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A New Subclass of Analytic Functions Associated with the q-Derivative Operator Related to the Pascal Distribution Series

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Abstract: A new subclass $TX_q[\lambda, A, B]$ of analytic functions is introduced by making use of the q-derivative operator associated with the Pascal distribution. Certain properties of analytic functions in the subclass $TX_q[\lambda, A, B]$ are derived. Some known results are generalized.

Keywords: analytic function; q-derivative operator; differential subordination; Pascal distribution series; Hadamard product

MSC: Primary 30C45; 30C50

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1. Introduction

Let H be the class of analytic functions g of the following form

$$g(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (z \in D = \{z \in C : |z| < 1)\}). \tag{1}$$

Also, let T be the subclass of H consisting of analytic functions which have the following form

$$g(z) = z - \sum_{k=2}^{\infty} |a_k| z^k \quad (z \in D). \tag{2}$$

If $g \in T$ satisfies the condition:

$$\left| \frac{g'(z) - 1}{(\mu - \nu)\tau - \nu[g'(z) - 1]} \right| < 1 \quad (z \in D),$$

then g is said to belong to the class $R^\tau(\mu, \nu)$ $(\tau \in C \setminus \{0\}, -1 \le \nu < \mu \le 1)$. The class $R^\tau(\mu, \nu)$ was defined by Dixit and Pal [1].

Let P denote the class of Caratéodory functions that are analytic in D and have the following form

$$h(z) = 1 + \sum_{k=1}^{\infty} h_k z^k \tag{3}$$

so that

$$\mathrm{Re}\{h(z)\} > 0 \quad (z \in D).$$

We now recall here the principle of subordination between analytic functions [2]. If g and h are two functions in H, we say that g is subordinate to h, written $g \prec h$ or $g(z) \prec h(z)$, if there is a function v which is analytic in D with $v(0) = 0$ and $|v(z)| < 1$, such that $g(z) = h(v(z))$. Further, if h is univalent in D, then we have the equivalence: $g(z) \prec h(z)$ $(z \in D) \Leftrightarrow g(0) = h(0)$ and $g(D) \subset h(D)$.

Now we define q-derivative D_q $(0 < q < 1)$ for $g \in H$:

$$(D_q g)(z) = \begin{cases} \frac{g(qz)-g(z)}{(q-1)z} & (z \neq 0) \\ g'(0) & (z = 0). \end{cases} \tag{4}$$

From (4), one can find that

$$D_q g(z) = 1 + \sum_{k=2}^{\infty} [k]_q a_k z^{k-1} \tag{5}$$

and

$$[k]_q = \frac{1-q^k}{1-q}. \tag{6}$$

From (6), we can see that if $q \to 1-$, then $[k]_q \to k$.

In [3], Jackson first showed the application of q-calculus. Since then, many scholars studied the applications of q-analysis in physics and mathematics ([4–26]). Very recently, Srivastava [2] investigated systematically the applications and mathematical explanation of the fractional q-calculus as well as the fractional q-derivative in GFT.

By using the q-derivative operator D_q, we now define a new class $TX_q[\lambda, A, B]$.

Definition 1. *Let $g \in T$ and satisfy the following differential subordination:*

$$\frac{\lambda z^3 (zD_q g(z))''' + (1+2\lambda) z^2 (zD_q g(z))'' + z(zD_q g(z))'}{\lambda z^2 (zD_q g(z))'' + z(zD_q g(z))'} \prec \frac{1+Az}{1+Bz} \quad (z \in D), \tag{7}$$

where $-1 \leq B < A \leq 1$ and $0 \leq \lambda \leq 1$, then g is said to be in $TX_q[\lambda, A, B]$.

If

$$P(Y = k) = \begin{pmatrix} m-1+k \\ m-1 \end{pmatrix} (1-s)^m s^k, k = 0, 1, 2, \cdots$$

for parameters s and m, respectively; then, Y is called to be the Pascal distribution. In [27] El-Deeb, Bulboacă, and Dziok defined the following power series whose coefficients are probabilities of the Pascal distribution:

$$\Psi_s^m(z) := z + \sum_{k=2}^{\infty} \begin{pmatrix} m-2+k \\ m-1 \end{pmatrix} (1-s)^m s^{k-1} z^k \quad (z \in D),$$

where $0 \leq s \leq 1$ and $m \geq 1$.

