CHILDREN'S UNDERSTANDING OF NEGATION

by

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ABSTRACT

Children's concepts of negation were first investigated by Inhelder and Piaget (1964) and their research provided evidence that the operation of negation is related to classification in general. Two hypotheses which have evolved from children's performance of classificatory tasks have been examined in this study.

Inhelder and Piaget (1964) hypothesised that class inclusion relations, come prior to, and are a necessary prerequisite for the handling of negation as a logical operation. A series of studies by Feldman (1972), designed to test the assumption made by Inhelder and Piaget that negation and class inclusion are related phenomena and to specify the relationship between them, provided data that suggested that competence at class inclusion tasks developed later than an understanding of the simple negation concepts used in her experiments.

An attempt to establish the results of Feldman, with respect to the logical operation of negation, was undertaken. The present study also attempted to extend the data concerning the understanding of null classes and empty sets. Inhelder and Piaget (1964) had observed that this notion was incompatible with the child's idea of collection and their results indicated that the null set was rejected throughout the stage of concrete operations.

The sample consisted of 120 children; 20 of each sex from each of grades 1, 2 and 3. The four tests consisted of a test

of class inclusion, a partial replica of Feldman's negation test, Diamond's test on double negatives and the null set, and a test of incomplete sets.

The principal findings of this study, as they relate to the preliminary aims of the research were that

1. There was little support for Inhelder and Piaget's (1964) claim that negation and class inclusion were strongly related.

2. There was support for Feldman's (1972) thesis that an understanding of the notion of negation develops earlier than the emergence of class inclusion in children's reasoning. This was so when the test of negation was other than that used by Feldman in her study.

3. There was support for the thesis that class inclusion develops later than most other concrete operational skills.

4. There was no support for the thesis that comprehension of the null set or empty class is delayed until the stage of formal operational thinking.

5. There appeared to be a relationship between the understanding of the null class or empty set and the understanding of the incomplete set. There also appeared to be a relationship between the understanding of the null set and the ability to deal with double negatives. Thus in future research it seems necessary to test the existence of these suggested relationships and to establish whether any rank order exists.

Signed: [redacted]

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INTRODUCTION

Awareness that general concepts underlie much of our thinking has emerged from recent research into children's cognitive development. The leading exponents of such views are Jean Piaget and his Swiss collaborators whose programme of research is widely known. Piaget has claimed that the development of fundamental concepts springs spontaneously from the child's own logical operations.

One area to which extensive research has been applied is that of the learning of mathematical concepts by children. The mathematician Henri Poincare, and others, argued that the number concept is a product of primitive intuition preceding logical notions. Others, including Bertrand Russell, have taken the view that number is a purely logical concept. Piaget's experimentation is designed to elucidate the psychological processes in the development of number concepts in young children.

In considering the essential cognitive components of an adequate elementary concept of number, that is what a child must implicitly know about the properties of numbers in order to be accredited with such a concept, Piaget (1958) has come to believe that these cognitive components principally include a concrete operational mastery of classification and seriation. His results from extensive experiments have shown that children must grasp the principle of conservation of quantity before they can develop a concept of number; in addition this concept of number, and other mathematical concepts, develop independently through maturation in addition to formal instruction.

Piaget's argument is that conservation of quantity is not a numerical notion but a logical operation. He has therefore concluded that simple logical operations precede the development of the concept of number. In tracing the development of children's thinking in relation to the problems of number, quantity, and space, and again in dealing with the notion of chance and inductive reasoning, Piaget has recognised the formation of elementary logical structures.

The principle, that the operations which define intelligence, and which change with age, are logical structures, is basic to Piaget's theory of intellectual development. No logical discovery analysed in his experiments can be reduced to an isolated concept, and a fundamental feature of his theory is that of the existence of general structures. The development of these generalised mental structures underlying the acquisition of specific concepts has demanded a separate study from him.

Piaget (1958) has maintained that to comprehend mental development it is necessary not only to study the competence of children at different ages in certain cognitive fields, but also to understand the structuring processes underlying their development of concepts. The structures formed, and the ways they are integrated, depend on the stage of development. His hypothesis is that between 7 - 11 years of age a child acquires the ability to carry out concrete operations. Such operations are types of actions involving the manipulation of physical objects, as a means of mentally transferring data. Operations become bound up together in structures: each set of structures can be related to a particular group of logical forms. During the concrete operational stage a group structure develops. This grouping is a classificatory or relational system in terms of which the elementary logical and mathematical reasoning of a child may be described.

Piaget's studies have led to a renewal of interest in cognition and in theories of cognitive structures, and have focussed attention on the development of logical thinking in children. The notion that the nature of intelligence lies in the performance of logical operations is pertinent to this study. The general field of this thesis is the logico-mathematical development of the child at the stage of concrete thinking.

CHAPTER I

REVIEW OF LITERATURE ON DEVELOPMENT OF LOGICAL THINKING

A. Studies in Formal Reasoning

Some of the earliest studies in formal reasoning were carried out in Britain by Burt and Valentine. Burt (1955) introduced logical reasoning items of the type which posed such problems as "Jane is fairer than Lily: Jane is darker than Susan: Which of the three is the fairest?" Such a problem which demanded interest in the logical implication as opposed to the content was found to be much more difficult than the simpler "Jane is fairer than Lily: Lily is fairer than Susan. Which is the fairest?"

Valentine (1961) constructed reasoning tests which required problem solving by inductive reasoning and the detection of fallacies and/or approval of arguments. A demand for reasons for the subjects choice reduced the possibility of scoring by guessing. His evidence strongly supported the premise that an ability to think logically developed with age and had a high correlation with academic success. He therefore considered evidence of formal reasoning an indication of intellectual potential.

The primary focus of a study by Hill (1960) was the investigation of the relationship between logical ability and age in children of 6, 7 and 8 years. In order to measure logical ability a hundred item test was constructed. Each item consisted of two premises, a question and two possible responses, yes and no. It was necessary to discriminate between a necessary conclusion and the negation of a necessary conclusion. This study provided evidence that children of six, seven and eight were able to deal very effectively with verbal premises that called for hypothetical reasoning.

O'Brien and Shapiro, (1968) recognised the weakness of this study to be that no test of the logical necessity of the conclusion was called for. A study was devised to determine the differences between the ability of young children to discriminate between a logically necessary conclusion and its negation, and their ability to test the logical necessity of a conclusion. They used the Hill test, and a second test of adapted items from the Hill study, in which no necessary conclusion followed from the premises and a 'not enough clues' option was added to the responses. Their analysis of these results confirmed Hill's suggestion that children have considerable success in recognising logical conclusions. Children of the same age however were found to experience great difficulty in determining the necessity of a conclusion and showed slow growth in that ability. These findings of O'Brien and Shapiro implied that hypothetical deductive ability could not be taken for granted in children of this age.

More recently Wason and Johnson-Laird (1972) have examined the fundamental problem of the extent to which most individuals can be considered naturally rational thinkers. Their interest was in a) to what extent an individual, given a set of assertions, could appreciate all that followed from them by virtue of logic alone, and b) the factors which governed the performance of apparently simple deductive problems and made intelligent people fail to reflect logical competence. The experiments conducted yielded some unexpected results. The most important finding was the difficulty experienced in understanding negative statements when they did not occur in an appropriate context, while the ease of handling affirmative statements led subjects to make fallacious inferences. There was an overwhelming tendency

1) for inference to be made whether logically valid or not, and

2) for the content of the material to assume increasing significance.

Their dissatisfaction with logic as a standard for assessing performance and their finding that the structural variability of cognitive processes was radically affected by the actual content of the experimental material led them to conclude that any general theory of reasoning must include a crucial semantic component.

B. North American Research Into Reasoning

American behavioural theorists have argued that thought and language are the heart of reasoning. Their claim has been that, although there exist structural changes in the hierarchy in which thought units are organised, thought structures with content are the basic explanation for differences in reasoning and problem solving. They have argued that the reason for differences is different semantic structures, or different habits of perceptual analysis, whereas Piaget has considered different logical structures to be responsible for the qualitative differences and cognitive performance of three, seven and twelve year old children.

Major studies have included those of Bruner Goodnow and Austin (1956) who investigated concept formation, Brunswik (1956) who directed his research into differences between thinking and perception, and Bruner (1957) who considered perception as an inferential process. Long and Welch (1941) made an early study on the interdependence of perceptual discrimination and conceptual measurement which was reconsidered by Braine (1959). He asserted that it was intrinsically impossible to study how concepts developed with methods employing verbal cues to evoke the concepts, and developed appropriate tests using non verbal methods to test the breadth and depth of children's understanding of various concepts. His evidence did not bear out, with any degree of regularity, Piaget's contention that a child who has reached a particular stage of development, characterised by certain logical principles, should be able to master any problem involving those principles. Wohlwill (1960) has contributed a further study on individual patterns of mastery of conceptual tasks.

The results of these studies have indicated that the details of conceptual development are determined by the interaction of

a) cognitive units of rules and vocabulary resources organised hierarchically,

b) determinants of attentional involvement eg. motives and standards, and

c) habits for processing information such as evaluation and analytical attitudes.

In addition to the group of experiments which have been directed mainly towards isolating the characteristics which influence the formation of concepts, other researchers have considered the effects of intelligence, training and experience, socio-economic status and vocabulary on these characteristics. Vinacke (1952) in reviewing the major studies of the behaviourists has tentatively drawn the following conclusions.

a) Children's concepts change with increasing age but more in the form of a gradual progression toward greater adequacy or more informed understanding, than of definite stages. Where stages of conceptual learning are recognised however, claims are made that they are characterised by different hierarchical organisations, by rules, habits of processing information, and motives that facilitate or oppose attentional involvement.

b) The apparent difference between children's thinking and that of adults is less a matter of kind than of degree.

c) There is a lack of evidence that an ability to acquire and use concepts can be meaningfully defined apart from other intellectual functions.

This body of literature, which has dealt with the modification of the course of intellectual growth, is significant in describing in detail the cognitive capabilities of school children.

C. Piagetian Inspired Research

Research by Piaget has focussed widespread interest by psychologists on the development of concepts in children, and has generated many further studies. Piaget has found in many different areas that children's thinking is essentially different from that of the adult. He has concluded that concepts in children pass from diffuse, pre-logical, subjective forms to more differentiated, logical and objective forms, and that concepts of a particular kind evolve in a definite order. A second major conclusion drawn by Piaget from his experiments is that three general stages are identifiable.

The validity of these trends in children's reasoning, established by the Genevan school, has been well attested in numerous studies and researches all over the world. One extensive group of Piagetian inspired experiments has been concerned principally with discovering the general characteristics of children's thinking. Others have tested Piaget's hypothesis of the three stage development of children's logical judgements.

