BIOECONOMICS

An essay on the classic papers of H. Scott Gordon, Milner B. Schaefer and Harold Hotelling

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DMS-519-IR SEPTEMBER 1989

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Invited paper in the session "Classics in Mathematical Biology" at the I.M.A. conference on The Mathematical Theory of the Dynamics of Biological Systems. Oxford, July 1989.

Of the three authors of the papers I have been asked to discuss, one (Milner B. Schaefer) was a biologist, one (H. Scott Gordon) an economist, while the third (Harold Hotelling) earned recognition both as a statistician and as an economist even though his initial training was in journalism and his two graduate degrees were in mathematics. This diversity of backgrounds reflects the eclectic nature of bioeconomics, and the subject continues to draw its practitioners from all of the above disciplines with the exception perhaps of journalism. As its name suggests bioeconomics deals with the economics of the utilization of biological resources; obviously then the theory must contain components of the disciplines of biology and economics. Also it contains a great deal of applied mathematics. Indeed it is only through the use of mathematical models that a rational discourse beyond the elementary level is possible on bioeconomic matters.

To the biologist the biology employed in bioeconomics will seem for the most part incredibly simplistic. This is largely due to the fact that the basic biological entity considered is usually a population differentiated if at all only by age, although often completely undifferentiated. It also reflects the fact that bioeconomics is in essence more a branch of economics than a branch of biology. However even though the models may be biologically very simple the mathematics involved in analyzing them can be very complicated indeed. As the subject develops, with attempts to incorporate greater detail and realism into its models, so the level of difficulty of the mathematics increases, often to the point where the only way forward seems to be through the use of numerical techniques.

Of the three papers I shall discuss two employ relatively sophisticated mathematical methods (calculus of variations and dynamical systems) while the third uses only elementary mathematics, but nonetheless establishes a result of fundamental economic importance. Broadly speaking the three papers discuss the following issues:

Hotelling. What constitutes the socially optimal exploitation of a resource over time? and in consequence what is rational conservation? How does the behaviour of the monopolist deviate from the socially optimal pattern, and what are the consequences for conservation? Similarly what are the consequences of free competition? Hotelling deals mainly with exhaustible resources, such as fossil fuels, although he points out that similar results should apply to the "mining" stage of virgin biological resources such as forests. Moreover, and more importantly from the bioeconomic point of view, the mathematical methodology developed by Hotelling can be extended to cover renewable biological resources.

Gordon. Why in most "mature" fisheries are fisherman poor, generating in total very little economic rent, even though "the fishery resources of the sea are the richest and most indestructible known to man"? Why is this so even in fisheries which have been regulated so as to produce something close to maximum sustainable yield? What constitutes an optimal level of exploitation from the economic point of view and why is this situation not brought about by laissez-faire policies as it apparently is in so many other areas of economic life?

Schaefer. What can be learned of importance to the fisheries manager from analyzing dynamical models of an exploited fish population and the exploiting fishery and how the two interact? How can such models be used operationally to determine the status of an exploited fishery, and to set management policy? What can such models reveal about the nature of the development of a fishery on a virgin stock; and can observed time series of fishery catches and efforts be explained by such models?

I shall attempt to highlight the results important to bioeconomics in the three papers by putting them all in the context of a single model of a fishery, whose essential elements are all found in the paper of Schaefer. Details can be found in the 1976 book of Colin Clark, Mathematical Bioeconomics (Chapt. 2), which I can say without hesitation is the classic book of bioeconomics.

Let $X(t)^1$ denote the biomass of a fish population at time t, and let $E(t)$ denote the total fishing effort at that time. Suppose that the dynamics of the population can be described by the differential equation

$$\frac{dX}{dt} = F(X) - h(E,X) \tag{1}$$

where F is a function representing the natural productivity of the population, and $h(E,X)$ is the production function for the fishery describing the rate of harvest as a function of the current biomass and fishing effort. Schaefer uses the functional forms $F(X) = rX(1-X/K)$, the Verhulst–Pearl logistic model familiar in population ecology, and $h(E,X) = qEX$ where q is a constant of proportionality now known as the catchability coefficient. With these functional forms the model is often referred as the Schaefer model, and it forms the basis of much theoretical and statistical work in bioeconomics.

