate an understanding of the relationship between the placement of 0 and the integers.
6. To demonstrate the addition of integers have students add a tree on the mountain 750m above sea level, and label the elevation accurately (+ 750m).
    a. Add a small starfish 125m below the shark and label the depth of the starfish accurately (- 175m).
        - Model the addition of the integers in both the positive and negative context when adding these features. (-125m + - 50m = -175m)

b. Have the students repeat the procedure adding a bird, cabin, submarine and sea monster at specific, given amounts while moving about the class observing drawings and prompting students for explanation of their images.

7. This activity can progress into a representation of a thermometer using a manipulative or pictorial representation before making explicit connections from these pictorial representations to the structure and use of a number line when calculating operations with integers.

<u>Assessment Tools:</u> A simple checklist can be used to monitor students' understanding of the concept of integers throughout the lesson. The checklist can be completed using a simple Yes/No indicator in the columns while circulating throughout the classroom during independent work. A sample checklist that can be used for each student as follows:

	Yes	No
Accurate placement of +ve integers		
Accurate placement of -ve integers		
Accurate adding of +ve integers		
Accurate adding of -ve integers		

This lesson allows students to make real-world connections between integers. This creates relevance for the numeracy concept and honors the personal schemata individuals possess. Possible adaptations for this introductory lesson include having students do fewer representations if written output is difficult or having students work with a partner to create a larger picture that demonstrates understanding by both students. The lesson can be adapted to address any number of learning outcomes within the number strand when the beginning focus is representation of a real-world example or situation.

Cautions.

Despite this strategy possessing several benefits including multiliteracy opportunities (illustrations, technology support, written text), differentiation opportunities, and the presence of choice and creativity, there are some cautions that must be acknowledged. Some students have difficulty drawing the images and will require adaptations to complete the task such as an outline drawing of the mountain or pictures on cards to be glued into place. If technology is available, students can add integers to a picture that has been scanned into a document folder for them. For some students this process is not easily connected to algorithms and symbolic formulas. An explicit connection to number line representation, pictorial real-world examples and symbolic forms must be made through repeated instruction to ensure a comprehensive, accurate understanding of the concept. Ultimately, not every representation is beneficial for each student but one of the representations will most likely enable the students to build conceptual schema.

Lesson Example Two

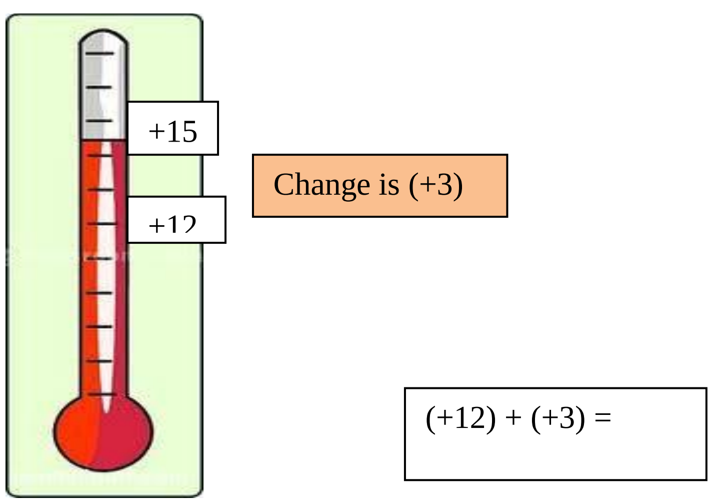
Learning Outcome: Students will be able to demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically (BCME, 2007, p. 40).

Suggested Achievement Indicator: "add two given integers using concrete materials or pictorial representations and record the process symbolically" (BCME, 2007, p. 52).

Teaching Considerations: This lesson can follow the previous lesson or be used as an introduction for the unit. Students will practice reading positive and negative integers in a real world context such as temperature within a model-guide-practice gradual release of responsibility model.