Very recently, Frasin and Darus [28] introduced a class $C_q(\lambda, \alpha)(0 \leq \alpha < 1, 0 \leq \lambda \leq 1)$ associated with the q-derivative operator. They considered the following series:

$$\Phi_s^m(z) := 2z - \Psi_s^m(z) = z - \sum_{k=2}^{\infty} \begin{pmatrix} m-2+k \\ m-1 \end{pmatrix} (1-s)^m s^{k-1} z^k \quad (z \in D) \tag{8}$$

and the linear operator $I_s^m : H \to H$ by the Hadamard product:

$$I_s^m g(z) := g(z) * \Psi_s^m(z) = z + \sum_{k=2}^{\infty} \begin{pmatrix} m-2+k \\ m-1 \end{pmatrix} (1-s)^m s^{k-1} a_k z^k \quad (z \in D), \tag{9}$$

where $0 \leq s \leq 1$ and $m \geq 1$.

Now, we recall the following Lemmas.

Lemma 1 ([1,29])**.** *Let $g \in R^{\tau}(\mu, \nu)$, then*

$$|a_k| \le (\mu - \nu)\frac{|\tau|}{k}, \quad k = 2, 3, \cdots.$$

Lemma 2. *Let $g(z) = z - \sum_{k=2}^{\infty} |a_k| z^k \in T$. If*

$$\sum_{k=2}^{\infty} k[k]_q(\lambda k - \lambda + 1)\left[k - 1 + \frac{A-B}{1+|B|}\right]|a_k| \le \frac{A-B}{1+|B|} \quad (z \in D),$$

where $0 \le \lambda \le 1$ and $-1 \le B < A \le 1$, then $g \in TX_q[\lambda, A, B]$.

Lemma 2 can be proved by using the same way as in [13].

In this article, we will derive certain properties of analytic functions in $TX_q[\lambda, A, B]$. Some known results are also generalized.

In order to facilitate our calculations and proofs, we derive several identities which hold for $m \ge 1$ and $0 \le s < 1$ as the following:

$$\sum_{k=0}^{\infty}\binom{m-1+k}{m-1}s^k = (1-s)^{-m},$$

$$\sum_{k=2}^{\infty}\binom{m-2+k}{m-1}s^{k-1} = \sum_{k=0}^{\infty}\binom{m-1+k}{m-1}s^k - 1 = (1-s)^{-m} - 1, \tag{10}$$

$$\sum_{k=2}^{\infty}(k-1)\binom{m-2+k}{m-1}s^{k-1} = sm\sum_{k=0}^{\infty}\binom{m+k}{m}s^k = s\binom{m}{m-1}(1-s)^{-(m+1)}, \tag{11}$$

$$\sum_{k=3}^{\infty}(k-2)(k-1)\binom{m-2+k}{m-1}s^{k-1} = 2s^2\binom{1+m}{m-1}(1-s)^{-(m+2)}, \tag{12}$$

$$\sum_{k=4}^{\infty}(k-3)(k-2)(k-1)\binom{m-2+k}{m-1}s^{k-1} = 6s^3\binom{2+m}{m-1}(1-s)^{-(m+3)}, \tag{13}$$

and

$$\sum_{k=5}^{\infty}(k-4)(k-3)(k-2)(k-1)\binom{m-2+k}{m-1}s^{k-1} = 24s^4\binom{3+m}{m-1}(1-s)^{-(m+4)}. \tag{14}$$

2. Main Results

Theorem 1. *Let $m \ge 1$. If*

$$\Psi(m, \lambda, s, A, B) \le \frac{A-B}{1+|B|}, \tag{15}$$

then the function Φ_s^m belongs to $TX_q[\lambda, A, B]$, where

$$\begin{aligned}\Psi(m, \lambda, s, A, B) :=\ & 24\lambda\frac{\binom{3+m}{m-1}s^4}{(1-s)^{m+4}} + 6\left[1 + \lambda\left(\frac{A-B}{1+|B|} + 8\right)\right]\frac{\binom{2+m}{m-1}s^3}{(1-s)^{m+3}} \\ & + 2\left[5 + \frac{A-B}{1+|B|} + \lambda\left[5\left(\frac{A-B}{1+|B|}\right) + 14\right]\right]\frac{\binom{1+m}{m-1}s^2}{(1-s)^{m+2}} \\ & + \left[4 + 3\left(\frac{A-B}{1+|B|}\right) + 4\lambda\left[1 + \frac{A-B}{1+|B|}\right]\right]\frac{\binom{m}{m-1}s}{(1-s)^{1+m}}.\end{aligned}$$

