IRREVERSIBLE INVESTMENT AND OPTIMAL FOREST EXPLOITATION

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ABSTRACT

A forest harvest scheduling model which includes as activities the level of investment in harvest capacity and the accumulated harvest capacity in each period, is presented. The inclusion of these activities, in addition to the harvest activities, allows for the removal of harvest-flow constraints found in more typical Model II formulations of the harvest scheduling problem. The optimal harvest and investment policy can be determined by linear programming or quadratic programming methods, depending on whether prices are constant or supply-dependent. The new model better reflects economic reality than existing models, and provides a method for determining the optimal economic development of a forest industry, and the optimal draw-down of old growth forest. Numerical examples are given.

1. INTRODUCTION

Models which enable the projection of yields from a forest are in great abundance today and are being actively used to assist in forest management decisions. These models range from so-called stand-level models, which focus on the yield stream from repeated rotations of timber on a single site, to forest-level models, which deal with collections of sites carrying timber of varying ages and species. In the case of forest-level models, one is able to impose forest-wide constraints (for example, to control the age profile of the forest for hydrological or wildlife purposes or to reflect budgetary restrictions on reforestation etc.). Perhaps the most controversial of such forest-wide constraints are those which control the nature of the stream of harvests. These harvest-flow constrains have arisen mainly as a result of a desire on the part of forest managers to maintain a reasonable even flow of timber from a given forest unit, with the objectives of maintaining the stability of local forest-based economies; of providing an assured wood supply to companies investing in the forest sector; and for general purposes of conservation. Despite these commendable objectives, the use of harvest flow constraints has received considerable criticism from economists, both on the grounds that the objectives have little economic basis and on the grounds that flow constraints may be an ineffective or inefficient means of achieving them (see, e.g. Dowdle (1976), Hyde (1977), Waggener (1977), Parry, Vaux and Dennis (1983)).

In this paper we introduce modifications to the well-known Model II forest-level harvest scheduling model (Johnson and Scheurman, 1977), which make it into a more realistic mode, from the economic point of view, and at the same time eliminate the need for harvest flow constraints. To make these modifications we use the approach suggested by Reed and Errico (1986), which explicitly uses the dynamics of the forest and formulates the harvest scheduling problem as one of dynamic control.

If harvest flow constraints are not employed with Model II, the optimal harvest policy would involve the very rapid draw-down of old-growth forest, limited only by growing stock constraints (e.g. to ensure the existence of sufficient forest for wildlife and recreational purposes) and by the demand for timber, as reflected in a downward-sloping demand curve. In practice however, the size of the harvest in a given period is limited by the capacity of logging operations, mills, etc., i.e. inputs which are not susceptible to short term variation but can be varied in the long term, and referred to as "quasi-fixed factor inputs" (e.g. Treadway 1970). Such capital would be "non-malleable" (Arrow & Kurtz, 1970) in the sense that the possibilities for disinvestment would be limited - the investment would be "irreversible". In this paper we include explicitly, as a control variable, the level of capital investment in each period. The accumulated level of capacity (investment net of depreciation) is included as a state variable. We seek a policy of investment and harvesting which will maximize the present value of the stream of harvests net of investment costs. The inclusion of a capacity-investment component in the optimization model allows the simultaneous solution of the optimal harvesting problem and the optimal capital policy problem (see Arrow 1968). The solution to the optimal capital policy problem is in itself interesting. However, we do not emphasize this aspect in this paper.

The model is directly analogous to that solved analytically by Clark, Clarke & Munro (1979) for the problem of optimal fisheries investment and is derived ultimately from the ideas of K. Arrow (1968) on the problem of optimal capital policy with irreversible investment.

The inclusion of a capacity-investment component in the optimization model makes the imposition of harvest flow constraints unnecessary, and leads, we claim, to an economically more realistic approach to the problem of optimal harvest scheduling.