Peel and Lodwick (1959) questioned on the content of passages of historical interest. Intuitive thought was characterised by only a single aspect of a situation being focussed upon and the others ignored. Such answers concentrated on physical details and contained inconsistencies. The concrete thought level was recognised by the capacity to combine simple relations between concrete objects, but answers were constructed in terms of their given data rather than in a wider context. At the stage of formal reasoning the subject was able to judge logically and inferentially, thought was reversible, and argument consistent. The evidence obtained emphasised the sequential development of logical thinking as suggested by Piaget.

Lunzer (1965) undertook several studies to arrive at a clearer understanding of the advances in reasoning that appear in children as their age increases. His investigations were carried out by tests composed of verbal analogies, numerical analogies and series completions. His findings confirmed that three observable stages exist and that children progress from intuitive awareness, through the ability to examine two judgements simultaneously and arrive at a conclusion, to a stage of formal reasoning which does not develop before the age of 11 years and often considerably later.

The interrelation of Piagetian tests themselves were studied by Smedslund (1964) who attempted to determine the interrelations of various aspects of concrete reasoning as measured by a battery of Piagetian tests. He obtained detailed results from a large number of individual tests but it was found difficult to relate these to an overall theory.

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Beard (1963) in a number of studies in concept development concluded that for number concepts the order of attainment closely followed that found by Piaget but this was not borne out on all tasks.

The most famous of Piaget's investigations dealt with the conservation of quantity and, in this area, replications carried out by Almy, Chittenden and Miller (1966) Beard (1963) Dodwell (1960 and 1961) Elkind (1964) and Hood (1962) reported observable and distinct preoperational and concrete operational behaviours.

PIAGETIAN THEORY

From his study of the formation of concepts and intellectual operations Piaget has formulated a theory of development. Four basic principles have emerged. These are that

a) intelligence is defined as the ability to perform and codify operations,

b) development is associated with passage from one stage of operations to another,

c) passage from one stage to another is a function of both experience and maturation, and

d) the operations that define intelligence and which change with age are logical structures that are neither dependent on, nor derivative from, language.

Three operational levels in the cognitive development of the child have been recognised by him: each one characterises the attainment of a major stage of development and, within each one sub-stages can be distinguished.

Stage 1, sensory motor operations.

Stage 2, concrete thinking operations.

Stage 3, formal thinking operations.

Piaget therefore claims that development follows a sequential order. Each of its stages is considered necessary to the next one. Intellectual development is dependent on maturation, experience of the physical environment and the interaction of the social environment. The effect of maturation consists essentially in the opening of new possibilities for development by giving access to structures which could not be evolved earlier.

The three aspects of experience are

1. involvement with objects on which action is exerted but without the implication that any knowledge is necessarily extracted from the objects,

2. physical experience which consists of extracting information from the objects themselves, and

3. logico-mathematical experience which involves discovery by manipulation and involves acting upon objects.

Stages of development occur earlier or later in the average chronological ages according to the child's cultural and educational environment. The aim of Piaget's theory of development may then be considered to be the definition and analysis of the structures of operations which are specific to intelligence.

This study has been concerned with that part of Piaget's theory emphasizing concrete operations. This second developmental stage has been described as extending from the middle of the second year to the eleventh or twelfth year. At first it is a forerunner to concrete operational thinking and is based essentially on the activity of the child. Two types of activities have been distinguished; the logico-mathematical type of activity such as bringing together, dissociating, ordering and counting, for which objects are no more than a support, and exploratory activities aimed at extracting information from objects themselves.

The first level of the stage of concrete operations is concluded at about seven years of age. By this time the average child has acquired the ability to use certain logical operations in his thinking, and has reversibility of concrete thought processes. The remainder of this period is a long process of structuring these logical operations and leads the child towards the capability of working in formal logic. The theory of concrete operations is pivotal in Piaget's system since these operations provide the foundation for the development of reasoning about class membership, number concepts and future logical deductions.

Piaget's work on the development of a concept of class has related closely to his claim that concrete operations develop at 6 or 7 years of age. In investigating the properties of these concrete operational structures, the natural and spontaneous development in the child's thought of how to classify and seriate was observed. Piaget realised that a child's overt behaviour was often considerably removed from the child's beliefs and assumptions and that what a child really understands can not be inferred from the conventional numerical operations that he is called upon to learn in the classroom.

REVIEW OF THE LITERATURE ON CLASSIFICATION

A. Piaget's Research

In keeping with his adoption of a logical model, Piaget has carried out extensive, detailed work into the origins and growth of classification and seriation (Inhelder and Piaget 1964). This focusses upon the psychological mechanisms underlying the development of classificatory behaviour in a large number of situations, and has attempted to explain such behaviour in developmental terms. It is an essential part of their thesis that logical reasoning, in relation to classification and seriation, develops before formal reasoning which involves the production of systematic hypotheses and their verification.

In the opinion of Inhelder and Piaget classification has certain properties which characterise the operations that are actually involved in the subjects' actions, and Piaget has predicted the sequence of development of these skills. It is contended for instance that classification begins when a child can group together two objects that are equivalent because they look alike in some way. This is resemblance sorting. As the child develops he extends the scope of his sorting to more than two (consistent sorting) and then to all the objects that could be considered equivalent in some respect (exhaustive sorting). Acceptable categories for grouping become recognised and experience in constructing one class at a time prepares the child for forming successive and simultaneous classification and for understanding class inclusion. Gradual recognition that objects do not belong exclusively in different categories (multiple class membership) leads to active experimentation with different groupings with first one, and then another single attribute, as a focus for this horizontal classification. As logical abilities develop this method of choosing criteria becomes more complex; the choice of single attributes and then combinations of attributes leads to the construction of hierarchical classification.

Inhelder and Piaget have argued that it is the child's use of this classificatory structure that enables him to form classes that are related to each other hierarchically. Class inclusion is thus the product of experiences in which the child has found diverse attributes to guide his grouping and diverse schemes of attribute combination. A second group of experiences enable the child to understand class inclusion. When children recognise that objects belong to more than one class they begin to describe ways in which classes overlap by using terms like "some" and "all". With such verbal tools for comparison they begin to construct and can experiment with simple class and sub-class relationships. Only when the child has mastered the feat of recognising a common feature on which to group, can he learn to divide classes into constituent parts by focussing on the distinguishing attribute. When the child has understood that combination and division are reversible and opposite processes he can be said to have mastered the classificatory skills.

a) He can combine class A with another A' to obtain B. In set theoretic symbols $A \cup A' = B = A' \cup A$.

b) He can understand that the order in which combinations are made does not effect the result $(A \cup A') \cup B = A \cup (A' \cup B)$.

c) He can understand that adding nothing to class A does not alter class A and can acknowledge the existence of the null set. $A \cup 0 = A$.

Principles a) and b) are clear. The child however may not have grasped the implication of c). There is no immediate understanding that the union of two non null sets is necessarily larger than either of the subsets. $A = 0$ and $B = A \cup A'$ then $A \subset B$. This is the principle of class inclusion which Piaget has claimed is not understood before 7 - 8 years of age. Class inclusion operations then relate to the child's ability to manipulate part/whole relationships within a set of categories; when this can be done systematically false generalisations are no longer made.

Attention has been drawn to the difficulty, for the child, of grasping the notion of a class with a single element and it has been hypothesised that no intensive classification is involved until the stage of concrete operations when a singular class is treated in the same way as other classes.

Inhelder and Piaget claim that an understanding of order relation is logically prior to the notion of complementary classes because the difference relation can be perceived while a class as such can not. A complementary class is one which may be combined with a given class to yield the next higher ranking class. They have argued further that complementarity precedes inclusion and is a more elemental notion psychologically than negation. Thus they suggest that the development of negation depends on that of class inclusion.

In their analysis of negation the claim has been made that initially the operations involved are undifferentiated. This lack of differentiation has been described as being akin to the intuitive or preoperational understanding of "otherness" as opposed to the operational understanding which depends on inclusion. Two types of negation have been recognised as having general meaning in the hierarchical system of class inclusion. The first is that of negation with respect to the whole i.e. not A in the absolute sense. Young children think that the strongest negation is that which is most meaningful because it points to a wide difference and Piaget gives an example of a boy who, having identified a man and a ladder as "not animal" expanded his answer:

> "Ladder is more true. The ladder is made of wood. The man has legs; he's more like an animal than the ladder is."

The second is negation with respect to the next including class i.e., A' = B - A. Older children draw the conclusion that the most useful negations are those which indicate differences between neighbouring classes. A more mature boy in Piaget's sample recognised both the man and the ladder as not animals but he said:

> "The man is more right. A man is a little like animals. He has legs. He has more or less the same body."

Inhelder and Piaget's study of complementarity and negation was followed up by an attempt to find out how children, at different levels, have dealt with a situation where a complementary class has existed as a class, but has contained no objects and has been, therefore, an empty class or null set. They confirmed that the objects to which they applied were necessarily associated with concrete operations. This supposed that the objects did positively exist and so the notion of an empty class was excluded as it was incompatible with the child's notion of collection. The order of difficulty was thus even greater and the results have indicated that the null set was rejected right up to the time when the structure of inclusion relations began to be separated from their concrete content at the stage of formal thinking. This was held to be true in the whole realm of classification.

B. Other Researchers Into Classification

The fact that some kind of classification is implicit in a great many actions, and in every judgement, has led psychologists to recognise the importance of consistent classification and categorization. Ideas of Gestalt psychology were of influence on early workers who investigated the development of classification. Goldstein and Scheerer's work (1941) in the field, and their analysis of categorization from the point of view of the abstractions involved, led to the thesis of a pre-categorization level in children's thinking. This has much in common with Piaget's distinction between operational and pre-operational thinking.

The researches of Annett (1959) and others, were designed to determine the extent to which children were able to classify at different ages. Bruner and Kenney (1966) investigated the way in which children learn to grasp double classification, that is classification of objects according to two of their attributes at the same time. Their conclusions were that children were unable to set up a matrix before the age of seven and could deal with only a single dimension or grouping for a period of almost two years before that. They have suggested that this may be because language development does not allow a child to represent the array verbally before that time.

One basic Piagetian contention is that as the logical complexity of a task increases, the average age at which the task is mastered rises. However insufficient data is available from his studies, on the performance of individual subjects on a wide variety of tasks, to prove that the order of difficulty is the same for all learners. Kofsky (1966) has examined the stage sequence of classification behaviour and, by the construction of eleven experimental tasks based on the reports of Inhelder and Piaget, has attempted to determine whether the order of difficulty corresponded to that described by Piaget. She has applied scalogram analysis to the question as to whether mastery of a particular classification rule reflected mastery of previous rules. This has indicated that the invariance of the task mastery sequence was only partially substantiated, although a significant correlation was found between the age of children and the number of successful completions.

Piaget has also claimed that parallel stage sequences exist for classification and seriation. Lovell, Mitchell and Everett (1962) attempted to identify the relationship between classification and seriation by the duplication of many of the experiments originally reported by Inhelder and Piaget. Their results agreed fairly well with those of the originators. They were also able to indicate the limited ability of educationally retarded pupils to develop logical structures.