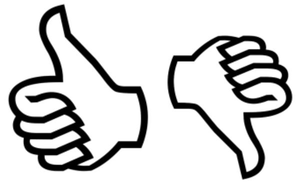
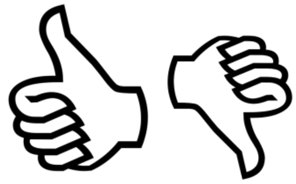
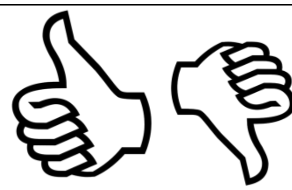
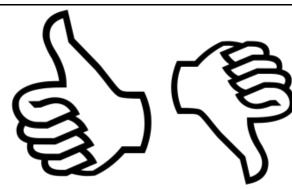
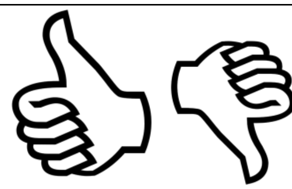
Lesson Procedure:

1. Teacher demonstrates reading the temperature on a thermometer at the front of the class. Use a beaker that is half-full of warm water and read the temperature. Record on the board. Students record the temperature on a representation of a thermometer in their notebooks.
2. Add some hot water to the beaker and decide how many degrees the temperature increases. Have students record the change in temperature as an integer (+3) on their paper.
3. Model the addition equation of integers on the board for the class (+12) + (+3) = (+15)
4. Students record the change in temperature and the result on their thermometer representation. They copy the equation from the board to their representation. Including a scale of degrees and the numerical labels shifts the representation along the continuum of development towards schematic imagery.



5. Place an ice cube in the water and record the temperature change on the board as a negative integer (-5). Students record the change in their notebooks.
6. Model the addition equation of integers on the board for the class, (+15) + (-5) = (+10)
7. Students record the change in temperature and the result on their thermometer representation. They copy the equation from the board to their representation.
8. Give the students the hypothetical scenario, “The temperature is +3 and rises 4 degrees. What is the new temperature?” They must complete the question using another thermometer representation to demonstrate the change and the numerical equation, (+3) + (+4) = (+7). Circulate to check for understanding by asking students to verbally explain their thinking.
9. Give students the hypothetical scenario, “The temperature is +8 and drops 6 degrees. What is the new temperature?” They must complete the question using another thermometer representation to demonstrate the change and numerical equation, (+8) + (-6) = (+2). Circulate to check for understanding by having students verbally explain their thinking.
10. Students work in partners to create 3 additional scenarios based on the questions modeled. They must include a thermometer representation for each and a numerical equation.

<u>Assessment:</u> Students complete a peer and self assessment checklist based on their understanding and confidence with the material. A simple thumbs up/thumbs down technique works well to fill in the boxes. The same checklist can be used for both assessments. Students colour or circle the image that applies to them or their partner for each statement.

I can find and draw the starting temperature.	
I can find and draw the ending temperature.	
I can show the temperature change on the thermometer.	
I saw that adding a positive meant the temperature rose.	
I saw that adding a negative meant the temperature dropped.	

This lesson shows students a second example of real world connections for integers. Students who have written output issues or for classes with time constraints, outlines of thermometers can be supplied by the teacher. Students can colour the thermometers and label the temperatures with numerical (integer) values. As an adaptation to increase the complexity of the problem, students can add integers that move the temperature across the zero line. For example, start at -2 and increase the temperature 10 degrees. The equation is (-2) + (+10) = (+8). This small change in the numerical values challenges students to understand the abstract concept of negative and positive values.

Numeracy Strand: Number. Topic: Decimals.

Visual strategy: Graphic organizer

Lesson Example Three

Learning Outcome: Students will be able to compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using benchmarks, place value and equivalent fractions and or decimals (BCME, 2007, p. 40)

Suggested Achievement Indicator: "order the numbers of a given set that includes positive fractions, positive decimals and/or whole numbers in ascending or descending order, and verify the result using a variety of strategies" (BCME, 2007, p. 53).

Teaching Considerations: It is important to determine the assessment tool and what the assessment goal will be for the activity when planning the lesson so that students are clear about the learning focus; by doing so, the activity is made relevant to their learning progression. There are several traditional ways to teach this learning outcome using a variety of manipulatives, number line representations and pictorial representations. I will provide an example of a word problem application using the specific graphic organizer developed by Arthur Hyde (2006) which is based on Donna Ogle's strategy of KWL (Know-Want to Know-Learn).

Lesson Procedure:

1. Provide the students with a blank copy of the graphic organizer below (Appendix B) and a copy of the word problem.
2. Read the word problem aloud, instructing the students to focus on creating an image in their mind that describes the context of the problem.

"The school raised $8500.00 for the Tour de Rock last year. Each pod in the school participated and raised a sum of money. Red pod raised 3/20, Blue pod raised 0.18, Yellow pod raised 2/10, Orange pod raised 0.35 and Green pod raised 0.12 of the total amount. Who raised the largest portion of the total? Arrange the pods and the fractions/decimals from greatest to least according to how much money they raised."