In sections 2, 3 and 4 we introduce the model. The emphasis is on the modifications introduced. The interested reader should refer to Reed and Errico (1986) for explanations of the basic model. In section 5, we compare the effects of the downward sloping demand curve and that of the investment activities on the optimal solution. We also relate investment activities to flow constraints. In section 6, we present an example using data from the Fort Nelson timber supply area in north-eastern British Columbia. Section 7 presents a short discussion of the results and points to possible refinements of the model.

2. THE DYNAMIC MODEL

As in Reed and Errico (1986), we shall use a discrete time model, with equal time periods (of say 10 or 20 years). We describe the forest by a <u>state-vector</u> $\underset{\sim}{X}_t = (x_1^t, x_2^t, \ldots, x_k^t)$. For $i = 1, \ldots, k-1$, x_t^i represents the area of forest with stands belonging to age-class i (i.e. of age between $(i-1)$ periods and i periods); in addition, x_k^t represents the area of the forest with stands belonging to age-class k (i.e. of age $(k-1)$ periods or more). We assume that the period is sufficiently short so that all the stand in age-class i produce approximately the same volume of timber per unit area, v_i. The vector $\underset{\sim}{V}' = (v_1, \ldots, v_k)$ is referred to as the <u>volume-at-age</u> vector.

The <u>harvest vector</u> $H_t = h_1^t, h_2^t, \ldots, h_k^t)$ represents the area harvested from each age-class.

For $i = 1, \ldots, k-1$, areas uncut belonging to age-class i in period t move to age-class $i+1$ in period $t+1$. The area of uncut stands in age-class k remains in age-class k. Any (clear-)cut area moves to age-class 1. This gives rise to the following dynamic equations:

(1) $$\underset{\sim}{X}_{t+1} = R\underset{\sim}{X}_t - S\underset{\sim}{H}_t$$

where

(2) $$R = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \cdot & \cdot & & & \\ 0 & 0 & \cdots & 1 & 1 \end{bmatrix} \qquad S = \begin{bmatrix} -1 & -1 & \cdots & -1 & -1 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \cdot & \cdot & & & \\ 0 & 0 & \cdots & 1 & 1 \end{bmatrix}$$

In this paper we introduce two new variables. Firstly the variable K_t will represent the harvest or mill capacity remaining at the end of period $t-1$ (or the initial capacity for $t=1$), measured in units of timber volume per period. We shall refer to K_t as the <u>carried-over capacity</u> variable. Secondly the variable I_t will represent the new capacity built at the beginning of period t, again measured in units of timber volume per period. We shall refer to I_t as the <u>investment</u> variable. The total capacity at the start of harvesting in period t with be $K_t + I_t$. This capacity provides an upper bound on the volume of timber that can be harvested in period t. Thus we have the constraint:

(3) $$\underset{\sim}{V}'\underset{\sim}{H}_t \leq K_t + I_t$$

It will be assumed that capacity depreciates at a constant rate $1 = \gamma$ during each period. Thus we have the following dynamic equation for capacity

(4) $$K_{t+1} = \gamma(K_t + I_t)$$

3. THE OBJECTIVE FUNCTION

We wish to maximize the total discounted net return from exploitation of the forest. Let c denote the unit cost of establishing new capacity. We shall assume that the price of timber is determined by the supply corresponding to a downward-sloping demand curve. Specifically when the total vaolume of timber harvested during period t is $\underset{\sim}{V}'\underset{\sim}{H}_t'$, let $p(\underset{\sim}{V}'H_t)$ denote the gross return on one unit produced. (More accurately, $p(V'H_t)$ should denote the per-unit return <u>net of production costs but not of capital costs</u>). We will assume that the function p is linear:

(5) $$p(\underset{\sim}{V}'\underset{\sim}{H}_t) = a - b\underset{\sim}{V}'\underset{\sim}{H}_t$$

The total net return during period t is obtained by subtracting the costs of new investment from the gross return i.e. net return in period t is

(6) $$(a - b\underset{\sim}{V}'\underset{\sim}{H}_t)\underset{\sim}{V}'\underset{\sim}{H}_t - cI_t$$

It would be possible to consider more general functions $p(\underset{\sim}{V}'\underset{\sim}{H}_t)$. However, the maximization problem (see below) would be more difficult to solve and would require large computing resources, for nonlinear price schedules.