Consider now the population in equilibrium with a constant rate of fishing effort E. For the Schaefer model the equilibrium biomass level is

$$\bar{X}(E) = \frac{K}{r}(r-qE) \tag{2}$$

and the corresponding equilibrium rate of catch is

$$\bar{Y}(E) = qE\bar{X}(E) = \frac{qK}{r} \cdot E(r-qE) \tag{3}$$

a quadratic in E. Suppose now that the unit price of fish is p and the unit cost of fishing effort is C. In equilibrium with fishing effort E, the total revenue generated per unit time is

$$TR = p\bar{Y}(E) = \frac{pqK}{r} E(r-qE)$$

while the total cost of fishing per unit time is

$$TC = CE$$

These are graphed in Figure 1. The principal result of Gordon's paper is that <u>in a fishery with open access, the total fishing effort will reach an equilibrium at the level E_0 at which total revenue equals total cost</u>. Thus no sustainable net economic rent will be generated in an open-access fishery. Gordon calls this situation the <u>bionomic equilibrium</u> and justifies it by observing that at effort levels lower than E_0, the average revenue net of costs is positive. Thus there will be a tendency for new fishermen to enter the fishery perhaps switching from less productive fishing grounds. Similarly if the total effort exceeds E_0, there will be some fishermen making a net loss since the average revenue is negative. These fishermen will tend to leave the fishery. Thus the total effort will either expand or contract until the equilibrium level E_0 is reached, at which point, on average, the revenue earned by an individual fisherman just covers his costs.

It is the open-access, common-property aspect of the fishery that causes fishermen to be poor. As Gordon notes, "everybody's property is nobody's property. Wealth that is free for all is valued by none because he who is foolhardy enough to wait for its proper use will only find that it has been taken by another."[2]

At the time Gordon wrote there had been a number of attempts to regulate fisheries so as to produce maximum sustainable yield (see Figure 1), by imposing closures once a specified catch quota had been realized. However Gordon points out that from an economic point of view this would not alleviate the dissipation of economic rent. A situation in which net profits are to be earned from an open-access fishery is inherently unstable from a bionomic point of view, since the presence of profits will attract new entrants to the fishery, which in turn will force management to impose more restrictions on fishing. Fishermen would then find themselves in a race to catch the available fish before their competitors do, and would thus be under great incentive to invest in larger more efficient boats and gear. Their unit costs would therefore increase since they would be idle for much of the year. This process would continue until a new bionomic equilibrium is reached, one in which the fishery has greater excess capacity than that which prevailed before regulation.[3] True the fish stock would now be in better shape, but apart from an option value, this would be of little benefit to society. As Gordon points out regulatory policies should be "evaluated in terms of human, not piscatorial objectives".

In order to achieve economically optimal exploitation (which Gordon suggests should be at the level E^* where marginal revenue equals marginal cost thereby maximizing total net revenue) some limitations on entry into the fishery are clearly required. Gordon suggests private ownership or unified control by a regulatory agency. Besides closures regulatory instruments at the hand of management include gear limitations, licences, taxes on effort or catch, transferable quotas etc. A considerable literature has developed in this area but I will defer discussion of it until the socially optimal exploitation in a dynamic framework has been discussed.

Gordon's paper is significant mainly because of its insight into the wasteful nature of open-access exploitation of common-property resources. This has now become a standard paradigm of economic thought (see also G. Hardin's article, 'The Tragedy of the Commons') and is invoked in discussions of issues as far removed from fisheries management as environmental pollution and urban congestion. However it has been slow to penetrate government fisheries departments which sometimes in the past have sought to "solve" the problem of poverty among fishing communities by offering subsidies to fishermen, rather than trying to limit the total effort in an economically efficient manner.

