

# Change point detection of flood events using a functional data framework

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21 **Abstract**

22           Change point detection methods have an important role in many hydrological and  
23 hydraulic studies of river basins. These methods are very useful to characterize changes  
24 in hydrological regimes and can, therefore, lead to better understanding changes in  
25 extreme flows behavior. Flood events are generally characterized by a finite number of  
26 characteristics that may not include the entire information available in a discharge time  
27 series. The aim of the current work is to present a new approach to detect changes in  
28 flood events based on a functional data analysis framework. The use of the functional  
29 approach allows taking into account the whole information contained in the discharge  
30 time series of flood events. The presented methodology is illustrated on a flood analysis  
31 case study, from the province of Quebec, Canada. Obtained results using the proposed  
32 approach are consistent with those obtained using a traditional change point method, and  
33 demonstrate the capability of the functional framework to simultaneously consider  
34 several flood features and, therefore, presenting a comprehensive way for a better  
35 exploitation of the information contained in a discharge time series.

36 **Keywords:** Functional data analysis, Change point detection, Hydrology, Flood.

37

## 38 **Introduction**

39           Detection of changes in hydrological data is of interest to better understand  
40 hydrological regimes, and separate events. Changes in a series can occur in numerous  
41 ways, gradually or abruptly, and can affect the mean, median, variance, autocorrelation,  
42 or any other aspect of the data. In the future, regions that are relatively sheltered from  
43 wind storms, heat waves, droughts and floods, may no longer be in a warmer climate  
44 (Goudie 2006). Detection of changes in long time series of hydrological data is an  
45 important and difficult issue, of increasing interest. Change point detection in hydrology  
46 are essential to characterize the impacts of the climate disturbances on hydrological  
47 regimes (Kingston et al. 2011). It is then very important, particularly where we observe  
48 changes in the frequency and/or in the intensity of various forms of extreme weather  
49 events. Detection of eventual changes in collected data of hydrologic time series sets is  
50 thus obviously an important step before performing any descriptive or predictive analysis.

51           Literature abounds with studies on change point testing in scalar or vector time  
52 series. For example, Kundzewicz and Robson (2004) gave a general guidance on the  
53 methodology for change detection in hydrological records. Wong et al. (2006) proposed a  
54 relational method for discrete data. Change point analysis is addressed in both classical  
55 and Bayesian statistics. Methods in classical statistics usually consist of performing  
56 several kinds of tests to either confirm or reject the hypothesis of change. Most of them  
57 address slope or intercept change in linear regression models (Solow 1987, Easterling and  
58 Peterson 1995, Vincent 1998). Bayesian statistics methods are performed to obtain a  
59 statistical distribution for the change point and eventually a distribution for the other  
60 model parameters. The inference on parameters was performed using Monte-Carlo

61 Markov Chain algorithms (MCMC). Seidou and Ouarda (2007) proposed a Bayesian  
62 method of multiple change point detection in multiple linear regression. This method is  
63 numerically efficient and does not involve the time-consuming Monte-Carlo Markov  
64 Chain simulations as opposed to other Bayesian change point methods. The procedure  
65 was initially designed to detect a change in the relationship between a set of explanatory  
66 variables and the dependent variables. Using the time variable as an explanatory variable,  
67 this approach can detect the change point in a given time series.

68         The flood event is an integration of spatial and temporal variations in water input,  
69 storage and transfer processes within a catchment (Hannah et al. 2000). Particularly,  
70 discharge (rate of flow) time series is the main source of information for studying flood  
71 events. Arguably, the hydrograph of a flood event as a graph showing the discharge  
72 versus time has been the cornerstone of statistical hydrology, as it is directly related to the  
73 design of hydraulic infrastructures. In spite of considerable progress in the development  
74 of new statistical tools for change point analysis, researchers' previous efforts have been  
75 mainly focused on a single or few characteristics of the flood hydrograph ignoring the  
76 continuous behaviour of the flood event in time. Classical change point detection  
77 approaches involve a substantial simplification of the overall extreme hydrological event,  
78 through focusing on a single or few characteristics of the flood event such as the peak or  
79 the volume, and, therefore, fail to account for the whole information stored in flood  
80 hydrographs presented as continuous curves. Despite the extensive literature on change  
81 point methods, little recognition appears to have been given to a more general approach  
82 considering the entire information contained in the discharge time series. The overall  
83 objective of this paper is to present a new approach that attempts to handle this concern

84 by considering the discharge time series of the flood event as a continuous curve using a  
85 functional data analysis (FDA) framework.

86 The first application of FDA to the hydrological context refers to Chebana et al.  
87 (2012) introducing an exploratory analysis and outlier detection of hydrographs. Chebana  
88 et al. (2012) showed that FDA is more general, flexible and representative of the real  
89 hydrological phenomena. For classification of flood events, Ternynck et al. (2016)  
90 showed that obtained classes using functional approaches are more representative than  
91 those obtained using a traditional multivariate hierarchical classification method.  
92 Masselot et al. (2016) adapted a functional regression model for streamflow forecasting.  
93 Suhaila and Yusop (2017) employed the functional framework to study the spatial and  
94 temporal variability of precipitation in Peninsular Malaysia. More recently, Requena et  
95 al. (2018) proposed a functional multiple regression for flow duration curves estimation  
96 while Larabi et al. (2018) developed a stepwise multicriteria for rainfall-runoff model  
97 calibration defined on the basis of FDA.

