A TEST OF TWO MINIMAX MODELS FOR PREDICTING THE SCALING OF PARTITIONS OF STIMULUS SETS

by

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ABSTRACT

The organizing of a set of objects by partitioning them into groups is an aspect of many familiar tasks. A distinction is drawn between the sorting version of the partitioning tasks in which the basis for sorting a set of objects into groups is explicitly specified and the classification version of the tasks in which no objective is explicitly specified. In contrast to such traditional problems as concept identification in which the partitioning is prescribed by the experimenter and the subject is required to identify the underlying concept, the subject in either a classification or a sort situation has the freedom to partition a set of objects in any way that he chooses.

The present research is directed towards the ultimate goal of developing a model that would predict the partitions made by subjects when sorting objects on the basis of perceived physical similarity. In sorting on the basis of similarity, subjects presumably attempt to maximize the similarity within groups and the difference between groups. This sorting objective is called the "minimax" objective. It is argued that, when sorting on similarity, subjects compare possible partitions in terms of the degree to which they meet the minimax objective. Two competing models were

developed to predict subjects' judgments of the degree to which a given partition of the stimulus objects meet the minimax objective. The proposed models are not models of sorting but models of the judgmental processes that are assumed to be involved in the process of sorting.

The specific purpose of the two dissertation experiments was, first, to test whether subjects can make minimax judgments and, second, to test the two models of the judgmental process. Although both judgmental models could, with the addition of supplementary assumptions, have been tested in a sorting situation, the experiments provided a more direct test of the judgmental assumptions. The relevance of these experiments and models to the problem of sorting is discussed in the dissertation. In each experiment, subjects were required, first, to judge the similarity between all possible pairs of objects and, second, to judge the degree of similarity within and difference between groups for each of a number of selected partitions. Consistency was high among subjects both in their judgments of similarity and in their judgments of minimax. On the basis of the pairwise similarity judgments, predictions of the minimax judgments for partitions were generated from each model and compared with the corresponding minimax judgments actually made by the subjects. The weighted-means model was supported to a degree over the unweighted-means model; the informal verbal reports of subjects tended to confirm this superiority of the weighted model. However, both models were very good predictors of the obtained minimax judgments.

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INTRODUCTION

The organizing of a set of objects by partitioning them into groups is an aspect of many familiar tasks. To remember a shopping list one might group the items on the basis of the type of product or the location in a store. Likewise in organizing a home library one might group the books on the basis of subject matter, book size, author, etc.

In recent years this kind of organizing activity has attracted the attention of experimental psychologists. It is, of course, a kind of inverse of the long-studied process called concept identification. In experiments dealing with concept identification the subject typically is presented with an already classified set of stimuli, and he is asked to discover the rules that account for it (Bruner, Goodnow, & Austin, 1956). The concern of this dissertation is the "opposite" process, where a subject is given the stimuli and asked to group them in any way he chooses. The questions of interest now are: (a) what partitions are actually produced, (b) what rules govern the partitions that are produced, and (c) are these rules derivable from an objective adopted by the subject?

The experimental study of partitioning behaviour has usually employed one of the procedures generated by two dichotomous variables expressed as instructions to the subject (Figure 1). The two variables are (a) the number of groups into which the stimuli must be partitioned and

(b) the objective that the subject must meet in partitioning the set of stimuli. With regard to the former, Garner and his colleagues (Imai, 1966; Imai & Garner, 1965) distinguished between free and restricted classification. With regard to the latter, a distinction must be made between sorting and classification tasks. Although the distinction between the sort and classification tasks is not made clearly in the literature, it is a distinction that is important to make. This distinction is made here.

Under the free-classification condition the subject is required to classify a set of stimuli into an unspecified number of groups, and no basis for classifying is stated by the experimenter. An experiment described by Garner (1974, p. 103) provides an example of free-classification instructions. In this study each subject was "simply given a set of stimuli and told to divide them into as many classes as he likes, and of whatever sizes." Under the free-sort condition the subject is instructed to sort the stimuli into an unspecified number of groups, and to do so according to a specified objective. In a study reported by Miller (1969), the subjects were instructed to sort words into groups "on the basis of similarity of meaning." Under the restricted-classification condition the subject is instructed to classify the stimuli into a specified number of groups, and no basis for classifying is stated. Mandler (1967), in his Experiment B, instructed his subjects to use between two and seven groups in classifying a set of words, and no

		NUMBER OF GROUPS	
		Any number acceptable	A specified number
OBJECTIVE OF PARTITIONING	On any basis	FREE CLASSIFICATION	RESTRICTED CLASSIFICATION
	On a specified basis	FREE SORT	RESTRICTED SORT

Figure 1. Proposed nomenclature for classification tasks.

basis for the classification was provided. Under the <u>restricted-sort</u> condition the subject is required to sort the stimuli into a specified number of groups, and to do so according to a specified objective. In his second experiment, Anglin (1970, p. 30) instructed his subjects to sort 20 words into exactly four piles "on the basis of similarity of meaning."

It is clear that the essential difference between the classification and the sort tasks lies in the freedom that the subjects have in deciding the basis for their partitioning. In both the free-sort and the free-classification tasks, subjects have the freedom to choose the number and the composition of groups, but only the free-classification subjects have the additional freedom to select the objective for their classification. In both the restricted-sort and the restricted-classification tasks, subjects have the freedom to place a given stimulus in any one of the specified number of groups, but only the restricted-classification subjects have the additional freedom to select their own objective.

It is proposed that in classification situations subjects always adopt an objective, although the objective chosen need not be the same for each subject. Therefore, sorting and classification are equivalent tasks if the subjects in a classification situation happen to adopt the objective explicitly specified in the sort instructions.

Investigations of sorting will be relevant to the problem of classification, particularly if classifiers adopt the specified sorting objective. However, there are two difficulties associated with studying classification which are not associated with studying sorting: first, the objective adopted by each subject in a particular classification situation is unknown to the experimenter and, second, even in the same classification situation subjects may adopt different objectives. Mandler's (1967) difficulties with classification tasks are typical. In Experiment A, for example, 26% of his subjects classified a set of 100 words on the basis of physical similarity (i.e., alphabetical sorting), and the other subjects apparently sorted on similarity of meaning. Mandler was forced to discard all the non-content sorters from his analysis, for the analysis presupposed that all subjects used content categories. Thus the basic classification instruction may allow several different interpretations on the part of the classifier.

The sort and classification tasks have been employed extensively for such uses as diagnosing brain damage, assessing developmental stages of cognitive functioning, assessing dimensional dominance, studying the organization of lexical information in memory, and investigating the category-recall function. A form of classification task, the Weigl-Goldstein-Scheerer Object Sorting Task, has served as a diagnostic test in neuropsychological studies of brain damage (Goldstein & Scheerer, 1941). Vygotsky (1934 / 1962) employed the free-classification task to identify stages in the cognitive development of children. Different types of classification were held to reflect different stages in the development of the cognitive ability of abstraction. Following a suggestion by Trabasso and Bower (1968, p. 172), Ozioko (1976) employed the method of free classification to assess dimensional dominance, which is the tendency of subjects to attend to a particular property or dimension of a stimulus such as colour or shape. The order in which subjects employed the different dimensions as classification rules in successive free classifications of a set of objects was taken to represent the hierarchy of preferences among the dimensions. The method of sorting has been used to study how lexical information might be organized and stored in memory. Miller (1969) required judges to sort a set of words into groups on the basis of similarity in meaning, and the proportion of judges placing a given pair of words into the same group was taken as a measure of proximity. He then investigated patterns of organization in the pooled data matrix of proximities. For Mandler (1967) also, the subjective organization of verbal material in memory was held to be reflected in the classifications of words made by subjects. Assuming that organization determines recall, Mandler inferred that the number of words recalled should be a direct function of the number of categories into which subjects classify the set of stimulus words.

Whereas the above investigations have been concerned with the utilization of the partitions in a variety of ways, Garner and his colleagues (Garner, 1974; Handel, 1967; Handel & Imai, 1972; Handel & Preusser, 1969; Imai, 1966; Imai & Garner, 1968; Preusser & Handel, 1970) have been particularly concerned with discovering the rules that govern the formation of classifications by subjects. From their investigations, Garner and his colleagues have concluded that a number of different factors govern classification behaviour--dimensional preferences, number of stimuli, number of groups, the set of stimuli, and others. There have been no corresponding investigations of sorting behaviours.

The present research was directed towards the ultimate goal of developing a model that could predict the partitions made by subjects in sorting objects on the basis of perceived physical similarity. It can be argued that, in sorting on the basis of similarity, subjects attempt to maximize the similarity within groups and the difference between groups. This particular sorting objective will be called the "minimax" objective in this dissertation. During the process of sorting, sorters would have to make judgments of the degree to which various partitions of the stimulus set met the minimax objective. It was assumed that an adequate model of sorting on the basis of similarity would have to incorporate assumptions that predict how minimax judgments are made. In this dissertation two competing models, described in a subsequent section, are proposed to predict subjects' judgments of the degree to which a partition of objects meets the minimax objective. The proposed models are models of the judgmental processes that are assumed to be involved in the process of sorting. It must be emphasized, however, that the proposed models are _not_ models of sorting. To predict sorting behaviour, the models must be supplemented by other assumptions. For example, the sorts that subjects make can be predicted by one of the proposed judgmental models together with the assumption that, when sorting, subjects consider all possible partitions of the stimulus set.

The specific purpose of the dissertation experiments was, first, to test whether subjects can make minimax judgments and, second, to test the two competing models of the judgmental process. Thus, these experiments did _not_ employ sorting tasks. Although both judgmental models could, with the addition of supplementary assumptions, have been tested in a sorting situation, the dissertation experiments provided a more direct test of the judgmental assumptions.

The two experiments that were conducted utilized a scaling method. The procedure was very similar in both experiments. In the first experimental session a subject judged the physical similarity that he perceived between all possible pairs of 16 stimulus objects. In a subsequent session the same subject judged the degree to which objects were similar within and different between groups for 13 (or 16) partitions. Predicted minimax judgments were generated from each model based on either the individual subject's own pairwise similarity judgments or on the averaged pairwise similarity judgments. These predicted judgments were compared with the corresponding minimax judgments actually made by the subjects.

Definitions

The following list of definitions will apply to these terms as they are used in this proposal:

Group. A group of stimuli is a subset of the given set of stimuli.

Partition. A partition of a set of stimuli is a division of the stimuli into disjoint subsets (groups) whose union is the given set. The arrangement of groups and of stimuli within groups is irrelevant.

Classify. To classify a set of stimuli is to partition the set on any basis.

Sort. To sort a set of stimuli is to partition the set according to a specified objective.

Minimax objective. The minimax objective is to maximize both the similarity within groups and difference between groups. The version of this objective that was employed in the dissertation experiments was to minimize the physical similarity perceived between stimulus objects in different groups while simultaneously maximizing the physical similarity perceived among objects within each group.

Optimal sort. The optimal sort of a set of objects is that partition, of all partitions that are possible, in which the similarity within groups is maximized and, simultaneously, the similarity between groups is minimized. A sort is optimal when, of all possible partitions, it has satisfied the minimax objective to the greatest degree.

MODELS

The purpose of this section is to describe in detail two competing models that were developed for predicting the minimax judgments made by subjects. A measure of the degree to which the objects in a given partition are similar within groups and different between groups was derived from each of two sets of plausible assumptions about a subject's perception of within and between-group similarity.

Minimax Model 1: Weighted Means

The weighted-means model specifies the operations subjects are assumed to perform when comparing different partitions of a set of objects on the minimax objective. The model has four major characteristics. First, the direct perception of the physical properties of objects forms the basis for judgments of similarity between pairs of objects. Second, judgments of similarity within and between groups are based on these pairwise similarities. Third, partitions of objects are judged in terms of the extent to which the similarity within groups is maximized relative to the similarity between groups. Fourth, a quantitative measure of minimax is derived from the model. Different partitions can be compared on the minimax dimension.

The subject's judgment of minimax is assumed to be based on three distinct components. The similarity perceived between a pair of objects is the first and most elementary component. A critical assumption of the model is that the pairwise similarities be known for all possible pairs of objects. The empirical measurement of the pairwise object similarities is discussed in a later section titled Similarity Scaling of Object Pairs. The pairwise similarities form the basis for the remaining two components.

The second component of the subject's judgment of minimax is the average similarity within groups, denoted $\bar{S}_{WG}$. It is computed as the average of the similarities for all the within-groups pairs of objects in a given partition. For a set of N objects there are $\binom{N}{2}$ different pairs of objects. In any given partition each pair of objects is either a within-group or a between-group pair. The pair of objects is said to be within groups when both objects are sorted into the same group. When two objects are sorted into different groups, the two objects are said to be a between-groups pair.

The third component, the average between-groups similarity $\bar{S}_{BG}$, is the average of the similarities perceived for all the between-groups pairs of objects in a given partition. The latter two components, $\bar{S}_{WG}$ and $\bar{S}_{BG}$, are determined by the pairwise similarities and the partitioning of a set of objects.

The subject's judgment of minimax is assumed to be based on $\bar{S}_{WG}$ and $\bar{S}_{BG}$. The ratio of $\bar{S}_{WG}$ to $\bar{S}_{BG}$ is taken to be the measure of the degree to which within-groups similarity has been maximized relative to the between-groups similarity, that is, $M1 = \bar{S}_{WG} / \bar{S}_{BG}$, where small values of $\bar{S}_{WG}$ represent high within-groups similarity. The relationships can be expressed mathematically as:

$$(1) \quad \bar{S}_{WG} = \frac{1}{\tilde{N}} \sum_{k=1}^{m} \sum_{i<j} s_{ij,kk} \qquad 1 \le i < j \le n_k;\ n_k > 1;\ 1 \le k \le m$$

$$(2) \quad \bar{S}_{BG} = \frac{1}{\binom{N}{2} - \tilde{N}} \sum_{h<k} \sum_{(i,j)} s_{ij,hk} \qquad 1 \le i \le n_k;\ 1 \le j \le n_h;\ 1 \le h < k \le c;\ c > 1$$

$$(3) \quad M1 = \bar{S}_{WG} / \bar{S}_{BG} \qquad \bar{S}_{BG} > 0$$

where k,h = subscripts denoting groups $\underline{k}$ and $\underline{h}$, respectively;

i,j = subscripts denoting stimulus objects;

c = number of groups in partition;

m = number of groups for which $n_k > 1$ and $1 \le m \le c$;

n_k, n_h = number of stimuli in groups $\underline{k}$ and $\underline{h}$, respectively;

N = total number of objects, $N = \sum_{k=1}^{c} n_k = n_1 + n_2 + \ldots + n_c$;

$\tilde{N}$ = total number of within-groups pairs of objects,

$$\tilde{N} = \sum_{k=1}^{m} \binom{n_k}{2} = \binom{n_1}{2} + \binom{n_2}{2} + \ldots + \binom{n_m}{2} ,$$

where $n_1 \geq n_2 \geq \ldots. \geq n_c$;

$S_{ij,kk}$ = the judged similarity between objects $\underline{i}$ and $\underline{j}$, both of which are in group $\underline{k}$;

$S_{ij,hk}$ = the judged similarity between objects $\underline{i}$ and $\underline{j}$, one object chosen from each of groups $\underline{h}$ and $\underline{k}$, respectively;

$\bar{S}_{WG}$ = the overall average similarity perceived within groups;

$\bar{S}_{BG}$ = the overall average similarity perceived between groups;

M1 = minimax measure for the weighted means model.

To illustrate the computation of M1, consider the following hypothetical example in which the stimulus set consists of four stimuli: red circle (RC), red square (RS), green triangle (GT), and black triangle (BT). For computational purposes, let the pairwise similarities between stimuli $\underline{i}$ and $\underline{j}$, denoted by $\underline{S}_{ij}$, be given by:

Stimulus Pair	Similarity Measure $\underline{S}_{ij}$
GT-BT	1
RC-RS	2
RC-GT	3
RC-BT	3
RS-GT	3
RS-BT	3

where low scores on the $\underline{S}_{ij}$ measure represent the perception of a high degree of similarity between two objects. The GT and BT which differ only in colour are perceived as being the most similar whereas objects differing only in shape (RC and RS) are seen as being less similar.

For these four objects there are, excluding division into four groups or one group, 13 distinguishable partitions. Seven of these partitions are shown in Table 1 along with the computations of M1 (p), the minimax measure for partition p. Each of the remaining six partitions are equivalent on this minimax measure to one of the partitions shown. The partitions are arranged in order from lowest to highest minimax as predicted by the weighted means model. This model predicts that the partition that subjects will judge to have the greatest similarity within groups and least similarity between groups is the first partition, with GT and BT together in one group, RC in a second group, and RS in a third group. In this partition there are five between-groups pairs of objects and one within-groups pair. The within-groups pair, GT and BT, has a similarity of 1 and, therefore, the average within-groups similarity $\bar{S}_{WG}$ also equals 1. The five between-groups pairs have similarities of 2, 3, 3, 3, and 3 for an average of 14/5 = 2.80, i.e., $\bar{S}_{BG} = 2.80$. The degree to which the similarity within groups is maximized relative to the similarity between groups is given by M1 = 1.00/2.80 = .36 for this partition.