How would the level of effort approach the bioeconomic equilibrium level E_0 in an unregulated fishery? To answer this a dynamic model for the growth of the fishery is required. Schaefer proposed a model in which the new investment in the fishery is proportional to the return to be expected. This leads to the equation

$$(4) \qquad \frac{dE}{dt} = kE(X-\bar{X}_0)$$

for the growth of effort, where $\bar{X}_0$ is the bionomic equilibrium population level. Equations (1) and (4) form a dynamical system for the population and fleet size, of the type analyzed by Lotka [1923] and Volterra [1926]. In fact with the specific functional forms used by Schaefer it is a classic Lotka-Volterra predator-prey model with crowding of the prey species (predators = fishing fleet, prey = fish stock). Schaefer shows that the bionomic equilibrium forms either a stable node or a stable focus. In either case the catch and total effort at first increase. Subsequently the catch decreases converging eventually (possibly in an oscillatory fashion) to the bionomic equilibrium level where the net rent is zero. This "boom-and-bust" pattern of development has been observed in many fisheries,[4] and Schaefer presents data from two fisheries whose development appears to follow this pattern.

Besides developing a model which explained the growth and decline of a fishery (and incidentally introducing dynamical systems methodology into fisheries modelling[5]) Schaefer considers the question of how the current state of a fish stock could be determined using available data on annual total catch and total effort, and further how effort should be adjusted to attain maximum sustainable yield. (Note that typical of biologists at the time his objective was in terms of biomass yield, rather than economic or social benefit which he felt was "not susceptible to quantitative reasoning"). Under the production function assumption of Schaefer $(h = qEX)$, the catch per unit of effort (c.p.u.e.) is proportional to population size. Furthermore c.p.u.e. is an observable quantity and so can be used as a proxy for the unobservable population size. Thus trends in the time series of c.p.u.e.'s may reveal underlying trends in the population size etc.

The problem of analyzing catch and effort data to assess the status and potential yield of a fish stock is of considerable importance in fisheries management and has received much attention since Schaefer's paper. Pella and Tomlinson (1969), Schnute (1977), and Jensen (1984) considered refinements of Schaefer's method. Deriso (1980), using a relatively sophisticated discrete–time model for the population dynamics, which included age–structure and continuous recruitment obtained a predictive equation in terms of catches and efforts from which model parameters can be estimated using non–linear least squares. Schnute (1983) considered a class of models which included the Deriso and other models as special cases. Another approach to the problem has been to eschew the specification of underlying population and production models in favour of the more pragmatic procedure of developing a statistically simple model relating an output catch series to an input effort series. Gulland (1961) and Fox (1970) used simple regression methods to obtain "black-box" models of this type, while Mendelssohn (1980) used the more sophisticated statistical techniques of Box-Jenkins transfer function models. Reed (1986a) suggested that randomness in the catch series may be due more to the stochastic nature of the process of catching fish than to any stochasticity in the population dynamics, and developed a method of determining maximum likelihood estimates of biological and production parameters based on a stochastic catch model. An attempt to incorporate randomness from both sources led Ludwig, Walters and Cooke (1988) to consider two approximate methods of maximizing the appropriate likelihood function. At root the problem is very difficult since one is dealing with one observable input series (effort) and two output series (catch and population size) both non-linear and with randomness present, but with only one series (catch) observable. Integrating out the unobservable size variable from the likelihood function presents a formidable problem. The paper of Ludwig, Walters, and Cooke represents the state of the art of the statistical problem of analyzing catch-effort data - addressed by Schaefer in 1954.

Let us return now to the question of what constitutes the socially optimal exploitation of a fishery described in the Schaefer model above. Gordon suggested the maximization of equilibrium economic yield[6]. A difficulty with this objective however is that it ignores the dynamic aspect of the problem - i.e. how the resource should be utilized over time. This is fundamental question present in all areas of resource management. It is particularly obvious in the case of exhaustible resources, and was first addressed in that field by Hotelling in 1931. Hotelling suggested that the social objective should be the maximization of the total present value of the flow of social surplus from the resource, and used the methods of the calculus of variations to find the optimal solution.