- This problem uses a real-world concept and connection for the students to build schemata about the relative quantity of fractions and decimals.

3. Students can then draw a quick sketch of the situation presented in the problem.
    - Reinforce that this representation should be a reflection of their mental image and should not yet include any numerical symbols.
4. Students are then guided through the use of the graphic organizer (Hyde 2006) as the teacher models the filling out of information in the columns.

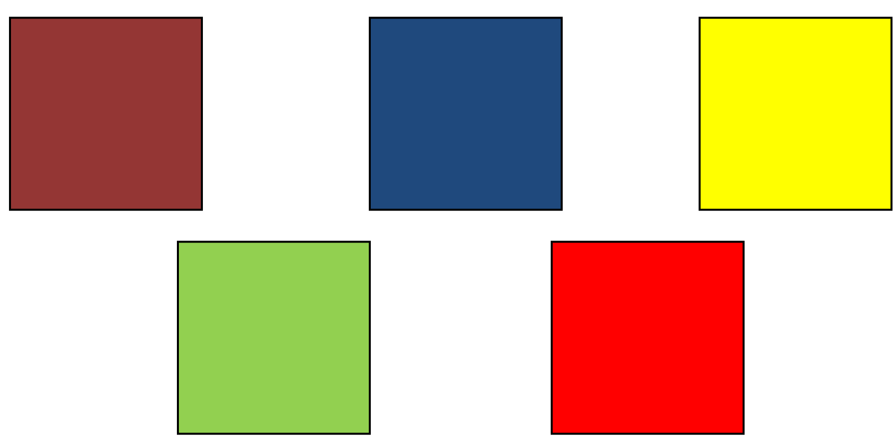
What do you know for sure? *-there are 5 pods* *-each pod raised some money* *-Red-3/20, Blue 0.18, Yellow 2/10, Orange 0.35, Green 0.12of the total*	What are you trying to find out? What will the answer tell us? *-Who raised the most money* *-the order of pods from greatest to least* Will the answer (estimate) be BIG or small?-*BIG 3850.00*	What are special conditions or assumptions? *-the amounts must equal 100% or 1.00 or 100/100* *-must know conversion of fractions-decimals or decimals-fractions* *-greatest=most=biggest*

a. The first column allows students to organize the information needed from the problem in a more succinct manner.
b. The second column requires students to actively set a goal for the solution and use mental estimation to predict the outcome. Depending on the time allowed for the

activity, students may vocalize or write down justifications for the estimate to clarify connections to schema.

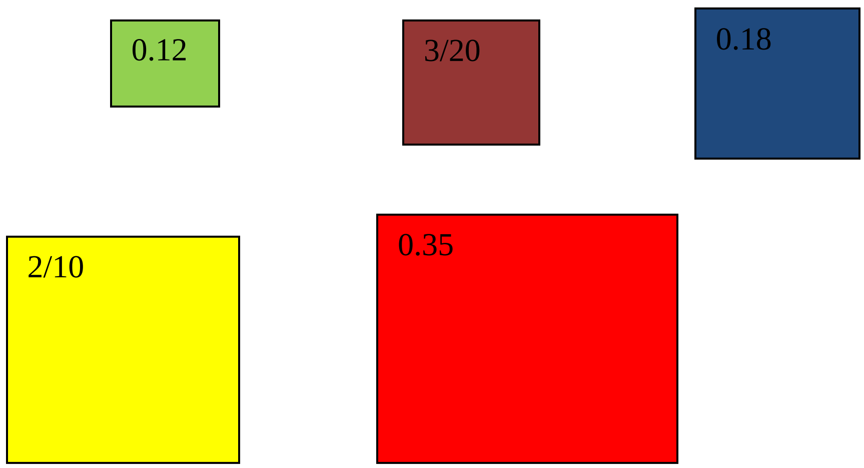
  c. The third column is where students make connections to prior leaning, clarify important vocabulary and make assumptions explicit.

5. Once the organizer is completed, students choose an appropriate strategy to solve the word problem.
   a. They must label numerical symbols to identify what each symbol represents and explain their thinking process in clear steps.
   b. Doing a teacher think-aloud during the first word problem will help students understand the depth and breadth of explanation needed to make procedural thinking clear.
   c. At this point students should be using representations and imagery of the problem find a solution
   d. A pictorial representation of the solution would include boxes to reflect the different pods and perhaps colours for identification



Students can then use this representation as a dynamic tool (Stylianou, 2011) to further their understanding by adding numbers for each of the pod colours and a total amount at the end.

e. A sophisticated schematic representation of the problem is shown below:



Red + Blue + Yellow + Green + Orange = total money

<u>Assessment Tool:</u> The final solution can be assessed according to the performance standards provided by the Ministry of Education (www.bced.gov.bc.ca/perf_stands/numerg7.pdf) or by means of a teacher created criteria sheet for each student such as the checklist below:

criteria	**All**	**Most**	**Some**	**None**
Are the amounts in correct order?				
Is there a representation shown for the amount?				
Is there evidence				

of conversion between fraction-decimal-whole numbers?				