98 A growing research area is being advanced focusing on the development of new  
99 statistical tools to analyze functional data. For instance, many existing tools in the  
100 univariate and multivariate statistical literature have been adapted to the functional  
101 context (Dabo-Niang et al. 2010, Fischer 2010, Chebana et al. 2012). Some authors  
102 investigated the change point detection method in the FDA context for testing the  
103 assumption of a common functional mean for independent functional data (Aue et al.  
104 2009, Berkes et al. 2009). Thereafter, Zhang et al. (2011) adapted this work to the case of  
105 functional dependent data.

106 The aim of the present paper is to introduce and adapt the FDA framework to change  
107 point detection of flood events. The present paper is structured as follows: a brief  
108 presentation of the data set and the study area is provided in section 1, the proposed  
109 functional change point detection approach is presented in section 2. Results of the  
110 application of the proposed method to the case of flood events in two stations from the  
111 province of Quebec, Canada, are illustrated in section 3. Discussion and conclusion of the  
112 main findings are given in sections 4 and 5, respectively.

### 113 **1. Data Description**

114 Daily flow data recorded at two hydrological stations in the province of Quebec,  
115 the Romaine River and the Moisie River stations, are considered (Figure 1). The  
116 available data series for the Romaine river station covers the period from 1961 to 2000  
117 recorded over a drainage area of  $13000 \text{ Km}^2$ . For the Moisie river station, with a  
118 drainage area of  $19000 \text{ Km}^2$ , daily flow records between 1968 and 1991 are used. Given  
119 the nature of most of the flooding events that characterize the area, mainly caused by  
120 snow melting in spring and summer, only flood events occurring between March 1<sup>st</sup> and  
121 August 31<sup>st</sup> are considered in the current analysis.

122 The selection of these two stations is mainly based on previous finding about the  
123 inhomogeneity of their flood regimes (Ternynck et al. 2016). Furthermore, previous  
124 results on flood event behaviour for both Romaine river and Moisie river stations  
125 demonstrate an apparent change in annual maxima discharges time series (Seidou and  
126 Ouarda 2007). Thus, it is expected that these two case studies may represent  
127 comprehensive examples to test and validate the proposed approach.

128 While the proposed approach is general and can be applied to entire annual  
129 discharge series, a prior knowledge about the season on which major flood events occur  
130 can be helpful to primarily focus on possible changes in the flood event of interest. This  
131 allows avoiding misleading conclusion in change point results that are due to changes  
132 affecting streamflow not related to the major flood event. Although climate change might  
133 shift the timing of flood events (Blöschl et al. 2017), this should not be a concern since  
134 our choice of the spring-summer period is long enough to account for this fact.

## 135 2. Functional change point detection method

136 Consider  $n$  years of daily flow series recorded from March 1<sup>st</sup> to August 31<sup>st</sup> at a  
137 given station corresponding to flood events occurring in the spring-summer period. Let  
138  $x_i = (x_i(t_1), \dots, x_i(t_j), \dots, x_i(t_T))$ ,  $i = 1, \dots, n$  be the set of  $n$  discrete observations where  
139 each  $t_j \in \mathbf{T} \subset \mathfrak{R}^+$  and  $j = 1, \dots, T$  is the  $j^{\text{th}}$  record time point corresponding to the day  $j$   
140 from time subset corresponding to the  $\mathbf{T}$  from March 1<sup>st</sup> to August 31<sup>st</sup> which include the  
141 set  $\{1, \dots, T\}$ . For instance, discrete observations  $x_i$  are daily flow within a given  $i^{\text{th}}$  year  
142 for the spring-summer period with  $T = 181$ . For a given year  $i$ , each set of measurements  
143  $(x_i(t_1), \dots, x_i(t_T))$  will be converted to a functional data denoted  $\{X_i(t), t \in \mathbf{T} \subset \mathfrak{R}^+\}$  using  
144 a smoothing technique.

145 In order to build functions, Ramsay and Silverman (2007) presented two main  
146 basis systems namely: the Fourier system and the B-spline system. Those systems are  
147 now well-established in the statistical literature of FDA. Actually, most of theoretical  
148 developments have been made based on them. As suggested by Ternynck et al. (2016),

149 we use the B-spline basis system for smoothing spring and summer daily discharge data.  
 150 The Fourier system is commonly used for periodic data, while the B-spline system is  
 151 rather used for non-periodic data. Fourier basis functions have been used by Chebana et  
 152 al. (2012) for smoothing daily streamflow that cover the entire year to obtain annual  
 153 streamflow curves. Since the present application considers only the spring and summer  
 154 period, the Fourier basis appears, however, to be less suited.

155 The main idea of the change point detection, here, is to test whether the mean of  
 156 the functional observations  $X_1, \dots, X_n$  remains constant over time. We assume that  
 157  $X_i(t) = \mu_i(t) + \varepsilon_i(t)$ ,  $i = 1, \dots, n$  where  $\mu_i(t)$  denotes the functional mean and  $\varepsilon_i(t)$  is a  
 158 zero-mean functional sequence. We wish to test the null hypothesis  
 159  $H_0 : \mu_1(t) = \mu_2(t) = \dots = \mu_n(t)$  against the alternative  $H_a$  that there is an unknown change  
 160 point  $k^*$  in the mean, i.e.  $H_a : \mu_1(t) = \mu_2(t) = \dots = \mu_{k^*}(t) \neq \mu_{k^*+1}(t) = \dots = \mu_n(t)$ . The  
 161 change can occur at any point  $i$  and we want to test whether it occurs or not. The  
 162 existence of change points means that the data can be divided into several consecutive  
 163 segments, with a constant mean within each segment. Berkes et al. (2009) proposed an  
 164 approach to test the assumption of a common functional mean for independent data. This  
 165 approach is based on the following quantity (which measures a deviation between the  
 166 mean of the functional observations  $X_1, \dots, X_k$  and that of  $X_{k+1}, \dots, X_n$ ):

$$167 \quad P_k(t) = \frac{k(n-k)}{n} \{ \hat{\mu}_k(t) - \tilde{\mu}_k(t) \}, k = 1, \dots, n \quad (1)$$