In the Schaefer fishery model the total present value is

$$(5) \qquad PV = \int_0^\infty e^{-\delta t}[pqX(t) - C]E(t)dt$$

where δ is the discount rate used to compare benefits earned at different times. The socially-optimal pattern of exploitation $\tilde{E}(t)$ is that which maximizes (5) subject to the constraints $E(t) \geq 0$ and the dynamic equation (1).

Like the Hotelling problem this problem can be solved using the methods of the calculus of variations (see Clark, 1976, 2.5), but it is more conveniently solved using its modern successor, optimal control theory. The application of optimal control theory to dynamic problems in economics has proved to be spectacularly successful. With so many problems in resource management relating to resource use over time such methods now play a central part in the theory of bioeconomics and resource management. Indeed virtually the whole of the modern theory of exhaustible resources takes the Hotelling paper as its starting point (see Devarajan & Fisher, 1981 for a review and assessment of Hotelling's contribution to this field), and if there can be said to be a mathematical theme to Clark's seminal 1976 book on bioeconomics it is the Pontryagin maximum principle of optimal control theory. I shall discuss some other applications of optimal control theory in bioeconomics later but first let us examine the fishery optimization problem above.

Clark (1976) shows that the optimal solution involves driving the population as rapidly as possible to the level $\tilde{X}$ which satisfies

$$(6) \qquad F'(\tilde{X}) - \frac{c'(\tilde{X})F(\tilde{X})}{p - c(\tilde{X})} = \delta$$

(where $c(X)$ is the unit cost of harvesting when the population is at level X, and for the Schaefer production function is equal to C/qX) and subsequently keeping the population at that level by applying effort $F(\tilde{X})/q\tilde{X}$. The optimal biomass level $\tilde{X}$ lies between the bionomic equilibrium level $\bar{X}_0$ and that level X^*, advocated by Gordon which maximizes equilibrium economic yield. As the discount rate increases so the optimal biomass level decreases, with the bionomic equilibrium level $\bar{X}_0$ corresponding to an infinite discount rate and the equilibrium-yield-maximizing level X^* corresponding to a zero discount rate. Thus the maximization of equilibrium yield can be seen as equivalent to the extreme position of assigning equal weight to future revenues and current revenues, while the bionomic equilibrium would be the optimal solution if future revenues were given no value – an attitude which, as Gordon observed, would logically be adopted by fishermen in an open-access fishery.

Another area of economics which has flourished with the use of modern optimal control methods is capital theory, and a certain formal identification of a resource stock as a capital asset can be made via the resulting mathematics. (The concept of a resource stock as a capital asset however goes back much further – indeed one of the standard examples of capital theory is the problem of when to cut down a growing stand of trees. (See Samuelson, 1976)). Clark (1976, Chapter 3) discusses the capital-theoretic aspects of the fishery problem. One interesting result is that if the maximum growth rate that the population can achieve is smaller than that of capital invested elsewhere (i.e. smaller than the interest or discount rate) then it may be optimal from the point of view of maximizing present value <u>for the sole owner</u> to harvest the population to extinction (see also Clark 1985, p. 113). Thus private ownership of a fishery resource, as suggested by Gordon, will not necessarily guarantee conservation.

We saw earlier how Schaefer derived from his model the pattern of fleet development and catch in an open-access fishery. What constitutes the optimal pattern of fleet development? This has been studied as an optimal control problem with two state variables (population size and fleet size) and two controls (fishing effort and investment) by Clark, Clarke and Munro (1979). The solution is quite complicated but it can involve the creation in the early stages of exploitation, of fleet capacity in excess of that which is optimal in the long-run (see Clark 1985, Section 3.3).

We have seen how Gordon concluded that regulation of a fishery by imposing closures would not alleviate the dissipation of economic rent in an open-access environment. The same is true of vessel and gear restrictions. These measures are sometimes called "regulation by maximization of inefficiency". What form of regulation would overcome the economic distortions brought about by open-access? This question is discussed in detail in Clark (1985, Chapter 4). Clark's conclusions (similar in spirit to those of Gordon) are that only through some method of attaching property rights, such as transferable quotas, or a tax on catch, can the deleterious effects of competition for a common-property resource be overcome.