Proof. From Lemma 2 and (8), we need to show that

$$P_q := \sum_{k=2}^{\infty} [k]_q k \left[\frac{A-B}{1+|B|} + k - 1 \right] (\lambda k + 1 - \lambda) \binom{k+m-2}{m-1} (1-s)^m s^{k-1} \le \frac{A-B}{1+|B|}.$$

□

Letting $q \to 1-$, we obtain from (6) that $P_q \le P_1$ and

$$\begin{aligned} P_1 &= \sum_{k=2}^{\infty} k^2 \left[k - 1 + \frac{A-B}{1+|B|} \right] (\lambda k + 1 - \lambda) \binom{k+m-2}{m-1} (1-s)^m s^{k-1} \\ &= \sum_{k=2}^{\infty} \left[k^3 + \left(\frac{A-B}{1+|B|} - 1 \right) k^2 \right] (\lambda k + 1 - \lambda) \binom{k+m-2}{m-1} (1-s)^m s^{k-1} \\ &= \sum_{k=2}^{\infty} \left\{ \lambda k^4 + \left[1 - \lambda \left(2 - \frac{A-B}{1+|B|} \right) \right] k^3 - (1-\lambda) \left(1 - \frac{A-B}{1+|B|} \right) k^2 \right\} \binom{m-2+k}{m-1} (1-s)^m s^{k-1}. \end{aligned}$$

By using the identities

$$k^2 = 1 + 3(k-1) + (k-2)(k-1), \tag{16}$$

$$k^3 = 1 + 7(k-1) + 6(k-2)(k-1) + (k-3)(k-2)(k-1), \tag{17}$$

$$k^4 = 1 + 15(k-1) + 25(k-2)(k-1) + 10(k-3)(k-2)(k-1) + (k-4)(k-3)(k-2)(k-1) \tag{18}$$

and (10)–(13), we have

$$\begin{aligned} P_1 &= \sum_{k=2}^{\infty} \left\{ \lambda k^4 + \left[1 - \lambda \left(2 - \frac{A-B}{1+|B|} \right) \right] k^3 + (\lambda - 1) \left(1 - \frac{A-B}{1+|B|} \right) k^2 \right\} \binom{m-2+k}{m-1} s^{k-1} (1-s)^m \\ &= \lambda \sum_{k=5}^{\infty} (k-4)(k-3)(k-2)(k-1) \binom{m-2+k}{m-1} s^{k-1} (1-s)^m \\ &+ \left\{ 10\lambda + \left[1 - \lambda \left(2 - \frac{A-B}{1+|B|} \right) \right] \right\} \sum_{k=4}^{\infty} (k-3)(k-2)(k-1) \binom{m-2+k}{m-1} s^{k-1} (1-s)^m \\ &+ \left\{ 25\lambda + 6 \left[1 - \lambda \left(2 - \frac{A-B}{1+|B|} \right) \right] + (\lambda - 1) \left(1 - \frac{A-B}{1+|B|} \right) \right\} \cdot \\ &\sum_{k=3}^{\infty} (k-2)(k-1) \binom{m-2+k}{m-1} s^{k-1} (1-s)^m \\ &+ \left\{ 15\lambda + 7 \left[1 - \lambda \left(2 - \frac{A-B}{1+|B|} \right) \right] + 3(\lambda - 1) \left(1 - \frac{A-B}{1+|B|} \right) \right\} \cdot \\ &\sum_{k=2}^{\infty} (k-1) \binom{m-2+k}{m-1} s^{k-1} (1-s)^m \\ &+ \left\{ \lambda + \left[1 - \lambda \left(2 - \frac{A-B}{1+|B|} \right) \right] + (1-\lambda) \left(\frac{A-B}{1+|B|} - 1 \right) \right\} \sum_{k=1}^{\infty} \binom{m-2+k}{m-1} s^{k-1} (1-s)^m \\ &= 24\lambda \frac{\binom{3+m}{m-1} s^4}{(1-s)^4} + 6 \left[1 + \lambda \left(\frac{A-B}{1+|B|} + 8 \right) \right] \frac{\binom{2+m}{m-1} s^3}{(1-s)^3} \\ &+ 2 \left[5 + \frac{A-B}{1+|B|} + \lambda \left(5 \left(\frac{A-B}{1+|B|} \right) + 14 \right) \right] \frac{\binom{1+m}{m-1} s^2}{(1-s)^2} \\ &+ \left[4 + 3 \left(\frac{A-B}{1+|B|} \right) + 4\lambda \left(\frac{A-B}{|B|+1} + 1 \right) \right] \frac{\binom{m}{m-1} s}{1-s} + \left(\frac{A-B}{|B|+1} \right) (1 - (1-s)^m) \\ &= (1-s)^m \Psi(m, \lambda, s, A, B) + \left(\frac{A-B}{|B|+1} \right) (1 - (1-s)^m). \end{aligned}$$

