

THE ROLE OF EXPECTATIONS AND RISK AVERSION IN  
THE TERM STRUCTURE OF INTEREST RATES  
- EVIDENCE FROM THE TREASURY BILL MARKET

by

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### ABSTRACT

Market rates of interest for various types of debt instruments may differ for a number of reasons. One of the major causes of differences is the term to maturity, that is, the length of time over which a debt instrument is scheduled to be redeemed by the issuer. The relationship between interest rate (yield) and term to maturity on debt instruments that have otherwise identical characteristics is termed the 'term structure of interest rates'. Various theories have been advanced to account for this relationship, namely, the pure expectations theory, the market segmentation theory, the risk-premium theory and the preferred habitat theory.

This thesis is devoted to the study of the role and relative importance of expectations and risk aversion in the determination of the term structure. Focusing on the very short end of the Canadian bond market, the Treasury bill market, the empirical findings provide strong evidence in support of the importance of interest-rate expectations in the term structure of interest rates. In particular, results

of different tests confirm that expectations are shaped by regressive forces, that is, interest rates are expected to regress toward a long-run normal level based on past experience. The empirical findings also indicate that the market does not price debts of different maturities in such a way that the forward rates implied in the empirical term structure are equivalent to the expected future rates. Instead, forward rates are found to be upward biased estimates of expected future rates, exceeding them by a positive risk premium. Additional tests on the risk premium reveal that it is inversely related to the current interest rate but directly related to the interest rate of the previous period. However, there is a lack of sufficient evidence that it increases with the term to maturity. While it is difficult to quantify the amount of influence of market segmentation on the term structure based on the empirical results, its impact on the term structure does not appear to be substantial.

The implications of these findings to policy-makers in the pursuit of maintaining a certain degree of control over the term structure is that it is possible for the government to shift and to alter the shape of a yield curve by open market operations. However, in view of the general conclusion that interest-rate expectations are the dominant and a

more important factor than both risk aversion and market segmentation in determining the term structure, it is easier for the monetary authorities to shift than to alter the shape of a yield curve appreciably, and the effects of monetary operations will be transmitted through the entire maturity continuum without significant lags.

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## Chapter I

### INTRODUCTION TO THE TERM STRUCTURE OF INTEREST RATES AND AN OVERVIEW OF THE MAJOR THEORIES

#### 1.1 Introduction

Economists often speak of 'the interest rate' as if there is only one interest rate prevailing in the market at any given time. In fact, a wide spectrum of interest rates is observed in the financial markets, which makes it very difficult to select a 'representative' rate of interest.

Market rates of interest for various types of debt instruments may differ because of a number of reasons. The major cause of differences is the credit risk of the instruments, that is, the risk of default of the promised interest and principal payments. The observable differences in interest rates may also be due to differences in the degree of marketability of the instruments, in the cost of administering the instruments, in the provision of special tax treatment, whether the instruments are convertible into common stocks, and so forth.

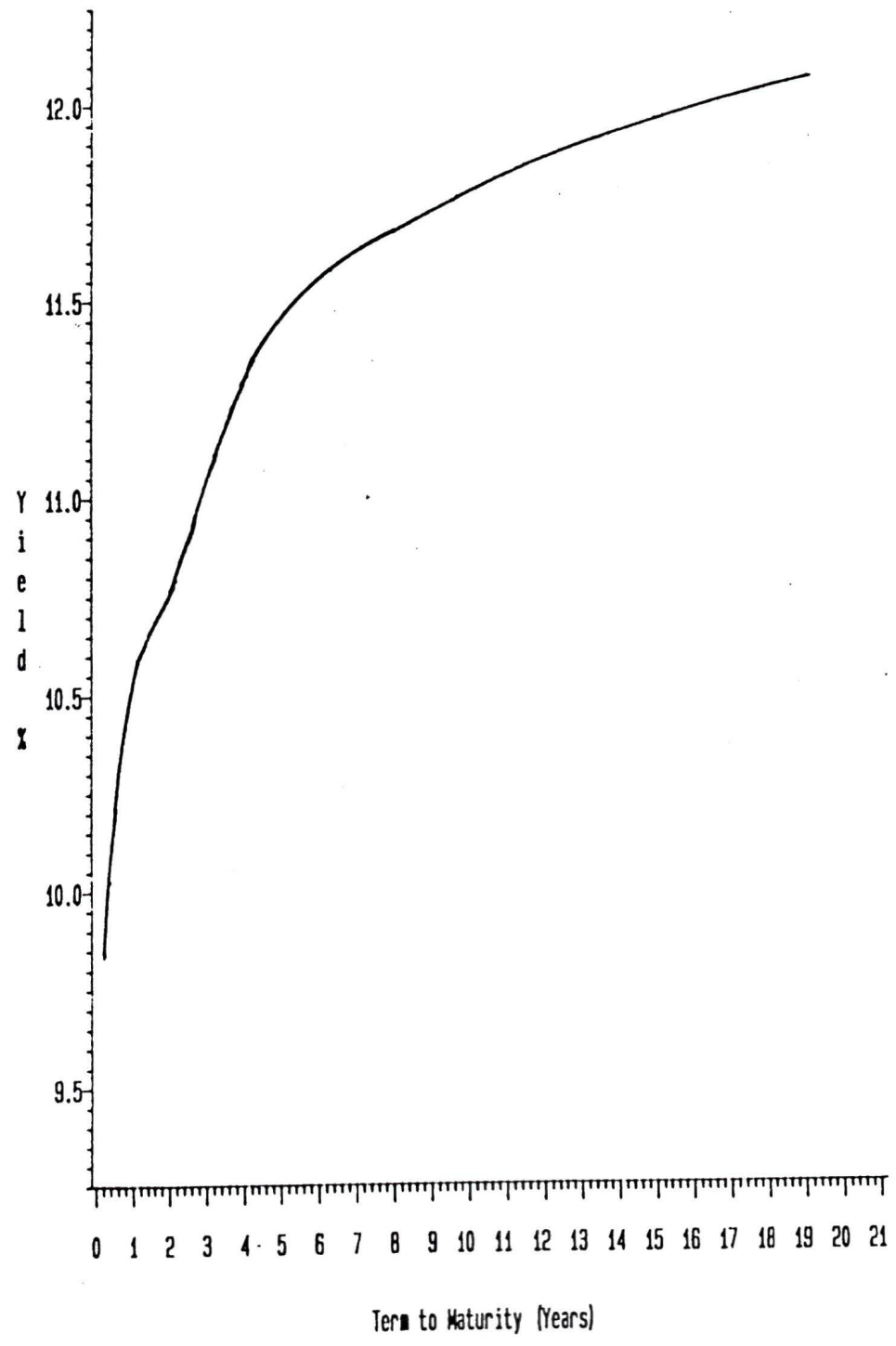
Another important determinant of the observed interest rate differentials is the term to maturity, that is, the length of time until the principal amount of the instrument is due. The relationship between the interest rate, that is, the yield, and the term to maturity for instruments that are homogeneous with respect to all other characteristics is called the 'term structure' or 'maturity structure' of interest rates. This relationship is best illustrated by a yield curve which records the yields calculated from the market prices and the maturity terms of a security at a particular time. Maturity is plotted on the horizontal axis and yield to maturity on the vertical axis. An example is given in Figure 1 where the yield curve for Government of Canada bonds at the end of 1984 is given.

An understanding of the determinants of the term structure of interest rates is essential to government officials in the conduct of monetary policy, to bond-portfolio managers responsible for the selection of an appropriate maturity composition, and to bond issuers concerned with the selection of an optimal financing decision.

The most common way to analyze the term structure is to consider an  $n$ -period loan as a one-period loan successively renewed for  $n$  periods. That is, any  $n$ -period loan is

# FIGURE 1

Yield Curve for Government of Canada Bonds at the End of 1984



theoretically equivalent to a spot one-period loan, plus (n-1) forward one-period loans upon the reinvestment of the funds. Hence, all long-term spot rates can be expressed in terms of current one-period spot rate and (n-1) forward one-period rates, as in the following equation.

$$(1.1) \quad 1+R_{n,t} = [(1+r_{1,t})(1+{}_{t+1}r_{1,t}) \dots (1+{}_{t+n-1}r_{1,t})]^{1/n}$$

where  $R_{n,t}$  is the yield for an n-period loan at time t,  $r_{1,t}$  is the current one-period rate at time t, and  ${}_{t+i}r_{1,t}$  is the forward one-period rate for time t+i implicit in the term structure at time t.

Under the condition of perfect foresight, all future rates are known with certainty. This, together with rational behavior, efficient markets and no transaction costs, implies that the implicit forward rates in the above equation can be replaced by future rates which are known to prevail over the (n-1) periods. Arbitrage would ensure that the yields from investing funds for a given period of time were equal regardless of the terms to maturity of the bonds purchased. If this were not the case, arbitrage activities would bid up the prices of the bonds which offered a higher return, thus lowering the rate of return. At the same time,

the prices of the bonds which offered a lower return were bid down, thus increasing the rate of return. In this way, the rates of return on bonds of different maturities were equalized. Unfortunately, perfect foresight does not exist in the real world. Hence, future rates are not known with certainty and yields on bonds of different maturities may differ.

Various theories, with the first expressed by Irving Fisher (1930), have been developed to account for the term structure of interest rates. Any theory that attempts to do this should be able to explain the shape of the yield curve, the tendency for all rates to move together, and the fact that long rates fluctuate less widely than short rates.

This thesis is devoted to the testing of the major alternative term structure theories (to be discussed in Sections 1.2.1 to 1.2.4) using Canadian data. As we shall see, the major differences between these theories lie in the behavioral assumptions of the market participants. Thus, these theories should not be considered as incompatible with each other and their validity is essentially an empirical question. This also implies that any test of the term structure is, to a certain extent, a 'nested' test which has different implications to different theories.

The structure of this thesis is outlined as follows. The remaining sections of this chapter will give an overview of the alternative term structure theories, namely, the pure expectations theory, the market segmentation theory, the risk-premium theory and the preferred habitat theory. Emphasis will be placed on the key assumptions associated with the theories and their implications on the yield differentials between securities of different maturities. A theoretical account for the shape of the yield curve as provided by each theory is also included in this chapter. Chapter 2 provides a survey of the past empirical work designed to test the hypotheses derived from the theories. Chapter 3 states the hypotheses to be tested, and describes the data and testing procedures of the empirical study to be undertaken in this thesis. The empirical results are presented and analyzed in Chapter 4. The policy implications of the findings are also discussed in this chapter.

## 1.2 Main Alternative Theories

Four theories have been advanced to explain the term structure of interest rates, namely, the pure expectations theory, the market segmentation theory, the risk-premium theory and the preferred habitat theory. The following discussion will start from the pure expectations theory where

the term structure is explained entirely by interest-rate expectations. Using this theory as a building block, we then consider the other alternative theories.

### 1.2.1 Pure (Unbiased) Expectations Theory

The pure expectations theory was first expressed by Irving Fisher (1930) and further developed by Frederick Lutz (1940). In its basic form, it claims that the implicit forward rates in equation (1.1) are essentially determined by market expectations.<sup>1</sup> After identifying forward rates with expected rates, the theory further argues that the long-term rate of interest is an unbiased average of the current short-term rate and future short-term rates expected to rule over the holding period. Accordingly, equation (1.1) can be rewritten as

$$\underline{1+R_{n,t}} = [(1+r_{1,t})(1+t+1r^e_{1,t}) \dots (1+t+n-1r^e_{1,t})]^{1/n}$$

<sup>1</sup> As stated by R.S. Masera in The Term Structure of Interest Rates - An Expectations Model Tested on Postwar Italian Data (Oxford: Clarendon Press, 1972), page 12

If ... forward rates have no significant relation to expected rates, the above relationships [term structure equations] do not carry functionality and should be considered as truisms, identical equations ... However, if forward rates are identified with expected rates, the above formulae immediately provide the explanation of the term structure: the differences between spot yields are functions of expected rates.

where  ${}_{t+i}r_{1,t}^e$  is the expected future one-period rate at time  $t$  to prevail at time  $t+i$ .

The rationale for this theory is based on arbitrage activities in which market participants exhaust all the opportunities for making profit. If the forward rates implicit in the current empirical term structure are higher than the short-term rates expected to prevail in the future, arbitrageurs will take advantage of this opportunity and make profit by borrowing short and lending long. That is, they will borrow short-term funds at the current spot rate and lend the funds in the form of long-term bonds to earn a higher return. On the other hand, when the forward rates are lower than the expected future interest rates, arbitrageurs will make a profit by borrowing long and lending short. These activities will continue until the forward rates implied in the empirical term structure are equal to the expected future short-term rates. As a result, implicit forward rates are unbiased estimates of expected future short-term rates. The yield differentials between securities of different maturities are determined exclusively by the market's expectations of future interest rates.

Furthermore, securities of different maturities are perfect substitutes. The expected holding-period return of an investment is the same regardless of the length of the investment period and the combination of securities it is composed of.

#### **1.2.1.1 Key Assumptions of the Pure Expectations Theory**

The above theory is based upon several simplifying assumptions. The theory first assumes that the behaviour of all market participants is motivated by profit maximization. Under this assumption, lenders of funds act according to their expectations of future short-term rates to maximize their holding-period yields from bonds of different maturities. Similarly, borrowers of funds, based on their expectations, try to minimize the interest cost of issuing bonds.

The second assumption asserts that market participants formulate a uniform set of expectations of future short-term rates. In addition, it is assumed in the third assumption that these expectations (forecasts) are accurate so that expected holding-period yields must be equal to the holding-period yields subsequently realized for all maturities.

In the fourth assumption, the theory hypothesizes the market as one in which speculative activities are predomi-

nant. Even though some of the participants are risk-averse hedgers,<sup>2</sup> the market is dominated by speculators who are neutral to risk. This ensures that there is a sufficient amount of arbitrage activities that bring forward rates equal to expected future short-term rates, and that speculators are willing to bear extra risk involved in longer term bonds without demanding a premium. Driven by the profit motive, speculators will exploit all profit opportunities in the market until they are exhausted. Hence, bonds of different maturities are perfect substitutes for one another, that is, expected rates of return for all maturities are the same.

The fifth assumption, a zero coupon rate, abstracts from complications which will arise from the reinvestment of the coupon payments.<sup>3</sup> Finally, the assumption of no transaction costs further simplifies the picture.

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<sup>2</sup> The risk that we are concerned with here is the risk associated with the dispersion of the probability distribution of the rate of return. The greater the dispersion, the more risky is a financial asset, and vice versa. The conventional measure of risk is the standard deviation of the expected rate of return. For more details, see Van Horne, Financial Market Rates and Flows (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1978), page 56.

<sup>3</sup> More discussion on this assumption is found in Section 1.2.1.2.

### 1.2.1.2 General Relaxation of the Assumptions

It can easily be recognized that some of the assumptions of the pure expectations theory mentioned in the previous section are unrealistic when compared with real world situations. In this section, these assumptions are gradually relaxed. It will be seen that all of the assumptions can virtually be relaxed without infringing on the general validity of the theory except for one assumption. It is the assumption that the market is dominated by speculators who are neutral to risk. This assumption is crucial for the theory to hold.

(i) Perfect forecasting. The relaxation of this assumption brings the theory closer to reality. This assumption, being a sufficient condition for the theory to hold, is by no means necessary. If imperfect forecasting is allowed, ex-ante forecasts on yields to maturity may turn out to be different from ex-post yields. However, the theory will hold as long as market participants act according to their expectations to maximize their holding-period yields. The term structure is still determined by expectations of future spot rates, as hypothesized by the theory.

(ii) Profit maximization. This assumption is closely related to that of perfect forecasting in which there exists

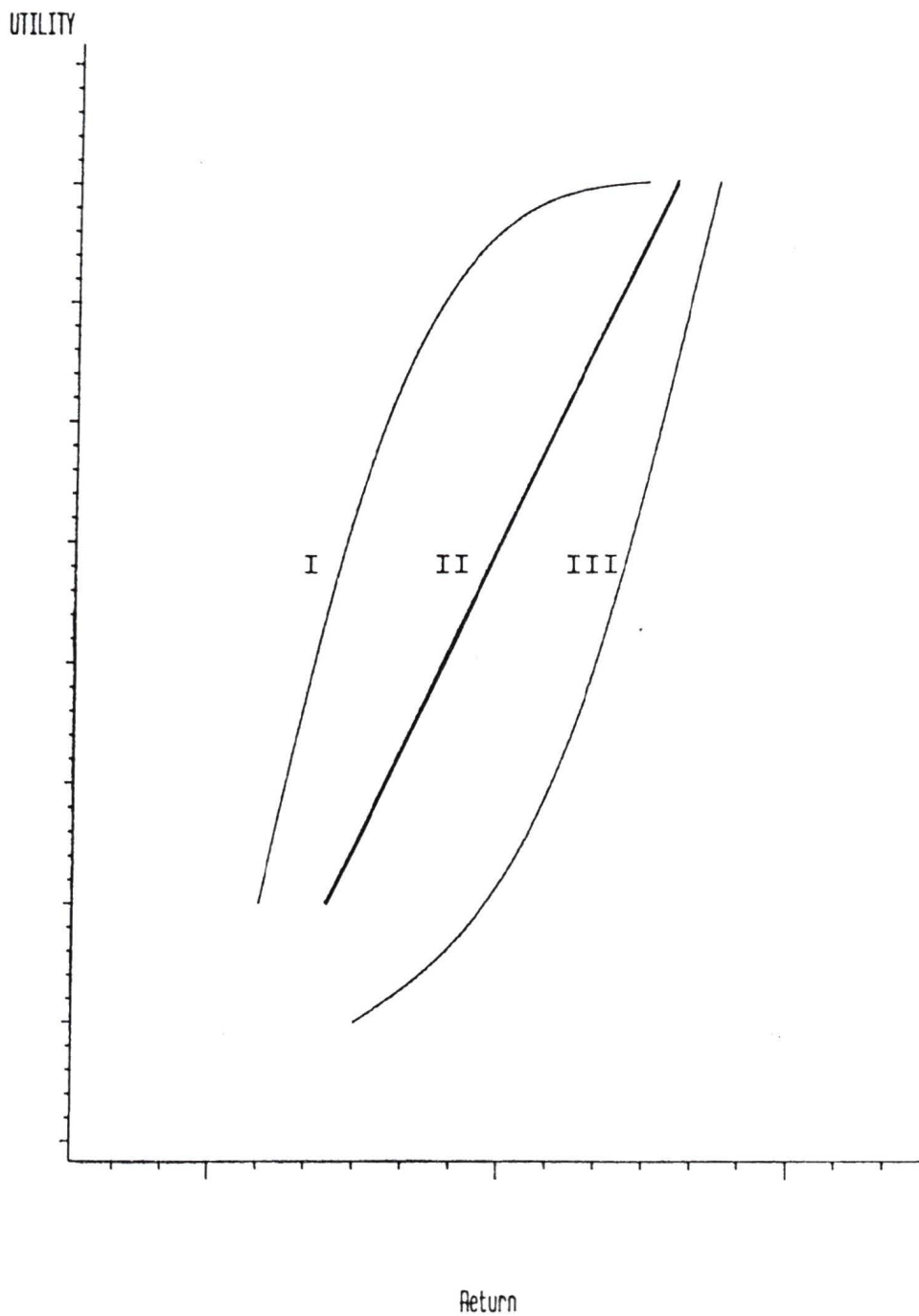
perfect certainty in the rate of return of an investment. If uncertainty is introduced, it seems more plausible to assume that market participants are motivated by the maximization of the expected utility of the investment outcome rather than the maximization of expected profit (or return). However, under the pure expectations theory, it is postulated that the market is dominated by participants (speculators) who are indifferent to risk, that is, their utility-of-return function is linear. Under the assumption of a linear relationship between utility and return, the maximization of expected utility is equivalent to the maximization of expected return. However, if universal risk aversion is assumed, investors' utility-of-return function will be concave in shape.<sup>4</sup> Maximization of expected utility of investment outcome will not be equivalent to maximization of expected return because investors may demand a risk premium to compensate for the extra risk perceived to be involved in long-term obligations. In this event, implicit forward rates will be biased estimates, rather than unbiased estimates of expected future short-term rates as assumed in the pure expectations theory.

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<sup>4</sup> Utility-of-return functions of a risk-neutral individual, a risk-avorter and a risk-seeker are shown in Figure 2. For more detailed explanation for the shape of the curves, see Allen, David E., Finance: A Theoretical Introduction (Oxford: Martin Robertson, 1983), pp. 63-65.

## FIGURE 2

Utility of Return Functions



I : Risk-averse

II : Risk-neutral

III : Risk-seeking

(iii) Uniform expectations. If different expectations prevail in the market, an investor's action may not be based upon his own expectations. Instead, investors will act according to what they think other investors' expectations are and how they will behave. Hence, the term structure is no longer determined by a unique set of expectations, nor can we say anything about the expectations of the market participants. Opponents of the pure expectations theory argue that the existence of non-uniform expectations may invalidate the theory.<sup>5</sup>

In reply, proponents of the pure expectations theory defend their position by arguing that the prevalence of non-uniform expectations poses no threat to the theory even in the short run. As long as the market is dominated by people sharing the same expectations and who act according to these expectations, and as long as the action of the risk-neutral speculators is strong enough, the theory is applicable for the long run as well as for the short run.

(iv) Transaction costs. While the assumption of zero transaction costs is unrealistic, the relaxation of this assumption does not affect the general validity of the theory. An investor purchasing a long-term bond incurs two

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<sup>5</sup> See Lockett, D., "Professor Lutz and the Structure of Interest Rates", Quarterly Journal of Economics 74 (February 1959).

trading costs, one in buying and the other in disinvesting. Or, the investor can invest successively in short-term bonds, which will entail a series of reinvestment costs. A choice between the two alternatives clearly depends on the relative transaction costs entailed in buying several short-term bonds versus a single long-term bond. According to a study by Burton Malkiel (1966),<sup>6</sup> transaction costs in the secondary market are found to increase with term to maturity such that no significant differences exist between the two alternatives. Hence, the impact of transaction costs on the term structure can be considered as neutral.

