SOME INTEGRALS INVOLVING A GENERAL CLASS OF POLYNOMIALS AND THE MULTIVARIABLE H-FUNCTION

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Motivated by a recent work of A. Gueran [3], the authors establish two general integral formulas associated with the *H*-function of several variables, which was introduced and studied in a series of papers by H.M. Srivastava and R. Panda (cf., e.g., [12] and [13]; see also [10]). Each of these integral formulas involves a product of the multivariable *H*-function and a general class of polynomials with essentially arbitrary coefficients. By assigning suitable special values to these coefficients, the main results can be reduced to the corresponding integrals involving the classical orthogonal polynomials including, for example, Hermite, Jacobi [and, of course, Gegenbauer (or ultraspherical), Legendre, and Tchebycheff], and Laguerre polynomials, the Bessel polynomials considered by H.L. Krall and O. Frink [4], and such other classes of generalized hypergeometric polynomials as those studied earlier by F. Brafman [1], H.W. Gould and A.T. Hopper [2], and M. Lahiri [5]. Furthermore, the multivariable *H*-function occurring in each of our main results can be reduced, under various special cases, to such simpler functions as the generalized Lauricella function of H.M. Srivastava and M.C. Daoust [9], which indeed includes (as its particular cases) a great many of the useful functions of hypergeometric type in one and more variables. A specimen of some of these interesting applications of our main integral formulas is presented briefly.

1. INTRODUCTION AND DEFINITIONS

We begin by recalling the definition of the general class of polynomials $S_n^m(x)$ introduced by Srivastava [7, p. 1, Equation (1)]:

$$(1.1) \qquad S_n^m(x) = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k}\, x^k \qquad (n = 0,1,2,\dots),$$

where m is an arbitrary positive integer, the coefficients $A_{n,k}$ $(n,k \geqq 0)$ are arbitrary constants, real or complex, and $(\lambda)_n$ denotes the Pochhammer symbol defined by

$$(1.2)\qquad (\lambda)_n = \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} = \begin{cases} 1, & \text{if } n = 0, \\ \lambda(\lambda+1)\cdots(\lambda+n-1), & \forall n \in \{1,2,3,\ldots\}. \end{cases}$$

By suitably specializing the coefficients $A_{n,k}$, the polynomials $S_n^m(x)$ can easily be reduced to the classical orthogonal polynomials including, for example, the Hermite polynomials $H_n(x)$, the Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$, and the Laguerre polynomials $L_n^{(\alpha)}(x)$, and indeed also to several familiar particular cases of the Jacobi polynomials such as the Gegenbauer (or ultraspherical) polynomials $C_n^{\nu}(x)$, the Legendre polynomials $P_n(x)$, and the Tchebycheff polynomials $T_n(x)$ and $U_n(x)$ of the first and second kinds (see, for details, [7], [14, pp. 158-160], and [15]).

Other interesting special cases of the polynomials $S_n^m(x)$ include such generalized hypergeometric polynomials as the Bessel polynomials $y_n(x,\alpha,\beta)$ considered by Krall and Frink [4, p. 108, Equation (34)], the generalized Hermite polynomials $g_n^m(x,h)$ considered by Gould and Hopper [2, p. 58], and the Brafman polynomials [1, p. 186] which contain $g_n^m(x,h)$ as a particular case. Furthermore, since (cf. [8, p. 458, Equation (1.7)] and [11, p. 77, Equation (10)])

$$(1.3)\qquad H_{n,m,\nu}(x) = \nu^n\, g_n^m(x,-1/\nu^m) = g_n^m(\nu x,-1),$$

the Gould-Hopper polynomials $g_n^m(x,h)$ contain, as a special case, the generalized Hermite polynomials $H_{n,m,\nu}(x)$ considered by Lahiri [5, p. 118, Equation (3.2)].

