Some Methods of Analysis of Fishery Catch-Effort Data

by

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B.Sc., University of Ghana, 1988

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

in the Department of Mathematics and Statistics

We accept this thesis as conforming to the required standard

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Abstract

Most existing methods of analyzing fishery catch-effort data either implicitly assume that the main source of randomness is in the stock dynamics while ignoring randomness in the catching process or vice versa. In this thesis we consider the problem of estimating the parameters of a simple fishery model from a time series of catch and effort by two methods, one of which allows for randomness in both processes.

The Kalman filter approach incorporates a stochastic dynamic model with a stochastic catch model where the only unobservable variable is the stock size. We considered two models in this instance; one in which we assumed that there is randomness in the relationship between the escapement and the stock size and one in which there is none. The main idea is the use of the Kalman filter to obtain the likelihood function in order to estimate model parameters.

Unlike the Kalman filter approach, the contagious distribution method uses a stochastic catch model coupled with a deterministic stock dynamic model. The basic idea followed in this case is the assumption that the probability that a given unit of fish is captured in a given time interval is influenced by other units also being captured.

We applied both methods on real data sets. Data sets were divided into two parts. One was used to fit the model and the other used to test the one-step ahead prediction. In both methods model parameter estimates are numerically obtained by the method of Maximum Likelihood, and where possible we use the likelihood ratio test procedure to obtain approximate confidence intervals for the parameters. We also estimated the 'optimal effort' using the Kalman filter approach and maximum substainable yield (MSY) using the contagious distribution method.

In terms of the prediction, the methods did reasonably well. Residual plots did not show any sign of model inadequacy. However, for the Kalman filter approach, sometimes we were not able to obtain interval estimates for

some of the parameters. Also we were not able to estimate the ratio of the variances and needed to assume it. The method also only estimates stock sizes as fraction of 'equilibrium' stock size.

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Contents

List of Figures

List of Tables

Acknowledgements

I wish to express my sincere gratitude to my Supervisor, Dr. W. J. Reed of the Department of Mathematics and Statistics, University of Victoria, for his guidance and encouragement and for his unlimited patience. Without him I could not possibly have finished this thesis. I would also like to thank the other members of my committee for the time they took to read this thesis. Finally, I would like to express my gratitude to my friends, especially Didit, for their moral support and encouragement throughout my studies.

Dedications

This thesis is dedicated to my parents: *Foli and Nako.*

Chapter 1

Introduction

For many fisheries detailed biological statistics are not available, and the only reliable data for assessment of potential yields and for management purposes are in the form of a bivariate time series of annual aggregated catch $\{C_t\}$ and aggregated effort $\{E_t\}$, $t = 1, \ldots, n$. The units of C_t would usually be units of mass, while units of E_t would be units of aggregated effort such as boat days or net hours etc., where standardized "boats" or "nets" are understood. In some cases aggregated effort data are not available; in others observations of catch per unit of effort (CPUE) from survey trawls are available for analyzing catch-effort data.

Basically two approaches have been used in analysing catch-effort data:

1. **Equilibrium based models** (Gulland [1961], Fox [1970], Mendelssohn [1980]), in which catch $\{C_t\}$ is related directly to effort $\{E_t\}$ without explicit consideration of transient changes in stock size over time.

   Gulland [1961] considered the regression of $\frac{C_t}{E_t}$ on $\frac{1}{m}(E_{t-1}, \ldots, E_{t-m})$.

Fox [1970] fitted the regression of $\ln(\frac{C_t}{E_t})$ on $\frac{1}{m}(E_{t-1}, \ldots, E_{t-m})$.

Chayes [1949] and Eberhardt [1970] have shown that scaling the dependent variable catch C_t by the independent variable effort E_t introduces artificial correlation into the data, thereby biasing the fit. Also ordinary least squares produces biased estimates and inflated F-statistics when used with variables lagged on themselves. (Johnston [1984]). Mendelssohn [1980] used a Transfer Function model (Box and Jenkins [1976]) on the C_t and E_t series to forecast the catches. He fitted a Transfer Function model of the form $(1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r)C_t = (\omega_0 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_s B^s)E_t + \eta_t$, where η_t is assumed to be an autoregressive moving average process of normally distributed disturbances, and B is the backward shift operator, i.e. $B^r y_t = y_{t-r}$. These methods ignore biological details for the sake of statistical simplicity. They provide no information on population abundance and its growth.

2. **Dynamic based models** (Deriso [1980], Pella and Tomlinson [1969], Schnute [1977,1985], Reed [1986], Ludwig, Walters and Cooke [1988], Berck and Johns [1991]), in which there is an attempt to account for the effect of stock changes over time on the observed catch-effort series.

   Deriso [1980], Pella and Tomlinson [1969], and Schnute [1977,1985] all assumed a deterministic catching process of the form $C_t = qE_tX_t$ where q is a constant called the **catchability coefficient** and X_t the stock (population) size at the start of the fishing season in year t.

Randomness was implicitly assumed to occur only in the population dynamics. Their mathematical models describing the population dynamics are manipulated so as to obtain equations in the observable quantities (C_t, E_t) and the parameters of the model. The parameters are then estimated by Ordinary Least Squares or Non-Linear Least Squares. These methods tend to ignore the time series structure of the data and could produce badly biased estimates of production parameters.

Reed [1986] allowed error in the catching process but assumed a deterministic reproduction process. He assumed that the total number of captures C_t is a random variable having a Poisson distribution with parameter $\lambda = qE_tX_t$. He then fitted a root-normal approximation to the Poisson model and to the (C_t, E_t) series. He however stated that the assumption of a deterministic reproduction process is as unrealistic as that of a deterministic catching process, and suggested that the "ideal" method should allow randomness in both the catching process and the population dynamics; however this leads to a model which is statistically very complicated.

Ludwig et.al. [1988] used two approximate methods to account for errors in both the catching process and the population dynamics in the analysis of catch-effort data. Their first method (Total Least Squares) involved augmenting the process error sum of squares with a sum of squares of estimated observation errors in the effort. Their second method (Approximate Likelihood) estimated the first and second moments of the distribution of the stock variable and used this to describe the joint distribution of the observed catches $\{C_t\}$, using a linear approximation from the series expansion of the stock equations. Berck and Johns [1991] used the extended Kalman filter to estimate the stock biomass of the Pacific Halibut fishery. They regarded stock size in two management areas as the state variables and effort to be the endogenous variable.

In this thesis we attempt to analyse catch-effort data by two methods:

1. **Kalman Filter Approach**

   We investigate the use of a Kalman filter to obtain maximum likelihood (ML) estimates of the parameters of a simple fishery model, in which effort observations are regarded as exogenous variables.

2. **Contagious Catch Model**

   We broaden the model of Reed [1986] to include a more sophisticated stochastic model of the catching process in which there is a "contagion" effect, i.e. units of fish are not treated independently. Instead the event that some fish are captured will be assumed to increase the probability that others are likewise captured. However the assumption of deterministic stock dynamics will be retained. This is not the case for the

Kalman filter approach where stochastic dynamics will be assumed.

Chapter 2

Kalman Filter Model

We begin with a brief description of the main ideas of the use of the Kalman filter to obtain the likelihood function for a given statistical model. Most of the following material can be found in Harvey [1989] (Chapter 3).

2.1 State Space Form

The general state space form (SSF) applies to a multivariate time series y_t containing N elements. These observable variables are related to an $m \times 1$ vector α_t known as the state vector by a **measurement equation.**

$$y_t = Z_t \alpha_t + d_t + \varepsilon_t, \quad t = 1, \ldots, T \tag{2.1}$$

where Z_t is an $N \times m$ matrix, d_t is an $N \times 1$ vector and ε_t is an $N \times 1$ vector of serially uncorrelated normally distributed disturbances with mean zero and covariance matrix H_t, i.e.

$$E(\varepsilon_t) = 0 \quad \text{and} \quad \mathrm{Var}(\varepsilon_t) = H_t. \tag{2.2}$$

In general the elements of α_t are not observable. However, they are known to be generated by the process

$$\alpha_t = T_t \alpha_{t-1} + c_t + R_t \eta_t, \; t = 1, \ldots, T \tag{2.3}$$

where T_t is an $m \times m$ matrix, c_t is an $m \times 1$ vector, R_t is an $m \times g$ matrix and η_t is a $g \times 1$ vector of serially uncorrelated normally distributed disturbances with mean zero and covariance matrix Q_t, i.e.

$$E(\eta_t) = 0 \quad \text{and} \quad \text{Var}(\eta_t) = Q_t. \tag{2.4}$$

Equation 2.3 is called the **transition equation.** The specification of the state space system is completed by two further assumptions:

1. The initial state vector, α_0, has a mean of a_0 and a covariance matrix P_0.

2. The disturbances ε_t and η_t are uncorrelated with each other in all time periods and uncorrelated with the initial state vector.

Equations 2.1 to 2.4 with the above assumptions form a model in what is called **the state space form** (SSF).

2.2 The Kalman Filter

The Kalman filter can be applied to models in the state space form. It is a recursive procedure for computing the optimal estimator of the state vector at time t based on the information available at time t. If the initial state vector, α_0, and disturbances are normally distributed, application of the Kalman filter enables the computation of the likelihood function; this opens the way for the estimation of unknown parameters in the model.

Let a_{t-1} denote the optimal (in the sense that it minimises the mean square error) estimator of α_{t-1} based on the observations up to y_{t-1}. Let P_{t-1} denote the $m \times m$ covariance matrix of the estimation error. Denote the estimate of the state vector α_t at time t given all observations on y up to and including time $t-1$ by $a_{t|t-1}$ and its covariance matrix by $P_{t|t-1}$. Given a_{t-1} and P_{t-1}, the optimal estimator $a_{t|t-1}$ of α_t is given by

$$a_{t|t-1} = T_t a_{t-1} + c_t. \tag{2.5}$$

while the covariance matrix of the estimation error is

$$P_{t|t-1} = T_t P_{t-1} T_t' + R_t Q_t R_t', \quad t = 1, \cdots, T \tag{2.6}$$

(where a $(')$ denotes the transpose of the matrix).

Equations 2.5 and 2.6 are known as the prediction equations.

Once a new observation becomes available at time t the estimator $a_{t|t-1}$ of α_t and its covariance matrix can be updated. The updating equations are

$$a_t = a_{t|t-1} + P_{t|t-1} Z_t' F_t^{-1} (y_t - Z_t a_{t|t-1} - d_t) \tag{2.7}$$

and

$$P_t = P_{t|t-1} - P_{t|t-1} Z_t' F_t^{-1} Z_t P_{t|t-1} \tag{2.8}$$

where

$$F_t = Z_t P_{t|t-1} Z_t' + H_t \tag{2.9}$$

and its inverse is assumed to exist. The estimators a_t and P_t are optimal given observations up to and including time t.

Taken together Equations 2.5 to 2.9 make up the Kalman filter. Given initial conditions, the Kalman filter delivers the optimal estimators of the state vector as each observation becomes available.

2.3 Maximum Likelihood Estimation

From the definition of the conditional probability density function, it is possible to write the joint density function of the observations as

$$p(y_1, y_2, \ldots, y_T) = \prod_{t=1}^{T} p(y_t|Y_{t-1}) \tag{2.10}$$

where $p(y_t|Y_{t-1})$ denotes the distribution of y_t conditional on the information set at time $t-1$, that is, on $Y_{t-1} = (y_{t-1}, y_{t-2}, \ldots, y_1)$. Regarded as a function of the parameter vector Θ, this gives the likelihood function

$$L(\Theta; y) = p(y_1, y_2, \ldots, y_T) = \prod_{t=1}^{T} p(y_t|Y_{t-1}). \tag{2.11}$$

If the disturbances and the initial state vector in the state space model are normally distributed, the distribution of y_t conditional on Y_{t-1} is also normal and its mean and covariance matrix are given directly by the Kalman filter.

Its mean is

$$\hat{y}_{t|t-1} = Z_t a_{t|t-1} + d_t \tag{2.12}$$

and its covariance matrix is F_t given in Equation 2.9.

The log likelihood function can thus be written as

$$l = \log L = -\frac{NT}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log|F_t| - \frac{1}{2}\sum_{t=1}^{T} v_t' F_t^{-1} v_t \tag{2.13}$$

where

$$v_t = y_t - \hat{y}_{t|t-1}, \qquad t = 1, \cdots, T \tag{2.14}$$

and $\hat{y}_{t|t-1}$ given in Equation 2.12 is the minimum mean square estimator of y_t. This form is sometimes known as the **prediction error decomposition of the likelihood.**

Once the likelihood function has been found it can be numerically maximized with respect to the unknown parameters in the model.

2.4 The Simple Fishery Model

In this and the next section, two models of a fishery with observed catch-effort data are formulated in state space form.

Model One

Since we cannot observe the actual stock of fish in the sea, we use an observable statistic as a proxy. Such an observable proxy is catch-per-unit effort(CPUE). Using catch-per-unit effort as a proxy for stock size has a long pedigree in the fisheries literature (e.g. Schaeffer [1954], Gulland [1961]).

Suppose that the population dynamics can be described by the **log-linear form**

$$X_{t+1} = aS_t^b e^{z_t}, \quad t = 1, \cdots, T$$

where X_t denotes total fish biomass at the start of the fishing season in year t and S_t denotes the escapement in that year; z_t is a random error term

$$z_t \sim N(0, \sigma_z^2)$$

and a and b are unknown parameters with $0 < b < 1$, $a > 0$. Thus we are assuming multiplicative, log-normal randomness in the population dynamics. Suppose also that the escapement S_t is related to X_t through the equation

$$S_t = e^{-qE_t} X_t.$$

This model assumes that mortality is proportional to fishing effort. It ignores natural mortality during the fishing season but is appropriate for a fishery with a relatively short fishing season. The constant q is known in the fisheries literature as the **catchability coefficient.** Substituting the escapement equation into the stock equation we have

$$X_{t+1} = ae^{-bqE_t} X_t^b e^{z_t}. \tag{2.15}$$

Taking the natural logarithm of Equation 2.15 and writing x_{t+1} for $\ln X_{t+1}$ we get

$$x_{t+1} = bx_t + \ln a - bqE_t + z_t. \tag{2.16}$$

This is a Gaussian linear model.

We shall think of Equation 2.16 as the state equation because it describes the evolution of the state variable, (log) fish stock x_t, whose value is never directly observed.

Although we cannot observe X_t directly, at least for past years we can estimate it using the standard proxy catch-per-unit effort, $U_t = \frac{C_t}{E_t}$ Suppose we assume that U_t is related to X_t through the equation

$$U_t = ke^{\varepsilon_t} X_t$$

where $k > 0$ and

$$\varepsilon_t \sim \quad N(0, \sigma_\varepsilon^2).$$

Then

$$y_t = \ln U_t = x_t + \ln k + \varepsilon_t. \qquad (2.17)$$

Equation 2.17 is the **measurement equation** because the dependent variable is observable and depends on the unobservable variable x_t.

The constant $\ln k$ in the measurement equation could be treated as part of the state x_t (Harvey [1989], p104). We however maintain it as a separate term. Thus, our measurement equation is

$$y_t = x_t + \ln k + \varepsilon_t, \qquad t = 1, \cdots, T \qquad (2.18)$$

and the state equation is

$$x_{t+1} = bx_t + \alpha - bqE_t + z_t \tag{2.19}$$

where $\alpha = \ln a$.

Suppose we further assume that ε_t and z_t are uncorrelated with each other in all time periods and the initial unobservable log stock x_0 has mean a_0 and variance P_0, then Equations 2.18, 2.19 and the above assumptions provide a fishery model in state space form with observation y_t.

Following Harvey [1981,1989], we let

$$\sigma_\varepsilon^2 = \sigma_*^2$$

and

$$\lambda = \frac{\sigma_z^2}{\sigma_\varepsilon^2}$$

so that

$$\varepsilon_t \sim \; N(0, \sigma_*^2)$$

and

$$z_t \sim \; N(0, \lambda\sigma_*^2).$$

Thus, we have parameterized the model in terms of σ_*^2, the ratio λ of the variances and the other parameters of the model, (α, b, q, k).

In specifying the mean and variance of the initial stock we assume that prior to fishing, the stock population is in a stochastic equilibrium (i.e. stationary state) so that

$$\begin{aligned} E(x_{\tau+1}) &= E(x_\tau) &= a_0 \\ Var(x_{\tau+1}) &= Var(x_\tau) &= P_0 \end{aligned}$$

for negative values of τ (i.e. before the start of fishing).