(v) Coupon payments. The yield to maturity of an n-period bond is that interest rate which equates the present value of the stream of payments (coupon payments and principal) received from the bond to its market price.<sup>7</sup> If coupon payments are allowed for, the calculated n-period yield will equal the (true) expected holding-period yield only in special circumstances. One of these circumstances

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<sup>6</sup> For more details, see Malkiel, Burton, The Term Structure of Interest Rates (Princeton, New Jersey, Princeton University Press, 1966), pp.112-23.

<sup>7</sup> This proposition is demonstrated as follows. Letting P be the market price of an n-period bond, F be the face value (principal), C be the annual coupon payment and y be the yield to maturity, the formula is

$$P_n = [C/(1+y)] + [C/(1+y)^2] + \dots + [(C+F)/(1+y)^n]$$

arises when coupon payments are received only when the bond matures. Another is when all the intermediate coupon payments can be reinvested at the calculated yield to maturity. Both of these are special situations. It follows that using the conventionally calculated yield-to-maturity data will produce inaccurate estimates of (true) expected holding-period yields. Hence, the theoretical analysis of the term structure is usually made on the assumption that there are no coupon payments.<sup>8</sup>

It should be recognized that all the key assumptions of the pure expectations theory can be relaxed without threatening the underpinnings of the theory other than the assumption of risk neutral speculators. It is this assumption that virtually distinguishes the pure expectations theory from the other term structure theories. If the market is dominated by risk-averse hedgers, which results in insufficient speculative activities, or speculators are numerous in number but they are also risk-averse, then the pure expectations theory may collapse. Forward rates can no longer be claimed as unbiased estimates of future rates, nor does perfect substitutability exist between securities of different

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<sup>8</sup> For more details, see Macaulay, F., Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields and Stock Prices in the United States Since 1856, (New York: National Bureau of Economic Research, Publication No. 33, 1938).

maturities.

### 1.2.2 Market Segmentation Theory

The market segmentation theory (also called the hedging pressure theory) was first proposed by John Culbertson in 1957.<sup>9</sup> Basically, it claims that market participants do not act on the basis of expectations of future rates, and that the forward rates implicit in the term structure do not carry any economic implications.

Instead, the market segmentation theory envisions the market as being dominated by risk-averse hedgers, and that the term structure of interest rates is determined by 'hedging' or 'institutional' pressures in the market. Hedging refers to offsetting a liability with an asset of the same maturity, and vice versa. In order to hedge against uncertain fluctuations in the rate of return, investors manage their investments so that the maturity composition of the

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<sup>9</sup> His argument was as follows: if the pure expectations theory was valid, a positive yield gap between long and short rates should not persist since it would imply continuing lost opportunities to investors for making profit by arbitrage. By examining the realized holding-period yields across the maturity spectrum, he found that the yield on long-term bonds, for most of the time, exceeded that on short-term bonds. He concluded that this could not be due to random errors and that most of the activities (especially at the long end of the market) was determined by non-speculative activities. However, it should be noted that Culbertson used ex-post (realized) holding-period returns as the basis of his argument, that is, he presumed that expectations were all fulfilled.

portfolio matches the maturity composition of liabilities. Thus, different groups of investors who have different maturity needs will confine their investments to restricted segments of the maturity spectrum. Similarly, bond issuers will offer bonds with the same maturity as that of the asset to be financed. For example, the liabilities of commercial banks are primarily composed of deposits subject to pay on demand or short notice. As a result, they have a strong preference for short- and medium-term maturities which offer the liquidity they need. On the other hand, investors with long-term obligations such as life insurance companies and pension funds prefer longer maturities to match with the maturity of their liabilities.

Suppose a life insurance company, a long-term investor, wants to have its funds invested for 15 years. If it invests in a 25-year bond, it is assured of a steady flow of income during the holding period but is uncertain of the market value of the bond upon disinvestment. The risk associated with the uncertainty of the future market price of a bond is called the capital risk. Alternatively, if it invests in short-term securities, it is assured of receiving the principal at maturity but is uncertain of the return it is able to obtain upon reinvestment of its funds. This risk is called the income risk. This theory argues that both of

these risks are so strong that market participants will not leave their preferred maturity ranges to take advantage of higher returns in other maturities. Consequently, the bond market will be entirely segmented according to maturity, with the hedging pressures in the various maturity ranges determining the term structure. Referring to the examples given above, commercial banks will be active in the short end of the bond market while the investment activities of life insurance companies and pension funds will be concentrated at the long end of the market.

Clearly, given the behavior of risk-averse hedgers, expectations will have little or no role to play in the term structure. It is not that expectations are unimportant. Rather, the market segmentation theory contends that the existence of inconsistent expectations, transaction costs, and, most importantly, risk aversion on the part of the investors implies that the term structure cannot be accounted for by the expectations theory. Even if investors may form expectations of their own, they do not act upon them. Hedging dominates any effect that expectations may have on the term structure.

The primary prediction of this theory is that prices of securities are determined by supply and demand factors with-

in each market, and that these factors are predominantly the result of the hedging activities of the risk-averse market participants. In its extreme form, it claims that the rate in each market is independently and solely determined by the supply and demand conditions in that market, with no reference to the market conditions in the adjacent or distant maturity ranges. It implies that the term structure may be discontinuous, and that interest rates will not move together and converge. Securities of different maturities are very imperfect substitutes for each other.

### 1.2.3 Hicksian Risk-Premium Theory

This theory is built upon virtually the same assumptions as is the pure expectations theory. It retains the fundamental assumption that investors act in the market on the basis of their expectations of future spot rates. However, investors, who do not have perfect knowledge on the return of their investment, are assumed to be risk averters.

There are two kinds of investors: hedgers who stay in their preferred habitats and speculators (or arbitrageurs) who are willing to depart from their preferred habitats if they are compensated for the higher risk due to uncertainty. The risk of a bond, which may be measured by the standard deviation of the return of the bond, tends to increase with

its length to maturity. Hence, given universal risk aversion and uncertainty about future rates, holders of long-term bonds will demand a risk or liquidity premium to compensate for bearing the uncertainty. The size of the premium is expected to depend on the relative supplies of longer maturities and the degree of risk aversion on the part of the lenders. John Hicks (1939) also argued that there exists a 'constitutional weakness' on the long side of the market, resulting from the assumption that the average preferred term of the borrowers exceeds the average term preferred by the lenders.<sup>10</sup> This imbalance can be eliminated only if a risk premium is offered to speculators who supply long-term funds and bear the risk involved.

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<sup>10</sup> Hicks argued in Value and Capital (pp. 138-146) that

... other things being equal, a person engaging in a long-term contract puts himself in a more risky situation than he would be in if he refrained from making it ... Borrowers will have a stronger propensity to borrow long... The forward rate of interest for any particular future week ... is thus determined ... at that level which just tempts a sufficient number of 'speculators' to undertake the forward contract. It will also have to be higher than the short rate expected by these speculators to rule in that week...; it will, indeed, have to exceed it by a sufficient amount to induce the marginal speculator to undertake the risk.

Under this hypothesis, forward rates will be upward biased estimates of future spot rates, exceeding them by the amount of the risk premium. It implies that securities of different maturities are no longer perfect substitutes and that the expected rate of return of an investment depends on the maturities of the securities it is composed of. The Hicksian model can be expressed as follows:

$$R_{n,t} = [(1 + r_{1,t})(1 + {}_{t+1}r^e_{1,t} + {}_{t+1}P_{1,t}) \dots (1 + {}_{t+n-1}r^e_{1,t} + {}_{t+n-1}P_{1,t})]^{1/n}$$

where  ${}_{t+i}P_{1,t}$  is the risk premium for period  $t+i$ .

Risk premiums are generally considered to be a monotonically increasing function, but at a decreasing rate, of the term to maturity. A widely held view also suggests that they are a stable function of the overall level of interest rates.<sup>11</sup> When interest rates are low (usually during periods of economic downturn), investors, anticipating a rise in interest rates and a corresponding drop in bond prices, will demand a relatively high risk premium to invest in risky bonds. The opposite will occur at times of high interest rates (usually during periods of high-level economic activi-

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<sup>11</sup> See Keynes, John Maynard, The General Theory of Employment, Interest and Money (New York: Harcourt, Brace & World, Inc., 1936), pp. 201-202.

ty). Investors expect a drop in interest rates and a corresponding increase in bond prices. In the prospect of a capital gain, they are less concerned with safety and more willing to bear risk. This suggests that risk premiums vary inversely with the level of interest rates relative to a level which is considered normal.

R.A. Kessel(1965), however, claims that the relationship is direct. As interest rates increase, people incur a higher opportunity cost in holding money versus income-earning assets. This results in an increase in the demand for short-term securities (a close money-substitute). Hence, short-term rates decrease relative to long-term rates and the risk premiums in forward rates must rise. This also explains why securities of different maturities in the risk-premium theory are imperfect substitutes.

In brief, if the level of interest rates has an influence on risk premium, the type of relationship is determined by whether market participants are more concerned with capital gain/loss or the opportunity cost of holding money.

#### **1.2.4 Preferred Habitat Theory**

The preferred habitat theory was initially developed by Modigliani and Sutch (1966, 1967) and later extended by

Modigliani and Shiller (1973). It simultaneously incorporates the factors emphasized by the pure expectations theory, the market segmentation theory and the Hicksian risk-premium theory. In other words, it synthesizes the above three theories into one.

The expectations component of the preferred habitat theory is built upon the earlier work by Frank De Leeuw (1965). He suggested that expectations of future interest rates were determined by two components: an extrapolative component and a regressive component of expectations. Over the short run, market participants expect the present trend in interest rates to continue in the future. However, in the long run, interest rates are expected to regress toward a normal level.

The theory also hypothesizes that the actual return of an n-period bond exceeds the one determined by the pure expectations model by a premium. Since this premium arises from an imbalance between the volume of funds supplied by lenders and that demanded by borrowers in the n-period market, it need not be a monotonically increasing function of the term to maturity, and may be positive or negative. To eliminate this imbalance, it is necessary to bring into the n-period market lenders (borrowers) from neighbouring maturities by a

positive (negative) risk premium if there is a shortage (excess) of funds supplied in the  $n$ -period market.

The major prediction of this theory is that the term structure of interest rates is not solely determined by expectations of future spot rates, but also by the relative supplies of funds in different maturities, and the degree of risk aversion of the investors. While investors have their preferred habitats, they may leave them if sufficient inducement is offered in the other maturities. In the absence of sizable inducements, investors, being risk-averse, will stay in their preferred habitats. This causes the bond market to be partially segmented, in which securities of different maturities are imperfect substitutes.

### 1.3 The Shape of the Yield Curve

According to the pure expectations theory, the shape of the yield curve can be explained by the market's expectations of future interest rates. For example, a horizontal yield curve implies that the market expects future short rates to remain the same as the current short rate. An upward sloping yield curve is formed when the market expects rates to rise in the future, whereas a descending yield curve occurs when rates are expected to fall.

The presence of risk premiums in the Hicksian theory implies a bias toward upward sloping yield curves. A descending yield curve is formed only when expected future short-term rates are lower than the current short-term rate by amounts exceeding the premiums. Risk premiums tend to cushion the downward slope in the yield curve at cyclical peaks (that is, when short rates exceed long rates) and to accentuate the upward slope in the yield curve at cyclical troughs (that is, when long rates exceed short rates). However, risk premiums in the context of the preferred habitat theory do not necessarily imply an upward bias as the risk premiums can be positive or negative.

In the market segmentation theory, the yield on one maturity bears no direct relationship with those on other maturities. It, therefore, does not offer a systematic explanation for the shape of the yield curve as a whole. Nevertheless, if commercial banks dominate security markets, their demand for short-term securities will keep short-term rates below long-term rates. On the other hand, if life insurance companies and pension funds dominate security markets, their demand for long-term issues will result in a downward sloping yield curve.

#### 1.4    Summary

The term structure of interest rates is concerned with the relationship between yield and maturity on securities, with all other factors held constant. A number of theories attempt to explain the term structure. The pure expectations theory states that the term structure is determined solely by expectations. The market is assumed to be dominated by risk-neutral speculators who, through arbitrage activities, bring forward rates equal to short-term rates expected to prevail in the future. Consequently, forward rates are unbiased estimates of expected future short-term rates. The theory also implies that securities of different maturities are perfect expected substitutes in the sense that the expected return is the same.

On the other hand, the market segmentation theory assumes that the market is dominated by risk-averse hedgers. Their maturity preferences are so strong that they will not leave their preferred maturity range to take advantage of yield differentials. This denies the possibility of arbitrage activities, and implies that substitutability between securities of different maturities is zero. As a result, there exists a number of markets, with interest rates determined by supply and demand factors within each market. Changes in

the term structure will arise only from changes in relative asset supplies and demands.

The risk-premium theory is built upon the pure expectations theory, but modified by the presence of risk premium. In this theory, market participants are concerned with the uncertainty of long-term bonds. This uncertainty stems from the greater price volatility of long-term bonds compared with short-term bonds. Investors prefer to lend short unless they are compensated, in the form of a risk premium, for bearing the risk of lending long. Forward rates, therefore, are upward biased estimates of expected future rates. The degree of substitutability, depending on the degree of risk aversion, is limited.

It should be noted that the above three theories, each of which emphasizes one determinant of the term structure, are not mutually exclusive. A synthesis of the three theories is provided by the preferred habitat theory. This theory agrees that expectations play a role in determining the term structure. However, it argues that risk aversion does not necessarily lead investors to stay short. Rather, investors will stay in their preferred habitats unless sufficient yield inducement is offered in the other maturities. The risk premium for a given maturity can be positive or negative,

depending on whether the supply of funds exceeds, or is exceeded by the demand for funds in that maturity.

Term structure theories have important implications for the conduct of monetary policy. If the pure expectations theory holds, any government policy aimed at altering the shape of the yield curve by changing the maturity composition in the bond market will be fruitless. The only way the government can bring about a change in the term structure is by changing the market's expectations of future rates.

In contrast, the market segmentation theory suggests that the government can shift or twist a yield curve by open market operations. For example, the government may wish to keep short-term rates high to attract foreign capital inflow and, at the same time, maintain a low level of long-term rates to stimulate domestic investment. This 'twist' of the term structure can be achieved by simultaneously selling short-term securities and buying long-term securities in open market operations. A downward shift of the yield curve can be accomplished by buying securities across the whole maturity spectrum, whereas an upward shift can be accomplished by selling securities of all maturities.

As long as securities of different maturities are imperfect substitutes, monetary policy has a role to play in

influencing the term structure. However, the effectiveness of the policy will depend on the degree of segmentation in the market, more under the market segmentation theory and less under the risk-premium theory and the preferred habitat theory.

## Chapter II

### A REVIEW OF PAST EMPIRICAL STUDIES

In the past two decades, there have been numerous empirical tests of the term structure of interest rates. This chapter intends to provide a review of the major empirical tests on the term structure theories. Particular attention is given to the way researchers subject the theories to empirical testing and the way they structure their tests. Though not without exceptions, the tests generally support the importance of expectational elements in determining the term structure. There has been a lack of substantial empirical support for the market segmentation theory.

#### 2.1 Major Problems Encountered in Testing Term Structure Theories

In order to analyze the role of expectations in the pure expectations theory, the risk-premium theory and the preferred habitat theory, information on expectations of future rates is needed. Unfortunately, market expectations of future interest rates are not directly observable.

One way to generate information on market expectations is to conduct interviews with categories of investors. Through the analysis of the portfolio and debt management behaviour of the investors in each category, we can specify more accurately the variables which enter into the objective functions of the investors, determine the importance of expectations and the way by which expectations are formed. This kind of study, however, involves problems such as how to classify investors into appropriate categories, how to take into account that some categories overlap with each other, how to assess the role of expectations, and how to deal with the equity and property markets, which possibly affect the term structure.

More commonly, another way to deal with the data problem is to hypothesize a mechanism explaining the expectations formation process. This is not an invalid procedure and is often used in economics. However, if values of expectational variables are not market given but generated by a mechanism, such a test is in effect a joint test of two hypotheses. It is a test on the hypothesis on the formation of expectations, as well as the hypothesis making use of expectational variables (the pure expectations theory, for instance). If such a model is supported by the data, both hypotheses will have empirical content. However, if it turns out that the

data fail to support the hypotheses, we cannot reject the pure expectations theory since the failure may be attributed to misspecification of the expectations formation process.

It should be noted that the lack of directly observable market expectations also poses a problem to the testing of the risk-premium theory and the preferred habitat theory because both of these theories accept the view that expectations is one determinant of the term structure.

In testing the supply effects in the market segmentation theory, the major problem encountered is the difficulty involved in devising an accurate test for the supply effects. Since markets for different types of debt instruments are interdependent, an ideal supply variable is one which measures maturity composition of all instruments. However, there is a lack of refined data on the maturity composition of all private and public debt. The problem aggravates if the equity and real estate markets are also included to take into account their influence on the term structure. Also, the maturity composition of outstanding debt cannot be considered as an exogeneous variable as in the market segmentation theory. Term structure and maturity composition are jointly determined.

## 2.2 Empirical Tests on the Pure Expectations Theory and the Risk-Premium Theory

As mentioned in the previous section, one way to generate information on expectations is by surveying market participants. One of the studies using this method was undertaken by Edward J. Kane and Burton G. Malkiel (1967). They surveyed a large sample of commercial banks, life insurance companies and non-financial institutions. They found that not all investors formulated specific interest-rate expectations, and that investors did not hold a uniform set of expectations. Moreover, both the willingness to predict and the extent of disagreement varied with the futurity of the rate predicted.

Another way to test the pure expectations theory is to set up some mechanism by which expectations are formed by investors (already mentioned in Section 2.1). The two theories, the pure expectations theory and the theory of expectations formation, are tested jointly. This approach was adopted by Dobson, Sutch and Vanderford (1976). They undertook a thorough testing of alternative expectations formation processes (inertial, regressive, extrapolative, extrapolative-regressive, adaptive and convex).<sup>12</sup> Using monthly U.S. Treasury bill data for the period 1954-1968,

<sup>12</sup> These models will be discussed in detail in Chapter 3.

the authors evaluated the results both in terms of the internal consistency and the explanatory power of the models. They found only the linear regressive, the simple extrapolative and the extrapolative-regressive model to be internally consistent, with the latter having the highest explanatory power.

Besides the above two ways to test the pure expectations theory, another approach is to make use of the implied forward rates implicit in the empirical term structure. One of the better tests of the pure expectations theory using this approach was undertaken by David Meiselman (1962). He introduced an error-learning model in which market participants are assumed to revise their expectations on future rates whenever previously held expectations prove to be in error. Since the pure expectations theory claims that forward rates are unbiased estimates of expected short-term rates, Meiselman tested the pure expectations theory by investigating whether forward rates change in conformance with the error-learning model, that is, whether forward rates change on the basis of errors made in forecasting the current short-term rate. The error-learning model can be expressed as follows:

$${}_{t+i}r_{1,t} - {}_{t+i}r_{1,t-1} = f(r_{1,t} - {}_t r_{1,t-1}) + u_t$$

Or, it can be rewritten as

$$\Delta {}_{t+i}r_{1,t} = a + bE_t + u_t$$

where  ${}_{t+i}r_{1,t-1}$  is the forward one-period rate implicit at time  $t-1$  for a loan to begin at time  $t+i$ ;  $\Delta {}_{t+i}r_{1,t}$  is the change in forward rates;  $r_{1,t}$  is the spot one-period rate at time  $t$ ; and  $(r_{1,t} - {}_{t}r_{1,t-1})$  is the one-period forecasting error,  $E_t$ , and  $u_t$  is the error term.

Using the Durand basic corporate bond yield data from 1901-1954, Meiselman found that changes in forward one-year rates were highly correlated with the forecasting errors. Moreover, the constant term was not found to differ significantly from zero. Meiselman argued that the constant term was a measure of risk premium. Since the constant term was not significant, risk premiums were not present in the term structure, and the risk-premium theory was rejected. He therefore contended that the results were consistent with the pure expectations theory that forward rates were equivalent to expected future short-term rates.

However, Meiselman's assertion that a zero constant term was inconsistent with the risk-premium theory has been shown to be invalid. John H. Wood (1963) and Reuben Kessel (1965) independently demonstrated that an insignificant constant term could be consistent with the presence of risk premiums embodied in forward rates.

The error-learning model has been applied to other sets of data (James Van Horne (1965), Reuben Kessel (1965), Joseph Conard (1966) and A. Buse (1967) ). The results generally support the notion that interest-rate expectations are important in explaining the term structure, and that expectations are revised in the way hypothesized by the error-learning model. Furthermore, in most of the cases, the constant term was found to be significantly different from zero. This supports the risk-premium theory that risk premiums are embodied in forward rates.

The presence of risk premiums obscures the predictive power of forward rates and makes forward rates appear to be inaccurate estimates of future spot rates. This argument is supported by the study of Richard Startz (1982). He found that one to two thirds of the variation in the difference between forward rates and realized spot rates was due to variation in risk premium. The true forecasting component of the forward rate can therefore be clearer after the removal of risk premium.

Eugene Fama (1976), using a model for estimating the risk premiums embedded in forward rates, managed to net variations in risk premiums out of the time series of the forward rates. After proper adjustments for variations through time

in risk premiums, he found that forward rates were fairly accurate predictors of future spot rates and comparable to those obtained from the information contained in the time series of past spot rates.

If risk premiums are embodied in forward rates and affect the term structure, it is important to know whether they vary with the term to maturity, and with such factors as the level of interest rates, the degree of investors' certainty about future rate movements, and general business confidence. Studies on risk premiums suggest that, for early maturities, they increase monotonically but at a decreasing rate with maturity. For later maturities, the evidence is mixed as to whether premiums increase monotonically with maturity. Fama (1984), in his study on one-month U.S. Treasury bills, found evidence that risk premiums on bills did not increase monotonically with maturity but generally reached a peak at eight to nine months.

The evidence is also mixed as to whether risk premiums vary directly or inversely with the level of interest rates. Burton Malkiel (1966),<sup>13</sup> using Durand corporate yield data for the period 1900-1942, found evidence to support an inverse relationship between risk premiums and the level of

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<sup>13</sup> See Malkiel, Burton G., op. cit., Chapter 3.

interest rates. Charles R. Nelson (1972),<sup>14</sup> using the 1901-1958 Durand yield data, found further support to an inverse relationship. He isolated risk premiums by subtracting expected rates generated by a linear model from the corresponding forward rates. Regressing these risk premiums against the level of interest rates and an index of business confidence, he found that both of the coefficients were significantly negative, which supported an inverse relationship.