Assuming a per-period discount factor α, the total discounted net return (present value) is:

$$(7) \qquad J = \sum_{t=1}^{\infty} \alpha^{t-1} \left\{ [a - b\underset{\sim}{V}'\underset{\sim}{H}_t]\underset{\sim}{V}'\underset{\sim}{H}_t - cI_t \right\}$$

We seek a harvest policy and an investment policy to maximize this total discounted net return.

4. THE OPTIMIZATION PROBLEM

We are now ready to state the complete optimization problem. The initial state of the forest is specified by the vector $\underset{\sim}{X}_0 = (x_1^0, x_2^0, \ldots, x_k^0)$. The initial capacity is denoted by K_0 (in our examples, we set $K_0 = 0$). We seek to:

Maximize

$$(8) \qquad J = \sum_{t=1}^{\infty} \alpha^{t-1} \left\{ [a - b\underset{\sim}{V}'\underset{\sim}{H}_t]\underset{\sim}{V}'\underset{\sim}{H}_t - cI_t \right\}$$

subject to:

$$(9) \qquad \underset{\sim}{X}_1 = \underset{\sim}{X}_0$$

$$(10) \qquad K_1 = \underset{\sim}{K}$$

$$(11) \qquad \underset{\sim}{H}_t \leq \underset{\sim}{X}_t \quad \text{for} \quad t = 1, 2, \ldots$$

$$(12) \qquad K_t = \gamma(K_{t-1} + I_{t-1}) \quad \text{for} \quad t = 2, 3, \ldots$$

(13) $\underset{\sim}{V}'\underset{\sim}{H}_t \leq K_t+I_t$ for $t = 1, 2, \ldots$

(14) $X_t = R\underset{\sim}{X}_{t-1} - S\underset{\sim}{H}_{t-1}$ $t = 2, 3, \ldots$

The reader should note that in this formulation there are no harvest flow constraints, of the type which are typically found in harvest scheduling models (e.g. sequential flow constraints, arbitrary constraints etc.). The only constraint on the size of the harvest in any period is that imposed by the available harvest capacity (constraint (13)).

In practice, it is sufficient to solve the optimization problem for a finite number, N, of periods. If so desired terminal payoff values corresponding to the state of the forest and capacity after N periods may be employed. However realistic discount rates usually make these values insignificant for reasonable values of N.

The optimization problem with an N-period time horizon is a straightforward quadratic or linear (if $b = 0$) programming problem with activities $\underset{\sim}{X}_t$, $\underset{\sim}{H}_t$, K_t and I_t $(t = 1,2,\ldots)$. Standard methods of solution such as the complementary simplex algorithm (see e.g. Hartley, 1985) can be used.

5. OTHER FORMULATIONS AND COMPARISONS

In this section, we compare the effect of the downward sloping demand curve, the investment activities and of flow constraints on the optimal solution.

For this purpose a slightly different formulation of the model is useful. Let $K_t = L_t + M_t$ where L_t represents the capacity carried over from period t-1 and in use during period t and M_t represents the capacity carried over from period $t-1$ but idle during period t. Obviously, if $M_t > 0$, then $I_t = 0$ and if $I_t > 0$ then $M_t = 0$. Simple algebraic manipulations show that the objective function (7) can be expressed in either of the following forms:

$$(15) \qquad J = \sum_{t=1}^{\infty} \alpha^{t-1}\left\{[a-b\underset{\sim}{V}'\underset{\sim}{H}_t-c]\underset{\sim}{V}'\underset{\sim}{H}_t + cL_t\right\}$$

$$(16) \qquad J = \sum_{t=1}^{\infty} \alpha^{t-1}\left\{[a-b\underset{\sim}{V}'\underset{\sim}{H}_t - c(1-\alpha\gamma)]\underset{\sim}{V}'\underset{\sim}{H}_t - c(1-\alpha\gamma)M_t\right\}$$

It is easy to create a linear or quadratic programming problem equivalent to the one defined in section 4, but with activities L_t and M_t replacing activities I_t and K_t.