Taking expectation of Equation 2.19 for such values gives

$$a_0 = ba_0 + \alpha$$

i.e.

$$a_0 = \frac{\alpha}{1-b}$$

and taking variances, gives

$$P_0 = b^2 P_0 + \sigma_z^2$$

or

$$P_0 = \frac{\sigma_z^2}{1-b^2} = \frac{\lambda\sigma_*^2}{1-b^2}.$$

Now that we have put our simple fishery model into a SSF we can apply the Kalman filter. By letting the variance of the disturbances be proportional to a positive scalar σ_*^2 and similarly specifying the initial variance as proportional to σ_*^2 we can run the Kalman filter independently of σ_*^2. (Harvey [1981,1989])

If we let the best estimate of the stock at time t given all observations (on y) up to and including time $t-1$ be denoted by $x_{t|t-1}$ and its associated variance by $P_{t|t-1}$ then the prediction equations for the fishery model are

$$\begin{aligned} x_{t|t-1} &= bx_{t-1} + \alpha - bqE_t \\ P_{t|t-1} &= b^2 P_{t-1} + \lambda. \end{aligned}$$

The updating equations are

$$\begin{aligned} x_t &= x_{t|t-1} + P_{t|t-1} f_t^{-1}(y_t - x_{t|t-1} - \ln k) \\ P_t &= P_{t|t-1} - \tfrac{P_{t|t-1}^2}{1+P_{t|t-1}} \end{aligned}$$

where

$$f_t = 1 + P_{t|t-1}$$

is a scalar and

$$\hat{y}_{t|t-1} = x_{t|t-1} + \ln k$$

$$v_t = y_t - \hat{y}_{t|t-1}, \quad t = 1, \cdots, T.$$

Reparameterising the univariate model so that $Var(\varepsilon_t) = \sigma_*^2$ and $Var(z_t) = \lambda\sigma_*^2$ enables σ_*^2 to be concentrated out of the likelihood function. To see this we first observe that the prediction error decomposition yields

$$\log L = -\frac{T}{2}\log 2\pi - \frac{T}{2}\log \sigma_*^2 - \frac{1}{2}\sum_{t=1}^{T} \log f_t - \frac{1}{2\sigma_*^2}\sum_{t=1}^{T} \frac{v_t^2}{f_t}.$$

Since v_t and f_t do not depend on σ_*^2, differentiating the log likelihood with respect to σ_*^2 and equating to zero gives

$$\hat{\sigma}_*^2 = \frac{1}{T}\sum_{t=1}^{T} \frac{v_t^2}{f_t}.$$

Substituting back into the expression above for the log-likelihood yields **concentrated log likelihood** function

$$l(\hat{\sigma}_*^2; \Theta) = -\frac{T}{2}(\log 2\pi + 1) - \frac{1}{2}\sum_{t=1}^{T} \log f_t - \frac{T}{2}\log \hat{\sigma}_*^2$$

where Θ is a vector of the other parameters in the model ($\Theta' = (\alpha, b, q, k)$). To obtain maximum likelihood(ML) estimates one maximizes numerically the concentrated log likelihood l over the parameters Θ. Alteratively one can minimise

$$-2l(\hat{\sigma}_*^2; \Theta') = \sum_{t=1}^{T} \log f_t + T \log \hat{\sigma}_*^2.$$

Since one cannot compute derivatives of the concentrated log-likelihood one needs to use a minimization routine which does not depend on derivatives. One such routine widely used in statistics is the simplex algorithm of Nelder and Mead [1965].

2.5 A More Complex Fishery Model

Model Two

In **Model One** it was assumed that the escapement S_t was functionally related to the initial stock size X_t (i.e. no randomness in the relationship between S_t and X_t). We relax that assumption here and assume that

$$S_t = X_t e^{-[(q+\sigma_1 Z_t)E_t]} \tag{2.20}$$

where Z_t is a standard normal ($N(0,1)$) random variable and σ_1 is a standard deviation parameter.

We also assume that the catch C_t is related to X_t by

$$C_t = X_t[1 - e^{(-qE_t - \sigma_1 E_t Z_t)}]. \tag{2.21}$$

Thus since $C_t + S_t = X_t$, we are assuming that there is no natural mortality or growth over the fishing season. Such assumption would be reasonable for a seasonal fishery with short fishing season.

The stock dynamics will be assumed to be the same as in **Model One** i.e.

$$X_{t+1} = F(S_t)e^{\sigma_2 W_t}$$

where $F(S_t) = aS_t^b$ is the log-linear form, W_t is a standard normal random variable independent of Z_t and σ_2 is a standard deviation parameter. i.e.

$$W_t \sim \quad N(0,1)$$

$0 < b < 1;\ \sigma_2 \geq 0.$

This model is the same as the "normal error" model of Ludwig, Walters and Cooke [1988, p460] developed under the assumption that there are errors in the observation of the "effective effort", (the functional form F corresponds to one of their two possible forms).

Another way of justifying this model is through assuming that effort is measured accurately, but that there is randomness in the relationship between catch and effort (see Reed [1986]). In the above equation for the catch and escapement the catchability can be thought of as a random variable $q + \sigma_1 Z$. An intra-season stochastic model of the population dynamics which can yield the above relationship is given in Appendix A.

If we substitute the escapement equation into the stock equation we have

$$X_{t+1} = aX_t^b(e^{[(-q-\sigma_1 Z_t)bE_t]})e^{\sigma_2 W_t}.$$

Taking logs and writing x_t for $\ln X_t$ gives

$$x_{t+1} = \ln a + bx_t - bqE_t + \sigma_2 W_t - \sigma_1 Z_t E_t.$$

or

$$x_{t+1} = \ln a + bx_t - bqE_t + \eta_t \tag{2.22}$$

where

$$\eta_t = \sigma_2 W_t - \sigma_1 Z_t E_t.$$

This is our **transition equation** with

$$\eta_t \sim N(0, \sigma_2^2 + \sigma_1^2 E_t^2).$$

From the catch Equation 2.21 on dividing both sides by E_t we get

$$\frac{C_t}{E_t} = X_t \frac{1 - e^{(-qE_t - \sigma_1 E_t Z_t)}}{E_t}$$

as the equation relating the proxy catch-per-unit effort to the unobserved stock size X_t. Taking logs gives

$$y_t = \ln\left(\frac{C_t}{E_t}\right) = x_t + \ln\left(\frac{1 - e^{(-qE_t - \sigma_1 E_t Z_t)}}{E_t}\right).$$

If we assume that $\sigma_1 Z_t$ is small, then, if we take the first order Taylor series expansion of the second term about $Z = 0$ we get

$$y_t = x_t + \ln\left(\frac{1 - e^{-qE_t}}{E_t}\right) + \frac{E_t e^{-qE_t}}{1 - e^{-qE_t}} \sigma_1 Z_t.$$

or

$$y_t = x_t + A_t + B_t\sigma_1 Z_t.$$

where

$$B_t = \frac{E_t e^{-qE_t}}{1 - e^{-qE_t}}$$

and

$$A_t = \ln\left(\frac{1 - e^{-qE_t}}{E_t}\right).$$

If we let $\varepsilon_t = B_t\sigma_1 Z_t$ then our **measurement equation** is

$$y_t = x_t + A_t + \varepsilon_t \tag{2.23}$$

and

$$\varepsilon_t \sim \quad N(0, B_t^2\sigma_1^2).$$

However the measurement and transition equation disturbance ε_t and η_t are correlated

$$Cov(\varepsilon_t, \eta_t) = Cov(\sigma_1 B_t Z_t, \quad \sigma_2 W_t - \sigma_1 E_t Z_t) = -\sigma_1^2 B_t E_t.$$

Summarizing we have the SSF for this fishery model:

$$y_t = x_t + A_t + \varepsilon_t$$

as the measurement equation, and

$$x_{t+1} = \ln a + bx_t - bqE_t + \eta_t$$

as the transition equation, where

$$\varepsilon_t \sim \quad N(0, \quad \sigma_1^2 B_t^2)$$

$$\eta_t \sim \; N(0, \;\; \sigma_2^2 + \sigma_1^2 E_t^2)$$

$$E(\varepsilon_t, \eta_s) = \begin{cases} -\sigma_1^2 B_t E_t & t = s \\ 0 & \text{otherwise} \end{cases}$$

Following Chan et al. [1984] we can transform the SSF to an alternate SSF in which the measurement and transition equation disturbances are uncorrelated. Taking the transition equation and adding the measurement equation gives a new transition equation

$$x_{t+1} = \ln a + (b + \frac{E_t}{B_t})x_t - bqE_t - \frac{E_t}{B_t}(y_t - A_t) + \eta_t^* \tag{2.24}$$

where

$$\eta_t^* = \eta_t + \frac{E_t}{B_t}\varepsilon_t.$$

Our new system consists of the original measurement equation 2.23, together with the transition equation define above. The inclusion of y_t in the new transition equation does not affect the Kalman filter, as y_t is known at time t. By this construction $E(\eta_t^*, \varepsilon_s) = 0$ for all t, s and

$$\text{Var}(\eta_t^*) = \sigma_2^2.$$

Suppose we let $\sigma_2^2 = \sigma_*^2$ and $\sigma_1^2 = \lambda\sigma_*^2$. Then

$$\varepsilon_t \sim \; N(0, \;\; \sigma_*^2 \lambda B_t^2)$$

$$\eta_t^* \sim \; N(0, \;\; \sigma_*^2).$$

Using the same prior assumption as **Model One** the mean a_0 and variance P_0 of the initial stock is

$$E(x_0) = a_0 = \frac{\alpha}{1 - b}$$

and

$$\mathrm{Var}(x_0) = P_0 = \frac{\sigma_*^2}{1-b^2}.$$

We can now run the Kalman filter for the transformed SSF independently of σ_*^2. If we let $x_{t|t-1}$ and $P_{t|t-1}$ have the same meaning as that in **Model One** then the prediction equations of **Model Two** are

$$\begin{aligned} x_{t|t-1} &= \ln a + (b + \tfrac{E_t}{B_t})x_{t-1} - bqE_t - \tfrac{E_t}{B_t}(y_t - A_t) \\ P_{t|t-1} &= (b + \tfrac{E_t}{B_t})^2 P_{t-1} + 1. \end{aligned}$$

The updating equations are

$$\begin{aligned} x_t &= x_{t|t-1} + P_{t|t-1} f_t^{-1}(y_t - x_{t|t-1} - A_t) \\ P_t &= P_{t|t-1} - \tfrac{P_{t|t-1}^2}{f_t} \end{aligned}$$

where

$$f_t = \lambda B_t^2 + P_{t|t-1}$$

is a scalar and

$$\hat{y}_{t|t-1} = x_{t|t-1} + A_t$$

$$v_t = y_t - \hat{y}_{t|t-1}, \quad t = 1, \cdots, T.$$

Like **Model One** σ_*^2 can be concentrated out of the likelihood function yielding a concentrated log-likelihood of similar form to that of **Model One**. The simplex algorithm can again be used to obtain the MLEs of the parameters. Computations were performed by generating FORTRAN code for the concentrated log-likelihood using the simplex algorithm (Nelder and Mead [1965]), E04CCF in the NAG library. (Harvey [1981a, Chapter 4]).

Chapter 3

Results from The Kalman Filter Models

The method was applied on two real data sets; (1) **The North Atlantic Redfish** (*Sebastes marinus*) of ICNAF division 3L from 1959 – 1991 (Power[1992]) and (2) **Rock sole** (*Lepidopsetta bilineata*) from area 5C in British Columbia from 1956 – 1989 (Fargo and Leaman[1991]).

Plots of catch versus effort (Fig. C1 and Fig. C3) show a tendency not only for the occurrence of higher levels of catch at higher levels of effort but also for greater variability in catch at higher levels of effort.
Figs. C4 and C6 show time series plots of catch and effort for the two data sets. Solid lines join the effort and dashed lines join the catches.

It soon became apparent when attempting to maximize the concentrated log-likelihood that one could not maximize simultaneously over the parameters Θ and λ. Attempts to do this led to convergence to different maxima with similar values of the objective function using different starting values.

Further investigation indicated that simultaneous changes in the parameters α and λ led to very small changes in the concentrated log-likelihood. The parameter a is simply a *scale factor* and since direct observations on the stock-size variable X_t are not available, the parameter a (or $\alpha = \ln a$) is not estimable. Because of this, α was set to zero (a set equal to one) in both **Model One** and **Model Two**. This implies that the median equilibrium stock size before fishing was 1, and the estimates $\hat{X}_t$ of the stock size subsequently are expressed as proportions of this median equilibrium stock size. Thus the estimates $\hat{X}_t$ are dimensionless quantities. Using this method implies that for **Model Two** we need a multiplicative constant on the right hand side of the catch Equation 2.21 to change the units of the catches, thus there will be an extra term (log(constant)) in the measurement equation.

Attempts to maximize the concentrated log-likelihood with respect to λ led to a failure of the optimization routine to converge. It led to larger and larger values of λ with very small changes in the objective function. It is not surprising that λ cannot be estimated by Maximum Likelihood. A similar situation arises in estimating the parameters of the structural or functional relationship between variable X and Y when both are observed subject to error (Kendall and Stuart Vol2, 1967, p375). A customary resolution of this difficulty in such "error-in- variables" problems is to assume that the ratio of the two error variances is known (Kendall and Stuart ibid, p380) and subsequently to investigate the sensitivity of the estimates of the other parameters to the assumed value of this ratio.

This was the method followed by Ludwig, Walters and Cook [1988] in their 'Total Least Squares' method and it is this procedure which we shall follow here. It turns out that the value of the concentrated log likelihood is not very sensitive to the assumed value of λ (For example when we applied the model on redfish data for $\lambda = 10$ the value of the concentrated log-likelihood is 97.24 while for $\lambda = 100$ the value is 97.27). In reporting results estimates will be given for three values of λ $(0.5, 1.0, 10.0)$ which were considered to cover the plausible range of the ratio of the two variances.

The likelihood ratio test procedure (Cox and Hinkley [1974]) was used to obtain approximate confidence intervals for the parameters for various choices of λ. Details of the likelihood ratio test procedure are given in Appendix A1. This procedure, although computationally cumbersome, is more likely to produce better confidence intervals for shorter data series than approximate confidence intervals of the form

$$\text{Estimate} \pm \; Z_{\frac{\alpha}{2}}\text{SE(Estimate)}$$

(where $Z_{\frac{\alpha}{2}}$ is $\frac{\alpha}{2}$ precentage point of a standard normal distribution), since such intervals are based on a quadratic approximation to the likelihood function which may be far from accurate for shorter time series. Since the estimates of the various parameters may be correlated, and it is not feasible to present joint confidence regions for more than two parameters, the reported confidence intervals are for the given parameter ignoring the estimates of the other parameters. The correlation between the parameters b and q is stronger in one data set than in the other(see Fig. C10 for that of rock sole and Fig. 11 for that of redfish for **Model One** with $\lambda = 1$ for the profile log-likelihood contours of the two parameters). The contours in Fig. C10 and Fig. C11 are at levels of one, two, three, four and five units below the maximum value. They can thus be interpreted as approximate 63%, 86%, 95%, 98%, 99% confidence regions for parameters q and b. (Kalbfleisch [1985b, p121]) The numerical results reported in this chapter are for three choices of λ.

Each data set was split into two parts, one part was used to fit the model; one step predictions were then made for later years and then compared with actual values in the other part.

Estimates and standard errors of the stock biomass were constructed from the log biomass estimates and their variance which were obtained from the Kalman Filter. Since the conditional and updated estimates of the log of stock biomass in each time period is distributed normally with mean x_t and variance P_t, the conditional and updated estimates of the stock in each time period X_t, has a log-normal distribution with mean

$$E(X_t) = e^{x_t + \frac{1}{2}P_t}$$

and variance

$$V(X_t) = e^{2x_t + P_t}(e^{P_t} - 1).$$

The biomass estimates x_t obtained by using part of the data were used to obtain one step ahead predictions of future observations of catch-per-unit of effort (CPUE) which were compared with actual observations.

For each data set and with $\lambda = 1$ and for both models plots of the "residuals"($e_{t|T}$) were performed. The residuals for **Model One** were calculated using the equation

$$e_{t|T} = y_t - x_{t|T} - \ln \hat{k} \tag{3.1}$$

where $x_{t|T}$ is the fixed-interval smoother of x_t given by

$$x_{t|T} = x_t + P_t^*(x_{t+1|T} - \hat{b}x_t) \tag{3.2}$$

and

$$P_t^* = \frac{\hat{b}P_t}{1 + \hat{b}^2 P_t} \qquad t = T-1, \cdots, 1 \tag{3.3}$$

with $x_{T|T} = x_T$ and $P_{T|T} = P_T$. (Harvey [1989]). Similar equation was used for the calculation of the "residuals" of **Model Two**.