168 where  $\hat{\mu}_k(t) = \frac{1}{k} \sum_{1 \leq i \leq k} X_i(t)$  and  $\tilde{\mu}_k(t) = \frac{1}{n-k} \sum_{k+1 \leq i \leq n} X_i(t)$ . If the mean changes, the  
169 difference  $P_k(t)$  is large for some values of  $k$  and  $t$ . To deal with the infinite dimension  
170 of the observations (curves), we consider the projections of the functions  $P_k(\cdot)$  on the  
171 principal components of the data. In fact, principal component analysis represents  
172 functional data as  $X_i(t) = \mu(t) + \sum_{1 \leq l \leq \infty} \eta_{i,l} v_l(t)$ , where  $\mu(t)$  is the functional mean,  $\eta_{i,l}$   
173 are the scores and  $v_l(t)$  are the eigen-functions of the covariance operator (Hall and  
174 Hosseini-Nasab 2006). These projections can be expressed in terms of functional scores,  
175 which can be easily computed using the R package “fda”. We consider the estimated  
176 scores  $\hat{\eta}_{i,l}$  corresponding to the largest  $L$  eigenvalues given by:

$$177 \quad \hat{\eta}_{i,l} = \int \{X_i(t) - \bar{X}_n(t)\} \hat{v}_l(t) dt, \quad i = 1, 2, \dots, n \quad j = 1, 2, \dots, L \quad (2)$$

178 with  $\bar{X}_n(t)$  is the sample mean function and  $\hat{v}_l(t), l = 1, \dots, L$  are the estimated eigen-  
179 functions of the covariance operator. It is supposed that  $k = [n\alpha]$  where  $\alpha \in (0,1)$  and  
180  $[\cdot]$  denotes the integer part. Note that  $P_k(t)$  does not change if the  $X_i(t)$  are replaced by  
181  $X_i(t) - \bar{X}_n(t)$ . Hence,  $P_k(t)$  can be written as:

$$182 \quad P_k(t) = \sum_{1 \leq i \leq k} (X_i(t) - \bar{X}_n(t)) - \frac{k}{n} \sum_{1 \leq i \leq n} (X_i(t) - \bar{X}_n(t)) \quad (3)$$

183 Consequently, the projections are defined by  $\int P_k(t) \hat{v}_l(t) dt = \sum_{1 \leq i \leq n\alpha} \hat{\eta}_{i,l} - \frac{[n\alpha]}{n} \sum_{1 \leq i \leq n} \hat{\eta}_{i,l}$  and  
184 are used for testing whether the mean function remains constant. For this purpose, the  
185 following statistic is considered:

186 
$$S_{n,L} = \frac{1}{n^2} \sum_{l=1}^L \lambda_l^{-1} \left( \sum_{1 \leq i \leq n_z} \hat{\eta}_{i,l} - \frac{k}{n} \sum_{1 \leq i \leq n} \hat{\eta}_{i,l} \right)^2 \quad (4)$$

187 where  $\hat{\lambda}_1, \hat{\lambda}_2, \dots, \hat{\lambda}_L$  denote the  $L$ -estimated eigenvalues. The test rejects the hypothesis  
 188  $H_0$  if  $S_{n,L}$  is greater than the corresponding critical value, tabulated in Berkes et al.  
 189 (2009).

190 While this test does not take into account the temporal dependence it will be  
 191 considered here as a first simple step to introduce the functional change point detection  
 192 framework in hydrology. Few other researchers recognize this limitation and propose  
 193 some improvements. For instance, a more complex approach has been proposed by  
 194 Zhang et al. (2011) in order to take into account the temporal dependence. For sake of  
 195 simplicity, this latter will not be considered in the current analysis. In the interim, the  
 196 functional approach being used here may, nevertheless, serves as a stepping-stone  
 197 towards this more complex approach.

### 198 **3. Results**

199 Results of application of the proposed method to the above-mentioned data set are  
 200 compared with those obtained in Seidou and Ouarda (2007). In the latter, the change  
 201 point detection method has been applied separately to the peak, the duration and the  
 202 volume of flood events occurring in spring and summer. The first step to apply the  
 203 functional method consists on performing a functional principal component analysis  
 204 where the first principal components explaining large part of the data variance are,  
 205 therefore, to be retained. For the Romaine river station, we retained the first four  
 206 principal components as they represent 83% of the explained variance. In the hypothesis

207 testing, we set the first type error at 5%. By applying the functional method for the  
 208 Romaine river station, we obtain a change point at the year 1984. This suggests that we  
 209 can split the set of curves into the following two segments  $\overline{TD}_1^a$  : 1961-1984 and  $\overline{TD}_2^a$  :  
 210 1985-2000, of size 24 and 16, respectively as shown in Figure 2.a. We can see from  
 211 Figure 3 that based on the mean, the median and the modal curves, the two obtained  
 212 segments have two different peaks. The peak of the first segment is significantly higher  
 213 than that of the second. One can also note that changes affect not only the peak, but also  
 214 the duration, the volume and the peak date of the flood event as well. Indeed, in both  
 215 classes, flood events began at the same time, but last longer in  $\overline{TD}_1^a$  .