As mentioned earlier there is now a very considerable literature in bioeconomics using variational techniques which derive ultimately from Hotelling's introduction of the calculus of variations to solve problems of resource allocation over time. (It may be of some interest to those who have suffered at the hands of reviewers and editors to know that this classic paper was rejected by the Economic Journal because, according to Hotelling's later account, its mathematics was too difficult for that journal's readership.) Other areas in fisheries modelling which have employed optimal control techniques include: the Schaefer model with demand elasticity and with non-linear production functions

(Clark, 1976, Chapter 5) and with the possibility of random catastrophic collapse (Reed, 1988), cohort models (Clark, 1976, Chapter 6), multi-species models (Mesterton-Gibbons, 1987, 1988) and differential game theory models for the management of trans-boundary fisheries (Munro, 1979) and for the enforcement of fishery regulations (Sutinen and Andersen, 1985). In forestry some applications include forest thinning (Clark & de Pree, 1979) and expenditure and investment on fire protection (Reed, 1987). Strictly outside of bioeconomics, but still within the realm of resource management, optimal control techniques have been applied to problems of soil erosion, pest management and groundwater conservation.

Much of the most interesting work in bioeconomics over the past decade or so has involved the use of stochastic models. Through their use phenomena such as environmental fluctuation in population dynamics, uncertainty as to the current state of a resource stock, uncertainty in future resource prices and the essential stochasticity of search and capture, have been examined. The techniques of dynamic programming have proved more useful than "variational" methods (i.e. the stochastic maximum principle) for stochastic optimization problems both from the analytic and computational points of view. The excellent book of Marc Mangel (1985) deals largely with continuous-time problems and makes extensive use of stochastic dynamic programming. Applications in bioeconomics along with exhaustible resources and pest management are discussed and useful references are given. The review of Andersen and Sutinen surveys both continuous-time and discrete-time stochastic models in bioeconomics to 1984. Also Chapter 6 of Clark (1985) deals with stochastic models in fisheries management, stressing the essential difference between stochastic fluctuation and uncertainty.

Readers of this essay might very well get the impression that bioeconomics is concerned solely with fisheries. While undoubtedly much of the most interesting work has taken place in that area, there are nonetheless other areas of importance.

Apart from the management of wildlife resources (which share much in common with fisheries) the other main area of bioeconomics is forestry. Open access and/or the inability to effectively enforce property rights is a serious cause of over-exploitation of forests in many third-world countries, fully bearing out the conclusions of Gordon on open-access common-property resources.

In more developed countries the common-property phenomenon is less important. Nonetheless there are many other externalities of considerable effect associated with forest exploitation which make the formulation of realistic tractable models difficult. These include the destruction of habitat for wildlife, with possibly a resulting reduction in genetic diversity, the destruction of the natural regulation of water flows with flooding and silting of rivers as a consequence, changes in local climate and, perhaps, changes in global climate through aggravation of the greenhouse effect. Regrettably, though understandably, forest management models tend to focus on timber supply with other objectives entering, if at all, as constraints on harvest policies.

Because forest resources are utilized over time the results and methods developed by Hotelling are relevant to forest bioeconomics. However because trees grow slowly, the inclusion of age structure is important in forest management models. This leads in continuous time to difficult optimal control problems for partial differential equation systems; in discrete time the methods of linear and non-linear programming can be employed. Not surprisingly the discrete-time approach has, to date, been more successful.

