STOCHASTIC MODELS OF THE GROWTH OF FIRMS

by

Peter Gordon Mills
B.Sc., Queen's University, 1979
Graduate Diploma, University of Cambridge, 1990

A Thesis Submitted in Partial Fulfillment of the
Requirements for the Degree of

MASTER OF ARTS

in the Department of Economics

We accept this thesis as conforming
to the required standard

[redacted]

Dr. Serge Nadeau, Supervisor (Department of Economics)

[redacted]

Dr. Leonard Laudadio, Departmental Member (Department of Economics)

[redacted]

Mr. Thomas Shoyama, Outside Member (School of Public Administration)

[redacted]

Mr. William McNally, External Examiner (School of Business)

University of Victoria

Supervisor: Dr. Serge Nadeau

ABSTRACT

The purpose of this thesis is investigate the growth rates of large firms; specifically to test whether a stochastic model is able to adequately explain firm growth.

The objectives of the thesis are to specify five functional forms of the stochastic model, suitable for testing, and test whether those models can explain the growth rates observed in recent years within a large sample of Canadian firms.

Two static models are specified in which, at equilibrium, the distribution of individual firm sizes around the mean firm size in the sample would be Pareto and log-normal distributions, resulting from a stochastic growth process. A transitional model, which predicts the growth rates for firms from one period to the next before the stochastic process of growth reaches equilibrium, is also specified. The transitional model is subsequently modified to allow for and test any effects on the growth process due to either initial size or persistent growth.

Data was assembled on 735 large Canadian firms, covering the period 1986 to 1988, inclusive. The sample was divided into eight sub-samples, each consisting of those firms that were engaged in the same broadly defined area of commercial activity.

None of the five forms specified were able to fully explain the firm sizes or rates of growth in the sample data. Tests of the residuals indicated the presence of heteroskedasticity when the models were tested by Ordinary Least Squares regression, suggesting that a stochastic model may be a mis-specification of the growth process.

Examiners:

[redacted]

Dr. Serge Nadeau, Supervisor (Department of Economics)

[redacted]

Dr. Leonard Laudadio, Departmental Member (Department of Economics)

[redacted]

Mr. Thomas Shoyama, Outside Member (School of Public Administration)

[redacted]

Mr. William McNally, External Examiner (School of Business)

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1.1. Models of Firm Growth

Economic theory has yet to provide an accepted model of the behaviour of firms over time and, as a result, the growth of firms has not been fully explained. Economic literature offers four models to explain the firm growth, described as: classical, life cycle, managerial and stochastic.

None of these models has gained universal acceptance. Different utility functions underlie them, so they cannot be reconciled analytically. The acceptance of one model over the others therefore depends on the weight of empirical evidence. The predictions of firm growth provided by these schools are, for the most part, mutually exclusive. If empirical data supports the predictions offered by one model then the others can be rejected.

The stochastic model is based on a random process of growth and disallows the systematic influences that are inherent in the other three models. The stochastic model thus serves as a null hypothesis for the other three models. If it succeeds in explaining the observed growth of firms then the other models are disproved. The broad purpose of this study is to test the stochastic model as a predictor of firm growth.

The specification of the pure stochastic model is simple. It can be modified to include effects related to the initial size of the firm and persistent growth without much complication.

1.2 Growth as a Relevant Issue

Given the size of its output relative to the other G7 countries, Canada seems to have a high proportion of very large firms.

Table 1.1: Representation of Countries in the International Fortune 500

		Firms in	
Country	Rank in G7	Top 100	Top 500
Japan	2	27	152
F.R.Germany	3	17	53
France	4	11	41
Britain	5	10	72
Italy	6	4	9
Canada	7	5	31

source: Fortune (1987)

Italy, with a moderately higher level of output, has substantially fewer firms in the world's 500 largest. France, ranked fourth in output, has slightly more than Canada. The relatively high proportion of large firms in Canada suggests that Canadian data may demonstrate, more clearly than data from other countries, the behaviour of large firms.

Whether firms maximize profit or maximize size is an important practical issue for the private sector. The markets in which large Canadian firms compete are sometimes less than perfectly competitive and the ability to predict how a large firm would respond to certain market conditions would be valuable to investors and competing firms.

The growth of firms relates to some important issues of public policy. Large firms attempting to maximize their growth and size may, intentionally or otherwise, alter the structure of a market. This may give rise to oligopolistic or monopolistic conditions in domestic markets. There are public policy arguments for and against domestic monopolies for large firms that also compete in international markets. The issue here is the nature of the process by which firms grow and whether it may, if left unregulated, create an undesirable market structure.

Research into the growth of firms was commonplace in the 1950's and 1960's, following the development of Baumol's (1959) and Marris' (1964) models. Activity in this field waned in the late 1970's, however, as the empirical evidence of the previous 20 years failed to conclusively support any one of the models proferred during that time. The principal reason for following this line of research after a decade of dormancy is the availability of new data. In the last five years, data has been developed for Canadian firms which assigns firms to a few broadly defined areas of commercial activity. These new data are less limiting than data used in previous studies, categorized into narrow market definitions at the expense of representing the diversification of firms.

1.3 Objectives, Layout and Findings of the Thesis

The purpose, or general goal, of the thesis is to test the ability of stochastic models to explain the growth of firms. The specific objectives of the thesis are to test 5 different forms of the stochastic model to determine whether it can account for the observed growth of firms.

Five forms of the stochastic model are identified and defined in chapter two and then specified as econometric models in the third chapter. The data to be used are introduced in the fourth chapter. The results of fitting the models to the data are reported in the fifth chapter. The sixth chapter presents the conclusions of the thesis.

In general, the forms of the stochastic model tested in this thesis failed to fully explain the growth of firms that was observed in the data. There is heteroskedasticity in the data, introduced by either errors in the data or the forms of the stochastic model used being mis-specifications, which should include some of the behavioural elements of the classical, managerial or life-cycle models.

2.1. Differing Theories of Firm Growth

Several models attempt to explain the size and growth of firms. These models may be broadly grouped into four categories: classical, life cycle, managerial and stochastic. These models have sometimes conflicting predictions. Since differing utility functions underlie them, they cannot be reconciled analytically.

The classical model of a profit-maximizing firm predicts that the firm will grow at an indeterminate rate until it reaches its optimum size. Coase's (1937) model of transaction costs makes the same prediction. Transaction theory predicts that a profit-maximizing firm will grow as it identifies external transactions at its boundary which could be more profitably conducted within the boundary of the firm. The firm's boundary is pushed outward to "internalize" the transaction.

The life cycle model starts from the premise that a firm's costs will change over time. In Downie's (1958) model of dynamic competition, more innovative and therefore more efficient firms have lower unit costs, higher unit revenues and, thus, higher retained earnings with which to increase market share. More efficient firms will grow faster in pursuit of greater market share. Larger, older firms will find their original innovations are, in turn, eclipsed by newer technology. They will lose market share to a new generation of small and innovative firms.

In the managerial model, the utility function of the managers is not based on profits, which is the utility function of the owners of the firm. Baumol (1959) used a management utility function with performance measures familiar to managers, such as sales. The managerial model predicts that, when they control the firm, managers will increase their utility through increases in firm size. Larger firms are more likely to be insulated from the direct control of their profit-seeking owners[1] and managers can increase the size of those large firms more quickly.

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1 The divorce of ownership and control is postulated to be prevalent in large firms, which often have widely dispersed shareholders and a board of directors drawn from the ranks of professional managers, rather than the shareholders themselves.

The stochastic model rejects all of the systematic and behavioural influences of the models outlined above and stipulates that firm growth is a random process. In this chapter, some analytical forms of the stochastic model are reviewed.

2.2. Theoretical Foundations of Stochastic Models

The stochastic model is, in its most general form, a statement of a random process. First identified by Kapteyn (1903), stochastic models are used to model the process of change in many different populations found in both nature and society. This model rejects the influence of any deterministic effects on a population as a whole, while allowing the possibility of such effects on an individual member of the population.

The stochastic models was first applied to industrial economics by Gibrat (1931) as a counter-hypothesis to deterministic models of the firm. The model disallows three key factors of deterministic models from having a systemic effect on large groups of firms: market structure, firm cost curves and utility functions. In the micro-economics of the individual firm, these factors can come into play but only in a random fashion.

Two general assumptions are required to use the stochastic model in a general analytical form to explain growth from one period to the next in a dynamic population:

I. Each member of the population faces a probability distribution of growth from one period to the next; and

II. Each member's growth from one period to the next is chosen at random from that probability distribution.

The stochastic model can be analytically stated in general form as the Law of Proportionate Effect. Specific analytical forms can be derived from this law by adding assumptions about the probability distribution identified in the first general assumption.

In this chapter, the Law of Proportionate Effect is applied to the size and growth of firms in a market. Two analytical forms, a Pareto form and a log-normal form, are adopted and the two general assumptions above are restated as five more specific assumptions. Some of specific assumptions are then relaxed where economic plausibility suggests it and analytical simplicity permits it. In doing so three more specific forms, all variations on the log-normal form, are adopted.

Five specific analytical forms are thus derived here, each one requiring fewer of the five specific assumptions to maintain the underlying hypothesis that firm growth is a random process. If none of these forms can adequately explain the actual growth of firms in the sample then the underlying general assumptions, stated above, are probably invalid and the stochastic growth process has failed.

2.2.1. The Law of Proportionate Effect

Applied to the size and growth of firms, the Law of Proportionate Effect holds that the size of a firm will change over time according to the following random process:

$$S_t = S_{t-1}\, \varepsilon_t \qquad (2.1)$$

where S is the size of the firm measured as the value of sales and ε represents the stochastic element of growth, generated by a random process.[2] This model stipulates that the only cause of growth is the random process and, in the absence of it, the firm would not grow at all.

The Law of Proportionate Effect can be modified to include aggregate market growth as well as chance. The determinate portion of the Law of Proportionate Effect, therefore, asserts that the rate of growth of an individual firm will equal the growth of that firm's market or the economy (gross national product) as a whole:

$$S_t = \alpha S_{t-1}\, \varepsilon_t \qquad (2.2)$$

where α is a co-efficient of steady-state growth of market demand in that firm's industry from t-1 to t .

The Law of Proportionate Effect governs a random process growth which, over a sufficient period of time, transforms any initial size distribution of firms to a skewed distribution in a final equilibrium condition.[3] There are several formulations of skewed distributions, of which the Pareto and log-normal distributions are the most commonly

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[2] The time subscripts throughout, t, denote the value of the variable during or at the end of the time period t.

[3] In other words, the limiting distribution generated by a random growth process is a skewed distribution. At the general level of the Law of Proportionate Effect, economic literature usually illustrates the limiting distribution with a numerical example. Prais (1976) shows how the size distribution of a group of firms evolves from a uniform distribution to a skewed distribution over about six periods.

adopted. These distributions are very similar, both in the assumptions required for their application and the form of the distributions.

2.2.2. The Pareto Distribution as the Distribution of Firm Size

The Pareto size distribution of firms is derived from Pareto (1949), who shows that if the size distribution of firms is generated by a random process then the equilibrium relationship between the size and rank of each firm in a market takes the form:

$$S_i R_i^{\nu} = A \tag{2.3}$$

where S_i is the size of the ith firm, R_i is the rank of the ith firm in its market and A and ν are constants. The parameter ν is a measure of concentration; the higher the value of ν, the more concentrated the market.[4] The Pareto size distribution is a straight line on log-log scale, with a slope of ν:

$$\log S_i = \log A - \nu \log R_i \tag{2.4}$$

Using the following assumptions:

[1] each firm faces an identical distribution of growth probabilities;

[2] the distribution does not change over time;

[3] the growth rate of a firm in any period is independent of the firm's initial size;

[4] the firm's growth rate in any period is independent of the firm's previous growth; and

[5] the population of firms does not change over time;

Champernowne (1953) shows that if the growth process is a Markov process[5] then, in final equilibrium, the size distribution in a sample conforms to the Pareto Law regardless of the initial sample distribution. A random sampling of the probability distribution of growth possibilities faced by the firm, a process embodied in equation (2.2), will result in a

[4] Consider two firms, S_i and S_j, where $S_i > S_j$, implying $R_i < R_j$:

$$S_i R_i^{\nu} = A = S_j R_j^{\nu}$$

$$S_i = S_j (R_j/R_i)^{\nu}$$

because $(R_j/R_i) > 1$, the higher the value of ν, the larger S_i is relative to S_j and the more concentrated is the market.

[5] A Markov chain, or Markov process, is a random process in which the variable is permitted to change only at discrete and regular intervals of time.

equilibrium size distribution of firms conforming to the Pareto distribution (or a similar skewed distribution) if these five specific assumptions are met.

These five specific assumptions are, therefore, substitutes for the two general assumptions. This is an important substitution because the five specific assumptions are more readily used in the specification of testable, functional forms of models than are the two general assumptions.

The limiting distribution defined in equation (2.4) is easily tested. However, because it represents the final equilibrium of the growth process, the test is likely to fail. Specifically, a test of the Pareto distribution to a sample may fail for two reasons that cannot be differentiated: first, the final equilibrium has not been reached or, second, one or more of the five specific assumptions have been violated.