216 In a second step, we reiterate the procedure on the obtained two segments. We  
 217 therefore only find a change point on the segment  $\overline{TD}_1^a$  at the year 1968. Consequently,  
 218 we obtain the three following periods  $\overline{TD}_1^b$  : 1961-1968,  $\overline{TD}_2^b$  : 1969-1984 and  $\overline{TD}_3^b$  :  
 219 1985-2000 of respective size 8, 16 and 16. According to the Figure 4, we can see that  
 220 based on the mean curve, flood events of the segment  $\overline{TD}_2^b$  begin before those of the  
 221  $\overline{TD}_1^b$  , however floods in both segments end at the same time. Accordingly, the flood  
 222 durations for the segment  $\overline{TD}_2^b$  are larger than those of the segment  $\overline{TD}_1^b$  . While flood  
 223 events in both  $\overline{TD}_2^b$  and  $\overline{TD}_1^b$  have almost the same peak, the functional approach seems  
 224 to be able to detect the difference in the duration of the flood events. Flood events of the  
 225 segment  $\overline{TD}_2^b$  begin at the same time with the flood events of the segment  $\overline{TD}_3^b$  , and then  
 226 they take end at the same time with the segment  $\overline{TD}_1^b$  . Moreover, the two segments  $\overline{TD}_1^b$   
 227 and  $\overline{TD}_2^b$  have almost the same peak. Consequently, the segment  $\overline{TD}_2^b$  can be considered

228 as an intermediate period that enables the transition from the flood regime of the segment  
229  $\overline{TD}_1^b$  to the flood regime of the segment  $\overline{TD}_3^b$ .

230 In conclusion, for the Romaine river station, functional change point method, has  
231 detected two change points, the first at year 1984 and the second at years 1968 as shown  
232 in Figure 2.a. This result has divided flood events for the Romaine river station into three  
233 periods: the first with very large floods, which begins later, a second intermediate period,  
234 and a third period characterized by less important floods which starts early. For the  
235 comparison of the functional change point results with a traditional method approach we  
236 applied the Bayesian approach of Seidou and Ouarda (2007) to the peak, the volume and  
237 the duration of flood events separately. The method of Seidou and Ouarda (2007) based  
238 on the duration detects a change point at the year 1987. The same method, however,  
239 based on the volume and the peak detects a change point at the year 1985, which is closer  
240 to the first change point detected by the proposed functional approach (at year 1984). The  
241 Bayesian approach based on the volume and the peak separately was not able to detect  
242 the second change point in the segment  $\overline{TD}_1^a$ . This is due to the fact that this change does  
243 not affect the peak or the volume, but mainly affects the occurrence time of flood events.  
244 The functional approach allows detecting this change in the occurring time of flood  
245 events because it directly considers a large part of the information contained on the entire  
246 discharge series, including information on shape, peak time, duration, etc...

247 For the Moisie river station, using the functional approach, we choose the first  
248 four principal components since they represent 85% of the explained variance. In the  
249 hypothesis testing, we set the first type error at 5%. We obtain a change point at the year

250 1981 which suggests splitting the set of curves into two segments as follows,  $\overline{TD}_1^c$  : 1968-  
251 1981 and  $\overline{TD}_2^c$  : 1982-1991, of size 14 and 10, respectively. We, then, reiterate the  
252 procedure on the obtained two segments, but no change point was detected. Therefore,  
253 we can conclude that this method allows detecting just one change point at year 1981 as  
254 shown in Figure 2.b. Figure 5 shows the mean curve, the median curve and the modal  
255 curve of flood hydrograph corresponding to the two obtained segments. This figure  
256 shows that flood events in the two segments  $\overline{TD}_1^c$  and  $\overline{TD}_2^c$  occur at the same date, but  
257 those of the segment  $\overline{TD}_1^c$ , last longer and have a larger peak. For the Moisie river station  
258 we test the existence of a change point on the peak, the volume and the duration  
259 separately using the method of Seidou and Ouarda (2007). Only, the method based on the  
260 peaks detects a change point at the year 1978.

#### 261 **4. Discussions**

262 It is worth noting that the purpose of the comparison with the conventional  
263 approach is not to show that the functional approach performs better, but rather to check  
264 whether this approach gives results consistent with those obtained using a traditional  
265 approach. Note that, when the focus is only on one characteristic of the flood event, such  
266 as the peak, the volume or the duration, traditional univariate approach preferred.  
267 However, the functional approach takes into account all the characteristics of the flood  
268 event simultaneously, hence, if no preferences on the flood event characteristic, the  
269 functional approach is recommended. Then, the graphical representation of the median  
270 curve, the mode curve and the mean curve is helpful to summarize the differences  
271 between the different flood periods after the detection of the change point.