Dodwell (1960) gave a wide array of tasks to the same children and analysed the pattern of difficulties encountered. In his work with children of several age levels he was able to assess the performance differences among diverse age groups.

In a preliminary study to determine the relative difficulty of aspects of classification Lunzer (1970) used a battery of twenty-one Piagetian tests. He has shown class inclusion to be the seventh most difficult operation after

1. one to one exchange,
2. one to one correspondence,
3. "all" and "some",
4. reproducing order,
5. seriation,
6. cross classification.

He has also shown the age, at which 50% of children tested achieved a near operational level, to be 7 years. He anticipated that further studies would give abundant proof that the achievements of concrete reasoning do not mature simultaneously at the onset of this period. He further concluded that task difficulty is very much dependent on its articulation, on its content and on the manner of its presentation.

Elkind (1961) and Kohnstamm (1963) both of whom attempted to determine whether Piaget's stages were significantly related to age in class inclusion tests, found there was little agreement between their results and attention was drawn to socio economic background as an important variable.

Vernon (1965) investigated environmental handicaps in the development and retardation of intellectual development by comparing test scores of Piagetian items with an assessment of environment in white English, and negro West Indian immigrant, cultural groups. The West Indian children's performance was inferior, but negligibly so on both the test of conservation of quantity and on that of class inclusion.

Shantz (1967) aimed at testing the degree of association between certain logical operations and has offered direct evidence concerning the correlationship of three of these. Her results offer only moderate support for Piaget's claim of close interdependence of logical structures and related abilities. There was also considerable variation in the task inter-correlations across the age range tested.

Morf, (1959) a co-worker with Piaget, has demonstrated however that classification skills and logical subdivision do develop at the same time and are prerequisites for the learning of class inclusion. Flavell (1970) discussed this study.

The double classification problem, which is of major interest in Piagetian theory and which has appeared as an item in some intelligence tests, has required subjects to take into account simultaneously two different aspects of stimulus variation while inferring a logical relation. Jacobs and Vandeventer (1971) from a number of studies have shown that this skill can be taught and the learning retained. They further found that the skill could be transferred for use in tests involving new stimulus dimensions. They taught children to construct matrices in which objects were classified, for example, by both size and colour density. Extended training was found to produce significantly more transfer than the regular training. This training was carried out with grade 1 children just prior to the age at which they might have been expected to develop concrete operations.

Kohnstamm (1967) examined extensively the data on the appearance of the capacity to solve inclusion tasks. He expressed dissatisfaction with Piaget's analysis and interpretation of this quantitative comparison of two classes, one of which is included in the other. By subjecting children to an intensive learning programme, he demonstrated that the development of thought operations is not fixed in a certain order, independent of chance environmental variables.

Another researcher whose investigations have provided evidence contradicting the hierarchy of classificatory activities formulated by Piaget is Feldman (1972) whose study is now reported in detail.

CHAPTER II

BACKGROUND TO THE PROBLEM

A. Feldman's Aims

A series of studies by Feldman (1972) were designed

a) to discover what children understood by negation used as a logical connective,

b) to reveal the developmental progression of such understanding,

c) to study the influence on the understanding of negation of such variables as sex of subjects, idiom in which the negative was expressed, the familiarity of experimental materials and the concomitant case of labelling them, and the relative size of the classes involved, and

d) to relate negation to other logical abilities which Piaget and his collaborators suggested should emerge at the same period.

B. Feldman's Technique

A first study utilised a block giving situation to investigate the negation of a single attribute, to extend these findings to how children understood negation applied to two attributes, and to clarify issues regarding the choice of idioms and placement of the negative.

Commands requesting the subjects to give to the examiner all the blocks named and none of the others were again used in an experiment to determine whether the size of the complementary set influenced responses. The technique remained unchanged but three different sets of materials were used in a further test designed to provide evidence as to whether or not spontaneous verbal labelling of familiar materials facilitated thinking in a negation task.

Negation commands again involved responding to commands concerning attribute blocks, in the third study, when an attempt was made to specify the relationship between negation and class inclusion. Beads and pictures were used in the class inclusion test.

C. Feldman's Results

These studies have provided some interesting results both supporting and contradicting the findings of Inhelder and Piaget, already discussed.

Feldman's data were based on studies focussed to test the assumption made by Inhelder and Piaget, that negation and class inclusion are related phenomena, and attempted to specify the relationship between them. Her results show that hierarchical class inclusion develops later than the simple negation concepts used in her experiments. She has stressed that it is important to note that the operation of negation is almost certainly more complex than the operational measure of it used in her research, and has considered the possibility that negation preceded class inclusion in this study only because the negation test was too easy.

Further work by Feldman has shown the negation of two attributes to be more difficult than the negation of one attribute. In this study 37% of responses to negation commands by children in the 5 - 6 year age range were correct, whereas in class inclusion, considering just the crucial commands for the same aged subjects, only 20% of the responses were correct. A statistically significant, but theoretically weak correlation was thus obtained between negation and class inclusion. The results from the study in which two attributes were negated in both idiomatic and non idiomatic forms, have provided evidence that even quite difficult negation tasks in a classification situation were easier than class inclusion tasks, and that the form of expressing commands influenced the difficulty of carrying out negation.

Other conclusions which were drawn by Feldman from her data on the several hundred children aged between 3 - 7 years who participated in her studies were as follows.

1. Children's understanding of negation as a logical operation developed slowly with age and experience, reaching competent performance between the ages of 6 - 7 years on simple tasks of negation involving one attribute and a familiar idiom.

2. Negation involved a cognitive operation and was not merely a problem of semantics.

3. Common errors to negation commands revealed basic difficulties in carrying out the cognitive operation.

4. The form of the idiom had relatively little effect on one attribute commands although for two attribute commands the "except" idiom was easier than the "but not" idiom.

5. The familiarity of the material and the ease of labelling it was a highly significant variable, as was the number in the complementary set. (It was noticed that the smaller the complementary set, the more correct responses were given).

These studies have not answered the question as to what is the necessary prerequisite for understanding negation in a classification situation, if class inclusion as Inhelder and Piaget believe is not the prerequisite. Feldman has suggested that this might be the classification process itself.

CHAPTER III

DEFINITION OF SPECIFIC AREA OF RESEARCH

Negation is clearly a difficult operation for children. It has seemed important to establish and extend the results of Feldman since there has been no coherent theoretical position or systematic research with respect to the logical operation of negation. The present study was undertaken

a) to describe what children understood by negation used as a logical operation in a classificatory situation,

b) to assess the relationship between class inclusion and negation used as logical operations,

c) to test the assumption made by Feldman that hierarchical class inclusion develops later than negation concepts, and not before as hypothesised by Inhelder and Piaget,

d) to observe whether a developmental change occurred in the understanding of null classes and empty sets.

CONSIDERATIONS AND HYPOTHESES

A number of considerations were relevant to the formation of hypotheses in this study.

1. Piaget and Inhelder (1958) claim that children have a logic of their own which only gradually approximates to the logic of adults and is by no means identical to it. All further studies have provided supportive evidence. From this it has been assumed that errors on experimental tasks might be expected to decline with maturity.

2. Piaget and Inhelder (1958) observed, and Feldman (1972) has confirmed, that sex was not a significant variable in any of their experiments. Sex differences were therefore anticipated to be insignificant.

3. Piaget and Inhelder (1964) showed the natural development of the totality structure of classification in the child's thought to involve the operations of complimentarity, inclusion and negation. They concluded that class inclusion preceded negation. Although subsequent research (Feldman 1972) suggested that an understanding of negation preceded hierarchical inclusion it was assumed that a degree of association existed between the two. Based on the theorizing of Inhelder and Piaget (1964) an understanding of the null set, although another logical ability, was not expected to emerge until a later time.

4. Bellugi's investigation (1967) into the difficulty experienced by young children in placing the negative in their speech alerted Feldman (1972) to the ambiguities found in the English wording of negation tasks and a major focus of her

research had been of the effect of idiom, the way in which commands are expressed, on the understanding of negation. The relative ease with which children handle situations of numerical equivalence was noted by Piaget and Inhelder (1964) in their studies of class inclusion. They observed, however, striking differences in difficulty which appeared to depend on the particular properties involved, and which they attributed to the "intuitive or imaginal potential" of the property in question. Donaldson and Lloyd (1971) have postulated that children are vague so long as they rely purely on analysis of utterances. From these studies it has been assumed that the capacity for linguistic analysis and to resist circumstantial constraints is the principal variable affecting the understanding of negation as a logical connective.

5. The application of characteristics to classes was found by Feldman (1972) to be more difficult than their application to single instances. Young children were observed to change their criterion for selection when the complementary set was large. Bruner (1964) Vygotsky (1962) and Inhelder and Piaget (1964) had also reported this phenomenon. This resulted in partial responses involving incomplete categorization. It was assumed with the older subjects in this sample that under inclusive errors would not be more numerous than other errors.

6. Young children's preference for colour over shape, noted by Colby and Robertson (1942) was found by Feldman (1972) to relate to the performance of nursery school pupils of higher socio-economic status, measured by the educational

attainment of the parents, and kindergarten pupils of lower socio-economic status, measured by the same criterion. It was assumed however that more correct responses to commands concerning colour than to commands concerning shape would not be a significant factor with subjects of the age range being investigated in this study.

Following the execution of the experimental tasks, the data was analysed to test the following hypotheses

1. a) There is a significant difference in the number of correct responses made by subjects in each of the three age groups.

b) There is no significant difference between the scores of male and female subjects on any simple classificatory task.

2. a) There is a high positive correlation between individual scores on tests of class inclusion and negation.

b) No significant relationship between the understanding of null sets and the understanding of negation and class inclusion is expected to exist.

3. a) There is no qualitative difference between the errors made; they are common across children, different idioms and different commands. The greatest variable affecting understanding is one of linguistic significance.

b) There is little difference anticipated between the number of responses made by this sample which are incomplete and those which are over inclusive.

c) There is little difference expected in responses made by this sample to commands concerning colour rather than to commands concerning shape.

PROPOSED DATA ANALYSIS

When the data has been collected the following analyses will be carried out.

1. Descriptive statistics will be used to present the means and standard deviations of the total scores obtained on each of the four tests by each age group.

2. A two way analysis of variance based on total scores will be carried out to obtain information regarding any significant effects of the variables of age and sex.

3. The percentage of subjects passing items in each group will be computed.

4. Based on the above data a test for linear trends in items, using chi square analysis will be made.

5. A principal component analysis of the significant inter correlations between scores will be made with the expectation of a tendency to load on the same factor.

6. A quantitative analysis of error patterns will be made with respect to the anticipated invariance of errors across age groups.