Using the dynamic model the estimated optimal effort to maximize expected catch in equilibrium was calculated. If the parameters of the model were known, the optimal effort could be obtained as follows:
Assume a constant effort E in every year. Let $w = e^{-qE}$, then in the stock dynamic model we have

$$X_{t+1} = aw^b X_t^b e^{z_t}.$$

or

$$\ln X_{t+1} = b \ln X_t + \ln a + b \ln w + z_t.$$

(a stochastic difference equation).

If we write a few iterations we have

$$\begin{aligned}
\ln X_1 &= b\ln X_0 + \ln a + b\ln w + z_0 \\
\ln X_2 &= b^2 \ln X_0 + (b+b^2)\ln w + (1+b)\ln a + bz_0 + z_1 \\
&\vdots \\
\ln X_n &= b^n \ln X_0 + (b+b^2+\cdots+b^n)\ln w + (1+b+\cdots+b^{n-1})\ln a + \zeta \\
&= b^n \ln X_0 + \tfrac{b(1-b^n)}{1-b}\ln w + \tfrac{1-b^n}{1-b}\ln a + \zeta
\end{aligned}$$

where $\zeta = b^{n-1}z_0 + b^{n-2}z_1 + \cdots + z_{n-1}$ and

$$\zeta \sim N(0, \sigma_z^2 \frac{1-b^{2n}}{1-b^2}).$$

Alternatively we can write

$$\ln X_n = b^n \ln X_0 + \frac{b(1-b^n)}{1-b}\ln w + \frac{1-b^n}{1-b}\ln a + \sigma_z\sqrt{\frac{1-b^{2n}}{1-b^2}}Z$$

where Z is standard normal random variable.

Taking limits as n approaches infinity gives $\ln X_n$ converging in distribution to

$$\frac{b}{1-b}\ln w + \frac{1}{1-b}\ln a + \sigma_z\sqrt{\frac{1}{1-b^2}}Z,$$

i.e. when in a stationary state the stock size is a random variable given by

$$X = (w^b a)^{\frac{1}{1-b}} e^{\sigma_z\sqrt{\frac{1}{1-b^2}}Z},$$

the corresponding catch is also a random variable given by

$$C = (1-w)w^{\frac{b}{1-b}} a^{\frac{1}{1-b}} e^{\sigma_z\sqrt{\frac{1}{1-b}}Z}.$$

Its expected value is maximized with respect to w at

$$\bar{w} = b.$$

The corresponding effort which will maximize the steady state catch is

$$\bar{E} = -\frac{1}{q} \ln b. \tag{3.4}$$

The maximum likelihood estimate $\hat{E}$ of $\bar{E}$ is $-\frac{1}{\hat{q}} \ln \hat{b}$ in effort units. We obtain a confidence interval for the "optimal" effort by reparameterizing the log-likelihood in terms of $\hat{E}$, and use the profile likelihood to obtain the confidence interval for $\hat{E}$.

3.1 Results of Model Fitting

Model One

For both redfish data and rock sole data with λ fixed beforehand the maximization of the log-likelihood function converged satisfactorily.

(a) *Redfish*

Redfish data for the years 1959-1987 were used to fit the model. Observation (catch-per-unit effort) prediction for the years 1988-1991 were made later and compared with actual catch-per-unit effort. The parameter estimates and their approximate confidence intervals (in brackets) for the redfish data are given in Table 3.1. Residual plots were performed for $\lambda = 1$, and this showed no lack of fit.

We were not able to obtain a confidence interval for the parameter q using the likelihood ratio test procedure since the graph of the profile log likelihood of q was highly skewed and at the upper end did not drop below $1.92(=1/2\chi^2_{1,0.05})$ units below the maximum. (see Fig. C7 for the plot of the profile likelihood for q and compare with that for $\ln k$ (Fig. C8) for the redfish data). Thus the likelihood ratio interval for q is open-ended. However unboundedly large values of the catchability q are obviously not possible. In consequence we give little credence to this result. We could not obtain a confidence interval for the "optimal" effort since the graph of its profile likelihood has the same form as that of q.

	Redfish data		
	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 10.0$
$\hat{q}$	0.000037 [0.00001−)	0.000042 [0.00002−)	0.000053 [0.00001−)
$\hat{b}$	0.26845 [0.06 − 0.64]	0.23774 [0.07 − 0.58]	0.19009 [0.08 − 0.45]
$\ln \hat{k}$	7.3061 [7.22 − 7.45]	7.3042 [7.22 − 7.44]	7.3014 [7.24 − 7.44]
$\hat{\sigma}^2_*$	0.023179 [0.019 − 0.032]	0.01735 [0.013 − 0.026]	0.00317 [0.002 − 0.005]
$\hat{E}$	35543	34204	31326

Table 3.1: Maximum likelihood estimates for **Model One** parameters for redfish data and 95 percent confidence intervals (in square brackets). k is in CPUE units, σ^2 is dimensionless. Units of q are per thousand hours, while b is dimensionless.

(b) *Rock sole*

As with redfish data, the rock sole data for the years 1956-1985 were used to fit the model and data for the years 1986-1989 were used later to compare with the predicted CPUE. Residual plot for $\lambda = 1$ again showed no lack of fit. Using the likelihood ratio test procedure we obtain the interval $(2000 - 12400)$ as the 95% confidence interval for the "optimal effort" with $\lambda = 1$.

Table 3.2 shows parameter estimates and their confidence intervals in brackets for rock sole. A look at Tables 3.1 and 3.2 shows that for each data set and for each of the values of λ chosen, the parameter estimates are in fairly close agreement, except for the stock dynamic parameter b. Since, for the redfish data, large values of q are plausible, while for both data sets the confidence intervals for the estimate of b are wide estimation of "optimal effort" will not be very precise. For both data sets the narrower confidence interval for $\ln k$ seems to indicate that the CPUE seems to be a good proxy for the stock X. Because large values of q cannot be excluded and because estimate $\hat{E}$ of the "optimal effort" seems to be too big, we can infer that the data sets are not sufficiently informative for us to assign $\hat{E}$ with confidence. The presumption is that higher effort might be appropriate but we can give no indication of how high these should be chosen.

3.2 Predictions

Once the maximum likelihood estimates of the parameters based on data up to time $t = T$ have been obtained, we can obtained a one-step-ahead prediction for the unobservable stock size and for the CPUE. For our fishery model given the effort at time $T+1$ the one-step-ahead prediction of the log stock size is

$$x_{T+1|T} = \hat{b}x_{T|T} - \hat{b}\hat{q}E_{T+1}$$

with estimation error

$$P_{T+1|T} = \hat{b}^2 P_{T|T} + \lambda$$

where $x_{T|T} = x_T$ and $P_{T|T} = P_T$. Also a one-step-ahead prediction of the observation (log CPUE) is

$$\hat{y}_{T+1|T} = x_{T+1|T} + \ln \hat{k}$$

with the variance of the prediction error given by

$$MSE(\hat{y}_{T+1|T}) = (P_{T+1|T} + 1)\hat{\sigma^2}_*.$$

Here $\hat{y}_{T+1|T}$ is the conditional expectation of the observation (log CPUE) at time $T+1$.

A $100(1-\alpha)\%$ prediction interval for $\hat{y}_{T+1|T}$ is

$$\hat{y}_{T+1|T} \pm Z_{\frac{\alpha}{2}}\text{RMSE}(\hat{y}_{T+1|T})$$

where $Z_{\frac{\alpha}{2}}$ is $\frac{\alpha}{2}$ percentage point of the standard normal distribution and RMSE is the root mean square error. From the predicted log CPUE and its prediction interval the predicted CPUE and its corresponding prediction intervals can be computed.

The MSE given above under estimates the true MSE because it does not take into account the extra variation due to estimating the parameters of the model (i.e. it ignores sampling error). Therefore we do not necessarily expect the observed catch-per-unit effort to lie within plus or minus two estimated standard deviations of the predicted catch-per-unit effort with probability of about 0.95.

Tables 3.3 and 3.4 show one step-ahead prediction of CPUE and their 95% prediction intervals for **Model One**. Like the parameter estimates, the one-step ahead predictions and their prediction intervals did not change much for the different values of λ. The predictions and the prediction intervals on the whole are satisfactory except for the year 1991 for redfish and the year 1986 for rock sole where the observed CPUE does not lie within the estimated prediction interval; however if one takes into account the fact that sampling error in the estimates were not included in the calculation of the prediction interval, the overall the prediction performance may be better than they look at first sight.

MODEL TWO

Model Two was fitted to the same data sets as **Model One**. As in fitting in **Model One** it was assumed that λ was known beforehand. It became apparent that for different values of λ the objective and all the parameter estimates except σ_*^2 did not change however, $\lambda\sigma_*^2$ was always constant. The numerical results reported are for the value of $\lambda = 1$.

We were not able to obtain approximate confidence intervals using the likelihood ratio test procedure for the redfish data. The graphs of the profile likelihood for the parameters and the 'optimal' effort for this data set were basically flat to the right of the maximum point and did not drop below 1.92 units below the maximum.(see Fig. C7). Thus it appears that a large range of the parameters values are plausible.

The parameter estimates and their approximate confidence interval (where applicable) are in Tables 3.5 and 3.6. The parameter estimates for the redfish data using **Model Two** are different to those obtained using **Model One**, however the estimates of **Model Two** seem to indicate large values of effort are possible just like **Model One**. For both data sets the estimate of σ_*^2 using **Model Two** was 0.0[1], apparently indicating that there is a large negative correlation between the transition error and the state error or that the model is misspecified.

The estimates for rock sole using **Model Two** seem to agree to some extent with those of **Model One**. The estimate of the "optimal effort" using **Model Two** lies within the range of the observed effort series. We used the likelihood ratio test procedure to obtain a 95% confidence interval for the "optimal effort" for rock sole with $\lambda = 1$ by reparametering the model in

[1]estimated values are $0.120e - 12$ and $0.21e - 10$

terms of $\hat{E}$. The interval is $(180 - 3050)$ for **Model Two**.

It should be noted that the parameter $\ln k$ in Tables 3.5 and 3.6 for **Model Two** is the multiplicative constant that changes the units of the catches after the modification mentioned in the first part of this chapter. It is different from the constant k in the equation relating the (CPUE) U_t to the stock X in **Model One**.

For **Model Two** the one-step ahead prediction for the log stock is

$$x_{T+1|T} = (\hat{b} + \frac{E_{T+1}}{\hat{B}_{T+1}})x_{T|T} - \hat{b}\hat{q}E_{T+1} - \frac{E_{T+1}}{\hat{B}_{T+1}}(y_{T+1} - \hat{A}_{T+1} - \ln \hat{k})$$

with estimation error

$$P_{T+1|T} = (\hat{b} + \frac{E_{T+1}}{\hat{B}_{T+1}}^2)P_{T|T} + 1.$$

The one step prediction of the observation (log CPUE) is given by the formula

$$y_{T+1|T} = x_{T+1|T} + \hat{A}_{T+1} + \ln \hat{k}$$

with mean square error

$$\mathrm{MSE}(y_{T+1|T}) = (P_{T+1|T} + \hat{B}^2_{T+1})\hat{\sigma^2_*}.$$

Tables 3.7 and 3.8 show the one step predictions for the observation for **Model Two** and their prediction intervals for the two data sets. Like **Model One**, all of the observed CPUE except that of the year 1991 for redfish and the year 1986 for rock sole do lie within the one-step ahead prediction intervals, however the prediction intervals using **Model Two** are narrower than those of **Model One**. To compare the predictive performance of **Model One** and **Model Two** for $\lambda = 1$ we calculate a prediction error sum of squares statistic $H = \sum_{t=T+1}^{n}(u_t - \hat{u}_t)^2$ for both models and for each data set, where $\hat{u}_t$ is the one-step ahead prediction of CPUE.

In terms of prediction **Model One** performs better than **Model Two** for the redfish data. H is 0.399 for **Model One** and 0.416 for **Model Two**, however it is the other way around for the rock sole data. H is 0.064 for **Model One** and 0.047 for **Model Two**.

Rock sole data			
	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 10.0$
$\hat{q}$	0.000087	0.000101	0.000144
	[0.00004 – 0.0002]	[0.00004 – 0.0002]	[0.00001 – 0.0004]
$\hat{b}$	0.77684	0.71028	0.475699
	[0.58 – 0.90]	[0.46 – 0.87]	[0.21 – 0.73]
$\ln \hat{k}$	6.16613	6.12242	6.00935
	[5.96 – 6.48]	[5.99 – 6.35]	[5.86 – 6.28]
$\hat{\sigma_*^2}$	0.023420	0.018682	0.00406
	[0.018 – 0.035]	[0.015 – 0.028]	[0.003 – 0.006]
$\hat{E}$	2903	3387	5159
		[2000 – 12400]	

Table 3.2: Maximum likelihood estimates for **Model One** parameters for rock sole data and 95 percent confidence interval(in square brackets). k is in CPUE units, σ^2 is dimensionless. Units of q are per thousand hours, while b is dimensionless.

				Estimated CPUE		
Year	*Catch*	*Effort*	*Observed CPUE*	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 10.0$
1988	26267	20205	1.300	1.150[0.79 – 1.66]	1.161[0.80 – 1.68]	1.178[0.82 – 1.70]
1989	19847	12267	1.618	1.265[0.88 – 1.82]	1.242[0.86 – 1.79]	1.284[0.89 – 1.84]
1990	17704	17409	1.017	1.166[0.81 – 1.68]	1.274[0.88 – 1.84]	1.256[0.87 – 1.87]
1991	11642	14171	0.821	1.244[0.86 – 1.80]	1.236[0.85 – 1.79]	1.222[0.85 – 1.76]

Table 3.3: One step-ahead prediction of CPUE and their 95 percent prediction interval in parentheses of upcoming CPUE for the years 1988-1991 for redfish with **Model One**. Units of catch are in of metric tons. Units of effort are in hours.

				Estimated CPUE		
Year	*Catch*	*Effort*	*Observed CPUE*	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 10.0$
1986	86	497	0.173	0.362[0.24 – 0.54]	0.359[0.23 – 0.54]	0.347[0.22 – 0.53]
1987	209	496	0.421	0.274[0.17 – 0.44]	0.265[0.16 – 0.42]	0.257[0.15 – 0.41]
1988	189	545	0.347	0.351[0.22 – 0.57]	0.361[0.22 – 0.59]	0.357[0.21 – 0.58]
1989	406	1476	0.275	0.339[0.21 – 0.54]	0.343[0.21 – 0.55]	0.342[0.21 – 0.56]

Table 3.4: One step-ahead prediction of CPUE and their 95 percent prediction interval in parentheses of upcoming CPUE for the years 1988-1991 for rock sole with **Model One**. Units of catch are in metric tons. Units of effort are in hours.

	Redfish data
$\hat{q}$	0.00000171
$\hat{b}$	0.6207
$\ln \hat{k}$	20.5117
$\hat{\sigma}^2_*$	0.0
$\hat{E}$	278914

Table 3.5: Maximum likelihood estimates for model parameters in **Model Two** for redfish data. k is in CPUE units, σ^2 is dimensionless. Units of q are per thousand hours while b is dimensionless. Meaningful confidence interval were not possible for this data set

	Rock sole data
$\hat{q}$	0.000024
	[0.000009 – 0.00005]
$\hat{b}$	0.9546
	[0.86 – 0.99]
$\ln \hat{k}$	16.798
	[16.3 – 17.9]
$\hat{\sigma_*^2}$	0.0
$\hat{E}$	1957
	[180 – 3050]

Table 3.6: Maximum likelihood estimates for model parameters in **Model Two** for rock sole data and 95 percent confidence intervals (in square brackets). k is in CPUE units, σ^2 is dimensionless. Units of q are per thousand hours while b is dimensionless.

Year	*Catch*	*Effort*	*Observed CPUE*	*Estimated CPUE*
1986	86	497	0.173	0.327[0.224 – 0.478]
1987	209	496	0.421	0.279[0.184 – 0.424]
1988	189	545	0.347	0.303[0.197 – 0.467]
1989	406	1476	0.275	0.305[0.199 – 0.465]

Table 3.7: One step-ahead prediction of CPUE and their 95 percent prediction interval in parentheses of upcoming CPUE for the years 1988-1991 for rock sole with **Model B**. Units of catch are in metric tons. Units of effort are in hours.

Year	*Catch*	*Effort*	*Observed CPUE*	*Estimated CPUE*
1988	26267	20205	1.300	1.29[0.875 – 1.911]
1989	19847	12267	1.618	1.30[0.886 – 1.918]
1990	17704	17409	1.017	1.35[0.916 – 1.991]
1991	11642	14171	0.821	1.31[0.884 – 1.940]

Table 3.8: One step-ahead prediction of CPUE and their 95 percent prediction interval in parentheses of upcoming CPUE for the years 1988-1991 for redfish with **Model B**. Units of catch are in of metric tons. Units of effort are in hours.