.

This basic procedure can be adapted for many topics within numeracy and a variety of word problems. The use of an organizer to spatially reorganize elements of the problem was a successful method used in the research by Zahner and Corter (2010), who commented that "reorganization may aid in the abstraction of a problem schema from the text of a word problem in part by selecting out the critical problem information from the mass of superficial story detail" (p. 190). Students should then be able to choose an appropriate strategy for solving the problem based on their schema. As the graphic organizer becomes more familiar, other explicit connections can be made between the organizer and comprehension strategies including making connections, predicting, inferring and summarizing. Reinforcing these strategies in numeracy will help students develop the skills to transfer their use across the curriculum. The organizer can be adapted into an electronic format for students with output issues and specific IEP recommendations. Possible adaptations to make the organizer more challenging would be adding a fourth column to make connections to other problems, contexts or domains. The use of the chart will be familiar to students who have already used the KWL chart, but its particular application in numeracy will need to be practiced several times in multiple contexts before it will become an important part of their operating schema.

Lesson Example Four

Learning Outcome: Students will be able to demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions (BCME, 2007, p. 40).

Suggested Achievement Indicator: “Sort a given set of fractions as repeating or terminating decimals” (BCME, 2007, p. 52).

Teaching Considerations: Students must be able to successfully complete long division questions prior to introducing terminating and repeating decimals. Spending time reviewing the algorithms for long division (How many goups?- Multiply- Subtract- Bring down next digit- Repeat) will help students experience success with this lesson. Additionally, many students forget or have not been taught that the line in a fraction represents division. For example, ¾ can be read as 3 divided by 4. Reinforcing this concept with repeated practice of division questions prior to this lesson will enable students to be successful in writing the questions accurately.

Lesson Procedure:

1. Write two decimals on the board, 0. 234 and 0. 666666666666666… as examples of terminating and repeating decimals. Ask students what they notice about the decimal numbers without giving them the vocabulary terms. Record student observations on the board for later reference.
2. If students did not brainstorm the ideas of decimals as parts of a whole (fractions) then introduce the concept now. For example, 0.234 is the same as 243/1000. Have students use long division to verify the answer.
3. Ask students: “What is the fraction for the repeating decimal?” Some may answer that it is impossible, this is true when only considering denominators that are multiples of ten.

Ask the students: “What happens when the denominator is a 3?” Give them the fraction 1/3 to divide using long division. Have students try the question individually, check their answers with a partner and then complete as a class on the board. The answer should be 0.33333333…

4. Hand out the chart below which is a graphic organizer to create a comparison of decimals and fractions.

Fraction	Decimal	Repeating OR Terminating
1/2		
2/3		
3/4		
2/5		
5/6		
5/7		
5/8		
4/9		
8/10		

- Students use long division on a separate sheet of paper to divide (convert) the fractions to decimals. The complete the chart by writing in the decimals and the word repeating or terminating.

5. Students then shade in the rows of repeating decimals yellow and terminating decimals blue. These colours will help students see the differences in the rows and regroup the fractions and decimals to observe patterns. Below the chart regroup the

terminating decimal fractions (1/2, 3/4, 2/5, 5/8, 8/10) and repeating decimals (2/3, 5/6, 5/7, 4/9) together.

6. Students make a list of the terminating denominators: 2, 4, 5, 8, 10. Ask: "What do you notice? Is there a pattern here?" Students should be able to see that most are even numbers, all except 8 are factors of 100 and 8 is a factor of 1000.

7. Students write a rule for terminating decimals in relation to denominators. A possible rule could be: If the denominator is a factor of a multiple of ten, then the decimal is terminating.

8. Students test a partner and the rule by creating fractions with denominators of 20, 50, and 125. Partners must convert the new fractions into a decimal using long division. This task can be done on a second copy of the chart above.

9. Students list the repeating decimals denominators: 3, 6, 9, 7. Ask: "What do you notice? Is there a pattern here?" Students should be able to see that most are multiples of 3 or the prime number of seven.