272           It should be borne in mind that numerous caveats apply to our findings. First, the  
273 proposed functional framework suffers from an edge effects issue and therefore is unable  
274 to identify possible changes near the beginning and the end of the data record.  
275 Nevertheless, this is a common issue for the traditional change point approaches. Further  
276 theoretical studies using generated (known) functional data sets may help to quantify this  
277 issue, as well to answer many other questions such as the determination of the minimum  
278 record length in order to detect a change. Secondly, the proposed approach does not allow  
279 detecting multiple change points simultaneously, and thus need to be iterated for each  
280 segment until no further change point is detected. Finally, as problems in hydrology often  
281 involve missing data, the proposed functional approach lacks the ability of handling  
282 missing data, and thus unable to take full advantage of the whole data record that may be  
283 available. For instance, a complete data records are available for Romaine river station  
284 from 1957 to 2012 as well for Moisie river station from 1966 to 2012, while, in contrast,  
285 our analysis was mainly limited to data records from 1961 to 2000 for Romaine river  
286 station and from 1968 to 1991 which are the longest periods for which there is no missing  
287 data.

288           In change point analysis, if a significant change is detected in hydrological  
289 characteristics, then it is important to try to understand the physical reason behind.  
290 Change in hydrological characteristics may be caused by climatic factor such as climate  
291 variability or climate change, but there may be many other possible explanations, such as  
292 anthropogenic change (urbanization, water abstraction etc.), natural catchment changes,  
293 and problem linked to data. The best way to improve understanding of change is rather to  
294 gather as much information as possible, using, e.g., information about change in the

295 catchment. In addition, related variables, like temperature and precipitation can help to  
296 determine whether changes in flow can be explained by climatic factors. Indeed,  
297 streamflow depends strongly on the spatial distribution of precipitation in a watershed,  
298 and on the interactions between temperature and precipitation which determines whether  
299 precipitation falls as rain or snow (Ben Alaya et al. 2014).

300 In a warming climate it is expected that the atmosphere's water holding capacity  
301 will increase with warming according to the Clausius-Clapeyron (C-C) equation (Collins  
302 et al. 2013), which may lead to more intense precipitation events that may directly affect  
303 streamflow and flood events behaviours. In addition, climate variability through oceanic  
304 and atmospheric oscillations on a large scale known as teleconnections, such as the North  
305 Atlantic Oscillation (NAO), El Nino-Southern Oscillation (ENSO) and Pacific Decadal  
306 Oscillation (PDO), influences the variability and trends in the climate system (Hurrell  
307 and Van Loon 1997, Rogers 1997) and thus may in turn affect characteristics of flood  
308 events.

309 Based on the obtained results, the frequency of flood events which occur later has  
310 decreased at both Romaine river and Moisie river stations while earlier floods  
311 characterized by low peaks and volumes became more frequent. Given the short record  
312 length of the data series used, attributing this change to corresponding underlying  
313 processes is challenging. Another challenge is that signals such as trends and shifts are  
314 superposed on variability arising from the memory within the hydrological system.

315 While the proposed approach is not able to distinguish between shifts and trends  
316 that may be present in the data, the results for the Romaine river station reflect hints

317 about the presence of a trend in functional data. Note, however, that the “trend”  
318 terminology in case of the sequence of functional curves is not the same used as in case  
319 of random variables where sample elements are points. The definition of a trend in case  
320 of the sequence of functional curves requires, first, to define an extended notion of order  
321 that tell us in which case a curve can be considered to be higher than another. Such a  
322 definition may not be, however, uniquely determined. To the best of our knowledge, a  
323 first attempt for functional trend analysis has been proposed by Fraiman et al. (2014).  
324 Nevertheless, we think that a more comprehensive way to handle this concern is to  
325 account for the notion of autocorrelation in the sequence of functional curves that has  
326 been proposed by Zhang et al. (2011).

327         Note that change point analyses are only descriptive. Hence, they cannot answer  
328 questions about how the hydrological system works in a non-stationary climate, and,  
329 therefore, cannot be used to predict future conditions. Indeed, seeking answers to those  
330 questions requires a hydrological modelling. Nevertheless, the proposed functional  
331 change point framework, as a mathematical descriptive tool, can play a very important  
332 role for scientific investigations. It can help to get first quantitative clues about what  
333 happened in the past. Unlike traditional change point approaches, the conclusions reached  
334 from the proposed framework are enhanced by providing reach mathematical pictures  
335 summarizing the mean, mode and median curves describing flood regimes. Those  
336 pictures are obtained within a mathematical framework that rigorously explains how they  
337 were obtained and how the conclusions were reached. Those steps can be easily applied  
338 to outputs of hydrological models, whether deterministic or stochastic, to rigorously  
339 check and test whether they reproduce similar pictures and same conclusions that have

340 been drawn from original data. As suggested in previous works, the interpretation of  
341 hydrological models, particularly in a non-stationary climate is always challenging  
342 (Montanari and Koutsoyiannis 2014, Serinaldi and Kilsby 2015, Serinaldi and Kilsby  
343 2018). On the other hand, by following the first clues given by a reach descriptive  
344 analysis, we may achieve a deep understanding about the complexity of the real  
345 mechanism. Our finding can serve as a starting point toward an effective calibration of  
346 hydrological models, or can merely be used for model testing.

347         As recommended by Koutsoyiannis and Montanari (2015), including additional  
348 information from prior physical knowledge about the physical process involved is  
349 essential to build a successful hydrological model that can be used to predict the future.  
350 In this respect, our approach opens the door to think on how to take full advantage of the  
351 functional framework from a modelling perspective. This can be achieved by casting the  
352 hydrological modelling in a functional regression framework by including major factors  
353 that influence flood events as covariates. The implementation of such approach, however,  
354 is not straightforward and is outside the scope of the current paper. Nevertheless, we  
355 think that, slowly, over the course of several steps, starting from functional descriptive  
356 tools, a pathway can be paved for development of sound functional regression models  
357 and perhaps eventual inclusion of additional knowledge from key factors involved in  
358 generating flood events.