Consider equation (16). The first of the two terms inside the braces represents the return on the harvest for period t net of the costs of investment, if investment is such that there is never any idle capacity. For example suppose a constant voluem harvest at level $\underset{\sim}{V}'\underset{\sim}{H}_t \equiv Q$ were sustained, with the capacity maintained at the level Q. In this case the present value of revenues earned would be $(a-bQ)Q/(1-\alpha)$, while the present value of investment costs incurred would be

$$(17) \qquad c[Q + \alpha(1-\gamma)Q + \alpha^2(1-\gamma)Q + \ldots] = \frac{c(1-\alpha\gamma)Q}{1-\alpha}$$

Thus the net present value would be

$$(18) \qquad J_Q = \frac{1}{1-\alpha}[(a-bQ) - c(1-\alpha\gamma)]Q$$

which is exactly (16) with $\underset{\sim}{V}'\underset{\sim}{H}_t \equiv Q$ and $M \equiv 0$. The factor $a - b\underset{\sim}{V}'\underset{\sim}{H}_t - c(1-\alpha\gamma)$ in (16) represents the per period return, net of capital costs, of sustaining a unit volume of production. Note that this unit net return depends on the volume harvested in a given period. This of course reflects the downward sloping demand curve.

It is clear from (16) that no investment nor harvesting will take place unless

$$(19) \qquad a > c(1-\alpha\gamma)$$

for otherwise the net return on all units of investment will be negative. The condition (19) says that the marginal return on the first unit invested $(a + \alpha\gamma a + \alpha^2\gamma^2 a + \ldots = a/(1-\alpha\gamma))$ must exceed the marginal cost of investment c.

If we consider period t alone, the maximum net return is obtained when

$$(20) \qquad \underset{\sim}{V}'\underset{\sim}{H}_t = (a - c(1-\alpha\gamma))/2b$$

and $M_t = 0$. We shall call the volume in (20) the <u>per-period optimal harvest volume</u>. This volume of harvest will be favoured provided the timber supply is sufficient to support it for several periods. In such a case we can say that the downward sloping demand curve dominates the solution.

The second term inside the braces in (16) represents the cost of carrying idle capacity and reflects the cost of capacity. Removing this term would be equivalent to considering the investment as completely reversible. If the cost of capacity is high, an optimal solution will tend to minimize M_t. This can be achieved either by creating less capacity in previous periods or by conserving timber in the previous periods for harvest in period t. Both actions are equivalent to limiting harvest in previous periods. In such a situation, we can say that the investment activities dominate the solution.

Finally we wish to relate the investment component of the model to harvest flow constraints found in more typical harvest scheduling models (e.g. Johnson & Scheurman, 1977). First assume that we impose a sequential flow constraints of the form

$$(21) \qquad \underset{\sim}{V}' \underset{\sim}{H}_{t+1} \geq (1-\delta)\underset{\sim}{V}' \underset{\sim}{H}_t \qquad t = 1, 2, \ldots$$

in addition to the other constraints in the model. In this case if $\gamma < 1 - \delta$, then we shall have $M_t \equiv 0$ for $t = 2, 3, \ldots$ since the flow constraint (20) forces the harvest volume to be greater than the carried over capacity in every period. Consequently, a sufficiently strong sequential flow constraint which limits the rate of decline of harvest volumes, will force the values of the capacity and investment activities. They can effectively be removed from the model. Second, assume that the cost of capacity is very high relative to the net return. Then, we shall have that $M_t = 0$ for $t = 1, \ldots$ for any optimal solution. This implies that $\underset{\sim}{V}' \underset{\sim}{H}_{t+1} \geq \gamma \underset{\sim}{V}' \underset{\sim}{H}_t$ for $t = 1, \ldots$. Consequently, when the cost of capacity is very high, the effect of the investment activities is equivalent to that of a sequential flow constraint, which limits the rate of decline in harvest volumes. Note that when the cost of capacity is very high it is only in the case of zero depreciation $(\gamma = 1)$, that the economics of investment would impose a condition of non-declining flow.