10. Students write a rule for repeating decimals in relation to denominators. A possible rule could be: If the denominator is a multiple of 3 or a prime number, the decimal is repeating.

11. Students test a partner and the rule by creating fractions with the denominators of 30, 13, and 17. Partners must convert the fractions to decimals using long division. This task can be done on the same chart.

<u>Assessment:</u> Students hand in the charts with the fractions they created for their partners. Students are marked on their ability to see the patterns, to convert fractions to decimals accurately, and their overall participation in the lesson based on informal observations. A more

formal assessment can be achieved by including similar questions on a unit test for decimals at a later date.

Adaptations can be made for students with output issues by scanning the charts into a computer and placing in a documents file. For other IEP considerations related to math computations and chunking tasks, students can use a calculator to convert the fraction to a decimal. It is important to remember that most students will need guidance and modeling to show the correct steps to enter numbers into the calculator. If time permits it would be beneficial for all students to use a calculator to check their long division. This lesson can also be adapted to challenge students by having students check fractions with very large denominators. For example, 3333 or 129. Not only will students find this challenge very engaging but it also reinforces the concepts taught in the lesson.

Numeracy Strand: Variables and Equations. Topic: Algebra

Visual strategy: Schematic Diagram

Lesson Example Five

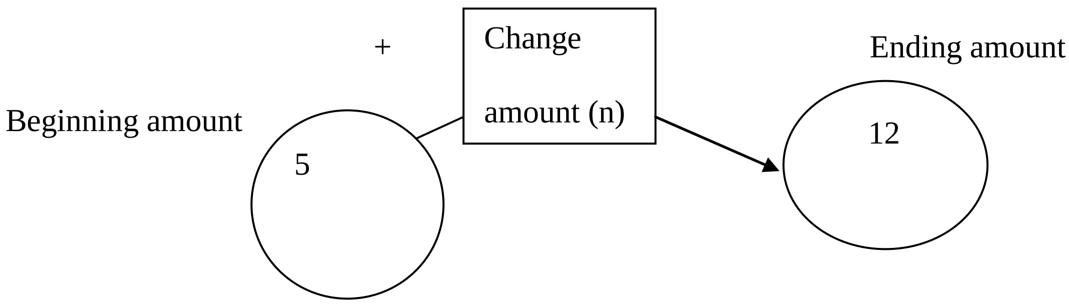
Learning Outcome: Students will be able to model and solve problems that can be represented by one-step linear equations of the form $x+a=b$, concretely, pictorially, and symbolically, where a and b are integers (BCME, 2007, p. 40)

Suggested Achievement Indicator: "draw a visual representation of the steps required to solve a given linear equation" (BCME, 2007, 55).

Teacher Considerations: Students in Grade 7 are familiar with the basic premise of algebra but have not seen the expression of an unknown quantity represented as a letter. This simple substitution of a letter in the expression or equation rather than an empty box or question mark can be daunting at first. Introducing students to schematic diagrams and reinforcing the diagrams through many opportunities of practice will allow the students to build schema for algebraic operations. These schematic diagrams (Appendix C) are based on diagrams used in the research studies of Jitendra, Hoff and Beck (1999) and Jitendra et al., (2009). In an equation where students are required to find a missing component of a change problem the specific schematic diagram can be used for organization of information.

Lesson Procedure:

1. Provide an algebraic example on the board: 5 + n = 12 Have students complete the diagram inserting the information in the appropriate places (see below).



2. Place the operational sign between the first two shapes to clarify what the variable 'n' represents.

- Many students will solve the mathematical equation by intuition based on their prior knowledge of addition number facts.

3. Do several examples of algebraic equations using the same schematic diagram to reinforce the process and visual representation of the problem.

4. Have students dictate to a partner the steps they are using to solve the equations as they explain the process.

- Incorporating this oral component addresses the concern that some students will not be able to breakdown the process in their minds and will rely on "I just knew the answer".

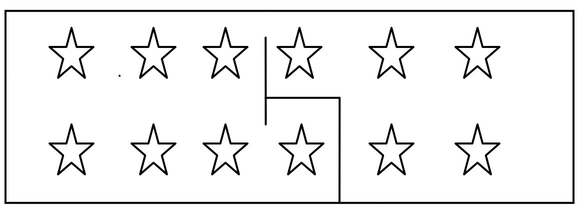
5. Partners then share with the class their strategies before attempting to formalize the steps in a visual representation.