Now, we can find that $P_1 \le \frac{A-B}{1+|B|}$ if (15) holds true. This proves the Theorem.

According to Theorem 1, the following Corollaries are derived.

Corollary 1. *Let $m \geq 1$. If*

$$6\frac{\binom{2+m}{m-1}s^3}{(1-s)^{m+3}} + 2\left[\frac{A-B}{1+|B|} + 5\right]\frac{\binom{1+m}{m-1}s^2}{(1-s)^{m+2}} + \left[3\left(\frac{A-B}{1+|B|}\right) + 4\right]\frac{\binom{m}{m-1}s}{(1-s)^{m+1}} \leq \frac{A-B}{1+|B|},$$

then the function Φ_s^m belongs to $TX_q[0, A, B]$.

Corollary 2. *Let $m \geq 1$. If*

$$24\lambda\frac{\binom{3+m}{m-1}s^4}{(1-s)^{4+m}} + 6(1+\lambda(9-\alpha))\frac{\binom{2+m}{m-1}s^3}{(1-s)^{3+m}}$$
$$+ 2(6-\alpha+\lambda(19-5\alpha))\frac{\binom{1+m}{m-1}s^2}{(1-s)^{2+m}} + (7-3\alpha+4\lambda(2-\alpha))\frac{\binom{m}{m-1}s}{(1-s)^{1+m}} \leq 1-\alpha,$$

then the function Φ_s^m belongs to $TX_q[\lambda, 1-2\alpha, -1]$, where $\alpha \in [0, 1)$.

Remark 1. *Making $\lambda = 0$ in Corollary 2, we have a result given by Frasin and Darus in [28].*

Theorem 2. *Suppose that $m \geq 1$ and $f \in R^\tau(\mu, \nu)$. If*

$$\Phi(\lambda, \mu, \nu, \tau, m, s, A, B) \leq \frac{A-B}{|B|+1}, \tag{19}$$

then $I_s^m f \in TX_q[\lambda, A, B]$, where the operator I_s^m is given by (9) and

$$\Phi(\lambda, \mu, \nu, \tau, m, s, A, B) := (\mu-\nu)|\tau|\Bigg[6\lambda\frac{\binom{2+m}{m-1}s^3}{(1-s)^3} + 2\left[1+\lambda\left(\frac{A-B}{1+|B|}+4\right)\right]\frac{\binom{1+m}{m-1}s^2}{(1-s)^2}$$
$$+\left[1+\left(\frac{A-B}{1+|B|}+1\right)(2\lambda+1)\right]\frac{\binom{m}{m-1}s}{1-s} + (1-(1-s)^m)\frac{A-B}{1+|B|}\Bigg].$$

Proof. By using Lemma 2, we need only to prove that

$$Q_q := \sum_{k=2}^{\infty}[k]_q k\left[\frac{A-B}{|B|+1} + k - 1\right](\lambda k - \lambda + 1)\binom{k+m-2}{m-1}(1-s)^m s^{k-1}|a_k| \leq \frac{A-B}{|B|+1}.$$