Recently, Gueran [3] evaluated certain integrals involving the product of the multivariable H-function (cf., e.g., [12] and [13]; see also [10, p. 251, Equation (C.1)]) with the generalized Hermite polynomials $H_{n,m,\nu}(x)$ or their associated polynomials. The object of the present paper is to establish two general integral formulas involving the product of the multivariable H-function with the polynomials $S_n^m(x)$ defined by (1.1) or their multidimensional analogue defined by

$$(1.4)\qquad S_n^{m_1,\ldots,m_r}(x_1,\ldots,x_r) = \sum_{k_1,\ldots,k_r=0}^{m_1k_1+\ldots+m_rk_r\leqq n} (-n)_{m_1k_1+\ldots+m_rk_r} \cdot A(n;k_1,\ldots,k_r)\, \frac{x_1^{k_1}}{k_1!}\cdots\frac{x_r^{k_r}}{k_r!},$$

where $m_1, \ldots, m_r$ are arbitrary positive integers, and the coefficients

$$A(n;k_1,\ldots,k_r)\quad (n,k_i\geqq 0;\ i=1,\ldots,r)$$

are arbitrary constants, real or complex.

For the multivariable *H*-function defined by Srivastava and Panda [12, p. 271, Equation (4.1) et seq.], we shall employ the contracted notations (due essentially to Srivastava and Panda [12]) which are used in a recent monograph by Srivastava, Gupta and Goyal [10, p. 251, Equation (C.1)]. Thus, following the various conventions and notations explained fairly fully in these earlier works ([12] and [13]; see also [10]), let

$$H[z_1,\ldots,z_r] \equiv H^{0,v:u_1,v_1;\ldots;u_r,v_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\;\middle|\; \begin{matrix} (a_j;\alpha_j',\ldots,\alpha_j^{(r)})_{1,p}: \\ \\ (b_j;\beta_j',\ldots,\beta_j^{(r)})_{1,q}: \end{matrix} \right. \tag{1.5}$$

$$\left. \begin{matrix} (c_j',\gamma_j')_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ \\ (d_j',\delta_j')_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{matrix}\right]$$

denote the *H*-function of r complex variables $z_1,\ldots,z_r$. Here, for convenience, $(a_j;\alpha_j',\ldots,\alpha_j^{(r)})_{1,p}$ abbreviates the p-member array

$$(a_1;\alpha_1',\ldots,\alpha_1^{(r)}),\ldots,(a_p;\alpha_p',\ldots,\alpha_p^{(r)}),$$

while $(c_j^{(i)},\gamma_j^{(i)})_{1,p_i}$ abbreviates the array of p_i pairs of parameters

$$(c_1^{(i)},\gamma_1^{(i)}),\ldots,(c_{p_i}^{(i)},\gamma_{p_i}^{(i)}), \qquad i = 1,\ldots,r,$$

and so on. Suppose, as usual, that the parameters

$$\begin{cases} a_j,\ j = 1,\ldots,p;\ c_j^{(i)},\ j = 1,\ldots,p_i; \\ \\ b_j,\ j = 1,\ldots,q;\ d_j^{(i)},\ j = 1,\ldots,q_i;\ \forall\ i \in \{1,\ldots,r\} \end{cases} \tag{1.6}$$

are complex numbers, and the associated coefficients

$$(1.7)\qquad \begin{cases} \alpha_j^{(i)},\ j=1,\ldots,p;\ \gamma_j^{(i)},\ j=1,\ldots,p_i; \\ \\ \beta_j^{(i)},\ j=1,\ldots,q;\ \delta_j^{(i)},\ j=1,\ldots,q_i;\ \forall\ i\in\{1,\ldots,r\} \end{cases}$$

are positive real numbers such that

$$(1.8)\qquad \Lambda_i \equiv \sum_{j=1}^{p} \alpha_j^{(i)} - \sum_{j=1}^{q} \beta_j^{(i)} + \sum_{j=1}^{p_i} \gamma_j^{(i)} - \sum_{j=1}^{q_i} \delta_j^{(i)} \leqq 0$$

and

$$(1.9)\qquad \Omega_i \equiv -\sum_{j=v+1}^{p} \alpha_j^{(i)} - \sum_{j=1}^{q} \beta_j^{(i)} + \sum_{j=1}^{v_i} \gamma_j^{(i)} - \sum_{j=v_i+1}^{p_i} \gamma_j^{(i)} + \sum_{j=1}^{u_i} \delta_j^{(i)} - \sum_{j=u_i+1}^{q_i} \delta_j^{(i)} > 0, \qquad \forall\ i\in\{1,\ldots,r\}$$

where the integers v, p, q, u_i, v_i, p_i and q_i are constrained by the inequalities $0 \leqq v \leqq p$, $q \geqq 0$, $1 \leqq u_i \leqq q_i$, and $0 \leqq v_i \leqq p_i$, $\forall\ i \in \{1,\ldots,r\}$, and the equality in (1.8) holds true for suitably restricted values of the complex variables $z_1,\ldots,z_r$.