The goal is have students understand that moving backwards through the diagram requires use of opposing operations to solve the problem. The explanation of the steps for solving an algebraic equation should be detailed and may include images. For example:

When I saw the question 5 + n = 12, I knew right away that n had to be 7. I could replace n with 7 in the equation to check my guess. 5 + 7 = 12

I then tried to use the opposing operation of subtracting to find n. 12 - 5 = n. I can solve this question easily, and found the answer 7.

If I draw a diagram of 12 stars and make a group of 5, then the leftover stars will be the answer



<u>Assessment Tool:</u> A simple rubric (see below) that assesses the level of detail in the written explanation and accuracy of the steps can be created by the teacher for this activity. Each rubric can reflect the different focus of the lesson.

Criteria	**Exceeding expectations**	**Fully meeting expectations**	**Minimally Meeting expectations**	**Not yet meeting expectations**
diagram	Fully completed and correct	Fully completed and correct	Fully completed may have a minor error	Not completed
explanation	-uses text and representations to explain thinking -specific details are provided -thinking is clear and logical	-uses text and representations to explain thinking -details are provided -thinking is clear but may miss expressing a "step"	-uses representations and some text -few details are provided -steps are missing in the explanation so thinking is unclear	-uses only a representation or text -few or no details are provided -thinking is confusing and missing several steps
Understanding of variable	-demonstrates understanding of variable placement	-demonstrates understanding of variable placement	-demonstrates limited understanding of a variable	demonstrates no understanding of variable placement

Using simple diagrams like the ones presented in Appendix C based on schema-building instruction research by Jitendra et al. (2009) helps students develop a general application for the diagrams and encourages transference across domains. Students can use the same diagrams for fractions, decimals, integers, and whole numbers. When the process is familiar to students, the cognitive load is eased and they can focus on the specific calculations of the numbers (Zahner & Corter, 2010). The application of visual imagery as a tool for developing conceptual understanding is reinforced through a variety of exposures. Despite the simple presentation of the diagrams, students must be explicitly taught its components and correct usage for the diagram to be effective. As students work with increasingly complex problems, the diagrams can be combined and additional shapes added to accommodate new terms. These simple adaptations make the diagrams appealing for a number of grades and abilities.

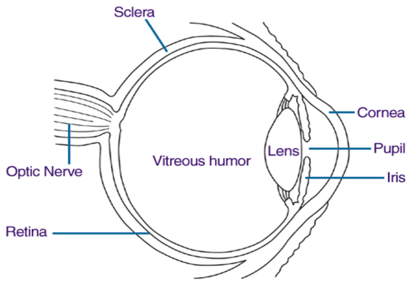
Final Thoughts on Visualization in Numeracy

Incorporating visualization strategies in the numeracy classroom has many potential benefits for students. These strategies, most commonly introduced to students in the context of literacy activities, are an important component of the multiliteracy approaches that complement the constructivist orientation to pedagogy that is being encouraged by recent government curriculum documents. Encouraging students to generate multiple visual representations of abstract numeracy concepts offers a way of scaffolding their learning and provides them with concrete tools with which to approach a diverse range of conceptual problems. Despite all the potential advantages of schema building and visualization in numeracy, there are some cautions that must be considered. First, not all visual representations are effective in developing concepts and schemata. In fact, pictorial imagery, images that depict literally what is described in the problem, has a negative impact on the success rate of problem solvers. Conversely, schematic imagery, images that show relationships and often include labels to clarify understanding, has a positive impact (Zahner & Corter, 2010). Numerous studies emphasized and highlighted the difference between pictorial and schematic imagery by stressing the importance of incorporating explicit connections within imagery and across symbol systems in numeracy. Second, visual imagery as a strategy for developing numeracy concepts has not been found to benefit all students in a single classroom (Woolner 2004). Several studies produced mixed results when determining the influence of visual imagery on the success rates of problem solvers. As with any strategy promoted in education, there must be the disclaimer that a strategy is merely a single tool in the instructional repertoire of teaching. Implementing schema building and visualization strategies in numeracy classrooms within a constructivist framework will not be the "holy grail" of teaching, but it will contribute to the development of conceptual understanding for many students.