Table 1

Minimax Judgments Predicted by the Weighted and Unweighted Means Models for All Possible Partitions of a Set of Objects

Partition Description[a]	Model					
	Weighted			Unweighted		
	$\bar{S}_{WG}$	$\bar{S}_{BG}$	M1	$\bar{S}_{WG}$	$\bar{S}_{BG}$	M2
(GT,BT)(RS),(RC)	1.00	2.80	.36	1.00	2.67	.38
(RC,RS),(GT,BT)	1.50	3.00	.50	1.50	3.00	.50
(RC,RS),(GT),(BT)	2.00	2.60	.77	2.00	2.33	.86
(GT,BT,RC),(RS)	2.33	2.67	.88	2.33	2.67	.88
(RC,RS,BT),(GT)	2.67	2.33	1.14	2.67	2.33	1.14
(BT,RS),(GT),(RC)	3.00	2.40	1.25	3.00	2.50	1.20
(BT,RS),(RC,GT)	3.00	2.25	1.33	3.00	2.25	1.33

[a]A partition description consists of an enumeration of all objects plus a specification of how the objects are divided into groups. Objects enclosed within parentheses are in the same group; objects separated by parentheses are in different groups. G = green, R = red, B = black, C = circle, T = triangle, S = square.

Also shown in Table 1 are the minimax computations for the unweighted model. Further discussion of the table is deferred until after the second model is presented.

<u>Minimax Model 2: Unweighted Means</u>

In the weighted model each within-groups pair of objects contributes directly to the measure of overall similarity within groups. The average is taken directly over the similarity judgments for all such pairs. In much the same way each between-groups pair of objects contributes directly to the measure of overall similarity between groups. The subject's perception of overall similarity, either within or between groups, may be a direct function, not of the pairwise similarities, but of the perceived similarity within and between individual groups.

The unweighted-means model assumes that the pairwise similarities contribute only indirectly to judgments of overall similarity within or between groups. Instead, the subject is assumed to base his judgments of overall similarity within groups on the average similarity perceived within each group. The similarity within each group is, in turn, based directly on the pairwise similarities which are averaged. Thus, the overall similarity within groups is based on the average of <u>average</u> within-groups similarities in which each group C_k contributes to the final index a single number obtained by averaging all similarities within

C_k. The overall similarity between groups is computed as the average of the similarity perceived between all pairs of groups. The similarity between any two groups is the average of the judged similarity for all pairs of objects chosen one from each group. Mathematically these relationships can be expressed as:

$$(4) \qquad \bar{S}_{WG} = \frac{1}{m} \sum_{k=1}^{m} \left(\frac{1}{\binom{n_k}{2}} \sum_{i<j} S_{ij,kk} \right) \qquad \begin{array}{l} 1 \le i < j \le n_k \\ n_k > 1 \\ 1 \le k \le m \end{array}$$

$$(5) \qquad \bar{S}_{BG} = \frac{1}{\binom{c}{2}} \sum_{h<k} \left(\frac{1}{n_h \; n_k} \sum_{(i,j)} S_{ij,hk} \right) \qquad \begin{array}{l} 1 \le i \le n_k \\ 1 \le j \le n_h \\ 1 \le h < k \le c \end{array}$$

$$(6) \qquad M2 = \bar{S}_{WG} / \bar{S}_{BG} \qquad \bar{S}_{BG} > 0$$

where M2 = the minimax measure for the unweighted model and the other terms are as defined for Equations (1) to (3).

The difference between the perceptual elements in the two models is reflected in the contribution made by each group of objects to the index of average within-groups similarity. In the weighted model each group in a partition contributes in direct proportion to the number of objects,

n_k, that it contains (that is, group C_k directly contributes $\binom{n_k}{2}$ similarity comparisons to the final index). In the unweighted means model each group contributes a single number to the final index. It can be seen that for the unweighted model the final index does not depend on the group sizes $n_1, \ldots, n_c$, in the sense that each group is weighted equally. This invariance property does not hold for the weighted model.

Except for this difference in perceptual elements the two minimax models are identical. Both models represent the subject as ultimately basing his judgments of minimax on the pairwise comparisons. For both, the judgment of minimax is based directly on the perception of overall similarity, within and between.

Illustrative computations of the minimax measure M2 are presented in Table 1. The value of M2 for the first partition, for example, is obtained in the following way: with only one pair of objects, $n_1 = 2$, the average similarity between pairs which are in group 1 is just the similarity between GT and BT, i.e., 1.00/1 = 1.00. The remaining two groups are ignored in computing the overall average within-groups similarity, $\bar{S}_{WG}$; they contain no pairs of objects. Therefore, $\bar{S}_{WG} = 1.00/1 = 1.00$. The average similarity between groups 1 and 2 is the average of the similarities between the pairs, GT-RS and BT-RS: (3+3)/2 = 3.00. The average similarity between groups 1 and 3 is (3+3)/2 = 3.00, and between groups 2 and 3 is 2.00/1 = 2.00. The overall average similarity between groups equals (3+3+2)/3 = 2.67. The measure of minimax for partition 1, M2 (1), is given by the ratio $\bar{S}_{WG} / \bar{S}_{BG}$ = 1.00/2.67 = .38.

A comparison of M1 and M2 for the partitions in Table 1 indicates that the two models generate identical predicted rankings of the partitions on the minimax dimension. Furthermore, the intervals between successive partitions are almost identical for the two theoretical measures of minimax. Predictions generated by the two models are virtually indistinguishable for this set of objects when the pairwise similarities are those given on page 14.

To summarize, both models represent the subject as operating on the pairwise similarities to make estimates of the similarity within and between groups for a given partition. They differ, however, in the way subjects are characterized as operating upon those pairwise similarities. In the weighted-means model, the subject is pictured as making a global or overall judgment of within-group similarity in which each within-group pair contributes equally and directly. In the unweighted-means model , by contrast, the subject is conceived as weighting some within-groups pairs much more heavily than others--in particular, each group contributes equally and directly to the overall perception of within-group similarity. A corresponding distinction between models is made for the similarity between groups.

EXPERIMENT 1

Two measures of the degree to which a given partition meets the minimax objective were derived, one from each dissertation model. The models should predict subjects' judgments of minimax for all possible partitions of a set of objects. Therefore, the models were tested for selected partitions by comparing the judgments of minimax made by subjects with those predicted by each of the models.

The experiment consisted of two parts: (a) a scaling of the similarity between pairs of objects from the most to the least similar pair and (b) a scaling of a set of partitions on the minimax dimension. The same subjects served in both parts of the experiment. The first part provided the information required by the models to generate the predictions, while the second part provided a test of the models.

This experiment was designed to serve three purposes: first, to assess whether subjects could make minimax judgments (whether or not they correspond to the models' predictions), second, to test the predictions of each model separately, and, third, to compare the models against each other.

Method

Similarity Scaling of Object Pairs

An assumption of both models was that the perceived similarity between all possible pairs of objects in the set be known. The purpose of this part of the experiment was to measure the pairwise similarities as perceived by individual subjects. These measured values were used by the models to generate predictions for the second part of the experiment. Partitions presented in the second part of the experiment were selected so that the predicted difference in minimax between adjacent partitions was as great as possible.

Stimuli. The stimulus objects are illustrated in Figure 2. They consisted of the 16 possible combinations of four colours (yellow Y, red R, green G, and black B) and four shapes (circle C, triangle T, square S, and hexagon H). The areas of the circle, equilateral triangle, square and regular hexagon were 2.39, 1.97, 2.52, 2.70 cm^2 respectively. The areas were chosen to give subjectively equal areas. In Munsell notation (hue/value/chroma), the four colours were 6.5Y/9/11, 5R/6/12, 2.5BG/5.5/8, and N3/.

The large set of 16 stimulus objects was selected to provide a wide variety of possible partitions for the partition scaling. The individual colours and shapes were selected to provide a wide range of perceived similarities.

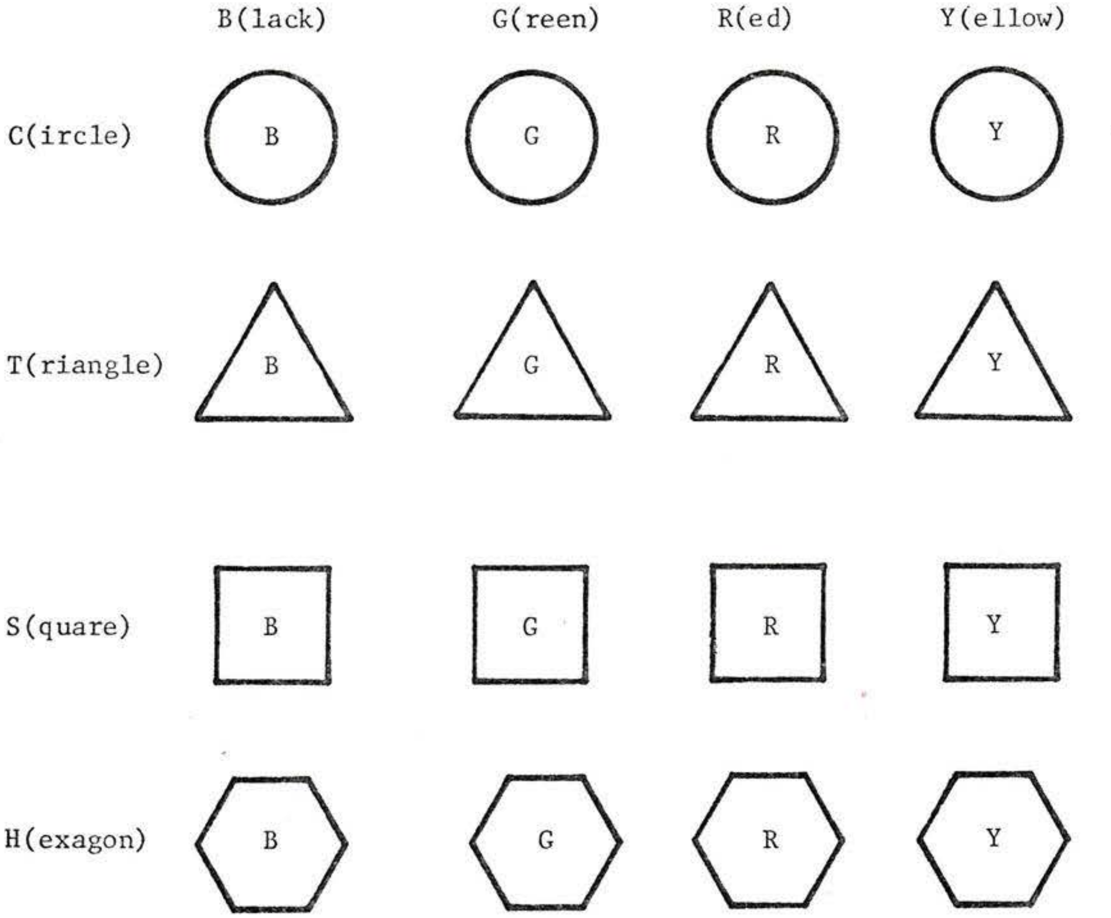

Figure 2. Schematic illustration of the 16 stimulus objects employed in Experiment 1.

The actual stimuli employed in the scaling of objects were the 120 pairs of different objects plus the 16 pairs of identical objects. The stimulus objects were drawn on 7.6 x 12.7 cm white file cards, two objects per card. An example is shown in Figure 3. As the left-right location of each pair of different objects could be reversed on a card, two sets of stimulus cards were constructed. The two sets were identical except that the location of each corresponding pair of objects was reversed. For example, if the pair RC-BT was in one set, then the complementary pair BT-RC was in the other. One set of stimuli was constructed by randomly assigning objects to the left or right positions on a card with the restriction that each object, colour, and shape occurred with equal (or near equal) frequency in each position. The second set of stimuli was generated by constructing the complements (position reversal) of each stimulus in the first set. The 16 pairs of identical objects were added to both sets to make two complete decks of stimuli.

Task. The experimental task was a spatial scaling of the perceived physical similarity of pairs of objects. This procedure was discussed in MacGregor (1975). The task was divided into four phases. First, the subject was permitted to thumb through the deck of 136 objects to familiarize himself with the stimuli to be scaled. Second, he was required to sort the cards into a number of piles that

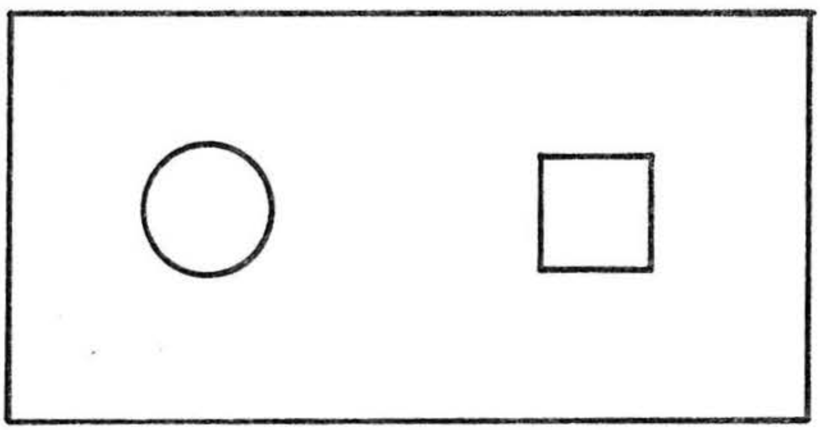

Figure 3: Example of a stimulus employed in the similarity scaling of object pairs in Experiment 1.

differed in the similarity between the two objects on a card--from very similar to one another, to one in which the pairs of objects were very different. Third, within each group he rank ordered the cards from the most to the least similar pair of objects. Fourth, the subject was shown a 457.2 x 71.1 cm rectangular cardboard strip marked off in intervals of 4.2 cm. The pair of objects ranked by the subject as being the most similar was placed on the extreme left interval on the cardboard scale. The subject was instructed to consider this pair of objects as being absolutely identical. He was required to place the remaining cards on the scale such that the distances between the cards reflected the differences in similarity that he perceived between each pair of objects.

Previous investigators (Handel & Imai, 1972; Imai & Garner, 1968) have employed a traditional magnitude estimation procedure. They presented the pairs of objects sequentially and required a subject to judge the similarity of one pair before presenting the succeeding pair. At most only a single pair was ever available to the subject for inspection. Such a procedure can make very heavy demands on a subject's memory, especially as the number of pairs of stimulus objects increases. Their scaling procedure may be insensitive in the sense that it would not be capable of fully reflecting a subject's discriminatory abilities. Under such conditions a subject may simplify the procedure he uses for making judgments. For example, the subjects in Imai and Garner's (1968) experiment judged a circle, a square, and a triangle as being equally similar to one another. However, it is possible that, with several pairs simultaneously available, subjects might have been able to make finer discriminations. For example, a triangle and square might have been judged as more similar to each other (because they are both angular) than either was to a circle.

The spatial scaling procedure has a distinct advantage over Imai and Garner's method. Memory requirements are minimized by making all previously scaled stimuli and the subject's own judgment of similarity for each available for inspection.

The customary procedure (Handel & Imai, 1972; Imai & Garner, 1968) has been to elicit similarity judgments on only the pairs of different objects. In the present experiment the 16 pairs of identical objects were included with the others to provide an anchor point for the similarity judgments. The anchor point served to establish a zero point for the similarity measure. With a zero point the spatial scaling procedure should produce a ratio level of measurement (Torgerson, 1960, p. 261).

Subjects. Eight subjects, selected from among the third and fourth year undergraduate students in a volunteer subject pool, were tested. The 5 female and 3 male subjects were between the ages of 18 and 25.

Procedure. Each subject was randomly assigned one of the two decks of 136 stimuli, with the constraint that four subjects were presented with each deck. The experiment took approximately 2 to 3 hours, and each subject was tested separately in a room with only the experimenter present.

The complete instructions to the subject are given in Appendix A. Because the scaling task was relatively complex, each subject was given a familiarization task. The stimuli for this task consisted of seven index cards with a circle inscribed on each. The diameters of the circles varied from 5.08 to 12.70 cm in 2.54 cm increments. Subjects were asked to rank order the stimuli from smallest to largest and then to lay them on the scale so that the differences between the cards reflected the differences in the perceived areas of the circles.

Minimax Scaling of Partitions

The purpose of this part of the experiment was to provide a test of the models. In this phase of the experiment the subjects were required to scale a selected set of partitions on the minimax dimension. The obtained judgments were compared with the judgments predicted by the models. This large body of metric information was intended to provide a strong, sensitive test of the models.

Stimuli. The stimuli presented to each subject consisted of 13 partitions selected from among all possible partitions of subsets of the 16 objects. Three criteria were used in selecting the partitions. First, partitions were selected such that the differences between partitions in predicted judgments of minimax were as great as possible for each model. The theoretical measures of minimax may be incapable of making very fine discriminations between some partitions, even if the models have captured the essence of the judgmental process. Second, the partitions were selected to produce as large as possible a difference between the theoretical scales predicted by the models. The task of testing between the two models should be simplified if the predicted scales are very different. Third, to justify generalization across size and content of subsets of objects, the partitions were selected from the 16 objects in subsets that varied in content and size.

Each partition was drawn on a 17.7 x 28.2 cm white card. Equally-spaced, vertical, black lines were drawn on each card to separate objects that were in different groups. Objects in the same group were drawn one below another. The objects were drawn as described in the previous section on similarity scaling. The 13 partition stimuli are illustrated schematically in Figures 4, 5, 6, and 7.

Subjects. The same subjects who scaled the object pairs on similarity also scaled the partitions on minimax.