Then it is known that the multiple Mellin-Barnes contour integral (cf., e.g., [10, p. 251, Equation (C.1) with $n = v$, $m_i = u_i$, and $n_i = v_i$ $(i = 1,\ldots,r)$]) representing the multivariable *H*-function (1.5) converges absolutely, under the conditions (1.9), when

$$(1.10)\qquad |\arg(z_i)| < \frac{1}{2}\,\Omega_i\,\pi, \qquad \forall\ i\in\{1,\ldots,r\},$$

the points $z_i = 0$ $(i = 1,\ldots,r)$ and various exceptional parameter values being tacitly excluded. Furthermore, we have (cf. [13, p. 131, Equation (1.9)]):

$$(1.11)\qquad H[z_1,\ldots,z_r] = \begin{cases} O(|z_1|^{\xi_1}\cdots|z_r|^{\xi_r}),\ \max\{|z_1|,\ldots,|z_r|\}\to 0 \\ \\ O(|z_1|^{\eta_1}\cdots|z_r|^{\eta_r}),\ v=0,\ \ \min\{|z_1|,\ldots,|z_r|\}\to\infty, \end{cases}$$

where, with $i = 1,\ldots,r$,

$$(1.12)\qquad \begin{cases} \xi_i = \min\{\mathrm{Re}(d_j^{(i)})/\delta_j^{(i)}\}, & j = 1,\ldots,u_i, \\ \\ \eta_i = \max\{\mathrm{Re}(c_j^{(i)}-1)/\gamma_j^{(i)}\}, & j = 1,\ldots,v_i, \end{cases}$$

provided that each of the inequalities in (1.8), (1.9) and (1.10) holds true.

We remark in passing that, throughout the present work, we shall assume that the convergence (and existence) conditions corresponding appropriately to the ones detailed above are satisfied by each of the various *H*-functions involved. Also, for convenience, we shall denote by

$$H^*[z_1,\ldots,z_r]$$

the special case of the multivariable *H*-function (1.5) when $v = 0$.

2. THE MAIN INTEGRAL FORMULAS

The main results to be established here are the following integral formulas:

$$(2.1)\qquad \int_0^\infty t^{\lambda-1}\, H^*\left[z_1 t^{\mu_1},\ldots,z_r t^{\mu_r}\right] S_n^m(xt^\sigma)\,dt$$

$$= \frac{1}{\mu_r} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k}\, x^k\, z_r^{-\rho_k/\mu_r}$$

$$\cdot\, H^{v_r,u_r:u_1,v_1;\ldots;u_{r-1},v_{r-1}}_{q_r+p,p_r+q:p_1,q_1;\ldots;p_{r-1},q_{r-1}} \left[\begin{matrix} z_1\, z_r^{-\mu_1/\mu_r} \\ \vdots \\ z_{r-1}\, z_r^{-\mu_{r-1}/\mu_r} \end{matrix} \right|$$

$$(1-d_j^{(r)}-\delta_j^{(r)}\,\rho_k/\mu_r;\ \delta_j^{(r)}\,\mu_1/\mu_r,\ldots,\delta_j^{(r)}\,\mu_{r-1}/\mu_r)_{1,q_r},$$

$$(1-c_j^{(r)}-\gamma_j^{(r)}\,\rho_k/\mu_r;\ \gamma_j^{(r)}\,\mu_1/\mu_r,\ldots,\gamma_j^{(r)}\,\mu_{r-1}/\mu_r)_{1,p_r},$$

$$\left.\begin{matrix} (a_j+\alpha_j^{(r)}\ \rho_k/\mu_r;\ A'_j,\ldots,A_j^{(r-1)})_{1,p}:\ (c'_j,\gamma'_j)_{1,p_1};\ldots;(c_j^{(r-1)},\gamma_j^{(r-1)})_{1,p_{r-1}} \\ \\ (b_j+\beta_j^{(r)}\ \rho_k/\mu_r;\ B'_j,\ldots,B_j^{(r-1)})_{1,q}:\ (d'_j,\delta'_j)_{1,q_1};\ldots;(d_j^{(r-1)},\delta_j^{(r-1)})_{1,q_{r-1}} \end{matrix}\right],$$