Glossary

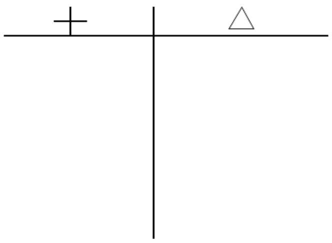
This glossary contains the relevant terms for the literacy strategy of visualization. These terms are included here because there is often confusion expressed by teachers and researchers with respect to the general definitions due to the variety of terms in use. In fact, Stylianou (2010) suggests that "it is noteworthy that researchers in mathematics education do not always agree on what representation means" (p. 326) or on many of the other important terms related to the topic. The following definitions are the ones used and cited throughout the handbook.

diagram: "a visual representation that presents information in a spatial layout" (Diezmann & English, 2001 as cited in Pantziara et al., 2009, p. 40)



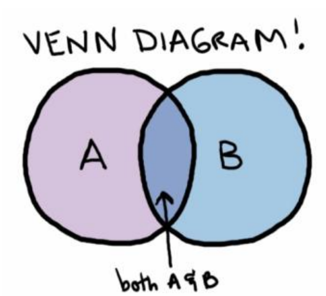
(www.nei.nih.gov)

external imagery: "manifestations of mathematical concepts that act as stimuli on the senses and help us understand these concepts" (Janvier, Giraradon & Morand, 1993 as cited in Pape & Tchoshanov, 2001, p. 119). Examples include: numerals, algebraic equations, graphs, tables, diagrams, or charts



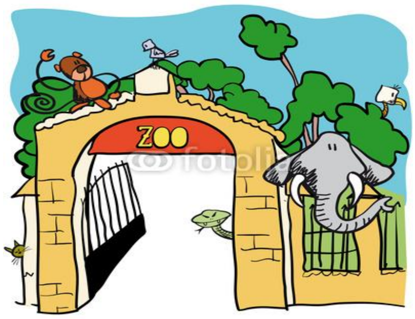
(www.better-leadership.com)

graphic organizer: "visual representations that help students identify, organize, and remember important ideas from what they read" (Boardman et al., 2008, p. 23). Examples include: Venn diagrams, KWL charts, concept maps, schematic diagrams or framed outlines



(www.streetbonersandtvcarnage.com)

illustration: a drawing or picture that reflects the text and the mental imagery associated with the text (Hibbing & Rankin-Erickson, 2003)



(www.us.fotolia.com)

imagery: “the ability to form mental representations of the appearance of objects and to manipulate these representations in the mind” (Hegarty & Kozhevnikov, 1999, p. 684)**internal imagery**: “abstractions of mathematical ideas or cognitive schema that are developed by a learner through experience” (Pape & Tchoshanov, 2001, p. 119)

manipulatives: concrete representations and tools used to teach abstract concepts. Examples include: base-ten blocks, fraction circles or integer tiles

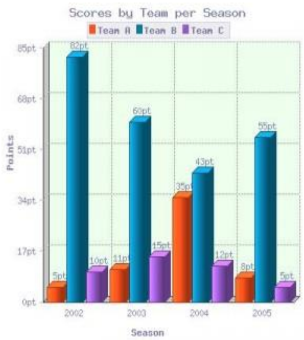

(www.mcruffy.com)

mental imagery: “the process of forming internal pictures of objects or events not present to the eye that can affect later recall and comprehension” (Douville, 2004, p. 36)

pictorial (iconic) imagery: “representations that depict the physical appearance of the elements described in the problem” (Zahner & Corter, 2010, p. 180)

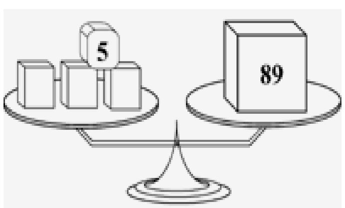

(www.hvnet.com)

representation: “a configuration that stands for something else” (Stylianou, 2010, p. 266). Examples include: symbolic expressions, drawings, written words, graphs, diagrams or numerals.



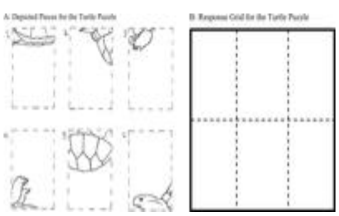
(www.hhsprincipalsoffice.wordpress.com)

schematic imagery: “representations that depict relationships described in the problem” (Zahner & Corter, 2010, p. 180)



(www.learnnc.org)

spatial imagery: “a representation of the spatial relationships between parts of an object and the location of objects in space or their movement” (Hegarty & Kozhevnikov, 1999, p. 685)


(www.psucnet.apa.org)

visualization: “the construction and formation of internal images (e.g. mental images) and/or external images (e.g. with the aid of pencil and paper) and then using those images effectively for mathematical discovery and understanding” (Zimmermann & Cunningham, 1991 as cited in van Garderen, 2003, p. 246), or “the act of bringing a visual onto the mind’s screen” (Mazur, 2003 as cited in Kotsopoulos & Cordy, 2009, p. 261)

visual imagery: “a presentation of the visual appearance of an object, such as its shape, color, or brightness” (Hegarty & Kozhevnikov, 1999, p. 685)

(www.recipes.wikia.com)

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Appendix B

What do you know for sure?	What are you trying to find out? What will the answer tell us?	What are the special conditions or assumptions?
	Will the answer (estimate) be BIG or small?	