CHAPTER IV

METHOD

A. Sample

The subjects were drawn from a cross sectional representative sample of pupils in the 6 - 9 age range. It was not a perfectly stratified sample; the probability of mean attainment was a little above average (See Appendix 1). One hundred and twenty pupils, forty from each of grades 1, 2 and 3 were tested. The two sexes were balanced with 20 male and 20 female subjects in each grade sample. The subjects' mean ages were 81 months (first graders), 94 months (second graders), and 106 months (third graders). All the pupils were drawn from an urban elementary school in Victoria, B. C. All were Caucasian.

B. Procedure

The subjects were individually tested by an experimenter who sat at a small table. Each testing lasted from 15 - 20 minutes. Independent variables which could influence responses, and which were controlled, were proportional balance of the sexes, the number of objects to be handled and familiarity of the materials. Each child was presented with four tests. All subjects attempted all items on all four tests. One half of the subjects in each age group had the order of the tests reversed.

A correct response was recorded

a) on the class inclusion test when the set had been identified as being greater than the subset,

b) on the negation test when all the blocks belonging to a requested set, and none of the other blocks had been handed to the examiner,

c) on the incomplete set test when all the incomplete sets in each item had been identified,

d) on the Diamond double negative and null set test when, by a verbal or motor response, the subject had indicated an understanding of what was being asked.

The instructions to the subjects were as follows:

Class Inclusion Test (See Appendix 2)

a) "I want you to look at this picture. Can you count how many dolls we have? Yes there are 6 dolls. Now, tell me, are there more dolls or more big dolls?" If the subject said "big dolls" the examiner explained it should have been dolls because all, big and little, were dolls.

b) "Now let us look at the picture of these cars. Tell me, are there more cars or more yellow cars?" If the subject was incorrect it was explained again.

c) "Here is a picture of some ducks. Are there more ducks or more black ducks?" Half of the subjects in each cell had the questions reworded so that the subset was referred to before the set i.e., more big dolls or more dolls; more yellow cars or more cars; more black ducks or more ducks.

Negation Test

With attribute blocks on a tray before her the examiner said: "I am going to ask you to give me some of these blocks. I want you to listen carefully and make sure that you give me the ones I ask for and none of the other blocks. Let's start." Six negation commands with varying idiom and number of attributes were then given. The six commands which constituted the experimental tasks were expressed in the following four ways. N equalled the number of commands of each type

a) not x (N = 1)

b) x not y (N = 1)

c) except (N = 1)

d) not x and not y expressed by different idioms (N = 3)

The commands were always given in the same order.

Negation Commands: "Give me. . . ."	No. of blocks in correct response
1. The set that are not blue.	12
2. The set of yellow ones but not the circles	4
3. The set of everything except red triangles	16
4. The set of circles but not the red ones and not the blue ones	2
5. The set that are neither squares nor circles	6
6. The set not squares and not yellow	8

Incomplete Sets Test

Using the attribute blocks, as in test 2, the examiner said "You have seen that there is a set of blue blocks, a set of yellow blocks and a set of red blocks. Can you see what other sets we have? Yes we have a set of circles, a set of squares and a set of triangles. Now we are going to play a game. I am going to ask you to close your eyes. While you do I am going to take one block off the board. . . . Now you may look. I want you to tell me which sets have a piece missing from them. Can you tell me which coloured set has a piece missing? Can you tell me which shaped set has a piece missing? Yes, when I take one block away there are two sets which are not complete. Now we will put that piece back. Close your eyes and I will take away a different block. . . . Now look again."

Blocks removed	Commands
1. one red circle	Which set of coloured blocks has a piece missing? Which set of shaped blocks has a piece missing?
2. one yellow square	As above.
3. two blue squares	Which sets are incomplete now?
4. two red triangles	As above.
5. two blue squares and one yellow triangle	As above.
6. two yellow triangles and one red circle	As above.

Diamond's Double Negatives Null Set Test

The aim of the procedure was to get the subjects to identify boxes in terms of two sets of negated attributes.

Although a classificatory activity no manipulation was necessarily involved.

With the boxes on the table before her the examiner said "Here are some boxes. They can be open like this (pointed to box without lid) or closed like this (pointed to box with lid). They can be empty like this (pointed to empty box) or have pencils in like this (pointed to box with pencil in). Now I want to ask you some questions about these boxes. Listen carefully."

<u>Diamond Null Set</u>

Test Commands	Verbal Examples	Motor
1. What would a box be like that was not closed and not empty?	"open with a pencil in"	point out appropriate box
2. What would a box be like that was not closed and had not got a pencil in?	a)"open and empty" b)"like this without a lid"	point out appropriate box
3. What would a box be like that was not open and had not got a pencil in?	a)"closed and empty" b)"like this with a lid"	point out appropriate box
4. What would a box be like that was not empty and not open?	a)"closed but with a pencil in; but you wouldn't get a pencil that size in; it's too big and the lid wouldn't fit on" etc.	"like that but" . . . must verbally qualify

C. Materials

Class Inclusion Test

3 pictures of a) 4 large and 2 small dolls
b) 6 white and 3 yellow cars and
c) 3 white and 5 black ducks

Negation and Incomplete Set Tests

18 plastic attribute blocks i.e.

6 red - comprised of 2 circles, 2 squares and 2 triangles

6 yellow - comprised of 2 circles, 2 squares and 2 triangles

6 blue - comprised of 2 circles, 2 squares and 2 triangles

Diamond Double Negatives and Null Set Test

10 plastic pill boxes, some empty, some with pencils in, some with lids on, some without lids.

CHAPTER V

RESULTS AND DISCUSSION

The major variable considered in the present study was that of age. The means and standard deviations of the total scores obtained on each of the four tests, by each age group, are shown in Table 1. A linear trend on the total scores of each test is observable.

Initial hypothesis 1 was concerned with age effects. It was hypothesised that there would be significant differences between the means of the correct responses made by subjects in each of the three age groups. This hypothesis was tested by an analysis of variance. The analysis of variance of total scores with the use of two independent factors of age and sex is shown in Table 2. The factor of age was the only significant effect on all four tests. Thus the data strongly supported the thesis that there would be significant age effects.

In addition to considering the main effect, the present study included sex as a factor, it being hypothesised that there would be no significant difference between the mean scores of male and female subjects on simple classificatory tasks. The two way analysis of variance test, carried out on the number of correct responses and shown in Table 2,

TABLE 1

MEANS AND STANDARD DEVIATIONS OF TOTAL SCORES OBTAINED ON EACH OF THE FOUR TESTS BY EACH AGE GROUP

TASKS		GRADES		
		1	2	3
Negation (6 items)	Mean	4.600	5.100	5.275
	S.D.	1.280	.800	.806
Incomplete Sets (6 items)	Mean	3.525	4.075	5.375
	S.D.	1.927	1.100	1.325
Double negatives and null sets (Diamond) (4 items)	Mean	2.500	3.100	3.275
	S.D.	.936	.949	.774
Class inclusion (3 items)	Mean	1.350	1.500	2.075
	S.D.	1.182	1.068	.814

TABLE 2

A TWO-WAY ANALYSIS OF VARIANCE BASED ON TOTAL SCORES

TASK	SOURCE	S.S.*	D.F.	M.S.*	F	P.
Negation	SSA (sex)	0.033	1	0.033	0.03	0.86
	SSB (age)	11.67	2	5.833	5.72	0.01
	SSAB	2.067	2	1.033	1.01	0.37
	ERROR	116.2	114	1.019		
Incomplete Sets	SSA (sex)	0.833	1	0.833	0.28	0.60
	SSB (age)	58.32	2	29.16	9.86	0.01
	SSAB	3.717	2	1.858	0.63	0.54
	ERROR	337.1	114	2.957		
Double Negatives	SSA (sex)	1.008	1	1.008	0.84	0.36
	SSB (age)	9.817	2	4.908	4.08	0.02
	SSAB	0.717	2	0.358	0.30	0.74
	ERROR	137.1	114	1.202		
Class Inclusion	SSA (sex)	0.408	1	0.408	0.54	0.47
	SSB (age)	11.72	2	5.858	7.69	0.01
	SSAB	4.617	2	2.308	3.03	0.05
	ERROR	86.85	114	0.762		

* Corrected to four figures.

indicated that in none of the tests given were sex differences found to be significant. A test for possible interaction effects between sex and age was made. The data from this analysis confirmed that sex was not a significant factor.

In her correlational study of negation and class inclusion based on the theorizing of Inhelder and Piaget (1964) Feldman obtained a statistically significant but theoretically weak correlation coefficient between negation and class inclusion. Hypothesis 2 was therefore concerned with the relationship which might exist between the two, and it was anticipated that there would be a positive correlation between individual scores on tests of class inclusion and negation.

Examination of the correlation coefficients between negation and class inclusion presented by Feldman has revealed no consistent pattern. In her study a moderate correlations (.54) was found in the 3 year old sample and was interpreted as reflecting the fact that these young children had difficulty in handling both class inclusion and negation. A low correlation was at age 4 (.14) but a moderate correlation (.50) at age 5 provided a conflicting result at a time when there was great variation in the negation scores. Low correlations at ages 6 and 7 (.33 and .22 respectively) were explained by her as a result of the fact that, although her subjects rarely made errors to negation commands at those age levels, there was still great variety on class inclusion scores. Similar evidence has been provided by this study. Correlations between the tests administered in this study are shown in Table 3. It would appear that only weak

TABLE 3

CORRELATIONS BETWEEN TEST SCORES AT EACH AGE LEVEL

Grade 1 (n=40)	Class Inclusion 1	Negation 2	Incomplete Sets 3	Double Negatives 4	Null Set 5
1	1.00	0.27		0.01	0.22
2	0.27	1.00		0.36	0.28
3			1.00		0.11
4	0.01	0.36		1.00	0.34
5	0.22	0.28	0.11	0.34	1.00
Grade 2 (n=40)	1	2	3	4	5
1	1.00	0.20		0.06	0.05
2	0.20	1.00		0.24	0.24
3			1.00		0.08
4	0.06	0.24		1.00	0.24
5	0.05	0.24	0.08	0.24	1.00
Grade 3 (n=40)	1	2	3	4	5
1	1.00	0.37		0.01	-0.12
2	0.37	1.00		-0.02	0.07
3			1.00		0.13
4	0.01	-0.02		1.00	0.08
5	-0.12	0.07	0.13	0.08	1.00
All Grades	1	2	3	4	5
1	1.00	0.33		0.08	0.13
2	0.33	1.00		0.29	0.27
3			1.00		0.19
4	0.08	0.29		1.00	0.34
5	0.13	0.27	0.19	0.34	1.00

relationships exist between the various measures used. The Feldman type negation test correlated a little more strongly with the class inclusion test than did the double negative test at each of the three levels. The present results have thus shown that class inclusion and negation do not appear to be closely related, and it is therefore necessary to reject the hypothesis that there is a high degree of positive correlation existing between them.