Sequential flow constraints of the form $\underset{\sim}{V}' \underset{\sim}{H}_{t+1} \leq (1+\delta)\underset{\sim}{V}' \underset{\sim}{H}_t$, which limit the rate of growth of the harvest have the effect of limiting the rate of growth of capacity and vice-versa. In consequence, it is harder to find economic justification for flow constraints of this type than for those which limit the rate of decline of harvest volumes.

6. EXAMPLE

We begin this section with some general remarks.

In all examples, a per-annum discount rate of 5% and a per-annum depreciation rate of 3% were used. The initial capacity is set equal to 0. The initial state of the forest is taken from the Fort Nelson Timber Supply Area in northeastern British Columbia, and represents stands of spruce (Picea glauca Moench Voss), on sites of site index 20+ (base age 100) of medium accessibility to mill centres. For age-classes of 20 years, the initial forest state vector is (in hectares)

$$\underset{\sim}{X}_0 = (241,\ 125,\ 1404,\ 2004,\ 9768,\ 16385,\ 22815,\ 61995)'.$$

The corresponding volume-at-age vector is (in m^3/ha)

$$V' = (.01,\ .01,\ 16,\ 107,\ 217,\ 275,\ 298,\ 306)'.$$

Four simulations are presented below. The return on the sale of one unit of timber (net of variable costs) and the cost of one unit of capital were chosen to provide a range of illustrative examples. The numerical values of these parameters have meaning only in relation with the length of the period, which is used to discretize the problem, and are of little interest in themselves.

The results are summarized graphically in terms of the annual rate of harvesting, Fig. 1. In fact, given the initial state of the forest, knowing the rate of harvesting is sufficient to create complete harvesting and capital investment schedules.

It is well known that the type of model discussed here generates large programming problems. For example, using 20 year periods, 8 age-classes and a 320 year time horizon gives rise to a programming problem with nearly 200 variables and as many constraints. Furthermore, the range of values involved leads easily to conditions of under- or over-flow. In order to limit our use of academic computing resources, the examples given were computed with 40 year period length. Such a long period length gives results which are too coarse for the beginning of the planning horizon, but satisfactory for the end. As a consequence, the results given are based on a 20 year period length for the beginning of the planning horizon. The process by which this refinement is obtained is not entirely trivial, but it enables a large saving of computing resources. We plan to describe it elsewhere. For the harvest, results from the fine (solid line) and coarse (<u>dotted</u> line) simulations are shown in Fig. 1.

In Table 1, we give the annual rate of harvesting as well as the various components of the annual capacity. The column headed 'existing' and 'constructed' show respectively the capacity carried-over from the previous period and that which is constructed at the beginning of the current period. The 'total' of the two is given in the following column. The 'idle' column represents the difference between the 'total' capacity and that actually needed as indicated in the 'annual harvest' column.

Figure and Table 1(a) show the optimal rate of harvesting obtained with the model without either investment activities or a downward sloping demand curve. The optimal harvest schedule consists of harvesting all the timber available at the beginning of the planning horizon and then, again, at regular time intervals determined by the optimal rotation period. The result of the optimization with a 20-year period length gives a higher rate of harvesting for a shorter length of time. However, this is only a consequence of the fact that our model is discrete. In a continuous model, the large harvests would be instantaneous bursts. As expected, idle capacity dominates between the periods of intensive harvesting.