Other studies, however, have found evidence supporting a positive relationship between risk premiums and the level of interest rates. The first of such studies was undertaken by R. Kessel (1965) using Treasury bill rates.<sup>15</sup> Edward Kane (1983), using data obtained from a questionnaire, found that risk premiums were positive, time-variant, and varied positively with the level of interest rates. Benjamin Friedman (1979), also using information from a survey, confirmed that risk premiums in U.S. Treasury bills were non-zero and varied systematically with interest rates. However, no evidence was found to show that risk premiums varied systematically with the maturity composition of outstanding U.S. Treasury

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<sup>14</sup> See Nelson, Charles R., The Term Structure of Interest Rates (New York: Basic Books, Inc., 1972), Chapter 6.

<sup>15</sup> See Kessel, R., The Cyclical Behavior of the Term Structure of Interest Rates (New York: National Bureau of Economic Research, 1965), pp. 22-26.

bills, nor did they vary with the general indicators of economic activities including unemployment rate, growth rate of real income, inflation rate and growth rate of money supply.

In summary, empirical studies have found considerable evidence concerning the importance of expectations in determining the term structure. There is also evidence, however, that expectations are not the sole determinant. The presence of risk premiums has been supported in most of the studies. However, tests on the risk premiums provide inconclusive evidence as to how risk premiums vary with the term to maturity, and whether risk premiums vary directly or inversely with the level of interest rates.

### 2.3 Empirical Tests on the Market Segmentation Theory

For the most part, empirical work on the market segmentation theory has been concerned with the major implication of the theory that changes in relative supplies of or demands for debt instruments will alter the term structure. Empirical tests have been conducted to study the effects of shifts in the relative supply of various maturity securities and shifts in the relative demand by investors.

One study which provides evidence for the existence of market segmentation was undertaken by Elliott and Echols

(1976). They used a special regression technique called the piecewise linear regression technique to investigate whether a given scatter of yield data consisted of a single regression fit or a number of regression lines. A finding of multiple line segments (that is, discontinuities in the yield curve) would provide evidence that the bond market was compartmentalized, and there was an insufficient amount of speculative and arbitrage activities to eliminate inconsistencies in yields. Their finding of significant discontinuities in the yield curve in the "over 8 years" range led them to conclude that there existed considerable market segmentation in the longer maturity range.

Another study which also provides empirical support to market segmentation was conducted by Neil Wallace (1969).<sup>16</sup> He tested the hypothesis that forward rates calculated from the term structure were influenced by the implicit supplies of forward loans. Though the results supported the above hypothesis, the influence of segmentation was found to be rather small.

Besides the above two studies, there have been few other studies on the extreme version of market segmentation (that is, market segmentation as the dominant force in the determination of the term structure). Moreover, these studies

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<sup>16</sup> His work was summarized by Telser.

fail to show institutional forces as the major factor in shaping the term structure. Other studies regarding market segmentation are interested in finding out whether market segmentation is one of the determinants of the term structure, as will be discussed in the following section.

#### 2.4 Empirical Tests on the Preferred Habitat Theory

Partly in response to the finding that the extreme version of market segmentation does not have much empirical support, researchers started to test the preferred habitat theory of Modigliani and Sutch, which is a synthesis of the pure expectations theory, the risk-premium theory and the market segmentation theory.

The model developed by Modigliani and Sutch (1966, 1967) contains three basic elements. One is that investors have a preferred habitat, that is, they attempt to match the maturity of their assets with that of their liabilities. The second element is that long-term interest rates depend on expected future short-term rates. The third component is that expectations of future interest rates are formed on the basis of the past history of interest rates. However, there are two influences in the past history. One is the recent trend in interest rates, which suggests that a rise in

interest rates will lead to an expectation of a further rise, and vice versa. The other is the normal level of interest rates based on past experience, which suggests that the market expects interest rates to regress toward the normal level.<sup>17</sup>

Combining the two expectational influences into a single expectations variable, a general expression of the preferred habitat theory is obtained:

$$R_{n,t} = P + w_0 r_{1,t} + \sum_{i=1}^{\infty} w_i r_{1,t-i}$$

where  $P$  is the risk premium for the  $n$ -period bond, and  $w_i$ 's ( $i$  from one to infinity) are the weights of the distributed lag.

The problem associated with the estimation of this equation is that there is an infinite number of observations on past short-term rates. To solve this problem, Modigliani and Sutch postulated that the first  $m$  lag weights lay on a polynomial of low degree and all other short rates beyond the  $m$  periods could be approximated by a constant. The distributed lag on the first  $m$  past short rates was estimated with the polynomial distributed lag estimator suggested by

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<sup>17</sup> It was first suggested by Frank de Leeuw (1965) that expectations contained both extrapolative and regressive elements.

Shirley Almon (1965). This procedure imposes very little restriction on the lag structure, and helps to avoid the problem of multicollinearity between the independent variables. Using quarterly data on U.S three-month Treasury bills and long-term government securities from 1952 to 1961, they experimented with second, third and fourth degree polynomials. It was found that a fourth degree polynomial with 16 quarter lags produced the most satisfactory result in terms of the fit, the plausibility of the lag structure, and predicted signs and order of magnitude of the lag coefficients. The lag structure resembled an inverted U, providing strong evidence to their hypothesis that the most recent trend was expected to continue for a while before interest rates moved back to a long-run normal level.<sup>18</sup> They also tested for the supply effects by adding a variable representing changes in the maturity composition to the basic equation. However, no evidence was found to support the argument that changes in the maturity composition had significant influence on the term structure.

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<sup>18</sup> As the most recent trend (the extrapolative component) is expected to dominate for a while, the lag structure will rise initially, possibly from negative values. Then it reaches a peak (when the extrapolative component declines to zero) before it falls to zero (due to the dying off of the regressive component). For more detailed discussion on the shape of the lag structure, see page 68.

Following the studies by Modigliani and Sutch in the 1960s, Modigliani and Shiller (1973) generalized the previous model by making allowance for inflationary expectations on the term structure.<sup>19</sup> They added to the previous equation a distributed lag of past inflation rates to account for the effects of Fisherian inflationary expectations.<sup>20</sup> They found that this additional variable was not only statistically significant but also helped to reduce serial correlation of the error term. This led them to conclude that the long-term rate was mainly determined by past short-term interest rates and inflation rates.

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<sup>19</sup> They stated the rationale for making this modification in their paper (page 19) as follows.

... the fit [the Modigliani-Sutch equation] suffers from one shortcoming: though the residuals are reasonably small, they exhibit a very pronounced positive serial correlation... Such a finding suggests that some systematic and presumably highly serially-correlated explanatory variable has been omitted from the list of independent variables. However, in an economy in which prices have been subject to considerable variability, and may, therefore, be expected to continue in the future, it may be more reasonable to suppose that the future short-term money rate will be expected to reflect the rate of change of prices prevailing then.

<sup>20</sup> The modified equation is

$$R_{n,t} = P + w_0 r_{1,t} + \sum_{i=1}^{\infty} w_i r_{1,t-i} + v_0 I_t + \sum_{i=1}^{\infty} v_i I_{t-i}$$

where  $\sum w_i r_{1,t-i}$  and  $\sum v_i I_{t-i}$  are the distributed lags of past short-term rates and past inflation rates, respectively.

The Modigliani-Sutch approach has gained wide acceptance among researchers who have adopted it, in one way or another, in their empirical studies of the term structure. Among these researchers are Steven Dobson (1973), James Pesando (1975) and James Van Horne (1979).

Dobson attempted to investigate whether the Conversion Loan of 1958 in Canada<sup>21</sup> provided evidence that supply effects also played a part in determining the term structure. He added to the M-S equation variables representing the maturity composition of outstanding Government of Canada debt to determine whether changes in relative supplies had a significant impact on relative interest rates. The regression results indicated that expectations were the major determinant. Supply effects were rather small and limited to the short end of the market.

Adopting the rational expectations hypothesis to create a synthetic series of rational forecasts of future interest rates, James Pesando tested the market segmentation theory by investigating whether term premiums were significantly

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<sup>21</sup> The Conversion Loan of 1958 in Canada involved the most dramatic change in the maturity composition of government debt in Canadian history. In two-month period, the Government of Canada refunded forty-five percent of the outstanding marketable debt into longer term issues. As a result, the average maturity of publicly held government debt increased from 96 months on June 30, 1958 to 177 months on September 30, 1958.

influenced by the massive debt alternations that occurred during the Conversion Loan period. His results confirmed that this refunding operation, an ideal incidence for investigating segmentation effects, failed to provide empirical evidence on the segmentation theory.

Using U.S. experience, James Van Horne used the M-S equation to test a partial market segmentation theory to see if segmentation determined the term structure in addition to the combined influences of expectations and risk aversion. Similar to the findings of the other studies, the evidence suggests that the term structure is determined by factors other than segmentation.

The M-S approach, however, has come under strong attack since its introduction, especially in the 1970s. Researchers who found empirical evidence casting doubt on the earlier work of Modigliani and Sutch include John Wood (1964), David Fand (1966), Michael Hamburger and Cynthia Latta (1969), Michael Hamburger and Elliott Platt (1975), Llad Phillips and John Pippenger (1976), and John Brick and Howard Thompson (1978). They argued that the basic hypothesis of the M-S model<sup>22</sup> could not hold in an efficient capital market.<sup>23</sup>

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<sup>22</sup> In the M-S model, it is hypothesized that long-term interest rates are primarily determined by the current short-term rate and a distributed lag of past short-term rates.

In other words, long rates would not depend on a distributed lag of past short-term rates.

Wood and Fand found that the adjustment process in the bond market was very rapid, and that the past history of short rates provided no useful information regarding the long rate. The best estimate of next period's short rate is the current short rate.

Hamburger and Latta, on the other hand, re-estimated the M-S model using a second degree autocorrelation transformation in order to correct for positive serial correlation in the error term. In comparing their specification with the first-difference equation of Wood's model,<sup>24</sup> they found that the M-S model was only marginally superior to Wood's model. They therefore concluded that the distributed lag of past short rates did not contribute sufficiently to justify its inclusion in the M-S model.

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<sup>23</sup> An efficient market theory states that the market's conditional density function for the future price of a bond, given the information used by the market, is equal to the true conditional density function, given all the information available to the market. That is, the market has fully utilized all the available information in forming the forecast.

<sup>24</sup> Wood's model is

$$R_{n,t} = a + br_{1,t}$$

where  $R_{n,t}$  and  $r_{1,t}$  are the long-term and short-term rates at time  $t$ , respectively.

In a reply to Hamburger and Latta, Modigliani and Sutch compared the results of their model with Wood's results after both of the models had been corrected for serial correlation using the Hildreth-Lu procedure. Even though the M-S model did not outperform Wood's model, Modigliani and Sutch defended their model by stating in their paper that

... when the distributed lag is dropped from the equation, the autoregressive parameter is able to absorb a great deal of the variance that the distributed lag was able to explain. ... our model would have to be judged superior.

The controversy remained unresolved. In 1975, Hamburger and Platt re-examined the empirical validity of the pure expectations theory in the context of an efficient market model. The latter model contends that forward rates, as predictors of expected future interest rates, should incorporate all publicly available information. The pure expectations theory together with the efficient market hypothesis implies that the difference between the forward rate and the corresponding realized spot rate is a random number, reflecting the receipt of new information arriving at the market after the formation of the forward rate. They not only found a close relationship between the two variables, but also found that the forward rate was not significantly different from the current short-term rate prevailing in the

market when the forward rate was formed. The constant, though not very significant, gave some indication of the magnitude of the risk premium.

The work of Phillips and Pippenger provided additional evidence to support the above studies. They argued that short-term and long-term capital markets were efficient since current interest rates in these markets had incorporated all the information available to the market participants. It follows that there was no useful information contained in past short-term rates that had not been fully reflected in last period's long-term rate. To prove their argument, they suggested a simplified efficient market model<sup>25</sup> and compared it with a modified version of the Modigliani-Shiller model.<sup>26</sup> From the results of their

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<sup>25</sup> Shown below is the final version of the simplified efficient market model suggested by Phillips and Pippenger (a first-difference transformation of the original model to correct for serial correlation in the error term).

$$R_t = a + bR_{t-1} + c \Delta D_t + d \Delta r_t + e \Delta I_t + \Delta E_t$$

where  $R_t$  and  $R_{t-1}$  are the long-term yield at time  $t$  and  $t-1$ , respectively.  $\Delta D_t$  is the change in the variability of short-term yields,  $\Delta r_t$  is the change in the current short-term yield,  $\Delta I_t$  is the change in the current rate of inflation, and  $\Delta E_t$  is the change in the error term.

<sup>26</sup> The final version of the Modigliani-Shiller model used by Phillips and Pippenger in their test is also a first-difference transformation of the original model.

$$R_t = a + bR_{t-1} + c \Delta D_t + d_0 \Delta r_t + \sum d_i \Delta r_{t-i} + e_0 \Delta I_t + \sum e_i \Delta I_{t-i} + \Delta E_t$$

tests, they concluded that past inflation rates as a whole did not provide additional information for explaining the movements of the long rates. Apparently, the high significance of the distributed lag of past short-term rates in the Modigliani-Shiller model was due to the 'spurious' correlation over time between long rates and lagged short rates. They therefore concluded that the relationship between them was much weaker than what the Modigliani-Shiller model had suggested.

Brick and Thompson also criticized the lead-lag relationship between long-term and short-term rates implied in the M-S model.<sup>27</sup> Their results indicated that the bond market was efficient in that the time series of long-term and short-term rates followed a random walk and that the relationship between the two series was unstable over time.<sup>28</sup> Virtually no evidence of a consistent lead-lag relationship

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where  $\sum (d_i \Delta r_{t-i})$  and  $\sum (e_i \Delta I_{t-i})$  are the distributed lags of past changes of short-term rates and inflation rates, respectively.

<sup>27</sup> According to the M-S model, long-term rates consistently lag behind short-term rates.

<sup>28</sup> Using the Box-Jenkins identification procedure, they found that the sample autocorrelation function of the first-difference transformation of the interest rate series was not only stationary, but also exhibited white noise properties. This led them to conclude that these interest rate series followed a random walk process, that is, the difference between successive observations was a random number.

was found.

Built upon the efficient market hypothesis that past short-term rates were irrelevant in predicting the current long-term rate, these studies contradicted the earlier work of Modigliani and Sutch and suggested a reappraisal of distributed lag models.

To summarise, there has been a lack of substantial empirical evidence to support market segmentation as the major force in shaping the term structure. Tests on the preferred habitat theory, which suggests that market segmentation also determines the term structure in addition to the influences of expectations and risk aversion, have been conducted using the model suggested by Modigliani and Sutch. These tests also did not find significant evidence to support the presence of segmentational force in the term structure. Recent studies, using the efficient market approach, question the usefulness of past interest rates in predicting future interest rates.

## 2.5 Conclusion of the Empirical Studies

Empirical studies reported above found considerable evidence to support the notion that interest-rate expectations play a prominent role in the explanation of the term struc-

ture. In addition, the results also suggest that forward rates are biased and high estimates of expected future rates. This is consistent with the risk-premium theory that investors in long-term bonds must be compensated, in the form of a risk premium, for accepting greater potential price fluctuation of long-term bonds. However, further tests on the shape of the risk premium profile produce inconclusive results concerning the relationship between risk premiums and such factors as term to maturity and the level of interest rates.

While evidence suggests that market segmentation is not likely to be the major determinant of the term structure, tests on the preferred habitat theory, a more moderate version of the segmentation theory, do not support the presence of segmentation in either the U.S. or the Canadian bond market. This lack of substantial empirical support can partly be attributed to the lack of sufficiently refined data on the maturity structure of private and public debt, making it very difficult to devise an adequate test for the theory.

## Chapter III

### EMPIRICAL TESTING

The purpose of the empirical study in this thesis is to examine the role of expectations and risk aversion in the term structure of interest rates. It is hoped that this will enhance our understanding of the determinants of the term structure.

This chapter will describe the empirical study in detail, including the theories to be tested, the choice of data, the testing procedures and the specifications of the models. The results of the empirical work will be presented and analyzed in the following chapter, Chapter 4.

#### 3.1 Theories to be Tested

With the exception of the market segmentation theory, the other three theories discussed in the previous chapter (the pure expectations theory, the risk-premium theory and the preferred habitat theory) agree that expectations play an important role in the explanation of the term structure. In order to assess more precisely the role of expectations, these theories are tested as follows.

First, the pure expectations theory is tested using two criteria: unbiasedness and consistency. The first test, the unbiasedness test, investigates the basic claim of the pure expectations theory that forward rates implicit in the empirical term structure are unbiased estimates of expected future short-term interest rates. The second test, the consistency test, determines whether implicit forward rates are consistent with the most widely accepted expectations formation processes. It is essentially a joint test of the pure expectations theory and the individual expectations formation process. Even though this test cannot provide a definite answer to the empirical validity of the pure expectations theory (as discussed in Section 2.1), it does provide indirect evidence as to whether the term structure is determined solely by expectations. To the extent that forward rates are consistent with the expectations formation processes, the more confidence we can have in the pure expectations theory.

Second, the Hicksian risk-premium theory is examined by investigating whether forward rates are upward biased estimates of expected future rates, that is, the differences between forward rates and expected future rates are consistently positive. To do this, the ARIMA model<sup>29</sup> is used to

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<sup>29</sup> The ARIMA model (the autoregressive integrated moving average model) is a time series model based on the

generate expected future rates. In order to ensure that the ARIMA model produces unbiased forecasts, the ARIMA forecasts are first subject to a test of unbiasedness. Then the ARIMA forecasts are subtracted from the forward rates, as in the following equation, to yield the risk premiums.

$${}_{t+i}P_{1,t} = {}_{t+i}r_{1,t} - {}_{t+i}r^e_{1,t}$$

where  ${}_{t+i}P_{1,t}$  is the risk premium for period  $t+i$ .

The isolated premiums will then undergo several tests to see if they are significantly positive, vary systematically with interest rate levels, and increase with the degree of risk and uncertainty about future rate movements.

Third, the M-S model is tested to determine what polynomial shape reveals about the regressive and extrapolative components of expectations,<sup>30</sup> and whether the distributed lag of past short-term rates contributes significantly in

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assumption that the current value of a variable is a linear function of its past values (the autoregressive part) and current and lagged random disturbances (the moving average part). An ARIMA model can be expressed as

$$\Phi(B)\nabla^d Y_t = x + \Theta(B)E_t$$

where  $\Phi(B)$  is the autoregressive function of degree  $p$ ,  $\Theta(B)$  is the moving average function of degree  $q$ ,  $d$  is the degree of homogeneity,  $x$  is the mean of the variable  $Y$ ,  $E$  is the random disturbance, and  $B$  is the backward shift operator.

<sup>30</sup> A detailed discussion on the possible shape of the lag structure is found on page 68.

explaining the movements of the long rate. The importance of market segmentation and risk premium will be inferred from the magnitude and degree of significance of the constant term.

### 3.2 The Choice of Data and Derivation of Forward Rates

The term structure of the Canadian Treasury bill market (three-, six- and twelve-month) will be studied. The reason for choosing this market for the empirical study is fivefold. First, all issues of Canadian Treasury bills are perfectly homogeneous with respect to credit-worthiness, safety, tax treatment, etc., apart from term to maturity. This is a desirable property for the study of the term structure. Second, Canadian Treasury bills have no coupon payments, thus avoiding the problem of determining the rate of return from reinvesting the coupon payments. Third, Treasury bills can be regarded as free from risk of default, that is, contractual payments will be made. This implies that risk premiums, if any, must arise from the uncertainty involved in longer maturities. Fourth, data on the yields of Canadian Treasury bills are of higher quality and accuracy because the yields are reported on specific maturities (three-, six- and twelve-month). Whereas for Canadian government bonds, yields are reported on maturity ranges (one

to three years, three to five years, five to ten years, etc.). Finally, the Treasury bill market is most suitable for the purposes of the present study. As market participants are more likely to form interest-rate expectations for the near future rather than the distant future, the influence of expectations on the term structure is expected to be most pronounced at the short end of the market. Therefore, the Treasury bill market is selected.

Data on the yields of Canadian Treasury bills (three-, six- and twelve-month) are available in the Bank of Canada Review. Monthly data from January 1975 to December 1984 will be employed (except for the twelve-month Treasury bill rates which will start from July 1977 when the Bank of Canada first published the data).

The data set consists of 120 observations of three- and six-month bill rates, and 90 observations of twelve-month bill rates.<sup>31</sup> These interest rate series are reproduced in Tables A1 - A3 in the Appendix.

To derive the implicit forward rates from the bill rates, the following formulae will be used.<sup>32</sup>

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<sup>31</sup> It is the data set used in the empirical study except for the generation of ARIMA forecasts where additional observations prior to 1975 are used.

<sup>32</sup> These two formulae are derived from equation (1.1).

$$(3.1) \quad {}_{t+3}r_{3,t} = \left\{ \frac{[(1+R_{6,t})^2]}{(1+r_{3,t})} \right\} - 1$$

$$(3.2) \quad {}_{t+6}r_{6,t} = \left\{ \frac{[(1+R_{12,t})^2]}{(1+r_{6,t})} \right\} - 1$$

where  ${}_{t+3}r_{3,t}$  and  ${}_{t+6}r_{6,t}$  are the implicit three-month Treasury bill rates three-month-hence and six-month Treasury bill rates six-month-hence, respectively.  $R_{6,t}$  and  $R_{12,t}$  are the six- and twelve-month Treasury bill rates (the long rates), and  $r_{3,t}$  and  $r_{6,t}$  are the three- and six-month Treasury bill rates (the short rates).

These two formulae allow calculation of the implied forward rates based upon actual bill rates prevailing in the market at a specific time. Using the present data set, two series of forward rates (three-month bill rates three-month-hence and six-month bill rates six-month-hence) are obtained. They are presented in Tables A4 and A5 in the Appendix. These forward rates, according to the pure expectations theory, are equivalent to expected future bill rates. Whereas in the risk-premium theory, they are upward biased estimates of future bill rates.

### 3.3 Hypothesis Testing and Model Specification

This section will describe in detail the tests to be performed on each of the theories mentioned in Section 3.1, and

discuss the expectations formation models used in the consistency test of the pure expectations theory.