Chapter 4

Use of a Contagious Distribution

Reed [1986] considered a method of analyzing catch-effort data which assumed randomness in the catching process, with deterministic stock dynamics. The rationale for this was, that in the past, randomness in the catch process had been ignored. In developing his model, Reed [1986] assumed that captures of distinct units of fish constitute independent events. Hence he proposed that the number of units of fish caught in a unit of time has a binomial distribution and that over the fishing season has a distribution well approximated by a Poisson distribution. In this thesis we broaden his model by dropping the independence assumptions.

For most fish species it is reasonable that the event that a unit of fish is caught in a given time interval will increase the probability that a unit "adjacent" to it will also be caught in that interval. The basic idea followed in this chapter is to employ a distribution that takes into account this contagion effect. One possibility is to use a distribution based upon Polyá's Urn scheme (Johnson and Kotz [1977]). This was used by Reed [1993] in a completely different context of estimating the historical forest fire hazard.

In its multivariate form, Polyá's Urn scheme assumes that initially there are $b_0, b_1, \ldots, b_k$ balls of colours $0, 1, \ldots, k$, respectively, in an urn. Balls are drawn at random with replacement. Furthermore, if a ball of colour j is drawn, c additional balls of that colour are put into the urn before the next draw. This of course increases the probability of drawing a ball of the same colour on the next draw - a contagion effect. The multivariate distribution of the proportions $y_0, y_1, \ldots, y_k$ of balls of colours $0, 1, \ldots, k$ drawn in the limit as the number of draws approaches infinity is a **Dirichlet distribution** (Johnson and Kotz [1977]) with parameters $b_0/c, b_1/c, \ldots, b_k/c$.
The distribution has joint p.d.f

$$f(y_0, y_1, \ldots, y_k) = \frac{\Gamma(1/\rho)}{\prod_{i=1}^{k} \Gamma(\theta_i/\rho)} y_0^{(\theta_0/\rho - 1)} y_1^{(\theta_1/\rho - 1)} \cdots y_k^{(\theta_k/\rho - 1)} \tag{4.1}$$

on the simplex $y_0 + y_1 + \cdots + y_k = 1, \;\; 0 \leq y_i \leq 1$, where $\Gamma(a)$ is the usual gamma function defined by

$$\Gamma(a) = \int_0^\infty t^{a-1} e^{-t} dt. \tag{4.2}$$

The parameter ρ is $c/\sum_{i=1}^{k} b_i$, the number of extra balls added as a fraction of the number of balls initially in the urn, and θ_i is $b_i/\sum_{j=1}^{k} b_j$, the initial proportions of balls of different colours. The parameter $\rho(> 0)$ represents the degree of contagion present. With ρ near zero, the probability of a ball of a given colour being drawn is only slightly affected by the results of the previous draws. If ρ is large, however, the contagion effect is strong. Thus the **Dirichlet distribution** can be thought of as representing the probability distribution of the proportions of objects falling into different categories, when there is dependence between the objects. Given that one object is in a given category increases the probability that there will be others in that category (a contagion effect).

4.1 A Fishery Model with Contagion

In Appendix 1 Reed [1986] showed using a simple model of a fishery that

$$pr(\text{unit caught in fishing season}) = 1 - e^{-qE_t} = \theta_1$$

$$pr(\text{unit not caught in a season}) = e^{-qE_t} = \theta_0.$$

He went on to derive the distribution of the total catch C_t, assuming that units are captured or not captured independently of one another. If we relax this assumption and instead assume that the probability of a given unit being captured is influenced by other units being captured, we could model the proportions captured and escaping by a **Dirichlet distribution** with $k = 2$. Here we ignore natural mortality and assume with this model that throughout the fishing season, fish are either captured or escape, this would be appropriate for a fishery with short fishing season. It follows that the univariate distribution of proportion $y_t = C_t/X_t$ captured has a **Beta distribution** with p.d.f

$$f(y_t) = \frac{\Gamma(1/\rho) y_t^{(\theta_1/\rho - 1)}(1-y_t)^{(\theta_0/\rho-1)}}{\Gamma(\theta_1/\rho)\Gamma(\theta_0/\rho)} \quad (4.3)$$

$0 \le y_t \le 1$; $\rho > 0$ is the contagion parameter reflecting the lack of independence. Using the properties of the Beta distribution the expected proportion caught is

$$E(y_t) = 1 - e^{-qE_t}$$

with variance

$$V(y_t) = \frac{\rho}{1+\rho}(1 - e^{-qE_t})e^{-qE_t}.$$

We use the method of maximum likelihood to estimate the unknown parameters in the model.

Given an observed sequence of catches and effort $(C_t, E_t)_{t=1,\cdots,T}$ and assuming the stock sizes X_t are known, the log likelihood function is

$$T \log\Gamma(1/\rho) - \sum_{t=1}^{T} \log\Gamma(e^{-qE_t}/\rho) - \sum_{t=1}^{T} \log\Gamma(\frac{1-e^{-qE_t}}{\rho})$$
$$+ \sum_{t=1}^{T}(\frac{e^{-qE_t}}{\rho} - 1)\log(\frac{X_t - C_t}{X_t}) + \sum_{t=1}^{T}(\frac{1-e^{-qE_t}}{\rho} - 1)\log(\frac{C_t}{X_t}) \quad (4.4)$$

We cannot simply maximize the log-likelihood over the parameters of the model since the likelihood involves X_t, an unobserved variable. If we however assume a deterministic model for the stock dynamics of the form

$$X_{t+1} = F(S_t; \Theta)$$

where $S_t = X_t - C_t$ is the escapement and Θ is a vector of parameters of the stock dynamic model, the log-likelihood can then be expressed as a function of ρ, q, Θ and the initial stock size. For a given data set and a given deterministic dynamic model we seek, numerically, values of the parameters that maximize the log-likelihood function subject to the constraint $C_t \leq X_t, \quad t = 2, 3 \cdots, T$. The simplex algorithm (Nelder and Mead [1965]) was used to obtain the numerical results.

4.2 Parameter Estimation and Results

Since this method is an extension of Reed's [1986] model, the model was applied to the same data set as that used by Reed, viz. **North Atlantic redfish** *(Sebastes marinus)* (Huang and Redlack [1981]) for ICNAF division 3P from 1955 – 1976. Even though the data is not up to date, the reason it was used was to see how well this model compared with that of Reed [1986]. Here also, plots of catch versus effort (Fig. C2) reveal the tendency not only for the occurrence of higher levels of catch at higher levels of effort but also greater variability in catch at higher levels of effort. Fig. C5 is a time series plot of catch and effort. Solid lines join the effort and dashed lines join the catches.

Since there was no prior information regarding the appropriate parametric form of the dynamic model, we used the Ricker [1954] dynamic model used by Reed. The Ricker model is

$$X_{t+1} = S_t e^{\beta(1-\frac{S_t}{X_e})} \tag{4.5}$$

where X_e represent the unfished equilibrium stock level and β is a parameter related to the intrinsic growth rate.

Estimates of the model parameters were made using data for the years 1955 – 1971. The data for the years 1972 – 1976 were reserved for comparison with the one step-ahead predictions. We again used the likelihood ratio procedure to obtain approximate confidence intervals for the parameter estimates. Using the invariance property of maximum likelihood estimates we can obtain the maximum likelihood estimate of the maximum sustainable yield (MSY). For the Ricker model, the MSY is given by

$$S_0 \ e^{\beta(1-\frac{S_0}{X_e})} - S_0 \tag{4.6}$$

where S_0 is a solution to

$$\frac{dF}{dS} = (1 - \beta S/X_e)e^{\beta(1-S/X_e)} = 1. \tag{4.7}$$

To obtain the maximum likelihood estimate for MSY, maximum likelihood estimates $\hat{\beta}$ and $\hat{X}_e$ were substituted for parameter values in the above equation.

Using the expected value and variance of the proportion for the Beta distribution, we can write

$$\mathrm{E}(C_t) = (1 - e^{-qE_t})X_t$$

with variance

$$V(C_t) = \frac{\rho}{1+\rho}(1 - e^{-qE_t})e^{-qE_t}X_t^2.$$

Therefore given data up to year T and the effort E_{T+1} and the estimated stock size $\hat{X}_{T+1}$ in year $T+1$, the maximum likelihood estimate of the expected catch in year $T+1$ is

$$\hat{E}(C_t) = (1 - e^{-\hat{q}E_{T+1}})\hat{X}_{T+1}$$

and the estimated variance of the catch in year $T+1$ is given by

$$\frac{\hat{\rho}}{1+\hat{\rho}}(1 - e^{-\hat{q}E_{T+1}})e^{-\hat{q}E_{T+1}}\hat{X}_{T+1}^2$$

where $\hat{q}$ and $\hat{\rho}$ are maximum likelihood estimates using the data up to year T. These estimates of the variance of the expected catch will underestimate the true variance because they do not take into account the sampling variation in estimating the parameters of the model.

A statistic analogous to the coefficient of determination R^2 in regression was calculated as

$$R^2 = 1 - \frac{\sum_{t=1}^{T}(C_t - \hat{C}_t)}{\sum_{t=1}^{T}(C_t - \bar{C})} \tag{4.8}$$

where $\hat{C}_t = (1 - e^{-\hat{q}E_t})\hat{X}_t$ and $\bar{C} = (1/T)\sum_{t=1}^{T} C_t$. As in linear regression, R^2 represents the proportion of the total variation in catch which can be explained by the variation in the effort. A large value of R^2 indicates that, through the model, much of the variation in catch can be explained by the variation in the effort. Plots of the residuals $(C_t - \hat{C}_t)$ against year, against effort, against X_t and against fitted value did not show any signs of model inadequacy.

Also we followed Reed and calculated the prediction error sum of squares for CPUE (also used in Chapter3)

$$H = \sum_{t=18}^{22} (\frac{C_t}{E_t} - \frac{\hat{C}_t}{E_t})^2 \tag{4.9}$$

to measure the predictive power of the model. Table 4.1 shows the maximum likelihood estimates and their approximate confidence intervals, in addition to values of the statistics R^2 and H.

The parameter estimates are comparable with those of Reed [1986]. However the confidence intervals are narrower than those obtained by Reed. The ML estimate of MSY is 31400 metric tons which tends to agree with Reed's estimates (34500 metric tons) and with the suggestion that the stocks might be under exploited during the period considered i.e $1955 - 1976$ since during those times annual catch exceeded 30000 only three times. The value of R^2 is high though not as high as that of Reed (97.4%). Since the 95% confidence interval for ρ does not contain zero, one would reject the null hypothesis $H_0 : \rho \approx 0$ at 5% level. The observed value[2] of the maximum likelihood ratio statistic yielded a significance level, $p < 0.005$, thus, assuming the model is correctly specified, there is evidence of a contagion effect. Table 4.2 gives the estimates of the mean and standard deviation of the catch in year $T + 1$ given effort in year $T + 1$, $\hat{X}_{T+1}$, and catch-effort data up until year T. For

[2]we tested $H_0 : \rho = 0.1e - 6$

the fitted model the observed catches all lie within two standard deviations of the estimated expected catch even though the sampling variation was not taken in account in the estimation of the upcoming catch.

Using H as a measure of prediction performance the model has a better predictive power for the years $1972 - 76$ than that of Reed (0.192) (see Fig. C14). Like Reed, we noticed that the years $1960 - 1964$ are somewhat anomalous since these years all produced negative residuals.

$\hat{q}$	0.0172 [0.0008 – 0.019]
$\hat{X}_1$	58.67 [55.1 – 97.0]
$\hat{X}_e$	51.62 [47.25 – 60.1]
$\hat{\beta}$	1.616 [1.32 – 1.87]
$\hat{\rho}$	0.0118 [0.008 – 0.026]
$M\hat{S}Y$	31.44
R^2	94.00%
H	0.1340

Table 4.1: Maximum likelihood estimates and 95% confidence limits (in square brackets) for the model parameters for redfish data. Units of X_e,X_1, and MSY are thousand of metric tons. Units of q are per thousand hours, while β is dimensionless

Year	*Effort*	*observed catch*	*Estimated expected catch*
1972	38.35	26.04	28.15(3.146)
1973	30.01	18.37	23.74(3.063)
1974	44.95	22.16	31.10(3.213)
1975	55.72	28.25	35.51(3.499)
1976	39.15	19.97	28.01(3.777)

Table 4.2: One-step-ahead estimates of the expected catch and standard deviation(in brackets) of the upcoming catch for the years $1972 - 75$ for redfish in division 3P. Units of catch are thousand of metric tons. Units of effort are thousand of hours.

Chapter 5

Conclusion and Discussion

In the first part of this thesis we have shown that filter methods provide a reasonable basis of estimation of the parameters of a simple fishery model. Though the method did not estimate the unobservable stochastic variable (stock size) directly, we were able to give a one-step ahead prediction of the CPUE which overall looked reasonably good. Both **Model One** and **Model Two** estimated the "optimal effort" for the redfish data as considerably higher than historical levels of effort. This might indicate that, over the period considered, the fishery was not fully exploited. If one looks at Figs. C1 and C3 there seems to be little evidence of a non-linear relationship between catch and effort. However the anomaly could be due to a weakness in the model. As noted in Chapter 3 the standard error of the estimate of the optimal effort will be large (because of the wide confidence interval for b)

Use of the contagious distribution (Chapter 4) assuming deterministic stock dynamic did not have much effect on the estimates of the other parameters which were comparable to those of Reed [1986]. The estimate of the contagion parameter ρ appears small but significant. Using the prediction error sum of squares statistic H as a measure of performance, the contagious distribution model performs better than that of Reed [1986]. One could therefore say that the contagious distribution model has promise as an improvement to that of Reed [1986].

5.1 Discussion

In the first part of this thesis we have used a combination of the methods of maximum likelihood and the Kalman filter to provide a way to estimate the parameters of the stochastic difference equation assumed to govern the evolution of stocks in fisheries. The method uses the log-linear relationship for the stock dynamics and accounts for both errors in the stock dynamics and the observable catches.

One weakness of the method is that it did not provide an effective way to estimate λ (the ratio of the variances). However a look at the one-step predictions and their prediction intervals obtained in Chapter 3 seems to indicate that the consequence of an incorrect choice of λ is not severe. Another limitation is the need to use a log-linear form of the stock dynamic model. However, the extended Kalman filter (Harvey [1989], and Speed [1993]) can be used with other forms of non-linear stock function. In using the Kalman filter method, we have assumed that prior to fishing the state variable (log stock) is stationary and that the initial stock x_0 has a proper prior distribution with known mean and variance. This is only to simplify the model since before the collection of data, fishing would have been going on for some time in the remote past and therefore the initial stock will have a distribution with unknown mean and variance. The initial conditions under such situations are normally given in terms of a diffuse prior. (Harvey [1989]). It might be interesting for future work to consider such a situation, and also to consider a case where the initial stock size is fixed. Unlike most other methods to date the Kalman filter method produces a procedure which allows explicitly for randomness in both the catching process and the population dynamics.

The Kalman filter method can also incorporate other stock abundance estimates (which for many fisheries are available from survey vessel sampling etc.). In this case the observations y_t would be a vector of (logarithms) of abundance estimates. This introduces no major difficulties into the model. In fact, use of the Kalman filter may provide the best way of combining abundance estimates from different sources. The Kalman filter approach can also be generalized to a system in which the disturbances are no longer normally distributed. (Harvey [1989], and Speed [1993]).

Using the assumption that units of fish are not caught independently of one another, we also used a method of analysis of catch-effort data that allows randomness in the catching process but not in the population dynamics. This method also used the method of maximum likelihood to provide estimates and confidence intervals for the parameters of the model. Unlike the Kalman filter approach, there is much flexibility in the specification of the deterministic stock dynamic equation; in particular models exhibiting overcompensation (Clark [1990], p215) such as the Ricker model can be used. On the basis of the estimated MSY and the catch predictions, the method performs comparably to that of Reed, however the method is computationally more complex than that of Reed.

For both methods, we have used a simple aggregated biomass model for stock dynamics. An alternative might be to use an age-structured or size-structured model (see e.g. Deriso [1980]). Of course, there is a gain in information if such data are available, but there is also a cost due to an increase in model complexity. In many areas of statistics, it has been shown that with respect to parameters, parsimony is often the best policy, and it is usually better to use the simplest model that will do the job. (Ludwig and Walters [1985])

The ultimate validation of any statistical model is to see how well it fits observed data and how well it can predict future observations. In this case the results (predictions and prediction intervals) obtained on the sample data sets examined indicate that the methods perform reasonably well. This might not be the case for other data sets, and therefore future work is needed to conduct performance trials on the methods on other data sets and to test the model with simulated data where the parameter values are known in advance, since accuracy and consistency cannot in principle be determined from real data where the parameters are unknown. For the Kalman filter method, one would want to do this to see how well the method performed under a variety of scenarios including various stock dynamic models, various values for the ratio of the variances, observation errors in catch and effort statistics, etc. Only in this way, using simulated data, could one really assess the seriousness of the limitations of the method, such as the assumption of log-linear dynamics, and a known value of the ratio of the variances. Also by varying the variance parameters when simulated data is used, one could get an idea of the point at which the method would break down, or in other words, for what ranges of noise the method would perform adequately. Similarly, as in Ludwig and Walters [1985], one could assess the use of age-structured models, etc.

