NOTES ON K–THEORY OF MULTIPLIER ALGEBRAS AND CORONA ALGEBRAS

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Abstract

We give an answer to the question when the unitary group of the corona algebra of a simple AF-algebra is connected. We also compute the K-groups of the multiplier algebras and corona algebras of certain simple C^*-algebras. For example, if A_θ is an irrational rotation C^*-algebra and A is a non-stable hereditary C^*-subalgebra of $A_\theta \otimes \mathbf{K}$, then we find that $K_1(M(A)) = K_1(M(A)/A) = \{0\}$ and $K_0(M(A)) = \mathbf{R}$.

Key words: K-theory, multiplier algebras, infinitesmal elements, corona algebras

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0.Introduction

In [Ell3,2.9], George Elliott showed that unitary groups of corona algebras $M(A)/A$ of finite matroid algebras A are connected. It is also known that for infinite matroid algebras A, $K_1(M(A)/A) \cong K_0(A)$. One of the question leads to this note is why the unitary group of corona algebras of finite matroid algebras behaved so differently from that of infinite matroid algebras. If A is a stable C^*-algbera, it is easy to see that

$$K_1(M(A)) = K_0(M(A) = \{0\},$$

where $M(A)$ is the multiplier algebra of A. From six-term exact sequence in K-theory

$$\begin{tikzcd}
K_0(A) \arrow[r] & K_0(M(A)) \arrow[r] & K_0(M(A)/A) \arrow[d] \\
K_1(M(A)/A) \arrow[u, "\gamma"'] & K_1(M(A)) \arrow[l] & K_1(A) \arrow[l]
\end{tikzcd}$$

One concludes that $K_1(M(A)/A) \cong K_0(A)$. So in particular, this is true for separable simple AF-algebra such that all traces are infinite. But is it true, as in the case of matroid algebras, that if A is a separable simple AF-algebra such that all traces on A are finite then the unitary group for its corona algebra is connected? Recent work on C^*-algebra extension also shows that it is of importance to know the homotopy type of the unitary groups of corona algebras of simple AF-algebras, or even for other simple C^*-algebras of real rank zero. It is the purpose of this note to expose the facts behind these above mentioned phenomena. We will work in a more general class of C^*-algebras than AF-algebras. However, we will consider only those separable simple C^*-algebras with real rank zero, stable rank one and $K_0(A)$ weakly unperforated. We refer the reader to [BP], [Rf] and [BH] for the definitions of real rank, stable rank and weakly unperforated. Every AF-algebra A has real rank zero, stable rank one and weakly unperforated $K_0(A)$. The inductive limit C^*-algebras of real rank zero classified by Elliott (see [Ell3]) have stable rank one and have weakly unperforated K-group when they are simple.

We will use the notation $M(A)$ for the multiplier algebra of A (see [P, 3.12]). If A is a unital C^*-algebra, we denote by $U(A)$ and $U_0(A)$ the unitary group of A and the connected component of $U(A)$ containing the identity of A, respectively.

1. Quasitraces

Let A be a C^*-algebra and $P(A)$ be the Pedersen Ideal. Suppose that τ is a function from $M_\infty(P(A))$, the matrices over $P(A)$, to $\mathbf{C}$ which is linear on commutative subalgebras and satisfies

$$0 \le \tau(x^*x) = \tau(xx^*) \text{ for } x \in M_\infty(P(A)).$$

Then τ is a quasitrace.

Suppose that τ is a lower semicontinuous quasitrace on A, for every $x \in M(A)_+$ define

$$\tau(x) = sup\{\tau(a) : 0 \le a \le x, a \in A\}.$$

Then we have the following:

(1) Let $\{e_\alpha\}$ be an approximate identity for A. Then

$$\tau(x) = lim_\alpha \tau(x^{1/2}e_\alpha x^{1/2}).$$

In fact, for any $a \in A_+$ $a \le x$,

$$\tau(x^{1/2}e_\alpha x^{1/2}) = \tau(e_\alpha^{1/2}xe_\alpha^{1/2}) \ge \tau(e_\alpha^{1/2}ae_\alpha^{1/2}) = \tau(a^{1/2}e_\alpha a^{1/2}).$$