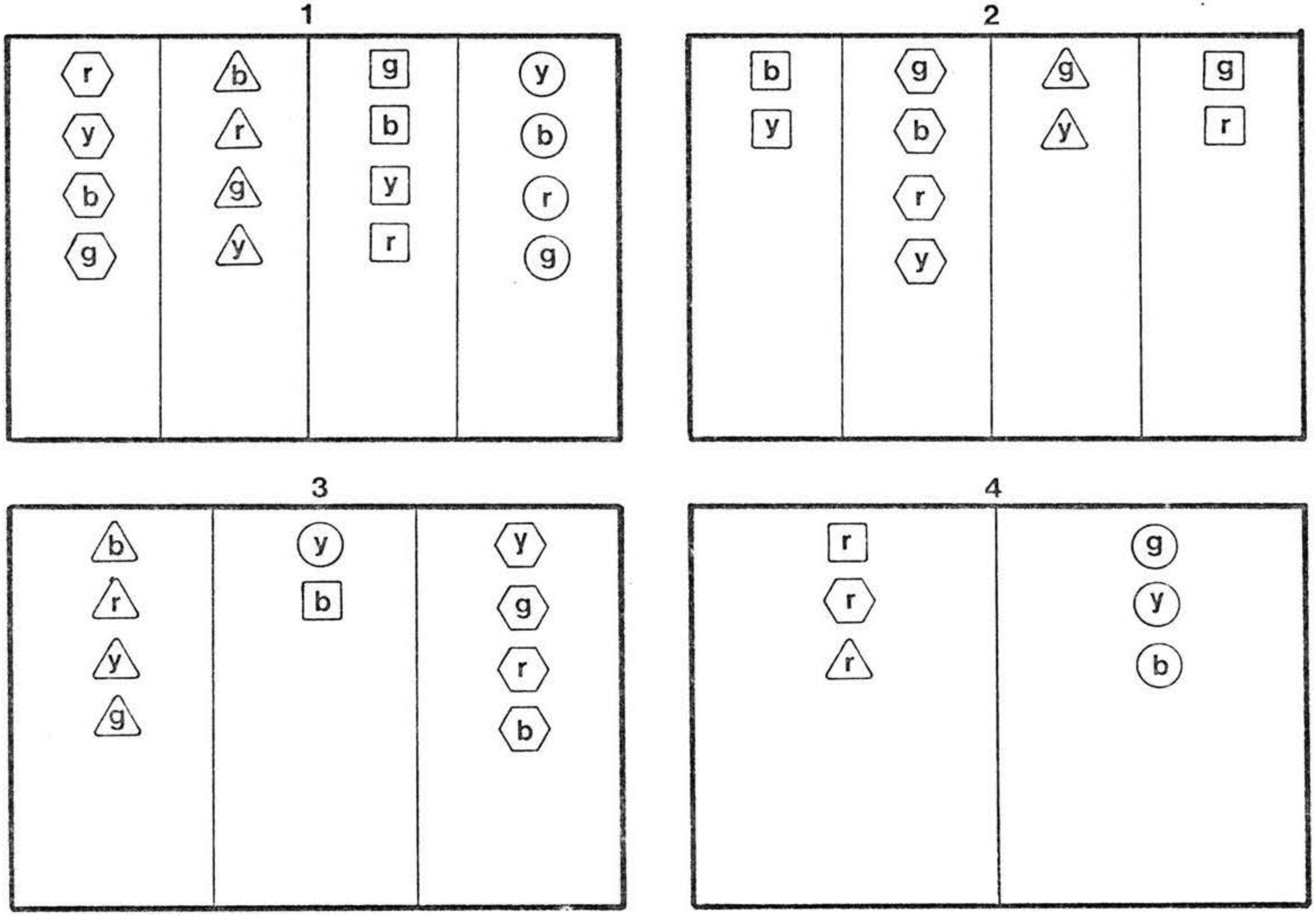

Figure 4. Schematic illustration of the stimuli employed in scaling minimax judgments in Experiment 1: partitions 1-4.

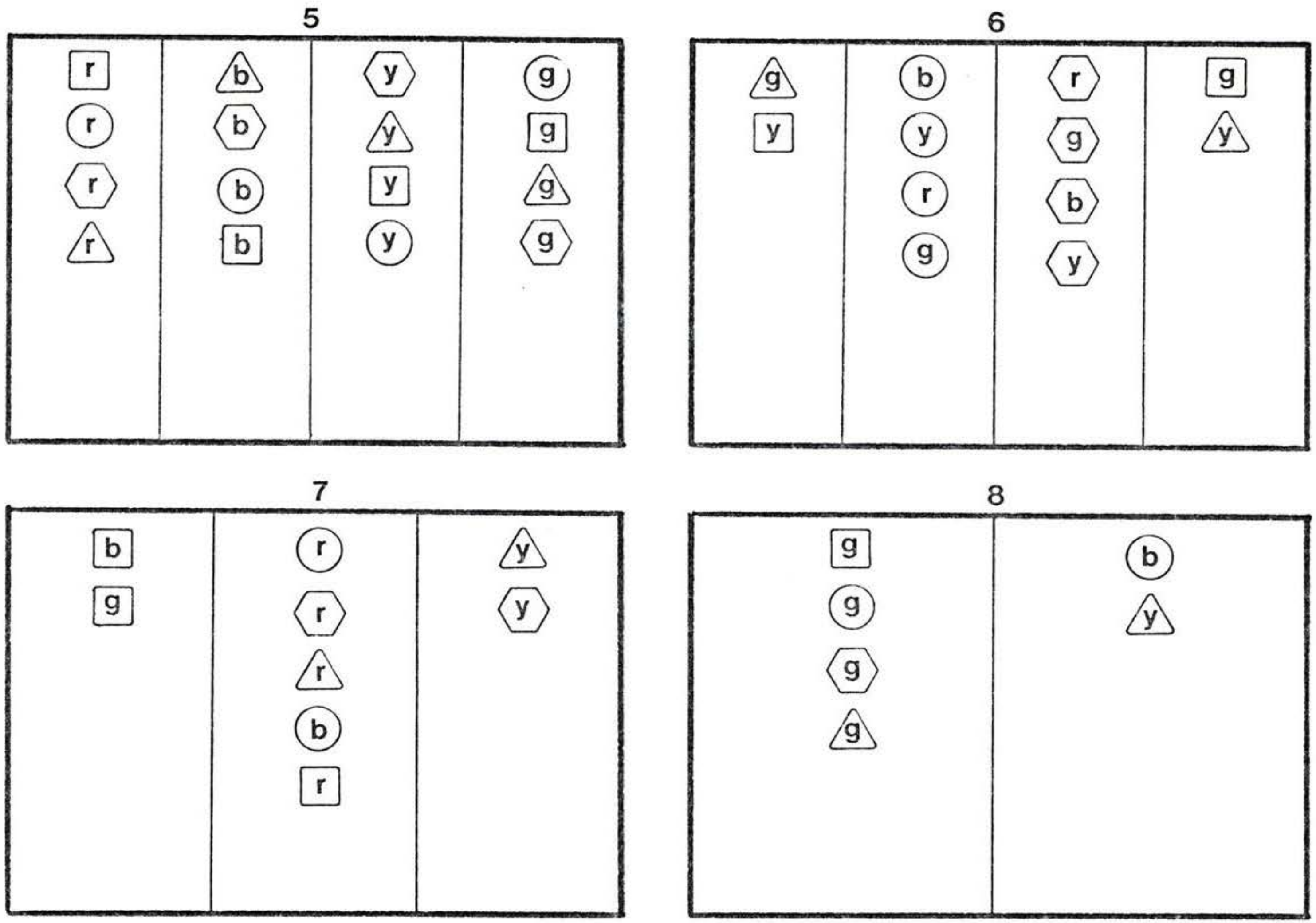

Figure 5. Schematic illustration of the stimuli employed in scaling minimax judgments in Experiment 1: partitions 5-8.

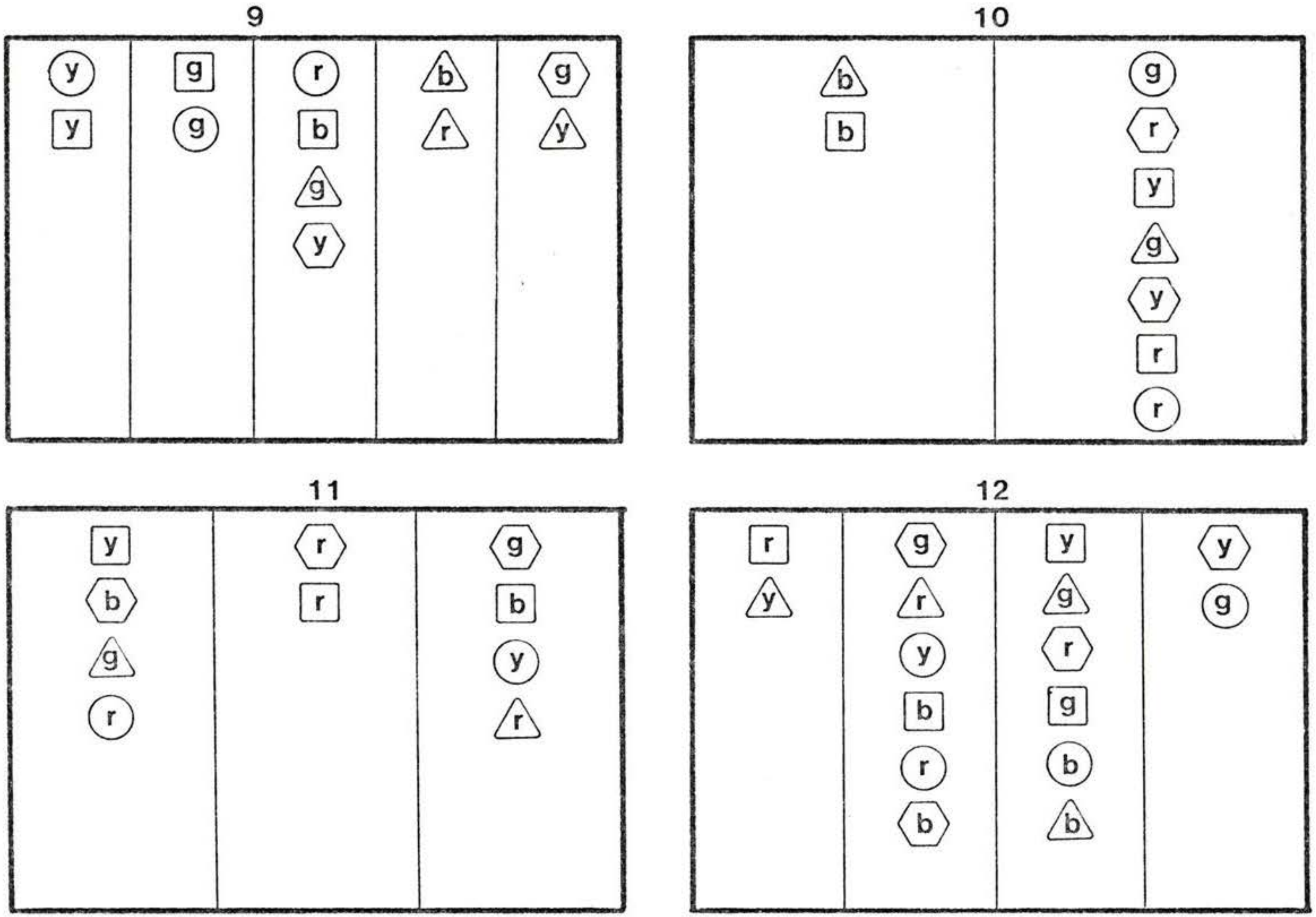

Figure 6. Schematic illustration of the stimuli employed in scaling minimax judgments in Experiment 1: partitions 9-12.

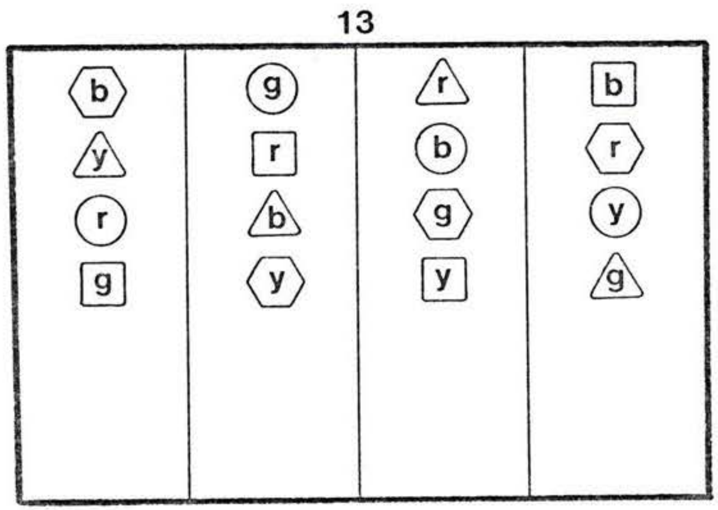

Figure 7. Schematic illustration of the stimuli employed in scaling minimax judgments in Experiment 1: partition 13.

Task. Subjects were asked to scale a set of 13 partitions on the minimax dimension, that is, on the degree to which the similarity within groups is maximized and, simultaneously, the similarity between groups is minimized. The method employed in scaling the partitions is described in the previous section on similarity scaling.

Procedure. The instructions for partition scaling are given in Appendix A. The scaling procedure was the same as that used in the similarity scaling of object pairs except that no familiarization task was given. The experiment took approximately 1 to 2 hours to complete. The time lag between the similarity scaling and the minimax scaling varied between 3-6 weeks.

Results

Similarity Scaling of Object Pairs

Similarity judgments by the eight subjects depended upon the relative weight assigned to differences in shape as opposed to differences in colour. For one subject differences in shape had little effect on judgments of similarity, that is, two objects differing only in shape were judged to be highly similar relative to two objects differing only in colour. In contrast, for another subject differences in colour had almost no effect on judgments of similarity, that is, identical colours were judged to be only marginally more similar than two different colours. The majority of subjects, however, judged shape identities to be only slightly more similar than colour identities. For a given subject the weighting of colour relative to shape was apparent from the effect that changing only the colour (or the shape) of one object had on the judgment of similarity between two identically coloured (shaped) objects. In contrast to the wide variation in the weights assigned to colour and shape across subjects, the intersubject consistency was actually relatively high when measured by the average Pearson correlation between the similarity judgments for the 28 possible pairs of subjects, average $r = .79$, normal $Z = 61.31$, $p \ll .001$; the correlations were averaged by Fisher's r to z transformation (Fisher, 1970, p. 199; McNemar, 1969, p. 158).[1]

The similarity judgments for each subject were transformed to a common range of 100 (see Appendix B for details) and then averaged across subjects to produce the average similarity judgments (see Table 2). The corresponding standard deviations of the averaged similarity judgments are presented in Table 16. In Table 2, the averages for stimulus comparisons having either the same shape or the same colour were computed by averaging over 4 similarity judgments for each of the 8 subjects or over 4 x 8 = 32 judgments in all. In this same table, the averages for stimulus comparisons having both different colours and different shapes were computed by averaging over 2 similarity judgments for each of the 8 subjects or over 2 x 8 = 16 judgments in all. Although the similarity judgments varied from subject to subject, the variation within a subject's 4 (or 2) corresponding similarity judgments was always zero. For example, each of the 8 subjects judged each of the stimulus pairs--GC-RC, GT-RT, GS-RS, GH-RH--to be of equivalent pairwise similarity.

Minimax Scaling of Partitions

The obtained minimax judgments for each individual were first transformed to a common range of 100 (see Table 3) and then those 8 individual scales were intercorrelated. Intersubject consistency in scaling the partitions was high with an average $\underline{r} = .80$, normal $\underline{Z} = 18.69$, $\underline{p} \ll .001$.[2] The obtained minimax judgments in Table 3 were averaged across

Table 2

Average Similarity Judgments of Object Pairs in Experiment 1

Colour	Shape						
	Same	S-H	C-H	T-S	C-S	T-H	C-T
Same	0	29.7	29.9	35.8	36.8	36.8	44.7
G-R	21.6	66.8	73.8	77.4	79.6	78.6	91.2
B-G	21.7	65.3	71.5	71.6	78.4	74.2	83.0
G-Y	22.6	65.9	69.0	71.3	76.6	72.8	83.3
R-Y	23.1	65.2	69.4	70.9	76.2	73.2	84.0
B-R	24.2	71.5	79.5	78.9	85.3	80.7	90.2
B-Y	29.8	78.2	83.0	84.1	85.0	87.3	96.5

NOTE. Each table entry is the average judgment of similarity across 8 subjects for any pair of objects with the shapes and colours specified in the column and row headings, respectively. B = black, G = green, R = red, Y = yellow, C = circle, T = triangle, S = square, H = hexagon.

Table 3

Minimax Judgments for the 13 Partitions for Each Subject in Experiment 1

Subject	Partition Number												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	15.3	15.3	27.8	37.4	0	25.0	52.8	62.5	41.7	89.0	69.4	82.0	100.0
2	0	4.6	12.0	28.7	35.2	16.7	62.0	44.4	60.2	75.0	84.3	90.7	100.0
3	22.7	10.5	13.4	0	22.7	46.5	31.4	33.7	68.6	39.0	83.7	86.0	100.0
4	.7	25.0	23.6	33.0	0	38.9	47.9	45.1	68.1	54.2	77.1	72.6	100.0
5	0	0	10.6	37.2	86.9	21.3	46.1	57.8	77.0	70.6	91.8	97.9	100.0
6	0	41.2	47.1	52.9	88.2	44.1	58.8	94.1	70.6	94.1	76.5	82.4	100.0
7	0	22.4	30.6	11.2	0	35.7	56.1	43.9	62.2	43.9	73.5	81.6	100.0
8	0	26.6	16.5	0	10.1	31.6	81.0	73.4	72.2	62.0	51.9	91.1	100.0

all subjects to give the mean judged minimax (see Table 4). The mean normalized rank, a measure derived from the obtained minimax judgments, is discussed in detail in Garner and Creelman (1970).

On the assumption that all individuals are representatives of the same population, minimax and/or similarity judgments can be averaged across subjects to provide a better estimate of the population mean.