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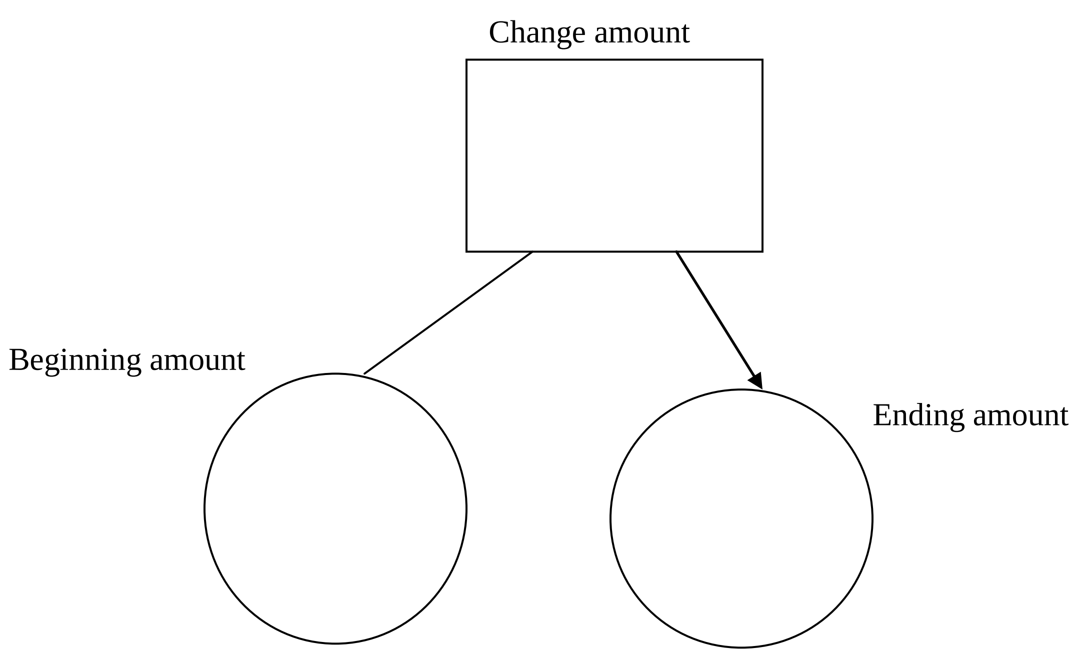
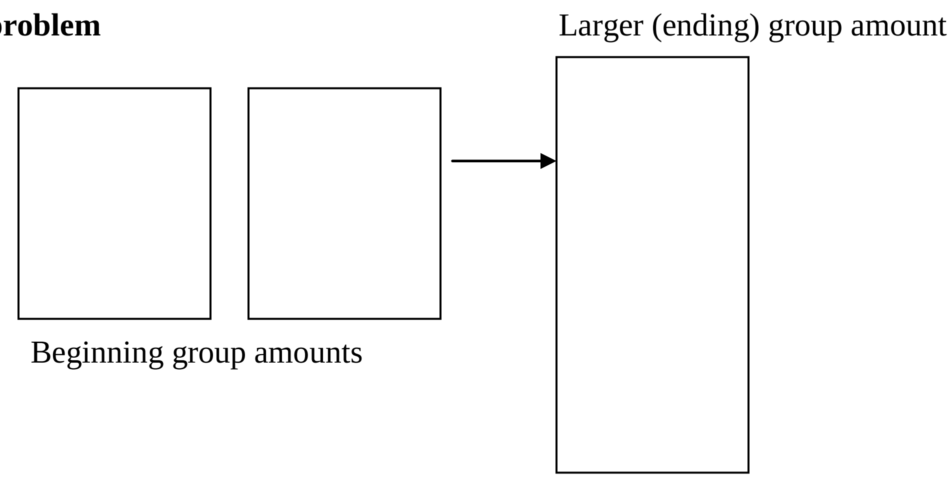
Appendix C- (Jitendra et al., 1999, p. 52)

Change problems



Group problem



Compare Problem