### 3.3.1 Tests on the Pure Expectations Theory

As mentioned in Section 3.1, there are two tests on the pure expectations theory: the test of unbiasedness and the test of consistency.

#### 3.3.1.1 Test of Unbiasedness

The first test on the pure expectations theory is a test of unbiasedness. Two series of forward rates (three-month Treasury bill rates three-month-hence and six-month Treasury bill rates six-month-hence) are first derived using formulae (3.1) and (3.2) given in Section 3.2. The two series of computed forward rates are then subject to the following two tests of unbiasedness.

Test 1: Theil's Decomposition Test<sup>33</sup>

The procedure first involves the calculation of Theil's inequality coefficient.<sup>34</sup> The properties of the coefficient

<sup>33</sup> For more details of the test, see H. Theil, Economic Forecasts and Policy (Amsterdam: North-Holland, 1961), pp. 30-37.

<sup>34</sup> Theil's inequality coefficient is defined as

$$\frac{[1/n \sum (R^f - R^a)^2]^{1/2}}{[1/n \sum (R^f)^2]^{1/2} + [1/n \sum (R^a)^2]^{1/2}}$$

are that if it is equal to zero, the forecast is perfect; if it is equal to one, the forecast is only as accurate as a forecast of no change; if it is greater than one, the forecast is worse than a forecast of no change. Then the coefficient is decomposed into three parts: a bias component, a regression component and a disturbance component.<sup>35</sup> The bias component measures whether a systematic bias is present in the forward rates. In general, a value above 0.1 or 0.2 is considered as an indicator that a systematic bias exists. The regression component indicates the ability of the forward rates to replicate the movements of the actual bill rates. The disturbance component measures the proportion of the discrepancies between the forward rates and the actual bill rates which is due to randomness. Since the three components add to one, unbiased forecasts will yield values of the first two components close to zero and that of the third component close to one.

Test 2: Perform the regressions

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where  $R^f$  is the forward rate and  $R^a$  is the corresponding realized bill rate.

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$$\begin{aligned} \text{Bias component} &= (M^f - M^a)^2 / [1/n \Sigma (R^f - R^a)^2] \\ \text{Regression component} &= (S_f - S_a)^2 / [1/n \Sigma (R^f - R^a)^2] \\ \text{Disturbance component} &= 2(1-p)S_f S_a / [1/n \Sigma (R^f - R^a)^2] \end{aligned}$$

where  $M^f$ ,  $M^a$ ,  $S_f$ ,  $S_a$  are the means and standard deviations of the series of forward rates and bill rates, respectively, and  $p$  is their correlation coefficient.

$$(3.3) \quad r_{3,t+3} = a + b_{t+3}r_{3,t} + u_t$$

$$(3.4) \quad r_{6,t+6} = a + b_{t+6}r_{6,t} + u_t$$

where  $r_{3,t+3}$  and  $r_{6,t+6}$  are the actual three- and six-month interest rates realized at time  $t+3$  and  $t+6$ , respectively, and  ${}_{t+3}r_{3,t}$  and  ${}_{t+6}r_{6,t}$  are the computed forward rates from above. The claim that implicit forward rates are unbiased estimates of future short-term rates is supported by the null hypothesis  $H_0 : (a,b) = (0,1)$ , and the error term  $u_t$  is serially uncorrelated.

### 3.3.1.2 Test of Consistency

The second test to perform on the pure expectations theory is a test of consistency. Five expectations formation processes which are frequently used in theoretical and empirical literature are chosen. The test seeks to determine whether the implicit forward rates are consistent with these expectations formation models. It is done by replacing the expected short-term rates ( ${}_{t+1}r^e_{1,t}$ ) in the alternative models by the computed forward rates (three- and six-month). The regressions are then performed for both the three- and six-month rates to see if the regression coefficients are significant and fall within the permissible range as imposed

by each model. Those models with coefficients outside the admissible range are discarded as inaccurate models for expectations formation.

Following this procedure, the acceptable models are evaluated by two criteria: goodness-of-fit and ability to forecast. The indicator for the goodness-of-fit criterion is the square root of the mean square deviation (M.S.S.D) of each model.<sup>36</sup> To assess the predictive power of the models, twenty-four forecasts (both three-month-ahead and six-month-ahead rates) are obtained for each model over the period July 1983 to June 1985.<sup>37</sup> The mean error, the absolute error and the root mean square error along with values of the Theil's inequality coefficient of the twenty-four predictions are computed for each model. These summary statistics will indicate the comparative performance of the alternative models of expectations formation.<sup>38</sup>

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<sup>36</sup> This criterion has been used by Alex Cukierman in his study of expectational processes (1977). He defined the square root of the mean square deviation (M.S.S.D.) as

$$[ \sum (\text{Regression residuals}^2) / (n - q) ]^{1/2}$$

where  $n$  is the number of observations, and  $q$  is the number of independent variables plus the constant (if any).

<sup>37</sup> Additional data for three- and six-month bill rates from January to June 1985 are used for comparing with the forecasts.

<sup>38</sup> Similar tests have been conducted by J. Walter Elliott and Jerome R. Baier (1979).

### 3.3.1.3 Expectations Formation Models

The literature of econometrics provides many models which purport to explain the formation of expectations. Here, five expectations formation models have been selected to perform the consistency test of the pure expectations theory. These models have been used in other research on expectations formation<sup>39</sup> and appear to be relevant to the study of interest rates.

1. Inertial Model: This simple model assumes that the expected one-period-ahead interest rate is identical to the currently prevailing rate. Thus, letting  ${}_{t+1}r^e_{1,t}$  be the expected one-period-ahead short-term rate and  $r_{1,t}$  be the current short-term rate, we have

$$(3.5) \quad {}_{t+1}r^e_{1,t} = br_{1,t}$$

where  $b$  should be insignificantly different from one.

2. Regressive Model: This model, suggested by Keynes, assumes that market participants expect interest rates to regress toward a 'normal' level based on past experience. When the current rate is above (below) the normal rate, it

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<sup>39</sup> These models have been used in the study by Steven D. Dobson, Richard C. Sutch and David E. Vanderford (1973) and in the study by Alex Cukierman (1977).

is expected to fall (rise). The gap between the current level and the normal level is assumed to close at a constant rate which falls between zero and one. Letting  $r^*$  be the long-run normal rate and  $a$  be a constant which falls between zero and one, we have

$$(3.6) \quad {}_{t+1}r^e_{1,t} = r_{1,t} + a(r^* - r_{1,t})$$

There are two versions of the regressive model, depending on how the value of  $r^*$  is defined. First, assuming that  $r^*$  is a constant, we have a linear regressive model.

$$(3.6.1) \quad {}_{t+1}r^e_{1,t} = c + (1-a)r_{1,t}$$

where  $c$  should be greater than zero.

Alternatively, defining  $r^*$  as a weighted average of  $n$  past short-term rates with the most recent past rates being weighted more heavily, a weighted regressive model is obtained.

$$(3.6.2) \quad \begin{aligned} {}_{t+1}r^e_{1,t} &= (1-a)r_{1,t} + a \sum_{i=1}^n w_i r_{1,t-i} \\ &= a_0 r_{1,t} + a_1 r_{1,t-1} + \dots + a_n r_{1,t-n} \end{aligned}$$

The coefficient of  $r_{1,t}$  should be positive and smaller than one, and all the other coefficients should be positive

and gradually decline to zero. The sum of all coefficients should be slightly less than or equal to one.

3. Extrapolative Model: Here, it is assumed that the market expects that a rise (or a fall) in interest rates will continue in the future. Thus, the expected one-period-ahead short-term rate is equal to the current short-term rate plus a constant fraction of the deviation of the current rate from a normal level,  $r^*$ . Letting  $b$  be a constant which falls between zero and one, we have

$$(3.7) \quad {}_{t+1}r^e_{1,t} = r_{1,t} + b(r_{1,t} - r^*)$$

There are two versions of the extrapolative model, depending on the specification of  $r^*$ . The first version is the simple extrapolative model, in which  $r^*$  is assumed to be last period's short-term rate. Thus, the change in interest rates expected in the next period is a constant fraction of the recent change in rates.

$$(3.7.1) \quad {}_{t+1}r^e_{1,t} - r_{1,t} = b(r_{1,t} - r_{1,t-1})$$

Alternatively, assuming that  $r^*$  is a weighted average of  $n$  past rates, we have the weighted extrapolative model.

$$\begin{aligned}
 {}_{t+1}r^e_{1,t} &= (1+b)r_{1,t} - b\left(\sum_{i=1}^n w_i r_{1,t-i}\right) \\
 (3.7.2) \quad {}_{t+1}r^e_{1,t} &= b_0 r_{1,t} + b_1 r_{1,t-1} + \dots + b_n r_{1,t-n}
 \end{aligned}$$

The coefficient of  $r_{1,t}$  should be greater than one and all the other coefficients should be negative and gradually approach to zero. The sum of all coefficients should be slightly less than or equal to one.

4. Extrapolative-regressive model: This model is a synthesis of the regressive model and the extrapolative model. It is expressed in the following equation, with the second term representing the regressive component and the third term representing the extrapolative component.

$${}_{t+1}r^e_{1,t} = r_{1,t} + a(r^* - r_{1,t}) + b(r_{1,t} - r^*)$$

Assuming that  $r^*$  is a weighted average of past short-term rates, the inertial 'pull' of the regressive component and the extrapolative 'push' of the recent trend can be combined into a single expectational variable. A general form of the model is obtained.

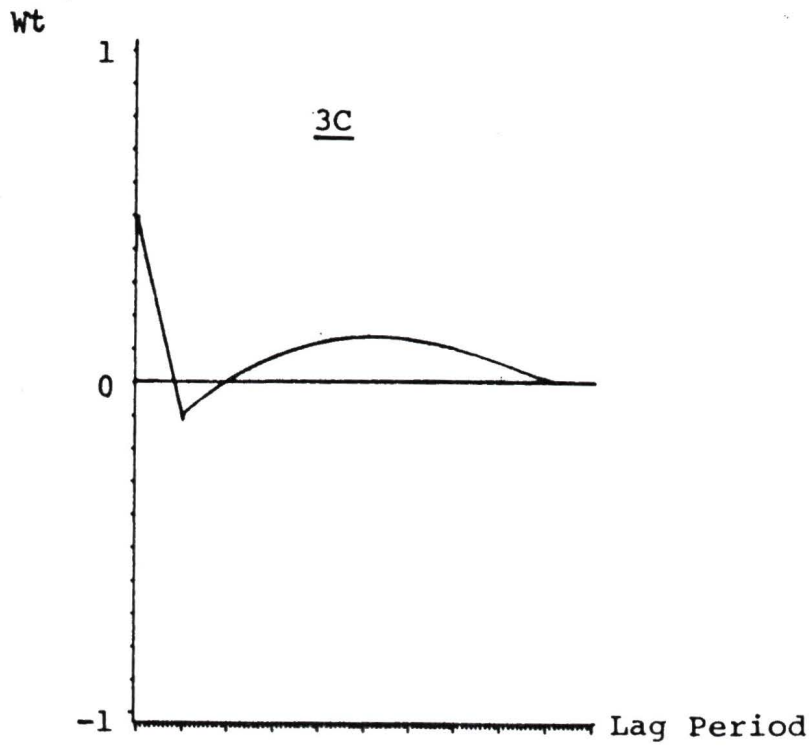
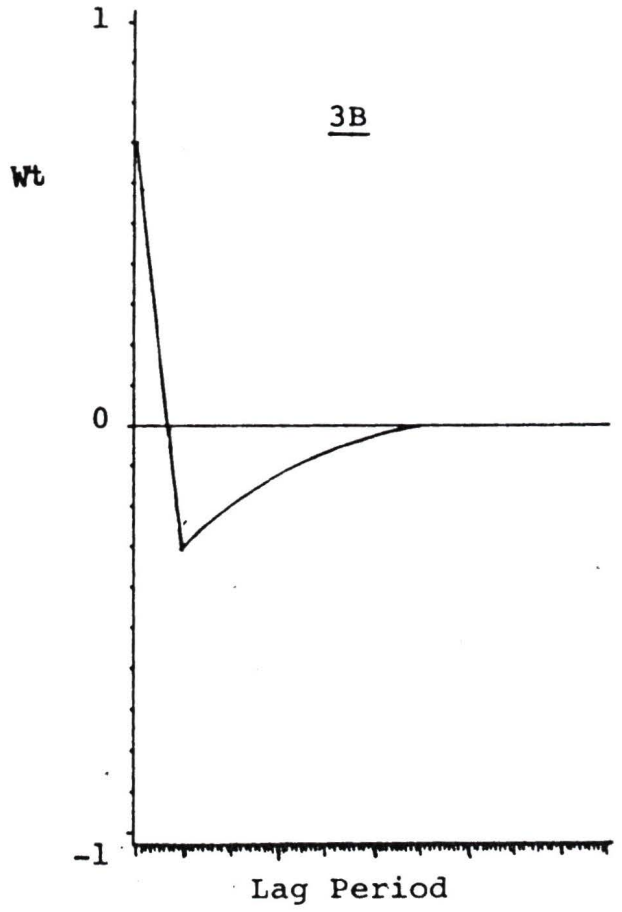
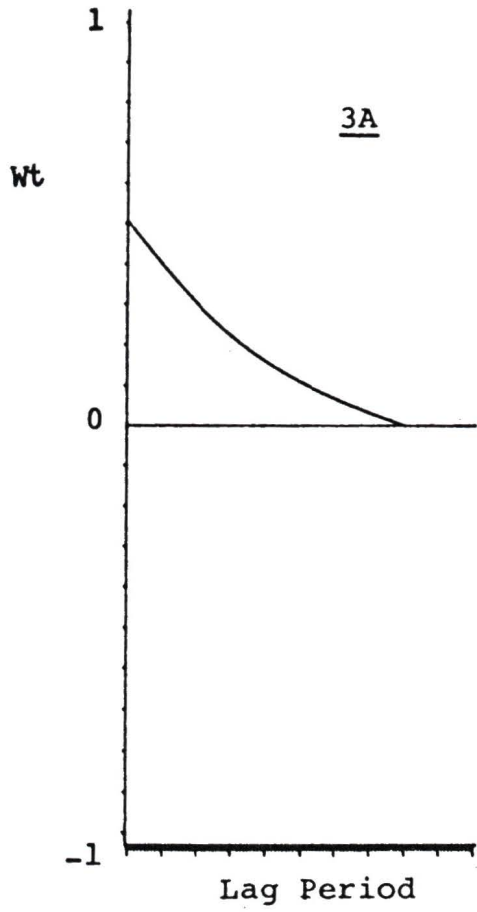
$${}_{t+1}r^e_{1,t} = C + w_0 r_{1,t} + \sum_{i=1}^{\infty} w_i r_{1,t-i}$$

In order to make this equation operational, it is assumed that the first  $m$  lag coefficients lie on a polynomial of low degree, and that rates further in the past can be approximated by a constant. The final version of the model is

$$(3.8) \quad {}_{t+1}r^e_{1,t} = C + w_0 r_{1,t} + \sum_{i=1}^m w_i r_{1,t-i}$$

The lag structure is estimated by the Almon interpolation technique. If the expectation of future rates is primarily determined by regression toward a long-run normal level, the lag structure will resemble the one shown in Figure 3A, where the lag structure should contain positive values only. However, if the extrapolative component is predominant, the lag structure will tend to approximate the one shown in Figure 3B; here, the lag structure starts from negative values and then gradually approaches zero. However, if both expectational forces are present simultaneously, then the lag structure will look like the one in Figure 3C. The lag structure first rises, possibly from negative values, reaches a peak as the extrapolative force dies off, and then falls toward zero as the influence of the regressive component dissipates.

Figure 3: Hypothetical Lag Shapes



It should be noted that the sum of the weights should be equal to or slightly smaller than one, and that  $w_0$  and the weights at the tail should be positive (refer to Figures 3A, 3B and 3C again).

5. Adaptive Model: This approach to expectations formation assumes that the error made in forecasting the current rate is used to adjust the previous period's forecast to obtain the forecast for the next period.<sup>40</sup>

$$(3.9) \quad {}_{t+1}r^e_{1,t} = {}_tr^e_{1,t-1} + a(r_{1,t} - {}_tr^e_{1,t-1})$$

The adaptive coefficient,  $a$ , should lie between zero and one.<sup>41</sup>

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<sup>40</sup> This model, an error-learning model, resembles the one suggested by David Meiselman (1962).

<sup>41</sup> The adaptive model can be expressed as a geometrically distributed lag of past rates through recursive substitution of the lagged values of  ${}_tr^e_{1,t-i}$ .

$${}_{t+1}r^e_{1,t} = a \sum_{i=0}^{\infty} (1-a)^i r_{1,t-i}$$

The adaptive coefficient,  $a$ , cannot fall outside the range  $(0,1)$ . Otherwise, the model is diverging.

### 3.3.2 Tests on the Hicksian Risk-Premium Theory

The presence of risk premiums is indicated by a significantly positive difference between forward rates (three-month-ahead and six-month-ahead) and the corresponding spot rates. To isolate risk premiums from the forward rates, the ARIMA model is used to produce forecasts of three- and six-month bill rates. Before the ARIMA forecasts are used to extract the risk premiums from the forward rates, they are subject to the same test of unbiasedness described in Section 3.3.1.1.

Risk premiums are obtained by subtracting the ARIMA forecasts from the corresponding forward rates. To determine whether the risk premiums are significant, the forward rates are regressed against the ARIMA forecasts, as in equations (3.10) and (3.11). The pure expectations theory corresponds to the null hypothesis:  $(a,b) = (0,1)$ . Support for the risk-premium theory is sought if the joint null hypothesis is rejected.

$$(3.10) \quad {}_{t+3}r_{3,t} = a + b {}_{t+3}r^e_{3,t} + u_t$$

$$(3.11) \quad {}_{t+6}r_{6,t} = a + b {}_{t+6}r^e_{6,t} + u_t$$

Two additional tests on the risk premiums will be performed. One is to find out whether a systematic relationship exists between risk premiums and interest rate levels (equations (3.12) to (3.15)). The other is to test whether risk premiums increase with the degree of risk and uncertainty, using the variance of the past six months' short rate ( $V$ ) as a proxy variable for the latter (equations (3.16) and (3.17)).

$$(3.12) \quad {}_{t+3}P_{3,t} = a + br_{3,t} + u_t$$

$$(3.13) \quad {}_{t+6}P_{6,t} = a + br_{6,t} + u_t$$

$$(3.14) \quad {}_{t+3}P_{3,t} = a + br_{3,t} + cr_{3,t-1} + u_t$$

$$(3.15) \quad {}_{t+6}P_{6,t} = a + br_{6,t} + cr_{6,t-1} + u_t$$

$$(3.16) \quad {}_{t+3}P_{3,t} = a + bV_3 + u_t$$

$$(3.17) \quad {}_{t+6}P_{6,t} = a + bV_6 + u_t$$

### 3.3.3 Test on the Preferred Habitat Theory

The preferred habitat theory, a theory which simultaneously incorporates the influences of expectations, risk aversion and segmentational forces on the term structure, will be tested using the model developed by Modigliani and Sutch. This model can be expressed as

$$(3.18) \quad R_{n,t} = P + w_0 r_{1,t} + \sum_{i=1}^m w_i r_{1,t-i} + u_t$$

where  $R_{n,t}$  is the yield on an  $n$ -period bond, the long-term rate;  $r_{1,t}$  is the yield on an one-period bond, the short-term rate;  $\sum w_i r_{1,t-i}$  is the distributed lag on  $m$  past one-period rates;  $P$  is the risk premium (or discount) and  $u_t$  is the error term. The risk premium (or discount),  $P$ , incorporates the influence of any factors which make the relationship between the long rate and the short rate diverge from that implied by the influence of expectations alone. It is basically a function of the maturity composition of outstanding debt and the degree of risk aversion of the market participants.

Here, the long-term rate is the twelve-month bill rate and the short-term rate is the three-month bill rate. The logic of this model requires that the short-term rate has a maturity equal to the period between observations. Since the three-month bill rate is chosen as the short-term rate, quarterly observations will be used for both the short-term and the long-term rates.

The model will be estimated by the Almon lag estimation technique. The degree of the polynomial and the length of the lag will be determined by experimentation. The shape of

the lag structure will reveal whether expectations are determined by regressive forces, extrapolative forces, or both. The magnitude and significance of the constant term,  $P$ , will indicate the extent of market segmentation and the degree of risk aversion of the market participants.

## Chapter IV

### EMPIRICAL RESULTS

This chapter will present and analyze the empirical results based on the testing procedures described in the previous chapter. It is, therefore, helpful to the reader to refer to the models and equations given in that chapter. As we shall recall from Section 3.2, two series of forward rates (three-month rates three-month-hence and six-month rates six-month-hence) have already been derived from the actual bill rates and they will be used in the following tests.

#### 4.1 Test Results of the Pure Expectations Theory

As described in Section 3.3.1.1, there are two basic tests applied to the pure expectations theory. The first basic test is concerned with whether forward rates are unbiased estimates of expected future rates. Here, there are two alternative methods of testing for bias: Theil's decomposition test and the test based on equations (3.3) and (3.4) given on page 62. The second basic test is concerned with

consistency between forward rates and alternative expectations formation models. Under the consistency test, the models are evaluated on the basis of three criteria: (1) whether the estimated coefficients fall within the admissible region imposed by each model; (2) the goodness-of-fit of the models; and (3) the ability of the models to produce accurate forecasts of future interest rates.

#### 4.1.1 Test of Unbiasedness

As mentioned above, there are two alternative tests of unbiasedness. The first is Theil's decomposition test. It involves the calculation of Theil's inequality coefficient which is a measure of fit between the forecasts and the actual interest rates. The smaller the coefficient, the more accurate are the forecasts. The coefficient is then decomposed into three components: a bias component, a regression component and a disturbance component. The ideal distribution of inequality over the three sources of prediction error is that the first two components equal zero and that the third component equals one.