provided that $\mu_i > 0 \quad (i = 1,\ldots,r)$,

$$A_j^{(i)} = \alpha_j^{(i)} - \alpha_j^{(r)}\ \mu_i/\mu_r > 0 \quad (i = 1,\ldots,r-1;\ j = 1,\ldots,p),$$

$$B_j^{(i)} = \beta_j^{(i)} - \beta_j^{(r)}\ \mu_i/\mu_r > 0 \quad (i = 1,\ldots,r-1;\ j = 1,\ldots,q),$$

$$\mathrm{Re}\left(\lambda + \sum_{i=1}^{r} \mu_i\xi_i\right) > 0, \quad \text{and} \quad \mathrm{Re}\left(\rho_{[n/m]} + \sum_{i=1}^{r} \mu_i\eta_i\right) < 0, \quad n \geqq 0,$$

where ξ_i and η_i are given by (1.12), and

$$\rho_k = \lambda + \sigma k \qquad (k = 0,1,2,\ldots,[n/m]);$$

$$(2.2) \qquad \int_0^\infty\cdots\int_0^\infty t_1^{\lambda_1-1}\cdots t_r^{\lambda_r-1}\ H^*\left[z_1t_1^{\mu_1},\ldots,z_rt_r^{\mu_r}\right]$$

$$\cdot\ S_n^{m_1,\ldots,m_r}\left(x_1t_1^{\sigma_1},\ldots,x_rt_r^{\sigma_r}\right)dt_1\cdots dt_r$$

$$= (\mu_1\cdots\mu_r)^{-1} \sum_{k_1,\ldots,k_r=0}^{m_1k_1+\ldots+m_rk_r\leqq n} (-n)_{m_1k_1+\ldots+m_rk_r}\ A(n;k_1,\ldots,k_r)$$

$$\cdot\ \Psi^*(-\omega_1,\ldots,-\omega_r) \prod_{i=1}^{r}\left\{\Phi_i(-\omega_i)\ z_i^{-\omega_i}\ \frac{x_i^{k_i}}{k_i!}\right\},$$

where $\Phi_i(\xi_i)$ and $\Psi(\xi_1,\ldots,\xi_r)$ are defined by Equations (C.2) and (C.3) in [10, pp. 251-252] with, of course, $n = v$, $m_i = u_i$, and $n_i = v_i \quad (i = 1,\ldots,r)$,

$$\Psi^*(\xi_1,\ldots,\xi_r) = \Psi(\xi_1,\ldots,\xi_r)\Big|_{v=0},$$

and

$$\omega_i = \frac{\lambda_i + \sigma_i k_i}{\mu_i} \qquad (i = 1,\ldots,r),$$

and, for convergence of the multiple integral (2.2),

$$\mu_i > 0,\ \sigma_i > 0,\ \mathrm{Re}(\lambda_i+\mu_i\xi_i) > 0, \quad \text{and} \quad \mathrm{Re}(\lambda_i+\mu_i\eta_i+[n/m_i]\sigma_i) < 0,$$

$$\forall\ i \in \{1,\ldots,r\},$$

ξ_i and η_i being given by (1.12).

Derivation of the Integral Formulas (2.1) and (2.2). Denote, for convenience, the left-hand side of the integral formula (2.1) by Ω. Substituting for the polynomials $S_n^m(xt^\sigma)$ from (1.1) into Ω, if we integrate the resulting (finite) series term-by-term, we readily have

$$(2.3) \qquad \Omega = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k}\, x^k \int_0^\infty t^{\rho_k-1}\, H^*\left[z_1 t^{\mu_1},\ldots,z_r t^{\mu_r}\right] dt,$$

where ρ_k is already defined with (2.1).

Now express the multivariable H-function occurring in (2.3) in terms of its multiple Mellin-Barnes contour integral given, for example, by [10, p. 251, Equation (C.1)] with, of course, $m_i = u_i$ and $n_i = v_i$ $(i = 1,\ldots,r)$, and $n = 0$. Changing the order of integration in such a manner that the innermost t- and ξ_r-integrals can be evaluated by appealing to the Mellin inversion theorem, and interpreting the resulting $(\xi_1,\ldots,\xi_{r-1})$-integral as an (obviously modified) H-function of $r - 1$ variables, we finally arrive at the right-hand side of (2.1) under the conditions stated already.