Since $\|a^{1/2}e_\alpha a^{1/2} - a\| \to 0$,

$$\tau(a) \le lim_\alpha \tau(x^{1/2}e_\alpha x^{1/2}).$$

(2) For any $x \in M(A)$, $\tau(x^*x) = \tau(xx^*)$.
In fact,

$$\tau(x^*e_\alpha x) = \tau(e_\alpha^{1/2}xx^*e_\alpha^{1/2}) = \tau((xx^*)^{1/2}e_\alpha(xx^*)^{1/2}).$$

So $\tau(x^*x) = \tau(xx^*)$.

(3) If $x, y \in M(A)_+$, $xy = yx = 0$, then

$$\tau(x+y) = \tau(x) + \tau(y).$$

This follows immediately from the definition.

It should be noted that it is not known in general every quasitrace is a trace though recently U. Haagerup shows that for exact C^*-algebras, every quasitrace is in fact a trace (see [H]).

3. Scales Let A be a separable simple C^*-algebras with real rank zero and stable rank one. Fix a nonzero projection $e \in A$. Let Δ be the set of positive homomorphisms τ from $K_0(A)$ into $\mathbf{R}$ such that $\tau(e) = 1$ and $QT(A)$ be the set of lower semicontinuous quasitraces τ such that $\tau(e) = 1$. It follows from [BH,III] that there is an (affine) homeomorphism $\chi : QT(A) \rightarrow \Delta$. We will identify these two compact sets. In fact Δ is a Choquet simplex ([BH,II.4.4]). We will use the notation $Aff(\Delta)$ for set of all real affine continuous functions defined on Δ. The map χ gives a homomorphism $\theta : K_0(A) \rightarrow Aff(\Delta)$ (see [BH,III. 1.3]). If $K_0(A)$ is weakly unperforated, then $K_0(A)$ has the strict ordering induced from θ. Moreover, since A has stable rank one, for any two projections in $M_\infty(A)$ if $\tau(p) < \tau(q)$ for all $\tau \in QT(A)$, then $q \succeq p$, i.e., there is a partial isometry $v \in M_\infty(A)$ such that $v^*v = p, vv^* \leq q$. Furthermore, $\theta(K_0(A))$ is dense in $Aff(\Delta)$. We say that $x \in K_0(A)$ is *infinitesmal* if $-\epsilon e \leq x \leq \epsilon e$ for all $0 < \epsilon \in \mathbf{Q}$(see [Eff, 4]). If $K_0(A)$ is weakly unperforated, by the proof of [Eff,4.2],

$$ker\theta = \{x \in \Delta : x \text{ is infinitesmal}\}.$$

We set

$$S = \{\tau \in \Delta : \hat{1}(\tau) < \infty\},$$

$$J = \{f \in \theta(K_0(A)) : f(\tau) = 0, \text{ if } \tau \in S\}$$

and

$$S_J = \{\tau \in \Delta : f(\tau) = 0, \text{ if } f \in J\}.$$

Moreover, $\theta(K_0(A))$ is a countable dense subgroup of $Aff(\Delta)$. Then S_J is closed and $S \subset S_J$. (If Δ is a Baue simplex, then S_J is the closure of S.) One can easily check that S_J is a face of Δ.

Now we define the following: A is said to have *finite scale* if $S = \Delta$ and *almost finite scale* if $S_J = \Delta$. It is clear that the definition of finite scale does not depend on the choices of the nonzero projection e. We will see later that the notion of almost finite scale does not depend on the choices of e either. The scales of a simple C^*-algebras are useful information for studying both corona and multipier algebras (see [Lin1] and [Lin2]).

If K is a compact convex space, we will use the notation $Aff^s(K)$ for the differences $f - g$, where both f and g are lower semicontinuous affine functions defined on K with possible value infinity.