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Appendix A

A Stochastic Model of Intra-Season Fishing Dynamics

We use lower case letters to denote variables within a season. Thus we let $n(t)$ denote the **biomass** of the population at time t. The initial value $n(0)$ is the population size X at the start of the season. If the season is of length T then $n(T)$ is the escapement S at the end of the season. We denote the **effective effort** at time t (number of boats fishing, or nets in the water etc. at time t) by $e(t)$. (To distinguish from the base of natural logarithms we shall always use the notation 'exp( )' for the exponentials in this Appendix)

A standard intra-season fishing model is

$$\frac{dn}{dt} = -(M + qe(t))n, \quad n(0) = X. \tag{A.1}$$

where M is **natural mortality rate**. The effort $e(t)$ is related to the **fishing mortality** by multiplication by the **catchability coefficient**, q, assumed constant.

The incremental catch in $(t, t + dt]$ is assumed to be

$$dc = qe(t)n(t)dt \tag{A.2}$$

Thus the model assumes a deterministic relationship between the level of effort applied and the catch. Clearly this is a fiction, as anyone who has ever gone fishing will testify. A more reasonable assumption would be that the incremental catch is a random variable with expected value equal to $qe(t)n(t)dt$ and a variance which grows with effort $e(t)$ (see Reed [1986, p176] for a discussion of this issue) Thus in place of Equation A.1 we shall assume

$$dn = -[M + q_0 e(t)]ndt - \sigma ne(t)dw \tag{A.3}$$

where $dw(t)$ is a **"white-noise" process** (see e.g Karlin and Taylor [1981, p342]). In effect we are assuming that the catchability fluctuates randomly about a mean value q_0 i.e that

$$q(t) = q_0 + \sigma dw(t) \tag{A.4}$$

Equation A.3 is a **stochastic differential equation**(SDE). It can be solved explicitly by considering the transformed variable

$$p(t) = \ln(n(t)) \tag{A.5}$$

However in integrating an SDE a choice must be made as to the nature of the stochastic integral employed - the Itô integral or the Stratonovich integral (Karlin and Tyalor [1981, p346]). Here we choose the Stratonovich integral on the grounds that variations in the catchability are likely to be serially correlated in time (for example when fishing boat finds a school of fish it will experience good catches of over a period of time) and the white-noise model Equation A.3 which assumes independent increments is only a convenient approximation (see Turelli [1977] for the discussion of the choice of stochastic integral in modelling).

Using the Stratonovich calculus we have

$$dp = -[M + q_0 e(t)]dt - \sigma e(t)dw \tag{A.6}$$

which can be integrated to give

$$p(t) - p(0) = -\int_0^t [M + q_0 e(s)]ds - \sigma \int_0^t e(s)dw(s) \tag{A.7}$$

which is also equal to

$$-(Mt + q_0 \int_0^t e(s)ds) - \sigma u(t)$$

where

$$u(t) \sim N(0, \int_0^t e^2(s)ds) \tag{A.8}$$

Thus

$$n(t) = n_0 \exp\{-(Mt + q_0 E_0) - \sigma u(t)\} \tag{A.9}$$

where

$$E_0 = \int_0^t e(s)ds \tag{A.10}$$

is the total effort over the season.

We note that the median of $n(t)$ (which is log-normally distributed) is $n_0 \exp\{-(Mt + E_0)\}$ which is the solution to A.1. In other words the deterministic model A.1 is correct for the median of the stochastic model.

The biomass dc caught in time $(t,\ t + dt]$ is

$$dc = (q_0 + \sigma dw)e(t)n(t) \tag{A.11}$$

Using A.3 and A.11 we get

$$dn + dc = -Mn(t)dt \tag{A.12}$$

This is essentially a conservation of biomass equation expressing the fact that the loss in biomass ($-dn$) is equal to that lost through fishing (dc) plus that lost through natural mortality ($Mn(t)$).

From Equation A.12 we can find an expression for the accumulated catch $c(t)$ by time t

$$c(t) = n_0 - n(t) - M \int_0^t n(s)ds \tag{A.13}$$

the integral on the r.h.s involves the integration of a geometric Brownian motion(Karlin and Taylor [1981, p359]) which cannot be evaluated in general in closed form. In order to proceed we shall assume **that the fishing season is short**. This will enable us to

1. ignore natural mortality. (i.e set $M = 0$)

2. neglect variation in effort over the season (i.e set $e(t) = \bar{e}$)

Using these assumption gives (from Equation A.9)

$$n(t) = n_0 \exp\{-q_0 E_0 + \sigma \bar{e} w(t)\} \tag{A.14}$$

where $\{w(t)\}$ is a **standard Wiener process** (The integral of the white-noise process $\{dw(t)\}$ with $E(w(t)) = 0$, $\mathrm{Var}(w(t)) = t$ i.e $w(t) \sim N(0,t)$); and from Equation A.13

$$c(t) = n_0(1 - \exp\{-q_0 E_0 + \sigma \bar{e} w(t)\}) \tag{A.15}$$

Thus the total catch C $(= c(T))$ and the escapement S $(= n(T))$ can be expressed as

$$C = X(1 - \exp\{-(q_0 + \sigma_1 Z)E_0\} \tag{A.16}$$

$$S = X \exp\{-(q_0 + \sigma_1 Z)E_0\} \tag{A.17}$$

where $Z \sim N(0,1)$ and the variance parameter σ_1 is equal to $\sigma/\sqrt{T}$. These are the relationship used in the **Model Two**

A.1 Maximum Likelihood Ratio Test Procedure for Obtaining Confidence Intervals

The method (Cox and Hinkley [1974]) is based on the fact that for testing $H_0 : \theta = \theta_0$ vs. $H_1 : \theta \neq \theta_0$, with or without nuisance parameters present, asymptotically the statistic

$$W = -2 \ln \Lambda \tag{A.18}$$

has a χ_1^2 distribution under H_0, where Λ is the maximum likelihood ratio statistic:

$$\Lambda = \frac{\max_{\Omega_0} L(\theta;\mathbf{X})}{\max_{\Omega} L(\theta;\mathbf{X})} \tag{A.19}$$

where Ω is the parameter space and Ω_0 is the subset of Ω for which $\theta = \theta_0$.

Suppose the model has k parameters,i.e. $\Theta = (\theta_1, \theta_2, \cdots, \theta_k)$ then a $100(1-\alpha)\%$ confidence interval for say θ_1 is given by the set of null values of θ_1 which would not be rejected at the level α using the maximum likelihood ratio test. From the asymptotic distribution of W, it follows that an approximate $100(1-\alpha)\%$ confidence interval for θ_1 is given by

$$\{\theta_1 : l(\theta_1, \tilde{\theta}_2, \ldots, \tilde{\theta}_k) \geq l(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k) - \frac{1}{2}\chi^2_{1,\,\alpha}\} \tag{A.20}$$

where $\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k$ are the maximum likelihood estimates of $\theta_1, \theta_2, \ldots, \theta_k$ and $\tilde{\theta}_2, \ldots, \tilde{\theta}_k$ are the maximum likelihood estimates of $\theta_2, \ldots, \theta_k$ when θ_1 is fixed at a given valua, $\chi^2_{1\,\alpha}$ is the $100\alpha\%$ point of the χ_1^2 distribution.

Computationally this procedure is fairly lengthy because it involves choosing a grid of values of θ_1, and for each obtaining the ML estimates of the other parameters. The function $l(\theta_1, \tilde{\theta}_2, \ldots, \tilde{\theta}_k)$ can be plotted against θ_1 and the set of values of θ_1 satisfying A.20 can be found graphically. A similar procedure can be used to obtain confidence intervals for the other parameters. It should be noted that the method is not appropriate in the case where the likelihood surface is irregular. The advantage of the method is that it is invariant under parameter transformation, thus if there is a parameter transformation which makes the likelihood surface close to a quadratic, then the method will provide accurate confidence interval.

Appendix B

Data Sets

YEAR	*CATCH*	*EFFORT*
1959	34107	30648
1960	11463	9040
1961	8349	4652
1962	3425	2638
1963	8191	5024
1964	3898	1912
1965	9451	8379
1966	6927	5833
1967	7684	4265
1968	2348	1480
1969	927	680
1970	1029	794
1971	10043	7294
1972	3095	2059
1973	4709	2684
1974	11419	14503
1975	3838	3986
1976	15971	12450
1977	13452	11205
1978	6318	6386
1979	5584	4775
1980	4367	2984
1981	9407	6622
1982	7870	5339
1983	8657	5794
1984	2696	2063
1985	3677	2539
1986	27833	16856
1987	33917	25799
1988	26267	20205
1989	19847	12267
1990	17704	17409
1991	11642	14171

Table B.1: Total annual catch and effort for redfish for ICNAF division 3L for the years 1959-91. Units of catch are in metric tons and units of effort are in hours.

YEAR	*CATCH*	*EFFORT*
1956	397	794
1957	726	2305
1958	368	814
1959	249	616
1960	471	1066
1961	110	345
1962	322	638
1963	155	408
1964	244	528
1965	539	1485
1966	961	2355
1967	948	1615
1968	811	1959
1969	1053	3510
1970	694	2810
1971	376	1614
1972	134	439
1973	186	514
1974	288	766
1975	383	1178
1976	277	745
1977	272	829
1978	356	1319
1979	647	1668
1980	482	1511
1981	126	457
1982	70	326
1983	60	210
1984	64	149
1985	28	92
1986	86	497
1987	209	496
1988	189	545
1989	406	1476

Table B.2: Total annual catch and effort for rock sole in area 5C for the years 1956-89. Units of catch are in metric tons and units of effort are in hours.

YEAR	*CATCH*	*EFFORT*
1955	4.601	4.800
1956	3.275	3.211
1957	2.387	2.392
1958	3.510	3.853
1959	3.774	5.149
1960	9.225	16.099
1961	9.776	16.000
1962	13.439	26.248
1963	13.747	20.157
1964	13.807	19.952
1965	18.733	20.699
1966	20.868	23.876
1967	31.991	39.789
1968	13.884	17.800
1969	32.051	43.548
1970	37.370	52.716
1971	27.500	44.426
1972	26.037	38.346
1973	18.368	30.013
1974	22.158	44.945
1975	28.250	55.720
1976	19.967	39.151

Table B.3: Total annual catch and effort for Redfish for ICNAF in division 3P for the years 1955-76. Units of catch are in thousand metric tons and units of effort are in thousand of hours.

Appendix C

Graphs

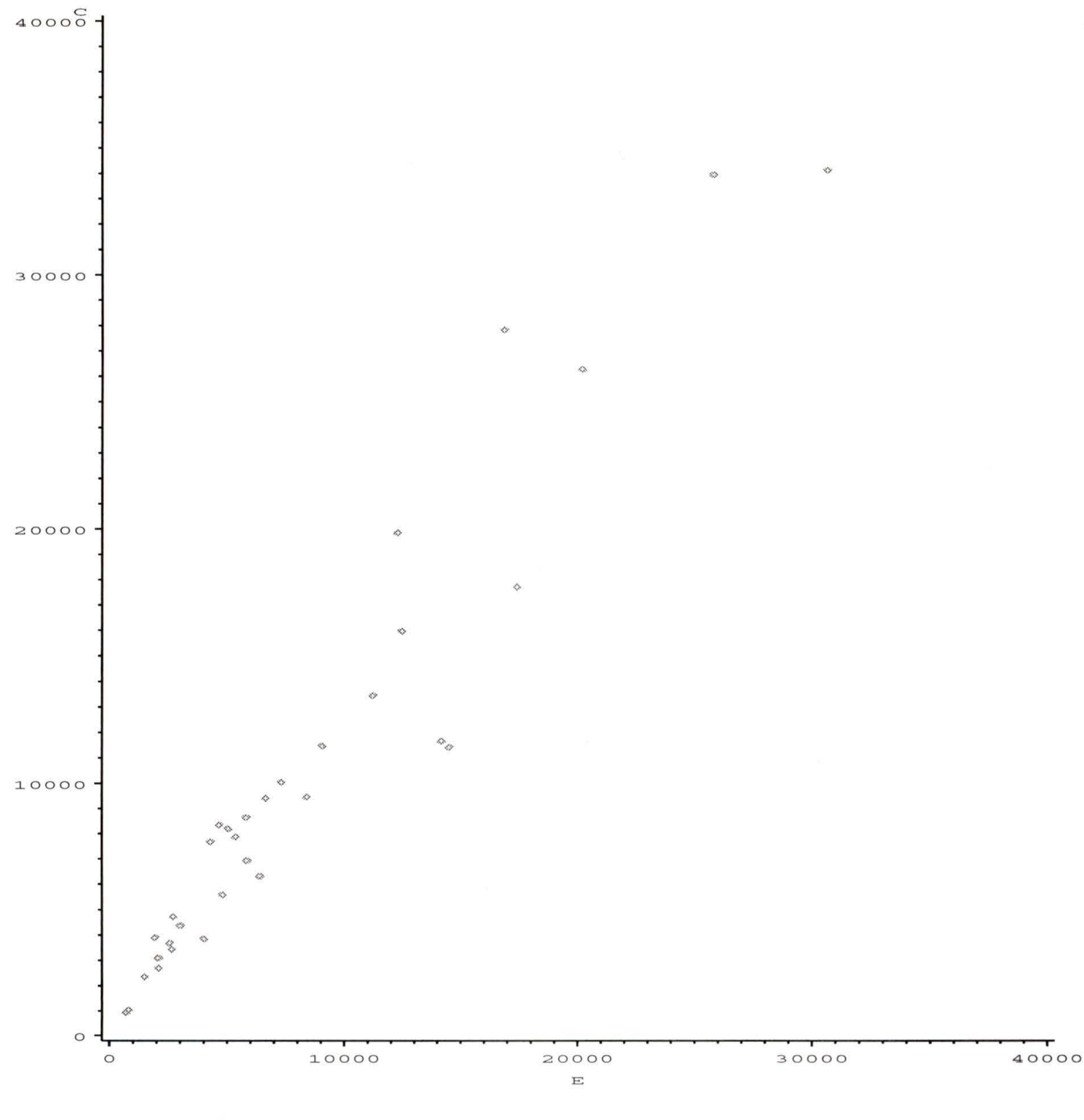

Figure C.1: Plot of catch versus effort for redfish in area 3L. Units of catch are in metric tons and units of effort in hours.