For each model predictions of the minimax judgments were generated from the average similarity judgments and compared with the average of the obtained minimax judgments. The weighted-means model was a good predictor of the mean minimax judgments; the Pearson correlation of .94 between predicted and obtained judgments is obviously significant and accounts for 88% of the variance. The corresponding correlation of .88 for the unweighted-means model is also highly significant. These correlations were not significantly different by Hotelling's (1940) test of the difference between correlated correlations, $t(10) = 1.10$, $p > .10$. The relationships between the mean minimax judgments and model predictions are plotted in Figures 8 and 9.

Two scales such as predicted and obtained minimax judgments may correlate perfectly and yet the predicted values can deviate considerably from the obtained values. The statistic employed to test the significance of the deviation of obtained from predicted minimax judgments (based on the

Table 4

Mean Predicted and Obtained Minimax Judgments for

Each of the Partitions in Experiment 1

Partition Number	Mean Judged Minimax	Mean Normalized Rank	Predictions Based on Average Similarity Judgments	
			Weighted	Unweighted
1	0	0	0	0
2	14.1	11.7	5.6	7.4
3	18.8	20.7	5.6	27.8
4	21.2	19.1	0	0
5	26.9	21.4	18.5	18.5
6	29.1	31.1	11.1	39.8
7	52.2	48.9	31.5	16.7
8	54.7	51.5	31.5	63.9
9	63.3	54.4	69.4	44.4
10	64.3	58.6	42.6	29.6
11	74.8	64.7	98.1	76.9
12	84.8	75.7	75.0	75.9
13	100.0	100.0	100.0	100.0

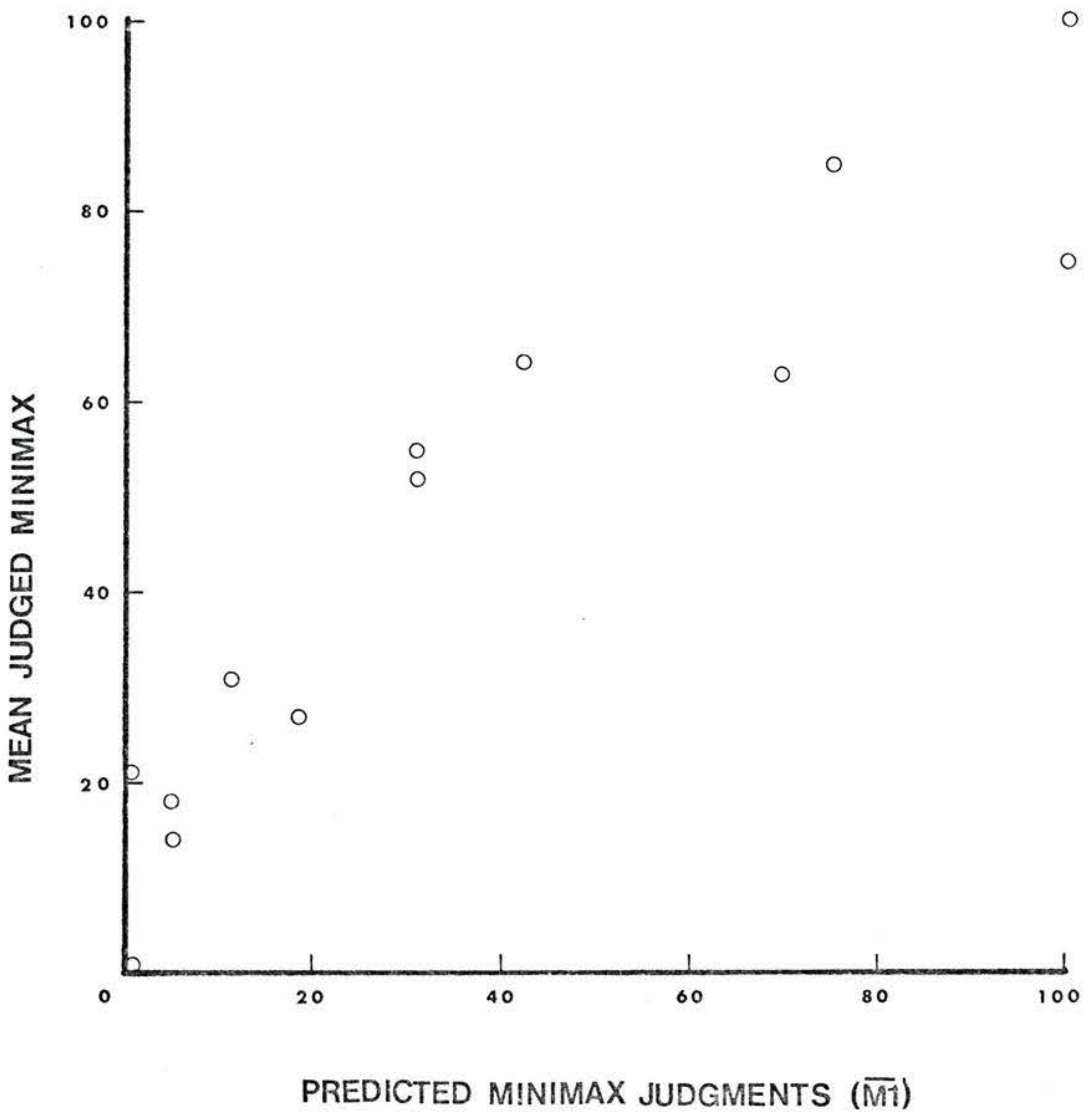

FIGURE 8. Mean judged minimax as a function of the minimax judgments predicted by the weighted means model ($\overline{MI}$) for Experiment 1 (N=8).

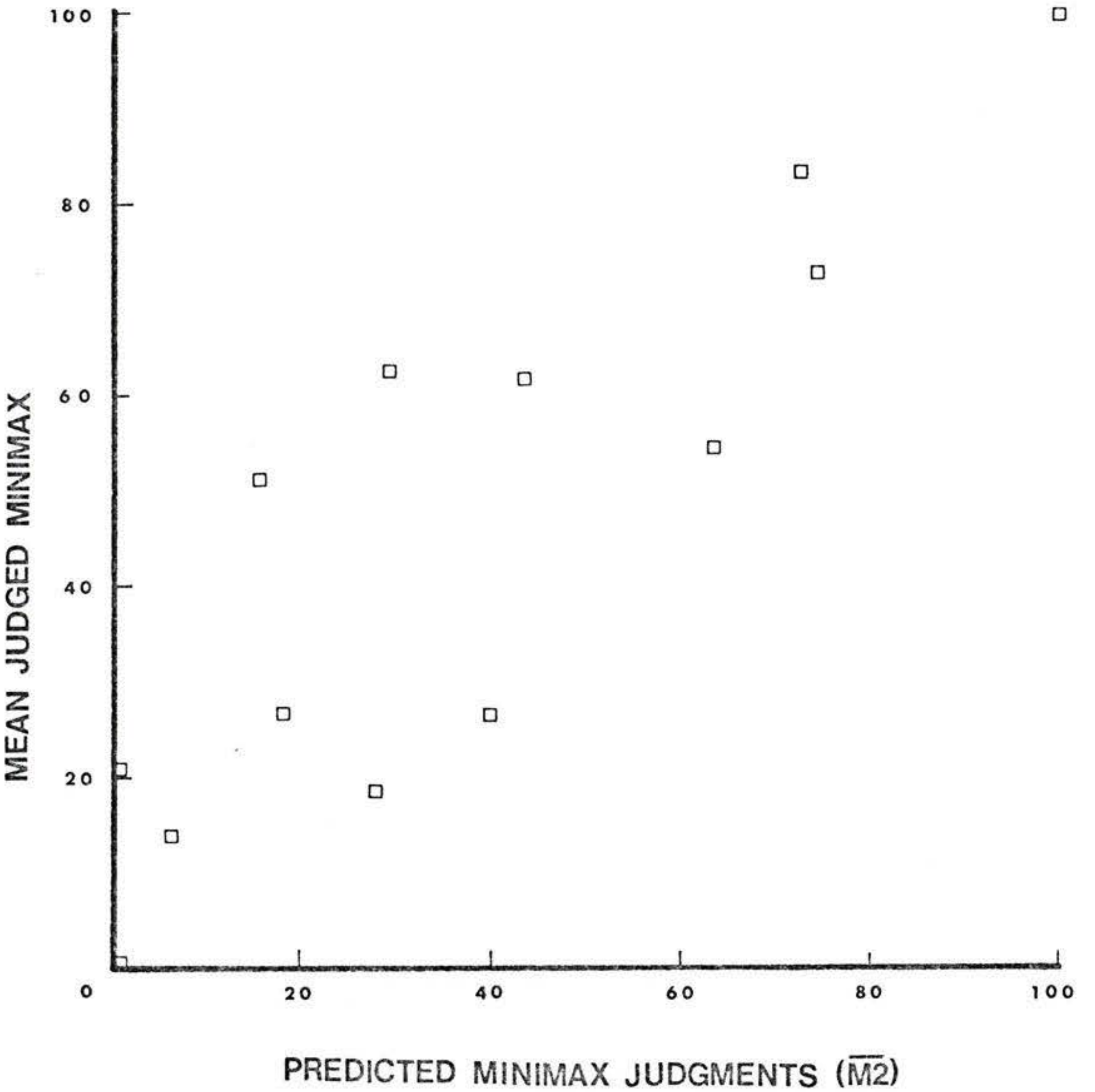

FIGURE 9. Mean judged minimax as a function of the minimax judgments predicted by the unweighted means model ($\overline{M2}$) for Experiment 1 (N=8).

average similarity judgments) was Lewis' (1960) Equation 10.46, an *F*-test for related samples. By this test, the obtained minimax judgments deviated significantly both from the judgments predicted by the weighted-means model, $F(13,104) = 8.74$, $p < .001$, and from the unweighted-means model, $F(13,104) = 11.44$, $p < .001$. A measure of the degree to which the predictions of a model deviate from the obtained minimax judgments for each subject is the sum of the squared deviations between predicted and obtained values for the 13 partitions (see Table 5). By this measure predictions for the weighted-means model (based on the average similarity judgments) were not significantly closer to the obtained minimax judgments than the predictions for the unweighted-means model, correlated $t(7) = 1.27$, $p > .10$.

The correlation of an individual's minimax judgments and that predicted by either model from his own similarity judgments was highly significant for 7 of the 8 subjects (see Table 6). The atypical subject had switched from treating colour differences as having the greatest weight in her similarity judgments to shape differences. This change was apparent both from the subject's written report and from the ranking of the partitions. Furthermore, the correlation between the obtained minimax judgments and that predicted by each model for the average similarity judgments was highly significant for all eight subjects (see Table 6). By Fisher's *r* to *z* transformation, the average correlation

Table 5

Sum of Squared Deviations Between Predicted and Obtained Minimax Judgments for Experiment 1

Subject	$\Sigma(E-M1)^2$	$\Sigma(E-M2)^2$	$\Sigma(E-\overline{M1})^2$	$\Sigma(E-\overline{M2})^2$
1	13,976.47	15,845.41	7,962.87	7,190.83
2	4,701.20	6,315.31	3,843.89	6,910.57
3	2,266.12	5,597.05	2,217.75	2,745.17
4	3,824.46	4,950.16	3,941.87	4,264.61
5	6,664.80	9,534.84	8,524.51	11,106.59
6	10,783.78	8,260.47	19,574.17	16,763.47
7	3,519.23	3,222.65	3,441.26	3,234.00
8	8,929.85	8,046.60	8,034.41	7,137.51
Sum	54,665.91	61,772.44	57,540.73	59,352.75

NOTE. E = the minimax judgments for an individual subject; M1 and M2 = the predicted minimax judgments (based on the similarity judgments for a given subject) for the weighted and unweighted models, respectively; $\overline{M1}$ and $\overline{M2}$ = the predicted judgments based on the average similarity judgments.

Table 6

Pearson Correlations Between Predicted and Obtained Minimax

Judgments for Each Subject in Experiment 1

Subject	EM1	EM2	M1M2	$\bar{E}$M1	$\bar{E}$M2	E$\overline{M1}$	E$\overline{M2}$
1	.440	.345	.838	.473	.351	.769	.746
2	.891	.830	.945	.861	.851	.914	.776
3	.906	.755	.803	.756	.624	.943	.899
4	.898	.813	.876	.940	.862	.898	.840
5	.862	.738	.869	.922	.844	.840	.725
6	.789	.820	.940	.943	.937	.648	.669
7	.891	.884	.886	.936	.894	.902	.877
8	.764	.763	.884	.932	.888	.781	.764
Median *r*	.876	.788	.880	.927	.856	.869	.770
Mean *r*[a]	.836	.771	.886	.886	.827	.857	.800

NOTE. All correlations based on *n* = 13 partitions. A correlation coefficient must exceed the critical value of .553 to be significant at the .05 level and .684 at the .01 level. E = individual minimax judgments; $\bar{E}$ = mean judged minimax; M1 and M2 = the predicted minimax judgments for the weighted and unweighted means models, respectively (both M1 and M2 are based on individual similarity judgments); $\overline{M1}$ and $\overline{M2}$ = predictions based on the average similarity judgments.

[a]Computed using Fisher's *r* to *z* transformation.

of an individual's minimax judgments with the weighted-model predictions for individuals was .84, normal $Z = 10.80$, $p << .001$, and with the unweighted-model predictions for individuals it was .77, normal $Z = 9.15$, $p << .001$. The average Fisher z score, and therefore the average correlation, for the weighted model was significantly higher than that for the unweighted model by the correlated t-test, $t(7) = 2.58$, $p < .05$. Predictions by the weighted model correlated more highly with the obtained minimax judgments than the unweighted model for all but one of the subjects.

To test the degree to which averaging increases the predictive power of a given model, the correlation based on an individual's own similarity and minimax judgments can be compared with (a) the correlation between the averaged minimax judgments and the predictions generated from each individual's similarity judgments and (b) the correlation between each individual's minimax judgments and predictions generated from the averaged similarity judgments. If there are large individual differences in the perception of similarity and/or minimax, then correlations based on averaged judgments should be lower than those based solely on individual judgments. Averaging judgments across subjects actually increased the Fisher z-score equivalents of the correlations, although never significantly so by the correlated t-test, $t(7) = 1.41$, .90, 1.79, and .67. Thus, individual differences in the perception of minimax and similarity were small.

The written reports that were elicited at the conclusion of the experimental session gave little indication as to whether a subject was operating in a manner consistent with one model or the other. However, unsolicited verbal reports made by the subjects during the scaling process were more informative. Most of these verbal comments were made in response to a question posed by the experimenter when the subject indicated that he had completed the scaling task. The question, posed for each of several pairs of partitions, was "Why is this partition to the left of that partition on the scale?" The original purpose for asking this question was to ensure that the subjects had understood the instructions, but the subjects' answers to this question were generally very clear statements of how the minimax judgments were made. Their answers were clearly more consistent with the weighted than the unweighted model. For example, in comparing partition 6 to other partitions, subjects would often point out that the objects in partition 6 were extremely similar within groups with only a little dissimilarity in the first and fourth groups. A subject operating in accordance with the unweighted model would have said the objects in partition 6 were of only medium similarity within groups, but no subjects made such a statement. However, the interpretation of these verbal reports must be qualified by the fact that they were not systematically recorded until after the completion of the experiment.

Although predictions were very accurate overall, the obtained rankings of several partitions deviated considerably from those predicted. The fourth partition, for which there was no overlap between groups in either colour or shape, was ranked lower by the subjects than was predicted by either model. That partition is an example of a bidimensional classification in which all the objects within a given group are identical on one dimension, but the common dimension changes from group to group. In bidimensional classification the levels of one dimension do not define the groups. The lower obtained ranking may reflect a lowered perception of similarity within groups because the nature of that similarity changes across groups. Such an explanation is, however, unsatisfactory because several partitions which contained partial bidimensional groups were predicted accurately.

Of the 13 partitions, 5 were particularly critical in the test between the models because the predicted ranks differed by two or more. The weighted-means model was more accurate in predicting all 5 partitions; a difference in rank of one was the largest between the obtained and the weighted-model rankings whereas the smallest difference in ranking getween the obtained and the unweighted rankings was two.

Conclusions

Both the scales obtained by averaging, similarity and minimax, have a high degree of generality across subjects, as evidenced by the high intersubject consistency in judgments. Averaging across subjects can be justified on this basis. The high correlations between the predicted and obtained minimax judgments represent considerable support for the models. Nevertheless, predictions by both models deviated a small but significant amount from the obtained minimax judgments. The difference between the two models in accuracy of prediction is not significant when the data are averaged.

The weighted-means model, however, was a better predictor of the individual minimax judgments than the unweighted-means model. The verbal reports support the conclusion that subjects were operating in accordance with the weighted-means model, that is, subjects appear to have made the perceived pairwise similarities the basis for judgments of minimax and to have done so by obtaining global impressions of within-group and between-group similarity.

EXPERIMENT 2

Experiment 2 was designed to minimize the time lag between the two scaling tasks that each subject performed. In Experiment 1, subjects made the minimax judgments 3-6 weeks after making the similarity judgments. If the two models had failed to predict the minimax judgments in Experiment 1, then failure could have been attributed to changes over time in the perception of pairwise similarity rather than to any deficiency of the models themselves. Experiment 2 was designed to rule out the alternative explanation of time lag. Although neither model actually failed to predict the minimax judgments in Experiment 1, Experiment 2 had been designed and testing begun before the completion of Experiment 1.