The intercorrelations between scores on each item of each test were computed and are shown on Table 4. Those which were significant statistically have been asterisked. Weak correlations are seen to have existed generally between items on the class inclusion test and items on the negation test. Nor did the items on these two tests correlate highly with each other. Only the incomplete sets test and the Diamond test show significant inter-item correlations.

On the null hypothesis it was predicted that no significant correlation would be expected between these two logical operations, and the understanding of null sets. The final item of the Diamond test probed an understanding of the null set. Comprehension of the null set has appeared to develop earlier than Inhelder and Piaget claimed. The relationship between this understanding and negation, although weak, appears closer than between negation and class inclusion. The low correlation observed between the understanding of the null set and the other logical abilities sampled was unexpected and may, it is suggested, repay further investigation.

TABLE 4

INTER-CORRELATIONS BETWEEN SCORES ON INDIVIDUAL ITEMS

		Class Inclusion			Negation (items 2, 5 and 6)			Incomplete Sets	
		1	2	3	4	5	6	7	8
Class	1.	1.000	0.167	0.312*	0.013	0.165	0.179	-0.060	0.067
Inclusion	2.	0.167	1.000	0.233	0.150	0.154	-0.009	0.081	0.042
	3.	0.312*	0.233	1.000	0.140	0.241	0.370*	0.036	-0.086
Negation	4.	0.013	0.150	0.140	1.000	0.004	0.179	0.081	0.042
	5.	0.165	0.154	0.241	0.004	1.000	0.251	0.181	-0.096
	6.	0.179	-0.009	0.370*	0.179	0.251	1.000	0.206	0.110
Incomplete	7.	-0.060	0.081	0.036	0.081	0.181	0.206	1.000	0.442*
Sets	8.	0.067	0.042	-0.086	0.042	-0.096	0.110	0.442*	1.000
	9.	0.049	0.193	0.055	0.039	0.012	0.080	0.220	0.217
	10.	0.234	0.199	-0.019	0.090	0.054	0.101	0.225	0.387
	11.	0.070	0.213	0.035	0.165	0.013	0.177	0.219	0.265
	12.	0.180	0.132	0.073	0.234	0.240	0.206	0.246	0.207
Diamond	13.	0.026	0.055	0.019	0.107	0.206	0.124	0.208	0.094
	14.	0.092	-0.062	0.056	0.093	0.063	0.067	0.024	0.129
	15.	0.128	0.010	0.023	0.074	0.069	0.103	0.015	-0.029
	16.	0.014	0.166	0.117	0.166	0.063	0.246	0.149	0.063

TABLE 4 (Continued)

INTER-CORRELATIONS BETWEEN SCORES ON INDIVIDUAL ITEMS

		Incomplete Sets				Diamond			
		9	10	11	12	13	14	15	16
Class Inclusion	1.	0.049	0.234	0.070	0.180	0.026	0.092	0.128	0.014
	2.	0.193	0.199	0.213	0.132	0.055	-0.062	0.010	0.166
	3.	0.055	-0.019	0.035	0.073	0.019	0.056	0.023	0.117
Negation	4.	0.039	0.090	0.165	0.234	0.107	0.093	0.074	0.166
	5.	0.012	0.054	0.013	0.240	0.206	0.063	0.069	0.063
	6.	0.080	0.101	0.177	0.206	0.124	0.067	0.103	0.246
Incomplete Sets	7.	0.220	0.225	0.219	0.246	0.208	0.024	0.015	0.149
	8.	0.217	0.387*	0.265	0.207	0.094	0.129	-0.029	0.063
	9.	1.000	0.494*	0.463*	0.460*	-0.026	-0.153	-0.128	0.096
	10.	0.494*	1.000	0.500*	0.480*	0.045	-0.058	0.022	0.147
	11.	0.463*	0.500*	1.000	0.558*	-0.043	-0.081	-0.046	0.135
	12.	0.460*	0.480*	0.558*	1.000	-0.037	-0.218	-0.085	0.186
Diamond	13.	-0.026	0.045	-0.043	-0.037	1.000	0.417*	0.346*	0.287*
	14.	-0.153	-0.058	-0.081	-0.218	0.417*	1.000	0.236	0.150
	15.	0.128	0.022	-0.046	-0.085	0.346*	0.236	1.000	0.321*
	16.	0.096	0.147	0.135	0.186	0.287*	0.150	0.321*	1.000

p .01 critical value of r = .254

TABLE 5

FACTOR ANALYSIS OF INTER-CORRELATIONS

		Communalities	(Unrotated Factor Matrix) 1	2	3	4	5	6
Variables	1.	0.791	0.297	0.212	-0.425	0.011	-0.651	0.230
	2.	0.462	0.375	0.053	-0.287	0.350	-0.122	-0.315
	3.	0.654	0.254	0.344	-0.639	-0.142	0.003	0.206
	4.	0.632	0.310	0.198	-0.056	0.315	0.473	0.413
	5.	0.762	0.275	0.344	-0.368	-0.385	0.036	-0.531
	6.	0.670	0.420	0.366	-0.262	-0.340	0.274	0.316
	7.	0.684	0.501	0.072	0.351	-0.498	0.189	-0.144
	8.	0.723	0.472	-0.095	0.513	-0.314	-0.228	0.278
	9.	0.560	0.628	-0.368	0.040	0.092	-0.049	-0.130
	10.	0.700	0.715	-0.229	0.153	0.132	-0.308	-0.002
	11	0.616	0.703	-0.289	0.056	0.167	0.046	0.069
	12.	0.651	0.739	-0.251	-0.124	0.031	0.145	-0.069
	13.	0.661	0.180	0.657	0.363	0.003	-0.040	-0.253
	14.	0.599	-0.019	0.613	0.327	-0.031	-0.257	0.220
	15.	0.545	0.065	0.602	0.193	0.345	-0.119	-0.087
	16.	0.567	0.375	0.444	0.141	0.357	0.270	-0.096
Latent roots		10.276	3.239	2.173	1.605	1.156	1.102	1.001
PERCENT OF TOTAL VARIANCE								
		64.223	20.246	13.581	10.031	7.222	6.887	6.256

Principal component analysis (unity in diagonals: latent roots greater than unity)

Variables 1 - 3 class inclusion
4 - 6 negation
7 - 12 incomplete sets
13 - 15 double negatives
16 null set

A principal component analysis of all the intercorrelations was carried out. The data are shown on Table 5. As might be expected from the intercorrelations the principal component analysis produced a rather fragmented factor structure, there being six components with latent roots greater than unity. With the exception of the double negatives all the tests loaded substantially on a weak general factor which accounted for 20% of the variance. This might be interpreted as a logic factor. However, the other factors seem to represent contrasts in the content of the items, e.g., factor 2 is bipolar; incomplete sets contrasting with double negatives and other items.

Responses To Tasks

An examination of the distribution of correct responses to tasks, shown on Table 6, indicates that class inclusion tasks presented the greatest difficulty to this sample at all three age levels with 13%, 18% and 35% responding correctly at grade levels 1, 2 and 3 respectively.

A total of only 43 subjects, or 36% of the sample, responded correctly to all six negation commands in the block giving test. The percentage of totally correct responses on items 1 - 5 was 58% at grade 1 level, 80% at grade 2 level and 83% at grade 3 level. Thus on items 1 - 5, 87 subjects or 78% made no errors, but difficulty with interpretation of the final command reduced the proportion of totally correct scores to 30% at first grade level, 32% at second grade level and 45% at grade 3 level. Thirty-three per cent of all the

TABLE 6

FREQUENCY WITH WHICH SUBJECTS AT EACH AGE LEVEL MADE CORRECT RESPONSES TO COMPLETE TASKS

TASK	GRADE 1		GRADE 2		GRADE 3	
	boys n=20	girls n=20	boys n=20	girls n=20	boys n=20	girls n=20
Class Inclusion	3	2	3	4	4	10
Negation	9	3	7	6	9	9
Incomplete sets	4	5	7	7	14	13
Double negatives	11	14	13	16	13	14
Items 16 - 19 (Diamond)	4	5	7	11	10	8

subjects failed to respond correctly to item 6 and a total of 120 errors were made out of the 720 responses.

On the double negative items of the Diamond test the percentage responding correctly was 62% at grade 1 level, 72% at grade 2 level and 67% at grade 3 level. Comprehension of the empty set, as measured by item 4 of the Diamond test, developed earlier than had been suggested with 30% of the pupils at first grade level, 50% at second grade level, and 65% at third grade level answering correctly.

The percentage of correct responses rose from 23% at the youngest level to 35% at second grade and 63% at third grade on the incomplete sets test. Table 6 shows little variation between the performance of boys and girls at each level, sex differences being least marked on this test.

Responses To Individual Items

The number of subjects passing each item of each test is shown on Table 7. The superior scores on item 2 of the class inclusion test can be accounted for by the fact that the question concerned the numerical disparity between the class and the smaller of the two subordinate classes. The improved performance on item 3, over item 1, in five of the six cells concerned, reflects the effect of the training feedback.

The negation test, composed of six items drawn from, or similar to, those used by Feldman principally in her studies with nursery school and kindergarten pupils was inappropriate for this age group. As a measure of an understanding of negation it was found to be too simple. Three of the items,

TABLE 7

FREQUENCY WITH WHICH SUBJECTS AT EACH AGE LEVEL MADE CORRECT RESPONSES TO INDIVIDUAL ITEMS

ITEM		GRADE 1		GRADE 2		GRADE 3	
		boys n=20	girls n=20	boys n=20	girls n=20	boys n=20	girls n=20
Class Inclusion	1	3	4	6	4	4	14
	2	14	16	20	13	18	20
	3	8	9	8	9	13	13
Negation	4	20	19	18	20	20	20
	5	16	14	17	19	19	17
	6	17	16	20	19	20	19
	7	19	18	19	20	19	19
	8	14	17	20	18	17	20
	9	10	4	7	7	10	11
Incomplete Sets	10	13	15	11	12	16	20
	11	12	18	13	17	18	20
	12	11	12	17	12	17	16
	13	10	14	13	15	20	18
	14	9	9	12	13	19	15
	15	11	9	13	17	18	16
Double Negatives	16	12	13	15	17	15	16
	17	17	18	18	18	19	20
	18	15	15	17	18	17	17
Null Set	19	6	6	8	12	15	11

the simple negation of one attribute command, the "except" command, and the "not . . . and not" command, i.e., items 4, 6 and 7, were passed by over 90% of the sample. The subjects worked quickly and confidently frequently using both hands as shown in the photograph (appendix iii). A number expressed satisfaction with their own performance. "This is easy." and "I am quick at this." or "I got all that right didn't I?" were frequent comments even from those who were not being completely successful. The poorer performance on the final item of this test showed that although similar to an original command used by Feldman, this item was more difficult and eliminated the ceiling effect for negation tests for subjects over 6 - 7 years of age, noted by her.