Another approach to forest modelling is at the level of a single stand (see Reed, 1986b for a survey of forest-level and stand-level optimization models). Stand-level models are more appropriate for land used for intensive forestry and are frequently employed to assess the benefits of silvicultural practices such as thinning, fertilization and weeding. Such models all derive ultimately from the classic 1849 paper of Martin Faustmann (see Gane, 1968 for an English translation) which deals with the question of the valuation of forest land and of immature stands by computing the total present value of the sequence of revenues earned from successive forest rotations. In evaluating an infinite sequence of revenues by computing total present value the Faustmann approach is similar to that of Hotelling, although of course the determination of the optimal rotation for a forest is much simpler (requiring only ordinary calculus) than the problem of determining the optimal rate of extraction of an exhaustible resource, which requires the calculus of variations.

To close I will attempt to summarize the importance of the three papers under discussion. Gordon's work is important primarily for its identification of the wasteful nature of open-access exploitation of common-property resources and the recognition that only through attaching property rights in some way can this trap be avoided. Schaefer's main contribution lies in the introduction of a dynamical model for the behaviour of a fish population and the exploiting fishing fleet which could be used to explain the development of a fishery and furthermore could be used operationally by fisheries managers. The model has subsequently formed the basis of much theoretical and statistical work in bioeconomics. While Schaefer introduced dynamical aspects he did not consider the important question of resource use over time. <u>It is the dimension of time that lies at the heart of questions of conservation and consumption</u>. Hotelling had recognized this and addressed and solved the problem for exhaustible resources using variational methods twenty years before Gordon and Schaefer wrote. It is Hotelling's formulation of the problem of intertemporal resource utilization and the introduction of the calculus of variations to solve it that makes his paper so relevant to bioeconomics. A solution, comparable to Hotelling's, for the renewable resource problem using the Schaefer model, was not obtained until the early seventies, twenty years after Schaefer's paper and forty years after Hotelling's. This was accomplished by Clark in 1973[7], and in many ways Clark's work represents a synthesis and completion of the work begun by Gordon and Schaefer. Since the publication of Clark's 1976 book the field of bioeconomics has flourished with much of the subsequent developments depending in one way or another on the pioneering work of Hotelling, Gordon and Schaefer.

Acknowledgements.

I wish to thank Professor Ray Beverton and Dr. Harry R. Clarke for many useful comments and suggestions. In particular I am indebted to Professor Beverton for bringing to my attention the pioneering work of Michael Graham. Also I would like to point out that the papers cited in this essay represent only a small sample of the corpus of published work in bioeconomics. Much excellent work has been omitted for reasons of space and style alone.

Footnotes

[1] Unfortunately notation differs in the three papers. The notation employed here differs in some respects from each of them, but has become somewhat standard since the publication of Clark's book.

[2] Michael Graham in his book The Fish Gate (Faber and Faber, London) first published in 1943 had also identified an equilibrium situation which he referred to as "The melancholy stability of depression (in which) average profit = nil". He stated what he called The Great Law of Fishing – that "fisheries that are unlimited become unprofitable". Graham attributed increases in effort mainly to attempts by fishermen already in the fishery to maintain their level of profits in the face of the inevitable decline in the stock size (and in consequence in catch per unit of effort) that accompanies the activity of fishing. This "Urge to Expand" which is forced upon the fishermen by the falling stock in the sea can take many forms e.g. buying more nets, building a bigger boat or investing in new more efficient fishing gear. Graham pointed out the irony associated with the introduction of a new technology which in the short run may solve the problems faced by an individual fisherman by increasing his fishing power, but in the long run, when all fishermen have been forced to adopt it, causes them all to be worse off than before its introduction. The fishermen are thus victims of what a later author, Fred Hirsch, (Social Limits to Growth, Harvard University Press, 1976) would call "the tyranny of small decisions".

[3] In contrast, Graham, writing a decade earlier had lauded the introduction of catch quotas in the Pacific Halibut fishery, predicting that "limiting the effort

would restore profit to a fishery". While he had correctly identified the process of increased fishing power in boats in an unregulated fishery, he failed to see that a similar phenomenon would occur in a fishery regulated by closures alone. Gordon with the advantage of hindsight recognized this and specifically pointed out the error in Graham's conclusions. It is perhaps surprising in view of the fact that Gordon was aware of The Fish Gate, that he should make no mention of the ideas expressed therein by Graham as the Great Law of Fishing since this was very similar to the notion of bionomic equilibrium discussed by Gordon.