In Experiment 1 a set of partition stimuli was selected on intuitive grounds for presentation to all subjects. On the assumption that an individual's similarity judgments would be a better estimate of his perception of similarity than the averaged judgments, the partitions in Experiment 2 were selected for each individual from a pool of 180 possible partitions according to the following criteria: first, the predicted minimax values had to span the range of possible theoretical values for each model. Failure to predict the empirical scaling of the partitions could not, then, be attributed to restriction in range. Second, for each partition the value of minimax predicted by one model was to be as different as possible from that predicted by the other model to provide a strong test between the two models. A random selection of partitions was unacceptable because it was likely to produce a set of partitions for which the predictions differed little between models. Third, for a given model the selected partitions were to be as different as possible from one another on the theoretical measure of minimax.

A computer program was used to compute, on the basis of an individual's similarity dictionary, the two theoretical minimax values for each of the 180 possible partitions. Since in practice all three criteria could not be satisfied simultaneously, the partitions were selected such that some subset of them satisfied each of the criteria.

Interpretation of the results of this experiment are, however, problematic. These problems are discussed at length in the Conclusion section.

Method

Similarity Scaling of Object Pairs

The purpose and procedure in this part of the experiment were essentially the same as those in Experiment 1.

Stimuli. To increase generality across stimulus sets, a new set of stimuli was employed. The stimulus objects consisted of the 15 possible combinations of 5 colours (red R, gold G, purple P, orange O, and yellow Y) and 3 shapes (circle C, square S, and hexagon H). Red and gold were glossy, the others flat. In Munsell notation the 5 colours were 6R/4.5/14, 2Y/6.2/10, 1.5RP/5.4/11, 3YR/6.5/13, and 7.5Y/9/9. The areas of the 3 shapes were 5.06, 6.45, and 4.83 cm^2, respectively. The objects were gummed cut-outs.

The actual stimuli consisted of the 105 pairs of distinct objects plus the 15 pairs of identities. Each stimulus was constructed by pasting two objects on a 7.6 x 12.7 cm white file card. As the left-right position of each pair of objects on a card could be reversed by inverting the card, only a single set of stimulus cards was used. The left-right position of each pair was determined independently for each subject by inverting a randomly selected half of the stimuli. The 15 pairs of identical objects were not added to that deck of 105 stimulus cards.

Task. The task was the same as that described in Experiment 1, except that the 15 identical pairs were not presented to the subject. Instead, they were placed directly on the zero point of the scale.

Subjects. Nine subjects, selected from among third and fourth year undergraduate students in a volunteer subject pool, scaled the stimuli. All 9 subjects were tested for colour blindness (American Optical Society, 1940), and 1 of those subjects was eliminated from the analysis for colour blindness. None of these subjects had previously served in Experiment 1.

Procedure. The procedure was essentially the same as that in Experiment 1. The complete instructions are given in Appendix A.

Minimax Scaling of Partitions

Stimuli. A pool of 180 partitions was formed by (a) randomly generating 80 distinct partitions and (b) generating 100 distinct partitions on the basis of expectations of the kind of partition that best met the criteria described earlier. The random partitions were generated by randomly selecting the number of groups in a given partition ($2 \leq c \leq 5$) and then randomly selecting the number of objects in each of the c groups ($2 \leq c_i \leq 6$), and, finally, randomly assigning c_i objects to the i^{th} group. It is important to note that the partitions were chosen from the set that includes all possible partitions of the 15 objects, all possible partitions of every subset of 14 objects selected from among the 15 objects, etc.

For each subject, 16 partitions were selected from the set of 180 partitions according to the three criteria discussed earlier. Selection of the 16 partitions were based on predicted minimax values generated from a given individual's similarity judgments.

Subjects. The same 8 subjects who scaled the object pairs on similarity scaled the partitions on minimax.

Procedure. The instructions, which were essentially the same as those used in the first experiment, are given in Appendix A. In addition, the American Optical (1940) test for colour blindness was given to each subject at the conclusion of the experiment.

Results

Similarity Scaling of Object Pairs

As in Experiment 1, the 8 subjects employed widely varying weighting schemes. Differences in shape had little effect on the judgments of similarity of 4 subjects, that is, two objects differing only in shape were judged to be highly similar relative to two objects differing only in colour. In contrast, differences in colour had little effect on the judgments of similarity of the other 4 subjects. The similarity judgments for each subject were transformed to a common range of 100 and averaged across subjects (see Table 7). The consistency among subjects in their judgments of similarity was high with the average correlation equal to .65, normal $Z = 41.97$, $p \ll .001$.

Minimax Scaling of Partitions

The obtained minimax judgments for each subject are given in Appendix F. The correlation between an individual's minimax judgments and that predicted by a given model from his own similarity judgments was highly significant for 7 of the 8 subjects (see Table 8). The correlation between the atypical subject's minimax judgments and that predicted by the average similarity judgments was, however, significant. The average correlation of an individual's minimax judgments with the weighted model was .76, normal $Z = 10.29$, $p \ll .001$,

Table 7

Average Similarity Judgments of Object Pairs in Experiment 2

Description of Object Pair	Same Shape	C-H	S-H	C-S
Same Colour	0	17.0	26.2	42.5
O-Y	19.1	45.6	53.7	70.8
G-Y	21.0	48.2	55.6	70.8
R-O	22.4	47.6	54.6	72.0
R-P	25.8	49.0	59.3	74.4
G-O	29.3	54.7	63.6	78.9
P-O	31.7	58.2	68.7	83.0
P-Y	39.3	67.6	75.0	90.1
R-Y	40.1	69.6	77.7	92.7
G-P	40.3	71.9	79.2	94.9
R-G	42.6	74.2	80.8	97.2

NOTE. The entry in the intersection of a row and a column is the average judgment of similarity for any pair of objects with the shapes and colours specified in the column and row headings, respectively. C = circle, H = hexagon, S = square, G = gold, P = purple, R = red, O = orange, and Y = yellow.

Table 8

Pearson Correlations Between Predicted and Obtained Minimax Judgments for Each Subject in Experiment 2

Subject	EM1	EM2	M1M2	E$\overline{M1}$	E$\overline{M2}$
1	.800	.783	.976	.884	.852
2	.827	.944	.849	.667	.742
3	.835	.818	.905	.812	.825
4	.695	.939	.711	.563	.667
5	.786	.892	.913	.812	.927
6	.849	.832	.951	.892	.879
7	.757	.898	.690	.651	.722
8	.415	.198	.890	.627	.374
Median *r*	.793	.862	.898	.740	.784
Mean *r*[a]	.765	.850	.893	.763	.785

NOTE. All correlations based on *n* = 16 partitions. A correlation coefficient must exceed the critical value of .497 to be significant at the .05 level and .623 at the .01 level. E = individual minimax judgments; M1 and M2 = the predicted minimax judgments for the weighted and unweighted means models, respectively (both M1 and M2 are based on individual similarity judgments); $\overline{M1}$ and $\overline{M2}$ = predictions based on the average similarity judgments.

[a] Computed using Fisher's *r* to *z* transformation.

and with the unweighted model it was .85, normal $Z = 12.78$, $p \ll .001$. The average Fisher z scores (and therefore the average correlations) for the two models were not significantly different by the correlated t-test, $t(7) = 1.72$, $p > .10$; for 4 subjects the weighted model was the better predictor and for the other 4 subjects the unweighted model was better.

Predictions based on the average similarity judgments were not significantly better than those based on individual judgments for both the weighted model, $t(7) = .07$, $p > .50$, and the unweighted model, $t(7) = 1.09$, $p > .50$.

Conclusions

The superior accuracy of the weighted-model predictions was not replicated in this experiment despite a more rigorous selection of partitions designed to maximize the differences in predictions. Both models were highly predictive, but the difference between them was insignificant. Four alternative explanations of this discrepancy in experimental results will be proposed and their tenability discussed. First, since the partitions were selected to maximize the discrepancy between weighted and unweighted model predictions, failure to find a significant difference cannot reasonably be attributed to a lack of experimental sensitivity. Second, a more plausible explanation is that some subjects operated in accordance with one model whereas the remaining subjects operated in accordance with the other model. On this interpretation the superiority of the weighted means model in Experiment 1 would be spurious. While this explanation may appear credible two other alternatives should be considered. Both alternatives involve changes in the Experiment 1 instructions which were altered for use in Experiment 2. One change was the removal of the instruction to use an "intuitive" approach to scaling on the minimax dimension. Subjects in Experiment 1 were instructed to use their intuition in scaling the partitions if they found the task too complex, an observation that pilot subjects had made. In

Experiment 2, however, subjects were simply asked to make the minimax judgments. It could be argued that the task in Experiment 2 was more difficult and, consequently, induced many subjects to reduce the cognitive strain by adopting a less demanding cognitive strategy for scaling--namely, the unweighted-means model. Data gathered in a pilot study (see Appendix D for details) argue against such an explanation. In that study four subjects received the "intuitive" version of the instructions and four the "non-intuitive" instructions. If the non-intuitive instructions induce subjects to adopt a less demanding strategy, then more non-intuitively than intuitively instructed subjects should make minimax judgments in accordance with the unweighted model. This was not the case. One subject under each instructional condition was better predicted by the unweighted model. Clearly, the non-intuitive instruction did not induce more subjects to adopt the unweighted strategy. Although not conclusive, these data argue against such an interpretation.

A second change made in the instructions in Experiment 2 was that the subjects were instructed not to consider the similarity in number between two groups "because that kind of similarity is not based on the physical properties of the objects themselves." The purpose of the instruction was to prevent subjects from using the number of objects within a group as a property like shape or colour in evaluating the overall similarity between groups. This change may have had the inadvertent effect of inducing some subjects to weight each group equally in assessing the overall similarity within groups. If so, then of the two models proposed the unweighted-means model would tend to be supported more highly than the weighted-means model. An additional subject (not included in the Experiment 2 data) who was tested with the Experiment 2 instructions provided support for this interpretation. In post-experimental testing this particularly verbal subject insisted that she had been instructed "to weight each group equally," a procedure she would not otherwise have followed.

Of all the alternative explanations of the Experiment 2 results, the last one discussed appears to be most plausible. It could be easily tested by rerunning Experiment 2 but deleting the instruction to ignore number.

DISCUSSION

A distinction has been drawn between sorting and classifying objects: in sorting, a basis for placing a set of objects into groups is clearly specified whereas in classification no such objective is specified. I have argued that it is highly probable that a classifier will always adopt some objective and that, therefore, the two tasks are essentially equivalent if the classifier happens to adopt the objective explicitly specified for the sort task. This implies that the study of sorting is an investigation of the way in which subjects sort according to specified objectives while the study of classification becomes an investigation of the kinds of objectives that classifiers adopt under various conditions. If they are indeed the same task, then a prerequisite for understanding classification must be an understanding of sorting according to specified objectives. Once a theory has been developed that successfully explains sorting behaviour for a given objective, it can then be used to predict classification behaviour on the assumption that the classifiers have adopted the given objective. When a theory has not been pretested in the appropriate sorting situation, failure of that theory to explain classification can be attributed to either of two alternatives: (a) the classifiers adopted an objective other than that implied by the theory, or (b) the classifiers adopted the objective implied by the theory, but the theory was an incorrect representation of subjects' behaviour in partitioning on the basis of that objective.

The work of Imai and Garner (1968) provides an example of the experimental test between two theories of free classification in which the rejection of one of the theories could be attributed to either alternative (a) or (b). The two theories which they proposed imply different objectives. In the case of their dimensional theory, the objective would be to classify on the basis of one of the stimulus dimensions such as colour, and in the case of the similarity theory, the objective would be to classify on the basis of similarity such that the objects were similar within and different between groups. In their experiment the free classification of two particular sets of objects was intended to provide a critical test between the theories, both of which accounted equally well for the classification of 12 other sets of objects. The test used by Imai and Garner for a given set of objects was a comparison of the proportion of classifiers producing the optimal dimensional classification(s) with the proportion producing the optimal similarity classification. They found that for the two critical sets of objects the larger proportion of subjects classified as predicted by their dimensional theory. On the basis of this test, Imai and Garner rejected the class of <u>all</u> theoretical explanations of free classification that are based on similarity.

However, it can be argued that it is at most their own similarity theory plus the subset of the class of similarity theories that are "equivalent" to their theory which can justifiably be rejected. It is possible that their experimental data could be accounted for by a similarity theory that is not equivalent to theirs. (The weighted-means model used in the present research, for example, predicts the preferred classification better than their similarity theory.)

An alternative method for the investigation of free classification can be based on the assumption that classifiers adopt an objective, and that, therefore, the two tasks are equivalent when the classifier happens to adopt the objective explicitly specified for the sorter. In the first of two stages in this method the partitions produced by the classifiers are compared directly with those made by sorters given specified objectives. If, for example, sorters instructed to sort on the basis of similarity did not partition objects in the same way that classifiers did, then there would be some justification for rejecting the entire class of similarity theories. In contrast, the method of Imai and Garner can justify the rejection of only a subset of all possible similarity theories.

The proposed method of directly comparing empirical proportions of subjects cannot justify the inference that both classifiers and sorters have exactly the same objective. But if, for several sets of objects, both the classification and a particular sorting instruction produce the same partitions, then it can be argued that the classifiers have adopted an objective that is "equivalent" to the given sorting objective. Thus, the two operations--sorting and classifying--may be equivalent in the sense that the same output is produced over a range of stimulus sets. Similarly, the method of Imai and Garner cannot justify the inference that a given theory represents the actual process of classification. But if the classifiers partition sets of objects as predicted by a theory, there is some justification for claiming that the theoretical representation of the classification process is equivalent to the actual classification process.

The second stage of the proposed method consists of developing a theory to predict the process of sorting on the objective identified in the first stage of free classification. A theory that can successfully account for sorting on the explicitly specified objective also applies to the corresponding free-classification situation (on the assumption that the classification and sort tasks are equivalent when the classifiers happen to adopt the objective explicitly specified for the sorter). Also, the two stages in this method for investigating free classification can be reversed with the first stage becoming a search for a theory that predicts sorting on a given objective.

The present research was directed towards the second stage of this proposed method, that is, to develop a model that could successfully predict the partitions made by subjects in sorting a set of objects into groups on the basis of perceived physical similarity. Subjects can sort on the basis of perceived similarity by placing very similar objects together within the same group and placing very different objects in separate groups. It can be argued that subjects attempt to maximize the similarity within and difference between groups. This is the minimax objective.

Two groups of investigators (Handel & Imai, 1972; Imai & Garner, 1968) have proposed methods of predicting the optimal classification in free classification on the assumption that subjects classify according to the minimax objective. In their investigation of free classification Imai and Garner (1968) introduced the hypothesis that subjects classify objects according to the minimax objective. They proposed that the optimal classification would be that which "minimizes intra-class differences and maximizes inter-class differences" (p. 171). Specifically, the optimal classification would minimize the number of dimensions on which stimuli within a group differ and maximize the number of dimensions on which stimuli in separate groups differ. However, there are two difficulties with using this method to identify the optimal classification: first, the method is restricted to the very limited set of stimulus situations in which the perceived differences depend solely on the number of dimensions by which a given pair of stimuli differs, and, second, prediction of the optimal classification is ambiguous because it is not clear how the within-group differences are to be weighted relative to the between-group differences. In a subsequent article, Handel and Imai (1972, footnote 3) proposed a quantitative method for identifying the optimal classification in free classification. Equation 3 in this dissertation is mathematically equivalent to their proposed measure of minimax.

Both Imai and Garner (1968) and Handel and Imai (1972) rejected the similarity explanation of classification. However, they did not provide any empirical evidence to suggest that subjects sorting on the basis of similarity would do so in a way predictable by either of their methods. Their subjects may have (a) adopted an objective other than minimax or (b) classified according to the minimax objective but in a way not predicted by either of their methods.

Common to both methods (Handel & Imai, 1972; Imai & Garner, 1968) of predicting the optimal classification in free classification is the implicit assumption that subjects make minimax judgments during the process of classifying. There are three additional assumptions that are implicit in Handel and Imai's (1972) method of predicting the classification of objects: first, subjects make minimax judgments according to the weighted-means model; second, subjects consider all possible partitions of the stimulus set; and, third, subjects classify according to the optimal partition.

In this dissertation, two competing models--weighted and unweighted means--are proposed to predict subjects' judgments of minimax for partitions of the stimulus set, the minimax measure derived from the weighted model being mathematically equivalent to the formula proposed by Handel and Imai (1972). Both dissertation models assume that subjects make minimax judgments when sorting. They differ in their representation of how subjects make those judgments.

The specific purpose of the dissertation experiments was, first, to test whether subjects can make minimax judgments and, second, to test the two competing models of the judgmental process. Although both minimax models could, with the addition of several assumptions, have been tested in a free-sorting situation, the dissertation experiments provided a more direct test of the minimax assumptions. These experiments are relevant to the problem of sorting only to the extent that sorters make minimax judgments during the process of sorting. Furthermore, these experiments are relevant to the problem of classification only to the extent that classifiers adopt the minimax objective and make minimax judgments during the process of classifying.

The two experiments that were conducted utilized a scaling method. The procedure was very similar in both experiments. In the first experimental session a subject judged the physical similarity that he perceived between all possible pairs of 16 stimulus objects. In a subsequent session the same subject judged the degree to which objects were similar within and different between groups for 13 (or 16) partitions. Predicted minimax judgments were generated from each model based on either the individual subject's own pairwise similarity judgments or on the averaged pairwise similarity judgments. These predicted judgments were compared with the corresponding minimax judgments actually made by the subjects.