2.2.3. The Log-Normal Distribution as the Probability Distribution of the Process

Since there is no assurance that the final equilibrium has actually been reached, it is advantageous to base a test on the transitions that take place in the growth process from one period to the next, before the process reaches equilibrium. To do this, a formulation of the growth process itself is necessary, so that transitions over a single period can be predicted and observed.

Gibrat (1931) formulated a specific form of random growth. He began with the general statement of the stochastic process, shown in (2.2) in logarithmic form:

$$\log S_t = \log \alpha + \log S_{t-1} + \log \varepsilon_t \qquad (2.5)$$

Gibrat then assumed that the distribution of $\log \varepsilon_t$ was normal.[6]

[6] The distribution of growth rates can be reasonably assumed to be normal only if positive and negative growth rates can be infinite ($-\infty \leq \log \varepsilon_t \leq \infty$). While infinitely high growth rates are possible, the lowest possible value is a termination of sales, i.e. a growth rate of -100%. Also, if ($-\infty \leq \log \varepsilon_t \leq \infty$) then ($0 \leq \varepsilon_t \leq \infty$), which precludes a decrease in absolute firm size.

Gibrat's assumption adds a significant restriction to the probability distribution of growth rates faced by the firm. The Pareto distribution does not require any restrictions on the form of that probability distribution but does require that the form remain constant over time.

Gibrat's assumption defines one of the random growth processes that would result in a log-normal equilibrium distribution, very similar to the Pareto distribution. After T periods, the size of the firm will be:

$$\log S_T = \log \alpha + \log S_0 + \log \varepsilon_0 + ... + \log \varepsilon_t + ... + \log \varepsilon_T \qquad (2.6)$$

The size distribution of those firms after T periods will be identical to the probability distribution of $\sum_{t=0}^{T} \log \varepsilon_t$, which is normal.

In summary, Gibrat's log-normal distribution requires the same five specific assumptions as the Pareto distributions, plus the addition of assumption [2A] below:

[1] each firm faces an identical distribution of growth probabilities;

[2] the distribution does not change over time;

[2A] the distribution is normal;

[3] the growth rate of a firm in any period is independent of the firm's initial size;

[4] the growth rate of a firm in any period is independent of the firm's previous growth; and

[5] the population of firms does not change over time.

Gibrat's additional restriction, the specification of the form of the generating distribution, allows more feasible testing. With the process of growth defined, the distribution of log ε_t in a single period can be predicted and tested. Such a test does not require that the sample be in final equilibrium.

2.3. Relaxing the Assumptions of the Stochastic Model

Referring to the five specific assumptions, modified with Gibrat's restriction, assumptions [1] and [2] imply that growth will be generated by random sampling from a probability distribution which is stable over time. These two assumptions are necessary to specify any form of the stochastic model and , therefore, cannot be relaxed. Assumption [2A] permits observation of the dynamic condition prior to the achievement of final equilibrium and cannot be relaxed. However, the specific assumptions [3], [4] and [5] can be relaxed without violating the two general assumptions underlying the Law of Proportionate Effect.

In the next two sections of this chapter, the log-normal model of stochastic models is modified by relaxing assumptions [3] and [4]. Assumption [5] is relaxed by modifying the sample of firms. If the tests derived by relaxing those last three assumptions are not empirically successful then their failure is due to a violation of assumption [1] or assumption [2]. If one of the first two assumptions is violated then the process which generates the growth of firms cannot be purely stochastic.

2.3.1. Initial Size as a Variable in the Stochastic Process

The Law of Proportionate Effect can be adapted to explain firm size and growth when the third assumption of the stochastic process is relaxed, allowing the probability distribution of growth rates to be related to firm size.

The inclusion of the effect of firm size effects into the log-normal form alters the form of equation (2.2) to:

$$S_t = \alpha \, S_{t-1}^{\beta} \, \varepsilon_t \qquad (2.7)$$

or in logarithmic form,

$$\log S_t = \log \alpha + \beta \log S_{t-1} + \log \varepsilon_t \qquad (2.8)$$

where β represents that element of growth that is directly related to the initial size of the firm. If $\beta = 1$ then initial size has no effect on firm growth. If $\beta < 1$ then small firms grow faster than larger ones in a regressive process, where the size of individual firms tends towards a mean growth rate. If $\beta > 1$ then large firms growth faster than small ones in a progressive process. The assumption of log-normal distribution for ε_t is unchanged.

In this form, the third of the five specific assumptions of the stochastic model has been relaxed.

The introduction of a size-growth variable into the stochastic model could violate one of the assumptions of the classical linear regression model: that the covariance of the error term and the independent variable (firm size) is zero. If it is not, then may be mis-specified. The explicit inclusion of the size-growth relationship in the form shown below is intended to ensure that covar($\log S_{t-1}, \mathcal{E}_t$)= 0. Hannah and Kay (1977) show[7] that when $\beta \neq 1$, the growth process is:

$$\log S_t = (1-\beta)\kappa + \beta \log S_{t-1} + \log \mathcal{E}_t \qquad (2.9)$$

where κ is a constant. This can be restated as a geometrically lagged function of the random variable[8] :

$$\log S_t = \kappa + \log \mathcal{E}_t + \beta \log \mathcal{E}_{t-1} + \beta^2 \log \mathcal{E}_{t-2} + ... + \beta^t \log \mathcal{E}_0 \qquad (2.10)$$

showing that $\log S_t$ remains a weighted sum of the independent random variable $\log \mathcal{E}_t$. It is therefore possible to integrate a systematic size-growth process into the stochastic model and test that model using linear regression.

2.3.2. Persistent Growth in the Stochastic Process

The log-normal form of the stochastic model, with or without a size-growth component, may be subject to serial correlation, i.e. $\text{cov}((S_t - S_{t-1}), (S_{t-1} - S_{t-2})) \neq 0$. Two notions suggest the presence of serial correlation. First, firms may have runs of luck,

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[7] Equation (2.8) can be re-arranged in the form of a generalized difference equation, which represents the growth process:

$\log S_t - \beta \log S_{t-1} = (1-\beta)\kappa + \log \mathcal{E}_t$

This collapses to the Gibrat equation (2.5), with $\log \alpha = 0$, when $\beta = 1$. The rearrangement only requires the assumption that $\log \alpha = (1-\beta)\kappa$. As κ is an unspecified constant, it is reasonable to assume that $\kappa = 0$.

[8] If we assume that the effect of initial firm size is a geometric lag, then $\log S_t$ and $\log S_{t-1}$ are themselves functions of lagged values of $\log \varepsilon$ and take the forms:

$\log S_t = \kappa + \log \mathcal{E}_t + \beta \log \mathcal{E}_{t-1} + \beta^2 \log \mathcal{E}_{t-2} + ... + \beta^t \log \mathcal{E}_0$

and

$\log S_{t-1} = \kappa + \log \mathcal{E}_{t-1} + \beta \log \mathcal{E}_{t-2} + ... + \beta^{t-1} \log \mathcal{E}_0$

then

$\beta \log S_{t-1} = \beta\kappa + \beta \log \mathcal{E}_{t-1} + \beta^2 \log \mathcal{E}_{t-2} + ... + \beta^t \log \mathcal{E}_0$

and

$\log S_t - \beta \log S_{t-1} = (\kappa + \log \mathcal{E}_t + \beta \log \mathcal{E}_{t-1} + \beta^2 \log \mathcal{E}_{t-2} + ... + \beta^t \log \mathcal{E}_0)$

$\log S_t - \beta \log S_{t-1} = (1-\beta)\kappa + \log \mathcal{E}_t$

Therefore, the assumption of a geometric model is consistent within the specifications of size and growth contained in equations (2.8) and (2.9).

when an event which spurs growth in one period may itself extend over subsequent periods. Second, an event that occurs in a single period may alter growth in more than one subsequent period. Both of these notions, or serial correlation generated by the random process itself, weaken the fourth specific assumption; that the growth rate of a firm in any period will be independent of the firm's growth in previous periods.

Such persistence in the rate of growth can be included in the log-linear form of the stochastic model, thus relaxing the fourth of the five specific assumptions:

$$\log S_t = \log \alpha + \log S_{t-1} + \omega(\log S_{t-1} - \log S_{t-2}) + \log \varepsilon_t \qquad (2.11)$$

In this expression, ω measures the persistence in growth rates between one period and the next. This is an accelerator model, stating that the growth rate in period t is a function of the change in the growth rate from period t-2 to period t-1. The larger the acceleration of the growth rate from t-2 to t-1, the larger the rate of growth in period t. This expression can be re-arranged as[9]:

$$\log S_t = \log \varepsilon_t + (1+\omega)\log \varepsilon_{t-1} + (1+\omega+\omega^2)\log \varepsilon_{t-2} + \ldots$$
$$+ (1+\omega+\omega^2+\ldots+\omega^t)\log \varepsilon_0 \qquad (2.12)$$

The probability distribution of S_t is still log-normal, since $\log \varepsilon_t$ is still a normally distributed random variable.

2.3.3. Births and Deaths in the Stochastic Process

If the sample of firms is to include all firms in a market over time, then all those firms that enter the market (births) or leave the market (deaths) must also be included. Their inclusion, however, violates the fifth specific assumption, that the population of firms does not change over time.

It is possible to modify the probability distributions of growth faced by firms to include births and deaths and show that the resulting size distributions of firms would follow the Pareto distribution.[10] However, these modified probability distributions

[9] The proof is done by induction, so an illustration of the three-period case is more useful. Such an illustration is appended.

[10] See Simon (1955), Steindl (1965) or Ijiri and Simon (1974).

include parameters which define the probabilities of new firms gaining sales or old firms losing them . These parameters cannot be estimated *ex ante* and empirically fitting a curve to size distribution data without those parameters gives no insight into how the growth process is influenced by births and deaths.

The alternative is to construct a sample containing only those firms that do not enter or leave the market during the sample period. Such a sample will be biased, as discussed in chapter four.

2.4. Economic and Theoretical Implications

Stochastic models are generically applied to any growth problem. They have, in themselves, no reference to economic theory. There are some difficulties in reconciling the stochastic models with economic theory. However, the ease of testing stochastic models makes them useful.

2.4.1. Economic Limitations of the Stochastic Models

The stochastic model does not exclude the possibility of changes in firm size being caused by economic factors such as cost curves, market structures and utility functions. However, it requires the strong assumption that these factors do not work in a systemic fashion, that is, their influence on one firm will be independent of their influence on all other firms.

The assumption that cost curves of a firm have no systematic effect on its rate of growth the size of firms has limited validity. As long as the long-run average and marginal cost curves of the firm are "U"-shaped, there is an optimum size of the firm which will be found at the production quantity where the upward-sloping marginal cost curve intersects the level or downward-sloping marginal revenue curve. This profit-maximizing size will be known to the managers of firms and they will make deliberate decisions to grow or shrink towards that profit-maximizing size. All firms of the same approximate size relative to the optimum size can be expected to make similar decisions. Their behaviour would then be systematically similar. Also, firms faced with declining average and marginal cost curves will face similar incentives to grow to the point of monopoly domination of their markets.

Only horizontal cost curves, with constant average and marginal costs, will satisfy the requirement that cost curves have no systematic effects. Therefore, the cost curves of each firm in a sample must be constant across the size distribution of all the firms in the sample. In practical terms, this condition may be met (approximately) if managers perceive their cost curves to be near-horizontal over the range of actual firm sizes in the sample.

If firms are deliberately moving towards an optimum size, a regressive process may be taking place in which smaller firms will grow faster than larger ones. Thus there are two possible explanations for such a regressive process: "U"-shaped cost curves or a managerial model of firm behaviour. A regressive process does not, in itself, allow any discrimination between the two models.

2.4.2. Strengths of the Stochastic Models

In terms of economic theory, stochastic models are considered weak because of their exclusion of two behavioural factors: the cost curves faced by the firm and the utility functions of the firm's owners and manager. A third behavioural factor, market structure, is ignored also and in this lies the convenience of the stochastic model. It treats market structure as a result, rather than as a determinant, of firm size.

A stochastic growth process will, over time, alter the size distribution of firms in two ways: the variance will increase and the distribution will be transformed to a log-normal or Pareto form. Since these forms are negatively skewed, more sales will be made by the larger firms and concentration among will increase among the largest firms in a market. A progressive process of growth will re-enforce the stochastic increase in market concentration and a regressive process will mitigate it.

Stochastic models have value as a contradiction to structure-conduct-performance (SCP) models. The SCP models predict that firms will react to perceived oligopolistic market conditions with strategic efforts to improve their position. Winners and losers may be expected in the resulting game, resulting in a smaller number of firms holding a larger share of the market. The notion that the strategic interaction of firms can account for increases in market concentration is theoretically appealing but very difficult to verify empirically. Stochastic models, with their assertion of random growth being the sole generator of size distribution, is a contradictory model which is simple to test.

In the previous chapter, four specific forms of the stochastic process were proposed as explanations for the size distributions of firms. They were:

1. the basic Pareto distribution, equation (2.4);
2. the basic log-normal distribution, equation (2.5);
3. the log-normal distribution including initial firm size as a determinant, equation (2.9); and
4. the log-normal distribution including previous growth as a determinant, equation (2.11).

In this section, those forms are translated into tests. For each test, a functional form, restrictions on the variables (if appropriate) and procedures for testing goodness of fit are identified. Previous empirical work is reviewed.

3.1 Functional Forms

3.1.1. Equilibrium Test of the Basic Pareto Distribution

The first test will check the goodness of fit, at a point in time, between the size distribution in a sample of firms and the Pareto distribution specified in equation (2.4).

The test of the Pareto distribution takes the form:

$$\log \mathbf{S_i} = \widehat{\log}\ a - \Diamond \log \mathbf{R_i} + e \tag{3.1}$$

where $\widehat{\log}\ a$ and $\Diamond$ are estimates of log **A** and ν specified in equation (2.4). The error term e_i is an estimate of unexplained deviation from the Pareto distribution. Any variation of e_i from zero indicates the presence of some determinant of firm size other than a random growth process. The objective of the test is to determine whether $var(e_i)$ is significantly greater than zero.

Equation (3.1) will be computed using OLS on the latest year sample described in the chapter four. The estimated parameters $\widehat{\log}\ a$ and $\Diamond$ are not relevant to the study.

For an initial test of goodness of fit, equation (3.1) is modified with higher order polynomial terms to determine if there is significant curvature:

$$\log S_i = \widehat{\log} a - \hat{v} \log R_i + \hat{c}(\log R_i)^2 + \hat{d}(\log R_i)^3 + e_i \qquad (3.2)$$

The hypothesis that the sample follows a Pareto distribution is rejected if either $\hat{c}$ or $\hat{d}$ is significantly different than zero.

For a finer test of goodness of fit, the estimated parameters $\widehat{\log} a$ and $\hat{v}$ are used to calculate expected values of $\log S_i$:

$$\widehat{\log} S_i = \widehat{\log} a - \hat{v} \log R_i \qquad (3.3)$$

The sample is divided into k sub-samples, by firm size. The data $\widehat{\log} S_i$ and $\log R_i$ are sorted into the sub-samples and counted. The number of firms in each sub-sample when sorted by $\widehat{\log} S_i$ is the expected frequency E_i. The number of firms in each sub-sample when sorted by $\log S_i$ is the observed frequency O_i. The statistic[11]:

$$\chi^2_{(k-1)} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \qquad (3.4)$$

is calculated and, if higher than the appropriate critical value, the hypothesis that the distribution of firm size is a Pareto distribution is rejected.

3.1.2. Equilibrium Test of the Basic Log-Normal Distribution

This is a test of the proposition that the probability distribution of growth possibilities faced by the firms in a sample is normally distributed and that the growth process has progressed to final equilibrium. As indicated by equation (2.5), the distribution of $\log S_i$ in equilibrium would be normal under this proposition. The objective of this test is to determine whether the distribution of $\log S_i$ is significantly different than a normal distribution.

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[11] This is the χ^2 test, a description of which is appended.

The mean and variance of the sample values of log S_i, $\overline{\log S_i}$ and $\mathrm{var}(\widehat{\log} S_i)$, are calculated. They are assumed to be accurate estimates of the values $\mu_{\log S}$ and $\sigma^2_{\log S}$ for the normally distributed population. These parameters, with the number of firms **N**, are used to calculate the expected frequency of firms E_i in k sub-samples. The sample is sorted into the same sub-samples to determine the actual frequency of firms O_i in each sub-sample. If the calculated χ^2 is higher than its appropriate critical value, the hypothesis that the sample follows a normal distribution is rejected.

Two assumptions underlie this test. The first is that the distribution of the error term is normal, which is evaluated with the χ^2 test. The second assumption is that the distribution is homoskedastic, that is, the variance of the distribution remains constant over the sample. The second assumption must also be tested.

The test of homoskedasticity used here is suggested by White et al (1987). First, an OLS estimate of the functional form is made and the error term estimated. In this study, all the functional forms will contain an independent variable (log S_{t-1}), a measured dependent variable ($\widehat{\log} S_t$) and an estimated error term (log e_t). The estimated error term is then regressed against the independent variable and dependent variable in the following forms:

$$(\log e_t)^2 = a + b\,(\widehat{\log} S_t)$$

$$(\log e_t)^2 = a + b\,(\widehat{\log} S_t)^2$$

$$(\log e_t)^2 = a + b \log S_{t-1}$$

Whether $(\log e_t)^2$ varies significantly across the sample is tested by calculating the χ^2 statistic and if it is higher than the appropriate critical value, the hypothesis that $\mathrm{var}(\log \varepsilon_t) = \sigma^2$, a constant, is rejected.

Empirical evidence alone can lead to confusion between heteroskedasticity and mis-specification of the functional form of the model. The omission of a relevant variable from the functional form may produce certain patterns in the residuals, i.e. heteroskedasticity. There are other causes of heteroskedasticity (e.g. measurement or sampling errors) but the observed patterns in the residuals may not, in themselves, be sufficient to diagnose the cause.

Also, stochastic models offer no a priori grounds on which to conclude that heteroskedastic patterns in the residuals are not due to a omitted (behavioural) variable. Here, where the model is a null hypothesis, the omission of a variable would defeat the null hypothesis. The presence of heteroskedasticity is, in itself, sufficient grounds for rejection of the stochastic models.

3.1.3. Transition (Non-Equilibrium) Test of the Log-Normal Distribution

The test of the basic log-normal distribution takes the form:

$$\log S_t = \widehat{\log}\,\alpha + \log S_{t-1} + \log e_t \qquad (3.6)$$

where $\widehat{\log}\,\alpha$ and $\log e_t$ are estimates of $\log \alpha$ and $\log \varepsilon_t$ specified in equation (2.5). Log ε_t is hypothesized to be normally distributed, i.e. $N(0, \sigma^2)$. Unfortunately, $\log \varepsilon_t$ cannot be observed, so it is estimated ($\log e_t$). The parameters of the distribution of $\log e_t$.and $\text{var}(\widehat{\log}\varepsilon_t) = \dfrac{\Sigma(\log e_i)^2}{n-1}$, are used to test the hypothesis that $\log \varepsilon_t \rightarrow N(0, \text{var}(\widehat{\log}\varepsilon_t))$.

These parameters, with the number of firms **n**, are used to calculate the expected frequency of residuals $\mathbf{E_i}$ and the actual frequency of firms $\mathbf{O_i}$ in each of **k** sub-samples. The χ^2 statistic is calculated to accept or reject the hypothesis, that $\log \varepsilon_t$ is normally distributed.

The assumption that the distribution is homoskedastic is also tested. The stochastic model is a contradiction of the behavioural models and it would be defeated by the presence of a behavioural variable. The presence of heteroskedasticity admits the possibility of a behavioural variable and is sufficient to reject the hypothesis.

3.1.4. Testing Initial Size of the Firm as a Determinant

The third specific assumption of the stochastic model is that the size of each firm in period t is independent of the firm's initial size. The purpose of this test is to determine if, by relaxing the third assumption, the fit between the log-normal distribution and the distribution of firm size in the sample can be improved. If so, then the influence of initial firm size will have contributed some explanation to the unexplained variance observed in the previous test.

The test of firm size as a determinant of growth takes the same form as the previous test of the basic log-normal distribution, shown in equation (3.6). In this test, the coefficient β is inserted to permit influence from initial firm size. Following equation (2.9), the test is in the form:

$$\log S_t = \hat{\alpha} + \hat{\beta} \log S_{t-1} + \log e_t \tag{3.7}$$

where $\hat{\alpha} = (1 - \hat{\beta})\hat{\kappa}$.

No restriction can be placed on $\hat{\alpha}$. It contains an estimate of κ, a parameter of which there is no *a priori* knowledge.

The parameter $\hat{\beta}$ is an indication of effect of initial firm size. $\hat{\beta} > 1$ indicates a progressive process in which large firms grow faster than small firms and the variance of the size distribution of firms is higher than would be generated by a pure stochastic process. $\hat{\beta} < 1$ indicates a regressive process, in which large firms grow slower than small firms and variance the size distribution tends to collapse around its mean over time. The variable could be restricted to $\hat{\beta} \neq 1$, rejecting the possibility that initial firm size has no effect.

The first objective is to test the hypothesis that $\beta = 1$.

Again the error term $\log e_t$ is an estimate of $\log \varepsilon_t$, which is hypothesized to be normally distributed. The second objective of the test is to accept or reject that hypothesis. This is done using the χ^2 statistic. If it is higher than the appropriate critical value, the hypothesis that $\log \varepsilon_t \rightarrow N(0, \sigma^2)$ is rejected.

Estimation of equation (3.7) by least squares methods requires that, by assumption, the parameters of the distribution of growth rates faced by each individual firm in a sample (or a sub-sample) be identical. This is a rigourous assumption but it is also a economically plausible one. Each firm in a sample may have deterministic influences that are internal to the firm, such as cost curves and management behaviour. However, the influences external to each firm (proposed to be stochastic in this study) are the same, provided each firm is in a single market The sub-samples used in this study are each associated with a single market.

The assumption of homoskedasticity is also tested. Again, the presence of heteroskedasticity admits the possibility of a deterministic variable and is sufficient to reject the hypothesis.

3.1.5. Testing Persistence of Growth as a Determinant

The fourth specific assumption of the Pareto and log-normal distributions is that the size of each firm in period t is independent of the firm's previous growth. The purpose of this test is to determine if the relaxation of the fourth assumption improves the fit of the distribution to the size distribution in the sample. If the fit is significantly better than the fits of the earlier tests, then the influence of previous growth will have contributed to reducing some of the unexplained variance in those earlier tests.

Following equation (2.11), the test takes the form:

$$\log S_t = \widehat{\log} \alpha + \log S_{t-1} + \hat{\omega}(\log S_{t-1} - \log S_{t-2}) + \log e_t \qquad (3.8)$$

where $\hat{\omega}$ is a measure of the persistence of growth. Equation (3.8) can be restated as:

$$(\log S_t - \log S_{t-1}) = \widehat{\log} \alpha + \hat{\omega}(\log S_{t-1} - \log S_{t-2}) + \log e_t \qquad (3.9)$$

The objective of the test is to determine whether $\hat{\omega}$ is significantly different than zero.

The assumption of homoskedasticity must be tested in the context of this specification since the presence of heteroskedasticity admits the possibility of a deterministic variable and is sufficient to reject the hypothesis.

3.3. Empirical Results of Previous Studies

Empirical tests of the stochastic model have followed three very general methods. The first compares the size distribution of firms to expected equilibrium distributions. Three studies reviewed here follow this method: Quandt (1966), Silberman (1967) and Ijiri and Simon (1974). The second measures the changes in firm size over a limited time and examines the pattern of transition from one period to the next. This method is followed in two studies reviewed here: Hymer and Pashigian (1962) and Singh and Whittington (1975).

The third general method is to make some tangential predictions from the stochastic model and test them. One clear prediction of the stochastic model is that mean and variance of the distribution of growth rates will not be related to firm size. If data is divided into sub-samples by firm size, then the distribution of growth rates should not significantly differ from one sub-sample to another.

3.3.1. Quandt (1966)

This work remains one of the most comprehensive studies undertaken which follows the first general method of comparing the sample distribution to a theoretical equilibrium distribution.

Quandt chose a number of different theoretical forms of a skewed distribution in equilibrium:

1. The basic Pareto distribution, $F(x) = 1 - (\frac{k}{x})a$;
2. An alternate form of the Pareto distribution, $F(x) = 1 - \frac{k}{(x+c)a}$;
3. Champernowne's (1953) distribution, $F(x) = 1 - \frac{ke\text{-}bx}{xa}$; and
4. A log-normal distribution, $F(x) = \int_0^x \frac{e -(1/2)[(\log x - m)/s]2/s}{xs\sqrt{2p}}\,dx$;

where F(x) is the probability density function of x and a,b,c, k, m, p and s are parameters.

Three samples were compiled. To compile the first sample, Quandt chose 30 industries in the United States at the four-digit SIC level[12]. The assets of every firm found listed in Moody's Manuals were included in a sub-sample for each industry. This list omitted private firms and subsidiary firms, with the probable result being the exclusion of many smaller firms. Each of the other two samples was the U.S. Fortune 500 listing for

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[12] The Standard Industrial Classification. All industry as a whole is first divided into 9 general categories. This, the highest level of division, is called the one-digit level. Each category at the one digit level is subdivided again and assigned a two digit code. As an example, all financial institutions are placed in the same one-digit level category, coded as 6000. All insurance companies are placed in the same two-digit level category, 6300. The similarities and differences of one industry to another are reflected in the individual code assigned to each firm, which shows its classification through 4 levels of subdivision.

one year. This also excluded private and subsidiary firms and also excluded all but the 500 largest firms. In the 1960's, Fortune 500 data was not classified by industry.

Quandt assessed the goodness of fit of each sub-sample to each of the distribution forms listed above. The result in each case was assessed first for overall goodness of fit and second for randomness in the residuals. The two Pareto distributions were a poor fit to the sub-samples, being rejected in at least half of the sub-samples. The Champernowne distribution fared better, being rejected in about one-third of the sub-samples. Quandt concluded that the log-normal distribution, which was rejected in one-fifth of the sub-samples, could be considered to have an acceptably close fit.

Quandt then measured the randomness of the residuals resulting from the fit of all the distributions above to all sub-samples, using three tests, :

1. A runs test, counting negative and positive residuals;
2. Fitting high-degree orthogonal polynomials to the residuals; and
3. Comparing observed and expected power spectra.

The second test indicated that the residuals of the log-normal distribution were the closest to being random, failing the randomness test in about one-third of the sub-samples. Of interest was the fact that the sub-samples in which log-normal distribution failed the closeness test were different that the sub-samples in which it failed the randomness test.

Quandt declined from nominating any distribution as being acceptable. His results added to the evidence that a pure stochastic model in an equilibrium condition was not an adequate model of firm size and growth. The singular contribution of the study was to develop and demonstrate the tools required to discriminate amongst subtly different forms of the stochastic process.

Quandt (1966) had a narrower scope than this study. Quandt (1966) was restricted to equilibrium tests of four distributions, of which two (Pareto and log-normal) are duplicated here. This study moves beyond Quandt (1966) to test non-equilibrium models with and without modifications. Also, this study does not use some of the more sophisticated methods to determine goodness of fit. Quandt's objective was to discriminate among the four distributions, where this study seeks to accept or reject one distribution in each test.

3.3.2. Silberman (1967)

This was also a test of goodness of fit between a sample and a theoretical equilibrium distribution. Silberman favoured the log-normal distribution not for any superior ability to reflect economic realities but because it could be transformed into linear form and subjected to standard tests of significance. A two-parameter log-normal distribution was modeled.

Silberman pointed out that previous studies had been corrupted to some unknown extent by the exclusion of some firms from the sample. Because of the nature of the sources, data on smaller and private firms were less readily available. Silberman's immediate objective was to use a statistical procedure that did away with the requirement for data on the individual firms in the population.

Silberman devised such a test, which compared expected and observed values of firm ranks in the sample. In approach, it was not significantly different than the χ^2 tests used in this study. However, it was configured to require only the concentration ratios[13] in each sample and the mean and variance of firm size in each sample. Information on the individual firms in the sample was not required.

The goodness of fit test compared the actual sample concentration ratios with hypothetical concentration ratios expected for a log-normal distribution. The hypothetical values were calculated from the mean and variance of the samples, assuming that the sample mean and variance were that of a log-normal distribution. The actual and hypothetical concentration ratios were compared. If they were significantly different, the hypothesis of a log-normal distribution was rejected. This test was first applied to the upper tail of the distribution, among the largest firms, where the differences between the various skewed distributions is most pronounced.

Data was taken from the U.S. Census Bureau reports for two years, 1947 and 1958. Sales was used as the measure of size. Ninety sub-samples were chosen, each comprising an industry at the SIC four-digit level.

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[13] The market share held by the largest firms in that market. The percentage share held by the four largest and the eight firms are most commonly reported, denoted as CR_4 and CR_8 respectively.

The hypothesis had to pass the following tests, in order:

1. There are more than 20 firms in the sub-sample;
2. No significant difference between the actual and hypothetical CR_4 ratios; and
3. No significant difference between the actual and hypothetical CR_8, CR_{20} and CR_{50} ratios.

The 90 sub-samples were tested and categorized as follows:

Table 3.3.2.

	Number of Sub-Samples	
	1947	**1958**
The distribution is log-normal at all levels (acceptance)	36	42
Only the smaller firms are log-normal (rejection)	28	31
The smaller firms are not log-normal (rejection)	16	9
Insufficient data to perform tests (rejection)	10	8
Total	90	90

Silberman did not find, on the whole, that the acceptance or rejection of the hypothesis varied significantly between the two years tested. Silberman concluded that the log-normal distribution was not an appropriate model of firm size distribution. However, he did note that the log-normal distribution was most successful in those samples with a large number of firms.

Silberman (1967) contributed a statistical method which did not require data on the individual firms in an industry sub-sample. This useful tool was necessary due to the paucity of data available at the time for small firms. However, the data used in this study[14] is fairly complete in that it includes individual firm data for a large number of firms. The data spans most of the firms in each of the industry groups identified.

While recent improvements in available data render Silberman's (1967) method largely unnecessary, the results are still relevant.

14 The "Report on Business 1000" is used in this study. It is specified in fourth chapter.

3.3.3. Ijiri and Simon (1974)

Ijiri and Simon observed that the divergence of sample data from the Pareto distribution was quite systematic; following a pattern of concavity in the general form:

$$\log S_i = \log A - v\log R_i + c(\log R_i)^2 \qquad (3.12)$$

They noted that the value of **c** was usually positive and the resulting distribution was concave upward.

Ijiri and Simon believed that the simple log-linear Pareto distribution was economically plausible and statistically appealing. Their study had the objective of finding an explanation for the quadratic form within economic plausibility. They explored two contributing causes, serial correlation in firm growth and the effect of mergers.

They modified the form of the distribution to account for serial correlation in the growth of firms. Their justification for the inclusion of such persistence of growth was the notion that good luck in one period will yield benefits that persist into subsequent periods[15]. With no independent means to estimate the degree of serial correlation, ω, they did not test their model.

The second plausible explanation for concavity was the effect of mergers and acquisitions. The authors had previously made analytical modifications[16] to the stochastic process to include mergers and acquisitions according to two assumptions:

1. The probability of the firm disappearing is independent of its size; and
2. The assets of the merged or acquired firm would be distributed to other firms in proportions that were independent of the sizes of those other firms.

However, it was clear to the authors that the experience of mergers and acquisitions did not follow these two assumptions. The authors drew data from the Fortune 500 listing.

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15 Their modifications to include the persistence of growth in the stochastic process are used in this study and described above in the previous section.

16 See Simon (1955).

Using an independent history of merger and acquisition activity[17], they reversed all the mergers and acquisitions, and created an adjusted "mergerless" database.

Comparing the adjusted and original databases, the authors made two observations. First, the rate of disappearance was significantly higher among smaller firms. Second, the mid-sized firms seemed to acquire a disproportionate share of the disappearing firms' assets.

The authors fit the adjusted database to the Pareto distribution and found that the fit had improved significantly. The coefficient **c**, while still greater than zero, had been significantly reduced by removing the effects of mergers and acquisitions. Also, the v coefficient was reduced. Since this coefficient is a measure of concentration, Ijiri and Simon concluded that mergers and acquisitions had an independent effect of increasing concentration.

Ijiri and Simon's (1974) method is used in this study, in the equilibrium test of the basic Pareto distribution.

3.3.4. Hymer and Pashigian (1962)

There are fewer examples of previous work on the non-equilibrium tests. This is one of the earliest. The authors performed their regressions with some skepticism, having somewhat discredited the Law of Proportionate Effect in the first part of their paper by showing that the variance of firm growth rates from their industry mean was inversely related to the size of the firm.

The authors tested the effect of initial size on firm growth, G_i, considering both the absolute size of the firm, $\mathbf{S_i}$ and its rank or relative size, $\mathbf{R_i}$:

$$\mathbf{G_i} = a + b\mathbf{S_i} + c\mathbf{R_i} + d\mathbf{G_I} \qquad (3.13)$$

where $\mathbf{G_I}$ is the mean growth rate of the industry group, determined independently. Results were determined for a sample of US firms first disaggregated at the two-digit level and then at the three-digit level. The coefficient b was always positive but never statistically significant. R^2 values ranged from 0.249 to 0.400.

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[17] U.S. Federal Trade Commission (1970). *Large Mergers in Manufacturing and Mining, 1948-1969.*

The authors observed that the variance of growth rates in sub-samples comprised of smaller firms was greater than the variance of growth rates in sub-samples comprised of larger firms. They hypothesized that this may be due to high variance among industry mean growth rates for industries with a relatively large proportion of small firms. Otherwise the variance of individual firm growth rates must be greater in industries with small firms. Hymer and Pashigian tested the latter hypothesis by regressing the standard deviation of each industry sub-sample, **D**, on the average size of the firms in that sub-sample, $\bar{S}$:

$$D = a + b\bar{S} + cG_i + dC \qquad (3.14)$$

where **C** was a measure of concentration in the industry sub-sample.

This regression indicated that the standard deviation of growth rates of firms in each sub-sample was not negatively related to the average size of the firms in that sub-sample. The authors also found that the standard deviation of firm growth rates was positively related to the mean rate of growth in the industry. R^2 values ranged between 0.425 and 0.667. Concentration was not a significant variable. The authors concluded that the greater variability of growth rates in smaller firms was not due to their industry grouping but occurred simply because of their small size.

Hymer and Pashigian's (1962) method is used in this study, in the test of the persistence of growth. This study, however, does not include rank or concentration as variables. Their inclusion becomes a test for the presence of behavioural influences on firm growth, which is beyond the scope of this study. The test for correlation between the variance of size sub-samples and the size of the firms in the sub-samples is also beyond the scope of this study.

3.3.5. Singh and Whittington (1975)

Singh and Whittington used a large and comprehensive database, 2000 British firms from 1948 to 1960 divided into 21 industry groups, to perform non-equilibrium tests and therefore their work is frequently quoted.

Singh and Whittington began with a test of equation (3.7) across the full cross-section of 21 industry sub-samples and the sample as a whole. For the entire sample they measured:

$$\log S_t = 0.41 + 1.06 \log S_{t-1} \qquad R^2 = 0.82$$

The result $\hat{\beta} = 1.06$ was found (with a t-ratio at the 5% level of significance) to be greater than 1, which would confirm a progressive process in which large firms grow faster than small firms. However, the dissagregated measurement of $\hat{\beta}$ in the sub-samples was significantly greater than 1 in only 3 of the 21 cases. It is difficult to see these results as strongly supporting the notion that the initial size of the firm influences its future growth.

The authors made a passing observation that $\hat{\beta}$ appeared to be higher in those industry sub-samples that had lower mean rates of growth, implying that , in relatively stagnant industries, larger firms grew relatively quickly and small firms grew relatively slowly. To investigate this observation, the sample was divided into two intervals, 1948-1954 and 1954-1960. The Spearman rank correlation coefficient, r_S, was calculated between $\hat{\beta}$ and the industry sub-sample mean growth rate across the 21 sub-samples, with the following results:.

Table 3.3.5.

	r_S
1948-1954	– 0.553*
1954-1960	– 0.574*
1948-1960	– 0.198

with * indicating a result less than 0 with a significant t-ratio at the 10 % level. Such a result tends to confirm that β is higher in industry groups with lower mean growth rates.

To test the persistence of firm growth, Singh and Whittington measured the growth rate of each firm during each period, g_{it}. The growth rate from 1954 to 1960, g_{i2}, was estimated as a function of the growth rate in period 1948 to 1954, g_{i1}:

$$g_{i2} = a + bg_{i1} + e_{it} \qquad (3.15)$$

which is equivalent to equation (3.9)[18] and b is a measure of ω, an indicator of the persistence of growth . The estimate could not prove that there was any significant correlation between g_{i2} and g_{i1} (R^2 was less than 0.05) and the values of the coefficient b were not reported. However, while it appears they had no evidence to support them, the authors reported that the values of b determined across the entire sample and across most of the industry sub-samples were significantly greater than zero and considerably below one.

Although they found no acceptable evidence of persistent growth, the authors conducted a third test, which, in effect, conjunctively tested the hypotheses that growth was influenced by initial firm size and previous growth:

$$(\log S_t - \log S_{t-1}) = a + b \log S_{t-1} - c(\log S_{t-1} - \log S_{t-2}) + e_{it} \qquad (3.16)$$

The addition of persistent growth as a variable weakened the relationship between growth and initial size, as determined above, to the point of statistical insignificance. Singh and Whittington were forced to conclude that the positive relationship between growth and initial size was largely determined by serial correlation of growth.

Singh and Whittington's (1975) method is used in this study, in the non-equilibrium tests of the log-normal distribution, modified for effects of initial firm size and the persistence of growth.

3.3.6. Other Studies

A number of other studies have investigated the hypothesis that a stochastic process of firm growth will result in a skewed distribution of firm size in a market and thus an increase in concentration over time. The results of some of those studies are summarized here.

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18 Because $g_{i1} = (\log S_{1954} - \log S_{1948})$ and $g_{i2} = (\log S_{1960} - \log S_{1954})$, then

$g_{i2} = a + bg_{i1} + e_{it}$ is equivalent to

$$(\log S_{1960} - \log S_{1954}) = \widehat{\log \alpha} + \hat{\omega}(\log S_{1954} - \log S_{1948}) + \log \varepsilon_t$$

Investigating the mean and variance of growth rates across sub-samples, Eatwell (1971) found insufficient evidence to prove Gibrat's stochastic growth process, finding sufficient evidence to confirm that only the mean rate of growth was not significantly different across sub-samples of different firm sizes.

Empirical evidence finds, with few exceptions, that the variance of growth rates is lower in sub-samples of larger firms. Singh and Whittington (1975) and Marcus (1969) found this same result using two different approaches. Singh and Whittington studied over 1,000 firms in the U.K. at highly aggregated (first and second digit S.I.C. groups) industry levels, while Marcus studied a smaller number of Canadian firms at a more disaggregated market level. Both these studies spanned periods of 10 to 20 years. Hart and Prais' (1956) study covers a much longer period, namely 1885 to 1950, and their results are among the few that show no correlation between firm size and the variance in growth rates.

4.1. Requirements of the Data

4.1.1. Variables

The tests specified in chapter three require time series data for a large number of individual firms. Little data is required for each firm: three sequential and discrete measurements of firm size, namely S_t, S_{t-1} and S_{t-2}. The firms in the sample can be arranged by rank, generating the variable R_i. Also, each firm is categorized into a broadly defined industry group, allowing the division of the sample into industry sub-samples at a level equivalent to the SIC first digit level. This permits the tests to be run on sub-samples of similar firms.

4.1.2. Choice of Variable to Measure Firm Size

The choices of available data to measure firm size are sales, assets and number of employees. Interchangability of these measures is usually accepted on the same grounds as instrumental variables, that is, a strong correlation among the independent variables to be replaced and each having a high degree of independence from the error term.

Smyth et al (1975) test the conditions under which one available measure of firm size can be treated as representative of another. They conclude that the relationship between two alternative measures of firm size must be log-linear and proportional[19] in order for those two measures to generate the same results in empirical studies. Their empirical investigation of U.S. and U.K. data for large firms over the period 1968-1973 shows that none of the measurement variables are proportionately related to any other in log-linear form and that using sales as a measure of growth will overstate firm growth and concentration.

Shalit and Sankar (1977), in a more detailed study, develop theoretical conditions under which measurement interchangability may occur. They also conclude that the relationship between two measurement variables must be log-linear and proportional.

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[19] For example, log(sales) = m log(assets), where m is a constant.

Using the U.S. Fortune 1,000 in 1971-72, Shalit and Sankar test the relationships among pairs of measurement variables. They find the best and only acceptable interchange, in log-linear form, occurs between assets and shareholders' equity. Sales, when paired against other measures, interchanges progressively better with the following (in ascending order): market value of stock, shareholders' equity, employees and assets. Sales and assets were the second-best of all the pairs.

Sales[20] is the chosen measure in this study. The number of employees in a firm must be adjusted for labour productivity before it can be used as a universal measure of firm size. Also, service firms and firms in different industries will have different assets. These must be adjusted for the productivity of capital in the appropriate industry group. The use of assets introduces a difficulty in finding a common basis for the valuation of assets.

4.2. Specification of the Data

4.2.1. Source of Data

Data for individual Canadian corporations are published in the *Report on Business Magazine* by the *Globe and Mail.* This data covers the 1,000 largest public and 200 largest private corporations in Canada. It is available in sufficient detail from 1978 to 1989 to provide the annual sales of the individual corporations for three consecutive years and categorize those corporations by their principal markets and activities.

4.2.2. Choice of Study Period

The choice of the period covered in this study is governed by two criteria:

[1] The extent to which the period represents business cycles or characteristic conditions in the world economy; and

[2] The recency of the data.

The first criteria is best met in three consecutive periods of steady growth in the Canadian economy and the other national economies into which Canadian firms export.

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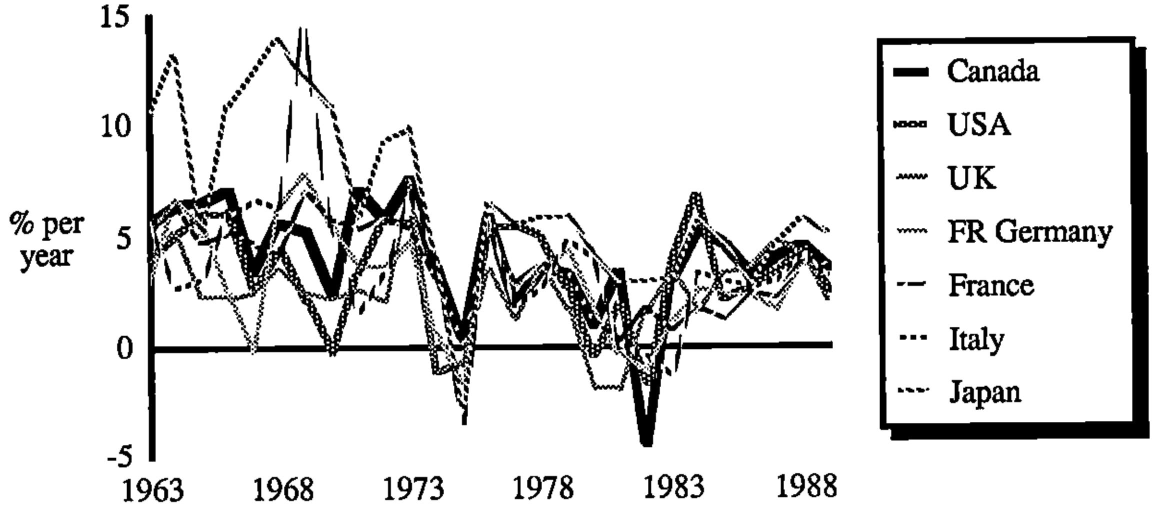
20 The reported sales figures were in nominal dollars. They were converted to constant dollars for this study using the Bank of Canada's calculation of the GDP deflator.

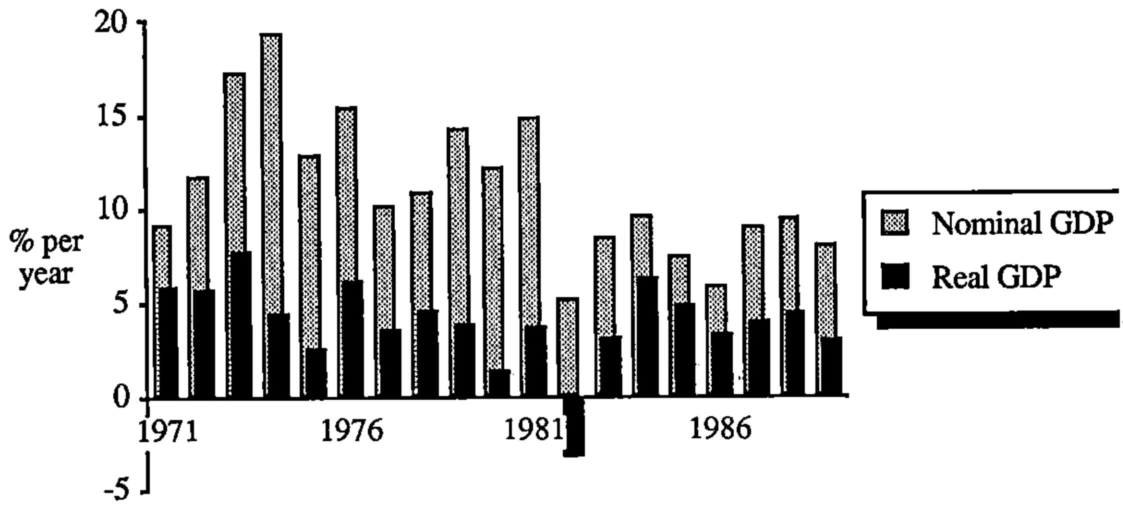
The two graphs below indicate that such a three-year period most recently occurred in the years 1986, 1987 and 1988.

Graph 4.2.2.1: % Annual Change in Constant Price GDP of the G7[21]



sources: I.M.F. Financial Statistics, various issues to December 1990.

Graph 4.2.2.2: % Change in Canadian Nominal GDP and Real GDP



source: Bank of Canada Review, January 1991.

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21 The constant price GDP data over the time period of the graph was based on three base years, generally one in the early 1970's and two in the 1980's. Data reported in terms of the subsequent base years were converted to the first base year as follows:

data in first base units = data in second base units * $\frac{\text{subsequent base year in first base units}}{\text{subsequent base year in subsequent base year units}}$

These graphs present data on economic growth in Canada and other G7 countries. This data identifies business cycles and periods during which some economic condition may have been characteristic. Figure 1 shows the period prior to the 1975 "recession"[22] to be one of high but erratic growth. The period 1975 to 1982 contains a cycle of growth that is similar amongst the seven countries, as is the post-1982 recovery.

Figure 2 displays the growth rates of real and inflationary gross domestic product in Canada. Real GDP growth again shows the 1975-1982 cycle and the resumption of higher growth in more recent years. Price inflation was legislated to relatively low levels in 1977 and 1978.[23] The 1975-1982 period was characterised by higher inflation than occurred in the subsequent period.

The period 1986 to 1988 appears to be the most recent period of positive and approximately log-linear macro-economic growth.

4.3. Limitations of the Data

In general terms, there can be three inaccuracies in the data which cause the stochastic hypotheses to fail. First, a firms may be improperly identified or reported in the sample. Second, firms may be improperly classified and placed in an incorrect sub-sample. Third, the firms selected for the sample may not be representative of the population. These inaccuracies could manifest themselves in this study through the problems laid out below.

4.3.1. Diversification

A firm in a market may be a subsidiary of a larger firm which has diverse activities across a number of different markets. The size and degree of diversification of the parent firm may change while the market share of the subsidiary firm under study remains constant. Such a change may, through the availability of resources and the strategic interests of the parent firm, change the market power and behaviour of the subsidiary firm in its market.

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22 A recession is two or more consecutive quarters of negative real GDP growth. This did not occur in all the G7 countries.

23 This was accomplished through the Anti-Inflation Board, established in 1976.

Antoniou (1988) proposes an adjustment to the market share of the subsidiary firm to account for the market share held by the parent firm in all other markets. He demonstrates the improved predictive power of the adjusted market share on simulated data. However, the data requirements of such a measure, namely all the relative market shares of every market in which the parent operates, make it very difficult to apply.

In any event, this study examines markets at a high level of aggregation and the effects of diversification are minimized.

4.3.2. Aggregation and Inter-Industry Effects

It is plausible that all firms within a particular industry may have cost curves which have more in common with one another than with cost curves of firms in different industries. It is therefore advantageous to construct sub-samples with firms that operate in substantially identical markets. In practice, such neat groupings of individual firms with near-identical market profiles are difficult to achieve.

Many large firms sell their outputs into multiple markets. Some firms will sell in related markets, all centered around a core business activity, and be vertically integrated. Other firms may sell products in diversified markets but the business activities of the firm are related with a common thread of management expertise. A common example in Canada are resource companies which apply their resource extraction experience across the mining, petroleum and forest sectors. Conglomerate firms carry on functionally unrelated activities in independent markets.

There is no universally accepted rule for categorizing such diversified firms into single industry groups. *Globe and Mail* classifies a firm as being in a particular industry or market if more than 50% of its sales are in that market. For firms whose single largest market makes up less than 50% of its total sales, *Globe and Mail* makes a qualitative judgment as to what comprises the core activity of the firm. Often, core activity is chosen on the basis of management expertise and not market exposure. Imperfect as this judgment is likely to be, it is accepted here as the best guess.

If the nature of a firm's business activity changes over time, it should be reclassified from one industrial group to another. There may be some lag in recognizing such a transition and, when reclassification does occur, the simplest solution is to exclude the firm from the sample. Such exclusions may bias the results if transitional firms tend to grow faster or slower than the mean.

Covering the spectrum of the sample with a few sub-samples will result in a high aggregation of data within each sub-sample. Each sub-sample is more likely to contain firms facing different market structures. The stochastic model does not, *a priori*, accept the hypothesis of market structure effects on firm performance. However, higher variations in market structure within each sub-sample may provide more opportunity for the stochastic hypothesis tested here to fail.

4.3.3. Small Firms with Low Growth Rates

The *Globe and Mail* reports data for the largest 1000 public and 200 private firms. A firm whose initial size puts it near the bottom of the sample may be smaller than the 1000th firm in subsequent years. Since only firms that are reported by the *Globe and Mail* for all three years are retained in the sample, some slow-growing small firms are eliminated and the estimates of small firm growth rates will be biased upward. The perscribed treatment for these biases is using a reduced sample. Thus, the possible samples are:

(A). The sample described above with an upward bias in the growth rates of small firms; and

(B). A smaller sample, in which all small firms are eliminated regardless of their growth rate.

Two concerns arise with the use of the smaller sample. First, number of firms in some sub-samples may be reduced to a number which does not permit the application of the χ^2 test. Second, the elimination of all smaller firms may not eliminate biases if smaller firms have growth rates which are significantly different than larger firms.

4.3.4. Merger and Divestiture

When firms in the sample merge together within the time frame of the sample, the sales of the acquiring firms and all the firms that it acquires are added together to create a single measure of size that is consistent before and after the merger. If one of the merged firms was absent from the *Globe and Mail* data prior to the merger, all are excluded from the sample. If a firm that was not originally included in the data is acquired by a larger firm, its acquisition is considered to be additional growth of that larger firm.

If a firm in the *Globe and Mail* data divests a subsidiary and the subsidiary remains as an independent company in the data, the former subsidiary is excluded from the sample. If the divested subsidiary is acquired by another firm in the *Globe and Mail* data, then it is considered as merged to the new parent and its growth is transferred from the old to the new parent as additional growth.

This asymmetrical treatment of mergers and divestments is necessary due to the likelihood of a parent firm using the proceeds of the divestment in new acquisitions or other growth activity. If a separated parent and subsidiary continued to have their growth combined in subsequent years, as the principle applied to mergers would suggest, the measurement of the parent firm's growth rate would be biased upward. That parent firm would be assigned the growth rate of itself, the former subsidiary and the new activities it financed with the funds gained through divestiture.

4.3.5. Other Limitations

The *Globe and Mail* has assured access to data from publicly traded corporations but private or government-owned firms may or may not respond to the *Globe and Mail's* requests for data.

Companies are permitted a wide latitude within that scope for reporting sectoral or world-wide sales, which introduces distortions in comparisons among companies in a given year.

4.4. Characteristics of the Data

The specified data were loaded into a computer spreadsheet. Of the 1,000 firms present in 1988, 12 were lost to transcription errors, leaving 988 firms available for analysis.

4.4.1. Industrial Classification

The *Globe and Mail* uses 42 industry or market classifications and assigns one to each firm. There are too few firms in the sample to ensure that each of 42 industry sub-samples would contain enough firms for the χ^2 test. The *Globe and Mail's* classifications were aggregated into eight industry categories, as shown below:

Table 4.4.1 The Industrial Classification of Firms

1. Finance and Management: banks, finance, management , property and casualty insurance, life insurance, trust companies.

2. Resources: forests, pulp and paper, base metal mining, precious metals, oil and gas production, oil and gas production services, integrated oil companies.

3. Chemicals: chemicals. medical and pharmaceutical, textiles, cement.

4. Manufacturing: automotive, consumer products, engineering, home furnishings, industrial products, steel, technology products, other manufactured products.

5. Media: broadcasting, entertainment, publishing.

6. Services: retailing, warehousing, real estate development, other services.

7. Transport: transportation, telecommunications, pipelines, utilities.

8. Food: beverages, food production, food distribution.

4.4.2. Continuing Presence of Firms

Many firms do not remain within the largest 1,000 firms for all of the three years 1986, 1987 and 1988. Only 735 firms are reported for all three of those years, 860 were reported for both 1988 and 1987 and 988 firms are reported for 1988 only.

There are two general reasons why a firm available in 1988 was not available in 1987 or 1986. First, the reporting practices of corporations change as firms merge and divest themselves of subsidiaries; refinancing may change their public reporting requirements. Second, small and fast-growing firms or new firms may appear in 1988 which were too small to be included in 1986 or 1987.

All the instances of absence in previous years have been classified according to these general reasons. These are shown below, along with the industrial grouping of firms in the sample.

Table 4.4.2. The Absence of Firms

Industrial Grouping	Present in 1988	Previously Absent due to Reporting	Previously Absent due to Age/Growth	Remaining
1. Finance/Management:	127	14	18	95
2. Resources:	335	38	70	227
3. Chemicals:	25	1	3	21
4. Manufacturing:	163	18	24	155
5. Media:	60	6	5	49
6. Services:	163	18	24	121
7. Transport:	60	6	5	49
8. Food:	44	4	3	37
Total	988	99	154	735

The high proportion of firms previously absent in the resource sector, particularly due to recent incorporation or fast growth, reflects the volatility of the mining industry.

In this chapter, the results of the tests are reported.

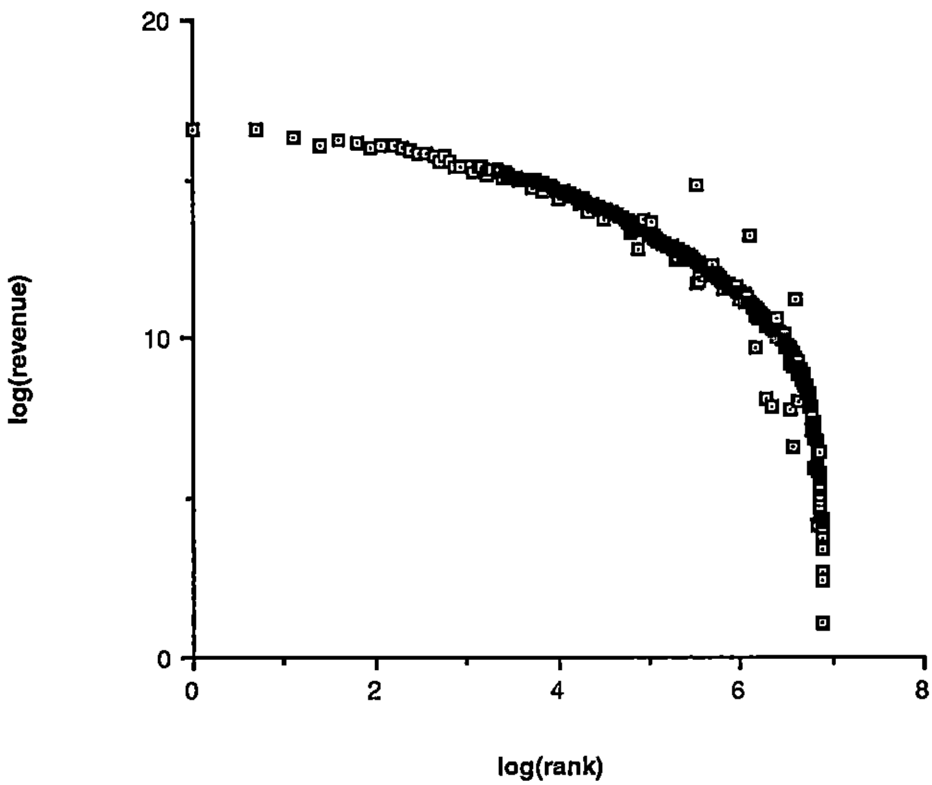
5.1 Equilibrium Test of the Basic Pareto Distribution

The test of the Pareto distribution takes the form:

$$\log S_i = \widehat{\log}\, a - \Diamond \log R_i + e_i \qquad (3.1)$$

As pointed out in chapter three, any variation of e_i from zero will indicate the presence of some determinant of firm size other than the random growth. The objective the test is to determine whether var e_i is significantly greater than zero. Casual inspection of the following graph indicates that this is likely the case.

Graph 5.1.1: Plot of 1988 Revenues against 1988 Rank



Equation (3.1) is estimated using OLS on the 1988 data; first for the entire sample and then by each industrial group. The estimated parameters $\widehat{\log}\, a$ and $\Diamond$ are, in themselves, not directly relevant but $\hat{v}$ is presented as an indicator of the market concentration in each industrial group:

Table 5.1.1: Regression Results for Basic Pareto Distribution

Industrial Grouping	$\hat{v}$	R^2	Measured $\chi^2_{(k-1)}$	Critical (10%) $\chi^2_{(k-1)}$
Full Sample	n/a	0.74	$1683_{(26)}$	35.6
1. Finance/Management:	2.01	0.83	$88_{(8)}$	13.4
2. Resources:	2.80	0.71	$329_{(26)}$	35.6
3. Chemicals:	2.42	0.79	$18_{(2)}$	4.6
4. Manufacturing:	1.83	0.76	$234_{(16)}$	23.5
5. Media:	1.96	0.93	$19_{(2)}$	4.6
6. Services:	2.00	0.83	$168_{(11)}$	17.3
7. Transport:	1.28	0.87	$41_{(2)}$	4.6
8. Food:	1.51	0.92	$9_{(2)}$	4.6

The basic Pareto distribution fails in the full sample and in every industrial sub-sample.

Equation (3.1) is then modified with higher order polynomial terms to determine if there is systematic or significant curvature:

$$\log \mathbf{S_i} = \widehat{\log a} - \hat{v} \log \mathbf{R_i} + \hat{c}(\log \mathbf{R_i})^2 + \hat{d}(\log \mathbf{R_i})^3 + e_i \qquad (3.2)$$

Table 5.1.2: Regression Results for the Higher Order Polynomial

Industrial Grouping	$\hat{c}$	$\hat{d}$	R^2	Measured $\chi^2_{(k-1)}$	Critical (10%) $\chi^2_{(k-1)}$
Full Sample	1.56	-0.16	0.91	$841_{(26)}$	135.6
t-ratio	$16_{(983)}$	$-21_{(983)}$			
1. Finance/Management:	2.01	-0.18	0.96	$64_{(6)}$	10.6
t-ratio	$6.5_{(122)}$	$-8.3_{(122)}$			
2. Resources:	5.53	-0.43	0.90	$273_{(26)}$	35.6
t-ratio	$11_{(331)}$	$-13_{(331)}$			
3. Chemicals:	20.18	-1.28	0.94	$19.7_{(4)}$	7.8
t-ratio	$4.3_{(21)}$	$-4.6_{(21)}$			
4. Manufacturing:	0.73	-0.09	0.92	$212_{(14)}$	21.1
t-ratio	$5.9_{(198)}$	$-8.6_{(198)}$			
5. Media:	1.73	-0.14	0.99	$4.4_{(4)}$	7.8
t-ratio	$4.8_{(28)}$	$-6.1_{(28)}$			
6. Services:	1.26	-0.12	0.96	$72_{(9)}$	14.7
t-ratio	$7.2_{(159)}$	$-9.7_{(159)}$			
7. Transport:	0.31	-0.05	0.99	$29_{(4)}$	7.8
t-ratio	$5.4_{(56)}$	$-9.5_{(56)}$			
8. Food:	0.40	-0.05	0.99	$26_{(4)}$	7.8
t-ratio	$3.2_{(40)}$	$-5.6_{(40)}$			

In the full sample and in all of the industrial sub-samples both $\hat{c}$ and $\hat{d}$ are significantly different than zero. The hypothesis that the distribution of firm size in the sample follows a Pareto distribution is rejected.

5.2. Equilibrium Test of the Basic Log-Normal Distribution

This is a test of the proposition that the probability distribution of growth possibilities faced by the firms in a sample is normally distributed and that the growth process has progressed to final equilibrium. As indicated by equation (2.5), the distribution of log S_i in equilibrium would be normal under this proposition. The objective of this test is to determine whether the distribution of log S_i is significantly different than a normal distribution.

The mean and variance of the sample values of log S_i are measured to be:

$\overline{\log S_i} = 10.7$ and $\mathrm{var}(\widehat{\log} S_i) = 6$. These values are used with the number of firms N = 987 to calculate $\chi^2 = 62$ with 26 degrees of freedom. Since the critical value, at the 10% level of significance, of $\chi^2_{(26)} = 35.6$, the hypothesis that the sample follows a normal distribution is rejected.

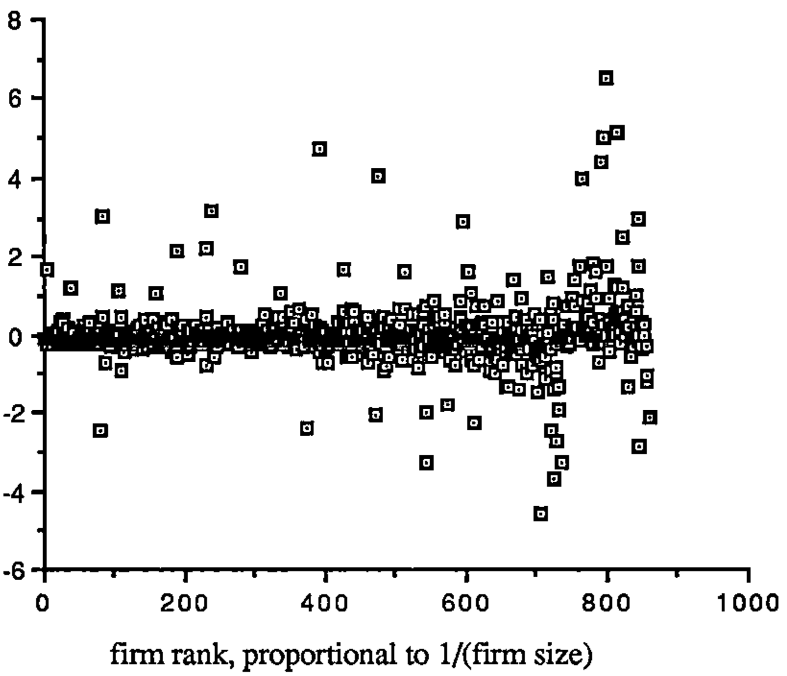
5.3. Transition (Non-Equilibrium) Test of the Normal Distribution

The test of the transitional log-normal distribution takes the form:

$$\log S_t = \widehat{\log}\,\alpha + \log S_{t-1} + \log e_t \qquad (3.6)$$

and the objective of the test is to accept or reject the hypothesis that log e_t is normally distributed.

Graph 5.3.2: Residuals of Log-Normal Transition Model Regression



5.3.1. Heteroskedasticity

Casual inspection of residual displays should not be given significant analytical weight. However, it provides some perspective on any heteroskedasticity detected. In this case, the failure of the tests for normal distribution of log e_t. might be attributable to an increasing variance of log e_t in the smaller firms.

The estimated error term, determined by OLS regression, is regressed against the independent and dependent variables and the significance of these relationships is tested by calculating the χ^2 statistic.

Table 5.3.1: Heteroskedasticity Results for Transitional Log-Normal Distribution

	Measured $\chi^2_{(1)}$	
Industrial Grouping	$(\log e_t)^2 = a + b\,(\log \hat{S}_t)$	$(\log e_t)^2 = a + b\,(\log \hat{S}_t)^2$
Full Sample	65.7	43.5
1. Finance/Management:	4.1	3.4
2. Resources:	23.1	16.1
3. Chemicals:	0.1	0.1
4. Manufacturing:	2.9	2.7
5. Media:	6.3	4.8
6. Services:	11.0	6.8
7. Transport:	3.2	2.8
8. Food:	10.4	7.3

For the full sample and all sub-samples except the chemical industry, the χ^2 statistic is higher than the critical value (10%) of $\chi^2_{(1)} = 2.7$ so the hypothesis that var(log ε_t) = σ^2, a constant, is rejected. These are sufficient grounds to reject the hypothesis that log e_t is normally distributed.

5.3.2. OLS Regression Results

Had heteroskedasticity not been present, the OLS regression of the model would have yielded the best, least squares, unbiased estimate of log e_t. In this instance it is not but the R^2 results are presented in any event.

Table 5.3.2: Regression Results for Transitional Log-Normal Distribution

Industrial Grouping	R^2	Measured $\chi^2_{(k-1)}$	Critical (10%) $\chi^2_{(k-1)}$
Full Sample	0.90	$1126_{(26)}$	35.6
1. Finance/Management:	0.91	$66_{(6)}$	10.6
2. Resources:	0.86	$420_{(16)}$	23.5
3. Chemicals:	0.98	$1.5_{(2)}$	4.6
4. Manufacturing:	0.94	$48_{(11)}$	17.3
5. Media:	0.92	$11_{(2)}$	4.6
6. Services:	0.85	$157_{(8)}$	13.4
7. Transport:	0.97	$30_{(2)}$	4.6
8. Food:	0.85	$38_{(2)}$	4.6

5.4. Testing Initial Size of the Firm as a Determinant

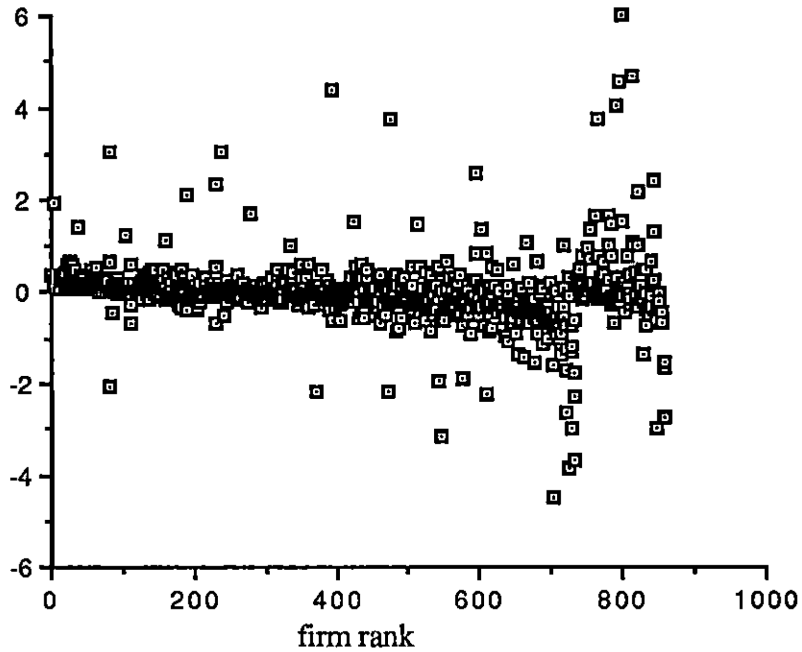
The third assumption of the Pareto and log-normal distributions is that the size of each firm in period t is independent of the firm's initial size. The purpose of this test is to determine whether including the firm's initial size as a variable improves the fit .

The test of firm size as a determinant of growth takes the same form as the test of the basic log-normal distribution shown in equation (3.7):

$$\log S_t = \hat{\alpha} + \hat{\beta} \log S_{t-1} + \log e_t \quad (3.7)$$

The first objective is to test the hypothesis that $\beta = 1$. The second objective is to test the hypothesis that log e_t is normally distributed.

Graph 5.4.2: Residuals of the Regression of Initial Size



This graph shows an increasing variance of log e_t in the smaller firms similar to that in graph 5.3.2. However, two other observations can be made. First, removing the restriction $\hat{\beta} = 1$ appears to have resulted in some heteroskedasticity, with a trend to negative residuals among the smaller firms, and a quantum increase in the value of the residuals around the 700th firm.

5.4.1. Heteroskedasticity

Log e_t is determined by OLS regression and then is regressed against the dependent variable and the significance of these relationships is tested by calculating the χ^2 statistic.

Table 5.4.1 Heteroskedasticity Results for the Test of Initial Size

	Measured $\chi^2_{(1)}$	
Industrial Grouping	$(\log e_t)^2 = a + b(\log \hat{S}_t)$	$(\log e_t)^2 = a + b(\log \hat{S}_t)^2$
Full Sample	62.9	45.0
1. Finance/Management:	4.1	3.5
2. Resources:	19.9	15.1
3. Chemicals[24]:	0.5	0.5
4. Manufacturing:	2.6	2.5
5. Media:	7.6	6.2
6. Services:	10.1	6.7
7. Transport:	3.7	2.3
8. Food:	11.8	9.1

Where the χ^2 statistic is higher than the critical value (10%) of $\chi^2_{(1)} = 2.7$, the hypothesis that $\text{var}(\log \varepsilon_t) = \sigma^2$, a constant, is rejected. This is the case for the full sample and all sub-samples except chemicals and manufacturing. Hence the hypotheses that $\log e_t$ is normally distributed is rejected.

5.4.2. OLS Regression Results

Had heteroskedasticity not been present, an OLS regression of the model would have provided the best, least squares, unbiased estimator of β. The results are presented in any event, although they cannot be used to test the hypothesis that $\beta = 1$.

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[24] The hypothesis of heteroskedasticity is not rejected for the chemical industry sub-sample. This is also the case for the transitional test of the log-normal model and for the test of persistent growth. The chemical sub-sample contains only 21 firms. This is a small number,relative to the other sub-samples, and its small size renders the χ^2 test less robust.

Table 5.4.2: Regression Results for the Test of Initial Size

Industrial Grouping	$\hat{\beta}$	t statistic Ho $\hat{\beta}=1$	R^2	Measured $\chi^2_{(k-1)}$	Critical (10%) $\chi^2_{(k-1)}$
Full Sample	0.93	$-7.3_{(857)}$	0.91	$716_{(26)}$	35.6
1. Finance/Management:	1.01	$2.9_{(106)}$	0.96	$64_{(6)}$	10.6
2. Resources:	0.89	$-4.7_{(265)}$	0.87	$168_{(16)}$	23.5
3. Chemicals:	1.01	$0.57_{(21)}$	0.98	$1.5_{(2)}$	4.6
4. Manufacturing:	0.97	$-1.3_{(184)}$	0.93	$63_{(11)}$	17.3
5. Media:	0.87	$-3.0_{(29)}$	0.94	$11_{(2)}$	4.6
6. Services:	0.88	$-4.1_{(146)}$	0.86	$89_{(8)}$	13.4
7. Transport:	0.95	$-2.3_{(53)}$	0.97	$19_{(2)}$	4.6
8. Food:	0.82	$-3.8_{(39)}$	0.89	$5.5_{(2)}$	4.6

Even if the test had indicated that $\hat{\beta}\neq 1$, that result would be negated by the rejection of the hypothesis that $\log \varepsilon_t \rightarrow N(0, \sigma^2)$. The χ^2 statistics shown above are, for the full sample and seven out of eight industrial sub-samples, higher than the appropriate critical values.

Despite the allowance for effects of initial firm size, the stochastic model is not able to fully account for the growth of firms.

5.5. Testing Persistence of Growth as a Determinant

The fourth assumption of the Pareto and log-normal distributions is that a firm's rate of growth in any period t is independent of growth experienced by that firm in previous periods. The purpose of this test is to determine whether relaxing the assumption that the size of each firm in period t is independent of the firm's previous growth improves the fit.

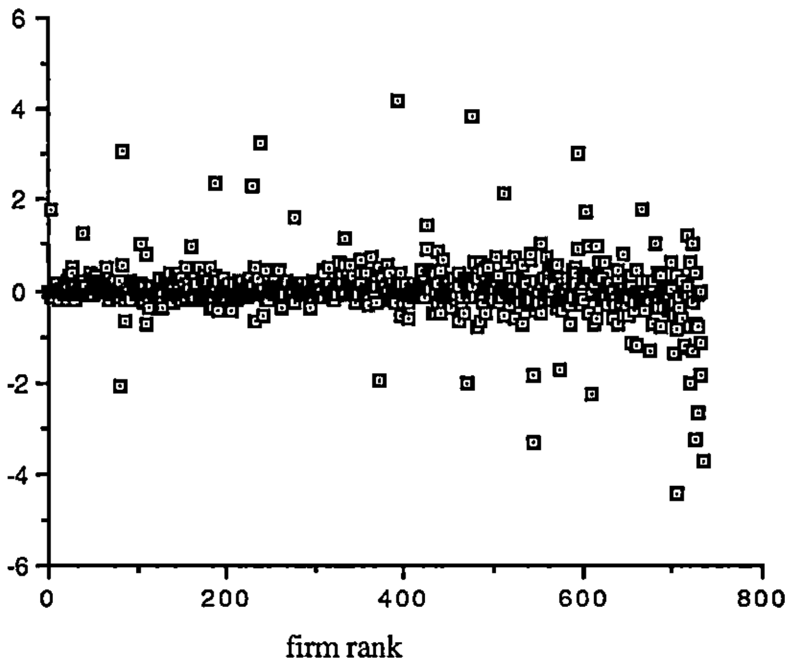
Following equation (2.10), the test takes the form:

$$\log S_t = \widehat{\log}\,\alpha + \log S_{t-1} + \hat{\omega}(\log S_{t-1} - \log S_{t-2}) + \log e_t \qquad (3.8)$$

where $\hat{\omega}$ is a measure of the persistence of growth. The objective of the test is to determine whether $\hat{\omega}$ is significantly different than zero.

The distribution of the error term is shown in the following graph. The horizontal axes shows the rank of the firm (and therefore its size: the larger the rank value, the smaller the firm). The vertical axis shows the residual, log e_t.

Graph 5.5.2: Residuals of the Regression of Persistent Growth



Bearing the limitations of casual inspection in mind, this third graph also shows increasing variance of log e_t. in the smaller firms.

5.5.1. Heteroskedasticity

The estimated OLS error term is regressed against the independent and dependent variables and the significance of these relationships is tested by calculating the χ^2 statistic.

Table 5.5.1: Heteroskedasticity Results for the Test of Persistent Growth

	Measured $\chi^2_{(1)}$	
Industrial Grouping	$(\log e_t)^2 = a + b(\log \hat{S}_t)$	$(\log e_t)^2 = a + b(\log \hat{S}_t)^2$
Full Sample	23.6	18.9
1. Finance/Management:	1.0	1.0
2. Resources:	7.6	6.7
3. Chemicals:	0.0	0.0
4. Manufacturing:	2.1	1.9
5. Media:	0.9	1.2
6. Services:	0.3	0.3
7. Transport:	1.2	1.1
8. Food:	10.0	7.6

Where the χ^2 statistic is higher than the critical value (10%) of $\chi^2_{(1)} = 2.7$, the hypothesis that $\text{var}(\log \varepsilon_t) = \sigma^2$ is rejected. This is the case for the full sample but not for 6 of the 8 industry sub-samples.

5.5.2. OLS Regression Results

Had the assumption that var(log ε_t) = σ^2 been maintained, an OLS regression of the model would have determined whether $\hat{\omega}$ is significantly different than zero. The results are presented in any event, although they cannot be used to test the hypothesis that $\hat{\omega} \neq 0$.

Table 5.5.2: Regression Results for the Test of Persistence

Industrial Grouping	$\hat{\omega}$	t ratio Ho $\hat{\omega}=0$	R^2	Measured $\chi^2_{(k-1)}$	Critical (10%) $\chi^2_{(k-1)}$
Full Sample	-0.14	$-4.4_{(732)}$	0.93	$720_{(25)}$	34.4
1. Finance/Management:	-0.05	$-0.06_{(92)}$	0.93	$19_{(1)}$	2.7
2. Resources:	-0.19	$-2.9_{(225)}$	0.91	$194_{(15)}$	22.3
3. Chemicals:	-0.09	$1.2_{(19)}$	0.98	$4.3_{(1)}$	2.7
4. Manufacturing:	-0.002	$-0.05_{(153)}$	0.94	$44_{(10)}$	16.0
5. Media:	-0.74	$-5.8_{(28)}$	0.96	$10_{(1)}$	2.7
6. Services:	-0.14	$-1.7_{(119)}$	0.91	$74_{(7)}$	12.0
7. Transport:	0.03	$0.4_{(47)}$	0.99	$13_{(1)}$	2.7
8. Food:	-0.48	$-3.9_{(35)}$	0.88	$15_{(1)}$	2.7

The tentative result is that ω is less than zero, implying that the effect of previous growth is that a stabilizer instead of an accelerator, e.g. with faster growth in the previous two years tending to retard growth in the current year.

Despite the allowance persistent growth, the stochastic model is not able to fully account for the growth of firms.

6.1. Summary Analysis of the Results

The first two tests, the equilibrium tests, found that the size distribution of firms was not a Pareto or log-normal distribution at equilibrium. It is not known whether the equilibrium size distribution of firms is something other than Pareto or log-normal, implying the models were mis-specified, or whether the sample was not at equilibrium.

The last three tests, the transition tests, found that the distribution of growth rates from one period to the next was not a log-normal distribution, even when the log-normal distribution was modified to allow for the effects of initial firm size and the persistence of growth. These tests failed due to the detected presence of heteroskedasticity. Whether the heteroskedasticity was present due to biases in the sample data or due to the stochastic model being a mis-specification is unclear.

The tactic underlying the successive tests was to relax those assumptions of the stochastic model that did not compromise the theoretical basis of the model itself. The tests would determine whether the explanatory power of the stochastic model was improved as the unessential assumptions were relaxed. The following table summarizes the progressive improvement in the proportion of variance explained as successive assumptions are removed.

Table 6.1: Summary of R^2 values from Regression Results

	Pareto	Log-Normal		
	Equilibrium	Transition	Initial Size	Persistence
Equation Number	(3.1)	(3.6)	(3.7)	(3.8)
Industrial Grouping				
Full Sample	0.74	0.90	0.91	0.93
1. Finance/Management:	0.83	0.91	0.96	0.93
2. Resources:	0.71	0.86	0.87	0.91
3. Chemicals:	0.79	0.98	0.98	0.98
4. Manufacturing:	0.76	0.94	0.93	0.94
5. Media:	0.93	0.92	0.94	0.96
6. Services:	0.83	0.85	0.86	0.91
7. Transport:	0.87	0.97	0.97	0.99
8. Food:	0.92	0.85	0.89	0.88

The table indicates improvement when the equilibrium models were abandoned and the transitional model was introduced. However, the subsequent modifications of the transitional model, to include the effects of initial firm size and previous growth brought no marked improvement.

The presence of a systematic variable in the error term requires that the general hypothesis that firm growth is fully accounted for by a stochastic process be rejected.

6.1.2. Comparison with Other Studies.

Quandt (1966), Silberman (1967) and Ijiri and Simon (1974) all found that, in general, the size distributions of their firms did not fit equilibrium distributions that would be generated by a stochastic process. This study has the same finding.

Although they did not identify it as such, Hymer and Pashigian (1962) also found heteroskedasticity in their sample. Further, the basic form of heteroskedasticity reported by them was also found in this sample: an increase in the variance of the error term among the smaller firms in the sample.

Singh and Whittington (1975) did considerable work to test the influence of the initial size of firms. Their study found weak evidence that β was significantly different than 1. This study was unable to make any conclusion about the value of β. These authors also suggested that, while they did not have a clear statistical basis to say so, that the value of ω was somewhere between 0 and 1. This study produced estimates of ω which fell in the range between 0 and -1 but that result is not given any significance.

6.2. Conclusions

6.2.1. Consequences of the Failure of the Stochastic Models

The stochastic model serves as a null hypothesis for the behavioural models: classical, managerial and life-cycle. If these four groups of models represent all possible models of firm growth then the failure of the stochastic models indicates that one or more of the others has some influence on firm growth.

6.2.2. Reasons for Failure - Review of the Assumptions

Two general assumptions underlie stochastic models:

I Each member of the population faces a probability distribution of growth from one period to the next; and

II Each member's growth from one period to the next is chosen at random from that probability distribution.

The tests began with five specific assumptions, derived from the general assumptions above:

[1] each firm faces an identical distribution of growth possibilities;

[2] that distribution does not change over time;

[3] a firm's growth rate will be independent of the firm's initial size;

[4] a firm's growth rate in a period will be independent of its previous growth; and

[5] the population does not change over time.

The tactics of testing were to eliminate the last three assumptions through construction of the sample and modification of the tests. The failure of all the tests implies that one or both of the first two assumptions were violated. The first two assumptions are necessary conditions for the application of the stochastic model.

As to which of the two specific assumptions failed, the empirical evidence provides no conclusive evidence. However, the increasing variance of the residuals in the smaller firms and the short time span used (one, two and three years) supports speculation that the distribution of growth possibilities faced by smaller firms differs from that faced by larger firms.

6.2.3. Other Conclusions - Policy Implications

There is a preponderance of large firms in Canada, outlined in chapter one. As a result, Canadian data could demonstrate, with relative clarity, the presence of managerial discretion and its use to maximize the growth of firms. The results of the tests which included initial size failed to indicate that large firms grow significantly faster than small firms. If large firms have a broad degree of managerial discretion, it does not appear to have been successfully used to increase rates of growth.

Large firms growing at the same rate as small firms suggests that either large firms are not able to achieve monopolistic control of markets or that they have already done so.

The failure of the stochastic model and the suggestion that firm growth is influenced by systematic factors creates some incentive to investigate those factors. Definition of those factors would allow the growth of firms to be predicted. The growth of individual firms is of interest to those who trade in investment and product markets. The aggregate growth of firms has an impact on market structure which, in turn, is of interest to public policy makers.

6.3. Directions for Further Work

The results of this study point in two directions for further work. The first is to perform more comprehensive tests on the stochastic models, staying within the scope of this study to find more empirical evidence on the power of stochastic models. The second is to proceed on from the scope of this study to examine other models of firm growth. The consistency with which the tests in this study failed suggests the second direction to be more productive.

6.3.1. Within the Scope of This Study

Further work on the models presented in this study would require a conclusion from this study that the stochastic models presented here fully specified the correct models of firm growth but that the empirical results were unsound. If that is true, then further tests of the models would be required using econometric methods that are capable of finer discrimination.

A logical next step in this direction would be to apply more comprehensive analysis to the residuals calculated in this study to gain some insight into the nature of the systematic influence that is present there.

6.3.2. Outside the Scope of This Study

This direction accepts the conclusion of this study, that the pure stochastic model is not the sole source of firm growth. The scope of further work would therefore be to examine one or more of the three deterministic models. Two methods could be used.

First, the specifications of the stochastic model could be further modified to combine them with certain systematic effects. These effects could be pursued through three avenues of influence on the stochastic model: market structure, firm cost curves and utility function of the firm's stake holders. This method would attempt to isolate the systematic effect in the error term of the stochastic model.

The second method would be to abandon the stochastic model entirely and specify tests based on one or more of the behavioural models. Such models would have to disallow the stochastic model entirely in order to meet econometric assumptions about the nature of their test residuals.

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Appendix One
Independent Error Terms in the Persistence of Growth Model

The proof is done by induction, so this illustration of the three-period case is more useful:

$\log S_t = \log S_{t-1} + \omega(\log S_{t-1} - \log S_{t-2}) + \log \varepsilon_t$

$\log S_{t-1} = \log S_{t-2} + \omega(\log S_{t-2} - \log S_{t-3}) + \log \varepsilon_{t-1}$

$\log S_{t-2} = \log S_{t-3} + \omega(\log S_{t-3} - \log S_{t-4}) + \log \varepsilon_{t-2}$

$$\begin{aligned}
\log S_t &= \log S_{t-2} + \omega(\log S_{t-2} - \log S_{t-3}) + \log \varepsilon_{t-1} + \omega(\log S_{t-2} + \\
&\qquad \omega(\log S_{t-2} - \log S_{t-3}) + \log \varepsilon_{t-1} - \log S_{t-2}) + \log \varepsilon_t \\
&= \log S_{t-2} + \omega\log S_{t-2} - \omega\log S_{t-3} + \log \varepsilon_{t-1} + \omega\log S_{t-2} + \omega^2\log S_{t-2} - \\
&\qquad \omega^2\log S_{t-3} + \omega\log e_{t-1} - \omega\log S_{t-2} + \log \varepsilon_t \\
&= \log S_{t-2} + \omega\log S_{t-2} + \omega\log S_{t-2} + \omega^2\log S_{t-2} - \omega\log S_{t-2} - \omega\log S_{t-3} + \\
&\qquad \log \varepsilon_{t-1} - \omega^2\log S_{t-3} + \omega\log e_{t-1} + \log \varepsilon_t \\
&= \log S_{t-2} + \omega \log S_{t-2} + \omega^2\log S_{t-2} - \omega\log S_{t-3} - \omega^2\log S_{t-3} + (1+\omega)\log \varepsilon_{t-1} \\
&\qquad + \log \varepsilon_t \\
&= (1 + \omega + \omega^2)\log S_{t-2} - \omega\log S_{t-3} - \omega^2\log S_{t-3} + (1+\omega)\log \varepsilon_{t-1} + \log \varepsilon_t \\
&= (1 + \omega + \omega^2)(\log S_{t-3} + \omega(\log S_{t-3} - \log S_{t-4}) + \log \varepsilon_{t-2}) - \omega\log S_{t-3} - \\
&\qquad \omega^2\log S_{t-3} + (1+\omega)\log \varepsilon_{t-1} + \log \varepsilon_t \\
&= (1 + \omega + \omega^2)(\log S_{t-3} + \omega(\log S_{t-3} - \log S_{t-4})) - \omega\log S_{t-3} - \omega^2\log S_{t-3} + \\
&\qquad (1 + \omega + \omega^2)\log \varepsilon_{t-2} + (1 + \omega)\log \varepsilon_{t-1} + \log \varepsilon_t
\end{aligned}$$

If the process begins at S_{t-3} then $(\log S_{t-3} - \log S_{t-4}) = 0$ and

$$\begin{aligned}
\log S_t &= (1 + \omega + \omega^2)\log S_{t-3} - \omega\log S_{t-3} - \omega^2\log S_{t-3} + (1 + \omega + \omega^2)\log \varepsilon_{t-2} + \\
&\qquad (1+\omega)\log \varepsilon_{t-1} + \log \varepsilon_t \\
&= \log S_{t-3} + (1 + \omega + \omega^2)\log \varepsilon_{t-2} + (1 + \omega)\log \varepsilon_{t-1} + \log \varepsilon_t
\end{aligned}$$

Appendix Two
Tests of Significance and Goodness of Fit

Assessing the fit of data to a theoretical distribution requires a test that differs from the F test or t test. These tests assess hypotheses that the moments of the distribution of data in a sample is significantly different than the moments of another distribution, usually the parent population. These parametric tests will only succeed when the parameters, μ and σ^2, are either known or assumed. They are inappropriate for testing the fit of data to a theoretical distribution, as each parameter of that theoretical distribution would have to be restricted to some specific value. Such restrictions would defeat the purpose of the test, which is to assess how well data fits a theoretical distribution with variable (and often unknown) parameters. Other tests, called nonparametric tests, are required.

χ^2 Test

The commonly applied nonparametric test, the χ^2 test, is an interval test. It divides the sample into sub-samples, or intervals, and tests how closely the observed frequency , O_i , in each interval corresponds to the expected frequency in each sub-sample, E_i , given by the theoretical distribution. The value of:

$$\chi^2_{(k-1)} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

decreases as the fit of the sample to the specified theoretical distribution improves. For a specified confidence level and each (k -1) degree of freedom, there is a critical maximum value of χ^2. If the observed value is higher than the critical value, the hypothesis that the data conforms to the theoretical distribution should be rejected[1].

Being an interval test, the χ^2 test requires data to be at in at least nominal form; that is, the data must be grouped into ranked sub-samples but not necessarily ranked within each sub-sample.

Some problems arise in using the χ^2 test to asses the fit of firm size to a Pareto distribution. First, the χ^2 test requires a minimum expected frequency about five in each sub-sample to ensure that an actual frequency of zero is significantly different than the

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[1] The squared deviation term in the numerator implies the χ^2 test is a one-tailed test.

expected frequency. The Pareto distribution has a very long right tail, so the expected frequency in each sub-sample could be less than five. The right tail of Pareto distribution, where there are a few large firms, is the segment of the distribution which draws the greatest economic interest.

Second, the results of the test are influenced by the choice of intervals, i.e. fewer and larger intervals versus more and smaller ones. However, the specified models give no guidance to the choice of interval.

Third, the χ^2 test does not detect organized patterns in the residuals. The measured χ^2 statistic may be less than the appropriate critical value when there is a small but systematic divergence between the sample and theoretical distributions.

VITA

Surname: Mills Given Names: Peter Gordon

Place of Birth: Toronto, Canada Date of Birth: May 25, 1956

Educational Institutions Attended:

University of Victoria	1987 to 1993
University of Cambridge	1986 to 1990
Queen's University at Kingston	1975 to 1979

Degrees Awarded:

Gr. Dip. (Cantab)	University of Cambridge	1990
B.Sc. (Honours)	Queen's University at Kingston	1979

Honours and Awards:

Canadian Pacific Scholarship	1987-1989

Publications:

None.