In order to establish the integral formula (2.2), we apply the definition (1.4) and evaluate the resulting integral in each term by making use of a known result [6, p. 160, Equation (2.1)] which indeed provides the multidimensional Mellin transform of the H-function of r variables.

REMARK. The special rôle played by the H-function variable z_r in (2.1) can be assumed instead by any one of the remaining variables $z_1, \ldots, z_{r-1}$. Moreover, each of the multivariable H-functions involved in (2.1) and (2.2) must satisfy the conditions corresponding appropriately to (1.8), (1.9), and (1.10).

3. APPLICATIONS

Each of the integral formulas (2.1) and (2.2) possesses a two-fold generality. Not only do the general polynomials involved reduce, under special cases, to various known or new classes of polynomials in one variable or more, the multivariable *H*-function can also be suitably specialized to a remarkably wide variety of useful functions (or products of several such functions) which are expressible in terms of the *E*, *F*, *G*, and *H* functions of one, two or more variables. For example, if $p = q = 0$, the multivariable *H*-function occurring on the left-hand side of each of our formulas (2.1) and (2.2) would reduce immediately to the product of r different *H*-functions of Fox. Thus the table listing various further special cases of the *H*-function (given, for example, by Srivastava, Gupta and Goyal [10, pp. 18-19]) can be used with a view to deducing analogous integral formulas involving any of these simpler special functions desired.

In terms of the Gould-Hopper polynomials $g_n^m(x,h)$ for which (cf. [14, p. 161, Equation (1.15)])

$$A_{n,k} = 1, \quad S_n^m(x) \to (-1)^n \left(\frac{x}{h}\right)^{n/m} g_n^m\left(-\left(\frac{h}{x}\right)^{1/m}, h\right),$$

our integral formula (2.1) readily yields the special case:

$$(3.1) \qquad \int_0^\infty t^{\lambda-1} H^*\left[z_1 t^{\mu_1}, \ldots, z_r t^{\mu_r}\right] g_n^m(xt^\sigma, h)\,dt$$

$$= \frac{1}{\mu_r} \sum_{k=0}^{[n/m]} \binom{n}{mk} \frac{(mk)!}{k!} h^k x^{n-mk} z_r^{-\theta_{n,k}/\mu_r}$$

$$\cdot H^{v_r, u_r : u_1, v_1; \ldots; u_{r-1}, v_{r-1}}_{q_r+p, p_r+q : p_1, q_1; \ldots; p_{r-1}, q_{r-1}} \left[\begin{matrix} z_1 z_r^{-\mu_1/\mu_r} \\ \vdots \\ z_{r-1} z_r^{-\mu_{r-1}/\mu_r} \end{matrix} \,\middle|\, \begin{matrix} \ldots \\ \\ \ldots \end{matrix}\right],$$

provided that $\mu_i > 0 \quad (i = 1, \ldots, r)$,

$$\mathrm{Re}\left(\lambda + \sum_{i=1}^{r} \mu_i \xi_i\right) > 0, \quad \text{and} \quad \mathrm{Re}\left(\theta_{n,0} + \sum_{i=1}^{r} \mu_i \eta_i\right) < 0, \quad n \geqq 0,$$

the parameters of the multivariable *H*-function occurring on the right-hand side being those that are displayed in (2.1) with ρ_k replaced by

$$\theta_{n,k} = \lambda + \sigma(n-mk) \qquad (k = 0,1,2,\ldots,[n/m]),$$

and with $A_j^{(i)}$ and $B_j^{(i)}$ constrained as before.

By virtue of one of the relationships in (1.3), a further special case of (3.1), when $h = -1$ and x is replaced by νx, would correspond to a recent result due to Gueran [3, p. 463, Equation (2.1)]. It should be pointed out that, by making use of an elementary Eulerian integral representing the Beta function $B(\alpha,\beta)$, the <u>corrected</u> version of Gueran's other result [3, p. 464, Equation (2.2)] can also be extended to hold true for a general class of polynomials associated with the $S_n^m(x)$ defined by (1.1). However, such integral formulas over the finite interval $(0, 1)$ would be analogous to various obvious consequences of Theorems 1 and 2 of Srivastava and Singh [14, pp. 166-169].