The first experiment clearly supported the weighted over the unweighted-means model for the individual subject. The weighted-model predictions were more highly correlated with the obtained minimax judgments for every subject. The verbal reports tended to confirm this superiority of the weighted model. Outright rejection of the unweighted-means model is unwarranted, however, because that model was also highly predictive of the obtained minimax judgments. Moreover, the conclusion that the weighted model represents the actual process of judging minimax is not justified for three compelling reasons. First, predictions for the weighted model deviated a small, but significant, amount from the obtained judgments. Second, the predictive accuracy of the weighted model was demonstrated under only one set of experimental conditions. Third, the data of Experiment 1 are at least as well accounted for by a third model: If it is assumed that in Experiment 1 the subjects (a) ignored the instruction to minimize the similarity between groups and attempted only to maximize the similarity within groups, and (b) made global judgments of the similarity within groups, then the average similarity within groups can be employed as a predictor of minimax judgments (see Table 9). Predictions by this model that are based on the average similarity judgments are highly correlated with the mean judged minimax, $r(11) = .96$, $p \ll .001$ (see Figure 10).[3] With such a high correlation between mean predicted and obtained judgments, this similarity-within-groups model must be considered a serious alternative representation of the judgmental process. The correlation of .96 is significantly higher than the corresponding correlation of .88 for the unweighted model, $t(10) = 2.34$, $p < .05$. But it is not significantly higher than the corresponding correlation of .94 for the weighted model, $t(10) = 1.67$, $p > .10$. Predictions based on the within-groups model deviated a small but significant amount from the obtained judgments, $F(13,104) = 4.48$, $p < .01$.

Experiment 1 is not an adequate test among the <u>three</u> models for two reasons. First, that experiment was designed to test between only the two dissertation models. As a result, predictions by the weighted model and the third alternative, the within-groups model, differ in only minor ways for the 13 partitions. Moreover, the latter model represents a post hoc attempt to fit a model to the data

Table 9

Six Different Predictors of the Obtained Judgments for the 13 Partitions in Experiment 1

Partition Number	Mean Judged Minimax	Mean Normalized Rank	Theoretical Predictions					
			$\overline{M1}$	$\overline{M2}$	$\overline{M3}$	$\overline{M4}$	$\overline{M5}$	$\overline{M6}$
1	0	0	0	0	0	0	48.4	47.5
2	14.1	11.7	5.6	7.4	.5	1.1	79.7	84.4
3	18.8	20.7	5.6	27.8	8.9	38.3	47.2	46.6
4	21.2	19.1	0	0	10.4	10.4	0	0
5	26.9	21.4	18.5	18.5	22.1	22.1	58.6	57.5
6	29.1	31.1	11.1	39.8	12.7	44.5	61.7	70.3
7	52.2	48.9	31.5	16.7	41.4	22.6	52.3	45.3
8	54.7	51.5	31.5	63.9	38.4	80.0	62.1	61.0
9	63.3	54.4	69.4	44.4	75.2	48.6	81.5	75.2
10	64.3	58.6	42.6	29.6	61.1	42.6	41.5	40.8
11	74.8	64.7	98.1	76.9	94.0	70.9	100.0	100.0
12	84.8	75.7	75.0	75.9	77.9	81.9	87.1	80.7
13	100.0	100.0	100.0	100.0	100.0	100.0	94.5	92.8

NOTE. $\overline{M1}$ = weighted model, $\overline{M2}$ = unweighted model, $\overline{M3}$ = average similarity within groups (weighted), $\overline{M4}$ = average similarity between groups (weighted), $\overline{M5}$ = average similarity within groups (unweighted), and $\overline{M6}$ = average similarity between groups (unweighted).

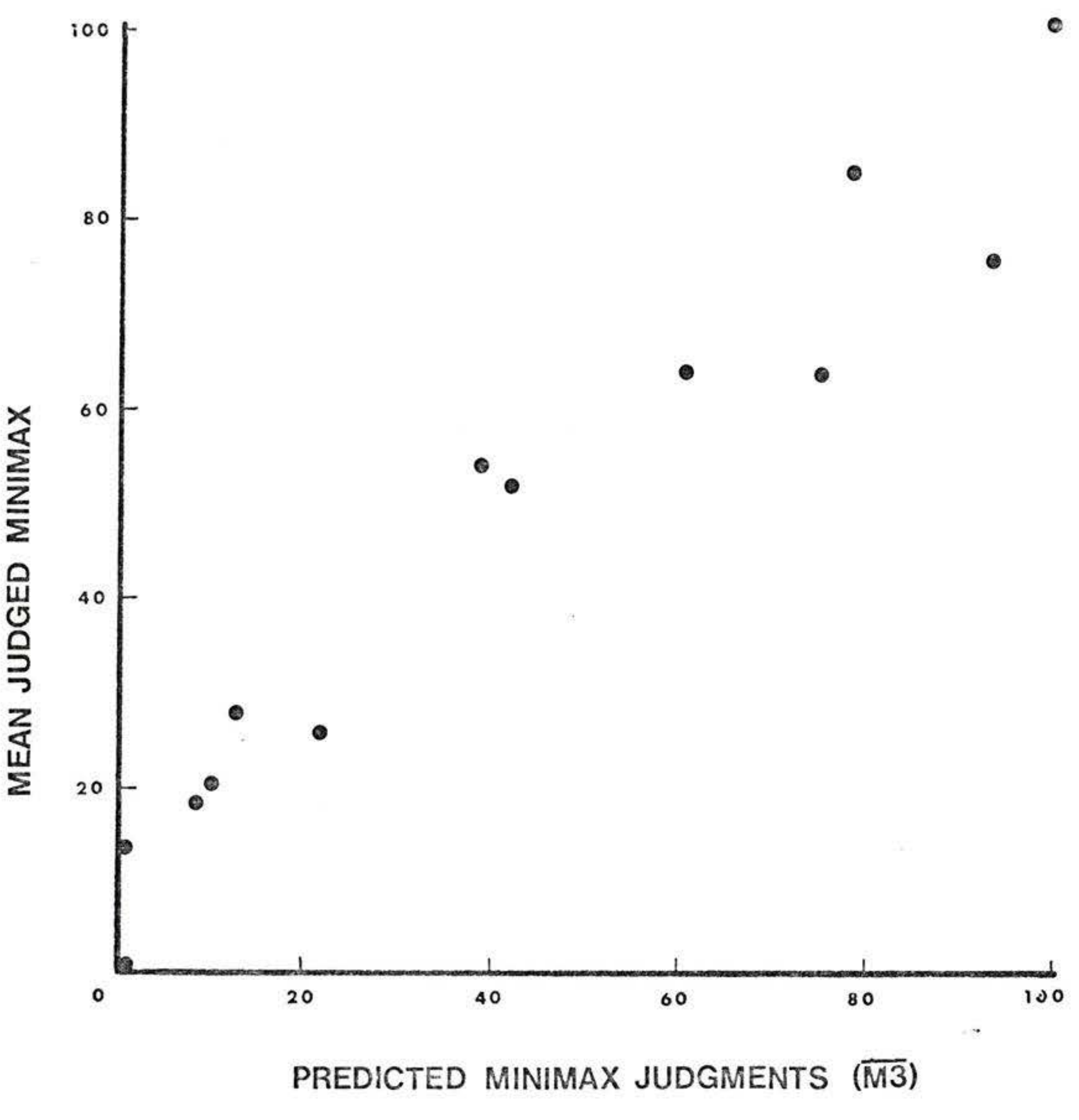

FIGURE 10. Mean judged minimax as a function of the minimax judgments predicted by the weighted within-groups model ($\overline{M3}$) for Experiment 1 (N=8).

not an a priori test between the models. The appropriate test among the three models would be one in which the partitions are selected to maximize the differences in predictions.

The three minimax models discussed thus far have all been good predictors of the obtained minimax judgments. It can be argued that <u>any</u> minimax model will also predict those judgments. This would be true if all minimax models were highly intercorrelated and any one of them predicted the obtained judgments. That all minimax models are not highly related is evident in Table 9 in which the predictions derived from six different models are presented together with the data obtained in Experiment 1. Whereas the obtained judgments correlate .94 with the predictions generated from model 1, the obtained judgments correlate only .52 with the predictions generated from model 6. The latter model is clearly an inadequate predictor of the obtained judgments.

Three basic assumptions were made in directly scaling the minimax dimension: first, that the dimension exists; second, that it has interval level measurement properties; and third, that this measurement property is directly reflected in judgments by subjects. With respect to the first assumption, Experiment 1 did not provide an estimate of intrasubject consistency, although intersubject consistency in judging minimax was high. The direct method of interval scaling did not provide evidence relevant to the last two assumptions. Although the subjects were instructed to use subjectively equal intervals, the scaling method provided no data to indicate whether they did. From the same data an alternative scale can be derived in which the third assumption is replaced by the weaker assumption that a subject's judgments reflect only an ordinal level of measurement. By making one additional assumption--namely, that the partition stimuli are normally distributed in scale value (that is, true differences in judged minimax between adjacent objects ranked near the extremes tend to be larger than differences between objects falling near the middle in rank), the indirect scaling method of normalized ranks can be used to derive an interval level scale (Garner & Creelman, 1970). The scale obtained by taking the average of the normalized ranks for each partition and transforming to a range of 100 is given in Table 4. That scale is closely related to the corresponding direct scale--mean judged minimax, $\underline{r}(11) = .993$, $\underline{p} << .001$. The relationship between the two obtained scales is plotted in Figure 11, and the relationship between the mean normalized rank and the minimax judgments predicted by the weighted and unweighted-means models are plotted in Figures 12 and 13, respectively.

Experiment 2 was designed to satisfy three objectives: (a) to maximize the difference between the weighted and the unweighted-model predictions by selecting the partitions for

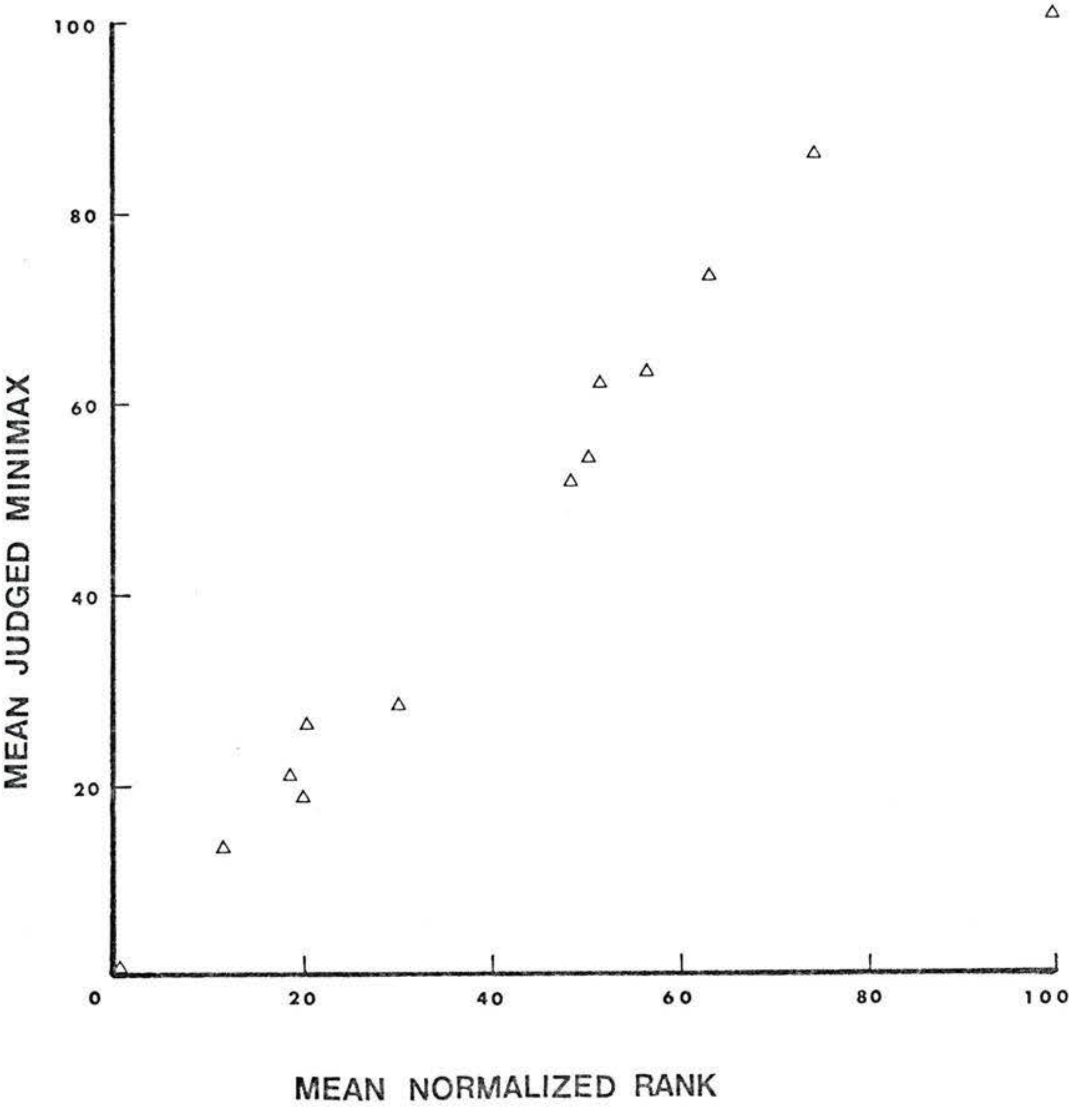

FIGURE 11. Mean judged minimax as a function of the mean normalized rank where both scales are based on the same data from Experiment 1 (N=8).

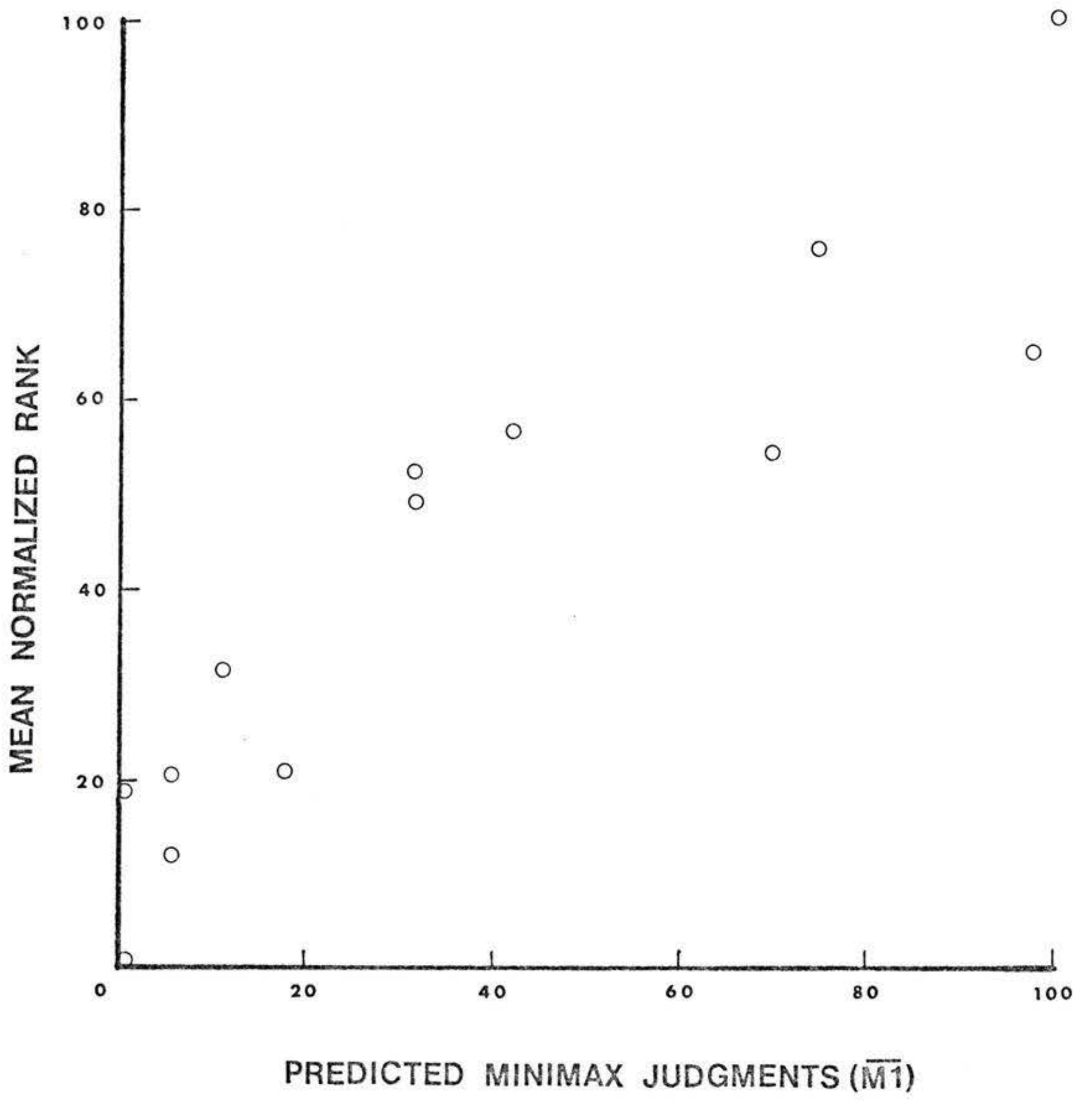

FIGURE 12. Mean normalized rank as a function of the minimax judgments predicted by the weighted means model ($\overline{M1}$) for Experiment 1 (N=8).