3. Lemma *Let A be a separable simple C^*-algebra with real rank zero and stable rank one. If furthermore, A has the following comparison property: for any two projections $e, f \in A$ if $\tau(e) > \tau(f)$ for all $\tau \in QT(A)$, then $e \sim f$, then two projections p, $q \in M(A) \setminus A$ are equivalent if and only if*

$$\tau(p) = \tau(q) \text{ for all } \tau \in T(A).$$

Proof : If there is a partial isometry $v \in M(A)$ such that

$$vv^* = p, \text{ and } v^*v = q,$$

then for all $\tau \in T(A)$, $\tau(p) = \tau(q)$.

Now suppose that

$$\tau(p) = \tau(q)$$

for all $\tau \in T(A)$. It follows from [Zh] that there are two approximate identities $\{e_n\}$ and $\{\epsilon_n\}$ for A consisting of projections such that

$$p = \sum_{n=1}^{\infty} a_n \text{ and } q = \sum_{n=1}^{\infty} b_n,$$

where a_n and b_n are projections in A and $a_n \le e_n$, $b_n \le \epsilon_n$, $n = 1, 2, \ldots$. Since A is simple and separable, pAp and qAq are stably isomorphic (see [Bn]).

Therefore $K_0(pAp) = K_0(qAq)$. Moreover, two C^*-subalgebras pAp and qAq have the same scale. So there is a projection $a \leq p$ such that

$$\tau(a) > \tau(b_1) \text{ for all } \tau \in T(A).$$

By our assumption, there is a projection $b_1' \in pAp$ such that $b_1' \sim b_1$. There is an integer n_1 such that

$$\|p_{n_1} b_1' p_{n_1} - b_1'\| < 1/2,$$

where $p_k = \sum_{n=1}^{k} a_n$. It follows from [Eff,A8] that there is a projection $b_1'' \leq p_{n_1}$ such that

$$b_1'' \sim b_1' \sim b_1.$$

Hence there is a partial isometry $u_1 \in A$ such that

$$u_1^* u_1 = b_1, \; u_1 u_1^* = b_1'' \leq p_{n_1}.$$

Without loss of generality, we may assume that $p_{n_1} - b_1'' \neq 0$. Similar to the above, there is $u_2 \in A$ such that

$$u_2 u_2^* = p_{n_1} - b_1'', \; u_2^* u_2 \leq b_{m_2}$$

for some $m_2 > 1$.

By continuing this way, we construct a sequence of partial isometries $\{u_k\}$ in A satisfing the following:

$$\sum_{i=1}^{2k-1} u_i^* u_i = \sum_{i=1}^{2k-1} b_i, \; \sum_{i=1}^{2k-1} u_i u_i^* \leq p_{n_{2k-1}}$$

and

$$\sum_{i=1}^{2k} u_i^* u_i \leq \sum_{i=1}^{2k} b_i, \; \sum_{i=1}^{2k} u_i u_i^* = p_{n_{2k}}.$$

Set $u = \sum_{i=1}^{\infty} u_i$. For any n,

$$e_n \sum_{i=1}^{k} u_i = \sum_{i=1}^{n} u_i$$

and

$$\sum_{i=1}^{k} u_i \epsilon_n = \sum_{i=1}^{n} u_i$$

if $k \geq 2n$. So $\sum_{i=1}^{k} u_i$ converges to u in the strict topology. Therefore $u \in M(A)$. It is clear that

$$u^*u = p \text{ and } uu^* = q.$$

Q.E.D.

4. Remark It follows from [BH] that if A is a simple C^*-algebra with real rank zero, stable rank one and weakly unperforated $K_0(A)$ then A satisfies the comparison condition in the lemma.

5.Theorem *Let A be a separable simple C^*-algebra with real rank zero, stable rank one and weakly unperforated $K_0(A)$. Then*
(1) $K_1(M(A)) = \{0\}$;
(2) $K_1(M(A)/A) = \theta^{-1}(J)$;
(3) $K_0(M(A)) \cong Aff^s(S_J)$.

Proof :

(1) This is contained implicitely in [Lin6]. It follows from [Lin6] that $cer(A) \leq 1+\varepsilon$. Then it follows from [Lin4] that the unitary group of multiplier algebra of every separable simple C^*-algebra with real rank zero and stable rank one is connected. In particular, the unitary group of $M_n(M(A)) \cong M(M_n(A))$ is connected for all positive integer n. Therefore $K_1(M(A)) = \{0\}$.