Figure and Table 1(b) show the result of using a model with investment activities but with a flat demand curve (b = 0). In this example the quotient a/c is between $1 - \alpha\gamma$ and 1. The investment activities dominate the transitional phase of exploitation of old growth at the beginning of the planning horizon (0 to 60 years). It can be seen that idle capacity can occur in an optimal solution.

Figure and Table 1(c) show the result of optimizing with both investment activities and a downward sloping demand curve. However, in this example, the downward sloping demand curve completely dominates the solution. Indeed, the per-period optimal harvest volume is only slightly higher than the maximum sustainable yield. As a consequence, the per-period optimal harvest volume can be sustained during the whole planning horizon. The capacity, of course, remains constant. New investment is required to make up for the effect of depreciation.

Figure and Table 1(d) again show the result of optimizing with both investment activities and a downward sloping demand curve. In this case, however, the downward sloping demand curve dominates the solution for the first forty years only, since the per-period optimal harvest volume is much larger than the maximum sustainable yeild. Subsequently, the investment activities dominate during a transitional phase until the forest becomes approximately normal and the rate of harvesting becomes close to that which maximizes sustainable yield.

7. DISCUSSION

The main purpose of the modifications proposed is to demonstrate an economic alternative to the harvest flow constraints found in earlier models.

Such harvest flow constraints have been developed out of the concepts of sustained yield and even-flow and have received much criticism from forest economists, who argue that such concepts have no economic basis. In spite of this critique the use of flow constraints seems to persist in practice. Indeed their use is enshrined in legislation in the United States in the U.S. National Forest Management Act of 1976, which requires that harvests from national forests do not exceed the sustained yield level.

While the economists' case against flow constraints has been strong, it has not carried the day. The reason for this is, we suspect, that no suitably convincing alternative has been proposed. If harvest flow constraints are not employed with Model II, the optimal policies would involve a very rapid draw-down of old-growth forest, limited only by growing-stock constraints (to ensure the existence of sufficient forest for wildlife and recreational purposes etc., or to preserve the forest ecosystem itself), by demand and by the ability of the forest industry to liquidate the forest. This last aspect has important economic implications which have been ignored in harvest scheduling models to date, but are addressed, in part, in our model. Other aspects not included here relate to the ability of industry to procure the capital and labour etc., necessary to expand processing capacity within the particular time frame. It is interesting to note that an optimal harvest schedule can imply periods of excess capacity. Excess capacity in itself can also be a source of difficulties for policy makers.

It is obvious that a downward sloping demand curve could prevent the irregular harvest flow resulting from models without flow constraints. However, unless the forest unit under consideration is very large it is doubtful whether the volume output would have much effect on prices, especially where large supplies are available from many sources, and a free market is operative.

Finally we would like to point out some simple but useful generalizations of our model. Firstly, it would be easy to include a time-dependent volume-at-age curve. Such a time dependence would be useful when a future change in yield is expected. For example, it is suspected that acid rain could have a significant effect on yield. Secondly it would be easy to include a time-dependent price function. This would be useful in exploring the effect of future changes in the value of timber (in real terms). Thirdly, the risk of catastrophic loss through fire, pest infestation etc. (Reed & Errico, 1986) can be included in the model. While these aspects can equally well be handled in existing forms of Model II, the purpose of including them in the model of this paper would be to investigate the effects on investment decisions and resulting harvest flows. Finally, a slight modification of the model would allow for quasi-malleable (i.e. partly reversible) capital investment.

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Figure 1. Optimal harvesting rate:

(a) in the absence of sequential flow constraints and of investment activities. The forest is completely harvested when it reaches the optimal rotation age.

(b) when investment activities are present. Old growth is drawn down gradually over the first 60 years.

(c) when investment activities and a supply dependent price schedule are present. Here, the per-period optimal volume can be sustained for the entire planning horizon. The investment activities do not influence the harvest schedule.