A second measure of the understanding of negation as a logical operation was Diamond's double negative test. A high proportion of correct responses was given to these double negative commands. On each item at each age level, girls scored well on this test although they did not markedly excel over the boys, as might have been anticipated, for girls superiority in language development is well documented (Maccoby 1966). Correct responses to the incomplete set tasks and the null set item showed the most noticeable developmental trends.

The data for each item was tested for significance by chi square analysis. Item 5 of the negation test which called for response to the "neither . . . nor . . ." command, and the last three items of the incomplete sets test all

TABLE 8 CHI SQUARE VALUES FOR INDIVIDUAL ITEMS

TEST	ITEM	b_{xy}	$V(b_{xy})$	χ_1^2 (df = 1)	Departure from regression line	Original χ^2 (df = 2)	Yates correction on original χ^2 values
Class	1	.14	.0025	7.84***	- .10	7.74	6.28*
inclusion	2	.10	.0016	6.30**	.21	6.51	4.88
items	3	.11	.0031	4.00*	1.40	5.40	4.28
Feldman type	2	.08	.0016	3.50	1.21	4.71	3.28
negation	5	.08	.0013	4.30*	9.00	13.30	10.99***
items	6	.09	.0031	2.44	1.00	3.44	2.58
	1	.10	.0025	4.00*	5.6	9.6	8.11**
Incomplete	2	.10	.0018	5.5**	1.9	7.4	5.67
sets	3	.06	.0016	2.25	3.85	6.1	4.85
items	4	.17	.0024	12.0***	1.9	13.9	11.80***
	5	.20	.0029	13.3***	.8	14.1	12.34***
	6	.17	.0026	11.1***	1.6	12.7	10.87***
Double	1	.075	.0025	2.25	1.35	3.6	2.60
negatives	2	.025	.0011	.56	- .01	.55	1.71
and null	3	.05	.0018	1.4	-	1.4	.74
set items	4	.24	.0031	6.7***	3.2	9.9	8.50**

b_{xy} = the regression co-efficient of age on correct response

$V(b_{xy})$ = the variance of the regression co-efficient

χ_1^2 = the amount of the total χ^2 accounted for by linear regression

$p < .05$ *
$p < .02$ **
$p < .01$ ***

showed that such data would arise by chance in less than 1% of all random samples ($p < .01$). Hence these were highly significant tasks. The null set item of the Diamond task and item 1 of the incomplete set tasks with $p < .02$, and item 1 of the class inclusion test with $p < .05$, were also significant.

Each task item was tested for a linear trend with age using regression methods for partitioning the overall chi square values. The component of the chi square due to a linear trend was separated out. An example of the calculation is shown in appendix 4. The results obtained and shown in Table 8 demonstrate linear trends in many items and the new χ^2 values show improved significance for these items. That the trend is linear rather than of a more complicated nature is shown by low values in the departure from the regression line column.

All three items of the class inclusion test show marked linear trends. Five of the six items on the incomplete set test also showed strong developmental trends, as did the null set item on the Diamond test. The double negative items yielded the lowest χ^2 values. Neither they nor the Feldman type negation items demonstrated that improved performance increased steadily with age and experience.

Response Errors

Hypothesis 3 was concerned with response errors. That the greatest variable affecting understanding and the performance of negation tasks is one of linguistic significance was the concern of this hypothesis. Table 9 shows the data which enables us to accept this hypothesis in a block giving situation. Eight categories of errors were identified from the responses. A total of 35 errors (29%) could be attributed to incorrect inference from the negation commands. The low proportion of totally correct scores reflected the difficulty encountered by pupils in all three age groups in inferring what was meant by the idiom "not . . . and not". The most frequent error arose from an inference that "not yellow and not square" meant "not yellow squares". Some, particularly among the younger children asked to have the command repeated. "That's harder"; "I nearly gave you the wrong ones just then"; "I am not sure if these are the ones", and "I think that can mean two different things" or "You nearly tricked me into giving you the wrong ones just then!" are examples of the reaction of some of the less shy children to the final task. Other subjects were obviously confused as they attempted to analyse the meaning of what had been said. The change in criterion for selection was reflected by their ignoring one of the two attributes and by the giving of over inclusive, under inclusive or jumbled responses.

Ignoring one attribute was observed to be a cause of error which was more common with the youngest children. It is not known whether this was due to faulty memory or intentional simplification of a command they felt too difficult to interpret. We must consider that the structure necessary to deal with two negative attributes might have been incomplete.

The error of misplacing the negative was observed by

TABLE 9

ERROR PATTERNS IN RESPONSES TO NEGATION COMMANDS

TYPES OF ERRORS	GRADES 1	2	3	ALL GRADES
Colour	0	1	0	1
Over Inclusive*	5	5	3	13
Under Inclusive**	8	6	3	17
Opposite	3	0	1	4
Wrong Inference	13	12	10	35
Ignored 1 Attribute	14	5	7	26
Misplaced Negative	5	3	2	10
Confused	7	4	3	14
Total Number of Errors	55	36	29	120

* An over inclusive response = the blocks requested + 1 extra block

** An under inclusive response = 1 block less than those requested

Feldman to occur more frequently with kindergarten than nursery school subjects. She noted that a correct interpretation of negation involved not only the coding of the negative and the attributes but also coding the order in which they appeared in the command. The frequency of this error appeared to decline with maturity.

It was predicted that there would be little difference between the number of responses which were incomplete and those which were over inclusive. The data showed 14% of the errors on tasks of negation as a logical operation were incomplete. Over inclusive responses accounted for 11% of the total. Thus the under inclusive responses were marginally greater in number than the over inclusive ones. The size of the complementary set did not appear to influence whether or not a response was under inclusive.

It was further considered that with the sample employed in this study there would be little difference in responses to commands concerning colours rather than to commands concerning shape. It should be noted that there were no errors concerning shape and only a single instance of an error concerning colour in all the responses to negation commands. It seems therefore that both colour and shape played little part in simple negation operations.

The Double Negative Tasks

A different measure of an understanding of negation was presented by the Diamond test. The first three items of this test probed the subjects' comprehension of expressions containing double negatives, and necessitated a response to the logical implication.

The Diamond test undoubtedly presented the greatest challenge to the subjects. One grade 2 girl asked if she might draw the answers; she was then able to verbalise her answers from her drawing. This response was accepted. Few children asked to have commands repeated. Most subjects preferred a motor response, as shown by the photograph in appendix 3. A few third grade pupils and one second grade boy verbalised in a manner which showed their appreciation of the problem. Wayne (8.5 yrs.) discussed each stage of his reasoning.

> "If it is not closed and not empty . . . is that what you said? . . . then it must be open, and that means without a lid, doesn't it? And it must be one that has a pencil in if it's not empty. . . . So you are talking about one of this kind here (indicating the appropriate box) aren't you?"

An examination of the response distribution to negation commands of the Feldman type, compared with responses concerned with an ability to correctly interpret the double negative commands of the Diamond test, yielded statistically weak correlation coefficients. A total of 31 subjects who were correct on all six items of the negation test were also correct in their responses to the three items on the Diamond test which called for an understanding of the use of the double negative. An unexpected result was that a further 35 subjects who were correct on all the block giving negation commands, except item 6, were successful in their responses to the Diamond tasks. This was of considerable interest as the idiom used in the

Diamond commands was the "not . . . and not . . ." expression which these subjects failed to interpret correctly in a different context. There was however in the Diamond context no possible alternative interpretation.

The Class Inclusion Tasks

The results of the class inclusion test showed a high rate of failure. Inhelder and Piaget (1964) noted that when questioned about the quantitive relation between a class and a subordinate class many children did not make the inference that the class is more numerous than its sub-class because the relation between the two subordinate classes was judged. That this behaviour continues until an age greater than that of the subjects in this sample has been the concern of Brainerd (1973). He has discussed the findings of Viner and Kronberg (1972) and Ahr and Youniss (1970) who have reported recently on the phenomenon that class inclusion develops later than most other post operational skills. The two positions regarding the nature of the factors responsible for the late emergence of class inclusion have been identified as performance and competence interpretations. Brainerd has investigated the two hypotheses concerned with the reason for this failure. His findings suggest that neither verbal facilitation (Wohlwill 1968) and relative size of subordinate classes (Ahr and Youniss 1970) nor misinterpretation of the experimenters questions can be held responsible for such juxtaposition errors. He suggests that juxtaposition errors are true mental errors which neither result from the performance factors nor are reduced by the elimination of them. This mental error appears to be presupposition. Lisa (7.7 years) was typical in her responses. After correctly counting the total number of dolls in the first class inclusion item she responded in this way.

Ex.	Are there more dolls or more big dolls?
Lisa	There are more big dolls than little dolls.
Ex.	Yes, that is right. There are more big dolls than little dolls, but that is not the question I asked you. I did not ask about the little dolls. Can you remember the question?
Lisa	Are there more big dolls or more dolls?
Ex.	Yes, the question is "are there more big dolls or more dolls?"
Lisa	More big dolls.
Ex.	More big dolls than what Lisa?
Lisa	More big dolls than little dolls.

Troy (7.1) is another example.

Ex.	Are there more dolls or more big dolls?
Troy	More big dolls.

It was explained it should have been "dolls" because all, big and little were dolls. Troy's attention was then directed to the picture of the cars.

Troy	I can count them. There are nine.
Ex.	Good. How many yellow ones are there?
Troy	Six white ones and three yellow ones.
Ex.	Yes there are only three yellow ones. Tell me are there more yellow cars or more cars altogether?
Troy	There are more white cars than yellow cars.
Ex.	Yes but that was not the question. Do you remember what I asked you? Shall I say it again? Are there more cars altogether or more yellow ones?
Troy	More white ones than yellow ones of course.

Such incidences were repeated many times over. The subjects presupposed the question that was going to be asked rather than misinterpreting what was said. Having seized upon the disproportionate balance between the subsets as the characteristic feature of the problem they reasoned only about this, and incorrect responses were given even when their attention was redirected to the wording of the question. This suggests support for Piaget's explanation that the formation of elementary logical structures is incomplete to accommodate questions about two levels of grouping at the same time.

The Null Set Task

Item 4 of the Diamond test investigated comprehension of the null set. Of those who failed to respond correctly less than half replied that they "didn't know". A few admitted that they did not have an example on the tray before them but were unable to verbalize what attributes the correct example possessed. Most selected from the physical array before them which disposed the subject to ignore one attribute of the verbal command. The younger subjects after recognising that they did not have an example which fitted the description chose to make a correct example by asking for something small they could put in the container so that the lid could be fitted. Several of the older boys would discuss the problem. One suggestion made to fit the lid onto the container with a projecting pencil, was that it should be sharpened till the lid fitted; another was that it should be snapped in two. Most suggested removing the projecting pencil and substituting something smaller.