Results obtained with Theil's decomposition method are shown in Tables 1 and 2. When the entire sample period is considered (1975 - 1984), the Theil's inequality coefficient obtained is 0.0758 and 0.1249 for the three- and six-month

Table 1: Theil's Decomposition Test on Three-Month Forward Rates

<u>Time Period</u>	<u>Inequality</u>	<u>Bias</u>	<u>Regression</u>	<u>Disturbance</u>
	<u>Coefficient</u>	<u>Component</u>	<u>Component</u>	<u>Component</u>
Jan.75 - Dec.84	0.0758	0.0024	0.0002	0.9974
Jan.75 - Dec.76	0.0231	0.0590	0.1659	0.7751
Jan.76 - Dec.77	0.0215	0.1887	0.0620	0.7493
Jan.77 - Dec.78	0.0301	0.0017	0.3968	0.6015
Jan.78 - Dec.79	0.0397	0.2723	0.2439	0.4838
Jan.79 - Dec.80	0.0836	0.0202	0.0135	0.9663
Jan.80 - Dec.81	0.1020	0.0206	0.0028	0.9766
Jan.81 - Dec.82	0.0937	0.0091	0.0237	0.9672
Jan.82 - Dec.83	0.0736	0.2997	0.0401	0.6602
Jan.83 - Dec.84	0.0528	0.1883	0.0717	0.7400

Table 2: Theil's Decomposition Test on Six-Month Forward Rates

<u>Time Period</u>	<u>Inequality</u> <u>Coefficient</u>	<u>Bias</u> <u>Component</u>	<u>Regression</u> <u>Component</u>	<u>Disturbance</u> <u>Component</u>
Jan.78 - Dec.84	0.1249	0.0027	0.0032	0.9941
Jan.78 - Dec.79	0.0825	0.6517	0.1842	0.1641
Jan.79 - Dec.80	0.1114	0.0980	0.0239	0.8781
Jan.80 - Dec.81	0.1414	0.1665	0.0411	0.7924
Jan.81 - Dec.82	0.1431	0.0002	0.0034	0.9964
Jan.82 - Dec.83	0.1340	0.5251	0.0870	0.3879
Jan.83 - Dec.84	0.1130	0.0667	0.0633	0.8700

forward rates, respectively. This indicates that the predictive power of forward rates is rather good. Decomposing the coefficient yields a very small bias component equal to 0.0024 and 0.0027, suggesting the absence of a systematic bias in the forward rates. The corresponding regression components are also very small (0.0002 and 0.0032), resulting in a large disturbance component which is very close to one in both cases. If forecasts are to be accurate, the Theil's inequality coefficient as well as the bias component and the regression component should approximate zero, and the disturbance component should be close to one. Therefore, the above results give an excellent and desirable distribution for the three sources of prediction error.

However, when the entire sample is subdivided into shorter overlapping subperiods, each of which consists of two years' observations, results reveal temporal variation in the magnitude of the bias component over the entire sample period.

Again in Tables 1 and 2, it is found that the bias component is generally larger in magnitude for each subperiod than for the entire sample. Of the nine subperiods for the three-month forward rates, four of them yield a bias component which is greater than 0.1. In the case of the six-month

forward rates, a greater bias component is obtained in some subperiods.

The difference between the sub-sample and full-sample results may be partly due to the hiking of interest rates during the early 1980s, producing fluctuations in interest rates, and thus increasing the numerator in the computation of the disturbance component (refer to Footnote 35 ). This results in a large disturbance component and a correspondingly small bias component. It is evidenced by examining the subperiods during the early 1980s, for which the bias component is generally smaller. Consequently, Theil's decomposition test produces a smaller bias component when it is performed on the entire sample.

From the above results, forward rates appear to be good predictors of future interest rates on average over long time periods, but less so over shorter time spans as indicated by the larger bias component for some of the subperiods. However, it should be noted that Theil's decomposition test is not a conclusive test. Without any precise critical value for the bias component, a value above 0.1 or 0.2 is only indicative of the existence of a systematic prediction error. Moreover, the Theil's inequality coefficient is always very small for both the entire sample and the sub-

periods (substantially below 0.1 for the three-month forward rates and less than 0.15 for the six-month forward rates), indicating an overall small prediction error. Therefore, even though the bias component in some subperiods is above 0.1, the magnitude of the bias, if any, is very small.

In summary, the evidence provided by Theil's decomposition test is mixed as to whether a bias exists in the forward rates. If a bias does exist, it is expected to be greater for the six-month forward rates than for the three-month forward rates as the Theil's inequality coefficient is always greater for the former than for the latter.

The second test of unbiasedness is a test on the null hypothesis that  $(a,b)$  in equations (3.3) and (3.4) are equal to  $(0,1)$ . Unbiasedness requires that the null hypothesis is not rejected and that the error term in each equation is not serially correlated.<sup>42</sup> Rejection of the null hypothesis or the presence of autocorrelation in the error term or both will indicate that forward rates are biased estimates of expected future bill rates.

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<sup>42</sup> If the error term is serially correlated, it means that the forecast fails to incorporate information available in past forecast errors.

Test results for both forward rate series are presented in Table 3. By comparing the critical F-values given in the table with the observed F-values, the joint null hypothesis as well as the individual hypotheses  $a=0$  and  $b=1$  are rejected at the 5% significance level for the three-month forward rates. For the six-month forward rates, the evidence is even stronger as all the above hypotheses are rejected not only at the 5% level but also at the 1% level of significance. A further indication of biasedness is the presence of serious positive serial correlation in the error term, as shown by the Durbin-Watson statistic. This violates one of the conditions of unbiasedness, that is, the residuals are independently and identically distributed.

While Theil's decomposition test does not provide conclusive evidence as to whether a bias exists, this test does indicate the existence of a bias in both forward rate series. In conclusion, a tentative interpretation of the evidence is that forward rates are biased forecasts of future interest rates, and that the bias tends to be greater for the six-month forward rates than for the three-month forward rates.

Table 3: Test of Unbiasedness on Forward Rates

$$r_{3,t+3} = a + b_{t-3}r_{3,t}$$

$$r_{6,t+6} = a + b_{t+6}r_{6,t}$$

Unrestricted Form:

	<u>Adj.R<sup>2</sup></u>	<u>D.W.</u>	<u>D/F</u>
$r_{3,t+3} = 1.3920 + 0.8663_{t+3}r_{3,t}$ (2.52)      (18.06)	0.74	0.45	1,115
$r_{6,t+6} = 6.4391 + 0.4799_{t+6}r_{6,t}$ (5.73)      (5.33)	0.25	0.24	1,82

(t-ratios in parentheses)

Hypotheses Testing:

	<u>Three-month</u>		<u>Six-month</u>	
	F-value	D/F	F-value	D/F
H <sub>0</sub> : (a, b) = (0, 1)	4.029	2,115	16.856	2,82
H <sub>0</sub> : a = 0	6.339	1,115	32.789	1,82
H <sub>0</sub> : b = 1	7.764	1,115	33.401	1,82
Critical F-values:	F(1,60,1%) = 7.08		F(1,60,5%) = 4.00	
	F(2,60,1%) = 4.98		F(2,60,5%) = 3.15	
	F(1,120,1%) = 6.85		F(1,120,5%) = 3.92	
	F(2,120,1%) = 4.79		F(2,120,5%) = 3.07	
Durbin-Watson:	D <sub>L</sub> (1,80,5%) = 1.61			
Durbin-Watson:	D <sub>L</sub> (1,100,5%) = 1.65			

#### 4.1.2 Test of Consistency

The consistency test attempts to determine whether the computed forward rates are consistent with the alternative expectations formation models. There are basically three criteria to evaluate the alternative models: whether the estimated coefficients are consistent with the requirements imposed by the formulations of the models, the goodness-of-fit and the predictive power of the models.

The estimation results of the models by the ordinary least squares (OLS) method show that all the models exhibit some degree of positive serial correlation, a condition which is often interpreted as a sign of misspecification of the models.<sup>43</sup> After correcting for autocorrelation by the Cochrane-Orcutt procedure, the results (shown in Tables 4 and 5) are found to be very similar to those obtained by OLS, except that the estimates of the standard errors of the coefficients obtained from OLS tend to be smaller than the true standard errors, thus making the estimated coefficients

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<sup>43</sup> Pindyck and Rubinfeld stated in Econometric Models and Economic Forecasts, page 153,

Positive serial correlation frequently occurs in time series analysis, either because of correlation in the measurement error component of the error term or more likely, because of the high degree of correlation over time present in the cumulative effects of the omitted variables in the regression model.

Table 4: Test of Consistency by Cochrane-Orcutt Procedure - Three-Month Forward Rates

<u>Model</u>	<u>Regression Equation</u>	<u>Adj.R<sup>2</sup></u>	<u>D.W.</u>
Inertial	$t+3r_{3,t} = 1.0112r_{3,t}$ (112.08)	0.96	2.01
Linear regressive	$t+3r_{3,t} = 0.8013 + 0.9442r_{3,t}$ (2.59) (34.76)	0.96	2.00
Simple ex- trapolative	$t+3r_{3,t} - r_{3,t} = 0.1846r_{3,t}$ (2.18)	0.17	2.08
Weighted regressive/ extrapo. 1	$t+3r_{3,t} = 1.1898r_{3,t} - 0.1799r_{3,t-1}$ (14.21) (-2.14)	0.96	2.04
Weighted regressive/ extrapo. 2	$t+3r_{3,t} = 1.2018r_{3,t} - 0.4244r_{3,t-1}$ (14.75) (-3.59)  $+0.2344r_{3,t-2}$ (2.87)	0.96	2.08
Adaptive	$t+3r_{3,t} - t+3r_{3,t-1}$ $= 1.2899(r_{3,t} - t+3r_{3,t-1})$ (17.29)	0.62	1.94

CONTINUE FROM TABLE 4

$$\begin{aligned}
 \text{Extrapolative-} & \quad t+3r_{3,t} = 0.5505 + 1.1170r_{3,t} & 0.93 & 1.97 \\
 \text{regressive 1} & & (2.39) & (14.76) \\
 & -0.2040r_{3,t-1} -0.0736r_{3,t-2} +0.0120r_{3,t-3} \\
 & (-2.99) & (-2.73) & (0.88) \\
 & +0.0528r_{3,t-4} + 0.0488r_{3,t-5} \\
 & (2.49) & (2.73)
 \end{aligned}$$

$$\begin{aligned}
 \text{Extrapolative-} & \quad t+3r_{3,t} = 0.4806 + 1.0971r_{3,t} & 0.93 & 1.98 \\
 \text{regressive 2} & & (2.24) & (15.03) \\
 & -0.1650r_{3,t-1} -0.0752r_{3,t-2} -0.0103r_{3,t-3} \\
 & (-2.87) & (-2.69) & (-0.92) \\
 & +0.0297r_{3,t-4} + 0.0447r_{3,t-5} +0.0348r_{3,t-6} \\
 & (2.14) & (2.63) & (2.75)
 \end{aligned}$$

Note: The weighted regressive model and the weighted extrapolative model have the same formulation.

The lag structure of the extrapolative-regressive model is of order 2.

Table 5: Test of Consistency by Cochrane-Orcutt Procedure - Six-Month Forward Rates

<u>Model</u>	<u>Regression Equation</u>	<u>Adj.R<sup>2</sup></u>	<u>D.W.</u>
Inertial	$t+6r_{6,t} = 1.0090r_{6,t}$ (72.89)	0.87	2.17
Linear regressive	$t+6r_{6,t} = 1.3370 + 0.9049r_{6,t}$ (2.21) (18.53)	0.88	2.12
Simple ex- trapolative	$t+6r_{6,t} - r_{6,t} = -0.5377r_{6,t}$ (-4.36)	0.23	2.14
Weighted regressive/ extrapo. 1	$t+6r_{6,t} = 0.4672r_{6,t} + 0.5456r_{6,t-1}$ (3.83) (4.46)	0.89	2.13
Weighted regressive/ extrapo. 2	$t+6r_{6,t} = 0.4792r_{6,t} + 0.7393r_{6,t-1}$ (3.96) (4.49)  $-0.2073r_{6,t-2}$ (-1.71)	0.89	2.19
Weighted regressive/ extrapo. 3	$t+6r_{6,t} = 0.4806r_{6,t} + 0.7088r_{6,t-1}$ (4.07) (4.23)  $-0.4686r_{6,t-2} + 0.2929r_{6,t-3}$ (-2.79) (2.46)	0.89	2.13

CONTINUE FROM TABLE 5

Adaptive  $t+6r_{6,t} - t+5r_{6,t-1}$   
 $= 0.5421(r_{6,t} - t+6r_{6,t-1})$  0.53 1.96  
 (9.06)

Extrapolative-  $t+6r_{6,t} = 0.7828 + 0.4506r_{6,t}$  0.86 2.03  
 regressive 1 (1.79) (3.67)  
 $+0.6120r_{6,t-1} -0.2667r_{6,t-2} -0.1097r_{6,t-3}$   
 (3.66) (-2.46) (-1.92)  
 $+0.2552r_{6,t-4}$   
 (3.30)

Extrapolative-  $t+6r_{6,t} = 0.6597 + 0.4306r_{6,t}$  0.87 2.04  
 regressive 2 (1.49) (3.54)  
 $+0.5973r_{6,t-1} -0.1566r_{6,t-2} -0.2115r_{6,t-3}$   
 (3.71) (-2.24) (-2.95)  
 $+0.0507r_{6,t-4} + 0.2484r_{6,t-5}$   
 (1.70) (4.17)

Note: The weighted regressive model and the weighted extrapolative model have the same formulation.

The lag structure of the extrapolative-regressive model is of order 3.

appear to be more significant than they actually are. However, when the equations are estimated by the transformed variables created by the Cochrane-Orcutt procedure, the goodness-of-fit statistics (adjusted R-square, Durbin-Watson statistic, etc.) are no longer comparable due to the transformation of the dependent variable.

The first criterion with which to evaluate the models is to check whether the estimated coefficients are consistent with the requirements imposed by the formulation of each model.

In terms of this criterion, both the inertial model and the linear regressive model are found to perform well. The estimated coefficient in the inertial model is always insignificantly different from one,<sup>44</sup> which is consistent with the basic assumption of the model that the best estimate of next period's rate is current period's rate. In the linear regressive model, the estimated coefficient not only falls within the admissible region (0,1), but is also highly significant. For this model, the implied long-run normal rate,

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Three-month bill rates:

$$t\text{-statistic} = (1.0112 - 1) / 0.0090^* = 1.24$$

Six-month bill rates:

$$t\text{-statistic} = (1.0090 - 1) / 0.0138^* = 0.65$$

\* It is the standard error of the estimated coefficient.

$r^*$ , is found to be 14.36% and 14.06% for the three- and six-month bill rates, respectively.<sup>45</sup> Even though they are higher than the means of the actual bill rates over the sample periods (10.89% and 11.96% for the three- and six-month bill rates, respectively), these estimates are consistent with the abnormally high interest rates experienced during the early 1980s.

Based on the belief that both the three- and six-month bill rates are generated by the same expectations formation process, which is quite reasonable, the adaptive model and the simple extrapolative model cannot be viewed as acceptable models. The adaptive coefficient in the adaptive model for the three-month bill rates not only falls outside the admissible region  $(0,1)$ , but is also significantly different from one, implying that the adaptive model is unstable and diverging. While the coefficient in the simple extrapolative model is expected to be positive and less than one, it is found to be significantly negative for the six-month bill rates. Therefore, these two models are discarded.

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<sup>45</sup> From equations (3.6) and (3.6.1),  $r^* = c/a$ .

For three-month bill rates:

$$a = (1-0.9442);$$

$$\text{therefore, } r^* = 0.8013/(1-0.9442) = 14.36\%.$$

For six-month bill rates:

$$a = (1-0.9049);$$

$$\text{therefore, } r^* = 1.3370/(1-0.9049) = 14.06\%.$$

The results of the weighted extrapolative model and the weighted regressive model<sup>46</sup> indicate that the lag is very short.<sup>47</sup> It is found that the number of past short rates included should be no more than two for the three-month bill rates and no more than three for the six-month bill rates. F-tests on longer lags indicate that the additional lags are no longer significant.

For the three-month bill rates, the estimated coefficient of the current short rate is significantly greater than one, and that of the lagged one-period rate is always negative with its absolute value smaller than one. They agree with the restrictions imposed on the coefficients in the weighted extrapolative model (equation 3.7.2).<sup>48</sup> However, for the six-month bill rates, the results support a weighted regres-

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<sup>46</sup> The formulations of these two models are the same except that they impose different restrictions on their parameters. See equations (3.6.2) and (3.7.2).

<sup>47</sup> The number of past short rates selected for the weighted regressive or the weighted extrapolative model is determined by a F-test which indicates whether an additional past short rate (or the addition of a group of past short rates) will contribute significantly in explaining the variance of the long rate.

<sup>48</sup> A model with two lagged past rates has been tried. The results are also presented in Table 4. The estimated coefficient of the lagged two-period rate is positive, which is inconsistent with the weighted extrapolative model. Moreover, since  $(1+b) = 1.2018$ ,  $b = 0.2018$ . The value of  $w_1$  is found to be 2.1 ( $0.4244 / 0.2018$ ) which falls outside the permissible range (0, 1). Thus, this specification is considered inferior.

sive model (equation 3.6.2) as the estimated coefficients of the current short rate and the lagged one-period rate are always positive and less than one.<sup>49</sup>

The results for the extrapolative-regressive model are quite striking.<sup>50</sup> The specification which produces the most satisfactory result is a second-degree polynomial with five (or six) lags for the three-month bill rates and a third-degree polynomial with four (or five) lags for the six-month

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<sup>49</sup> Models with two and three lags have also been tried. The results are presented in Table 5. For the two-lag specification, the coefficient of the second lag has the wrong sign (it should be positive and less than one). Moreover, since  $(1-a) = 0.4792$ ,  $a = 0.5208$ . Solving for  $w_1$ , it is found to be greater than one ( $0.7393 / 0.5208$ ). The same for the three-lag specification.

<sup>50</sup> There are basically two criteria for choosing the alternative specifications. One is the goodness-of-fit as indicated by the coefficient of determination (adjusted R-square). The other is the plausibility of the estimated lag structure. Unfortunately, the first criterion is not very useful. The adjusted R-square and the other regression coefficients are not sensitive either to the degree or to the length of the distributed lag. Moreover, it is not possible to do a F-test for determining whether a group of lag variables contributes significantly to the explanation of the dependent variable between polynomials of identical degree but different lengths. The reason is that the difference between the degrees of freedom of these polynomials is practically zero. Since toward the end of the lag structure, the weights should not fall from positive values to values below zero (shown in Figures 3A, 3B and 3C), those specifications with this lag shape are first eliminated (almost all of the fourth degree polynomials and some of the third degree polynomials). Those remaining are mostly second degree polynomials of various lengths. The length of the lag structure is then determined by the degree of significance of the lag coefficients.

bill rates. The lag structure is estimated with a zero restriction on the far end because toward the end the lag structure is expected to converge to zero. The length of the lag is found to be much shorter than those obtained in other studies involving longer maturities, and the coefficient of the current short rate for the three-month bill rate is always significantly positive and greater than one. These results are not necessarily inconsistent with the other studies,<sup>51</sup> but they tend to undermine the usefulness of distributed lag models in the study of short maturities.

By the first criterion of consistency, two models (the adaptive model and the simple extrapolative model) have been discarded as inappropriate expectations formation models under the pure expectations theory. An alternative criterion to discriminate the remaining models (inertial, linear regressive, weighted regressive, weighted extrapolative and extrapolative-regressive) is the goodness-of-fit criterion using the M.S.S.D. Unfortunately, this criterion does not prove to be very useful. As shown in Table 6,

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<sup>51</sup> In the paper by Dobson, Sutch and Vanderford (1976), they selected a third-degree polynomial with an eighteen-quarter lag for maturities 1-, 3-, 5-, 10-, 15- and 25-years. The coefficients of the lag structures for longer maturities were mostly significant. However, for one-year maturity, 16 out of the total 18 lags were insignificant, and the coefficient of the current short rate was very close to one (0.9764). They also stated that (pp. 1047) the shorter the maturity, the earlier did the lag coefficients become negative and insignificant.

Table 6: Square Root of Mean Square Deviation (M.S.S.D.)

<u>Model</u>	<u>Three-Month</u>	<u>Six-Month</u>
Inertial	0.719	1.182
Simple regressive	0.686	1.147
Simple extrapolative	0.716	1.091
Weighted regressive/ extrapolative 1	0.708	1.087
Weighted regressive/ extrapolative 2	0.694	1.106
Weighted regressive/ extrapolative 3	-----	1.055
Adaptive	0.814	1.060
Extrapolative- regressive 1	0.701	1.016
Extrapolative- regressive 2	0.701	0.989

Note: The adaptive model and the simple extrapolative model are included for reference.

when the entire sample is considered (1975 - 1984), the M.S.S.D's of all the models are insignificantly different from each other even though they are fairly small compared with the means of the dependent variables.<sup>52</sup>

To investigate the goodness-of-fit of the models in greater detail, the entire sample is again divided into overlapping subperiods, each of which consists of twenty-four monthly observations. This may reveal the relative performance of the models over different time periods. As shown in Table 7, both the linear regressive model and the extrapolative-regressive model appear to be better than the other models. However, since the difference in M.S.S.D. between the alternative models in most cases is less than 0.05, it is difficult to draw any strong conclusion as to which model is the best model.

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<sup>52</sup> The means of the three-month and six-month forward rates are 11.04% and 12.10%, respectively. The M.S.S.D. is in the neighbourhood of 6% of the former and 9% of the latter.

Table 7: Square Root of Mean Square Deviation (M.S.S.D.) of the Subperiods

Three-Month Forward Rates

JAN.	DEC.	INE	LIRE	ADAP	SIEX	WE1	WE2	WE3	EXR1	EXR2
75-76	0.36	0.38	0.42	0.35	0.34	0.39			0.33*	0.34
76-77	0.27	0.33	0.28	0.23	0.25	0.29			0.28	0.28
77-78	0.36	0.34	0.52	0.42	0.37	0.42			0.43	0.43
78-79	0.41	0.30	0.52	0.43	0.38	0.41			0.47	0.47
79-80	1.09	1.04	1.12	1.09	1.13	1.07			1.05	1.05
80-81	1.32	1.25	1.40	1.28	1.35	1.28			1.22	1.22
81-82	0.89	0.88	0.99	0.81	0.86	0.85			0.85	0.83
82-83	0.50	0.55	0.59	0.52	0.46	0.52			0.58	0.56
83-84	0.65	0.62	0.85	0.70	0.65	0.67			0.64	0.64

Six-Month Forward Rates

78-79	0.56	0.69	0.43	0.57	0.62	0.65	0.67	0.63	0.63	
79-80	1.38	1.40	1.21	1.10	1.14	1.16	1.19	1.13	1.07	*
80-81	1.82	1.50	1.62	1.48	1.57	1.66	1.58	1.36	1.32	*
81-82	1.61	1.57	1.41	1.56	1.57	1.65	1.51	1.43	1.37	*
82-83	1.18	1.24	1.09	1.21	1.16	1.15	1.08	1.20	1.15	
83-84	0.92	0.85	0.91	0.98	0.91	0.90	0.92	1.01	1.04	

Note:

\* indicates the smallest M.S.S.D. for each subperiod.