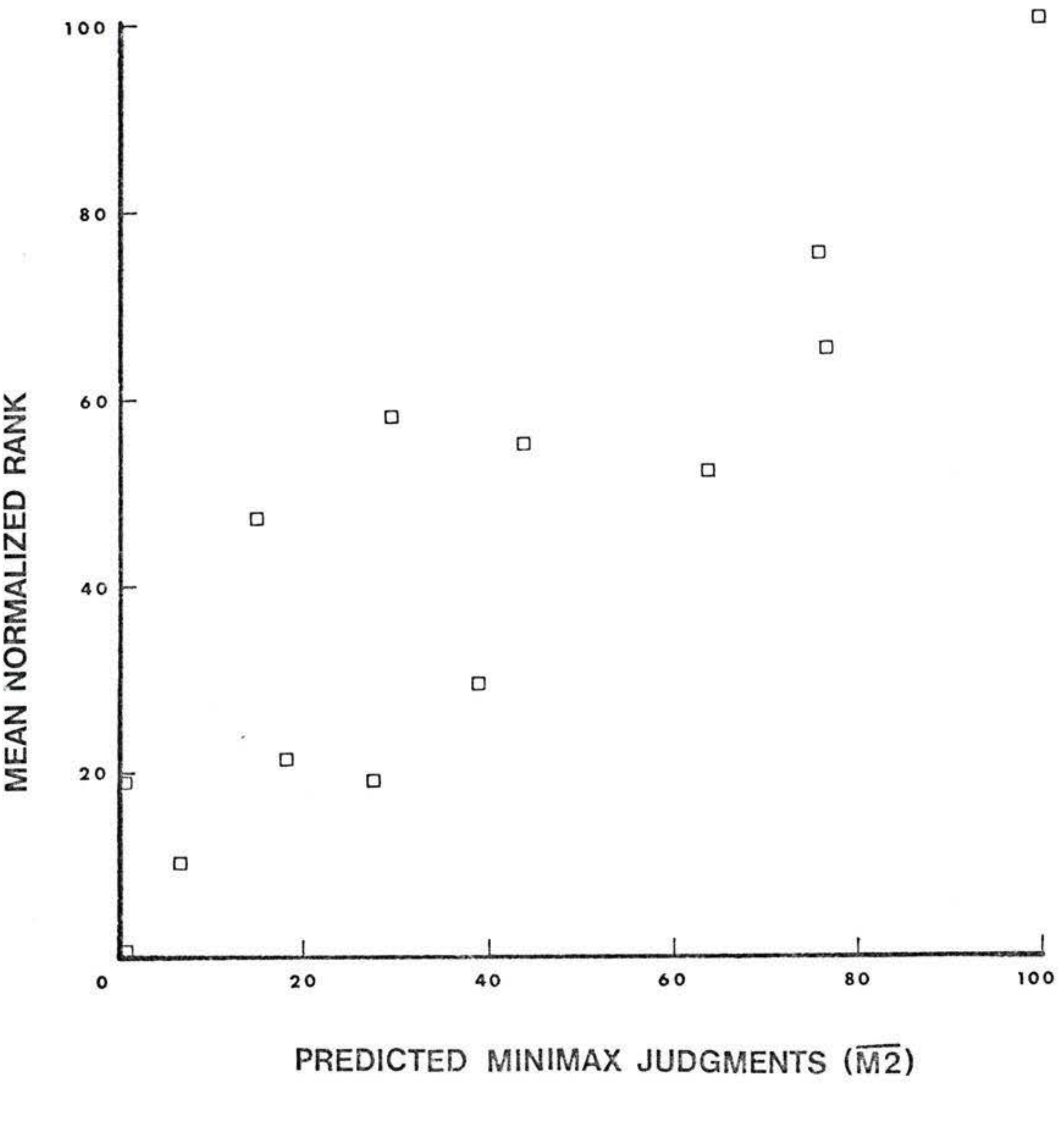

FIGURE 13. Mean normalized rank as a function of the minimax judgments predicted by the unweighted means model ($\overline{M2}$) for Experiment 1 (N=8).

each individual, (b) to minimize the time delay between the similarity and minimax scaling, and (c) to replicate the test between the models under slightly different experimental conditions. The interpretation of the results of that experiment must be deferred, however, until an additional experiment can be conducted to examine the hypothesis that the subjects in that experiment were inadvertently instructed to weight each group in a partition equally. The critical paragraph was in the instructions to Experiment 2 but not in the instructions to Experiment 1.

The procedure employed in both Experiments 1 and 2 required subjects to do both the similarity and the minimax scaling with the pairwise similarity judgments always being made prior to the minimax judgments. It can be argued that the similarity scaling task implicitly emphasized the need to pay attention to the similarity between pairs when doing the minimax scaling. The implication is that if subjects did not scale similarity first they might make their minimax judgments on some other basis than the pairwise similarities. The two dissertation models, both of which are based on pairwise similarities, could not then predict minimax judgments made on some basis other than the pairwise similarities. By this argument, Experiment 1 did not provide an adequate test of the two minimax models because the subjects were preset to attend to the pairwise similarities. However, the subjects in Pilot Study 1 (Appendix D) were not preset to attend to the pairwise similarities for they made the minimax judgments first. Nevertheless, the obtained minimax judgments were very accurately predicted from either hypothetical similarity judgments or similarity judgments elicited from the subjects after the completion of the minimax scaling. These data argue that the preliminary similarity scaling does not preset subjects to attend to pairwise similarity when doing the minimax scaling.

Two final comments should be made about the procedure used in this dissertation. The first is that although the sorting and scaling tasks may require subjects to make similar kinds of judgments, the two tasks do differ. For example, it is unrealistic to assume that sorters consider every possible partition when the set of objects is even moderately large. A more plausible assumption for sorting is that the sorter considers only a subset of all possible partitions and selects the optimal partition in that subset.

Second, although the experiments conducted in this dissertation were limited to objects varying on two well-defined dimensions with clearly discriminable levels, there is no a priori reason why the same judgmental processes should not also be operating if the dimensions of the stimulus objects are more complex and less well defined.

Directions for Future Research

Several questions remain that can only be answered by further experimentation. First, the interpretation of Experiment 2 can be clarified by altering the instructions as discussed earlier and replicating the experiment.

Second, consistent with the results of Experiment 1 is the alternative hypothesis that subjects are attending to only the average similarity within groups. That alternative explanation can be tested against the two dissertation models with a scaling task for which the partitions have been selected on the sole criterion that the predicted judgments are maximally different for the three models.

Third, in Experiments 1 and 2 the objects were multidimensional stimuli with two indpendently varying dimensions of shape and colour. Even if the replication of Experiment 2 supported the conclusions drawn from Experiment 1, generality over sets of objects would not be established. To establish generality would require that the weighted means model predict minimax judgments for sets of objects such as portraits which vary in a more complex way with many correlated dimensions. A further extension of the models could be to the use of words as objects which vary in meaning in complex ways. The method of sorting (on the basis of similarity in meaning) has been used to study the organization and storage of lexical information in memory on the assumption that the organization in memory is reflected in the organization in sorting (Miller, 1969). But no attempt has been made to predict the sorting or organization of a set of words from pairwise judgments of similarity in meaning. The assumption that subjects can make minimax judgments and do so according to a particular model can be tested by requiring subjects to scale selected partitions of words on the degree to which the words are similar within and different between groups in meaning. Accurate prediction of the minimax judgments for words would demonstrate considerable generality for a given minimax model.

And fourth, in testing the models with a scaling task, it was assumed that the empirical minimax scale was internally consistent. The manifest method of scaling employed in Experiment 1 does not provide, within a single experiment, a method of evaluating the validity of the assumption that the scale has interval level properties of measurement. There are two methods of evaluating this assumption. (a) The internal consistency of the scale derived from direct judgments of minimax can be checked by replicating Experiment 1 with different but overlapping sets of partitions. If the scale is internally consistent, the intervals should be equivalent between those partitions common to both experiments. (b) An alternative to the manifest method of scaling is the latent method of scaling in which an interval level scale can be derived from the judgments of several subjects by making the weaker assumption that a subject's judgments reflect only an ordinal level of measurement. Pair comparison scaling is a latent method for which the internal consistency of the resultant scale can be checked within a single experiment. Unfortunately, there is reason to believe that these two methods will not necessarily produce the same empirical scale (Garner & Creelman, 1967; Hays, 1967). Although both methods can be used to produce a scale with equal intervals, the assumptions required for scaling differ with the method. The possible lack of correspondence between scales derived by the two methods suggests that the testing of models is dependent to some extent on the method employed in constructing the empirical scale.

Establishment of the validity of a given model of sorting requires not only the prediction of minimax judgments by a scaling procedure but also the prediction of the optimal sort in sorting as well. The sorting task can be varied in several ways to provide a range of experimental situations which allow the selection or elimination of alternative models of sorting. Each variation of the sorting task can be used to test different implications of the models. (a) In the free-sort variation the sorter is free to partition a set of objects into any number of groups with any composition. This variation of the task can be used to test the power of several models to predict the partition that will be judged to be optimal among all possible partitions. (b) Compared to free sorting, the restricted-sort variation of the task, in which the sorter is restricted to using exactly _c_ groups, provides a slightly different test between the models. For each value of _c_, a different partition will be locally optimal with respect to minimax. Thus, the restricted-sort task requires that a model predict the locally optimal sort for each value of _c_, a requirement that can make greater demands than the free-sort task on the predictive power of a model. A characteristic of this procedure is the reduction in complexity of the subject's task achieved by a reduction in the number of logically possible partitions. (c) In the multiple choice variation of the task, the subject must choose from among several specified alternatives the sort that is optimal with respect to the minimax objective. This is not strictly a sort task, but it is a procedure that should be considered. Since this task provides an experimenter with complete control over the partitions which a subject must consider, partitions can be selected to discriminate maximally among several alternative models. (d) In sequential sorting, the objects are presented one at a time to a subject whose sorting objective is to maximize the similarity within groups and minimize the similarity between groups. A given object can be placed into one of the _c_ groups that are already available, or it can be used to start a new group. The first object to be sorted necessarily forms a new group. The second object can be placed in one of two ways: into the already available group along with the first object, or into a new group separate from the first object. Placement of the second object will depend upon the sorter's judgment of the degree to which each alternative sort meets the minimax objective. This procedure is repeated for each object in succession with the sorter considering only ($\underline{c}$ + 1) alternative sorts on each trial.

Two variations of this sequential-sort task are generated by permitting, or not permitting, reorganization of the objects at any time during sorting. If reorganization is not permitted, the final partition will not necessarily be equivalent to the optimal free sort of the same $\underline{N}$ objects. The succession of sorts will be locally optimal only and not optimal with respect to the partitioning of the total set of $\underline{N}$ objects. With reorganization permitted, the sorter can alter his partitioning of $\underline{N}$ objects if he perceives another arrangement of the objects that better meets the minimax objective. This reorganization behaviour should also be predictable by a model. The sequential-sort task with reorganization permitted provides a method for studying the process of sorting, a process that is obscured in free sorting with "simultaneous" presentation of the set of objects.

To account for sorting behaviour, the assumptions of the minimax models proposed in this dissertation must be supplemented by assumptions specifying how sorters actually use judgments of minimax in sorting a set of objects according to the minimax objective. For example, it can be assumed that, in free sorting a set of objects, subjects consider all possible partitions and sort according to the optimal partition. But this assumption is completely untenable even for sets with as few as five or six objects. A more plausible assumption is that subjects consider only a subset of all possible partitions and sort according to the locally optimal partition. Further assumptions appear to be necessary for several variations of the sorting tasks. Nevertheless, the concept of a minimax dimension and assumptions about how judgments of minimax are made appear to be indispensible parts of any model of the sorting tasks.

FOOTNOTES

1. The Spearman rank-order correlation coefficient gives essentially the same results, average rho = .74, which is highly significant by Kendall's coefficient of concordance, $W = .78$, $p \ll .001$ (Siegel, 1956).

2. Intersubject consistency, as measured by Spearman rho, was high with average rho = .78 which is highly significant by Kendall's coefficient of concordance, $W = .81$, $p \ll .001$.

3. The corresponding Spearman correlation coefficient between mean judged minimax and the within-groups measure is rho = .98, $p \ll .001$.

REFERENCES

American Optical pseudo-isochromatic plates for testing colour perception. New York: Beck Engraving, 1940.

Anglin, J. The growth of word meaning. Cambridge: MIT Press, 1970.

Bruner, J. S., Goodnow, J. J., & Austin, G. A. A study of thinking. New York: Wiley, 1956.

Fisher, Sir R. A. Statistical methods for research workers. Edinburgh: Oliver & Boyd, 1970. 14th edition.

Garner, W. R. The processing of information and structure. New York: Wiley, 1974.

Garner, W. R. & Creelman, C. D. Problems and methods of psychological scaling. In G. Summers (Ed.), Attitude measurement. New York: Rand McNally, 1970.

Goldstein, K. & Scheerer, M. Abstract and concrete behavior: An experimental study with special tests. Psychological Monographs, 1941, 53 (Whole No. 239).

Handel, S. Classification and similarity of multidimensional stimuli. Perceptual Motor Skills, 1967, 24, 1191-1203.

Handel, S. & Imai, S. The free classification of analyzable and unanalyzable stimuli. Perception and Psychophysics, 1972, 12, 108-116.

Handel, S. & Preusser, D. The effects of sequential presentation and spatial arrangements on the free classification of multidimensional stimuli. Perception and Psychophysics, 1969, 6, 69-72.

Hays, W. Quantification in psychology. Belmont, Calif.: Brooks/Cole, 1967.

Hotelling, H. The selection of variates for use in prediction with some comments on the general problem of nuisance parameters. Annals of Mathematical Statistics, 1940, 11, 271-283.

Imai, S. Classification of sets of stimuli with different stimulus characteristics and numerical properties. Perception and Psychophysics, 1966, 1, 48-54.

Imai, S. & Garner, W. R. Discriminability and preference for attributes in free and constrained classification. Journal of Experimental Psychology, 1965, 69, 596-608.

Imai, S. & Garner, W. R. Structure in perceptual classification. Psychonomic Monograph Supplement, 1968, 2 (9), 153-172.

Lewis, D. Quantitative methods in psychology. New York: McGraw-Hill, 1960.

MacGregor, J. N. Two tests of a coded-element model of perceptual processing. Unpublished Master's thesis, University of Victoria, 1975.

Mandler, G. Organization and memory. In K. W. Spence & J. T. Spence (Eds.), The psychology of learning and motivation. New York: Academic Press, 1967, Vol. 1.

McNemar, Q. Psychological statistics. New York: Wiley, 1969. 4th edition.

Miller, G. A. A psychological method to investigate verbal concepts. Journal of Mathematical Psychology, 1969, 6, 169-191.

Ozioko, J. Dimensional dominance and discrimination shift learning in adults. Unpublished Master's thesis, University of Victoria, 1976.

Preusser, D. & Handel, S. The free classification of hierarchically and categorically related stimuli. Journal of Verbal Learning and Verbal Behavior, 1970, 9, 222-231.

Torgerson, W. Theory and methods of scaling. New York: Wiley, 1960.

Trabasso, T. & Bower, G. H. Attention in learning. New York: Wiley, 1968.

Vygotsky, L. Thought and language. Cambridge: MIT Press, 1934/1962.

APPENDIX A

Instructions

Experiment I: Similarity Scaling of Object Pairs

Your task is to scale some pairs of objects in terms of the similarity that you perceive between them. First, however, I will show you what I mean by scaling. Look through these cards that have different sized circles drawn on them, and rank order the circles in terms of their area. (Hand deck of cards to subject.) Now lay the cards out on this scale in increasing order so that the distances between cards reflect the differences that you see between them in size; the bigger the difference in area between two cards, the bigger the distance between them. Please note that I am asking you to scale the circles in terms of area and not in terms of diameter. The distances between cards will be measured between the black marks at the centres of the cards. It is important to note that the cards can straddle the dividing lines like this. (Demonstrate.) In this way you can make any fractional distance you want. I am going to place the smallest circle in this position. (Place in extreme left position.) You place the other cards on the scale so that the distances between cards are proportional to the differences in size. If you need more divisions at the other end of the scale, just ask and I will add them. If, in your scaling, you find several cards out of order, feel free to change them around until you are satisfied. Any questions?

That's enough practice. These are the stimuli for your final task. (Give deck of cards to the subject.) Take a minute and look briefly through the cards so that you are familiar with them. As you can see, a different pair of objects is drawn on each card. The deck contains all possible pairs of the 16 different objects. Your job is to scale the cards on the <u>physical similarity that you perceive</u> between the two objects on each card. Notice that I am asking you to judge the similarity between these two objects, for example, and <u>not</u> the similarity between this card and that one. (Illustrate.) Ignore the position of the two objects on a card because position is irrelevant to the similarity between them. I decided by chance to draw an object on the left or the right side. The black lines mark the centre of each card. As you can see, the 16 objects differ only in shape and colour; they are all the same size. If you look really closely at the cards you can see very small differences in colour, for example, between this red circle and this red circle. I want you to ignore these small differences because the two red circles are supposed to be identical. Unfortunately, my drawing skills are not that good.

There are so many cards to scale this time that I want you to sort them first into a number of piles on the basis of similarity. For example, you can place the cards for which the two objects on a given card are very similar to one another in one pile here, those cards with very dissimilar pairs of objects over here, and other piles in between. In other words, arrange the piles in order from most to least similar pairs of objects. You can use any number of piles. You may want to divide the cards into three or four piles first and then subdivide some of those piles. In sorting these cards into piles you may have difficulty in deciding whether a given card belongs in one pile or in the adjacent pile. Don't worry about being too accurate at this stage of the task because later you will have the opportunity to rearrange the cards accurately on the scale. The only reason for sorting them into piles is to make the scaling easier.