The Incomplete Set Tasks

In the incomplete set test, it can be seen from Table 10 that incorrect responses involving colour accounted for 20%

TABLE 10

ERROR PATTERNS IN RESPONSES TO INCOMPLETE SET TASKS

TYPES OF ERRORS	GRADES 1	2	3	ALL GRADES
Colour	16	15	9	40
Shape	31	17	4	52
Single Attribute	8	7	3	18
Paired Attribute	11	6	4	21
"Did not know"	32	30	7	69
Total No. of Errors	98	75	27	200

of the total errors while those involving shape accounted for 26% of the total. Thus the attributes of colour and shape can not be considered insignificant in all tasks presented to this sample. At the grade 1 age level almost twice as many errors concerning shape as those concerning colour were recorded. At the grade 2 level the figures were almost equal and at the grade 3 level the pattern was reversed. I am unable to interpret this data in any meaningful way. It is necessary, however, to acknowledge this significant difference in the number of incorrect responses concerning colour rather than to those concerning shape.

That an understanding of the empty set was related to an understanding of incomplete sets was considered. Thirty subjects were correct on both item 4 of the Diamond test and all items of the researcher's incomplete set test. Forty-two subjects showed a lack of comprehension on both. Of the 27 subjects correct on item 4 of Diamond but not 100% correct on the incomplete set tasks 8 made an error on only one part of one item, 1 made an error on only one item, 5 made errors on only a part of two items. Therefore these 14 pupils may still be considered to have a good understanding of incomplete sets. It therefore seems possible that an understanding of empty sets and incomplete sets was related in 86 cases, or by 72% of the sample, if a less stringent measure of understanding than a 100% correct one was used.

CONCLUSIONS

One influence on children's reasoning to which the research has drawn attention is that of the construction of commands. This was noticeably so on the negation test. Donaldson and Lloyd (1971) have suggested that children experience difficulty with questions which if the linguistic constraints are not fully observed admit of more than one interpretation, as regards the statements involved. Responses to the most complex command of the block giving negation test fitted this claim. Feldman also noted the construction of commands to influence response to these commands which were significantly more difficult for her subjects. She has suggested that such a factor as idiom, or the way in which commands are expressed has more influence on developing schema than on well established ones. This notion fits with the claim by Inhelder (1962) that schemata are in a greater state of flux while developing, but that once they consolidate the variability across situations disappears. The order of the wording in the class inclusion test administered in this study however did not appear to influence the proportion of correct responses. Two alternative forms of wording the question were investigated. The number of subjects who judged the relationship between the two subordinate classes, instead of the relationship between the class and subordinate class, was almost equally divided between the two. Of those who had the question worded with the set referred to before the subset 38% made the error, and of those who had it worded with the subset referred to before the set 40% made the error. Thus the construction of the commands did not appear to influence responses on all tests.

A second claim made by Donaldson and Lloyd is that children experience difficulty with questions where the physical array disposes them to favour a wrong interpretation. This influence on children's reasoning was most obvious in responses to the null set item on the Diamond test, when 31 subjects (50%) selected from the examples on the tray before them. In the class inclusion test the picture may have reinforced in the children's minds the disproportionate balance between the subsets. It can not be ignored that the visual stimulus may have disposed them to judge a relationship they were not questioned about.

Overall it appears that the capacity for linguistic analysis and to resist circumstantial constraints is the principal variable affecting logical reasoning in young school age children.

The data presented in this paper has provided support for Feldman's claim that even quite difficult negation tasks in a classification situation are easier than class inclusion. Feldman has suggested the classification process itself as a prerequisite for an understanding of negation. She has suggested that logical connectives may be hierarchically acquired, and has indicated that the rank order of difficulty is, from least to most difficult, intersection, the positive set, exclusion and/or union, negation. No research has yet tested this speculation. The present finding that an understanding of incomplete sets and the null set appears to develop along side that of negation is provocative. The limited sample used in the research, and the need for further development of some of the measures used, clearly constrains the generalizability of the results. Further study into the developmental order of acquisition of these concepts is needed in view of the theoretical importance accorded to it by Piaget and his co-workers.

SUMMARY

The aims of this research were (a) to describe what children understood by negation used as a logical operation (b) to assess the relationship between class inclusion and negation used as logical operations (c) to test the assumption made by Feldman that hierarchical class inclusion develops later than negation concepts and not before as hypothesised by Inhelder and Piaget, (d) to observe whether a developmental change occurred in the understanding of null classes and empty sets.

One hundred and twenty children aged between 6 - 9 years of age participated in this study. A group of four tests were reported from which the following conclusions have been drawn.

1. Children's understanding of negation as a logical operation developed with age and experience. It did not reach asymptotic performance on negation tasks involving two attributes and double negative idioms until an age greater than that considered by this study.

2. Negation involved a cognitive operation and was not merely a problem of semantics. A capacity for linguistic analysis was the principal variables affecting the understanding of negation as a logical connective.

3. Contrary to the findings of Inhelder and Piaget, class inclusion did not appear to be strongly related to all measures of negation.

4. Evidence was provided in support of Feldman's claim that negation preceded class inclusion in children's reasoning.

5. The findings of this study supported the thesis that the emergence of class inclusion in children's reasoning developed later than the other concrete operational skills being considered.

6. Comprehension of the null set or empty class was shown to exist in a significant percentage of the subjects used in this sample. There was no support for the thesis that such comprehension was delayed until the stage of formal operational thinking.

7. A linear trend in the understanding of incomplete sets was observed.

8. There was some indication that a relationship existed between the ability to understand incomplete sets and the ability to understand the null set. The sample used in this study was too small, and the tests used were too short to provide conclusive evidence.

REFERENCES

Almy, Chittenden and Miller. Young children's thinking. (Studies of some aspects of Piaget's theory). N.Y.: Teachers College Press. 1966.

Annett, M. The classification of four common class concepts. British Journal of Educational Psychology. 1959. Vol. 29, page 223.

Ahr, P.R. and Youniss J. Reasons For Failure on the Class Inclusion Problem. Child Development. 1970. Vol. 41, p. 131.

Beard, R.M. The order of concept development; studies in two fields. Education Review. 1963. Vol. 15, No. 2, page 105-17.

Bellugi, U. The acquisition of negation (from D. McNeil, The development of language). Carmichael's Manual of Child Psychology. Wiley. 1970.

Braine, M.D.S. The ontogeny of certain logical operations. Psychology Monographs. 1959. 73 5.

Brainerd, C.J., and Kaszor, P. An analysis of two performance hypotheses about children's class inclusion errors. Unpublished paper, University of Alberta. 1972.

Bruner, Goodnow and Austin. A study of thinking. New York: Wiley. 1956.

Bruner, J. The course of cognitive growth. American Psychologist. 1964. Vol. 19, page 1.

Bruner, J. Going beyond the information given from Contemporary approaches to cognition. Harvard University Press. 1957.

Bruner and Kenney. On multiple ordering from Bruner (ed.), Studies in Cognitive Growth. New York: Wiley, 1966.

Brunswik, E. Perception and the representative design of psychological experiments. Berkley: University of California Press. 1956.

Burt, C. Evidence for the concept of intelligence. British Journal of Educational Psychology. 1955. Vol. 25, p. 158-177.

Dodwell, P.C. Children's understanding of number and related concepts. Canadian Journal of Psychology. 1960. Vol. 14, p. 191.

Donaldson, M. Positive and negative information in matching problems. British Journal of Psychology. 1959. Vol. 50, p. 253.

Donaldson and Lloyd. 1971.

Elkind, D. The development of quantitative thinking. Journal of Genetic Psychology. 1961. Vol. 98, p. 37.

Feldman, S.S. Children's understanding of negation as a logical operation. Genetic Psychology Monographs. 1972. Vol. 85, p. 3-50.

Flavell, J.H. The developmental psychology of J. Piaget. Princeton, N.J.: Van. Nostrand. 1968.

Flavell, J. Concept development in Mussen P. (ed.) Carmichael's Manual of Child Psychology. Vol. 1. Wiley. 1970.

Fogelman, K.R. Piagetian tests for the primary school. N.F.E.R. in England and Wales. 1970.

Goldstein, K. and Scheerer, M. Abstract and concrete behaviour. Psychology Monographs. 1941. Vol. LIII, p. 151.

Hood, H.B. An experimental study of Piaget's theory of the development of number in children. British Journal of Psychology. 1962. 53(3), p. 273.

Inhelder, B. Some aspects of Piaget's approach to cognition. From Kessen and Kuhlen (ed.) Thought in the young child. Monograph Society For Research in Child Development. 1962. Vol. 27, No. 83.

Inhelder and Piaget. The early growth of language in the child. New York: Harper Row. 1964.

Inhelder and Piaget. The growth of logical thinking. New York: Basic Books. 1959.

Jacobs and Vandeventer. The learning and transfer of double classification skills. Princeton, N. J.: Educational Testing Service. 1969.

Kofsky, E. A scalogram study of classification development. Child development. 1966. Vol. 37, No. 1, p. 191-205.

Kohnstamm, G.A. Piaget's analysis, right or wrong? The Hague: Mouton and Co. 1967.

Long, L. and Welch L. Reasoning ability in young children. Journal of Psychology. 1941. Vol. 12, p. 21-44.

Lovell, K., Mitchell, B. and Everett, I.R. An experimental study of the growth of some logical structures. British Journal of Psychology. 1962. Vol. 53, part 2, p. 175.

Lunzer, E.A. Recent studies in Britain based on the work of Piaget. N.F.E.R. in England and Wales. (occ. pub. no. 4.) 1961.

Lunzer, E.A. Problems of formal reasoning in test situations. From European research in cognitive development. Monograph of Society For Research in Child Development. 1965. Vol. 30, p. 19.

Lunzer, E.A. Construction of a standardised battery of Piagetian tests. From Research in Education. Manchester University Press. 1970.

Maccoby, E. Sex differences in intellectual functioning. From Maccoby (ed.) The development of sex differences. Stanford University Press. 1966.

Maxwell, A.E. Analysing qualitative data. Methuen and Co. 1961.

O'Brien and Shapiro. The development of logical thinking in children. American Educational Research Journal. 1968. Vol. 5, No. 4.

Peel and Lodwick. An investigation into the question whether the inferences children draw, on learning history, correspond with the stages of mental development that Piaget postulates. Unpublished dissertation. University of Birmingham. 1958. Cited by Peel, Conceptual learning and explainer thinking. From Lunzer and Morris (ed.) Development in Human Learning. London: Staples. 1960.

Shantz, C. and Sigel, I. Logical operations and concepts of conservation in children. Detroit: Merrill Palmer Institute of Human Development. 1967.

Smedslund, J. Concrete reasoning: a study of intellectual development. Monograph of the Society for Research in Child Development. 1964. No. 93, Vol. 29.