(2) Since A has real rank zero, by the proof of [Ell1,2.6], every unitary in A is lifted to a partial isometry in $M(A)$. Let $u \in M(A)/A$ be a unitary and $v \in M(A)$ is a partial isometry such that $\pi(v) = u$, where π is the quotient map. The index map $\gamma : K_1(M(A)/A) \to K_0(A)$ can be realised as

$$\gamma([u]) = [1 - v^*v] - [1 - vv^*].$$

Let $e = 1 - v^*v,\ g = 1 - vv^*$. Then for any $\tau \in S$,

$$\tau(e) = \tau(g).$$

So $\gamma([u]) \in \theta^{-1}(J)$.

On the other hand, if $f \in J$, there is $[x] \in K_0(A)$ such that $\hat{x} = f$. So there are projections $e_1, e_2 \in M_n(A)$ (for some integer n) such that

$$\tau(e_1) - \tau(e_2) = f(\tau) \ \ \tau \in \Delta.$$

Therefore

$$\tau(e_1 - e_2) = 0 \ \text{ for all } \tau \in S.$$

Now let $p = 1 - e_1$ and $q = 1 - e_2$. Then for all $\tau \in \Delta$,

$$\tau(p) = \tau(q).$$

Therefore, by Lemma 3, there is a partial isometry $v \in M(M_n(A)) \cong M_n(M(A))$ such that

$$v^*v = p, \ \ vv^* = q.$$

Since $e_i \in A$, $i = 1, 2$, $\pi(v)$ is a unitary in $M_n(M(A))$ and

$$\gamma(\pi(v)) = e_1 - e_2.$$

Morover, $\theta(e_1 - e_2) \subset J$. Since $K_1(M(A)) = 0$, this shows that

$$K_1(M(A)/A) = \theta^{-1}(J).$$

(3) Let p be a projection in $M_n(M(A)) \cong M(M_n(A))$. There are projections $p_k \in M_n(A)$ such that $p_k \leq p_{k+1}$, p_k converges to p in the strict topology (of $M(M_n(A))$). Hence we have

$$\tau(p) = lim_{k \to \infty} \tau(p_k).$$

So $\hat{p}$ is a lower semicontinuous affine function on Δ (with possible value infinity). Moreover, for any lower semicontinuous function f on Δ (with possible value infnity), since $\theta(K_0(A))$ is dense in $Aff(\Delta)$, there is a projection $p \in M(M_n(A))$ such that

$$\hat{p}(\tau) = f(\tau) \ \ \tau \in \Delta.$$

Now suppose that $\tau(p) = \tau(q)$, where $p, q \in M(M_n(A))$ and $\tau \in S_J$. Let $e' \in A$ be any nonzero projection. There is $f' \in Aff(\Delta)$ such that

$$\tau(e') = f'(\tau) \text{ for } \tau \in \Delta.$$

Define

$$f(\tau) = \begin{cases} f'(\tau) & \lambda\text{if } \tau \in S_J \\ \infty & \text{if } \tau \notin S_J \end{cases}$$

Since S_J is a face, one can easily check that so defined f is a lower semicontinuous function on Δ. As above, there is a projection $g \in M(A)$ such that, for any $\tau \in \Delta$,

$$\tau(p \oplus g) = \tau(q \oplus g).$$

By lemma 4,

$$p \oplus g \sim q \oplus g.$$

So $[p] = [q]$ in $K_0(M(A)$. Summarizing what we have established, we conclude that

$$K_0(M(A)) \cong Aff^s(S_J).$$

Q.E.D.