For the Bessel polynomials $y_n(x,\alpha,\beta)$ studied by Krall and Frink [4, p. 108, Equation (34)], we have [14, p. 160, Equation (1.13)]

$$m = 1,\ A_{n,k} = (\alpha+n-1)_k,\ S_n^1(x) \to y_n(-\beta x,\alpha,\beta),$$

and our integral formula (2.1) immediately yields the special case:

$$(3.2) \qquad \int_0^\infty t^{\lambda-1}\, H^*\left[z_1 t^{\mu_1},\ldots,z_r t^{\mu_r}\right] y_n(xt^\sigma,\alpha,\beta)\,dt$$

$$= \frac{1}{\mu_r} \sum_{k=0}^{n} \binom{n}{k} \binom{\alpha+n+k-2}{k} k! \left(\frac{x}{\beta}\right)^k z_r^{-\rho_k/\mu_r}\, H\begin{bmatrix} z_1\ z_r^{-\mu_1/\mu_r} \\ \vdots \\ z_{r-1}\ z_r^{-\mu_{r-1}/\mu_r} \end{bmatrix},$$

where the *H*-function parameters on the right-hand side and the conditions of validity are precisely the same as in the parent formula (2.1) with $m = 1$.

Further consequences of the integral formula (2.1), involving various other polynomial systems indicated above, can be similarly deduced.

Finally, we apply the integral formula (2.2) with a view to deducing the following result for the generalized Lauricella function of Srivastava and Daoust ([9, p. 454]; see also [11, p. 64, Equation (18)]):

$$(3.3)\qquad \int_0^\infty\cdots\int_0^\infty t_1^{\lambda_1-1}\cdots t_r^{\lambda_r-1}\, H^*\left[z_1 t_1^{\mu_1},\ldots,z_r t_r^{\mu_r}\right]$$

$$\cdot\, F^{1+E:U';\ldots;U^{(r)}}_{G:V';\ldots;V^{(r)}}\left[\begin{array}{l} [-n:m_1,\ldots,m_r],\ [(e):\theta',\ldots,\theta^{(r)}]:\ [(u'):\phi'];\ldots;[(u^{(r)}):\phi^{(r)}]; \\ \qquad\qquad\qquad\quad [(g):\psi',\ldots,\psi^{(r)}]:\ [(v'):\zeta'];\ldots;[(v^{(r)}):\zeta^{(r)}]; \end{array}\ x_1 t_1^{\sigma_1},\ldots,x_r t_r^{\sigma_r}\right] dt_1\cdots dt_r$$

$$= (\mu_1\cdots\mu_r)^{-1} \sum_{k_1,\ldots,k_r=0}^{m_1k_1+\ldots+m_rk_r\leqq n} (-n)_{m_1k_1+\ldots+m_rk_r}\, \Delta(k_1,\ldots,k_r)$$

$$\cdot\, \Psi^*(-\omega_1,\ldots,-\omega_r) \prod_{i=1}^{r} \left\{\phi_i(-\omega_i)\, z_i^{-\omega_i}\, \frac{x_i^{k_i}}{k_i!}\right\},$$

where, in addition to the notations, conventions, and conditions stated with (2.1),

$$\Delta(k_1,\ldots,k_r) = \frac{\prod_{j=1}^{E} (e_j)_{k_1\theta_j'+\ldots+k_r\theta_j^{(r)}} \prod_{j=1}^{U'} (u_j')_{k_1\phi_j'} \cdots \prod_{j=1}^{U^{(r)}} (u_j^{(r)})_{k_r\phi_j^{(r)}}}{\prod_{j=1}^{G} (g_j)_{k_1\psi_j'+\ldots+k_r\psi_j^{(r)}} \prod_{j=1}^{V'} (v_j')_{k_1\zeta_j'} \cdots \prod_{j=1}^{V^{(r)}} (v_j^{(r)})_{k_r\zeta_j^{(r)}}}.$$

It is not difficult to derive several further consequences of (3.3) associated with simpler functions of hypergeometric type in one, two or more variables.

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