INE = Inertial model

LIRE = Linear regressive model

ADAP = Adaptive model

SIEX = Simple extrapolative model

WE1 WE2 WE3 = Weighted regressive/extrapolative model

EXR1 EXR2 = Extrapolative-regressive model

Whether applied to the entire sample or subperiods, this criterion does not yield adequate evidence for discriminating the remaining models, nor does it indicate the relative performance of the models over different time periods. Therefore, it cannot be used to judge the performance of the models.

The third criterion to evaluate the models is concerned with the ability of the expectations formation models to predict future interest rates. Twenty-four forecasts (both three-month-ahead and six-month-ahead rates) are obtained for each model from July 1983 to June 1985. The forecasts are obtained with each model being re-estimated six times and each set of estimated coefficients used to generate four forecasts. A moving estimation period of sixty months is used.<sup>53</sup> Tables 8 and 9 give the mean, absolute mean and root mean square errors together with the values of the Theil's inequality coefficient of the forecasts of the remaining models.<sup>54</sup>

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<sup>53</sup> An illustration of this method is to use interest rate series (three- and six-month) over the period March 1980 to February 1985 to generate three-month-ahead and six-month-ahead forecasts for March to June 1985.

<sup>54</sup> Results for the weighted regressive/extrapolative model with more than one lag are also presented for comparison purposes.

Table 8: Predictive Ability - 24 Forecasts of Three-Month Bill Rate Three-Month-Hence from July 1983 to June 1985

	<u>MODELS</u>					
	<u>Inertial</u>	<u>Simple</u> <u>Regressive</u>	<u>Weighted</u> <u>Regressive/</u> <u>Extrapolative</u>		<u>Extrapolative-</u> <u>Regressive</u>	
			<u>1-lag</u>	<u>2-lags</u>	<u>5-lags</u>	<u>6-lags</u>
Mean error	0.650	0.343	0.665	0.650	0.307	0.321
Root-mean- square-error	0.851	0.678	0.826	0.835	0.575	0.590
Mean absolute error	0.662	0.448	0.665	0.674	0.424	0.410
Theil's inequality coefficient	1.167	0.750	1.201	1.171	0.773	0.790
-bias	0.732	0.334	0.771	0.712	0.343	0.367
-regression	0.097	0.221	0.071	0.106	0.278	0.256
-disturbance	0.171	0.445	0.158	0.182	0.379	0.377

Table 9: Predictive Ability - 24 Forecasts of Six-Month Bill Rate Six-Month-Hence from July 1983 To June 1985

	<u>MODELS</u>						
	<u>Inertial</u>	<u>Simple Re-</u> <u>gressive</u>	<u>Weighted</u> <u>Regressive/</u> <u>Extrapolative</u>			<u>Extrapolative-</u> <u>Regressive</u>	
			<u>1-lag</u>	<u>2-lags</u>	<u>3-lags</u>	<u>4-lags</u>	<u>5-lags</u>
Mean error	0.701	0.397	0.628	0.608	0.602	0.453	0.570
Root-mean- square-error	1.079	0.892	0.907	0.838	0.879	0.841	0.943
Mean absolute error	0.854	0.664	0.716	0.682	0.688	0.696	0.795
Theil's inequality coefficient	0.986	0.758	1.224	0.878	0.881	0.883	0.978
-bias	0.589	0.467	0.546	0.589	0.536	0.556	0.657
-regression	0.195	0.227	0.137	0.203	0.194	0.157	0.094
-disturbance	0.216	0.306	0.317	0.209	0.270	0.287	0.249

With the exception of the extrapolative-regressive model, the linear regressive model consistently produces the lowest mean, absolute mean and root mean square errors among all the remaining models for both interest rate series.<sup>55</sup> Comparing the linear regressive model and the extrapolative-regressive model, the results are inconclusive. The evidence fails to reveal that either one of them consistently outperforms the other. This result is consistent with the previous finding in the second test of consistency where the M.S.S.D.'s of the linear regressive model and the extrapolative-regressive model are, in general, marginally smaller than those of the remaining models.

However, the linear regressive model consistently produces the lowest Theil's inequality coefficient among all the models, including the extrapolative-regressive model. The decomposition of the coefficient further indicates that the forecasts of the linear regressive model always have a smaller bias component than those of all the other models. Although the bias component implies the existence of a systematic prediction error, the disturbance component of the linear regressive model is generally bigger than those of the other models, indicating that a larger proportion of the

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<sup>55</sup> The only exception is the root mean square error in the six-month interest rate series, which is slightly larger than those of the weighted regressive/extrapolative model.

forecast error is due to unsystematic or random disturbances.

Based on the Theil's inequality coefficient, it can be concluded that the linear regressive model is superior to the extrapolative-regressive model, and is the best among all the alternative models. The finding that the predictive ability of the linear regressive model in most cases is even higher than those of the models with one or more lagged variables implies that the lagged variables do not contain additional information, besides that already contained in the current short rate, that can improve the forecasting ability of the model.

The empirical support for a linear regressive model of expectations formation suggests that market participants expect future interest rates to regress toward some long-run 'normal' level. As the coefficient in the linear regressive model is found to be close to one (0.9442 and 0.9049 for three- and six-month forward rates, respectively), market participants appear to formulate expectations based primarily on the current short-term rate plus a constant which is determined by the long-term normal rate.

### 4.1.3 Summary of Test Results of the Pure Expectations Theory

As we have seen, Theil's decomposition test yields mixed evidence as to whether a bias exists in forward rates whereas the second test of unbiasedness shows that forward rates are biased estimates of future interest rates. On balance, there is more evidence to support the presence of a bias in forward rates. Therefore, a tentative interpretation of the empirical evidence of the unbiasedness test on the pure expectations theory is that forward rates are biased estimates of future short-term rates, and that the bias is time-variant which is indicated by the temporal variation in the bias component of the subperiods. Nonetheless, the bias is quite small as suggested by the low Theil's inequality coefficient for both the entire period and the subperiods. In other words, forward rates are fairly accurate, though biased, predictors of future bill rates.

In the test for consistency, the expectations formation models perform quite well. This is indicated by the fairly high adjusted R-square and the small M.S.S.D.'s of the models, as well as the relatively high accuracy of the forecasts produced by the models. However, the positive serial correlation in the error term in all the models indicates

that some relevant explanatory variables may be missing from the models. As we shall see, test results of the risk-premium theory show that forward rates are upward biased estimates of expected future interest rates, exceeding them by a risk premium. The autocorrelation problem of the models may be attributed to the omission of the risk premium from the models.

Among the alternative models, the linear regressive model, though naive, generally performs better than the other models. The lag structure of all the models with past short rates is found to be rather short (no more than six lags, and in most cases, just two or three lags), and none of these models is able to consistently outperform the linear regressive model. This seems to indicate that past short rates as a whole do not carry extra information for generating expectations on top of what is already contained in the current short rate.

The support for the linear regressive model implies that market participants believe that there exists a normal rate toward which interest rates converge. Such a finding is not particularly surprising even in view of abnormally high interest rates in the early 1980s. When interest rates first rose above the normal level based upon long-run experience,

market participants expected interest rates to fall and regress toward the normal level. It was only after they had experienced persistently high interest rates for some time before they anticipated the current upward trend to continue (that is, the extrapolative element of expectation). This explains why the linear regressive model generally performs better than the other models over the entire period studied, and why the extrapolative-regressive model also works well.

To conclude, the test results of the pure expectations theory can be considered as evidence to support the claim that expectations are an important but not the only factor in determining the term structure, and that these expectations tend to be regressive.

#### 4.2 Test Results of the Hicksian Risk-Premium Theory

As mentioned in Section 3.3.2, the first step to test the risk-premium theory is to isolate the risk premiums in the forward rates. It is done by constructing an ARIMA model for each interest rate series,<sup>56</sup> and using the model to gen-

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<sup>56</sup> The ARIMA model will relate the current interest rate to the past interest rates and to a weighted sum of current and lagged random disturbances in a linear specification. The formulation of the model is as follows:

$$\begin{aligned} \text{ARIMA } (p,d,q): \quad A(B)D^d r_{3,t} &= r^*_3 + M(B)E_t \\ A(B)D^d r_{6,t} &= r^*_6 + M(B)E_t \end{aligned}$$

erate forecasts of future three- and six-month bill rates. Risk premiums are obtained by subtracting the ARIMA forecasts from the corresponding forward rates.

While both of the original interest rate series are nonstationary, they exhibit stationarity after they have been differenced once (that is, the original series are homogeneous nonstationary of order one). The autocorrelation and partial autocorrelation functions of the new series (a first-difference transformation of the original series) are then used to determine the number of lags in the autoregressive portion and the number of disturbance terms in the moving average portion of each model. It is found that an ARIMA (0,1,1) model<sup>57</sup> is the correct specification for both interest rate series, as confirmed by a Chi-square test which indicates that the residuals are uncorrelated.

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where  $A(B)$  is the autoregressive function of degree  $p$ ;  $M(B)$  is the moving average function of degree  $q$ ;  $d$  is the degree of homogeneity;  $B$  is the backward shift operator;  $r_3^*$  and  $r_6^*$  are the respective means of the interest rate series; and  $E$  is the random disturbance.

<sup>57</sup> An ARIMA (0,1,1) model means that the order of homogeneity is one, the degree of the autoregressive part is zero and that of the moving average part is one, i.e.

$$r_{3,t} - r_{3,t-1} = E_t + bE_{t-1}$$

$$r_{6,t} - r_{6,t-1} = E_t + bE_{t-1}$$

Using this model, forecasts of three- and six-month bill rates are generated by a moving sixty month estimation interval, with each set of estimated coefficients used to generate the forecast for the next period. The forecasts are reproduced in Tables A6 and A7 in the Appendix. In order to ensure that the ARIMA forecasts are unbiased, they are subject to the same tests of unbiasedness previously performed on the pure expectations theory.

Tables 10 and 11 present the test results of the first test of unbiasedness, Theil's decomposition test. If the test is applied to all the forecasts, the Theil's inequality coefficient is found to be very small (0.032 and 0.037 for the three- and six-month forecasts, respectively). Furthermore, both the bias component and the regression component are practically zero, suggesting that the ARIMA model generates unbiased forecasts of future bill rates.

In order to confirm the above results, the sample period is again divided into subperiods, each of which consists of two years' observations. Results show that the Theil's inequality coefficient is always very small, and that the bias component never exceeds 0.1.<sup>58</sup> From these results,

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<sup>58</sup> The only exception is the subperiod from January 1978 to December 1979 for the three-month forecasts, for which the bias component is equal to 0.2178. But since a value above 0.1 or 0.2 for the bias component is only suggestive of the presence of a bias and it happens only for

Table 10: Theil's Decomposition Test on Three-Month ARIMA Forecasts

<u>Time Period</u>	<u>Inequality</u> <u>Coefficient</u>	<u>Bias</u> <u>Component</u>	<u>Regression</u> <u>Component</u>	<u>Disturbance</u> <u>Component</u>
Jan.75 - Dec.84	0.0324	0.0007	0.0026	0.9967
Jan.75 - Dec.76	0.0291	0.0014	0.0221	0.9765
Jan.76 - Dec.77	0.0252	0.0168	0.0041	0.9791
Jan.77 - Dec.78	0.0159	0.0972	0.1450	0.7578
Jan.78 - Dec.79	0.0232	0.2178	0.0055	0.7767
Jan.79 - Dec.80	0.0464	0.0482	0.0281	0.9237
Jan.80 - Dec.81	0.0425	0.0007	0.0069	0.9924
Jan.81 - Dec.82	0.0306	0.0674	0.0138	0.9188
Jan.82 - Dec.83	0.0243	0.0601	0.0127	0.9272
Jan.83 - Dec.84	0.0074	0.0007	0.0075	0.9918

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one subperiod, this exceptional case should not affect the general conclusion of the test.

Table 11: Theil's Decomposition Test on Six-Month ARIMA Forecasts

<u>Time Period</u>	<u>Inequality</u> <u>Coefficient</u>	<u>Bias</u> <u>Component</u>	<u>Regression</u> <u>Component</u>	<u>Disturbance</u> <u>Component</u>
July.77- Dec.84	0.0374	0.0002	0.0040	0.9958
Jan.78 - Dec.79	0.0262	0.0931	0.0011	0.9058
Jan.79 - Dec.80	0.0457	0.0099	0.0011	0.9890
Jan.80 - Dec.81	0.0473	0.0004	0.0036	0.9960
Jan.81 - Dec.82	0.0403	0.0242	0.0193	0.9565
Jan.82 - Dec.83	0.0325	0.0447	0.0108	0.9445
Jan.83 - Dec.84	0.0230	0.0076	0.0001	0.9923

we conclude that the ARIMA forecasts provide unbiased estimates of expected future bill rates.

The results of the second test of unbiasedness are presented in Table 12. Since the observed F-values are always smaller than the critical F-values, the joint null hypothesis of unbiased forecast as well as the individual hypotheses  $a=0$  and  $b=1$  are accepted at the 1% and 5% levels of significance. Also, the Durbin-Watson statistic indicates the lack of serial correlation in the error term, a condition which must hold for unbiasedness.

To conclude, the results of these two tests strongly support the hypothesis that the ARIMA forecasts are unbiased estimates of future bill rates.

Next, the ARIMA forecasts are subtracted from the corresponding forward rates, as in the following equations, to yield the risk premiums that are embedded in the three- and six-month forward rates.

$${}_{t+3}P_{3,t} = {}_{t+3}r_{3,t} - {}_{t+3}r^e_{3,t}$$

$${}_{t+6}P_{6,t} = {}_{t+6}r_{6,t} - {}_{t+6}r^e_{6,t}$$

where  ${}_{t+3}r^e_{3,t}$  and  ${}_{t+6}r^e_{6,t}$  are the ARIMA forecasts.

Table 12: Test of Unbiasedness on ARIMA Forecasts

$$r_{3,t+3} = a + b_{t+3}r^e_{3,t}$$

$$r_{6,t+6} = a + b_{t+6}r^e_{6,t}$$

Unrestricted Form:

	Adj.R <sup>2</sup>	D.W.	D/F
$r_{3,t+3} = 0.4853 + 0.9589r^e_{3,t}$ (1.95)      (43.60)	0.94	1.89	1,118
$r_{6,t+6} = 0.6986 + 0.9423r^e_{6,t}$ (1.92)      (31.92)	0.92	1.94	1,88

Hypotheses Testing:

	<u>Three-month</u>		<u>Six-month</u>	
	F-value	D/F	F-value	D/F
H <sub>0</sub> : (a, b) = (0, 1)	1.897	2,118	1.920	2,88
H <sub>0</sub> : a = 0	3.792	1,118	3.683	1,88
H <sub>0</sub> : b = 1	3.490	1,118	3.824	1,88

Critical F-value:	F(1,60,1%) = 7.08	F(1,60,5%) = 4.00
	F(2,60,1%) = 4.98	F(2,60,5%) = 3.15
	F(1,120,1%) = 6.85	F(1,120,5%) = 3.92
	F(2,120,1%) = 4.79	F(2,120,5%) = 3.07

The sample mean and standard deviation of the three-month risk-premium series are 0.168 and 0.755, respectively. Those of the six-month risk-premium series are 0.115 and 1.291, respectively. To test the null hypothesis of a zero risk premium, the following t-test is used.

$$t\text{-statistic} = (\bar{P} - 0)/s(P)$$

where  $\bar{P}$  is the mean and  $s(P)$  is the standard deviation of the risk premiums.

The t-statistics obtained are 0.22 and 0.1 for the three- and six-month risk premiums, respectively. On the basis of these statistics, it seems impossible to reject the null hypothesis that the risk premiums are insignificantly different from zero. However, simply looking at calculated means and standard deviations is an incomplete way to test whether risk premiums are significant. The proper way is to perform the following regressions.<sup>59</sup>

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<sup>59</sup> This test has been used by B. Friedman in his study of the U.S. Treasury bill market. For details, see Friedman, Benjamin M., "Interest Rate Expectations Versus Forward Rates: Evidence from an Expectations Survey", Journal of Finance 34 (September 1979), pp. 965-974.

$${}_{t+3}r_{3,t} = a + b{}_{t+3}r^e_{3,t} + u_t$$

$${}_{t+6}r_{6,t} = a + b{}_{t+6}r^e_{6,t} + u_t$$

The pure expectations theory claims that the joint null hypothesis  $H_0: (a, b) = (0, 1)$  holds and that the error term is serially uncorrelated. The presence of significant risk premiums will result in the rejection of the hypothesis.

Table 13 gives the results of the regression. For the three-month risk-premium series, the joint null hypothesis as well as the individual hypotheses ( $a=0$  and  $b=1$ ) are rejected at both the 1% and 5% levels of significance. The evidence is less strong for the six-month risk-premium series, where the hypotheses are rejected at the 5% but accepted at the 1% level of significance. Additional evidence to the existence of risk premiums is that the Durbin-Watson statistic in both cases indicates positive serial correlation in the error term.

Table 13: Test on the Significance of Risk Premiums

Risk premiums:	Three-month	Six-month
mean	0.168	0.115
std. dev.	0.755	1.291

$$t+3r_{3,t} = a + b_{t+3r}e_{3,t}$$

$$t+6r_{6,t} = a + b_{t+6r}e_{6,t}$$

Unrestricted Form:

	<u>Adj.R<sup>2</sup></u>	<u>D.W.</u>	<u>D/F</u>
$t+3r_{3,t} = 0.8100 + 0.9414t+3r e_{3,t}$ (3.49)      (46.50)	0.95	1.39	1,115
$t+6r_{6,t} = 1.3224 + 0.8990t+6r e_{6,t}$ (1.92)      (31.92)	0.85	1.52	1,82

Hypotheses Testing:

	<u>Three-month</u>		<u>Six-month</u>	
	F-value	D/F	F-value	D/F
H <sub>0</sub> : (a,b)=(0,1)	7.234	2,115	3.378	2,82
H <sub>0</sub> : a=0	12.186	1,115	6.734	1.82
H <sub>0</sub> : b=1	8.368	1,115	6.053	1,82
Critical F-values:	F(1,60,1%) = 7.08		F(1,60,5%) = 4.00	
	F(2,60,1%) = 4.98		F(2,60,5%) = 3.15	
	F(1,120,1%) = 6.85		F(1,120,5%) = 3.92	
	F(2,120,1%) = 4.79		F(2,120,5%) = 3.07	
Durbin-Watson:	D <sub>L</sub> =(1,100,5%) = 1.65			
	D <sub>L</sub> =(1, 80,5%) = 1.61			

If risk premiums are embodied in forward rates and affect the term structure, it is important to find out whether a systematic relationship exists between interest rate level and risk premium. Regression of risk premium on the current interest rate indicates that the relationship is negative and significant (see Tables 14 and 15). In both cases (three- and six-month), risk premium varies inversely with the current interest rate and directly with previous period's interest rate. This result is in support of the earlier findings of Malkiel (1966), Nelson (1972) and Van Horne (1978), but is in contrast to those of Kessel (1965) and Kane (1983) who found a significantly positive relationship between interest rate level and risk premium.

The finding of a negative relationship implies that market participants expect interest rates to regress toward a normal level, and they are concerned with capital gain from a rise in bond prices and capital loss from a fall in bond prices. When interest rates are low and are expected to rise with a corresponding drop in bond prices, market participants will demand a higher risk premium to compensate for a potential capital loss; the opposite will hold if interest rates are high and are expected to fall with a corresponding rise in bond prices.

Table 14: Tests on the Three-Month Risk Premiums

$${}_{t+3}P_{3,t} = 0.8851 - 0.0656r_{3,t}$$

$$(2.82) \quad (-2.40)$$

$$\text{Adj. } R^2 = 0.14 \quad \text{D.W.} = 2.07$$

$${}_{t+3}P_{3,t} = 0.8506 - 0.2607r_{3,t} + 0.1994r_{3,t-1}$$

$$(2.56) \quad (-2.88) \quad (2.22)$$

$$\text{Adj. } R^2 = 0.17 \quad \text{D.W.} = 2.04$$

$${}_{t+3}P_{3,t} = 0.9493 - 0.0754r_{3,t} + 0.0437V_3$$

$$(2.64) \quad (-2.31) \quad (0.70)$$

$$\text{Adj. } R^2 = 0.12 \quad \text{D.W.} = 2.09$$

Table 15: Tests on the Six-Month Risk Premiums

$${}_{t+6}P_{6,t} = 1.4487 - 0.1065r_{6,t}$$

$$(2.24) \quad (-2.03)$$

$$\text{Adj. } R^2 = 0.10 \quad \text{D.W.} = 2.14$$

$${}_{t+6}P_{6,t} = 0.9454 - 0.8565r_{6,t} + 0.7947r_{6,t-1}$$

$$(1.68) \quad (-7.03) \quad (6.58)$$

$$\text{Adj. } R^2 = 0.40 \quad \text{D.W.} = 2.11$$

$${}_{t+6}P_{6,t} = 1.8011 - 0.1622r_{6,t} + 0.2880V_6$$

$$(2.63) \quad (-2.85) \quad (2.68)$$

$$\text{Adj. } R^2 = 0.17 \quad \text{D.W.} = 2.16$$

Risk premium is also expected to increase with the degree of uncertainty investors perceive about the future course of interest rates. Using the variance of past six months' interest rates as a proxy for the degree of risk and uncertainty about future rate movements, a significantly positive relationship is found to exist between this proxy and the six-month risk premium but not the three-month risk premium.

In summary, the test results of the risk-premium theory suggest that risk premiums, though very small in size, are significant and time-variant. The two variables that have been identified to be related to risk premiums are the level of interest rate and, possibly, the amount of fluctuation in interest rate of the recent past. Since the adjusted R-square in the above equations is not very high, apparently, risk premiums are also determined by some other variables such as the level of business confidence, the degree of risk aversion of the investors, etc. The inclusion of these variables in the regressions should improve the goodness-of-fit.