6. Corollary *Let A be a simple C^*-algebra with real rank zero, stable rank one and weakly unperforated $K_0(A)$. Then*

$$U(M(A)/A)/U_0(M(A)/A) \cong K_1(M(A)/A) \cong \theta^{-1}(J).$$

Proof: It follows from [Lin6] that $M(A)/A$ has real rank zero. Then, by [Lin8,], the map from $U(M(A)/A)/U_0(M(A)/A)$ into $K_1(M(A)/A)$ is injective. Now let $u \in M_n(M(A)/A)$ for some n. As in the proof of 5, there is a partial isometry $v \in M(M_n(A))$ such that the image of v is u. Let $p = v^*v$, $q = vv^*$. Then, by Lemma 4,

$$\tau(p) = \tau(q) \text{ for } \tau \in \Delta.$$

It follows from [Zh, 3.2] that we may assume that $p = \sum_{i=1}^{n} p_i \otimes e_{ii}$, where $\{e_{ij}\}$ is a matrix unit for $M_n(\mathbf{C})$. We may further assume that $p_i \neq 0$. Since $\tau(q) > \tau(p_1)$ for all $\tau \in \Delta$ and A is simple, there is $q_1 \in M_n(M(A))$ such that $q_1 \leq q$ and $\tau(q_1) = \tau(p_1)$ for all $\tau \in \Delta$. So

$$\tau(q - q_1) = \tau(\sum_{i=2}^{n} p_i \otimes e_{ii}).$$

By repeating the above argument, we conclude that there are mutually orthognal projections $q_1, q_2, ..., q_n \in M_n(M(A))$ such that $q_i \leq q$ and

$$\tau(q_i) = \tau(p_i) \text{ for all } \tau \in \Delta$$

and $q = \sum_{i=1}^{n} q_i$. Furthermore, we may assume that $q = \sum_{i=1}^{n} q_i' \otimes e_{ii}$, where $q_i' \sim q_i, i = 1, 2, ..., n$. So here are partial isometries $v_i \in M(A)$ such that

$$v_i^* v_i = q_i' \text{ and } v_i v_i^* = p_i, \ \ i = 1, 2, ..., n.$$

Since both $1 - p$ and $1 - q$ are in $M_n(A)$, $(1 - p_i) \otimes e_{ii}$ and $(1 - q_i) \otimes e_{ii}$ are in A. So $\pi(v_i)$ are unitaries in $M(A)/A$. By the proof of Theorem 5,

$$[\pi(v_i) \oplus \pi(v_2) \oplus ... \oplus \pi(v_n)] = [u] \text{ in } K_1(M(A)/A).$$

We also know that

$$[\pi(v_1) \cdot \pi(v_2) \cdots \pi(v_n)] = [\pi(v_1) \oplus \pi(v_2) \oplus ... \oplus \pi(v_n)].$$

But $\pi(v_1) \cdot \pi(v_2) \cdots \pi(v_n) \in M(A)/A$. This proves the map from $U(M(A)/A)/U_o(M(A)/A)$ to $K_1(M(A)/A)$ is surjective.
Q.E.D.

7. Remark Many simple C^*-algebras satisfy the conditions in Theorem 5. Every simple AF-algebra satisfies the conditions in the theorem. The C^*-algebras of real rank zero classified by George Elliott (see [Ell2] and [Ell3]) also satisfy the conditions in theorem. The C^*-algebras of real rank zero considered in [G] satisfy the conditions in the theorem too. Other examples may also be found in [BJR].

The reader may also notice the absence of $K_0(M(A)/A$. What we have from the theorem is that $K_0(M(A)/A)$ is determined by the following exact sequence:

$$0 \to Aff^s(S_J)/\theta(K_0(A)) \to K_0(M(A)/A) \to K_1(A) \to 0.$$

In particular, if $K_1(A) = \{0\}$, then

$$K_0(M(A)/A) \cong Aff^s(S_J)/\theta(K_o(A)).$$

8. Corollary *Let A be a separable simple C^*-algebras with real rank zero, stable rank one and weakly unperforated $K_0(A)$. Suppose that A has almost finite scale. Then*

$$K_1(M(A)/A) \cong ker\theta = \text{infinitesmal elements.}$$

The converse is also true.

9. Remark It follows from the above corollary that the notion of almost finite scale does not depend on the choices of the element e.