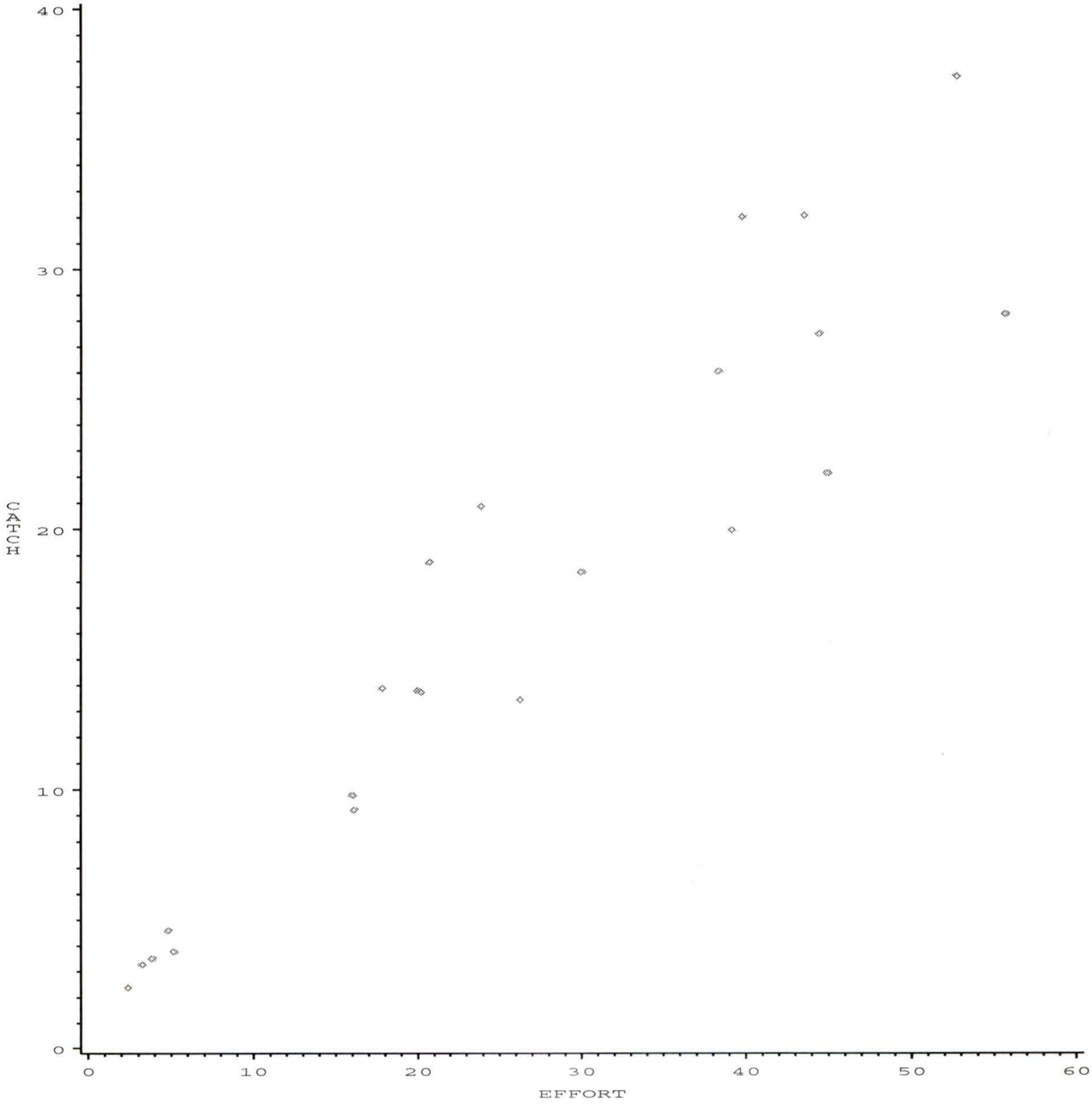

Figure C.2: Plot of catch versus effort for redfish in area 3P. Units of catch are in thousand metric tons and units of effort in thousand hours.

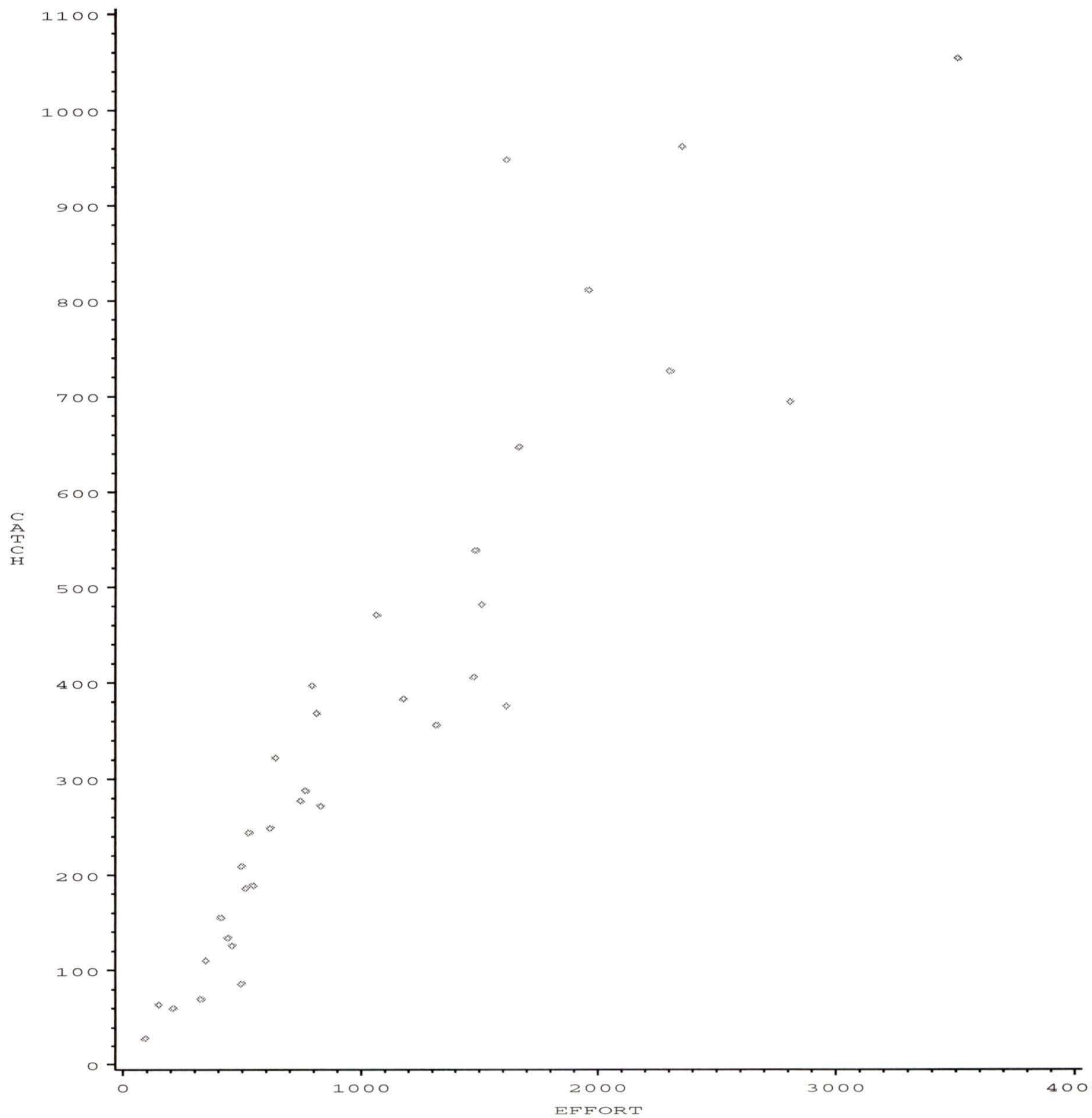

Figure C.3: Plot of catch versus effort for rock sole in area 5C. Units of catch are in metric tons and units of effort in hours.

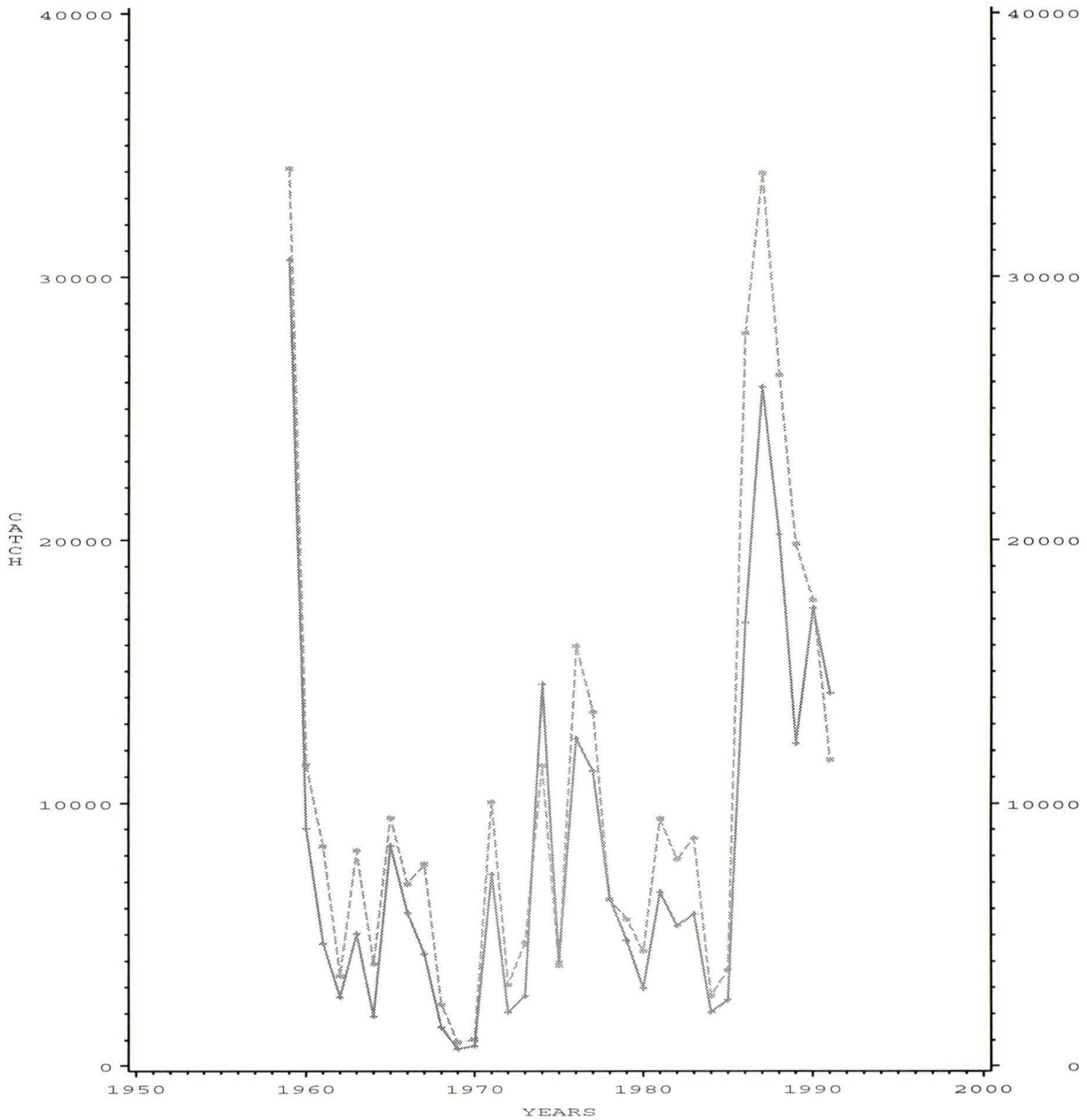

Figure C.4: Time series plot of catch(dashed lines) and effort(solid lines) for redfish in area 3L. Units of catch are in metric tons and units of effort in hours.

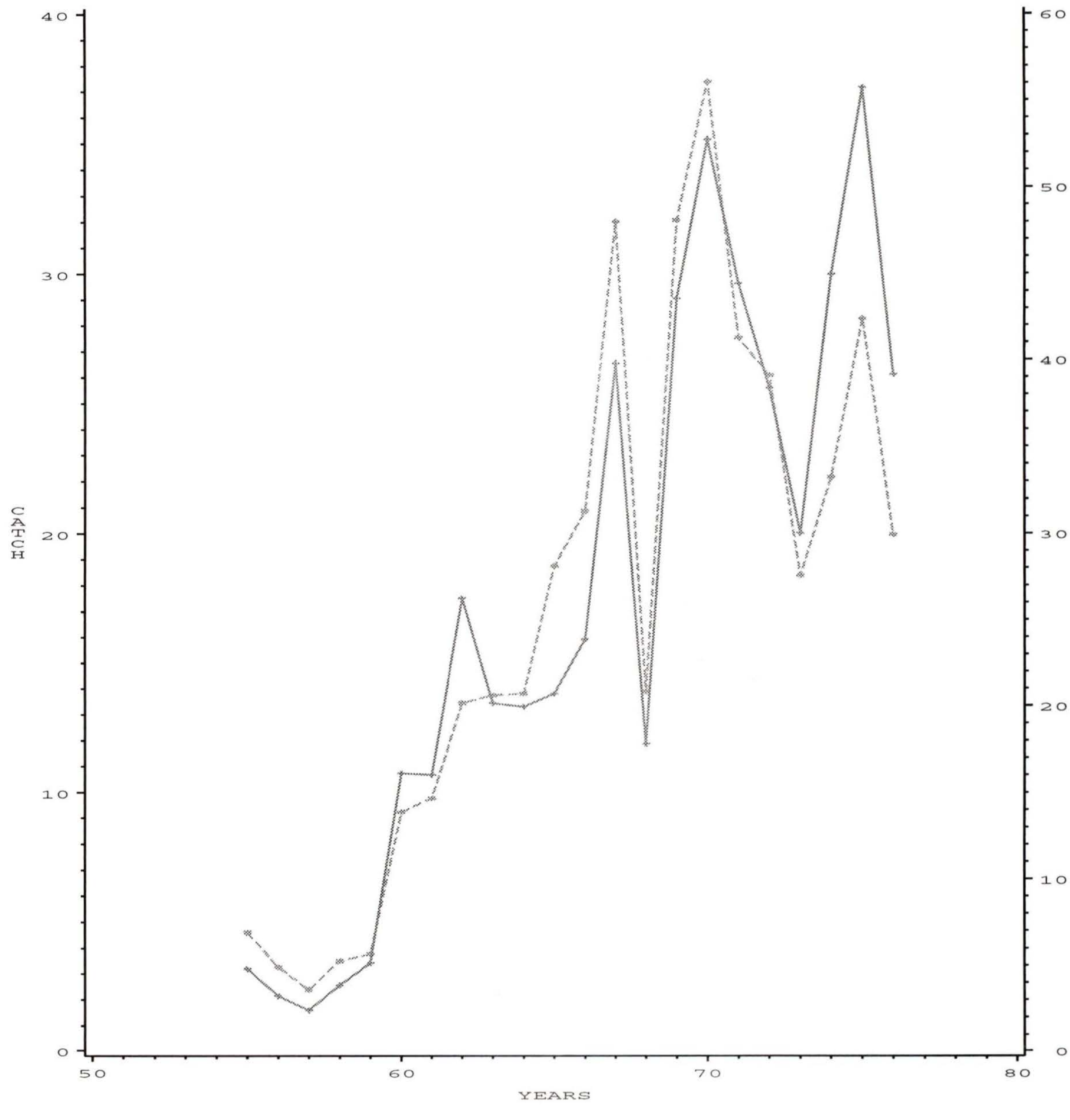

Figure C.5: Time series plot of catch(dashed lines) and effort(solid lines) for redfish in area 3P. Units of catch are in thousand metric tons and units of effort in thousand of hours.

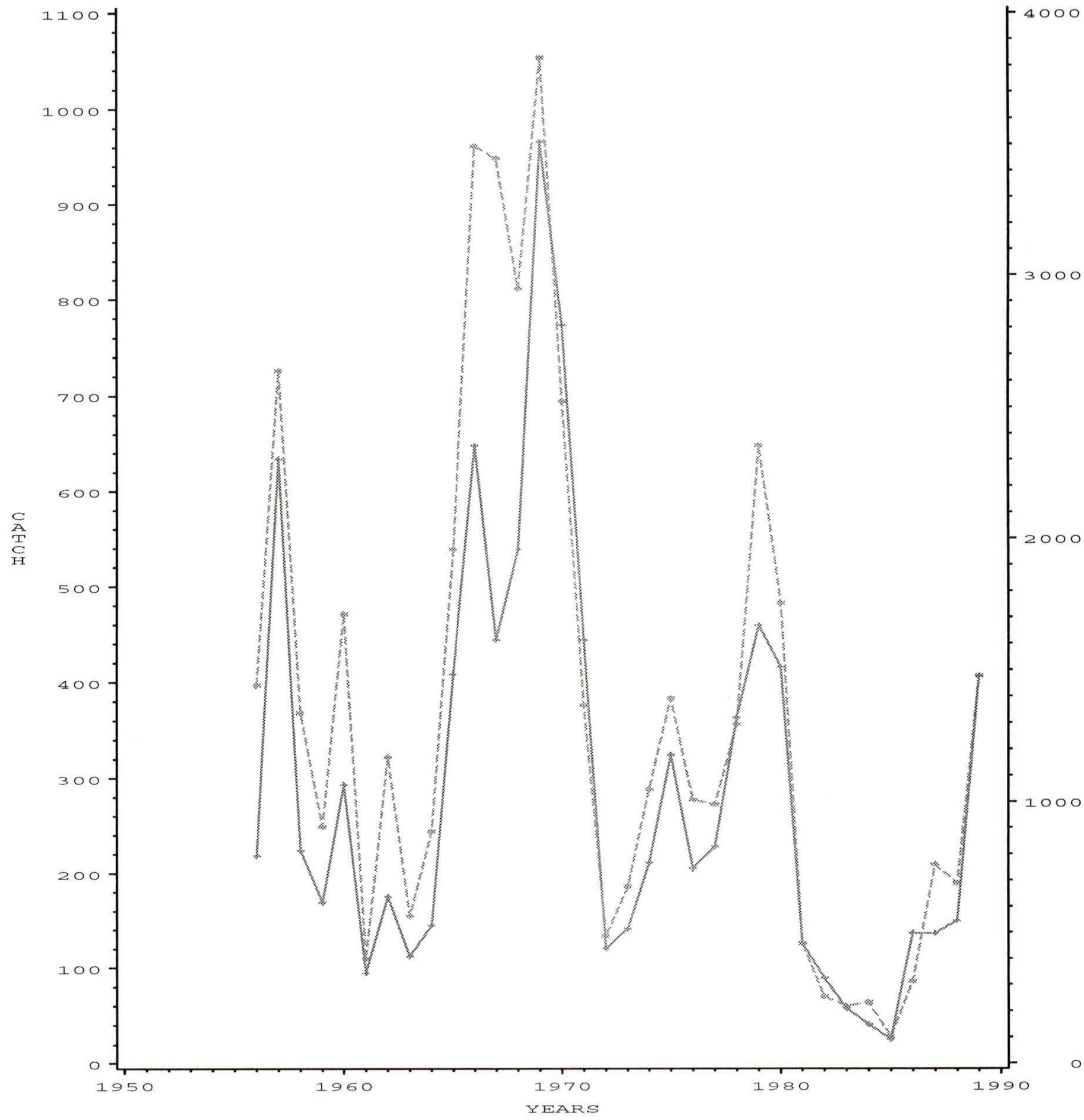

Figure C.6: Time series plot of catch(dashed lines) and effort(solid lines) for rock sole in area 5C. Units of catch are in metric tons and units of effort in hours.