### 4.3 Test Results of the Preferred Habitat Theory

As described in Section 3.3.3, the preferred habitat theory will be tested using the model introduced by Modigliani and Sutch (equation 3.18). Quarterly data of three- and twelve-month Treasury bill rates from July 1977 to December 1984 will be used as the short-term and long-term rate, respectively. The model is estimated by the Almon lag technique. The essence of this technique is to estimate the distributed lag in the model subject to the restriction that the lag weights lie on a polynomial of low degree.

Since the specification of lag structure depends on the nature of the problem studied, there are no specific rules of thumb and the selection of lag structure is an art. As a priori information on the degree of the polynomial ( $p$ ) and the length of the lag ( $n$ ) is unavailable, different values of  $p$  and  $n$  are tried. The final choice on the specification is primarily made on the basis of statistical and economic criteria. The statistical criterion involves the maximization of the adjusted R-square ( or minimization of the standard error of regression).<sup>60</sup> The second criterion is concerned with economic considerations and judges alternative

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<sup>60</sup> This 'minimum error variance' model-selection rule has been recommended by Theil (1961) and Schmidt and Waud (1973) for choosing the lag length. Later, Giles and Smith (1977) showed that this rule could also be used in models with autocorrelated error term.

specifications based on the degree of conformity with the hypothetical shape of the lag structure. For example, it is desirable that the shape of the chosen lag structure will approximate one of those given in Figures 3A, 3B and 3C. However, it is possible that these two criteria do not reinforce and that they contradict each other.

Accordingly, polynomials of degree two through four of varying lengths are tried.<sup>61</sup> Since it is expected that at the end the lag structure will approach zero, the polynomials are estimated with a zero end-point restriction at the far end of the lag structure. Unfortunately, results indicate that none of the specifications can simultaneously fulfill the two criteria. Those that minimize adjusted R-square are economically implausible, and those that conform to theoretically predicted lag structure have lower adjusted R-square. Therefore, one specification is selected for one criterion.

First, based on the statistical criterion, a third-degree polynomial with a ten-quarter lag is selected. Regression results, shown in Table 16, indicate that this specification explains over 97 percent of the total variance of the long-term rate. Even though most of estimated coefficients

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<sup>61</sup> Referring to Figures 3A, 3B and 3C again, a fourth degree polynomial should be flexible enough to produce the lag structure.

Table 16: Test Results of the Preferred Habitat Theory -  
Model 1

MODEL 1 (third degree polynomial with ten-quarter lag)

Lag period	Estimated coefficient	T-statistic
1	0.1361	3.86
2	0.0490	3.36
3	-0.0027	-0.26
4	-0.0261	-2.09
5	-0.0284	-2.64
6	-0.0168	-2.06
7	0.0016	0.26
8	0.0196	2.30
9	0.0301	2.85
10	0.0260	3.05
Constant	0.5017	1.13
Current rate	0.7915	18.44

Sum of lag coefficients = 0.1885

Sum of all coefficients = 0.9800

Mean lag = 0.589

Adjusted R-square = 0.9767

Durbin-Watson = 2.1019

Observed F(4,25) = 4.69

Critical values :

F(4,25,5%) = 2.76

F(4,25,1%) = 4.18

t(25,95%,2-tailed) = 2.06

are highly significant, the constant term is insignificantly different from zero. This implies that the twelve-month bill rate is determined by expectations of the future course of three-month bill rate, which, in turn, are generated by a distributed lag of past three-month bill rates. Also, the insignificance of the constant term implies that the maturity distribution for the relative demand and supply of securities and risk aversion do not exert a significant influence on the twelve-month bill rate. This result is inconsistent with the earlier result which supports the risk-premium theory.

The statistic labelled  $F(4,25)$  is distributed as the  $F$  distribution with 4 and 25 degrees of freedom and provides a test for the statistical significance of the whole distributed lag.<sup>62</sup> Comparing the critical  $F$ -value with the observed  $F$ -value, it is found that the distributed lag is significant at both the 1% and 5% levels of significance. The sum of the coefficients of the distributed lag and the coefficient of the current short-term rate is very close to one (0.9800), which is a desirable result in order for the model to give unbiased forecasts from a trendless series.

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<sup>62</sup> This is a joint  $F$ -test on a group of regression coefficients. The same test has been used by Dobson, Sutch and Vanderford (1976).

The lag structure, plotted in Figure 4, resembles the one shown in Figure 3C. However, if both extrapolative and regressive elements of expectation are present, the lag structure should start with a sharp dip at a low value (as shown in Figure 3C), which can be positive or negative, before rising to a peak and then declining to zero. However, this lag shape starts with two significantly positive values before it gradually reaching a minimum, rising to a peak and subsequently declining to zero. This lag shape suggests that expectations are determined by regressive forces in the first and second lag periods, and then dominated by extrapolative forces which subside later when the regressive forces reappear. This lag structure, selected according to the minimum-variance rule, suggests peculiar economic behaviour and is therefore economically implausible.

Turning to the economic criterion, a second order polynomial with a six-quarter lag is chosen (regression results shown in Table 17). This one is the most plausible economically among all the specifications. The shape of this lag structure (plotted in Figure 5) reveals that expectation of future rates is primarily regression toward a long-run average of past rates (compare Figure 5 with Figure 3A). Since only the first three lag coefficients are significant, three

## FIGURE 4

Estimated Lag Shape of Model 1

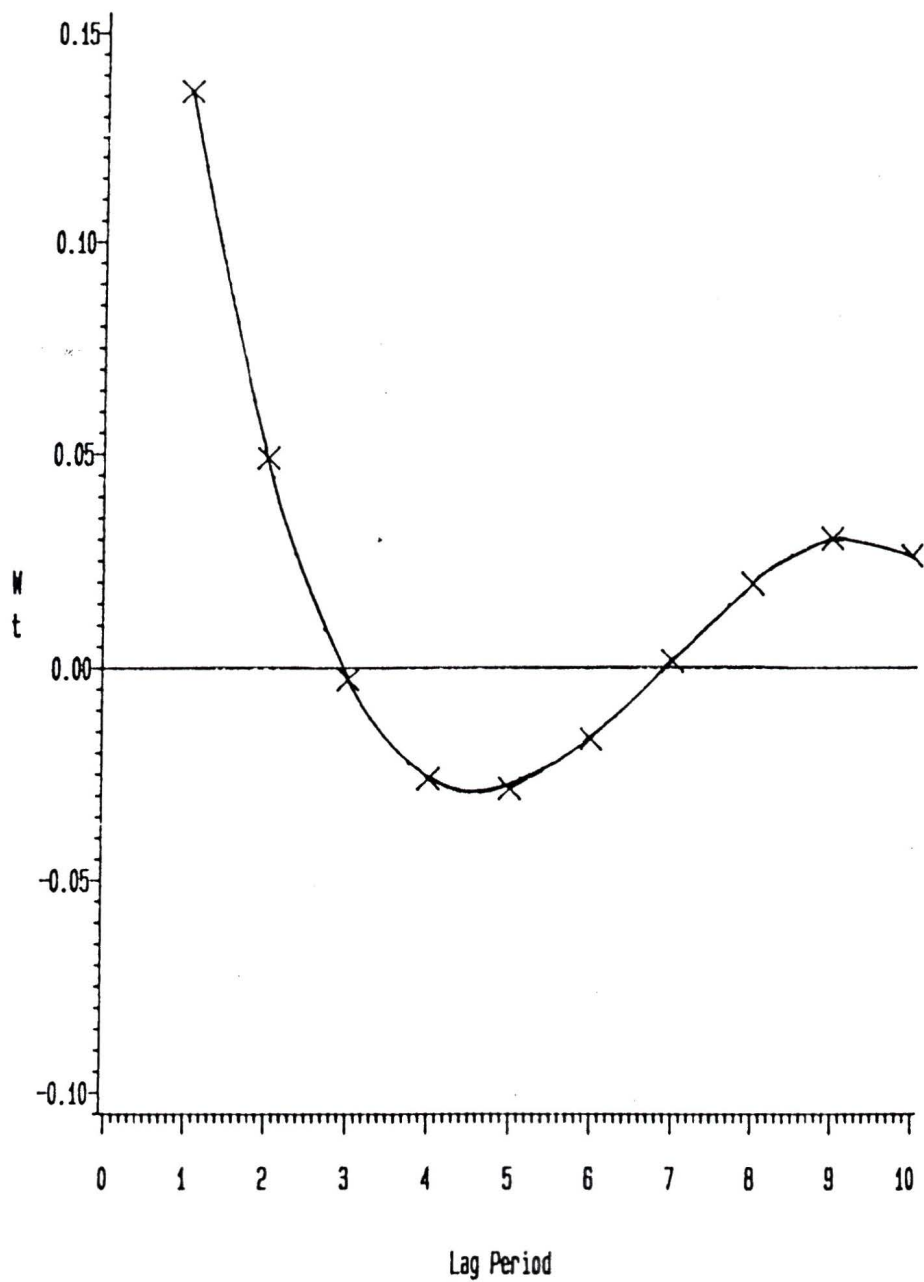


Table 17: Test Results of the Preferred Habitat Theory -  
Model 2

MODEL 2 (second degree polynomial with six-quarter lag)

Lag period	Estimated coefficient	T-statistic
1	0.0943	2.70
2	0.0504	3.02
3	0.0177	2.14
4	-0.0037	-0.33
5	-0.0137	-1.11
6	-0.0125	-1.40
Constant	1.1034	2.63
Current rate	0.7945	16.92

Sum of lag coefficients = 0.1325

Sum of all coefficients = 0.9270

Mean lag = 0.097

Adjusted R-square = 0.9712

Durbin-Watson = 1.65

Observed  $F(3,25) = 3.16$

Critical values :

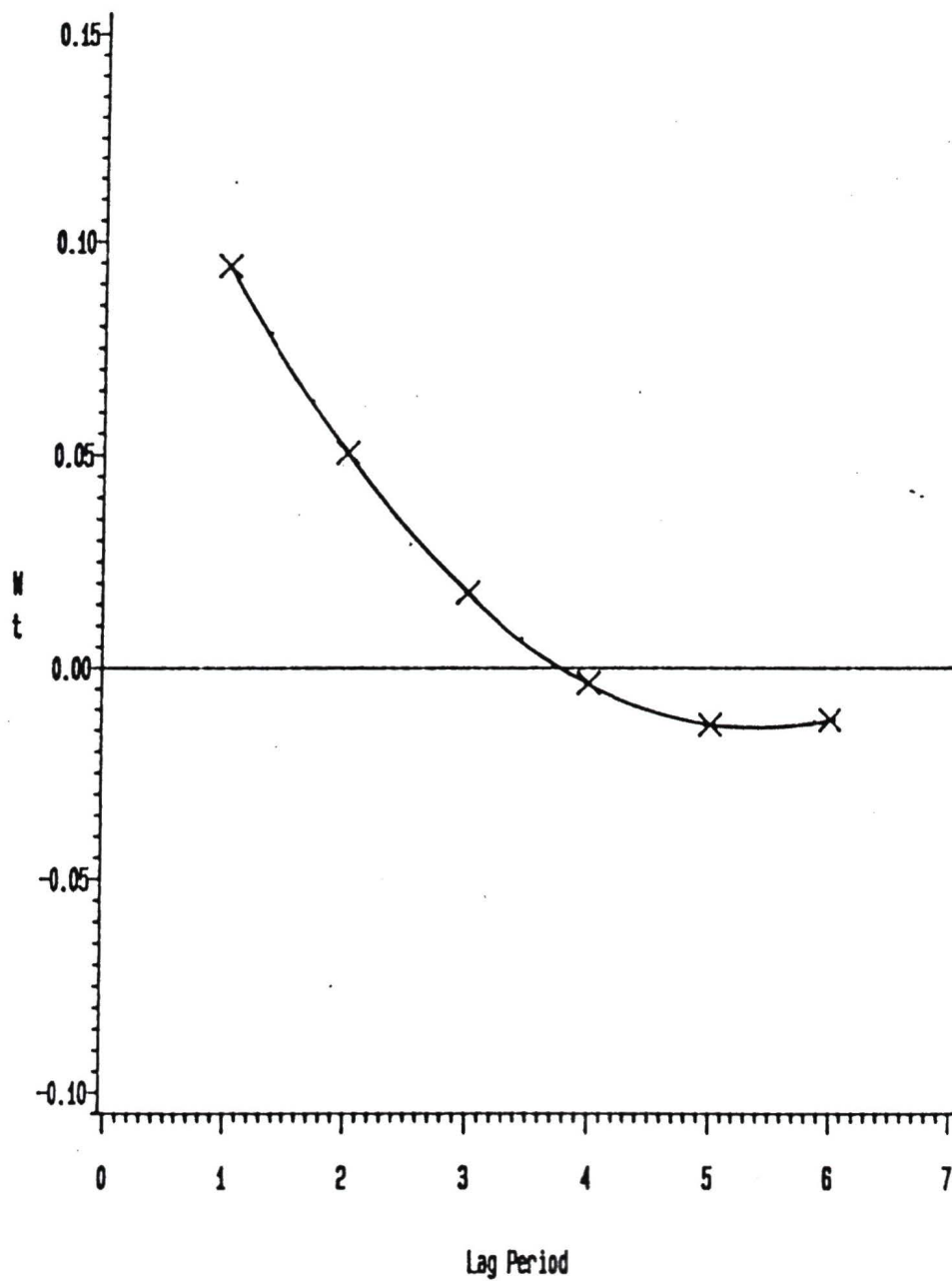
$F(3,26,5\%) = 2.99$

$F(3,26,1\%) = 4.68$

$t(26,95\%,2\text{-tailed}) = 2.06$

## FIGURE 5

Estimated Lag Shape of Model 2



-month bill rates beyond the recent past (in this case, nine months) no longer have an influence on the current twelve-month bill rate (that is, only the recent past matters). Moreover, the significance of the constant term indicates that market segmentation and risk aversion have an influence on the twelve-month bill rate. With respect to the statistical criterion, the regression explains about 97 percent of the variation of the long-term rate, which is just slightly lower than the first model. The sum of all the coefficients is close to one (0.9270) and the F-statistic indicates that the distributed lag is significant at the 5% level but not at the 1% level.

To solve this dilemma, there are four considerations which favour the choice of the second model. The first consideration is related to the time period of this test. Basically, it is a test on the relationship between three- and twelve-month bill rates as hypothesized in the preferred habitat theory during late 1970s and early 1980s.<sup>63</sup> During this period of time, interest rates rose to historical highs well above normal levels.<sup>64</sup> It is, therefore, reasonable to

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<sup>63</sup> Recall that data on twelve-month bill rates are not available until July 1977. Additional observations are lost due to the use of lags.

<sup>64</sup> The highest three- and six-month bill rates were 20.82% and 20.76%, respectively, which are well above the corresponding average rates of 10.89% and 11.96% over the time period under study.

find that the expectation of future rates at that time was mainly governed by regressive forces. Seeing that interest rates were much higher than the historical levels, market participants anticipated that future interest rates should fall. However, interest rates continued to rise, and remained at two-digit level for almost four years (from 1979 to the end of 1982). After experiencing rising interest rates for some time, market participants might change their expectations in favour of the extrapolative model in which a further rise in interest rates was expected. However, the second model which suggests regressive forces of expectation is a plausible model for the entire period.

Second, extrapolative elements of expectation, as suggested in the M-S model, are more likely to dominate in the short-run than in the long-run. Since quarterly data rather than monthly or weekly data are used in this test, any extrapolative force, if present, will be submerged if the duration of the extrapolative force is less than three months. Due to the use of quarterly data, the lag structure will reveal the operation of the longer term element of expectations formation, that is, the regressive element.

Third, support for the regressive model as the appropriate model of expectations formation has been found in the

consistency test of the pure expectations theory, where the linear regressive model performs better than all the other models.

Fourth, the finding of a significant negative relationship between interest rate level and risk premium in the test of the risk-premium theory, which implies that interest rates will regress toward a long-run normal level, is also consistent with the second model.

In view of the above considerations, the second specification based on economic criterion is more likely to be the true lag structure.<sup>65</sup> Therefore, for the particular time period we are studying, the regressive element of expectation is believed to be the predominant force in shaping the lag structure. Since the lag is very short, with only the first three lag coefficients significant, and the distributed lag as a whole is significant only at the 5% level but not at the 1% level, the current three-month bill rate is the dominant variable in determining the twelve-month bill rate. The presence of segmentational force and risk aversion of market participants in the Treasury bill market is indicated by the constant term which is signifi-

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<sup>65</sup> However, it should be noted that the lag structure, at least for Treasury bills, appears to be unstable. Different lag structures may be obtained for different time periods.

cantly different from zero.

#### 4.4 Conclusion of the Empirical Findings

The empirical results reported above indicate that, in the Canadian Treasury bill market, interest rate expectations play a prominent role in determining the rate structure. The market appears to forecast the future course of interest rates, and these forecasts are reflected in the forward rates implied in the empirical term structure. The role of expectations is revealed by the high adjusted R-square, small M.S.S.D.'s as well as the relatively high forecasting ability of the expectations formation models in the tests of the pure expectations theory. Based on the three criteria of consistency, the linear regressive model is found to have higher explanatory and forecasting power than all the other alternative models, including the models with one or more lags. Apparently, market participants form their expectations of future interest rates based on the current interest rate and a constant which is determined by the long-term normal rate.

This conclusion is also supported by the test results of the preferred habitat theory. Regardless of the lag structure specification, regression results using the Almon lag

technique show that the twelve-month bill rate is mainly determined by expectations of future three-month bill rates which, in turn, are shaped by a distributed lag of past three-month bill rates. For the particular time period under study, the regressive element of expectation appears to be the major force in the determination of the lag shape. In addition, the current three-month bill rate appears to be the dominant variable in determining the twelve-month bill rate. It is because the estimated coefficient of the current three-month bill rate is much larger than those of the lag variables, and only the first three lag coefficients are significant.

These results also tend to support the general view that the Treasury bill market is, by and large, efficient in the sense that information for forecasting future interest rates is contained in the current short-term rate. It is evidenced by the short lags of the expectations formation models, and the fact that the distributed lag obtained in the M-S model is short and is only significant at the 5% level but not the 1% level of significance.

The findings, however, also reveal that expectations are not the sole determinant of the rate structure. In the test of the pure expectations theory, forward rates are found to

be biased estimates of future short-term rates. However, the bias appears to be very small as indicated by the Theil's inequality coefficient.

The existence of bias in forward rates is consistent with the risk-premium theory that the rate structure is affected not only by expectations but also by risk premiums. The presence of risk premiums is also indicated by a significant constant term of the second model in the test of the M-S model. However, since the constant term is relatively small, the influence of risk aversion and market segmentation on the rate structure is not substantial.

Using the ARIMA model to produce unbiased forecasts of future short-term rates, the risk premiums are isolated from the forward rates. Both the three- and six-month risk premiums are found to be very small. However, there is no indication that the six-month risk premiums are greater than the three-month risk premiums on average. This result does not support Hicks' argument that risk premiums increase with maturity as risk increases with the remoteness of the future. However, it is consistent with the preferred habitat theory that risk premiums are not systematic in the sense of increasing with maturity, and that the bias in forward rates is also determined by supply and demand conditions in a particular maturity range.

Further tests on the risk premiums reveal that they vary negatively with the current interest rate, thus supporting the argument that interest rates are expected to regress toward some normal level. This, again, is consistent with the results cited earlier that the linear regressive model is found to be the best model in the test of the pure expectations theory, and that the regressive force of expectation is the major force in shaping the lag structure in the M-S model. Using the variance of past six months' interest rates as a proxy for the degree of uncertainty does not produce conclusive evidence as to whether risk premiums increase with the degree of uncertainty. It is possible that this proxy is not a good proxy for the degree of uncertainty as interpreted by the market participants. However, based on these results, it is difficult to draw any strong conclusions on what in fact affects the risk premiums besides the level of interest rates.

#### 4.5 Policy Implications of Empirical Results

Monetary authorities are normally interested in maintaining a certain degree of control over the term structure as a powerful instrument of economic policy. As mentioned in Chapter 1, alternative term structure theories have different implications to government authorities in the conduct of

monetary policy. Theoretical and empirical studies of the term structure can provide government officials with an understanding of the determinants of the term structure, and enable them to adopt the right policy to achieve the end.

Basically, there are two types of policy: 'bills only' policy and 'operation twist' policy. 'Bills only' policy refers to open market operation with its intervention confined to the Treasury bill market. Whereas for 'operation twist' policy, it refers to open market operation which often involves simultaneous sales and purchases of short-term and long-term securities aimed at altering the shape or 'twisting' the yield curve.

These two types of policy stand in contrast from a theoretical point of view. 'Bills only' policy is based on the pure expectations theory which implies perfect substitutability between securities of different maturities. According to this theory, the government can shift the yield curve downward (upward) simply by increasing (decreasing) the supply of Treasury bills. Since securities of different maturities are perfect substitutes, the influence of the policy will be transmitted through the whole maturity spectrum without significant lag or hinderance. Therefore, the increase (decrease) in the supply of Treasury bills will

result in a drop (rise) in interest rates across the entire maturity continuum. However, according to the market segmentation theory, 'bill only' policy only affects the Treasury bill market without any influence on the other maturity ranges.

On the other hand, 'operation twist' policy aimed at 'twisting' the yield curve is based on the market segmentation theory. Suppose the government wants to increase the short-term rate while maintaining a low long-term rate. By debt management policy, a large quantity of long-term debt is offered so as to depress the long-term rate and the short-term rate is raised by the government purchasing short-term bonds in open market operations. However, according to the pure expectations theory, these operations have no permanent effect on the term structure as long as market participants hold the same set of expectations. It is because risk-neutral speculators treat short-term and long-term securities as perfect substitutes.

In other words, if one accepts the pure expectations theory, no government policy can alter the shape of the yield curve unless it can influence the expectations of the market participants. In addition, 'bills only' policy is a viable policy for changing the level of the yield curve.

Alternatively, an upward (downward) shift of the yield curve can be accomplished by simultaneously buying (selling) securities in all maturity sectors. Conversely, if the market segmentation theory is accepted, 'operation twist' policy is the effective tool to influence the shape of the yield curve.