10. Corollary *Let A be a separable simple C^*-algebra with real rank zero, stable rank one and weakly unperforated $K_0(A)$. Then the unitary group of $M(A)/A$ is connected (or equivalently, $K_1(M(A)/A) = \{0\}$) if and only if A has almost finite scale and $K_0(A)$ has no infinitesmal elements.*

11. Remark From Corollary 10, we know when the unitary group of the corona algebra of a simple AF-algebra is connected. So there are simple C^*-algebras A with finite sacles, or bounded scales (equivalently, algebraically simple), or even continuous scales (equivalently, the corona algebras are simple) have nontrivial $K_1(M(A)/A)$. On the other hand, the unitary group of $M(A)/A$ can be connected even if A has no finite scale (the scale must be almost finite).

12. Examples

(1) Let B be the Bunce-Deddens algebra. Then B is simple and has real rank zero, stable rank one and unperforated $K_0(A)$. Moreover, $K_1(B) = \mathbf{Z}$.

Let A be a hereditary C^*-subalgebra of $B \otimes \mathbf{K}$, where $\mathbf{K}$ is the C^*-algebra of compact operators on the separable infinite dimensional Hilbert space. It follows from Theorem 5 that

$$K_0(M(A)) = \begin{cases} \mathbf{R} & \text{if } A \text{ is not stable} \\ \{0\} & \text{if } A \text{ is stable} \end{cases}$$

$$K_1(M(A)) = \{0\}$$

$$K_1(M(A)/A) = \begin{cases} \{0\} & \text{if } A \text{ is not stable} \\ K_0(B) & \text{if } A \text{ is stable} \end{cases}$$

(2) Let A_θ be an irrational rotation algebra. Then it is now known that A_θ has real rank zero and stable rank one (see [BJR], also [EE]). Moreover, $K_0(A) \cong \mathbf{Z} + \mathbf{Z}\theta$ and $K_1(A_\theta) \cong \mathbf{Z} + \mathbf{Z}\theta$. So $K_0(A_\theta)$ is unperforated. Let A be a (nonunital) hereditary C^*-subalgebra of $A_\theta \otimes \mathbf{K}$. Then it follows from Theorem 5 that

$$K_0(M(A)) = \begin{cases} \mathbf{R} & \text{if } A \text{ is not stable} \\ \{0\} & \text{if } A \text{ is stable} \end{cases}$$

$$K_1(M(A)) = \{0\}$$

and

$$K_1(M(A)/A) = \begin{cases} \{0\} & \text{if } A \text{ is not stable} \\ \mathbf{Z} + \mathbf{Z}\theta & \text{if } A \text{ is stable} \end{cases}$$

(3) Let $G = \mathbf{Q}^2$ with strict ordering from the first coordinate and A be a simple AF-algebra with $K_0(A) = G$. Then

$$K_0(M(A)) = \begin{cases} \mathbf{R} & \text{if } A \text{ is not stable} \\ \{0\} & \text{if } A \text{ is stable} \end{cases}$$

$$K_1(M(A)) = \{0\},$$

$$K_0(M(A)/A) = \begin{cases} \mathbf{R}/\mathbf{Q} & \text{if } A \text{ is not stable} \\ \{0\} & \text{if } A \text{ is stable} \end{cases}$$

and

$$K_1(M(A)/A) = \begin{cases} \mathbf{Q} & \text{if } A \text{ is not stable} \\ \mathbf{Q}^2 & \text{if } A \text{ is stable} \end{cases}$$

(4) Let $G = \mathbf{Q} \oplus \mathbf{Z}/n\mathbf{Z}$ with strict ordering from the first coordinate. By [Ell], there is a simple C^*-algebras of inductive limit of type I C^*-algebras which has real rank zero, stable rank one, $K_0(A) = G$ and trivial K_1-group. Then

$$K_0(M(A)) = \begin{cases} \mathbf{R} & \text{if } A \text{ is not stable} \\ \{0\} & \text{if } A \text{ is stable} \end{cases}$$

$$K_1(M(A)) = \{0\},$$

$$K_0(M(A)/A) = \begin{cases} \mathbf{R}/\mathbf{Q} & \text{if } A \text{ is not stable} \\ \{0\} & \text{if } A \text{ is stable} \end{cases}$$

$$K_1(M(A)/A) = \begin{cases} \mathbf{Z}/n\mathbf{Z} & \text{if } A \text{ is not stable} \\ G & \text{if } A \text{ is stable} \end{cases}$$

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