□

Since $f \in R^\tau(\mu, \nu)$, according to Lemma 1, we know that $|a_n| \leq (\mu-\nu)\frac{|\tau|}{n}$, $n \in N \setminus \{1\}$. Letting $q \to 1-$, we find from (6) that $Q_q \leq Q_1$ and

$$Q_1 \leq (\mu-\nu)|\tau|\sum_{k=2}^{\infty}k\left[k-1+\frac{A-B}{|B|+1}\right](\lambda k - \lambda + 1)\binom{k+m-2}{m-1}(1-s)^m s^{k-1}$$
$$= (\mu-\nu)|\tau|\sum_{k=2}^{\infty}\left[k^2 + k\left(\frac{A-B}{|B|+1} - 1\right)\right](\lambda k + 1 - \lambda)\binom{m+k-2}{m-1}(1-s)^m s^{k-1}$$
$$= (\mu-\nu)|\tau|\sum_{k=2}^{\infty}\left\{\lambda k^3 + k^2\left[1-\lambda\left(2-\frac{A-B}{|B|+1}\right)\right]\right.$$
$$\left.+(1-\lambda)k\left(\frac{A-B}{|B|+1} - 1\right)\right\}\binom{m+k-2}{m-1}(1-s)^m s^{k-1}.$$

By considering $k = 1 + (k-1)$, (16) and (17), we get

$$\begin{aligned}
Q_1 &\leq (\mu-\nu)|\tau| \sum_{k=2}^{\infty} \left[\lambda k^3 + k^2 \left[1 - \lambda \left(2 - \frac{A-B}{|B|+1} \right) \right] \right. \\
&\quad \left. + k(\lambda-1) \left(1 - \frac{A-B}{|B|+1} \right) \right] \left(\begin{array}{c} m-2+k \\ m-1 \end{array} \right) (1-s)^m s^{k-1} \\
&= (\mu-\nu)|\tau| \left\{ \sum_{k=4}^{\infty} (k-3)(k-2)(k-1)\lambda \left(\begin{array}{c} m-2+k \\ m-1 \end{array} \right) (1-s)^m s^{k-1} \right. \\
&\quad + \left[6\lambda + \left(1 - \lambda \left(2 - \frac{A-B}{|B|+1} \right) \right) \right] \sum_{k=3}^{\infty} (k-2)(k-1) \left(\begin{array}{c} m-2+k \\ m-1 \end{array} \right) (1-s)^m s^{k-1} \\
&\quad + \left[7\lambda + 3\left(1 - \lambda \left(2 - \frac{A-B}{|B|+1} \right) \right) + \left(1 - \frac{A-B}{|B|+1} \right) (\lambda-1) \right] \cdot \\
&\quad \sum_{k=2}^{\infty} (k-1) \left(\begin{array}{c} m-2+k \\ m-1 \end{array} \right) s^{k-1} (1-s)^m \\
&\quad + \left[\lambda + \left(1 - \lambda \left(2 - \frac{A-B}{|B|+1} \right) \right) + \left(1 - \frac{A-B}{|B|+1} \right) (\lambda-1) \right] \cdot \\
&\quad \left. \sum_{k=2}^{\infty} (k-1) \left(\begin{array}{c} m-2+k \\ m-1 \end{array} \right) s^{k-1} (1-s)^m \right\} \\
&= (\mu-\nu)|\tau| \left\{ 6\lambda \frac{\left(\begin{array}{c} 2+m \\ m-1 \end{array} \right) s^3}{(1-s)^3} + 2\left(\lambda \left(\frac{A-B}{|B|+1} + 4 \right) + 1 \right) \frac{\left(\begin{array}{c} 1+m \\ m-1 \end{array} \right) s^2}{(1-s)^2} \right. \\
&\quad \left. + \left((2\lambda+1)\left(\frac{A-B}{|B|+1} + 1 \right) + 1 \right) \frac{\left(\begin{array}{c} m \\ m-1 \end{array} \right) s}{1-s} + (1-(1-s)^m) \frac{A-B}{|B|+1} \right\}.
\end{aligned}$$

Thus we obtain that $Q_1 \leq \frac{A-B}{1+|B|}$ if (19) holds true. This proves the Theorem.

According Theorem 2, we have the following corollary.

Corollary 3. *Suppose that $m \geq 1$ and $f \in R^{\tau}(\mu,\nu)$. If the inequality*

$$(\mu-\nu)|\tau| \left[\frac{2\left(\begin{array}{c} 1+m \\ m-1 \end{array} \right) s^2}{(1-s)^2} + \left(2 + \frac{A-B}{|B|+1} \right) \frac{\left(\begin{array}{c} m \\ m-1 \end{array} \right) s}{1-s} + (1-(1-s)^m) \frac{A-B}{|B|+1} \right] \leq \frac{A-B}{|B|+1}$$

holds true, then $I_s^m f \in TX_q[0,A,B]$, where I_s^m is given by (9).