The empirical findings of this study show that interest-rate expectations are an important determinant of the term structure, indicating that 'bills only' policy will be effective in shifting the yield curve. The findings also support the presence of risk premium and some degree of market segmentation, which implies that there is limited substitutability between securities of different maturities. As long as market participants do not treat securities of different maturities as perfect substitutes, 'operation twist' policy can, to a certain extent, affect the term structure. Therefore, based on these findings, both types of policy will be effective: 'bills only' policy for influencing the level of interest rates and 'operation twist' policy for changing the shape of the yield curve.

However, both the risk premiums and the constant term in the M-S model are found to be small, indicating that the influence of risk aversion and market segmentation on the

term structure is relatively small compared with interest-rate expectations. This implies that 'bills only' policy will be relatively more effective than 'operation twist' policy. It is equivalent to say that it is easier for the government to change the level of the yield curve by 'bills only' policy than to change the shape of the yield curve by 'operation twist' policy.

Furthermore, the test results of the preferred habitat theory (Model 2) indicate that the twelve-month bill rate is primarily determined by the current three-month bill rate. The coefficients of the independent variables can be interpreted as follows. An increase of 1 percent in the current three-month bill rate will result in an increase in the twelve-month bill rate of 0.7945 percent in the first quarter, 0.0943 percent in the second quarter, 0.0504 percent in the third quarter and 0.0177 percent in the fourth quarter. In other words, 83 percent of the influence on the twelve-month bill rate is felt in the first quarter, that is, the same quarter. The effect of a change in the three-month bill rate on the twelve-month bill rate is not expected to be felt after four quarters as the lag coefficients beyond four quarters become insignificant and negative. The mean lag equal to 0.097 suggests that half the impact of a change in the three-month bill rate on the twelve-month bill rate

will be felt in 0.097 quarters (that is, about 0.3 months or 9 days). Under the condition that the above results not only hold for the Treasury bill market, but can also be generalized to the longer end of the maturity spectrum, it appears that the effect of open market operations undertaken in Treasury bills will be transmitted to the rest of the maturity spectrum fairly promptly, and that the policy lag is rather short.

## BIBLIOGRAPHY

1. Allen, David, E., Finance: A Theoretical Introduction (Oxford: Martin Robertson, 1983).
2. Almon, Shirley, "The Distributed Lag Between Capital Appropriations and Expenditures", Econometrica 33 (January 1965), pp. 178-196.
3. Barber, Clarence L. and John S. McCallum, "The Term Structure of Interest Rates and the Maturity Composition of the Government Debt: the Canadian Case", Canadian Journal of Economics 8 (1975), pp. 606-609.
4. Binhammer, H.H., Money, Banking and the Canadian Financial System (Methuen Publishers, 4th ed., 1982).
5. Brick John R. and Howard E. Thompson, "Time Series Analysis of Interest Rates: Some Additional Evidence", Journal of Finance 33 (March 1978), pp.93-103.
6. Buse, A., "Interest Rates, the Meiselman Model and Random Numbers", Journal of Political Economy 75 (February 1967), pp. 49-62.
7. Campbell, Tim S., "On the Extent of Segmentation in the Municipal Securities Market", Journal of Money, Credit and Banking 12 (February 1980), pp.71-83.
8. Christofides L.N., "An Empirical Analysis of Bond Markets and Their Implications for the Term Structure of Interest Rates", Manchester School of Economic and Social Studies 48 (June 1980), pp. 112-125.
9. \_\_\_\_\_ "Supply Variables in Term Structure Equations", Canadian Journal of Economics 8 (1975), pp. 276-281.
10. Conard, Joseph W., The Behavior of Interest Rates (New York: National Bureau of Economic Research, 1966).

11. Cukierman, Alex, "A Test of Expectational Processes Using Information from the Capital Markets - the Israeli Case", International Economic Review 18 (October 1977), pp. 737-752.
12. Culbertson, John, "The Term Structure of Interest Rates", Quarterly Journal of Economics 71 (November 1957), pp. 485-517.
13. De Leeuw, Frank, "A Model of Financial Behaviour", in Brookings Quarterly Econometric Model of the United States Economy (Chicago: Rand McNally & Company, 1965), pp. 465-530.
14. Dhrymes, P.J., Distributed Lags: Problems of Estimation and Formulation (San Francisco: Holden-Day, 1971).
15. Diller, Stanley, "Expectations in the Term Structure of Interest Rates", in Jacob Mincer (ed.), Economic Forecasts and Expectations (New York: National Bureau of Economic Research, 1969).
16. Dobell, Rodney and Thomas Sargent, "The Term Structure of Interest Rates in Canada", Canadian Journal of Economics 2 (1969), pp. 65-77.
17. Dobson, Steven W., "The Term Structure of Interest Rates and the Maturity Composition of the Government Debt: the Canadian Case", Canadian Journal of Economics 6 (1973), pp. 319-331.
18. \_\_\_\_\_, Richard C. Sutch and David E. Vanderford, "An Evaluation of Alternative Empirical Models of the Term Structure of Interest Rates", Journal of Finance 31 (September 1976), pp. 1035-1065.
19. Dodds, J.C. and J.L. Ford, Expectations, Uncertainty, and the Term Structure of Interest Rates (London: Martin Robertson and Company Ltd., 1974).
20. Elliott Walter J. and Jerome R. Baier, "Econometric Models and Current Interest Rates: How Well Do They Predict Future Rates?", Journal of Finance 34 (September 1979), pp. 975-985.
21. \_\_\_\_\_ and M.E. Echols, "Market Segmentation, Speculative Behaviour and the Term Structure of Interest Rates", The Review of Economics and Statistics 58 (February 1976), pp. 40-49.

22. Fama, Eugene F., "Forward Rates As Predictors of Future Spot Rates", Journal of Financial Economics 3 (1976), pp. 361-377.
23. \_\_\_\_\_ "The Information in the Term Structure", Journal of Financial Economics 13 (1984), pp. 509-527.
24. Figlewski Stephen and Paul Wachtel, "The Formation of Inflationary Expectations", The Review of Economics and Statistics 63 (February 1981), pp. 1-10.
25. Fisher, Irving, The Theory of Interest (New York, Macmillan, 1930).
26. Friedman, Benjamin M., "Interest Rate Expectations Versus Forward Rates: Evidence From an Expectations Survey", Journal of Finance 34 (September 1979), pp. 965-974.
27. Giles, D.E.A. and R.G. Smith, "A Note on the Minimum Error Variance Rule and the Restricted Regression Model", International Economic Review 18 (February 1977), pp. 247-251.
28. Hamburger, Michael J. and Cynthia M. Latta, "The Term Structure of Interest Rates: Some Additional Evidence", Journal of Money, Credit and Banking 1 (February 1969), pp. 71-83.
29. \_\_\_\_\_ and Elliott N. Platt, "The Expectations Hypothesis and the Efficiency of the Treasury Bill Market", The Review of Economics and Statistics 57 (1975), pp. 190-199.
30. Hicks, John R., Value and Capital (London: Oxford University Press, 1939).
31. Holden K., D.A. Peel and J.L. Thompson, Expectations: Theory and Practice (MacMillan Publishers Ltd., 1985).
32. Kane, Edward J., "The Term Structure of Interest Rates: An Attempt to Reconcile Teaching with Practice", Journal of Finance 25 (May 1970), pp.361-374.
33. \_\_\_\_\_ "Nested Tests of Alternative Term Structure Theories", The Review of Economics and Statistics 65 (1983), pp. 115-123.

34. \_\_\_\_\_ and Burton G. Malkiel, "The Term Structure of Interest Rates: An Analysis of a Survey of Interest Rate Expectations", The Review of Economics and Statistics 49 (1967), pp. 343-355.
35. Kessel, Reuben, The Cyclical Behaviour of the Term Structure of Interest Rates, Occasional Paper 91 (New York: National Bureau of Economic Research, 1965).
36. Keynes, John Maynard, The General Theory of Employment, Interest and Money (New York: Harcourt, Brace & World, Inc., 1936).
37. Lockett, D., "Professor Lutz and the Structure of Interest rates", Quarterly Journal of Economics 74 (February 1959).
38. Lutz, Frederick A., "The Structure of Interest Rates", Quarterly Journal of Economics 55 (November 1940), pp. 36-63.
39. Macaulay, F., Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields and Stock Prices in the United States Since 1856 (New York: National Bureau of Economic Research, Publication No. 33, 1938).
40. Malkiel, Burton, The Term Structure of Interest Rates (Princeton, New Jersey: Princeton University Press, 1966).
41. \_\_\_\_\_ The Term Structure of Interest Rates, Empirical Evidence and Applications (New York, General Learning Corporation, 1970).
42. Maser, R.S., The Term Structure of Interest Rates - An Expectations Model Tested on Postwar Italian Data (Oxford: Clarendon Press, 1972).
43. Meiselman, David, The Term Structure of Interest Rates (New York: Prentice-Hall, Inc., 1962).
44. Michaelsen, Jacob B., The Term Structure of Interest Rates (New York: Intext Educational Publishers, 1973).
45. Mincer, Jacob, "Models of Adaptive Forecasting", in Jacob Mincer (ed.), Economic Forecasts and Expectations (New York: National Bureau of Economic Research, 1969).

46. Modigliani, Franco and Richard Sutch, "Innovations in Interest Rate Policy", American Economic Review 56 (May 1966), pp.178-197.
47. \_\_\_\_\_ "Debt Management and the Term Structure of Interest Rates: An Empirical Analysis of Recent Experience", Journal of Political Economy 75 (August 1967), pp. 569-589.
48. \_\_\_\_\_ "The Term Structure of Interest Rates: A Reexamination of the Evidence", Journal of Money, Credit and Banking 1 (February 1969), pp. 112-120.
49. Modigliani, Franco and Robert J. Shiller, "Inflation, Rational Expectations and the Term Structure of Interest Rates", Economica 40 (February 1973), pp. 12-43.
50. Nelson, Charles R., The Term Structure of Interest Rates (New York: Basic Books, Inc., 1972).
51. Pesando, James E., "The Impact of the Conversion Loan on the Term Structure of Interest Rates in Canada - Some Additional Evidence", Canadian Journal of Economics 8 (1975), pp. 281-288.
52. Phillips Llad and John Pippenger, "The Term Structure of Interest Rates in the MIT-PENN-SSRC Model: Reality or Illusion?", Journal of Money, Credit and Banking 11 (May 1976), pp. 151-164.
53. Pindyck, Robert S. and Daniel L. Rubinfeld, Econometric Models and Economic Forecasts (New York: McGraw-Hill Inc., 2nd ed., 1981).
54. Schmidt, Peter and Roger N. Waud, "The Almon Lag Technique and the Monetary Versus Fiscal Policy Debate", Journal of the American Statistical Association 68 (March 1973), pp. 11-19.
55. Startz, Richard, "Do Forecast Errors or Term Premia Really Make the Difference Between Long and Short Rates?", Journal of Financial Economics 10 (1982), pp. 323-329.
56. Telser, Lester G., "A Critique of Some Recent Empirical Research on the Explanation of the Term Structure of Interest Rates", Journal of Political Economy 75 (August 1967), pp. 546-561.

57. Theil, H., Economic Forecasts and Policy (Amsterdam: North-Holland, 2nd ed., 1961).
58. Van Horne, James, "Interest-Rate Risk and the Term Structure of Interest Rates", Journal of Political Economy 73 (August 1965), pp. 344-351.
59. \_\_\_\_\_ Financial Market Rates and Flows (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1978).
60. \_\_\_\_\_ "The Term Structure: A Test of the Segmented Markets Hypothesis", South Economics Journal 46(4) (April 1980), pp. 1129-1140.
61. Wood, John H., "Expectations, Errors, and the Term Structure of Interest Rates", Journal of Political Economy 71 (April 1963), pp. 160-171.
62. \_\_\_\_\_ "The Expectations Hypothesis, the Yield Curve, and Monetary Policy", Quarterly Journal of Economics 78 (August 1964), pp. 457-470.

**APPENDIX**

Table A1  
Three-Month Treasury Bill Rates  
- January 1975 To December 1984

YR.	MON.						
75	1- 6	6.40	6.26	6.33	6.85	6.87	6.99
	7-12	7.44	7.87	8.41	8.16	8.52	8.64
76	1- 6	8.59	8.79	9.07	8.99	8.94	8.98
	7-12	9.07	9.13	9.11	9.01	8.59	8.14
77	1- 6	8.04	7.65	7.54	7.58	7.05	7.07
	7-12	7.14	7.14	7.10	7.24	7.26	7.17
78	1- 6	7.13	7.30	7.73	8.19	8.20	8.26
	7-12	8.66	8.80	9.17	9.85	10.36	10.46
79	1- 6	10.85	10.82	10.92	10.82	10.84	10.78
	7-12	11.24	11.45	11.64	13.61	13.62	13.66
80	1- 6	13.50	13.55	15.24	15.15	11.58	10.38
	7-12	10.06	10.49	10.95	11.91	13.70	17.01
81	1- 6	16.86	16.83	16.44	17.35	18.43	18.83
	7-12	20.29	20.82	19.35	17.96	15.07	14.41
82	1- 6	14.34	14.58	15.07	14.98	15.18	16.33
	7-12	15.25	13.70	12.73	11.21	10.72	9.80
83	1- 6	9.58	9.23	9.17	9.12	9.25	9.17
	7-12	9.24	9.32	9.24	9.24	9.48	9.71
84	1- 6	9.73	9.82	10.53	10.59	11.29	12.11
	7-12	12.73	12.13	12.02	11.42	10.50	9.84

Source: Bank of Canada Review, various issues.

TABLE A2  
Six-Month Treasury Bill Rates  
- January 1975 To December 1984

YR.	MON.						
75	1- 6	6.57	6.23	6.44	7.31	7.15	7.22
	7-12	7.68	8.22	8.65	8.25	8.73	8.83
76	1- 6	8.54	8.90	9.18	9.02	8.86	8.94
	7-12	9.04	9.04	8.95	8.91	8.50	7.93
77	1- 6	7.78	7.52	7.54	7.59	7.10	7.14
	7-12	7.25	7.29	7.21	7.39	7.41	7.36
78	1- 6	7.36	7.58	8.07	8.52	8.54	8.65
	7-12	8.86	8.97	9.34	10.10	10.61	10.71
79	1- 6	10.98	10.91	10.96	10.77	10.83	10.78
	7-12	11.24	11.82	11.90	13.93	13.49	13.60
80	1- 6	13.62	13.86	15.90	13.96	10.87	10.37
	7-12	10.32	11.12	11.52	12.40	14.17	15.30
81	1- 6	15.94	16.41	15.64	17.47	17.60	18.77
	7-12	20.72	20.76	19.47	18.19	14.41	14.51
82	1- 6	14.69	14.90	15.46	15.29	15.14	16.82
	7-12	15.37	13.47	12.30	11.29	10.90	9.39
83	1- 6	9.60	9.14	9.52	9.11	9.47	9.45
	7-12	9.59	9.62	9.33	9.36	9.72	9.86
84	1- 6	9.88	10.19	11.00	11.05	12.20	12.92
	7-12	13.02	12.44	12.04	11.48	10.47	10.16

Source: Bank of Canada Review, various issues.

TABLE A3

Twelve-Month Treasury Bill Rates

- July 1977 To December 1984

YR.	MON.						
77	7-12	7.39	7.43	7.40	7.60	7.58	7.54
78	1- 6	7.60	7.77	8.26	8.76	8.64	8.76
	7-12	8.89	9.00	9.35	10.17	10.49	10.54
79	1- 6	10.69	10.54	10.60	10.54	10.67	10.55
	7-12	10.44	11.42	11.90	14.00	13.42	13.42
80	1- 6	12.93	13.68	14.92	15.82	11.56	10.68
	7-12	10.56	11.68	12.36	12.23	12.56	15.13
81	1- 6	14.40	15.66	15.58	16.00	18.25	17.99
	7-12	19.35	20.59	19.36	18.33	14.35	15.60
82	1- 6	15.98	16.26	15.61	16.20	15.59	16.63
	7-12	15.88	13.46	12.95	11.00	10.89	9.79
83	1- 6	9.10	9.92	9.58	9.74	8.91	9.63
	7-12	10.10	10.42	9.76	9.37	9.68	10.12
84	1- 6	10.01	10.28	11.31	11.61	12.65	13.43
	7-12	13.85	12.54	12.50	12.05	10.83	10.71

Source: Bank of Canada Review, various issues.

TABLE A4

Three-Month Forward Rates Three-Month-Hence

- Jan. 1975 To Dec. 1984

YR.	MON.						
75	1- 6	6.74	6.20	6.55	7.80	7.44	7.46
	7-12	7.93	8.58	8.90	8.34	8.94	9.02
76	1- 6	8.49	9.01	9.29	9.05	8.78	8.90
	7-12	9.01	8.95	8.79	8.81	8.41	7.72
77	1- 6	7.53	7.39	7.54	7.60	7.15	7.21
	7-12	7.36	7.44	7.32	7.54	7.56	7.55
78	1- 6	7.60	7.87	8.42	8.86	8.89	9.06
	7-12	9.06	9.14	9.51	10.34	10.87	10.97
79	1- 6	11.11	11.00	11.00	10.72	10.82	10.78
	7-12	11.24	12.20	12.17	14.26	13.36	13.54
80	1- 6	13.74	14.17	16.59	12.86	10.20	10.36
	7-12	10.59	11.79	12.12	12.91	14.66	13.75
81	1- 6	15.01	16.00	14.88	17.59	16.81	18.71
	7-12	21.16	20.70	19.59	18.42	13.78	14.61
82	1- 6	15.05	15.23	15.86	15.61	15.10	17.32
	7-12	15.49	13.24	11.88	11.37	11.08	9.00
83	1- 6	9.62	9.05	9.88	9.10	9.69	9.74
	7-12	9.95	9.93	9.42	9.48	9.97	10.01
84	1- 6	10.03	10.57	11.49	11.53	13.18	13.78
	7-12	13.32	12.76	12.06	11.54	10.44	10.49

TABLE A5

Six-Month Forward Rates Six-Month-Hence

- July 1977 To Dec. 1984

YR.	MON.						
77	7-12	7.53	7.57	7.59	7.82	7.75	7.72
78	1- 6	7.85	7.96	8.45	9.01	8.74	8.87
	7-12	8.92	9.03	9.36	10.24	10.37	10.37
79	1- 6	10.41	10.18	10.25	10.31	10.51	10.32
	7-12	9.69	11.03	11.90	14.07	13.35	13.24
80	1- 6	12.27	13.50	14.00	17.91	12.29	11.00
	7-12	10.81	12.27	13.26	12.06	11.12	14.96
81	1- 6	13.00	14.94	15.52	14.65	18.92	17.24
	7-12	18.07	20.42	19.25	18.47	14.29	16.77
82	1- 6	17.38	17.74	15.76	17.16	16.05	16.44
	7-12	16.41	13.45	13.63	10.72	10.88	10.21
83	1- 6	8.62	10.76	9.64	10.41	8.38	9.81
	7-12	10.64	11.28	10.21	9.38	9.64	10.39
84	1- 6	10.14	10.37	11.63	12.20	13.12	13.96
	7-12	14.73	12.64	12.98	12.65	11.20	11.29

TABLE A6

ARIMA(0,1,1) Forecasts - Three-Month Bill Rates

YR.	MON.						
75	1- 6	7.061	6.006	6.404	6.289	7.156	6.726
	7-12	7.122	7.605	8.011	8.624	7.924	8.801
76	1- 6	8.571	8.594	8.878	9.156	8.916	8.949
	7-12	10.456	8.672	9.237	9.080	8.992	8.486
77	1- 6	8.048	8.038	7.546	7.538	7.591	6.904
	7-12	7.113	7.147	7.138	7.090	7.280	7.255
78	1- 6	7.147	7.125	7.346	7.831	8.286	8.178
	7-12	8.281	8.756	8.811	9.261	10.008	10.458
79	1- 6	10.461	10.960	10.784	10.942	10.797	10.848
	7-12	10.768	11.324	11.473	11.665	13.930	13.601
80	1- 6	13.665	13.485	13.556	15.406	15.136	10.466
	7-12	10.367	9.968	10.658	11.045	12.201	14.270
81	1- 6	18.275	16.367	16.980	16.251	17.742	18.679
	7-12	18.892	20.870	20.800	18.728	17.624	13.902
82	1- 6	14.612	14.228	14.729	15.216	14.878	15.310
	7-12	16.776	14.619	13.314	12.471	10.640	10.756
83	1- 6	9.375	9.669	9.040	9.226	9.072	9.326
	7-12	9.102	9.300	9.329	9.202	9.257	9.577
84	1- 6	9.768	9.714	9.866	10.819	10.491	11.631
	7-12	12.315	12.913	11.772	12.130	11.106	10.229

TABLE A7

ARIMA(0,1,1) Forecasts - Six-Month Bill Rates

YR.	MON.						
77	7-12	7.292	7.228	7.326	7.143	7.531	7.340
78	1- 6	7.373	7.353	7.704	8.275	8.658	8.473
	7-12	8.749	8.923	8.997	9.533	10.426	10.716
79	1- 6	10.706	11.144	10.772	11.012	10.651	10.912
	7-12	10.724	11.459	11.977	11.869	14.695	13.420
80	1- 6	13.613	13.621	13.878	16.155	14.173	10.133
	7-12	10.434	10.292	11.362	11.564	12.647	14.719
81	1- 6	15.526	16.112	16.535	15.272	18.123	17.487
	7-12	19.060	21.189	20.640	19.108	17.889	13.075
82	1- 6	14.842	14.650	14.971	15.601	15.200	15.123
	7-12	17.313	14.940	13.082	12.068	11.054	10.853
83	1- 6	8.942	9.786	8.963	9.670	8.962	9.602
	7-12	9.411	9.636	9.616	9.257	9.386	9.805
84	1- 6	9.874	9.882	10.269	11.189	11.014	12.500
	7-12	13.027	13.018	12.277	11.972	11.333	10.209

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Title of Thesis

THE ROLE OF EXPECTATIONS AND RISK AVERSION IN  
THE TERM STRUCTURE OF INTEREST RATES  
- EVIDENCE FROM THE TREASURY BILL MARKET

Author



Signature

IRENE NGA YI CHEUNG

Dec 23, 1985

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