[4] Graham in The Fish Gate describes a number of fisheries which have followed this pattern. Furthermore he offers convincing verbal arguments to explain the phenomenon.

[5] A claim for this honour could be made for Volterra whose 1926 predator-prey model was developed to explain periodic oscillation in the composition of fish catches in the Adriatic Sea.

[6] This is the sole-owner objective, but since no effect of supply on price is assumed here (unlike in the Hotelling paper) the socially optimal objective coincides with that of the sole owner.

[7] A partial solution was obtained by Crutchfield and Zellner in 1962. See Clark (1976) for references.

References

Andersen, P. and J.A. Sutinen, 1984. Stochastic bioeconomics: A review of basic methods and results. Marine Res. Econ. 1: 117–136.

Clark, C.W., 1976. Mathematical Bioeconomics: The Optimal Management of Renewable Resources. New York: Wiley–Interscience.

Clark, C.W., 1985. Bioeconomic Modelling and Fisheries Management. New York: Wiley–Interscience.

Clark, C.W., F.H. Clarke and G.R. Munro, 1979. The optimal exploitation of renewable resource stocks: problems of irreversible investment. Econometrica 47: 25–49.

Clark, D.W. and J.D. De Pree, 1979. A simple linear model for the optimal exploitation of renewable resources. App. Math. and Optim. 5: 181–196.

Deriso, R.B., 1980. Harvesting strategies and parameter estimation for an age–structured model. Can. J. Fish. Aquat. Sci. 37: 268–282.

Fox, W.W., 1970. An exponential yield model for optimizing exploited fish populations. Trans. Am. Fish. Soc. 99: 80–88.

Gane, M. (ed.), 1968. Martin Faustmann and the Evolution of Discounted Cash Flow. Commonwealth Forestry Institute, University of Oxford.

Gulland, J.A., 1961. Fishing and the stocks of fish at Iceland. U.K. Minist. Agric. Fish. Food Fish Invest. (Ser. 2) 23(4).

Hardin, G., 1968. The tragedy of the commons. Science 162: 1243–1247.

Jensen, A.L., 1984. Logistic surplus production model with explicit terms for growth, mortality and recruitment. Trans. Am. Fish. Soc. 113: 617–626.

Lotka, A.J., 1923. Contribution to quantitative parasitology. J. Wash. Acad. Sci. 13: 152–158.

Ludwig, D., C.J. Walters and J. Cooke, 1988. Comparison of two models and two estimation methods for catch and effort data. Nat. Res. Model. 2: 457–498.

Mangel, M., 1985. Decision and Control in Uncertain Resource Systems. New York: Academic Press.

Mendelssohn, R., 1980. Using Box–Jenkins models to forecast fishery dynamics: Identification, estimation and checking. Fish. Bull. 78: 887–896.

Mesterton–Gibbons, M., 1987. On the optimal policy for combined harvesting of independent species. Nat. Res. Model. 2: 109–134.

Mesterton–Gibbons, M., 1980. On the optimal policy for combining harvesting of predator and prey. Nat. Res. Model. 3: 63–90.

Munro, G.R., 1979. The optimal management of transboundary renewable resources. Can. J. Econ. 12: 355-376.

Pella, J.J. and P.K. Tomlinson, 1969. A generalized stock production model. Inter-American Tropical Tuna Comm. Bull. 13: 419-496.

Reed, W.J., 1986a. Analyzing catch-effort data allowing for randomness in the catching process. Can. J. Fish. Aquat. Sci. 43: 174-186.

Reed, W.J., 1986b. Optimal harvesting models in forest management – a survey. Nat. Res. Model. 1: 55-79.

Reed, W.J., 1987. Protecting a forest against fire: Optimal protection patterns and harvest policies. Nat. Res. Model. 2: 23-53.

Reed, W.J., 1988. Optimal harvesting of a fishery subject to random catastrophic collapse. IMA J. Math. Appl. Med. & Biol. 5: 215-235.