(d) when investment activities and a supply dependent price schedule are present. here, the optimal harvesting rate is determined by the downward-sloping demand curve for the first 40 years and by the investment activities for the following 80 years.

Table 1(a)

PERIOD From	To	ANNUAL HARVEST (000 m^3)	ANNUAL CAPACITY Constructed	Existing (000 m^3)	Total	Idle
0	20	1,630.0	1,630.0	-	1,630.0	-
20	40	7.6	-	886.6	886.6	879.0
40	60	.7	-	482.2	482.2	481.5
60	80	3.7	-	262.2	262.2	258.5
80	100	2.0	-	142.6	142.6	140.6
100	120	-	-	44.8	44.8	44.8
120	140	682.5	658.1	24.4	682.5	-
140	160	682.5	658.1	24.4	682.5	-
160	168	21.0	-	201.8	201.8	180.8
180	200	21.0	-	201.8	201.8	180.8
200	220	2.2	-	59.7	59.7	57.5
220	240	2.2	-	59.7	59.7	57.5
240	260	687.7	670.1	17.6	687.7	0
260	280	687.7	670.1	17.6	687.7	0

Table 1(b)

ANNUAL HARVEST (000 m^3)	ANNUAL CAPACITY Constructed	Existing (000 m^3)	Total	Idle
941.8	941.8	-	941.8	-
512.1	-	512.1	512.1	-
278.5	-	278.5	278.5	-
40.6	-	151.4	151.4	110.8
82.4	-	82.4	82.4	-
148.3	103.5	44.8	148.3	-
50.3	-	80.6	80.6	30.3
50.3	-	80.6	80.6	30.3
148.6	124.8	23.8	148.6	-
148.6	124.8	23.8	148.6	-
23.1	-	44.0	44.0	20.9
23.1	-	44.0	44.0	20.9
136.4	123.4	13.0	136.4	-
136.4	123.4	13.0	136.4	-

Table 1(c)

PERIOD From	To	ANNUAL HARVEST (000 m^3)	ANNUAL CAPACITY Constructed	Existing (000 m^3)	Total	Idle
0	20	220.0	220.0	-	220.0	-
20	40	220.0	100.4	119.6	220.0	-
40	60	220.0	100.4	119.6	220.0	-
60	80	220.0	100.4	119.6	220.0	-
80	100	220.0	100.4	119.6	220.0	-
100	120	220.0	175.2	44.8	220.0	-
120	140	220.0	100.4	119.6	220.0	-
140	160	220.0	100.4	119.6	220.0	-
160	168	220.0	154.9	65.1	220.0	-
180	200	220.0	154.9	65.1	220.0	-
200	220	220.0	154.9	65.1	220.0	-
220	240	220.0	154.9	65.1	220.0	-
240	260	125.0	59.9	65.1	125.0	-
260	280	125.0	59.9	65.1	125.0	-

Table 1(d)

ANNUAL HARVEST (000 m^3)	ANNUAL CAPACITY Constructed	Existing (000 m^3)	Total	Idle
518.4	518.4	-	518.4	-
497.2	215.3	281.9	497.2	-
442.8	172.4	270.4	442.8	-
291.1	50.3	240.8	291.1	-
184.2	25.9	158.3	184.1	-
130.0	85.1	44.8	129.9	-
82.1	11.4	70.7	82.1	-
82.1	11.4	70.7	82.1	-
95.7	71.5	24.3	95.8	-
95.7	71.5	24.3	95.8	-
82.1	53.8	28.3	82.1	-
82.1	53.8	28.3	82.1	-
95.8	71.5	24.3	95.8	-
95.8	71.5	24.3	95.8	-

Table 1. Optimal capacity corresponding to the situation described in Figure 1(a), (b), (c), and (d). The column headed 'Total' shows the total capacity available in each period. The column headed 'Existing' and 'Idle' show respectively the capacity carried over from the previous period and that which is idle in the current period. The 'annual harvest' column indicates the capacity which is used. Note that idle capacity occurs only in cases (a) and (b).