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362        **5. Conclusions**

363            The purpose of the present paper is to propose a new context of the change point  
364 detection of flood hydrographs using functional data framework. A functional change  
365 point approach is presented and adapted to flood events. An application is performed for  
366 two hydrological stations in the province of Quebec, Canada. The presented functional  
367 approach is compared to a classical Bayesian univariate approach applied to the peak, the  
368 volume and the duration of flood events separately. Based on this comparison, it has been  
369 shown that the functional approach gives results that are consistent with the traditional  
370 univariate approach. The functional approach has the benefit that it provides a  
371 comprehensive way to handle the flood event as a curve within a defined statistical  
372 framework and thus an opportunity for a better exploitation of the information contained  
373 in a discharge time series.

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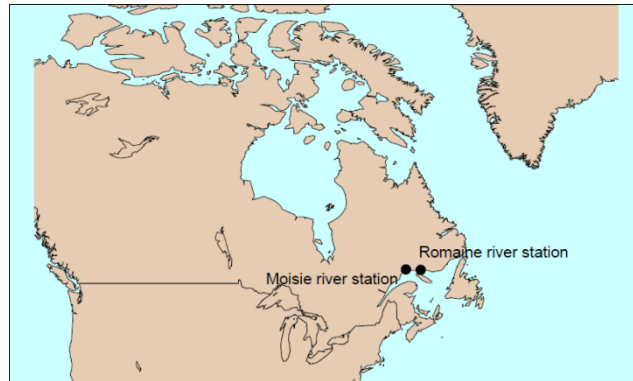
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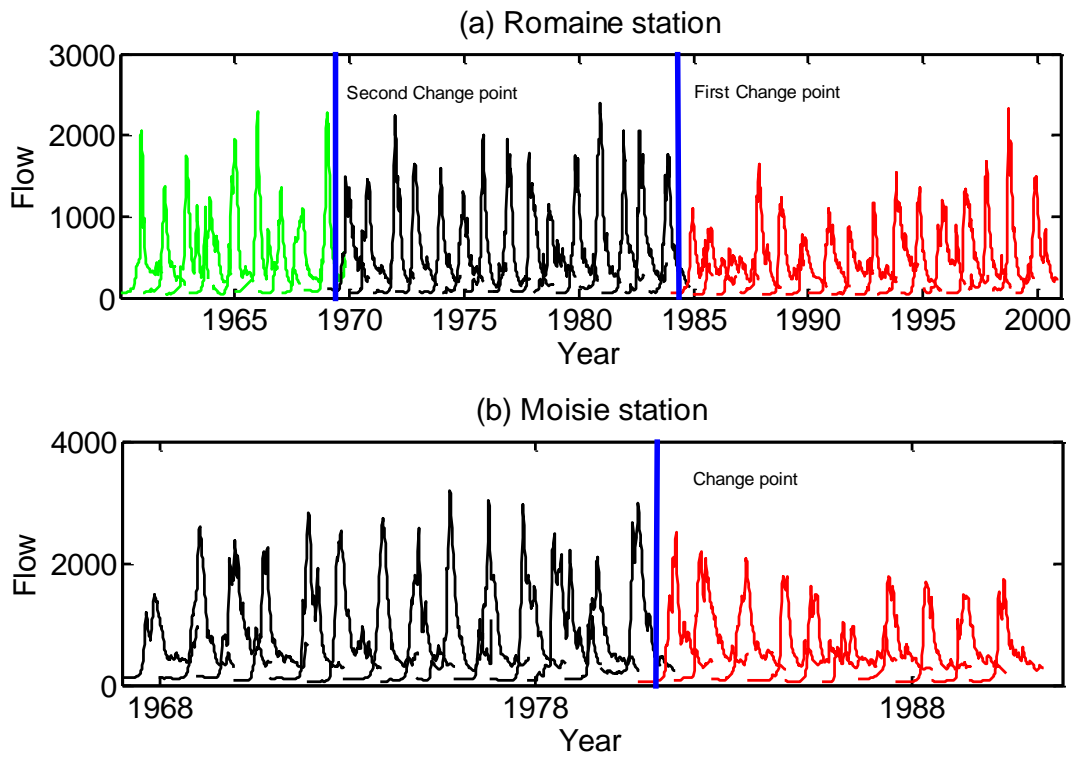
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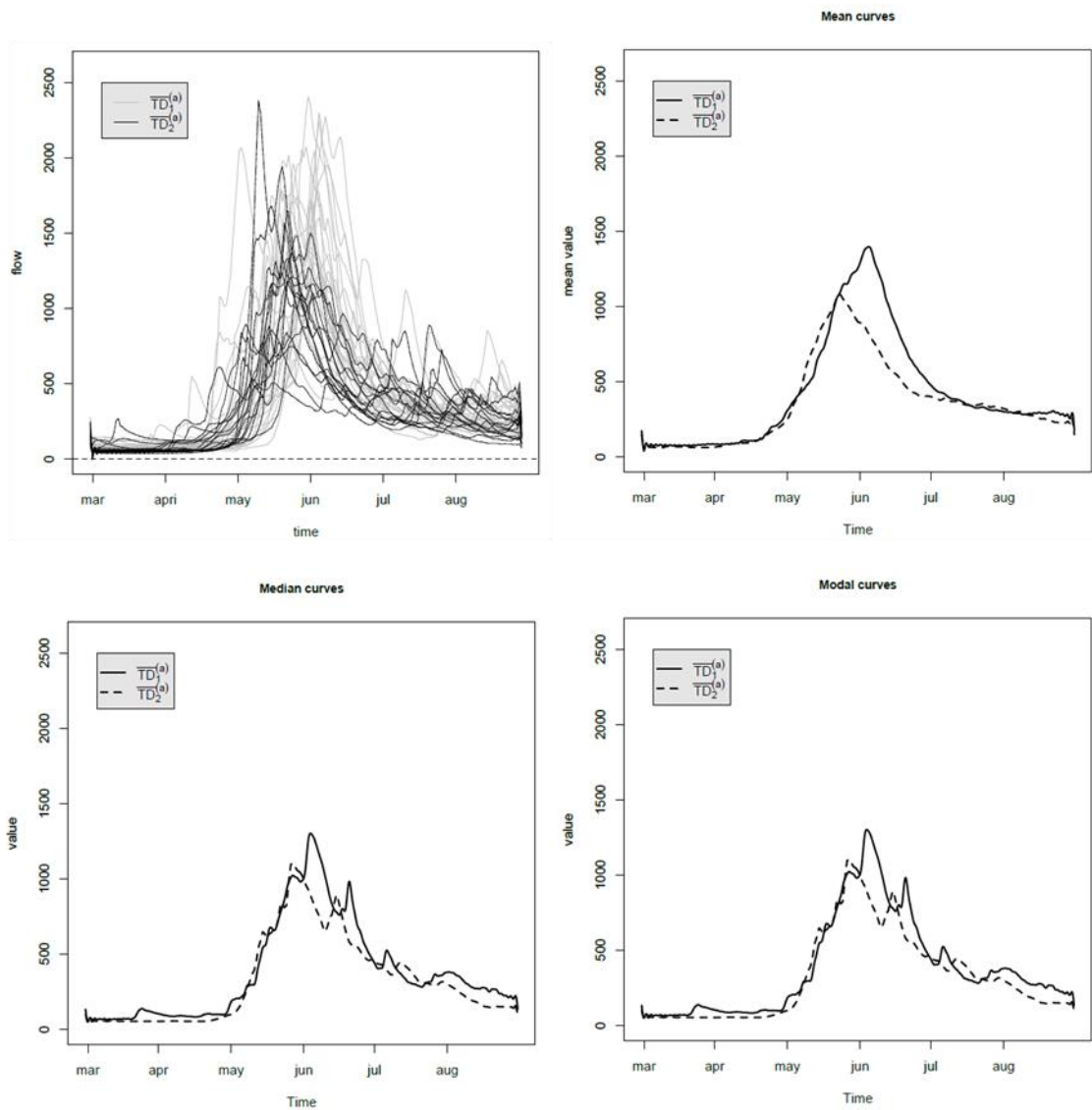