Samuelson, P., 1976. Economics of forestry in an evolving society. Econ. Inquiry. 14: 466-492.

Schnute, J., 1977. Improved estimates from the Schaefer production model: theoretical considerations. J. Fish. Res. Board Can. 34: 483-603.

Schnute, J., 1985. A general theory for the analysis of catch and effort data. Can. J. Fish. Aquat. Sci. 42: 414-429.

Sutinen, J.G., and P. Andersen, 1985. The economics of fisheries law enforcement. Land Econ., 387-397.

Volterra, V., 1926. Variazioni e fluttuazioni del numero d'individui in specie animali conviventi. Mem. R. Acad. Naz. dei Lincei., Ser. VI, Vol. 2.

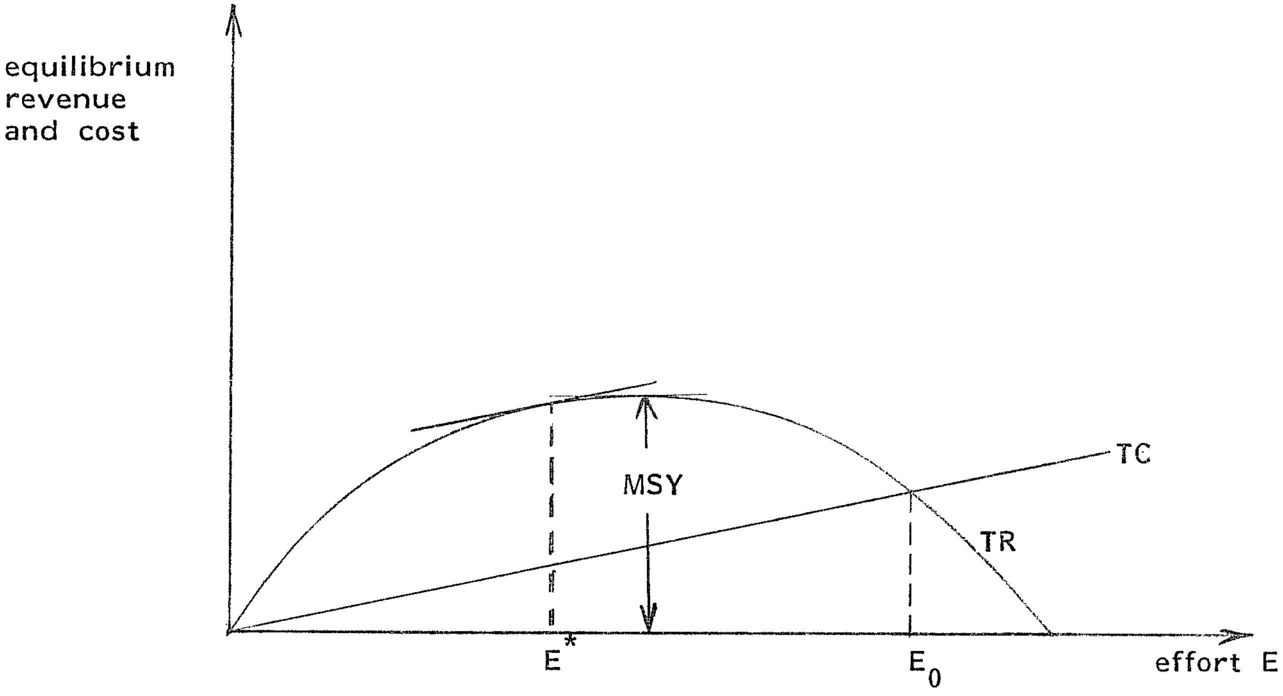

Fig. 1. Total revenue and total cost in equilibrium as a function of effort. The bionomic equilibrium is at level E_0. Equilibrium economic yield is maximized at effort level E^* at which marginal cost equals marginal revenue. Maximum sustainable yield (MSY) is also shown. Note that the level of effort which maximizes equilibrium yield is less than that which produces MSY.