Valentine, M. The use of a new reasoning test for selection of university and training college students. British Journal of Educational Psychology. 1961. Vol. 31, p. 227.

Vernon, P.E. Environmental handicaps and intellectual development. British Journal of Educational Psychology. 1965. Vol. 35, pt. 1, p. 9.

Vinacke, E. The Psychology of Thinking. New York: McGraw Hill. 1952.

Vygotsky, L.S. Thought and language. New York: Wiley. 1962.

Wason and Johnson Laird. The Psychology of Reasoning: structure and content. London: Batsford. 1972.

Winer and Kronberg. Responses from children, in grades kindergarten through six, to class inclusion questions presented verbally and pictorially. Unpublished manuscript, Ithaca College, 1972, cited by Brainerd.

Wohlwill, J.F. Children's responses to class inclusion questions for verbally and pictorially presented items. Child Development. 1968. Vol. 39, p. 449.

Wohlwill, J.F. A study of the development of the number concept. Journal of Genetic Psychology. 1960. Vol. 97, p. 345.

APPENDIX 1

DISTRIBUTION OF INTELLIGENCE IN THE SCHOOL POPULATION UTILIZED BY THIS STUDY

APPENDIX I

In May 1973 the Otis quick-scoring test of mental ability (Alpha test: short form) was administered to the first grade students used in this study.

The Alpha I.Q. were derived according to the method outlined in the Otis manual i.e., 100± difference of the score from the norm for age. The distribution fell between Alpha I.Q. scores of 89 and 145, the deviation of the scores being between -9 and +27.

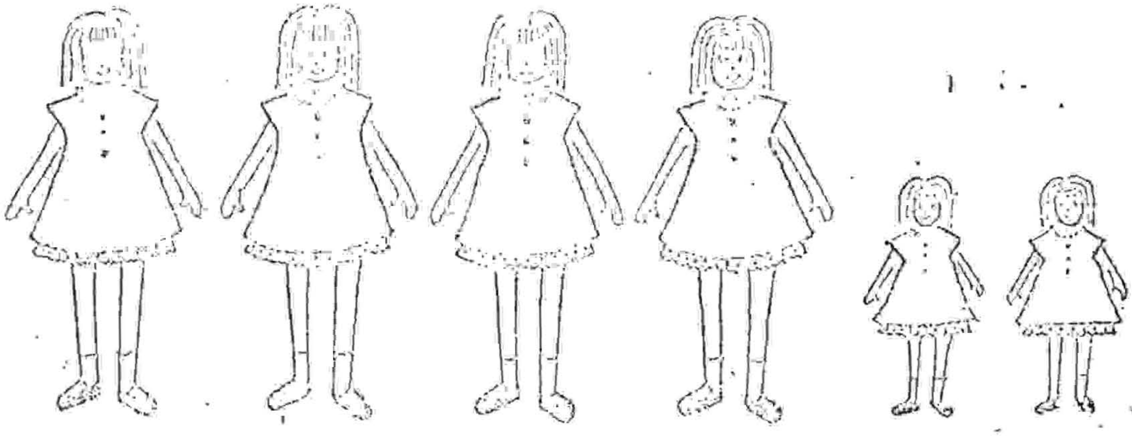
The mean Alpha I.Q. of the sample used in this study was 117 with a standard deviation of 6. This suggested a bunching effect in the high average range. Figures from previous years had shown the school population from which this sample was drawn to cluster round the mean and to be without the "tails" encountered in the normal distribution of intelligence.

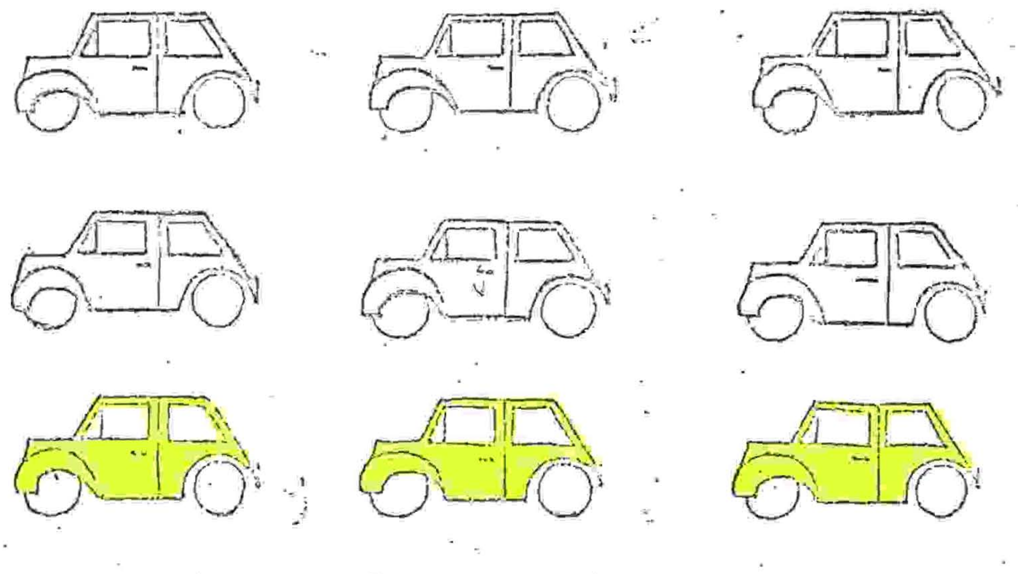
No provincial norms for grade 1 pupils are available. The most recent figures (1955-6) show the mean Alpha I.Q. to have been 102 at the third grade level.

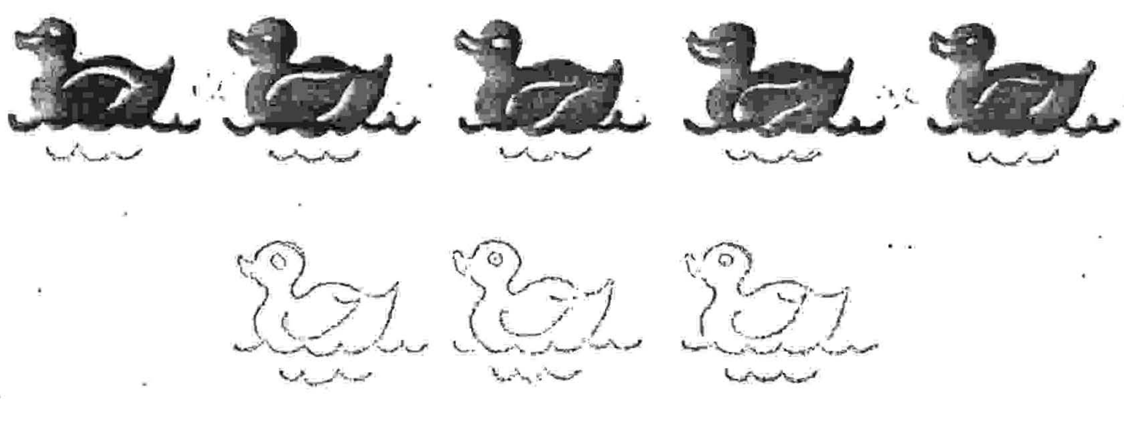
APPENDIX 2

STIMULUS MATERIAL USED IN THE CLASS INCLUSION TASKS

APPENDIX 2







APPENDIX 3

PHOTOGRAPHS OF A FIRST GRADE PUPIL ENGAGED IN THE FELDMAN TYPE BLOCK GIVING TEST AND A SECOND GRADE PUPIL ENGAGED IN RESPONDING TO THE DOUBLE NEGATIVE COMMANDS OF THE DIAMOND TEST

APPENDIX 3

APPENDIX 4

AN EXAMPLE OF THE CALCULATION INVOLVED IN PARTITIONING OF THE CHI SQUARE BY REGRESSION METHODS

APPENDIX 4

Steps in partitioning χ^2 by use of regression methods (Maxwell 1961)

1. The age groups were allotted numerical values

   grade 1 = -1; grade 2 = 0; grade 3 = +1.

2. The contingency table (using data from item 1 of the class inclusion test) arranged the results in a rectangular form.

	-1	0	+1		
Passing	7	10	18	=	35
Failing	33	30	22	=	85

3. The x value = age: the y value = response.

   The value +1 = correct response; the value 0 = incorrect response.

4. The frequency distribution and the data required to obtain the sum of the squares about the means was tabulated as follows:

The frequency distribution for y

y	f	fy	fy^2
1	35	35	35
0	85	0	0
totals	120	35	35

The frequency distribution for x

x	f	fx	fx^2
-1	40	-40	40
0	40	0	0
+1	40	40	40
totals	120	0	80

$$y^2 = \Sigma fy^2 - \frac{(\Sigma fy)^2}{\Sigma f} \quad \text{and} \quad x^2 = \Sigma fx^2 - \frac{(\Sigma fx)^2}{\Sigma f}$$

$$\therefore y^2 = 35 - \frac{35^2}{120} \qquad x^2 = 80 - \frac{0^2}{120}$$

$$y^2 = 35 - \frac{1225}{120} \qquad x^2 = 80 - 0$$

$$y^2 = 35 - 10 \qquad x^2 = 80$$

$$y^2 = 25$$

5. The frequency distribution for (x-y) was then obtained.

The frequency distribution for (x-y)

(x-y)	f	f(x-y)	$f(x-y)^2$
-2	7	-14	28
-1	43	-43	43
0	48	0	0
+1	22	22	22
totals	120	-35	93

$$(x-y)^2 = \Sigma f(x-y)^2 - \frac{(\Sigma f(x-y))^2}{\Sigma f}$$

$$(x-y)^2 = 93 - \frac{35^2}{120}$$

$$(x-y)^2 = 93 - 10$$

$$(x-y)^2 = 83.$$

6. The formula for estimating the regression coefficient of y(response) on x(age) is

$$b_{xy} = \frac{x^2 + y^2 - (x-y)^2}{2x^2} = \frac{80 + 25 - 83}{160} = \frac{22}{160} = .14$$

7. The variance of the regression coefficient is $V(b_{xy}) = \frac{s_y^2}{x^2}$

$$\text{but } s_y^2 = \frac{y^2}{f} \quad \therefore \quad V(b_{xy}) = \frac{y^2}{120\ x^2}$$

$$= \frac{y^2}{120\ x^2} = \frac{25}{120 \times 80} = \frac{25}{9600} = .0025$$

8. The regression coefficients based on df = 1 are distributed as

$$\chi^2 = \frac{(b_{xy})^2}{V(b_{xy})} = \frac{(.14)^2}{.0025} = \frac{.0196}{.0025} = 7.84.$$

9. This data has demonstrated that the highly significant overall χ^2 value is due almost entirely to linear trend. The probability of this result occurring in a random sample of the population is less than 1%.

VITA

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Educational Institutions Attended, with Dates of Entering and Leaving:

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		to	
		to	

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Teaching Diploma	1949	University of Sheffield
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Publications: