NUMERICAL MODELLING OF PRECIPITATION OVER COMPLEX TERRAIN

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Abstract

Numerical modelling of precipitation is one of the most important research areas in the development of numerical methods of weather analysis and forecasting. Due to the complexities of precipitation formation and topography, it is difficult to find a numerical model of precipitation suitable for all areas in the world. In different areas, we must consider their distinctive factors such as topogaphy and surface fluxes.

In this thesis, two models are described for calculating topographical effects on precipitation in complex terrain. One is a 3-dimensional variational model, and the other is a 3-dimensional initial value model. The three dimensional initial value model is the best in principle since it incorporates the most physics, but it requires an accurately defined initial state and is computationally expensive. The three dimensional variational model is comparatively simple and easy to compute. The 3-dimensional variational model is applied to calculate the precipitation for July 11, 1983 in the upper Columbia River watershed in southeastern B.C. as an experiment. The application of the variational model is the main contribution of this thesis.

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Chapter 1

Introduction

1.1 The Formation of Precipitation

In order to describe how precipitation is formed, this description should be prefaced by a brief discussion on the formation of precipitating clouds. The discussion is adapted from Saunders [17].

Most clouds are formed by the ascent of moist air into the more rarefied regions of the atmosphere. Such a motion is accompanied by an expansion of the air and a consequent cooling of it. If the ascent is sufficiently prolonged the cooling reduces the temperature of the air to that of its dew point, beyond which water vapour begins to condense out as tiny droplets. Further cooling now results in further condensation.

The vapour at first condenses onto particles of dust which are present in enormous numbers in the atmosphere. These particles, called condensation nuclei, are formed by the weathering of the earth's surface, by combustion, and by the breaking of sea waves.

These tiny cloud particles are transformed into raindrops, hailstones, and snowflakes.

This transformation usually takes place in one of two types of cloud - frontal cloud and shower cloud. In middle latitudes precipitation is from both cloud types.

Frontal cloud, which is associated with low pressure areas, is formed by the slow ascent of air (commonly 1 km in 3 hours) over an area perhaps as large as $10^5 km^2$; its passage gives continuous light or moderate precipitation lasting for several hours. Shower cloud, or cumulonimous cloud, is formed by the more rapid ascent of air (commonly 1 km in 5 min) over an area of the size of $10^2 km^2$; it gives light to heavy precipitation lasting perhaps half an hour.

Two theories explain the formation of precipitation. They are:

1. The coalescence theory

   Once cloud droplets have formed, continued ascent of the cloud air results in their growth by condensation. Cloud droplets settle slowly through the ascending air in which they are formed, and the larger fall faster and settle out through the others. In doing so, a larger drop collides with those in its path and incorporates many into itself; after each collision it becomes even larger and falls even faster. Although initially the growth is slow, in the later stages it is nearly explosive; it ceases when the drop falls out of the base of the cloud. Since the union of two colliding drops is termed coalescence, the process described above is termed the coalescence process of rain formation.

2. The theory of condensation on ice crystals

The ascent of cloud air results in its cooling at the rate of about 6°C per km. In middle latitudes, with temperatures at the cloud base about 10°C, the upper parts of the cloud are often colder than 0°C. Minute ice crystals may now appear and grow by condensation.

At temperatures not too far from 0°C, there occur a few ice crystals among a cloud of tiny water droplets, and these crystals may grow by condensation into large particles. The reason for this is that the growth of a particle by condensation depends on the difference between the rate at which molecules arrive at the surface of the particle from the vapour and the rate at which molecules escape from the surface. Since the molecules at the surface of an ice crystal are more firmly bound than those at the surface of a water drop, in the same circumstances an ice crystal grows more rapidly than a water drop.

Often ice crystals aggregate. The larger and faster falling sweep up the smaller and form snowflakes.

We have seen the physical processes responsible for precipitation are the coalescence of cloud droplets and condensation on ice crystals. In clouds which in their lower parts are warmer than 0°C, or indeed in cloud entirely colder than 0°C, both processes take place side by side.

1.2 Topographical Effects on Precipitation

The effect of topography on rainfall is fairly well known. It has been found that precipitation increases with altitude and greater rainfall occurs on the slopes facing the prevailing wind than on the lee slopes. However, the inter-relation between rainfall, height, aspect, wind-direction, etc., are so complicated that a satisfactory estimation of the orographic rainfall without the assistance of the reading of neighbouring gauges is unlikely to be possible without a physical understanding of the mechanism producing orographic rain.

To explain the amount and distribution of orographic rain on any particular day requires consideration of aspects of meteorology on three different scales. First, there are the large-scale synoptic factors which determine the characteristics of the air mass which crosses the hills, its wind-speed and direction, its stability and its humidity. Second, there is the dynamics of air motion over and around the hill or hills with which we are concerned; this determines to what depth and through what layers the air mass is lifted. Thirdly, there is the microphysics of the cloud and rain, which determines whether the water which is condensed as cloud will reach the ground as rain or snow, or whether it will be merely re-evaporated on the leeward side.

An extensive search for small scale precipitation effects was carried out by Bergeron [1] in the Uppsala region in Sweden and the results indicate that relatively low hills, of the order of 50m above the general level, may cause an increase of up to 25, sometimes even 50, percent in rainfall amounts, when extensive precipitation areas pass over the area.

Bergeron's explanation [2] is that the small scale terrain features cause formation of low level feeder clouds, with droplets too small for independent rain formation but large enough to be removed by precipitation falling from above.

Precipitation areas moving in a mountain area normally undergo some general strengthening when the air is forced against the mountain ridge. In addition the lower air layers are set in motion and some lifting occurs. The terrain-induced friction causes a general horizontal drift towards lower pressure, i.e. crossing the general wind and the direction of movement of the precipitation area. It is therefore well possible that these lower layers only take part in the precipitation formation as low feeder clouds, where droplets and impurities are removed by the raindrops falling from above.

Storebo [18] has done some computations to examine the Bergeron theory. These computations indicate that many small scale influences of topographic features on the rainfall pattern during passages of extensive precipitation systems may be due to droplet removal from fairly independent underlying clouds.

1.3 Contribution of this thesis

The development of numerical methods of weather analysis and forecasting is one of the most significant and spectacular advances in the practice of weather forecasting and the science of meteorology over their long history. In numerical prediction, we compute weather forecasts from numerical models made from primitive equations which describe the principles of atmospheric motions. Since many factors, such as topography and latent heat release, affect atmospheric motions, we must consider these factors in our numerical models. Two models are described for calculating topographical effects on precipitation in complex terrain. One is a 3-dimensional variational model, and the other is a 3-dimensional initial value model.

The reason for discussing the initial value model first is to provide an understanding of the complexity of weather prediction in general, and precipitation calculation in particular. This model is presented in Chapter 2.

Three dimensional initial value models are the best in principle since they incorporate the most physics. However, they require an accurately defined initial state and are computationally expensive.

In the 3-dimensional variational model discussed in Chapter 5, small-scale topographical effects are added to guess velocity fields by requiring that the winds satisfy the pressure tendency equation using observed pressures. The precipitation is computed from the integrated moisture tendency equation and combine calculated and observed precipitation using objective analysis. The winds are then adjusted to satisfy the moisture tendency equation using the objectively analyzed precipitation. The application of the variational model to an actual case is the main contribution of this thesis.

Chapter 2

A 3-dimensional Initial Value Model

2.1 Introduction

As pointed by Danard [9], since the earth's surface is approximately spherical, map projections inevitably have a spatially varying map scale (ratio of distance on map projection to true distance on earth). This variation is often ignored in applications to a local area, if velocities are small, or if integrations are carried out for short time periods. If integrations are performed for long periods, however, systematic errors due to neglect or improper application of the variable map scale may have significant effects.

In our two models for calculating topographical effects on precipitation in complex terrain, the spatial variations in the unit vectors have been included. The following discussion is adapted from Danard and Galbraith [10]. Previous versions of this model are given by Danard [4] [5] [7] [8] and Danard and Ellenton [6].

2.2 Primitive Equations

The 3-dimensional initial value model which provided the trial velocity fields employed in Chapter 5 uses σ coordinates in which the vertical coordinate is defined by

$$\sigma = \frac{p}{p_s} \tag{2.1}$$

The following eight prognostic and diagnostic equations(2.2 - 2.9) are integrated in time from an initial state to give predicted values of eight dependent variables u, v, T, r, p_s, ϕ', $\dot{\sigma}$ and ω:

$$\begin{aligned}\frac{\partial}{\partial t}(p_s u) &= -m_x m_y[\frac{\partial}{\partial X}(\frac{p_s u^2}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_s uv}{m_x})] \\ &\quad -p_s \frac{\partial}{\partial \sigma}(\dot{\sigma} u) + [f + (u\frac{m_y}{m_x}\frac{\partial m_x}{\partial Y} \\ &\quad -v\frac{m_x}{m_y}\frac{\partial m_y}{\partial X})]p_s v \\ &\quad -m_x p_s(\frac{\partial \phi'}{\partial X} + RT'_v\frac{\partial \ln p_s}{\partial X}) \\ &\quad +W_x + B_x + G_x + F_{hx} + F_{vx}\end{aligned} \tag{2.2}$$

$$\begin{aligned}\frac{\partial}{\partial t}(p_s v) &= -m_x m_y[\frac{\partial}{\partial X}(\frac{p_s uv}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_s v^2}{m_x})] \\ &\quad -p_s \frac{\partial}{\partial \sigma}(\dot{\sigma} v) - [f + (u\frac{m_y}{m_x}\frac{\partial m_x}{\partial Y} \\ &\quad -v\frac{m_x}{m_y}\frac{\partial m_y}{\partial X})]p_s u\end{aligned}$$

$$-m_y p_s(\frac{\partial \phi'}{\partial Y} + RT'_v \frac{\partial \ln p_s}{\partial Y})$$

$$+W_y + B_y + G_y + F_{hy} + F_{vy} \tag{2.3}$$

$$\begin{aligned} \frac{\partial}{\partial t}(p_s T) &= -m_x m_y[\frac{\partial}{\partial X}(\frac{p_s u T}{m_y}] + \frac{\partial}{\partial Y}(\frac{p_s v T}{m_x})] \\ &\quad -p_s \frac{\partial}{\partial \sigma}(\dot{\sigma} T) + \frac{RT_v \omega}{c_p \sigma} \\ &\quad +\frac{p_s H_L}{c_p} + D_{hT} \end{aligned} \tag{2.4}$$

$$\begin{aligned} \frac{\partial}{\partial t}(p_s r) &= -m_x m_y[\frac{\partial}{\partial X}(\frac{p_s u r}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_s v r}{m_x})] \\ &\quad -p_s \frac{\partial}{\partial \sigma}(\dot{\sigma} r) - p_s C + D_{hr} \end{aligned} \tag{2.5}$$

$$\begin{aligned} \frac{\partial p_s}{\partial t} &= -m_x m_y[\frac{\partial}{\partial X}(\frac{p_s u}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_s v}{m_x})] \\ &\quad -p_s \frac{\partial \dot{\sigma}}{\partial \sigma} \end{aligned} \tag{2.6}$$

$$\frac{\partial \phi'}{\partial \ln \sigma} = -RT'_v \tag{2.7}$$

$$\begin{aligned} p_s \dot{\sigma} &= -(1-\sigma)\int_0^\sigma m_x m_y[\frac{\partial}{\partial X}(\frac{p_s u}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_s v}{m_x})]d\sigma \\ &\quad +\sigma \int_\sigma^1 m_x m_y[\frac{\partial}{\partial X}(\frac{p_s u}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_s v}{m_x})]d\sigma \end{aligned} \tag{2.8}$$

$$\omega = \sigma(u m_x \frac{\partial p_s}{\partial X} + v m_y \frac{\partial p_s}{\partial Y}) - \int_0^{\sigma} m_x m_y [\frac{\partial}{\partial X}(\frac{p_s u}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_s v}{m_x})] d\sigma \tag{2.9}$$

The map scale factors (m_x,m_y) are defined by

$$m_x = \frac{dX}{dx} \tag{2.10}$$

$$m_y = \frac{dY}{dy} \tag{2.11}$$

where (dx,dy) are the true distances on the earth's surface of orthogonal horizontal infinitesimal line segments, and (dX,dY) are the corresponding distances on the projection. See Danard [9].

Eqs(2.2 - 2.9) are written in general form to permit easy transformation from one map projection to another. All derivatives in these equations are approximated by finite differences.

Given initial values for the eight dependent variables, the eight equations are numerically integrated forward in time to obtain values at a later time. The model has seven levels above the earth's surface. Eq(2.6) is applied at one level only; the other equations are applied at all levels.

2.3 Calculating Precipitation

The model considers precipitation as made of two components, the resolvable scale precipitation P_l and the sub-grid scale precipitation P_c. Thus

$$P = P_l + P_c$$

The units of P, P_l and P_c are kgm^{-2}(*i.e.*, mm water depth). Both P_l and P_c are "soft" in the sense of Krishnamurti *et al.* *[12]*. That is, they are assumed to occur over a fraction α of the grid area (area of size $dXdY$ centered on the grid point) defined here as

$$\begin{aligned} \alpha &= \frac{h - h_0(\sigma)}{1.0 - h_0(\sigma)}, \quad h > h_0(\sigma) \\ &= 0, \qquad\qquad\quad h \leq h_0(\sigma) \end{aligned} \tag{2.12}$$

where $h = r/r_s$ defines the relative humidity and h_0 is a threshold value for relative humidity for the occurrence of precipitation.

Resolvable scale precipitation, during a time-step Δt, is calculated from the equation

$$P_l = [-\frac{p_s}{g} \int \alpha\omega(\frac{dr}{dp})_s d\sigma]\Delta t \tag{2.13}$$

In (2.13), $(dr/dp)_s$ is the substantial rate of change of mixing ratio with pressure in the saturated state following the motion (*i.e.*, along a moist adiabat). See List[13]. The integration is done over all levels for which $\alpha > 0$ and for all upward motion, *i.e.*, $\omega < 0$.

Convective adjustment is used to calculate sub-grid scale precipitation. A fraction α of the grid area is said to be saturated in which the equivalent potential temperature is Θ_e where

$$\Theta_e = (T + \frac{Lr_s}{C_p})(\frac{p_0}{p})^{\frac{R}{C_p}}$$

The following criteria must be satisfied at all levels involved in the adjustment in order for convective overturning to take place at adjacent levels.

$$\frac{\partial \Theta_e}{\partial p} > 0 \tag{2.14}$$

$$\alpha > 0 \tag{2.15}$$

$$\omega < 0 \tag{2.16}$$

After overturning, the final state is assumed to satisfy

$$\frac{\partial \Theta_e}{\partial p} = 0 \tag{2.17}$$

If (2.14) initially holds over N adjacent levels, equation (2.17) gives (N-1) equations in 2N unknowns, the final temperature T_f and saturation mixing ratio at each of the N levels. (N+1) more equations are required. N of these equations are obtained from the Clausius-Clapeyron equation applied to the N levels.

$$\delta r = \frac{0.622 r_{si} L}{R T_i^2} \frac{p}{(p - e_{si})} \delta T \tag{2.18}$$

In (2.18), $\delta r = r_{sf} - r_{si}$ and $\delta T = T_f - T_i$ are changes from the initial to final states indicated by the subscripts i and f. The final equation is obtained from the relation below which holds provided the motion arising from convection is dissipated as heat.

$$-\frac{C_p}{g} \int \delta T dp = \frac{L}{g} \int \delta r dp$$

The integration is spread over all levels participating in the adjustment. This system of 2N linear equations is then solved simultaneously. The sub-grid scale precipitation resulting from the overturning is then given by

$$P_c = -\bar{\alpha} \int \delta r \frac{dp}{g}$$

where $\bar{\alpha}$ is the vertical average of α of the levels involved. The change in the resolvable scale mixing ratio and temperature are

$$\Delta r = \bar{\alpha} \delta r$$

$$\Delta T = \bar{\alpha} \delta T$$

Convective adjustment transports heat and moisture upward with Δr and ΔT both positive in the upper part of the convective cloud and negative in the lower part. In addition to providing sub-grid scale precipitation, convective adjustment is also necessary to prevent numerical instability in hydrostatic models.

Chapter 3

Variational Method

3.1 Variational Method

Sasaki [14][15] has developed an initialization method based on the calculus of variations in which differences between the observed values of the meteorological elements and the corresponding objectively modified values are minimized in the least-squares sense subject to some dynamical constraints.

It is desirable to review some aspects of calculus of variations first. Suppose $F(x, y, y')$ is a twice differentiable function. We wish to determine a function $y = f(x)$ such as that the following integral is a minimum(or maximum):

$$I = \int_a^b F(x, y, y')dx \tag{3.1}$$

Here the end points a and b are considered fixed with $f(a) = A$ and $f(b) = B$. Now suppose there is a small change from $y = f(x)$ to $y + \delta y = f(x) + \epsilon g(x)$, where ϵ is a parameter and $g(x)$ is arbitrary except that $g(a) = g(b) = 0$.

Then the integral (3.1) will be modified to

$$I + \delta I = \int_a^b (F + \delta F) dx \tag{3.2}$$

where δ is the variational operator,

$$\delta F = F_y \delta y + F_{y'} \delta y'$$

and the subscripts denote partial derivatives.

Thus a necessary condition for a minimum is

$$F_y - \frac{d}{dx} F_{y'} = 0$$

which is referred to as the Euler-Lagrange equation. Suppose F contains two functions, $y(x)$ and $z(x)$, that is, $F = F(x, y, z, y', z')$, and furthermore, the integral is subject to a constraint, say, $G(x, y, z) = 0$. In this case, a Lagrange multiplier λ may be introduced, and (3.1) is now changed to

$$I = \int_a^b [F(x, y, z, y', z') + \lambda G(x, y, z)] dx \tag{3.3}$$

The resulting Euler-Lagrange equations are

$$F_y - \frac{d}{dx}(F_{y'}) - \lambda G_y = 0$$

$$F_z - \frac{d}{dx}(F_{z'}) - \lambda G_z = 0$$

together with the constraint condition

$$G(x, y, z) = 0$$

3.2 The Need for Data Adjusting

The effect of incomplete and inaccurate observations on the quality of numerical weather forecasts may be regarded as the resultant effect of :

1. The dependence of the accuracy of initial analysis on the characteristics of the observing system - *e.g.*, station spacing, standard error of observation, standard deviation of the meteorological variables around their climatological mean values, and their characteristic time and space scales of fluctuation.

2. The dependence of the error of prediction (based on a hypothetically correct method of forecasting) on the characteristics of the initial analysis error.

The calculus of variations is used to derive formulations for optimally adjusting a numerical analysis scheme so as to minimize any analysis inconsistencies. Thompson [20] has worked out a sample variational problem in which he used as a dynamical constraint the vorticity equation for nondivergent barotropic flow. He was able to show, by variationally optimizing his scheme to adjust for errors, that the residuals in the adjusted analysis are significantly smaller than in the original analysis.

Since an adjustment routine of this kind is easily programmed into a computerized objective analysis scheme, it can be applied with a high degree of automation and efficiency.

There can be no question of the importance of deriving the appropriate optimization routines for any and all objective analyses of spatially distributed data.

3.3 Application in this Thesis

In the thesis a 3-dimensional variational model is designed in which small-scale topographical effects are added to adjust velocity fields by requiring that the winds satisfy the pressure tendency equation. Precipitation is computed from the integrated moisture tendency equation, and calculated and observed precipitation are combined using objective analysis. The winds are then adjusted by the variational method to satisfy the moisture tendency equation using the objectively analyzed precipitation. Details are given in Chapter 5. However, objective analysis will first be described in Chapter 4. This is because objective analysis is an important part of the variational model.

Chapter 4

Objective Analysis Techniques

4.1 Review of Relevant Objective Analysis Techniques

For a given situation, our knowledge about the state of the atmosphere is given by a large number of actual and recent observations, irregularly distributed in space and time. The procedure of combining these observed data to make conclusions about the total variation of the meteorological variables within the area of interest, has generally been called meteorological analysis. The term "objective analysis" has been used for analysis as a numerical procedure by the aid of a computer to distinguish from the manual or "subjective" analysis procedure. A more relevant terminology is "numerical analysis or automated analysis" versus "manual analysis".

The first numerical schemes proposed to do such an analysis were introduced in the 1940's. These schemes simply performed a two dimensional interpolation of the observed data onto a regular network of grid points. A short range numerical forecast or a "first guess" field was used as a preliminary field in order to obtain a better quality forecast. These two dimensional interpolation procedures were satisfactory for construction of initial analyses for the relatively simple numerical forecast models which were utilized at that time. Since then, however, the forecast models have been refined and have also become more sensitive to the quality of the initial analyses. Further, today we have a more mixed and complex observational network with very variable quality of data from various data sources. This progress in numerical modelling and observational techniques has necessitated significant development work in the field of numerical analysis methods. Today also numerical forecast models play a significant role in the analysis of observed data, and the term "4-dimensional data assimilation" has been introduced for this complicated procedure.

Over the past 30 years of development of numerical analysis schemes, most presented schemes belong to one of the following five classes of numerical analysis schemes, see Gustafsson [11]:

1. (Local) polynomial interpolation methods in which mathematical functions are locally, in the neighbourhood of the individual gridpoints, adjusted to fit the observed data.

2. Statistical interpolation methods (optimum interpolation methods).

3. Variational numerical analysis methods introduced by Sasaki (1958).

4. Successive correction methods.

5. Spectral analysis methods in which mathematical functions are globally (or hemispherically) adjusted to fit observed data.

4.2 Local Polynomial Interpolation Methods

The basic principle of the polynomial interpolation methods is to adjust polynomial functions to the observed data in the close vicinity of the gridpont, for which analyzed values are required. As an example we will consider two-dimensional analysis of a variable z. The variation of the variable z in the vicinity of the gridpoint is approximated by the following polynomial function

$$z(x,y) = \sum_{ij} a_{ij} x^i y^j \qquad i,j \geq 0 \qquad i+j \leq n$$

where x and y are coordinates of the gridpoint and n is the degree of the fitting polynomial. In order to determine the coefficients a_{ij} of the polynomial function $z(x,y)$, we utilize the least squares method to minimize the deviations between the observed data and the corresponding polynomial function values in the vicinity of the gridpoint. The minimizing procedure will result in a system of linear equations for determination of the coefficients a_{ij}.

In polynomial interpolation methods, the selection of interpolating functions is quite arbitrary and it is difficult to see how past experience on, *e.g.* atmospheric scales, enter into the analysis computations.

4.3 Statistical Interpolation Methods

In the statistical interpolation methods, past experience about the behaviour of the atmosphere is used as the main source of information for determination of the interpolation weights.

Assume that we want to utilize n observations f_{it}^{OBS} (i=1, ..., n) in the vicinity of a gridpoint g to compute the analyzed value f_{gt}^{NA} in the gridpoint at a certain time t. In statistical interpolation schemes, f_{gt}^{NA} is computed as a linear combination of a preliminary field value f_{gt}^{P} at the gridpoint and the observed deviations $f_{it}^{OBS} - f_{it}^{P}$ from the preliminary field:

$$f_{gt}^{NA} = f_{gt}^{P} + \sum_{i=1}^{n} \alpha_i (f_{it}^{OBS} - f_{it}^{P}) \tag{4.1}$$

The interpolation weights α_i (i=1, ..., n) are obtained by requiring that the mean square error of interpolation is a minimum:

$$Minimize(E = \overline{(f_{gt} - f_{gt}^{NA})^2}) \tag{4.2}$$

Insertion of (4.1) into (4.2) and evaluation of the square will give:

$$\begin{aligned} E &= \overline{(f'_{gt})^2} + \sum_{i=1}^{n}\sum_{j=1}^{n}(\overline{f'_{it}f'_{jt}} + \overline{\Delta f_{it} f'_{jt}} + \overline{f'_{it}\Delta f_{jt}} \\ &\quad + \overline{\Delta f_{it}\Delta f_{jt}})\alpha_i\alpha_j - 2\sum_{i=1}^{n}(\overline{f'_{gt}f'_{it}} + \overline{f'_{gt}\Delta f_{it}})\alpha_i \end{aligned} \tag{4.3}$$

where $f'_{it} = f_{it} - f_{it}^{P}$ denotes the deviations of the true values from the preliminary field values. In other words, the errors of the preliminary field values, $\Delta f_{it} = f_{it}^{OBS} - f_{it}$, denote the observational errors, including small-scale variations that we do not want to analyze.

Among the terms in the expression for E, terms of the type $\overline{\Delta f_{it} f'_{it}}$ denote the cross-covariance between preliminary field errors and observational errors. For most observing systems it is certainly correct to assume that there is no dependence between the observational errors and the preliminary field errors. This will simplify the expression for E

since

$$\overline{f'_{it}\Delta f'_{it}} = \overline{\Delta f_{it} f'_{jt}} = \overline{f'_{it}\Delta f_{jt}} = 0$$

If we introduce

$$m_{ij} = \overline{(f_{it} - f^{P}_{it})(f_{jt} - f^{P}_{jt})} = \overline{f'_{it} f'_{jt}}$$

$$d_{ij} = \overline{(f^{O}_{it} - f_{it})(f^{O}_{jt} - f_{jt})} = \overline{\Delta f_{it}\Delta f_{jt}}$$

we will have

$$E = m_{gg} + \sum_{i=1}^{n}\sum_{j=1}^{n}(m_{ij} + d_{ij})\alpha_i\alpha_j - 2\sum_{i=1}^{n} m_{gi}\alpha_i \tag{4.4}$$

Necessary conditions for a minimum of E with respect to the interpolation weights α_k (k=1, ..., n) are

$$\frac{\partial E}{\partial \alpha_k} = 2(\sum_{i=1}^{n}(m_{ik} + d_{ik})\alpha_i - m_{kg}) = 0 \qquad k = 1, \cdots, n \tag{4.5}$$

or

$$\sum_{i=1}^{n}(m_{ik} + d_{ik})\alpha_i = m_{kg} \qquad k = 1, \cdots, n \tag{4.6}$$

If we know the covariances of the m_{ik} and m_{kg} of the preliminary field errors and the covariances d_{ik} of the observational errors it is possible to solve the system of n linear equations (4.6) to obtain the interpolation weights α_k (k=1, ..., n) and by expression (4.1) it is then possible to compute the analyzed value f^{NA}_{gt}.

By multiplying equation (4.6) with α_k and summing for k=1, ..., n we will obtain

$$\sum_{k=1}^{n}\sum_{i=1}^{n}(m_{ik} + d_{ik})\alpha_i\alpha_k - \sum_{k=1}^{n} m_{kg}\alpha_k = 0 \tag{4.7}$$

By subtracting (4.7) from (4.4) we will obtain the minimized interpolation error

$$E^{MIN} = m_{gg} - \sum_{i=1}^{n} m_{ig}\alpha_i$$

Any practical application of the statistical interpolation method must include the following elements:

1. Computation of a preliminary field f^P_{gt} at all gridpoints and all observational points.

2. Modelling of the preliminary field error covariances $m_{ij} = \overline{(f_{it} - f^P_{it})(f_{jt} - f^P_{jt})}$

3. Modelling of the observational error covariances: $d_{ij} = \overline{(f^{OBS}_{it} - f_{it})(f^{OBS}_{jt} - f_{jt})}$

4. Selection of observed data f^{OBS}_{it} to influence the gridpoint value f^{NA}_{gt}.

5. Solution of the system of linear equations to obtain the interpolation weights α_k (k=1, ..., n).

Recently, Tanguay and Robert [19] have applied the optimum interpolation method to analyze precipitation amounts. In their proposed univariate analysis scheme, the auto-correlation function has been approximated by the second-order and fourth-order Taylor series expansion of the Gaussian Hill function. It is shown that their proposed schemes are faster than an optimum interpolation analysis scheme using the Gaussian autocorrelaction function. Also Bhargava [3] proposed another optimum interpolation analysis scheme to apply to the heavy rainstorm of 11-13 July 1983 in the Upper Columbia River Watershed. In her scheme, two assumptions are made: 1. Trial field errors and observational errors are independent of each others, 2. Observational errors and the deviations of the trial field values from the observations are uncorrelated. These two assumptions simplify the preliminary field error covariance and the observational field error covariance.

4.4 Spectral Analysis Methods

In spectral analysis methods, the actual state of the atmosphere is represented by an expansion in a series of space-dependent functions with time-dependent coefficients. The analysis procedure consists of the determination of those time-dependent coefficients which make the series expansion best fit the observed data. As an example of the spectral analysis technique, we will shortly describe the U.S. National Meteorological Center (NMC) spectral analysis method.

If ψ, ϕ and p denote longitude, latitude and pressure, respectively, the global representation of geopotential and wind components is given by

$$z(\psi,\phi,p) = \sum_{l=1}^{24}\sum_{m=1}^{24}\sum_{n=1}^{7}[a_{lmn}\cos(l\psi) + b_{lmn}\sin(l\psi)]H_{lm}(\phi)E_n(p)$$

$$u(\psi,\phi,p) = \sum_{l=1}^{24}\sum_{m=1}^{24}\sum_{n=1}^{7}[a_{lmn}\cos(l\psi) + b_{lmn}\sin(l\psi)]U_{lm}(\phi)E_n(p)$$

$$v(\psi,\phi,p) = \sum_{l=1}^{24}\sum_{m=1}^{24}\sum_{n=1}^{7}[a_{lmn}\cos(l\psi) - b_{lmn}\sin(l\psi)]V_{lm}(\phi)E_n(p)$$

The coefficients a_{lmn}, b_{lmn} are obtained by simultaneous minimization of integrals of the form

$$I_z = \int_V (z^{OBS} - z(\psi,\phi,p))^2 dv$$

$$I_u = \int_V (u^{OBS} - u(\psi, \phi, p))^2 dv$$

$$I_v = \int_V (v^{OBS} - v(\psi, \phi, p))^2 dv$$

with a weighting

$$\zeta \cdot I_z + I_u + I_v$$

where V denotes the total volume of the atmosphere from the surface up to 50 mb and ζ denotes the relative weight of height observations versus wind observation in the minimization procedure.

We will obtain the minimum value by application of the standard least-square technique

$$\frac{\partial}{\partial a_{lmn}}(\zeta \cdot I_z + I_u + I_v) = 0$$

$$\frac{\partial}{\partial b_{lmn}}(\zeta \cdot I_z + I_u + I_v) = 0$$

where l=1, ..., 24; m=1, ..., 24; n=1, .., 7

In the NMC spectral analysis system the computation of the average observed value in each integration box is a purely empirical weighting, based on numerical experiments. The net effect of the weighting gives less weight to satellite or aircraft data when radiosonde data are available in the same integration box, and reduces the weight of observations of any type when the data density is large. This weighting system assumes that satellite sounding data have larger error levels than do radiosonde data, but the weighting does not discriminate between random errors and errors with spatial correlation.

4.5 Successive Correction Methods

The first step in the successive correction method is to construct a preliminary field f_{gt}^P for all the gridpoints to be analyzed. The preliminary field could, *e.g.*, be computed as a weighted mean of a climatology field $\overline{f_g}$ and a valid numerical forecast field f_{gt}^{NP}.

The basic idea in the successive correction method is to correct this preliminary field iteratively during several analysis "scans". In each analysis scan the corrections are formally computed in the same way as in the statistical interpolation method, thus by interpolation of the deviations of the observed values from the preliminary field:

$$f_{gt}^{NA} = f_{gt}^P + \sum_{i=1}^{n} \alpha_i (f_{it}^{OBS} - f_{it}^p) \tag{4.8}$$

In the correction method the interpolation weights are computed explicitly without solving any systems of linear equations. For example, in the system of the Swedish Weather Service, the successive correction interpolation weights were given by formulas of the following form:

$$\alpha_i = \frac{\mu(r_{ig}) \cdot h(\rho_i)}{\alpha_p + \sum_{i=1}^{n} \mu(r_{ig}) \cdot h(\rho_i)}$$

The most important factor of the weight is the distance-dependent function $\mu(r_{ig})$, where r_{ig} is the distance between the gridpoint and the observational point. The $\mu(r)$ functions are very similar to the auto-correlation functions. Since the observations may be very unevenly distributed around the gridpoint, it is necessary to correct the weights for observations which are highly correlated. This is taken care of by the station-density function $h(\rho_i)$, where ρ_i is the number of influencing observations within a certain distance from observation i. Finally the weights are normalized by the sum of the weights. In order to give some weight to the preliminary field in data-sparse areas, a term α_p is also added to the normalizing factor.

One application of (4.8) for all gridpoints results in an analyzed field which in turn can be treated as a preliminary field (or a "second guess" field) and the whole procedure can be repeated. By a number of such correction scans it is possible to have a successive adjustment of the gridpoint-values to the observations.

The successive correction method is rather empirical in nature. In the statistical interpolation method all the empirical correction factors that are used in the successive correction method are taken care of in an automatic and optimum way. In this sense the successive correction method may be looked upon as an empirical approximation to the statistical interpolation method. The main advantage of the successive correction method is the relatively low number of computations compared with the statistical interpolation method.

4.6 An Example of a Successive Correction Objective Analysis Method

The following objective analysis technique is employed in the variational model described in Chapter 5.

Suppose we have randomly spaced observations of some quantity A (*e.g.*, 500 mb ht) and a guess field of A (*e.g.*, 12 hr prognosis) known at a regular array of points. We wish to combine the observations with the first-guess field to obtain a final analysed field.

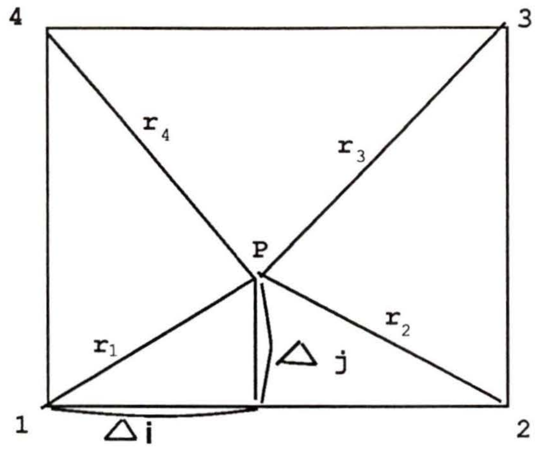

Figure 4.1: An observation station and the four grid points around it

Suppose we have an observation at P. Points 1, 2, 3 and 4 indicate the surrounding grid points as shown in Figure 4.1. Δi and Δj are expressed in units of grid length.

1. Compute A_g, the value of the guess-field interpolated at P.

2. Compute $D_p = A_{obs} - A_g$ where A_{obs} is the value observed at P.

3. Compute weights for correcting each of the 4 surrounding grid points

$$W_{mp} = \frac{1 - r_m^2}{\sum_{n=1}^{4}(1 - r_n^2)}, \qquad m = 1, \cdots, 4 \tag{4.9}$$

where

$$r_1^2 = \Delta i^2 + \Delta j^2$$

$$r_2^2 = (1 - \Delta i)^2 + \Delta j^2$$

$$r_3^2 = (1 - \Delta i)^2 + (1 - \Delta j)^2$$

$$r_4^2 = \Delta i^2 + (1 - \Delta j)^2$$

Don't let any r_i^2 exceed 1 (otherwise would get negative weights). Note that $W_{mp} \leq 1, \sum_m W_{mp} = 1$.

4. Compute the weighted difference to be added later to each of the four grid points surrounding the observation.

$$D_{mp} = W_{mp} \times D_p, \qquad m = 1, \cdots, 4 \tag{4.10}$$

Scan all observations. So far, the guess field has not been affected.

5. Adjust each grid point which has an observation in one or more of the four surrounding grid squares.

$$A_{adj} = A + \frac{\sum_p D_{mp}}{\sum_p W_{mp}}$$

where the summation extends over all observations affecting the grid point. In each term of the summation, m takes on one value only.

6. Holding all adjusted values fixed, compute the values at the unadjusted points by requiring that $\nabla^2 A$ be the same as in the guess field. This is a boundary value problem (both external and internal). In other words, the area between observations has the same Laplacian as the guess field.

7. The analysis is smoothed and regarded as a new guess field. The above procedure (1 - 6) is then repeated once.

4.7 The Application in Our Case

In the 3-dimensional variational model, we calculate the precipitation from the integrated moisture tendency equation and combine calculated and observed precipitation using the successive correction objective analysis method described in Section 4.6. We select the successive correction method because of its relatively low number of computations compared with the statistical interpolation method, although the statistical interpolation method can give better result than the successive correction method for a sparse data network.

Chapter 5

The 3-dimensional Variational Model

5.1 The Structure of 3-dimensional Variational Model

The three dimensional variational model consists of two hydrodynamical equations. One is the pressure tendency equation

$$\frac{\partial p_s}{\partial t} + \int_0^1 m_x m_y [\frac{\partial}{\partial X}(\frac{p_s u}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_s v}{m_x})] d\sigma \quad = \quad 0 \tag{5.1}$$

and the other is the integrated moisture tendency equation

$$g(P - E) + \frac{\partial}{\partial t}(p_s \bar{r}) + \int_0^1 m_x m_y [\frac{\partial}{\partial X}(\frac{p_s u r}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_s v r}{m_x})] d\sigma = 0 \tag{5.2}$$

where P is the precipitation , E is the evaporation from the earth's surface, and

$$\bar{r} = \int_0^1 r d\sigma$$

is the vertically averaged mixing ratio. Eq(5.1) is obtained by integrating Eq(2.6) from $\sigma = 0$ to $\sigma = 1$ and applying the kinematic boundary conditions $\dot{\sigma}(0) = \dot{\sigma}(1) = 0$. Eq(5.2) is obtained from Eq(2.5) by omitting $D_h r$, integrating from $\sigma = 0$ to $\sigma = 1$, and adding a turbulent moisture flux (E) at the lower boundary. Thus the 3-dimensional variational model may be regarded as a simplification of the 3-dimensional initial value model (*i.e.* only two of the initial value model's equations are employed).

Suppose there are N sets of data at times t_1, t_2,... $t_N = t_1 + \Delta t$. Write the pressure tendency equation(5.1) as

$$\frac{(p_{sN} - p_{s1})}{\Delta t} + \frac{1}{(N-1)} \sum_{i=1}^{N} \int_0^1 c_i m_x m_y [\frac{\partial}{\partial X}(\frac{p_{si} u_i}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_{si} v_i}{m_x})] d\sigma = 0 \qquad (5.3)$$

where $c_i = 0.5$ for i=1 or i=N, and c_i=1 otherwise.

Similarly, the moisture tendency equation(5.2) may be written as

$$g(P-E) + \frac{(p_{sN}\bar{r_N} - p_{s1}\bar{r_1})}{\Delta t} + \frac{1}{(N-1)} \sum_{i=1}^{N} \int_0^1 c_i m_x m_y [\frac{\partial}{\partial X}(\frac{p_{si} u_i r_i}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_{si} v_i r_i}{m_x})] d\sigma = 0 \qquad (5.4)$$

The 3-dimensional Variational model is much cheaper to run than the 3-dimensional initial value model and it is easy to "impose one's will" on them (*e.g.* make the calculated precipitation amounts agree with observed values) .

5.2 Applying the 3-dimensional Variational Model

We apply the 3-dimensional variational model to calculate the precipitation; procedures are the following:

1. Given N initial velocity fields(u_{m0}, v_{m0}), m=1,2,...,N at times t_1,t_2,...,$t_N = t_1 + \Delta t$. These may be provided by the initial conditions of the 3-dimensional initial value model (see Chapter 2).

2. Adjust the initial velocities by variational means so that the pressure tendency equation(5.3) is satisfied.

   The procedures of adjustment are the following:

$$I^2 = \int\int[\alpha^2 \int_0^1 [\sum_{i=1}^{N}(u_i - u_{i0})^2 + \sum_{i=1}^{N}(v_i - v_{i0})^2]d\sigma + \lambda J]dA \qquad (5.5)$$

   where J is the left side of Eq(5.3), α^2 is a Gauss precision modulus, λ is a Lagrange multiplier, and A is the area of the computational domain.

   We apply the variational operator δ to the two sides of the equation(5.5) to obtain the equation

$$\begin{aligned}\delta I^2 \;=\; & \int\int[2\alpha^2 \int_0^1 [\sum_{i=1}^{N}(u_i - u_{i0})\delta u_i + \sum_{i=1}^{N}(v_i - v_{i0})\delta v_i]d\sigma \\ & +\frac{\lambda}{(N-1)}[\int_0^1 (\sum_{i=1}^{N} c_i m_x m_y \frac{\partial}{\partial X}(\frac{p_{si}\delta u_i}{m_y}) + \sum_{i=1}^{N} c_i m_x m_y \frac{\partial}{\partial Y}(\frac{p_{si}\delta v_i}{m_x})d\sigma] \\ & +J\delta\lambda]dA \qquad (5.6)\end{aligned}$$

Now

$$\lambda\frac{\partial}{\partial X}(\frac{p_{si}\delta u_i}{m_y}) = \frac{\partial}{\partial X}(\frac{\lambda p_{si}\delta u_i}{m_y}) - \frac{p_{si}\delta u_i}{m_y}\frac{\partial\lambda}{\partial X}$$

If either λ or δu_i vanishes on the boundaries, then from Gauss's divergence theorm

$$\int\int\frac{\partial}{\partial X}(\frac{\lambda p_s\delta u}{m_y})dA = 0$$

and one obtains

$$\int\int[\int_0^1\lambda\frac{\partial}{\partial X}(\frac{p_{si}\delta u_i}{m_y})d\sigma]dA = -\int\int[\int_0^1\frac{p_{si}\delta u_i}{m_y}\frac{\partial\lambda}{\partial X}d\sigma]dA \tag{5.7}$$

Similarly, since

$$\lambda\frac{\partial}{\partial Y}(\frac{p_{si}\delta v_i}{m_x}) = \frac{\partial}{\partial Y}(\frac{\lambda p_{si}\delta v_i}{m_x}) - \frac{p_{si}\delta v_i}{m_x}\frac{\partial\lambda}{\partial Y}$$

one obtains

$$\int\int[\int_0^1\lambda\frac{\partial}{\partial Y}(\frac{p_{si}\delta v_i}{m_x})d\sigma]dA = -\int\int[\int_0^1\frac{p_{si}\delta v_i}{m_x}\frac{\partial\lambda}{\partial Y}d\sigma]dA \tag{5.8}$$

Substitution of Eqn(5.7) and (5.8) into Eqn(5.6) yields the following

$$\begin{aligned}\delta I^2 &= \int\int[\int_0^1(\sum_{i=1}^{N}[2\alpha^2(u_i - u_{i0}) - \frac{c_i p_{si} m_x}{N-1}\frac{\partial\lambda}{\partial X}]\delta u_i \\ &\quad + \sum_{i=1}^{N}[2\alpha^2(v_i - v_{i0}) - \frac{c_i p_{si} m_x}{N-1}\frac{\partial\lambda}{\partial Y}]\delta v_i)d\sigma]dA \end{aligned} \tag{5.9}$$

Using the least squares method, let $\delta I^2 = 0$. Without loss of generality we may set $2\alpha^2 = \frac{1}{N-1}$. A sufficient condition for (5.9) to vanish is that the integrand be zero everywhere. This gives

$$u_i = u_{i0} + c_i p_{si} m_x \frac{\partial \lambda}{\partial X}, i = 1, 2, \cdots, N \tag{5.10}$$

$$v_i = v_{i0} + c_i p_{si} m_y \frac{\partial \lambda}{\partial Y}, i = 1, 2, \cdots, N \tag{5.11}$$

Thus substituting (5.10) and (5.11) into (5.3), we obtain the equation

$$\begin{aligned} \int_0^1 m_x m_y [(\sum_{i=1}^N c_i^2 p_{si}^2 \frac{m_x}{m_y}) \frac{\partial^2 \lambda}{\partial X^2} + (\sum_{i=1}^N c_i^2 p_{si}^2 \frac{m_y}{m_x}) \frac{\partial^2 \lambda}{\partial Y^2}] d\sigma = -(N-1) J_0 \\ - \int_0^1 m_x m_y [(\sum_{i=1}^N c_i^2 \frac{\partial}{\partial X} (p_{si}^2 \frac{m_x}{m_y})) \frac{\partial \lambda}{\partial X} - (\sum_{i=1}^N c_i^2 \frac{\partial}{\partial Y} (p_{si}^2 \frac{m_y}{m_x})) \frac{\partial \lambda}{\partial Y}] d\sigma \end{aligned} \tag{5.12}$$

where

$$J_0 = \frac{(p_{sN} - p_{s1})}{\Delta t} + \frac{1}{(N-1)} \sum_{i=1}^N \int_0^1 c_i m_x m_y [\frac{\partial}{\partial X} (\frac{p_{si} u_{i0}}{m_y}) + \frac{\partial}{\partial Y} (\frac{p_{si} v_{i0}}{m_x})] d\sigma$$

Since p_s and λ are independent of σ, Eqn(5.12) may be written as

$$\begin{aligned} m_x m_y [(\sum_{i=1}^N c_i^2 p_{si}^2 \frac{m_x}{m_y}) \frac{\partial^2 \lambda}{\partial X^2} + (\sum_{i=1}^N c_i^2 p_{si}^2 \frac{m_y}{m_x}) \frac{\partial^2 \lambda}{\partial Y^2}] = -(N-1) J_0 \\ - m_x m_y [(\sum_{i=1}^N c_i^2 \frac{\partial}{\partial X} (p_{si}^2 \frac{m_x}{m_y})) \frac{\partial \lambda}{\partial X} - (\sum_{i=1}^N c_i^2 \frac{\partial}{\partial Y} (p_{si}^2 \frac{m_y}{m_x})) \frac{\partial \lambda}{\partial Y}] \end{aligned} \tag{5.13}$$

The λ's are evaluated by solving (5.13) using the overrelaxation method. Then u and v are evaluated by (5.10) and (5.11).

3. Using the winds produced in step 2, compute precipitation from the moisture tendency equation(5.4). Displace precipitation downwind because the precipitation, though formed in the atmosphere, is measured at the earth's surface. Combine the displaced precipitation with observed precipitation using the successive correction method described in Section 4.6. This procedure may be verified by randomly selecting some precipitation data, deleting them from the objective analysis, and comparing them with the objective analyzed precipitation.

At the completion of step 3, the actual precipitation has been analyzed. If one wishes to determine what the precipitation would be if the storm had occurred in another location, the following steps 4-6 are performed.

4. Displace the analyzed precipitation resulting from step 3 upwind, adjust the winds again by variational method to satisfy the moisture tendency equation. This can be done without changing the pressure tendency.

The procedures of the adjustment are the following:

$$I^2 = \int\int[\alpha^2 \int_0^1 [\sum_{i=1}^{N}(u_i - u_{i0})^2 + \sum_{i=1}^{N}(v_i - v_{i0})^2]d\sigma + \lambda K]dA \tag{5.14}$$

where α^2 is a Gauss precision modulus, λ is a Lagrange multiplier, K is the left side of the Eq(5.4), and A is the area of the computation domain.

We change $\vec{v}$ to $\vec{v} + \Delta\vec{v}$ which has the property that

$$\int_0^1 \nabla \bullet p_s \Delta\vec{v} d\sigma = 0 \tag{5.15}$$

Try

$$\Delta\vec{V} = \Delta\bar{\vec{V}}(\sigma - 0.5) \tag{5.16}$$

where $\Delta\bar{\vec{V}}$ has components $(U - U_0)$ and $(V - V_0)$.

Substituting (5.16) into (5.15), we obtain

$$\int_0^1 \nabla \bullet p_s \Delta\vec{V} d\sigma = \nabla \bullet p_s \Delta\bar{\vec{V}} \int_0^1 (\sigma - 0.5) d\sigma = 0 \tag{5.17}$$

Adjust velocities by $\Delta\vec{v}$ to satisfy moisture tendency equation(5.4). This won't affect the pressure tendency equation(5.3). From (5.16), we obtain

$$u_i - u_{i0} = (U_i - U_{i0})(\sigma - 0.5), i = 1, 2, \cdots, N \tag{5.18}$$

and

$$v_i - v_{i0} = (V_i - V_{i0})(\sigma - 0.5), i = 1, 2, \cdots, N \tag{5.19}$$

Substituting (5.18) and (5.19) into (5.14), we obtain

$$I^2 = \int\int[\int_0^1 \frac{\alpha^2}{12}[\sum_{i=1}^{N}(U_i - U_{i0})^2 + \sum_{i=1}^{N}(V_i - V_{i0})^2]d\sigma + \lambda K]dA \tag{5.20}$$

We apply the variational operator δ to the two sides of the equation(5.20), giving

$$\begin{aligned} \delta I^2 \;=\; & \int\int[\int_0^1 \frac{\alpha^2}{6}[\sum_{i=1}^{N}(U_i - U_{i0})\delta U_i + \sum_{i=1}^{N}(V_i - V_{i0})\delta V_i]d\sigma \\ & + \frac{\lambda}{(N-1)} \int_0^1 [\sum_{i=1}^{N} c_i m_x m_y \frac{\partial}{\partial X}(\frac{p_{si} r_i \delta U_i}{m_y}) \qquad (5.21) \\ & + \sum_{i=1}^{N} c_i m_x m_y \frac{\partial}{\partial Y}(\frac{p_{si} r_i \delta V_i}{m_x})]d\sigma \\ & + K\delta\lambda]dA \qquad (5.22) \end{aligned}$$

By analogy to the derivation of Eqns(5.7) and (5.8), since

$$\lambda\frac{\partial}{\partial X}(\frac{p_{si}r_i\delta U_i}{m_y}) = \frac{\partial}{\partial X}(\frac{\lambda p_{si}r_i\delta U_i}{m_y}) - \frac{p_{si}r_i\delta U_i}{m_y}\frac{\partial\lambda}{\partial X}$$

if either λ or δu_i vanishes on the boundaries, one thus obtains

$$\int\int[\int_0^1 \lambda\frac{\partial}{\partial X}(\frac{p_{si}r_i\delta U_i}{m_y})d\sigma]dA = -\int\int[\int_0^1 \frac{p_{si}r_i\delta U_i}{m_y}\frac{\partial\lambda}{\partial X}d\sigma]dA \qquad (5.23)$$

Similarly, since

$$\lambda\frac{\partial}{\partial Y}(\frac{p_{si}r_i\delta V_i}{m_x}) = \frac{\partial}{\partial Y}(\frac{\lambda p_{si}r_i\delta V_i}{m_x}) - \frac{p_{si}r_i\delta V_i}{m_x}\frac{\partial\lambda}{\partial Y}$$

if either λ or δv_i vanishes on the boundaries, one thus obtains

$$\int\int[\int_0^1 \lambda\frac{\partial}{\partial Y}(\frac{p_{si}r_i\delta V_i}{m_x})d\sigma]dA = -\int\int[\int_0^1 \frac{p_{si}r_i\delta V_i}{m_x}\frac{\partial\lambda}{\partial Y}d\sigma]dA \qquad (5.24)$$

Substitution of Eqn(5.23) and Eqn(5.24) into Eqn(5.22) yields the following equation

$$\begin{aligned}\delta I^2 &= \int\int[\int_0^1(\sum_{i=1}^{N}[\frac{\alpha^2}{6}(U_i - U_{i0}) - \frac{c_i p_{si} m_x \hat{r}_i}{(N-1)}\frac{\partial\lambda}{\partial X}]\delta U_i \\ &+ \sum_{i=1}^{N}[\frac{\alpha^2}{6}(V_i - V_{i0}) - \frac{c_i p_{si} m_y \hat{r}_i}{(N-1)}\frac{\partial\lambda}{\partial Y}]\delta V_i)d\sigma]dA \qquad (5.25)\end{aligned}$$

Using the least squares method, we let $\delta I^2 = 0$, and we set $\alpha^2 = \frac{6}{N-1}$, and then obtain

$$U_i = U_{i0} + c_i p_{si} \hat{r}_i m_x \frac{\partial \lambda}{\partial X}, i = 1, 2, \cdots, N \tag{5.26}$$

$$V_i = V_{i0} + c_i p_{si} \hat{r}_i m_y \frac{\partial \lambda}{\partial Y}, i = 1, 2, \cdots, N \tag{5.27}$$

where

$$\hat{r}_i = \int_0^1 r_i(\sigma - 0.5) d\sigma$$

Substituting from (5.18) gives

$$u_i = u_{i0} + (\sigma - 0.5) c_i p_{si} \hat{r}_i m_x \frac{\partial \lambda}{\partial X}, i = 1, 2, \cdots, N \tag{5.28}$$

$$v_i = v_{i0} + (\sigma - 0.5) c_i p_{si} \hat{r}_i m_y \frac{\partial \lambda}{\partial Y}, i = 1, 2, \cdots, N \tag{5.29}$$

Thus, substituting (5.28), (5.29) into (5.4), we obtain the equation

$$\int_0^1 m_x m_y [(\sum_{i=1}^N c_i^2 p_{si}^2 \hat{r}_i{}^2 \frac{m_x}{m_y}) \frac{\partial^2 \lambda}{\partial X^2} + (\sum_{i=1}^N c_i^2 p_{si}^2 \hat{r}_i{}^2 \frac{m_y}{m_x}) \frac{\partial^2 \lambda}{\partial Y^2}] d\sigma = -(N-1) K_0$$
$$- \int_0^1 m_x m_y [(\sum_{i=1}^N c_i \frac{\partial}{\partial X}(p_{si}^2 \hat{r}_i{}^2 \frac{m_x}{m_y})) \frac{\partial \lambda}{\partial X} - (\sum_{i=1}^N c_i^2 \frac{\partial}{\partial Y}(p_{si}^2 \hat{r}_i{}^2 \frac{m_y}{m_x})) \frac{\partial \lambda}{\partial Y}] d\sigma \tag{5.30}$$

where

$$K_0 = g(P - E) + \frac{(p_{sN} \bar{r_N} - p_{s1} \bar{r_1})}{\Delta t} + \frac{1}{(N-1)}$$
$$\sum_{i=1}^N \int_0^1 c_i m_x m_y [\frac{\partial}{\partial X}(\frac{p_{si} u_{i0} r_i}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_{si} v_{i0} r_i}{m_x})] d\sigma \tag{5.31}$$

Since p_s, $\hat{r}$ and σ are independent of σ, Eqn(5.30) may be written as

$$m_x m_y[(\sum_{i=1}^{N} c_i^2 p_{si}^2 \hat{r}_i^2 \frac{m_x}{m_y})\frac{\partial^2 \lambda}{\partial X^2} + (\sum_{i=1}^{N} c_i^2 p_{si}^2 \hat{r}_i^2 \frac{m_y}{m_x})\frac{\partial^2 \lambda}{\partial Y^2}] = -(N-1)J_0$$
$$-m_x m_y[\sum_{i=1}^{N} c_i \frac{\partial}{\partial X}(p_{si}^2 \hat{r}_i^2 \frac{m_x}{m_y})]\frac{\partial \lambda}{\partial X} - m_x m_y[\sum_{i=1}^{N} c_i^2 \frac{\partial}{\partial Y}(p_{si}^2 \hat{r}_i^2 \frac{m_y}{m_x})]\frac{\partial \lambda}{\partial Y} \quad (5.32)$$

The λ's are evaluated by solving (5.32) using overralxation method, then u, and v are evaluated by (5.28), (5.29).

5. Move storm to new location. Adjust winds by variational means to satisfy the pressure tendency equation.

6. Compute precipitation from the moisture tendency equation. Displace precipitation downwind.

Chapter 6

Numerical Experiments

6.1 Introduction

In Chapter 2 and 5 we described two models for calculating topographical effects on precipitation in complex terrain. The initial value model contains more physics than the variational model, but its success depends on being able to specify the initial conditions. This is difficult to achieve in data-sparse regions. Also, the initial value model is computationally very expensive. For these reasons, the variational model will be employed.

We apply the 3-dimensional variational model(described in Chapter 5) to calculate the precipitation for 11 July 1983 in the upper Columbia River watershed in southeastern B.C. as an experiment. The initial fields of velocity are obtained from the initial conditions of the 3-dimensional initial value model. These winds have already been adjusted to give zero pressure tendencies.

The data we have chosen to use are given on a regularly spaced grid covering the

Upper Columbia River watershed in southeastern B.C.. The grid area is 10' lat.× 20' long. (about 18×24 km).

At each grid point, we need the following data:

- The velocity component u in the x direction relative to earth and the velocity component v in the y direction relative to earth.
- The mixing ratio r.
- The evaporation from the earth's surface E.
- The vertical coordinate σ.
- The surface pressure p_s.

In our experiment, we use two sets of data at two times with an interval of 24 hours, so the variable N is equal to 2 in Eqn(5.3) and Eqn(5.4).

6.2 Some Computational Details

6.2.1 Adjusting the initial velocities by variational method

The initial velocity fields over a 24×24 grid covering the Upper Columbia River watershed come from the initial conditions of the 3-dimensional initial value model. The initial velocity fields in the boundary layer are shown in Figures 6.1 - 6.2. In order to calculate small-scale topographical effect on precipitation, we use observed pressures to adjust the velocity fields by requiring that the winds satisfy the pressure tendency equation using observed pressures. The adjusted winds are shown in Figures 6.3 - 6.4.

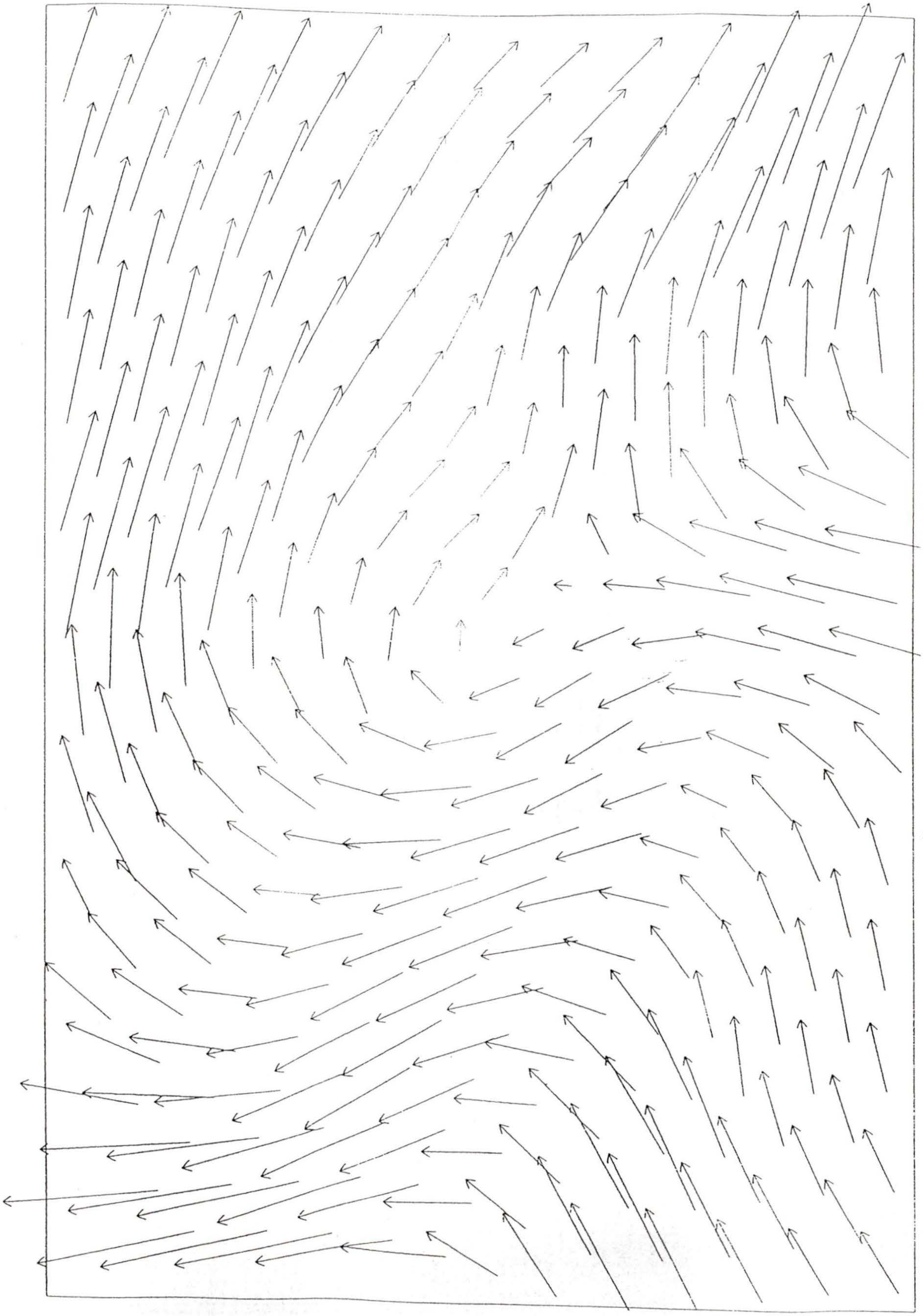

Figure 6.1: Wind velocity in boundary layer for 83/07/11

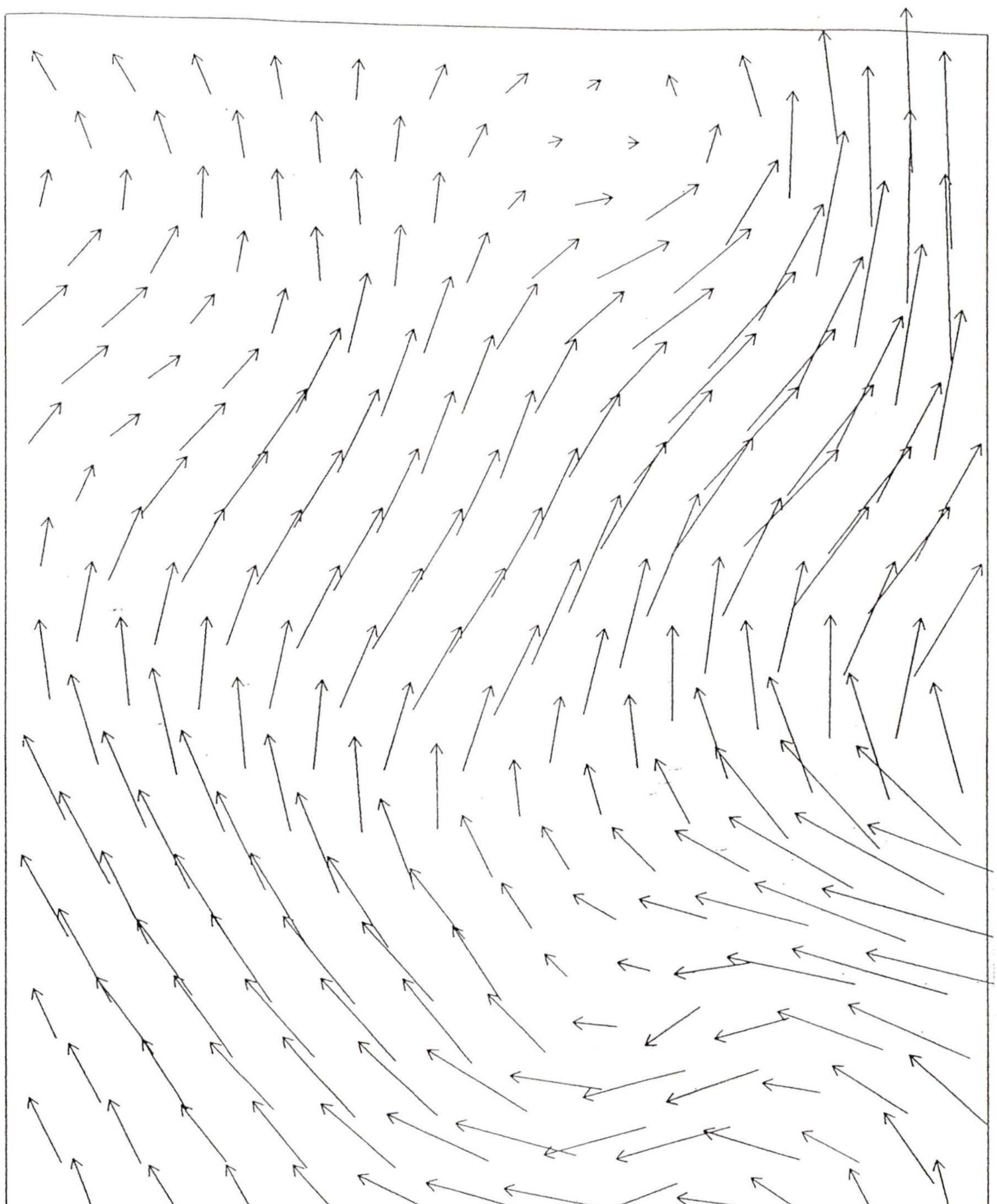

Figure 6.2: Wind velocity in boundary layer for 83/07/12

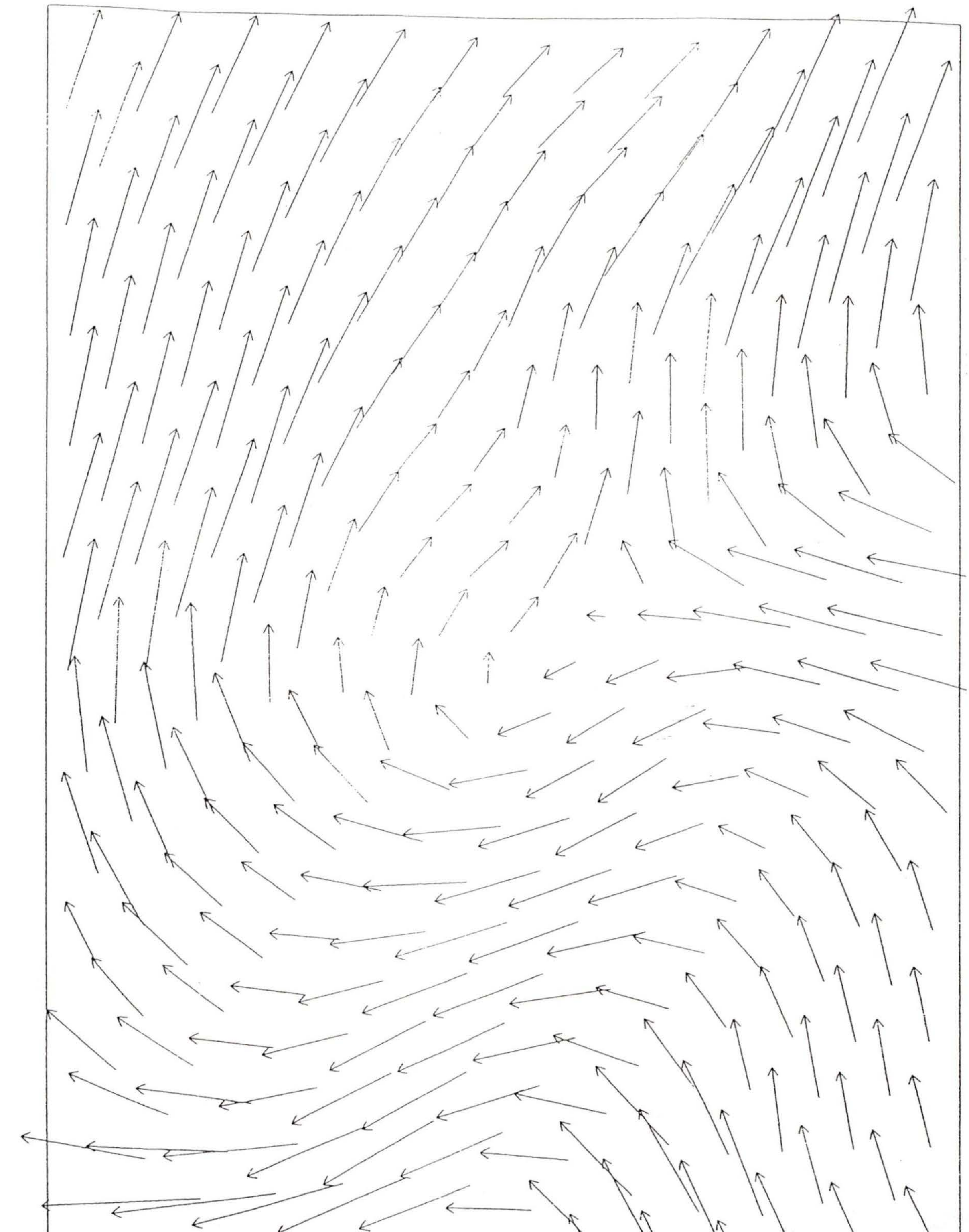

Figure 6.3: Adjusted wind velocity in boundary layer for 83/07/11

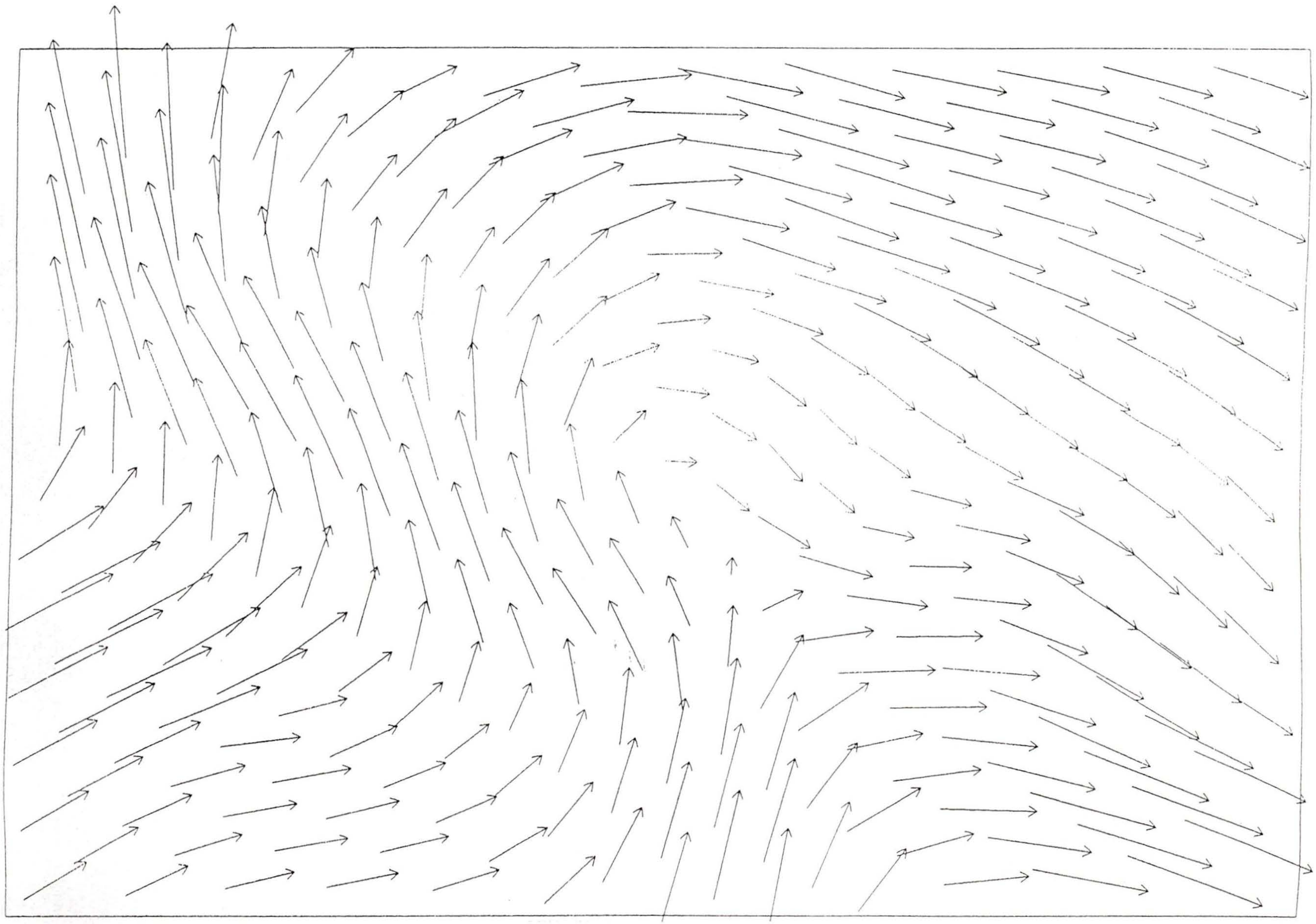

Figure 6.4: Adjusted wind velocity in boundary layer for 83/07/12

6.2.2 Precipitation Calculation

In our 3-dimensional variational model, we calculate precipitation by the integrated moisture tendency equation(5.4) using adjusted winds. These results in mm are shown in Figure 6.5. We then displace the precipitation downwind using bi-cubic splines for interpolation since the precipitation, though formed in the atmosphere, is measured at the earth's surface. From Figure 6.6 we can see the downwind displacement of precipitation. Combined with observed precipitation, we obtain the objective analysed precipitation using the successive correction method described in Section 4.6. The amounts of observed precipitation for July 11, 1983 are shown in Figure 6.7 and the amounts of the objective analysed precipitation for July 11, 1983 are shown in Figure 6.8.

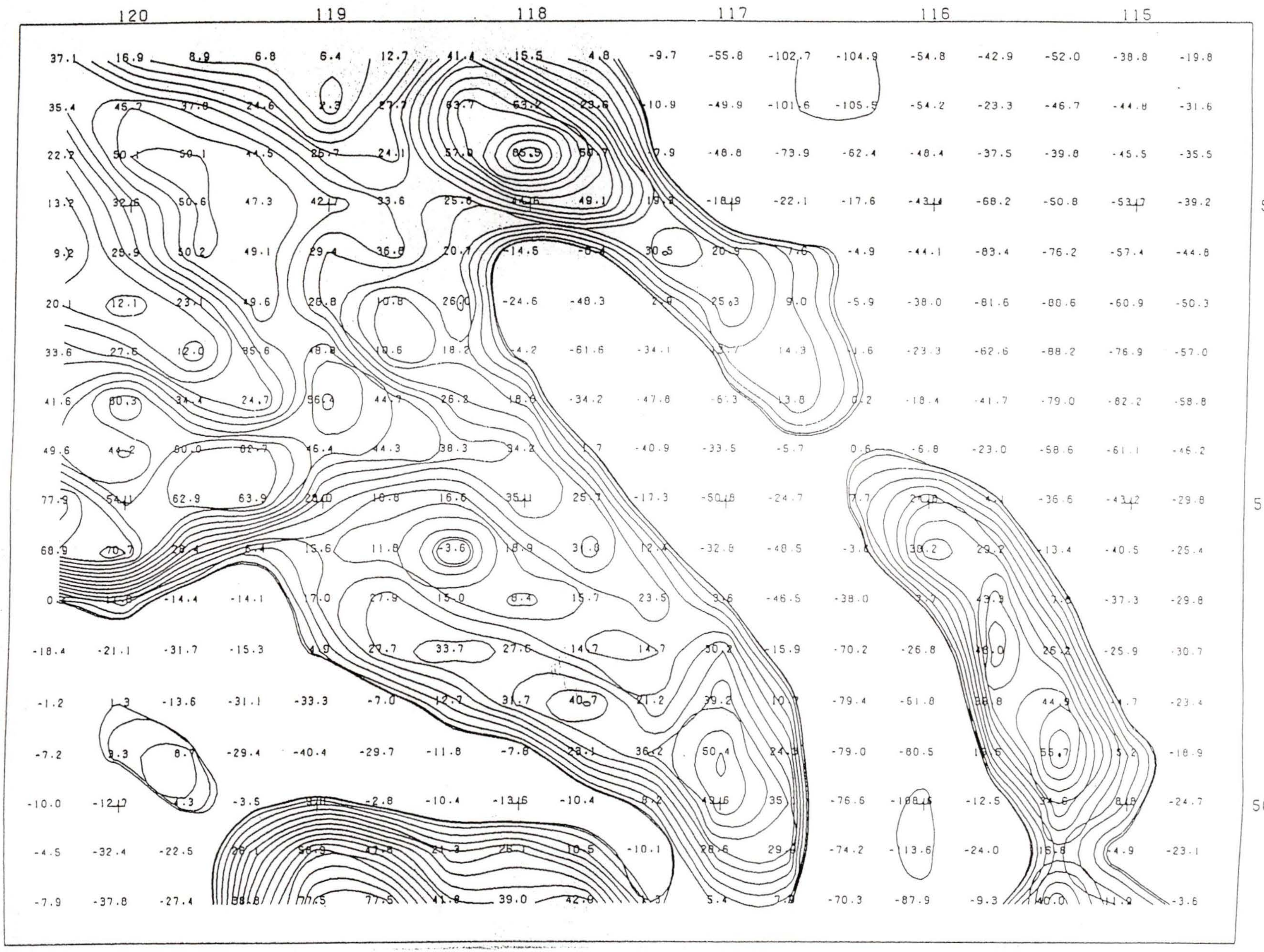

Figure 6.5: Undisplaced model precipitation (mm) for 83/07/11

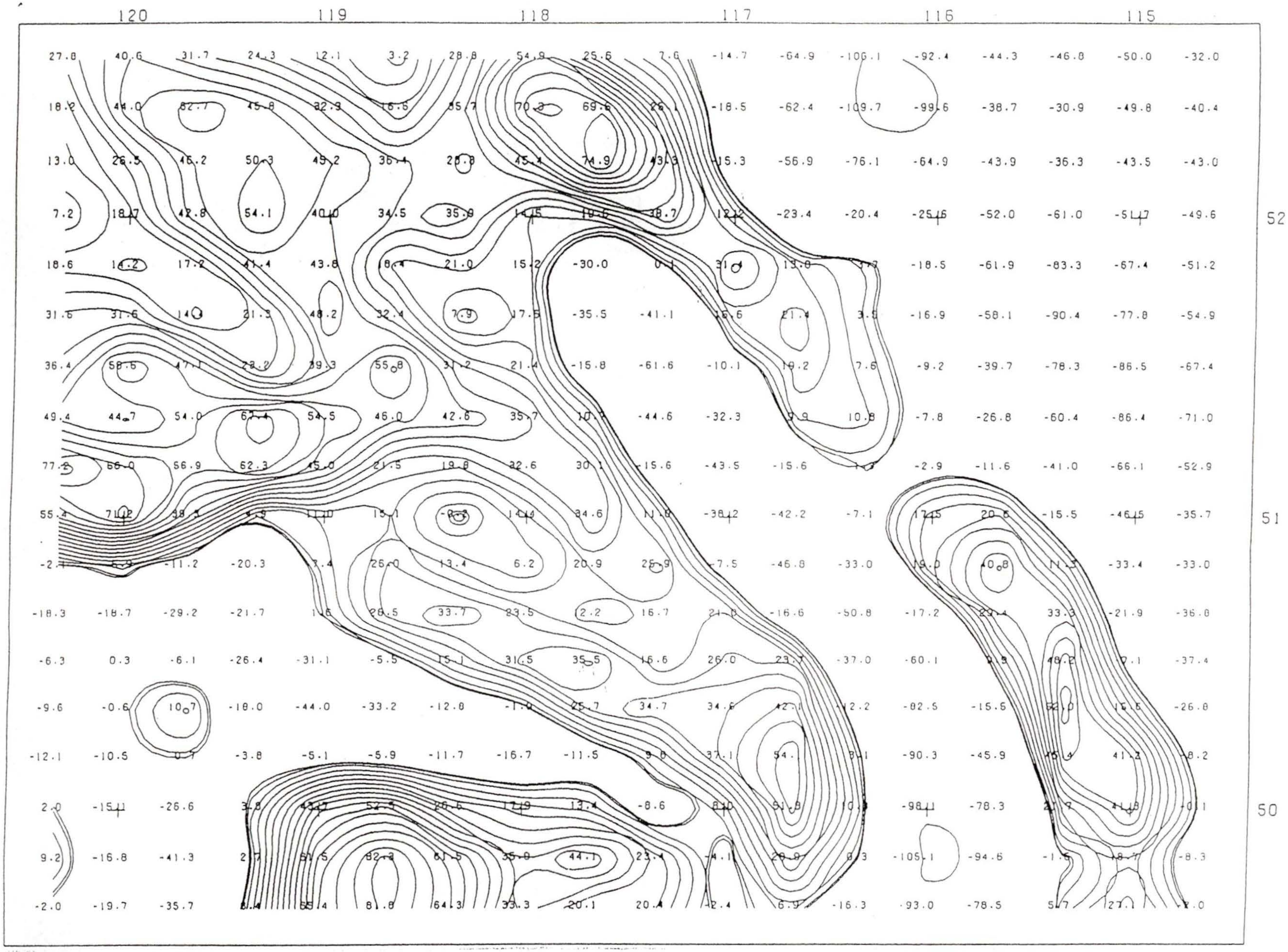

Figure 6.6: Displaced precipitation (mm) for 83/07/11

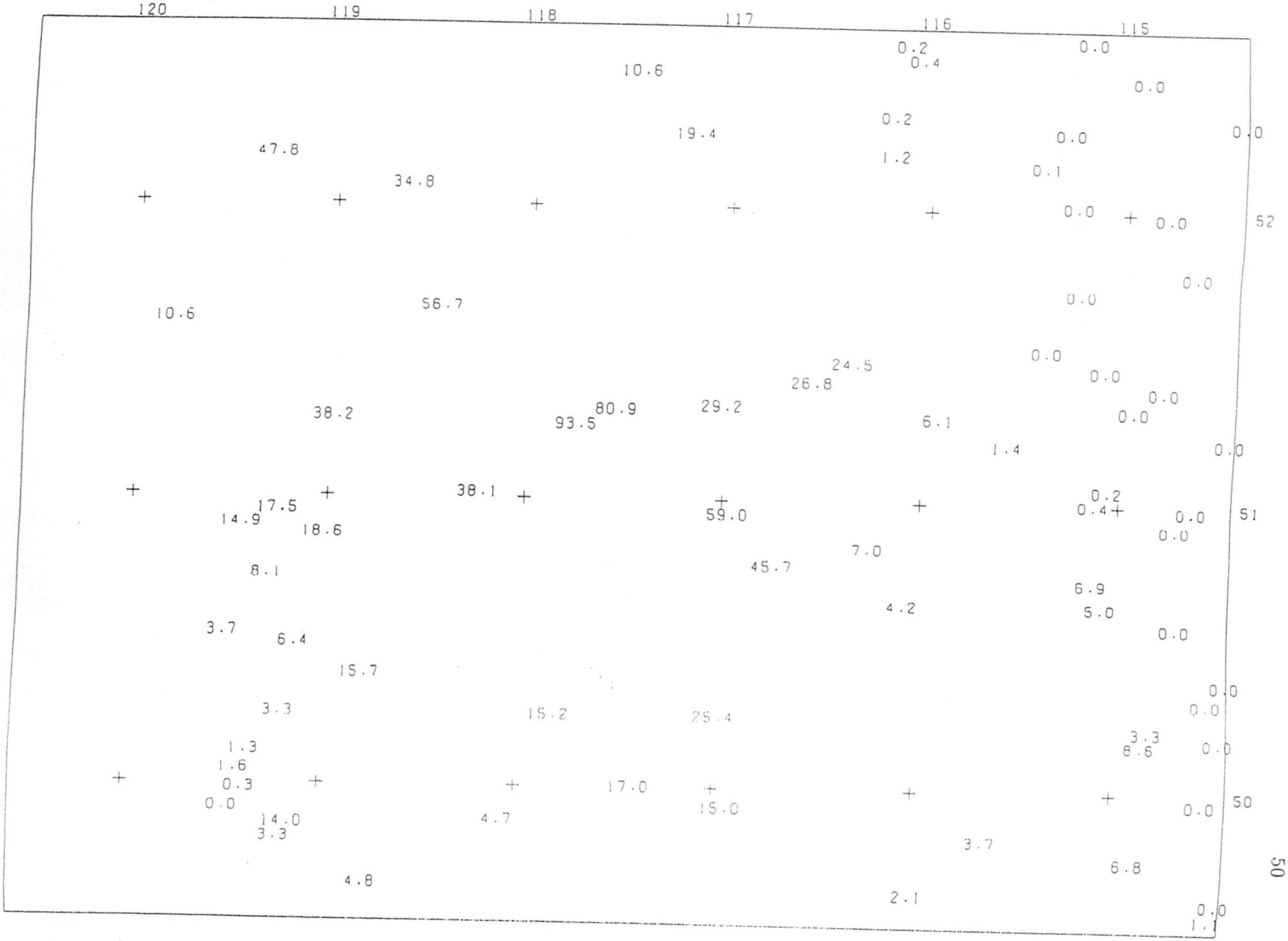

Figure 6.7: Observed precipitation (mm) for 83/07/11

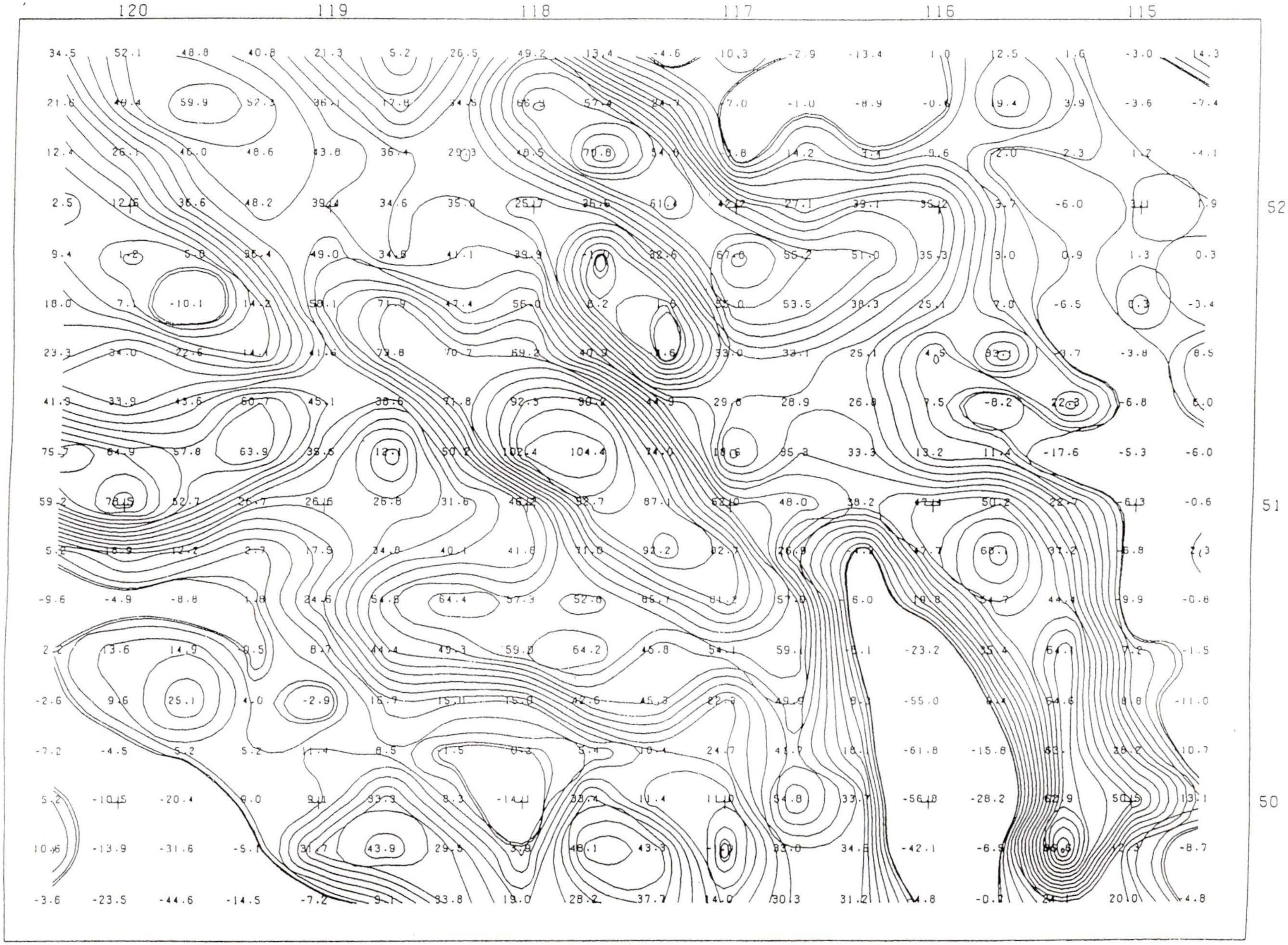

Figure 6.8: Objective analysed precipitation (mm) for 83/07/11

6.3 A Re-examination of the Pressure Tendency Equation

6.3.1 Adjusting the Initial Velocities by One Function

In Section 5.2, we have described how to use the pressure tendency equation (5.3) to adjust the initial velocities by variational means. From Eqns(5.10) and (5.11), it can be seen that the adjustments to the wind components, $(u_i - u_{i0})$ and $(v_i - v_{i0})$, are independent of σ. However, if the adjustments are multiplied by any function $f(\sigma)$, where

$$\int_0^1 f(\sigma)d\sigma = 1 \tag{6.1}$$

the adjusted winds (u_i,v_i) will still satisfy Eqn(5.3). The following describes how to use a function $f(\sigma)$ to adjust the initial wind to satisfy the pressure tendency equation.

Instead of Eqn(5.10) and (5.11), we let

$$u_i = u_{i0} + f(\sigma)c_i p_{si} m_x \frac{\partial \lambda}{\partial X}, i = 1, 2, \cdots, N \tag{6.2}$$

$$v_i = v_{i0} + f(\sigma)c_i p_{si} m_y \frac{\partial \lambda}{\partial Y}, i = 1, 2, \cdots, N \tag{6.3}$$

Substituting Eqn(6.2) and (6.3) into Eqn(5.3), we obtain

$$\int_0^1 m_x m_y [(\sum_{i=1}^{N} c_i^2 p_{si}^2 \frac{m_x}{m_y})\frac{\partial^2 \lambda}{\partial X^2} + (\sum_{i=1}^{N} c_i^2 p_{si}^2 \frac{m_y}{m_x})\frac{\partial^2 \lambda}{\partial Y^2}]f(\sigma)d\sigma = -(N-1)J_0$$
$$- \int_0^1 m_x m_y [(\sum_{i=1}^{N} c_i^2 \frac{\partial}{\partial X}(p_{si}^2 \frac{m_x}{m_y}))\frac{\partial \lambda}{\partial X} - (\sum_{i=1}^{N} c_i^2 \frac{\partial}{\partial Y}(p_{si}^2 \frac{m_y}{m_x}))\frac{\partial \lambda}{\partial Y}]f(\sigma)d\sigma \tag{6.4}$$

where

$$J_0 = \frac{(p_{sN} - p_{s1})}{\Delta t} + \frac{1}{(N-1)}\sum_{i=1}^{N}\int_0^1 c_i m_x m_y [\frac{\partial}{\partial X}(\frac{p_{si}u_{i0}}{m_y}) + \frac{\partial}{\partial Y}(\frac{p_{si}v_{i0}}{m_x})]d\sigma$$

Since p_s and λ are independent of σ, Eqn(6.4) may be written using Eqn(6.1)

$$m_x m_y[(\sum_{i=1}^{N} c_i^2 p_{si}^2 \frac{m_x}{m_y})\frac{\partial^2\lambda}{\partial X^2} + (\sum_{i=1}^{N} c_i^2 p_{si}^2 \frac{m_y}{m_x})\frac{\partial^2\lambda}{\partial Y^2}] = (1-N)J_0$$
$$-m_x m_y[(\sum_{i=1}^{N} c_i^2 \frac{\partial}{\partial X}(p_{si}^2 \frac{m_x}{m_y}))\frac{\partial\lambda}{\partial X} - (\sum_{i=1}^{N} c_i^2 \frac{\partial}{\partial Y}(p_{si}^2 \frac{m_y}{m_x}))\frac{\partial\lambda}{\partial Y}] \tag{6.5}$$

The interesting thing is that the Eqn(6.5) is the same as Eqn(5.13). This means that we can include the vertical change factor $f(\sigma)$ in our wind velocity adjustment without changing the λ equation - Eqn(5.13).

The following Tables 6.1 - 6.4 show some sample results.

Table 6.1: Comparison of computed precipitation and observed precipitation without adjusting wind

n	Nearest Grid Point	Unadjusted Wind Components (m/s)	Computed Precipitation	Observed Precipitation	Diff.
	i j k	u1 v1 u2 v2	(mm)	(mm)	(mm)
1	24 7 7	6.52 6.55 6.80 5.10	9.4	9.4	0.0
2	23 6 7	7.00 8.59 9.12 4.01	-7.7	5.8	-13.5
3	18 9 7	2.78 8.23 11.75 9.40	34.9	34.8	0.1
4	15 9 7	-0.79 12.72 10.71 10.06	18.5	57.7	-39.2
5	11 8 7	-4.18 13.65 3.14 5.32	9.0	17.4	-8.4
6	10 14 7	0.06 2.43 11.97 -2.99	-28.2	45.2	-73.4
7	6 20 7	13.47 -4.46 13.57 -12.33	19.5	1.8	17.7
8	5 12 7	2.11 10.13 13.38 13.18	-2.6	17.2	-19.8
9	2 10 7	7.41 4.36 6.53 24.52	8.7	5.4	3.3
10	1 12 7	5.04 9.16 12.17 14.83	-35.4	5.8	-41.2

Overall mean $\|\|diff\|\|$	21.65

Table 6.2: Comparison of computed precipitation and observed precipitation using function of $f(\sigma) = 1$ to adjust wind

n	Nearest Grid Point	Adjusted Wind Components (m/s)	Computed Precipitation	Observed Precipitation	Diff.
	i j k	u1 v1 u2 v2	(mm)	(mm)	(mm)
1	24 7 7	6.52 6.56 6.80 5.11	9.4	9.4	0.0
2	23 6 7	7.00 8.60 9.12 4.02	-7.8	5.8	-13.7
3	18 9 7	2.78 8.23 11.75 9.40	34.7	34.8	-0.1
4	15 9 7	-0.79 12.72 10.71 10.06	18.2	57.7	-39.5
5	11 8 7	-4.19 13.65 3.13 5.32	8.6	17.4	-8.8
6	10 14 7	0.06 2.43 11.97 -2.99	-28.5	45.2	-73.7
7	6 20 7	13.48 -4.46 13.58 -12.33	19.3	1.8	17.5
8	5 12 7	2.11 10.12 13.38 13.17	-2.7	17.2	-19.9
9	2 10 7	7.41 4.35 6.53 24.51	8.5	5.4	3.1
10	1 12 7	5.04 9.15 12.17 14.82	-35.5	5.8	-41.3

Overall mean $\|\|diff\|\|$	21.76

Table 6.3: Comparison of computed precipitation and observed precipitation using function of $f(\sigma) = -100 + 202\sigma$ to adjust wind

n	Nearest Grid Point	Adjusted Wind Components (m/s)	Computed Precipitation	Observed Precipitation	Diff.
	i j k	u1 v1 u2 v2	(mm)	(mm)	(mm)
1	24 7 7	6.52 6.96 6.80 5.51	9.4	9.4	0.0
2	23 6 7	6.93 8.96 9.05 4.38	-10.2	5.8	-16.0
3	18 9 7	2.69 8.36 11.66 9.53	32.2	34.8	-2.6
4	15 9 7	-0.87 12.78 10.63 10.12	13.9	57.7	-43.8
5	11 8 7	-4.35 13.60 10.63 10.12	3.0	17.4	-14.4
6	10 14 7	0.04 2.32 11.95 -3.11	-33.8	45.2	-79.0
7	6 20 7	13.62 -4.6 13.72 -12.47	15.9	1.8	14.1
8	5 12 7	2.05 9.84 13.32 12.88	-5.4	17.2	-22.6
9	2 10 7	7.40 3.96 6.52 24.12	5.5	5.4	0.1
10	1 12 7	5.04 8.74 12.17 14.41	-36.6	5.8	-42.4

Overall mean $\|\|diff\|\|$	23.5

Table 6.4: Comparison of computed precipitation and observed precipitation using function of $f(\sigma) = 3\sigma^2$ to adjust wind

n	Nearest Grid Point	Adjusted Wind Components (m/s)	Computed Precipitation	Observed Precipitation	Diff.
	i j k	u1 v1 u2 v2	(mm)	(mm)	(mm)
1	24 7 7	6.52 6.59 6.80 5.14	9.4	9.4	0.0
2	23 6 7	7.00 8.62 9.11 4.04	-8.0	5.8	-13.8
3	18 9 7	2.77 8.24 11.74 9.41	34.6	34.8	-0.2
4	15 9 7	-0.80 12.73 10.70 10.07	18.0	57.7	-39.7
5	11 8 7	-4.20 13.65 3.13 5.32	8.4	17.4	-9.0
6	10 14 7	0.06 2.42 11.97 -3.00	-28.7	45.2	-73.9
7	6 20 7	13.48 -4.47 13.58 -12.34	19.1	1.8	17.3
8	5 12 7	2.10 10.10 13.38 13.15	-2.9	17.2	-20.1
9	2 10 7	7.41 4.32 6.53 24.49	8.4	5.4	3.0
10	1 12 7	5.04 9.12 12.17 14.79	-35.5	5.8	-41.3

Overall mean $\|\|diff\|\|$	21.83

6.3.2 Adjusting the Initial Velocities by Two Functions

As shown in Section 6.3.1, we can use a function $f(\sigma)$ to adjust initial velocities and get different results compared with the results obtained without using a variable function $f(\sigma)$. Now we will describe how to use two functions of $f(\sigma)$ and $h(\sigma)$ to adjust initial wind to satisfy pressure tendency equation.

Instead of Eqn(5.10) and (5.11), we let

$$u_i = u_{i0} + f(\sigma)c_i p_{si} m_x \frac{\partial\lambda}{\partial X} + Ch(\sigma), i = 1, 2, \cdots, N \quad (6.6)$$

$$v_i = v_{i0} + f(\sigma)c_i p_{si} m_y \frac{\partial\lambda}{\partial Y} + Ch(\sigma), i = 1, 2, \cdots, N \quad (6.7)$$

where

$$\int_0^1 f(\sigma)d\sigma = 1 \quad (6.8)$$

$$\int_0^1 h(\sigma)d\sigma = 0 \quad (6.9)$$

and C is an arbitrary constant.

Substituting Eqn(6.6) and Eqn(6.7) into Eqn(5.3), we obtain the equation

$$\int_0^1 m_x m_y[(\sum_{i=1}^{N} c_i^2 p_{si}^2 \frac{m_x}{m_y})\frac{\partial^2\lambda}{\partial X^2} + (\sum_{i=1}^{N} c_i^2 p_{si}^2 \frac{m_y}{m_x})\frac{\partial^2\lambda}{\partial Y^2}]f(\sigma)d\sigma = -(N-1)J_0$$
$$-\int_0^1 m_x m_y[(\sum_{i=1}^{N} c_i^2 \frac{\partial}{\partial X}(p_{si}^2 \frac{m_x}{m_y}))\frac{\partial\lambda}{\partial X} - (\sum_{i=1}^{N} c_i^2 \frac{\partial}{\partial Y}(p_{si}^2 \frac{m_y}{m_x}))\frac{\partial\lambda}{\partial Y}]f(\sigma)d\sigma$$

$$-\int_0^1 m_x m_y [\sum_{i=1}^{N} c_i \frac{\partial}{\partial X}(\frac{p_{si}}{m_y})] Ch(\sigma) d\sigma - \int_0^1 m_x m_y [\sum_{i=1}^{N} c_i \frac{\partial}{\partial Y}(\frac{p_{si}}{m_x})] Ch(\sigma) d\sigma \qquad (6.10)$$

where J_0 is given by the equation following (6.4).

Since p_s and λ are independent of σ, Eqn(6.10) may be written using Eqn(6.8) and Eqn(6.9)

$$m_x m_y [(\sum_{i=1}^{N} c_i^2 p_{si}^2 \frac{m_x}{m_y}) \frac{\partial^2 \lambda}{\partial X^2} + (\sum_{i=1}^{N} c_i^2 p_{si}^2 \frac{m_y}{m_x}) \frac{\partial^2 \lambda}{\partial Y^2}] = (1-N) J_0$$
$$-m_x m_y [(\sum_{i=1}^{N} c_i^2 \frac{\partial}{\partial X}(p_{si}^2 \frac{m_x}{m_y})) \frac{\partial \lambda}{\partial X} - (\sum_{i=1}^{N} c_i^2 \frac{\partial}{\partial Y}(p_{si}^2 \frac{m_y}{m_x})) \frac{\partial \lambda}{\partial Y}] \qquad (6.11)$$

Again we see that the Eqn(6.11) is the same as Eqn(5.13). The following Tables 6.5 and 6.6 show some sample results.

Table 6.5: Comparison of computed precipitation and observed precipitation using functions of $f(\sigma) = -100 + 202\sigma$ and $h(\sigma) = 10 - 20\sigma$ to adjust wind

n	Nearest Grid Point	Adjusted Wind Components (m/s)	Computed Precipitation	Observed Precipitation	Diff.
	i j k	u1 v1 u2 v2	(mm)	(mm)	(mm)
1	24 7 7	2.15 2.59 2.65 1.36	9.4	9.4	0.0
2	23 6 7	2.56 4.59 4.89 0.23	-9.2	5.8	-15.0
3	18 9 7	-1.68 3.99 7.51 5.37	18.8	34.8	-16.0
4	15 9 7	-5.24 8.41 6.48 5.97	-0.7	57.7	-58.0
5	11 8 7	-8.72 9.23 -1.19 1.11	-11.8	17.4	-29.2
6	10 14 7	-4.33 -2.05 7.80 -7.26	-35.6	45.2	-80.8
7	6 20 7	9.25 -8.97 9.57 -16.62	-1.8	1.8	-3.6
8	5 12 7	-2.32 5.47 9.17 8.73	-8.4	17.2	-25.6
9	2 10 7	3.03 -0.41 2.37 19.96	0.4	5.4	-5.0
10	1 12 7	0.67 4.37 8.02 10.25	-39.4	5.8	-45.2

Overall mean $\|\|diff\|\|$	27.88

Table 6.6: Comparison of computed precipitation and observed precipitation using functions of $f(\sigma) = 3\sigma^2$ and $h(\sigma) = 1 - 3\sigma^2$ to adjust wind

n	Nearest Grid Point	Adjusted Wind Components (m/s)	Computed Precipitation	Observed Precipitation	Diff.
	i j k	u1 v1 u2 v2	(mm)	(mm)	(mm)
1	24 7 7	5.70 5.77 6.04 4.38	9.4	9.4	0.0
2	23 6 7	6.18 7.81 8.36 3.29	-7.9	5.8	-13.7
3	18 9 7	1.95 7.42 10.99 8.65	32.5	34.8	-2.3
4	15 9 7	-1.61 11.91 9.95 9.31	15.7	57.7	-42.0
5	11 8 7	-5.01 12.83 2.37 4.56	6.3	17.4	-11.1
6	10 14 7	-0.76 1.60 11.21 -3.76	-29.6	45.2	-74.8
7	6 20 7	12.67 -5.29 12.83 -13.10	16.3	1.8	14.5
8	5 12 7	1.29 9.29 12.62 12.40	-3.4	17.2	-20.6
9	2 10 7	6.59 3.51 5.77 23.73	7.7	5.4	2.3
10	1 12 7	4.22 8.30 11.41 14.04	-3.9	5.8	-41.7

Overall mean $\|\|diff\|\|$	22.30

6.3.3 Flattening the Earth

Because our sample initial winds have already been adjusted using variational methods so that the pressure tendency is zero (see second paragraph of Section 6.1), we can not see much difference between using one or two functions to adjust initial wind to satisfy pressure tendency equation and not using them. But if we flatten the earth after step 4 as described in Section 5.2, the pressure tendency is not equal to zero or even close to zero for the trial velocity fields obtained from step 4. We then adjust these trial velocity fields by variational means to satisfy the pressure tendency equation using different methods - one function, two functions, or constant method. After we adjust the trial velocity fields, we compute precipitation from the moisture tendency equation using the adjusted winds. The computed precipitation is compared with unadjusted winds' precipitation. These results are compared in the following Tables 6.7 - 6.9. From these tables, we can see a substantial difference between using one or two functions to adjust the initial wind to satisfy the pressure tendency equation and not using them.

Table 6.7: Comparison of adjusted winds' precipitation and unadjusted winds' precipitation using functions of $f(\sigma) = 1$ to adjust wind with flattening earth

n	Nearest Grid Point	Adjusted Wind Components (m/s)	Adjusted Winds' Precipitation	Unadjusted Winds' Precipitation	Diff.
	i j k	u1 v1 u2 v2	(mm)	(mm)	(mm)
1	24 7 7	6.52 4.29 6.80 2.97	9.4	-0.2	9.6
2	23 6 7	7.21 6.17 9.31 1.73	0.2	10.5	-10.3
3	18 9 7	4.02 6.98 12.96 8.18	37.7	16.1	21.6
4	15 9 7	1.00 12.14 12.49 9.49	55.5	42.6	12.9
5	11 8 7	-2.27 14.60 5.08 6.26	40.0	5.5	34.4
6	10 14 7	-0.51 4.29 11.41 -1.00	-0.2	63.0	-63.2
7	6 20 7	11.85 -3.57 11.74 -11.25	2.5	12.1	-9.5
8	5 12 7	2.58 12.36 13.88 15.56	15.5	19.2	-3.7
9	2 10 7	8.11 6.13 7.27 26.36	21.1	17.7	3.4
10	1 12 7	5.04 11.88 12.17 17.79	8.0	-38.9	46.9

Overall mean $\|\|diff\|\|$	21.55

Table 6.8: Comparison of adjusted winds' precipitation and unadjusted winds' precipitation using functions of $f(\sigma) = -100 + 202\sigma$ to adjust wind with flattening earth

n	Nearest Grid Point	Adjusted Wind Components (m/s)	Adjusted Winds' Precipitation	Unadjusted Winds' Precipitation	Diff.
	i j k	u1 v1 u2 v2	(mm)	(mm)	(mm)
1	24 7 7	6.52 -16.76 6.80 -18.27	9.4	-0.2	9.6
2	23 6 7	6.55 -19.29 8.65 -23.95	-104.2	10.5	-114.7
3	18 9 7	2.19 -1.67 11.11 -0.55	371.7	16.1	355.6
4	15 9 7	-2.07 7.63 9.39 4.93	548.1	42.6	505.5
5	11 8 7	6.89 30.64 14.32 22.46	660.1	5.5	654.6
6	10 14 7	-19.72 -0.17 -7.97 -5.49	-298.4	63.0	-361.5
7	6 20 7	-4.07 -6.02 4.31 -13.72	-145.2	12.1	-157.3
8	5 12 7	2.03 17.40 13.33 20.65	95.9	19.2	76.7
9	2 10 7	9.27 21.44 8.44 41.81	294.4	17.7	276.7
10	1 12 7	5.04 28.96 12.17 35.02	269.7	-38.9	308.6

Overall mean $\|\|diff\|\|$	282.08

Table 6.9: Comparison of adjusted winds' precipitation and unadjusted winds' precipitation using functions of $f(\sigma) = -100 + 202\sigma$ and $h(\sigma) = 10 - 20\sigma$ to adjust wind with flattening earth

n	Nearest Grid Point i j k	Adjusted Wind Components (m/s) u1 v1 u2 v2	Adjusted Winds' Precipitation (mm)	Unadjusted Winds' Precipitation (mm)	Diff. (mm)
1	24 7 7	2.15 -21.13 2.65 -22.43	9.4	-0.2	9.6
2	23 6 7	2.18 -23.66 4.49 -28.11	-104.8	10.5	-115.3
3	18 9 7	-2.18 -6.04 6.96 -4.70	362.5	16.1	346.4
4	15 9 7	-6.44 3.26 5.24 0.78	537.1	42.6	494.6
5	11 8 7	2.52 26.27 10.17 18.30	644.8	5.5	639.2
6	10 14 7	-24.09 -4.54 -12.12 -9.65	-286.7	63.0	-349.7
7	6 20 7	-8.44 -10.39 -8.47 -17.87	-148.9	12.1	-161.0
8	5 12 7	-2.34 13.03 9.18 16.49	93.7	19.2	74.5
9	2 10 7	4.90 17.07 4.29 37.66	288.9	17.7	271.2
10	1 12 7	0.67 24.59 8.02 30.87	263.5	-38.9	302.4

Overall mean $\|\|diff\|\|$	276.39

Chapter 7

Concluding Remarks and Future Work

In this thesis, we have applied the three dimensional variational model that was proposed by Dr. Danard to calculate precipitation in upper Columbia River watershed in southeastern B.C.. Topography is an influential factor and has been included in this model.

The above described numerical experiments strongly show that the three dimensional variational model is comparatively simple, easy to compute, and gives realistic results.

The numerical experiments also show that initial velocities can be adjusted by different methods, such as one variable function method, two variable functions method, and constant method. Theoretically speaking, adjusting velocities by two functions is the best, but this result can not be proven from our numerical experiments because the functions in our experiments are chosen randomly. How to find these functions to get a better result is one area of future work.

In our numerical experiments, the successive correction objective analysis method was used to compute the objective analyzed precipitation. Since the statistical interpolation method can be used for a better result, it is proposed in the future to use the statistical interpolation method to compute objective analyzed precipitation.

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Appendix A

List of Symbols

Symbol	Meaning	Equation number of first appearance
δ	Variational operator	(3.2)
μ	Lagrange multiplier	(3.3)
σ	Vertical coordinate ($= p/p_s$)	(2.1)
$\dot{\sigma}$	Substantial derivative of σ (vertical velocity)	(2.2)
ϕ'	Departure of geopotential from value in a reference atmosphere	(2.2)
ω	Substantial derivative of pressure	(2.4)
∇	Horizontal gradient operator	(5.15)

B_x, B_y	Components of term to smooth divergence tendencies	(2.2)(2.3)
C	Rate of condensation of water vapor	(2.5)
C_p	Specific heat of dry air at constant pressure	(2.4)
D_{hT}	Horizontal eddy diffusion of temperature	(2.4)
D_{hr}	Horizontal eddy diffusion of moisture	(2.5)
E	Evaporation from the earth's surface	(5.2)
f	Coriolis parameter	(2.2)
F_{hx}, F_{hy}	Components of term representing horizontal mixing of momentum	(2.2)(2.3)
F_{vx}, F_{vy}	Components of term representing vertical mixing of momentum	(2.2)(2.3)
g	The gravitational acceleration	(5.2)
G_x, G_y	Components of term to smooth divergence	(2.2)(2.3)
H_L	Latent heat of vaporization	(2.4)
m_x, m_y	Map scale factors in X and Y directions	(2.2)
p_s	Surface pressure	(2.1)
r	Mixing ratio	(5.2)
$\bar{r}$	Vertically averaged mixing ratio	(5.2)
R	Gas constant for 1 kg of dry air	(2.7)

T, T_v	Temperature, virtual temperature	(2.4)
T_v'	Departure of virtual temperature from value in a reference atmosphere	(2.7)
u, v	Horizontal velocity components	(2.2)
W_x, W_y	Components of term to nudge model vorticity tendencies towards observed large scale tendencies	(2.2)(2.3)

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