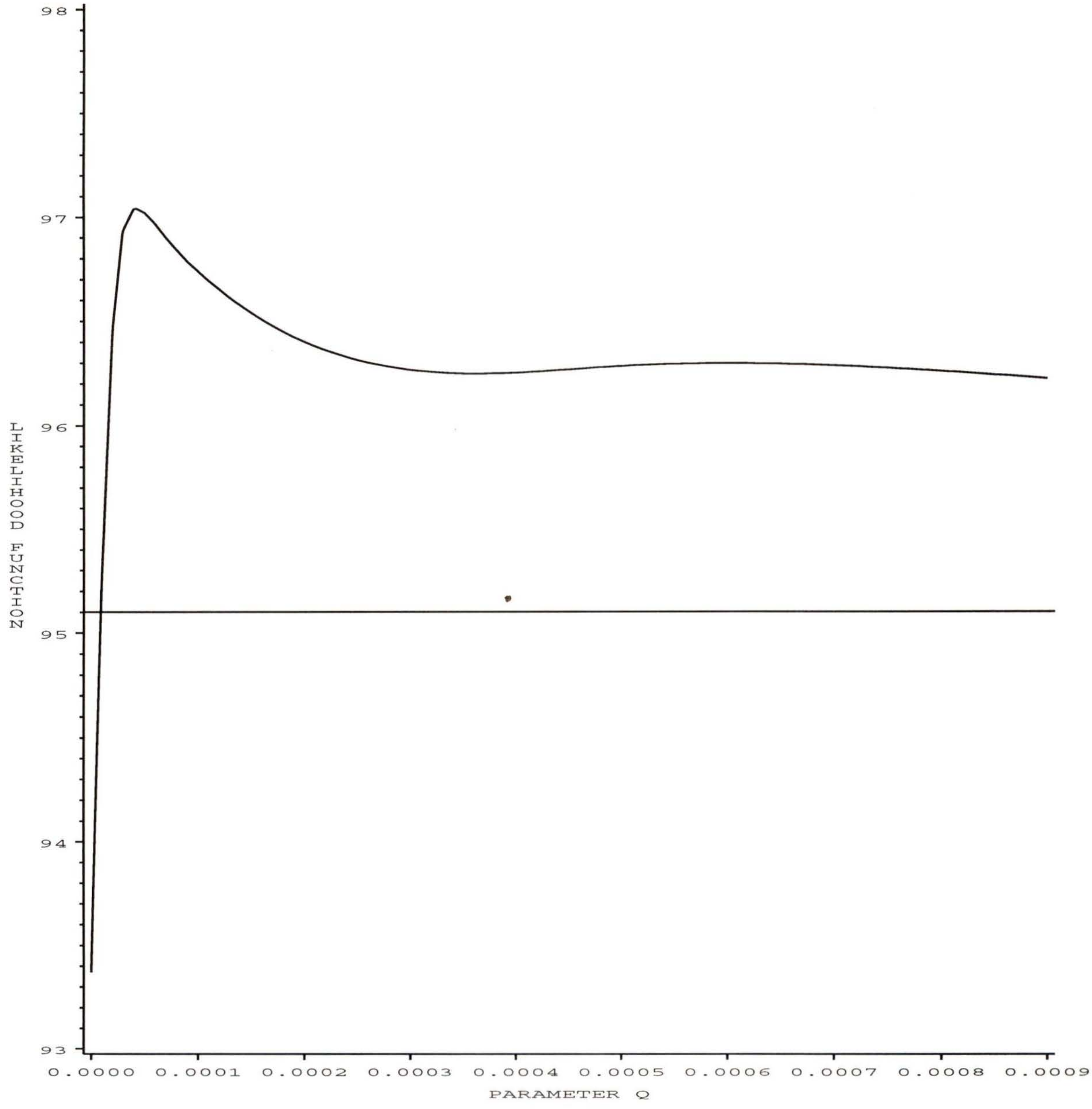

Figure C.7: Plot of log-likelihood maximized over the other parameters versus the parameter $\hat{q}$ for $\lambda = 1$ for **Model One** with redfish data in division 3L. The horizontal line is $1.92 (= \frac{1}{2}\chi^2_{1,\ 0.05})$ units below the maximum.

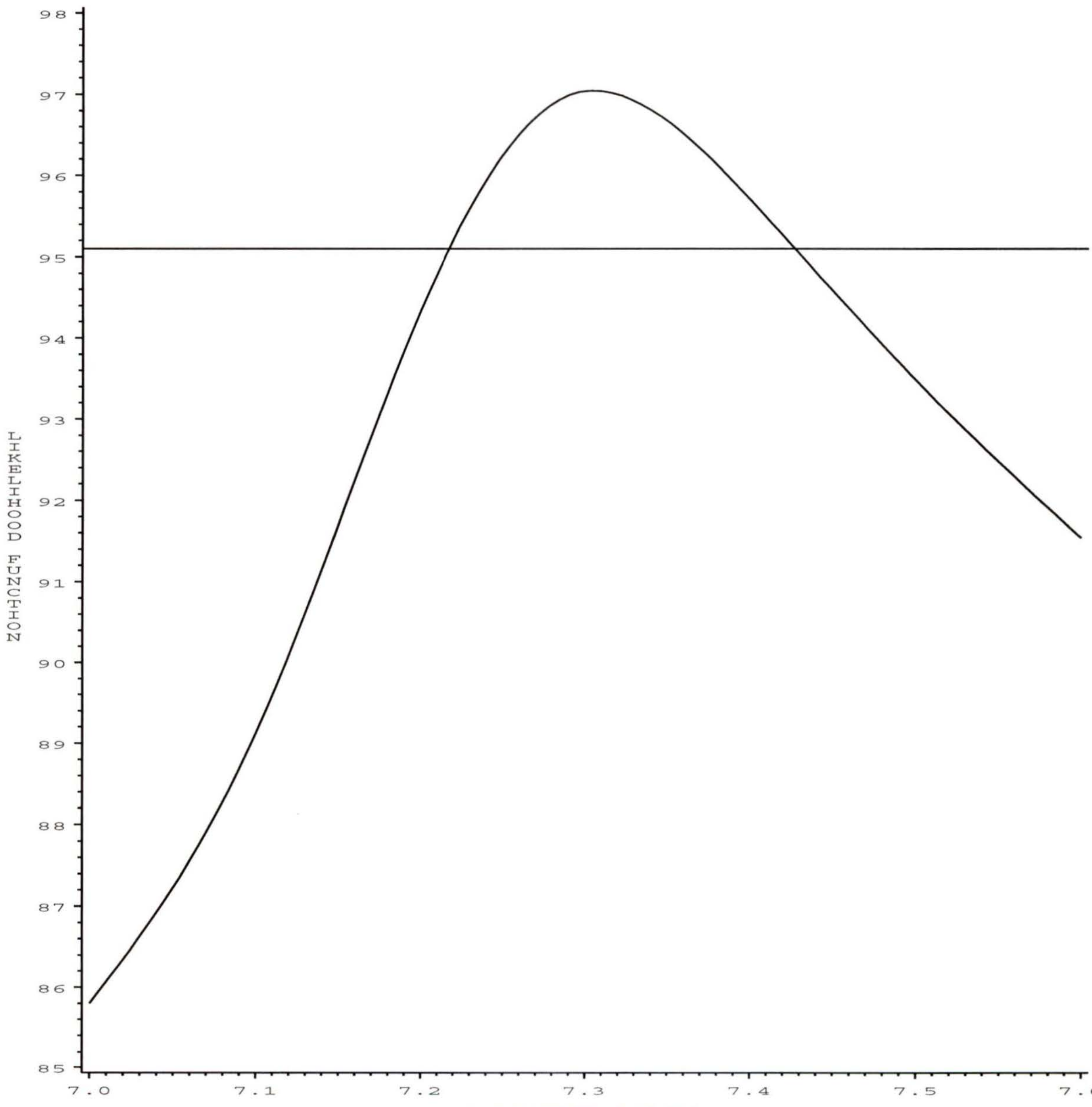

Figure C.8: Plot of log-likelihood maximized over the other parameters versus the parameter $\ln \hat{k}$ for $\lambda = 1$ for **Model One** with redfish data in division 3L. The horizontal line is $1.92 (= \frac{1}{2}\chi^2_{1,\ 0.05})$ units below the maximum.

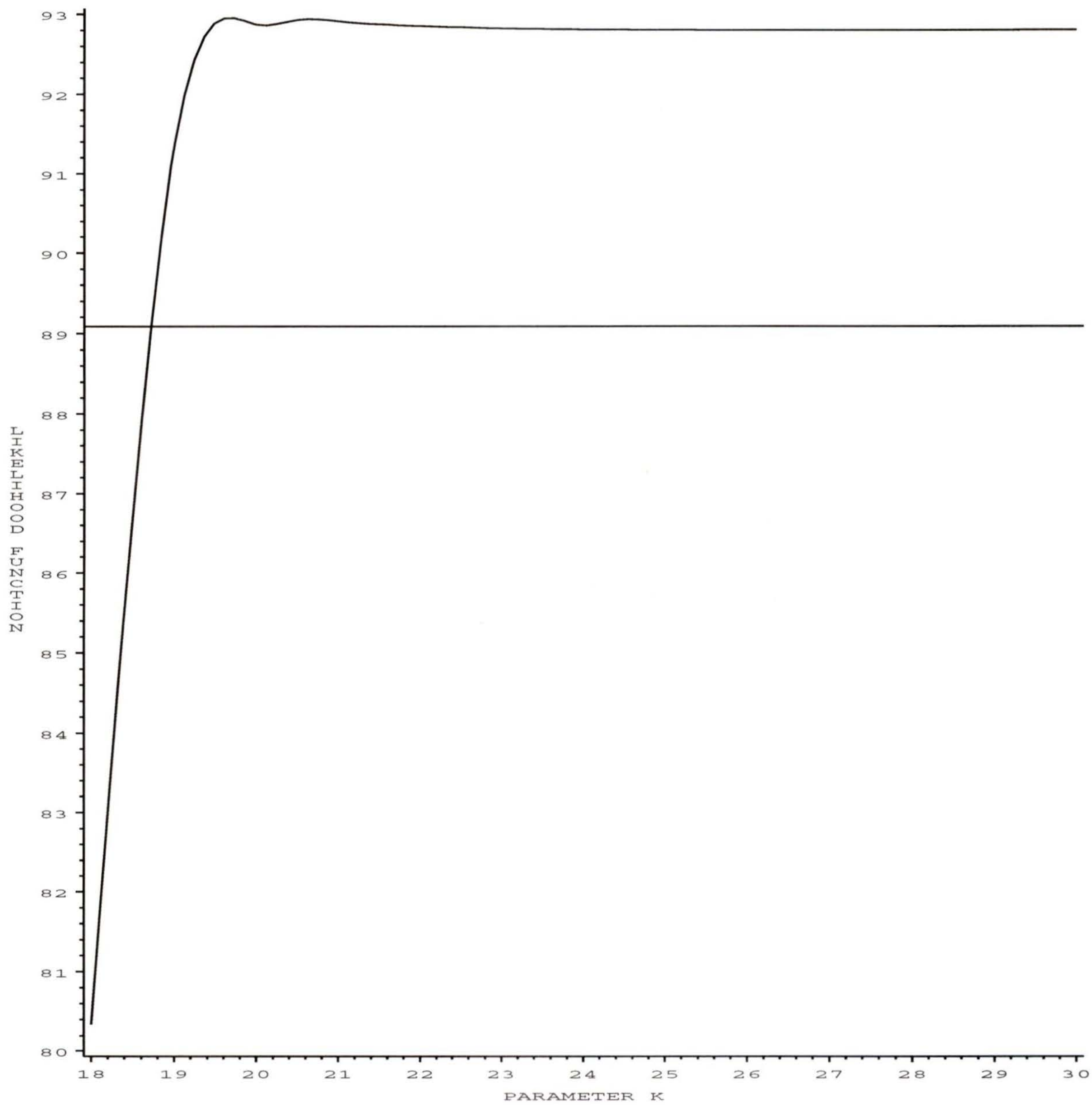

Figure C.9: Plot of log-likelihood maximized over the other parameters versus the parameter $\ln \hat{k}$. The horizontal line is $1.92(= \frac{1}{2}\chi^2_{1,\ 0.05})$ units below the maximum for **Model Two** for $\lambda = 1$ with redfish data in division 3L.

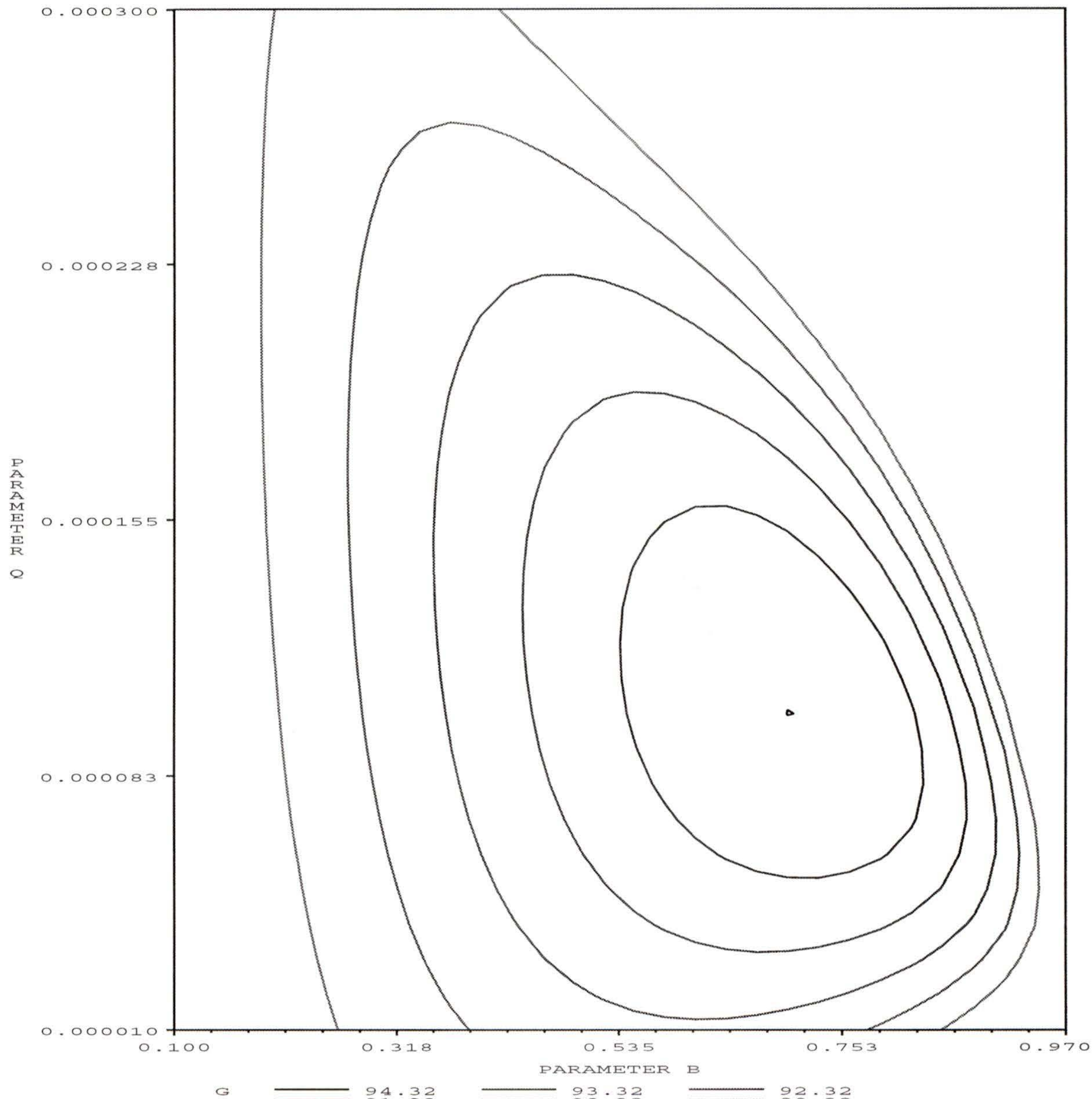

Figure C.10: Profile log-likelihood contours for q and b for **Model One** with rock sole data with $\lambda = 1$.

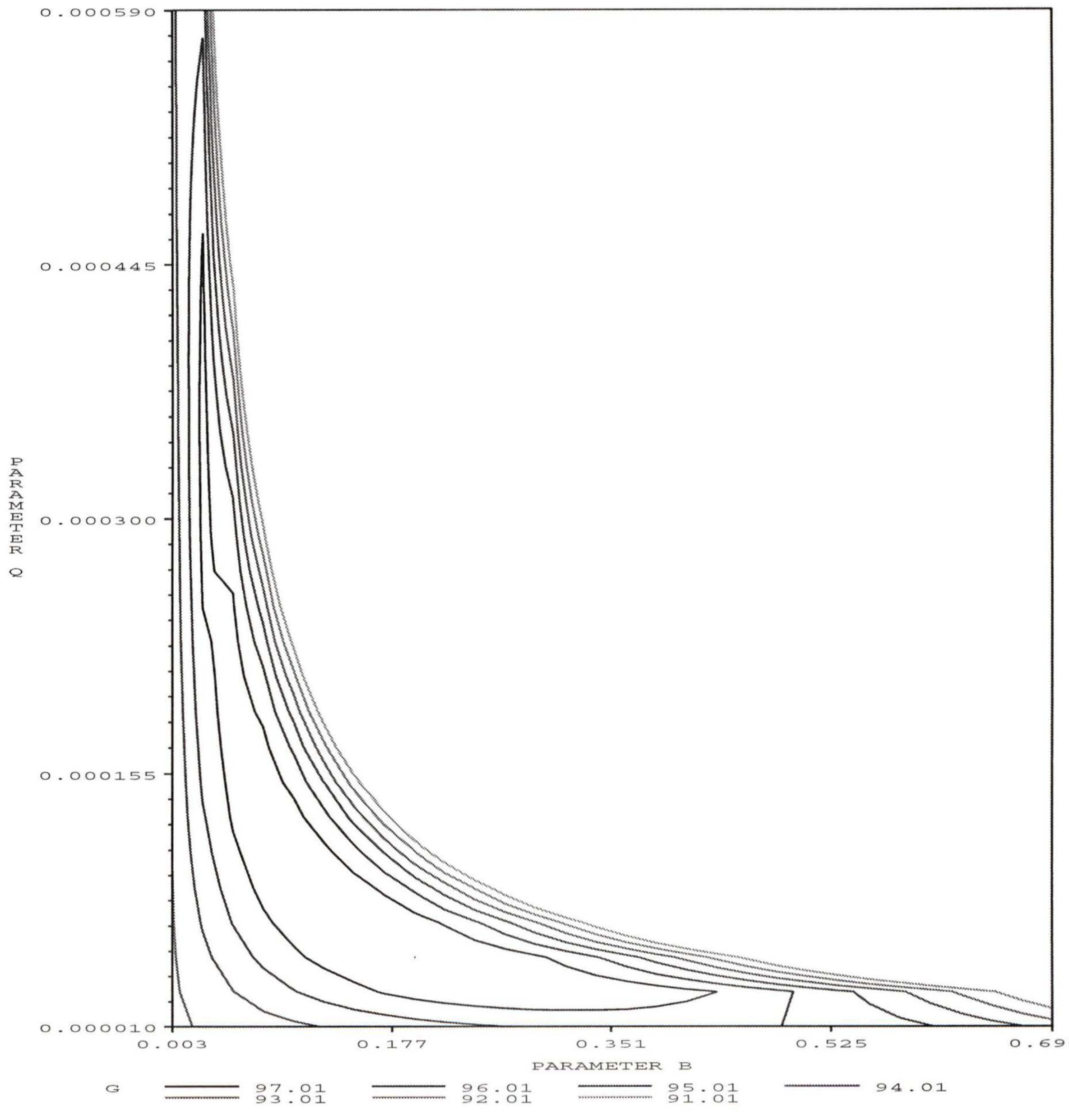

Figure C.11: Profile log-likelihood contours for q and b for **Model One** with redfish data with $\lambda = 1$.

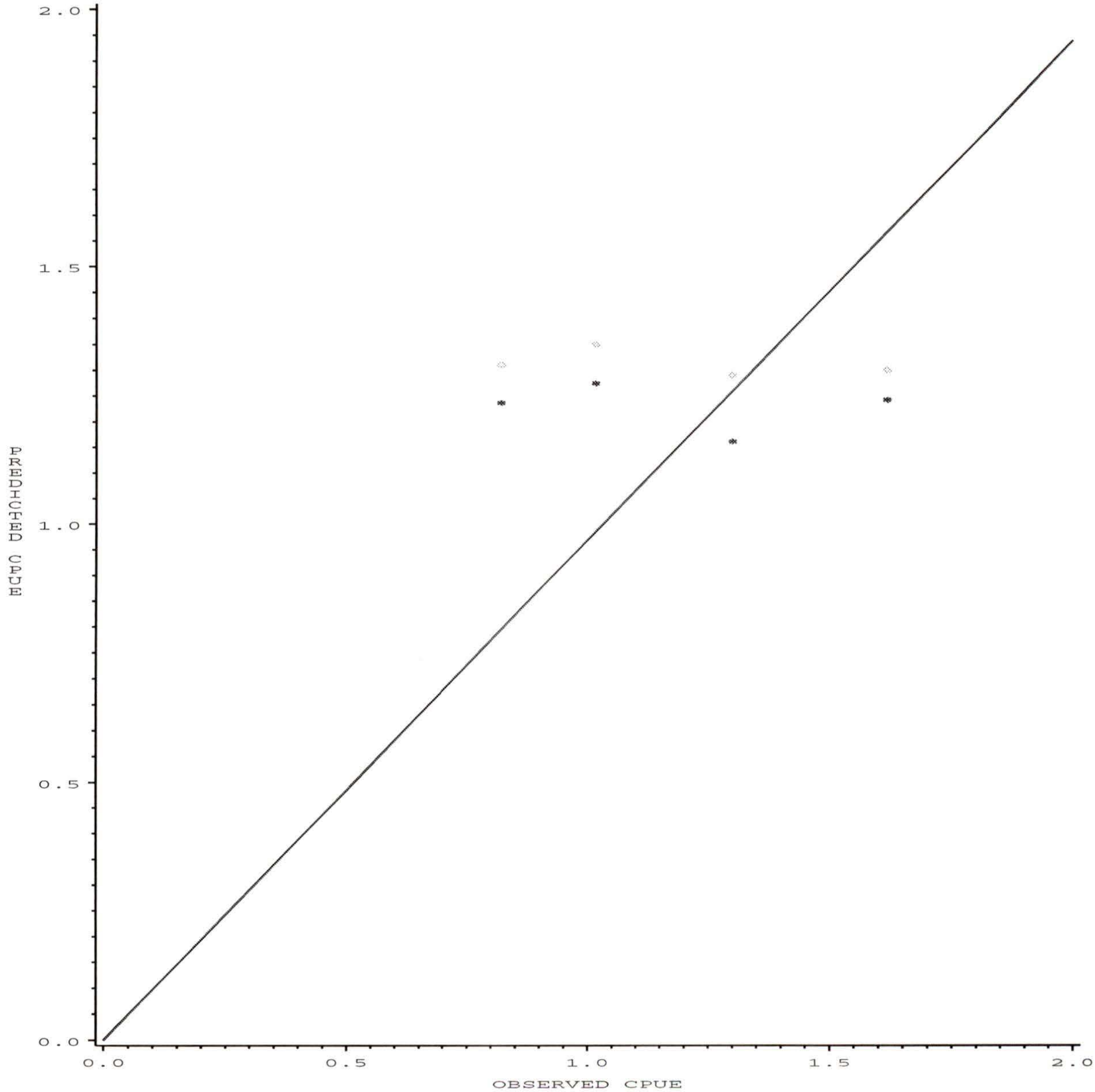

Figure C.12: Plot of predicted CPUE and observed CPUe the years 1988-91 for redfish for both models for $\lambda = 1$; **Model One**($*$), **Model Two**($\diamond$). Units of catch are in metric tons and units of cpue are in metric tons per hour.

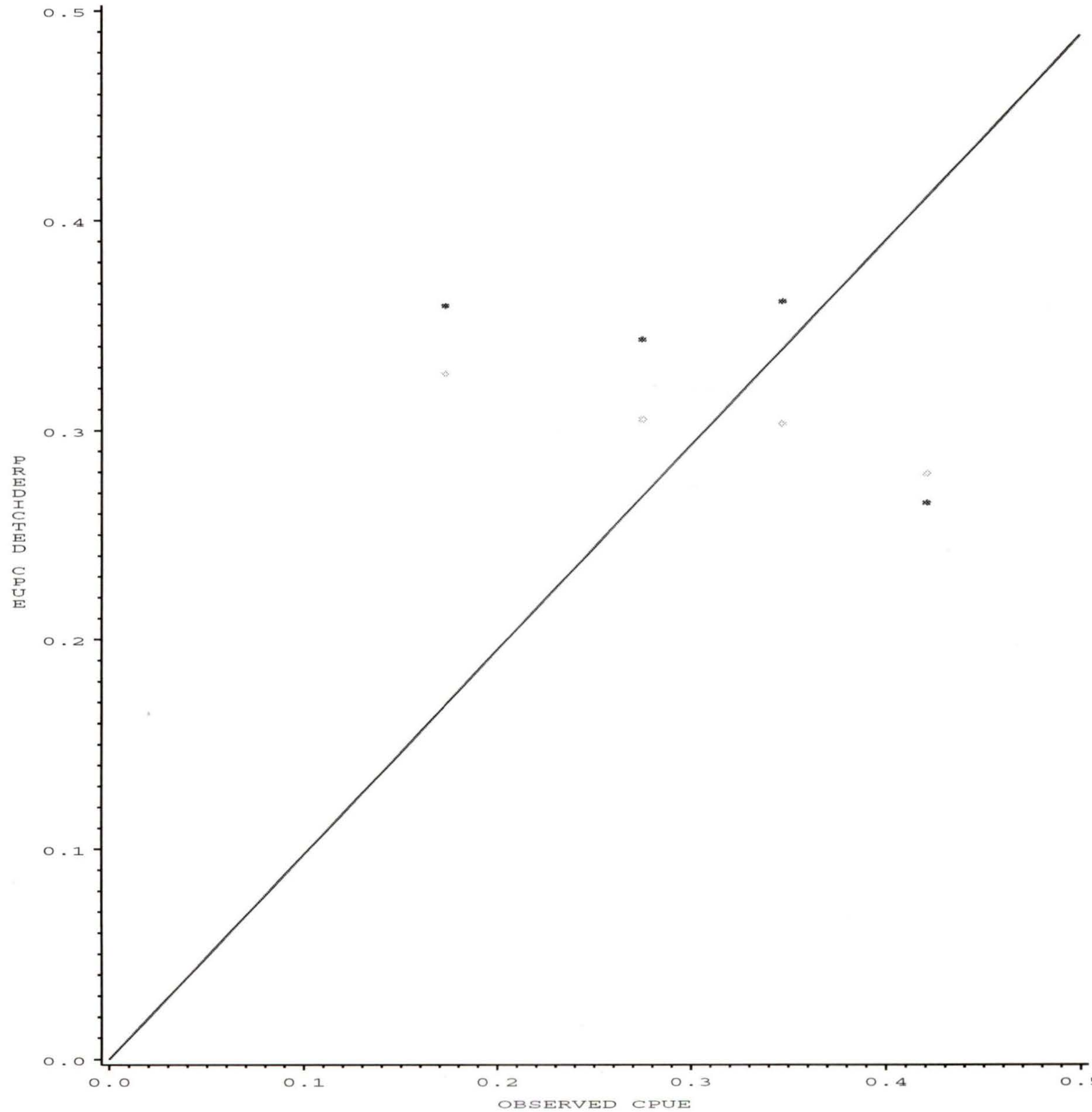

Figure C.13: Plot of predicted CPUE and observed CPUE the years 1986-89 for rock sole for both models for $\lambda = 1$; **Model One**($*$), **Model Two**($\diamond$). Units of catch are in metric tons and units of cpue are in metric tons per hour.

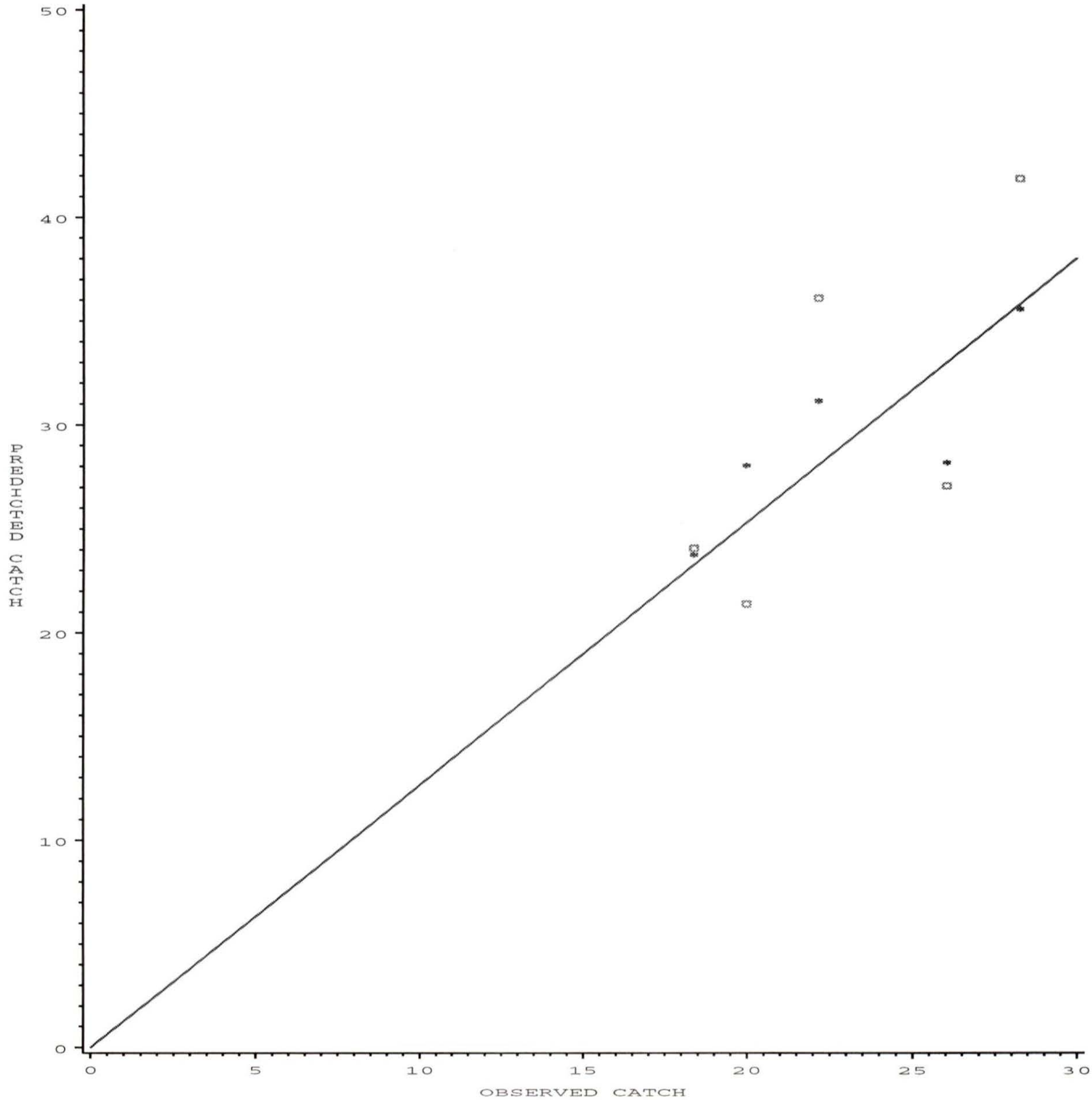

Figure C.14: Plot of predicted catch and observed catch the years 1972-76 for redfish in division 3P for both the contagious distribution model ($\star$) and that of Reed [1986]($\square$). Units of catch are in thousand of metric tons and units of effort in thousand of hours.

VITA

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