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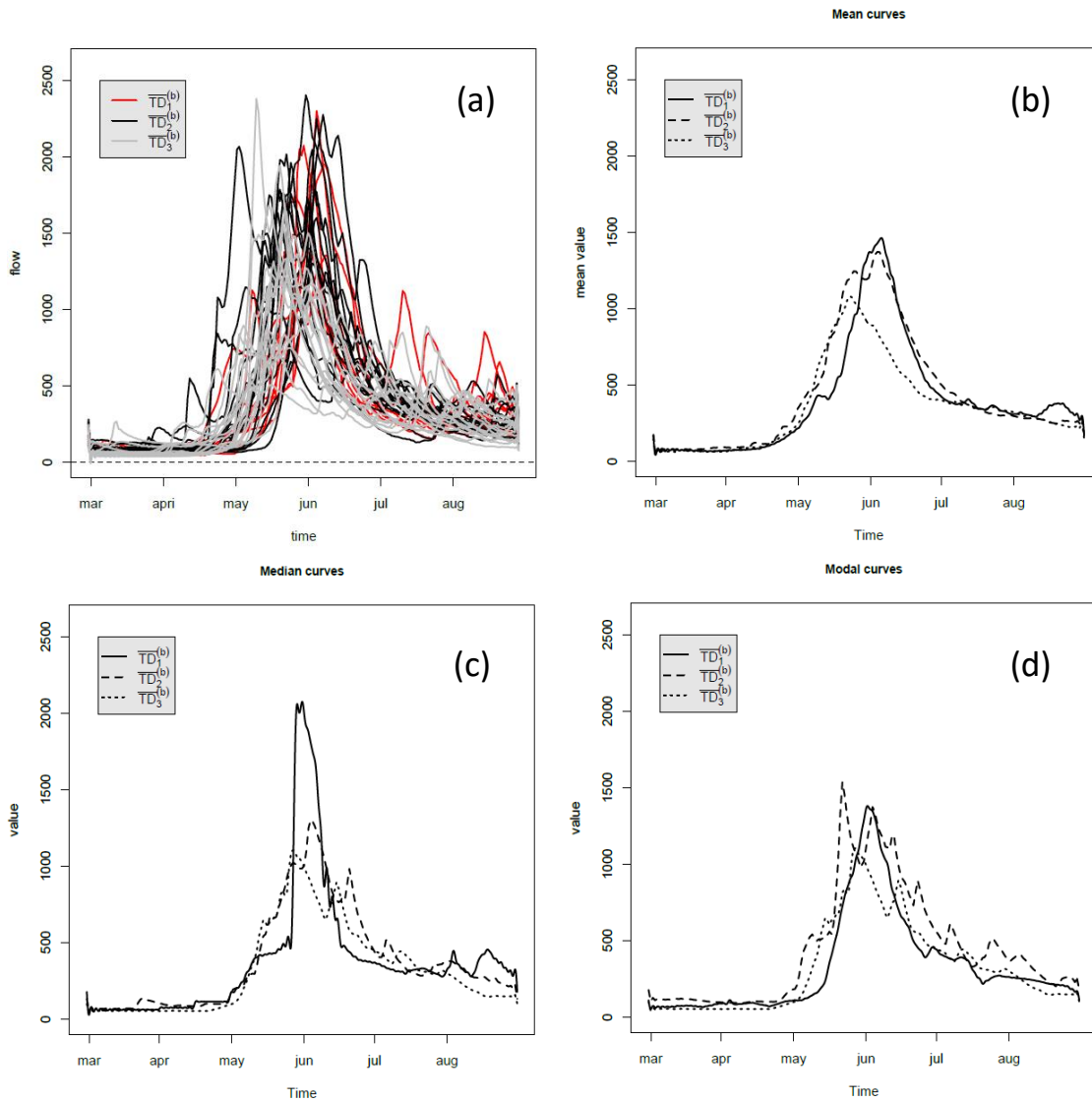
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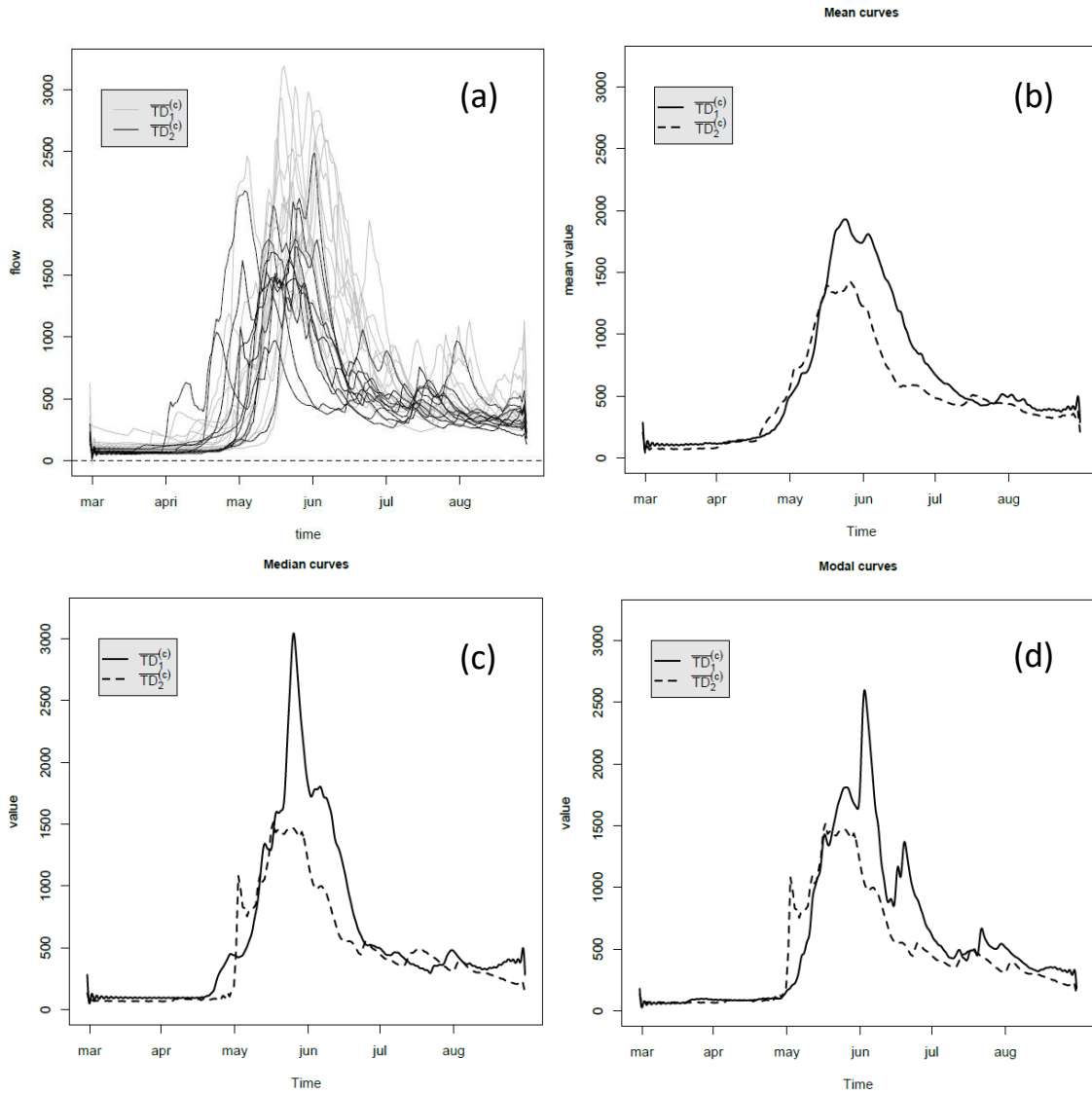
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