Theorem 3. *Let $m \geq 1$ and the function Γ_s^m be given by*

$$\Gamma_s^m(z) = \int_0^z \frac{\Phi_s^m(w)}{w} dw \quad (z \in D).$$

If

$$\Theta(m,\lambda,s,A,B) \leq \frac{A-B}{1+|B|}, \tag{20}$$

then $\Gamma_s^m \in TX_q[\lambda,A,B]$, where

$$\Theta(m,\lambda,s,A,B) := 6\lambda \frac{\begin{pmatrix} 2+m \\ m-1 \end{pmatrix} s^3}{(1-s)^{m+3}} + 2\left[1+\lambda\left(\frac{A-B}{1+|B|}+4\right)\right] \frac{\begin{pmatrix} 1+m \\ m-1 \end{pmatrix} s^2}{(1-s)^{m+2}}$$

$$+ \left(1+(2\lambda+1)\left(\frac{A-B}{1+|B|}+1\right)\right\} \frac{\begin{pmatrix} m \\ m-1 \end{pmatrix} s}{(1-s)^{m+1}}.$$

Proof. From (8), we get

$$\Gamma_s^m(z) = \int_0^z \frac{\Phi_s^m(t)}{t} dt = z - \sum_{k=2}^{\infty} k^{-1} \begin{pmatrix} m-2+k \\ m-1 \end{pmatrix} s^{k-1}(1-s)^m z^k \quad (z \in D).$$

□

According to Lemma 2, we find that $\Gamma_s^m \in TX_q[\lambda, A, B]$ if

$$R_q := \sum_{k=2}^{\infty} [k]_q k(\lambda k - \lambda + 1) \left[\frac{A-B}{|B|+1} + k - 1\right] \begin{pmatrix} m-2+k \\ m-1 \end{pmatrix} s^{k-1}(1-s)^m \leq \frac{A-B}{|B|+1}.$$

Now letting $q \to 1-$, we have from (6) that $R_q \leq R_1$ and

$$\begin{aligned} R_1 &= \sum_{k=2}^{\infty} k\left[k-1+\frac{A-B}{|B|+1}\right](\lambda k - \lambda + 1) \begin{pmatrix} m-2+k \\ m-1 \end{pmatrix} (1-s)^m s^{k-1} \\ &= \sum_{k=2}^{\infty} (\lambda k - \lambda + 1)\left[k^2 + \left(\frac{A-B}{|B|+1} - 1\right)k\right] \begin{pmatrix} m-2+k \\ m-1 \end{pmatrix} s^{k-1}(1-s)^m \\ &= \sum_{k=2}^{\infty} \left\{\lambda k^3 + \left[1-\lambda\left(2-\frac{A-B}{|B|+1}\right)\right]k^2 + k(\lambda-1)\left(1-\frac{A-B}{|B|+1}\right)\right\} \begin{pmatrix} m-2+k \\ m-1 \end{pmatrix} (1-s)^m s^{k-1}. \end{aligned}$$

By using the same method as in Theorem 2, we find that $\Gamma_s^m \in TX_q[\lambda, A, B]$ if (20) holds true. Thus, the Theorem is proved.

Corollary 4. *Let $m \geq 1$. If*

$$2\begin{pmatrix} 1+m \\ m-1 \end{pmatrix} s^2(1-s)^{-(m+2)} + \left(2+\frac{A-B}{1+|B|}\right)\begin{pmatrix} m \\ m-1 \end{pmatrix} s(1-s)^{-(m+1)} \leq \frac{A-B}{1+|B|},$$

then $\Gamma_s^m \in TX_q[0, A, B]$.

3. Conclusions

In recent years, many scholars (see, e.g., [30–32]) have been devoted to applications of q-analysis in physics and mathematics. In particular, Srivastava [2] systematically investigated the applications and mathematical explanation of the fractional q-calculus as well as the fractional q-derivative in GFT. In this paper, a new analytic function class $TX_q[\lambda, A, B]$ associated with the q-derivative operator and the Pascal distribution series is introduced and studied. Certain properties of functions in $TX_q[\lambda, A, B]$ are derived.

Scholars may consider some new q-analogous derivative operators and utilize these new operators to introduce new subclasses of analytic functions as potential avenues for future investigation. Also, the concepts given in this paper offer potential for extending to other operators such as the symmetric q-derivative operator. In particular, scholars could consider the symmetric q-derivative operator with differential subordination to define new subclasses of analytic functions.

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