When you have completed sorting the cards into piles, rank the cards within each pile from most to least similar pairs of objects. If at this time you want to change some cards from one pile to another, do so. If you find several cards for which the two objects on each card appear equally similar, then you can group them together without regard to order. Once again, the purpose of ranking the cards is to make the scaling easier. Any questions? Could you repeat your instructions back to me, please. (Give the subject time to sort and rank.)

Now you can place the cards on the scale so that the distances between the cards reflect the differences in similarity that you perceive between the pair of objects on each card. I am going to place these 16 cards in the extreme left position on the scale, and I want you to consider them to be pairs of identical objects. This provides a zero point for your scale--these 16 pairs do not differ; they are the most similar pairs of objects. First, I want you to place the other piles on the scale so that the distances between piles are proportional to the differences in similarity between pairs that are in different piles. Second, you can spread out the cards in each pile on the scale. The black lines on a card mark its centre. Judge your distances from the centre of one card to the centre of another, that is, from one black mark to another. It is important to remember that the cards can be placed at any point between the scale lines. (Demonstrate.) In this way you can make any fractional distances that you need. If you consider two or more cards to be of equal similarity, place them one above the other on the scale like this. (Demonstrate.)

This is the final scaling so be sure that all cards are in the right order from the most to the least similar and that the distances between cards reflect the differences that you see in the similarity between the pairs of objects. Arrange and rearrange the cards until you are satisfied. If you need more sections added to the scale, ask me. (Show extra sections to the subject.) Any questions? Would you repeat your instructions back to me, please?

Post-experimental Questionnaire

1. Did the subject place the cards on the scale so that the distance between any two cards is proportional to the difference in pairwise similarity? Check that the subject made distances proportional to differences in similarity. This can be accomplished by pointing to two pairs of cards with approximately equal distances between them and asking the subject whether the difference in similarity between one pair is the same as the difference between the other two cards.

2. Rank the pairs of colours from most to least similar.

3. Rank the pairs of shapes from most to least similar.

4. Do you think that you could scale these cards the same way if I asked you to do it again? Do the distances between cards reflect the differences that you perceive in similarity, or have you just arbitrarily decided that, say R and G, are different by this much and so on?

5. Do you have any colour vision problems?

Please do not discuss this experiment with anyone else for the next six weeks. Your discussion of this experiment might bias the responses of one of my future subjects.

<u>Experiment 1: Minimax</u>
<u>Scaling of Partitions</u>

This is another scaling task much like those you have already completed. These are the cards that I want you to scale this time. (Hand cards to subject.) As you can see, each card has a number of objects drawn on it, although the number of objects and the objects themselves may differ from card to card. There are 16 different objects in all. They differ from one another in shape and colour, but not in size. If you look really closely at the cards, you can see very small differences in colour, for example, between these two red objects. Please ignore these small differences because the two objects are supposed to be identical in colour. Unfortunately, I am not a skilled artist, and so I am not able to reproduce the colours and the shapes with great precision.

The objects on each card have been placed into groups. A group is just a collection of objects in which the position (the order) of the objects within the group is irrelevant. I've drawn black lines to separate the groups from one another. This card, for example, has __ groups, and this one has __ groups.

Your task is to rank order the cards in terms of the degree to which the objects are arranged on a card such that the similarity among the objects within each group is maximized and, simultaneously, the similarity between objects in different groups is minimized. To what extent are the objects the same within and different between? Another way of describing your task is this: Rank the cards from "best" to "worst" where the objects on the best card are the most similar within and the most different between and where the objects on the worst card are the most different within and the most similar between. In making this judgment you must consider, first, the similarity among objects in the same group and, second, the similarity between objects in separate groups. (Illustrate.) Your overall judgment for a card <u>must</u> take both factors into account.

You will probably find it easier to rely on your perception of the similarity within and between than it is to develop a very complex system for determining them. Trying to use a complex system can become very confusing. Several people who tried to do this earlier almost quit because it became so difficult. So trust your perception, your intuition, in judging the "goodness" of the cards.

In making your judgment I want you to ignore both the position of the objects within a group and the order that the groups are drawn on a card. I randomly arranged both. Your judgments should not depend on whether the red square was drawn before the red hexagon or vice versa. Nor should your judgments depend on whether this group occurs first on a card or last.

Take the top two cards as an example. On which card have the objects been grouped so that the objects within a group are most similar to one another and, at the same time, most different from objects in other groups? Now is the third card better than both of these cards, or worse, or in between? Just continue this procedure until all the cards are ranked.

Any questions? Would you repeat back to me your instructions, please?

Now I want you to lay the cards out on the scale so that the distances between the cards reflect the differences that you perceive between them in terms of the degree to which the similarity within groups is maximized and, simultaneously, the similarity between groups is minimized. I will place the first card in this position for you. (Place the highest ranked card in the extreme left box.) You can place the other cards relative to it. The distances between cards will be measured between the black marks at the centres of the cards. Remember that the centre black mark does not have to be placed directly on one of the scale dividing lines. (Illustrate.) If you need more divisions at either end of the scale, just ask and I will add more sections. If, in your scaling, you find several cards out of order, feel free to change them around until you are satisfied. Any questions?

Experiment 2: Similarity
Scaling of Object Pairs

Your task is to scale some pairs of objects in terms of the similarity that you perceive between them. First, however, I will show you what I mean by scaling. Look through these cards that have different sized circles drawn on them, and rank order the circles in terms of their area. (Hand deck of cards to subject.) Now lay the cards out on this scale in increasing order so that the distances between cards reflect the differences that you see between them in size; the bigger the difference in area between two cards, the bigger the distance between them. Please note that I am asking you to scale the circles in terms of area and not in terms of diameter. The distances between cards will be measured between the black marks at the centres of the cards. It is important to note that the cards can straddle the dividing lines like this. (Demonstrate.) In this way you can make any fractional distance you want. I am going to place the smallest circle in this position. (Place in extreme left position.) You place the other cards on the scale so that the distances between cards are proportional to the differences in size. If you need more divisions at the other end of the scale, just ask and I will add them. If, in your scaling, you find several cards out of order, feel free to change them around until you are satisfied. Any questions?

That's enough practice. These are the stimuli for your final task. (Give deck of cards to the subject.) Take a minute and look briefly through the cards so that you are familiar with them. As you can see, there is a different pair of objects on each card. The deck contains all possible pairs of the 15 different objects. Your job is to scale the cards on the physical similarity that you perceive between the two objects on each card. Notice that I am asking you to judge the similarity between these two objects, for example, and not the similarity between this card and that one. (Illustrate.) As you can see, the 15 objects differ in shape and colour. Although they also differ slightly in size, I want you to ignore those small differences. I couldn't obtain objects that were absolutely identical in size. Your judgments of similarity should be independent of the position of the two objects on a card. Whether an object is on the left or on the right side of a card should not affect your judgments. The black lines mark the centre of each card.

There are so many cards to scale this time that I want you to sort them first into a number of piles on the basis of similarity. For example, you can place the cards for which the two objects on a given card are very similar to one another in one pile here, those cards with very dissimilar pairs of objects over here, and other piles in between. In other words, arrange the piles in order from most to least similar pairs of objects. You can use any number of piles. You may want to divide the cards into three or four piles first and then subdivide some of those piles. In sorting these cards into piles you may have difficulty in deciding whether a given card belongs in one pile or in the adjacent pile. Don't worry about being too accurate at this stage of the task because later you will have the opportunity to rearrange the cards accurately on the scale. The only reason for sorting them into piles is to make the scaling easier.

When you have completed sorting the cards into piles, rank the cards within each pile from most to least similar pairs of objects. If at this time you want to change some cards from one pile to another, do so. If you find several cards for which the two objects on each card appear equally similar, then you can group them together without regard to order. Once again, the purpose of ranking the cards is to make the scaling easier. Any questions? Could you repeat your instructions back to me, please? (Give the subject time to sort and rank.)

Now you can place the cards on the scale so that the distances between the cards reflect the differences in similarity that you perceive between the pairs of objects on each card. I am going to place these 15 cards in the extreme left position on the scale, and I want you to consider them to be pairs of identical objects. This provides a zero point for your scale--these 15 pairs do not differ; they are the most similar pairs of objects. First, I want you to place the other piles on the scale so that the distances between piles are proportional to the differences in similarity between pairs that are in different piles. Second, you can spread out the cards in each pile on the scale. The black lines on a card mark its centre. Judge your distances from the centre of one card to the centre of another, that is, from one black mark to another. It is important to remember that the cards can be placed at any point between the scale lines. (Demonstrate.) In this way you can make any fractional distances that you need. If you consider two or more cards to be of equal similarity, place them one above the other on the scale like this. (Demonstrate.)

This is the final scaling so be sure that all cards are in the right order from the most to the least similar and that the distances between cards reflect the differences that you see in the similarity between the pairs of objects. Arrange and rearrange the cards until you are satisfied. If you need more sections added to the scale, ask me. (Show extra sections to the subject.) Any questions? Would you repeat your instructions back to me, please?

Experiment 2: Minimax
Scaling of Partitions

This is another scaling task much like those you have already completed. These are the cards that I want you to scale this time. (Hand cards to subject.) As you can see, each card has a number of objects on it, although the number of objects and the objects themselves may differ from card to card. There are 15 different objects in all. They differ from one another in shape and colour. Please ignore the small differences in size because I could not obtain ones absolutely identical in size.

The objects on each card have been placed into groups. A group is just a collection of objects in which the position (the order) of the objects within the group is irrelevant. I've drawn black lines to separate the groups from one another. This card, for example, has __ groups, and this one has __ groups.

Your task is to rank order the cards in terms of the degree to which the objects are arranged on a card such that the similarity among the objects within each group is maximized and, simultaneously, the similarity between objects in different groups is minimized. To what extent are the objects the same within and different between? Your overall judgment for a card must take both factors into account.

I am interested in your perception of the physical similarity among the objects. Consider this card, for example. Let us say that you are judging the physical similarity between the objects in this group and those in that group. (Illustrate.) Do <u>not</u> consider the similarity in number between these two groups because that kind of similarity is not based on the physical properties of the objects themselves.

In making your judgments I want you to ignore both the position of the objects within a group and the order that the groups are drawn on a card. I randomly arranged both. Your judgments should not depend on whether the red square was drawn before the red hexagon or vice versa. Nor whould your judgments depend on whether this group occurs first on a card or last.

Let us start with the top two cards. On which card have the objects been grouped so that the objects within a group are most similar to one another and, at the same time, most different from objects in other groups? Now is the third card better than both of these cards, or worse, or in between? Just continue this procedure until all the cards are ranked.

Any questions? Would you repeat back to me your instructions, please?

Now I want you to lay the cards out on the scale so that the distances between the cards reflect the differences that you perceive between them in terms of the degree to which the similarity within groups is maximized and, simultaneously, the similarity between groups is minimized. I will place the first card in this position for you. (Place the highest ranked card in the extreme left box.) You can place the other cards relative to it. The distances between cards will be measured between the black marks at the centres of the cards. Remember that the centre black mark does not have to be placed directly on one of the scale dividing lines. (Illustrate.) If you need more divisions at either end of the scale, just ask and I will add more sections. If, in your scaling, you find several cards out of order, feel free to change them around until you are satisfied. Any questions?

APPENDIX B

Transforming Judgments to a Common Range

In the two scaling experiments no standard interval or range was set, and subjects were free to choose their own size of interval. The judgments for each subject were scaled to a common range to make them directly comparable across subjects. The range was scaled to 100 by multiplying each distance by 100/range. If for one subject the distance on the scale was 5 for the extreme stimulus *x*, it was multiplied by $m = 100/5 = 20$. The value for all other partitions for a given subject were scaled by multiplying by the same constant *m*. This transformation preserved the sizes of intervals relative to each other.

APPENDIX C

Supplementary Analysis of Experiment 1

The correspondence between predicted and obtained minimax judgments was assessed at three different levels of measurement-rank order, ordered metric, and interval--where the levels make successively more restrictive assumptions about the measurement properties that are assumed to be reflected in the judgments made by subjects. At the rank-order level, the ranks for each partition were averaged over subjects to give the mean ranked minimax (see Tables 10 and 11). For each model, predictions were generated from the average similarity judgments. The correlation between mean ranked minimax and the predicted ranking for each model was high and obviously significant. The Spearman correlation was .96 for the weighted means model and .88 for the unweighted means model. The weighted means model was not a significantly better predictor of mean ranked minimax than the unweighted means model by Hotelling's (1940) test of the significance of the difference between correlated correlations, $t(10) = 1.96$, $p > .10$. By the sign test (Siegel, 1956), the mean ranking of the partitions predicted by the weighted means model was not a significantly better predictor of the mean minimax ranking than was the ranking predicted by the unweighted means model, $x(8) = 3$, $p > .10$. This

Table 10

Minimax Rankings of the 13 Partitions for Each Subject in Experiment 1

Subject Number	Partition Number												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	2.5	2.5	5	6	1	4	8	9	7	12	10	11	13
2	1	2	3	5	6	4	9	7	8	10	11	12	13
3	4.5	2	3	1	4.5	9	6	7	10	8	11	12	13
4	2	4	3	5	1	6	8	7	10	9	12	11	13
5	1.5	1.5	3	5	10	4	6	7	9	8	11	12	13
6	1	2	4	5	10	3	6	11.5	7	11.5	8	9	13
7	1.5	4	5	3	1.5	6	9	7.5	10	7.5	11	12	13
8	1.5	5	4	1.5	3	6	11	10	9	8	7	12	13
Sum of Ranks	15.5	23	30	31.5	37	42	63	66	70	74	81	91	104
Average Rank	1.94	2.88	3.75	3.94	4.63	5.25	7.88	8.25	8.75	9.25	10.13	11.38	13.00
Overall Rank	1	2	3	4	5	6	7	8	9	10	11	12	13

Table 11

Mean Predicted and Obtained Minimax Rankings for

Each of the Partitions in Experiment 1

Partition Number	Mean Ranked Minimax	Predicted Rank Based on Average Similarity Judgments	
		Weighted	Unweighted
1	1	1.5	1.5
2	2	3.5	3
3	3	3.5	6
4	4	1.5	1.5
5	5	6	5
6	6	5	8
7	7	7.5	4
8	8	7.5	10
9	9	10	9
10	10	9	7
11	11	12	12
12	12	11	11
13	13	13	13

result can be interpreted to mean that neither model predicts significantly better than the other, but such an interpretation is contradicted by a comparison of predicted and obtained rankings for individual subjects.

The average of the Spearman correlations between an individual's minimax ranking and the ranking predicted from the individual's similarity judgments was .80 for the weighted means model and .71 for the unweighted means model (see Table 12). For each of the eight subjects, the individual minimax ranking correlated more highly with the weighted than the unweighted means model, a difference that is highly significant by the sign test, $\underline{p} < .01$. Thus, the weighted-means model was a better predictor of minimax ranking across all subjects.

The rank-order correlation between an individual's minimax ranking and that predicted by either model from the individual's similarity judgments was highly significant for seven of the eight subjects. In scaling similarity the atypical subject judged pairs differing in colour only to be more similar than pairs differing in shape only, but reversed this judgment in scaling the partitions. This change was obvious both from the written report and the ranking of the partitions. Furthermore, the correlation between each individual's minimax ranking and that predicted by each model for the <u>average</u> similarity judgments was highly significant for all eight subjects.

Table 12

Spearman Correlations Between Predicted and Obtained Minimax

Judgments for Each Subject in Experiment 1

Subject	EM1	EM2	M1M2	$\bar{E}$M1	$\bar{E}$M2	E$\overline{M1}$	E$\overline{M2}$
1	.388	.234	.857	.478	.286	.793	.712
2	.853	.824	.900	.845	.846	.930	.765
3	.803	.688	.731	.692	.560	,902	.897
4	.885	.731	.879	.967	.819	.891	.801
5	.905	.762	.852	.945	.808	.890	.778
6	.839	.831	.956	.962	.945	.756	.675
7	.906	.860	.896	.951	.907	.910	.848
8	.814	.781	.889	.973	.902	.837	.747
Median *rho*	.846	.793	.884	.948	.832	.890	.772
Mean *rho*	.799	.714	.870	.852	.759	.864	.778

NOTE. All correlations based on n = 13 partitions. A correlation coefficient must exceed the critical value of .478 to be significant at the .05 level and .643 at the .01 level. E = individual minimax judgments; $\bar{E}$ = mean judged minimax; M1 and M2 = the predicted minimax judgments for the weighted and unweighted means models, respectively (both M1 and M2 are based on individual similarity judgments); $\overline{M1}$ and $\overline{M2}$ = predictions based on the average similarity judgments.

To assess the correspondence between predicted and obtained scales at the ordered metric level of measurement, the differences in minimax judgments between all possible partitions (i.e., the intervals) were ranked from smallest to largest for each of the three averaged scales given in Table 4: mean judged minimax and the minimax judgments predicted by each model. The rank-order correlation between the intervals in mean judged minimax and the intervals in the mean minimax judgments predicted by the weighted-means model was .74 (see Figure 14). The rank-order correlation between the intervals in mean judged minimax and the intervals in the mean minimax judgments predicted by the unweighted model was .54 (see Figure 15). Both correlations are highly significant at $p < .001$. The weighted-means model was, however, a significantly better predictor of the ranking of the actual interval sizes than the unweighted-means model, $t(75) = 3.04$, $p < .01$.

At the interval level of measurement, the correspondence between predicted and obtained scales can be assessed in several different ways. First, if there is to be correspondence at the interval level, then intervals between the obtained minimax judgments must vary as a linear function of the intervals between the predicted minimax judgments. From the plots in Figures 14 and 15, it is apparent that both the weighted and the unweighted intervals vary linearly with the obtained intervals, although there is considerably more

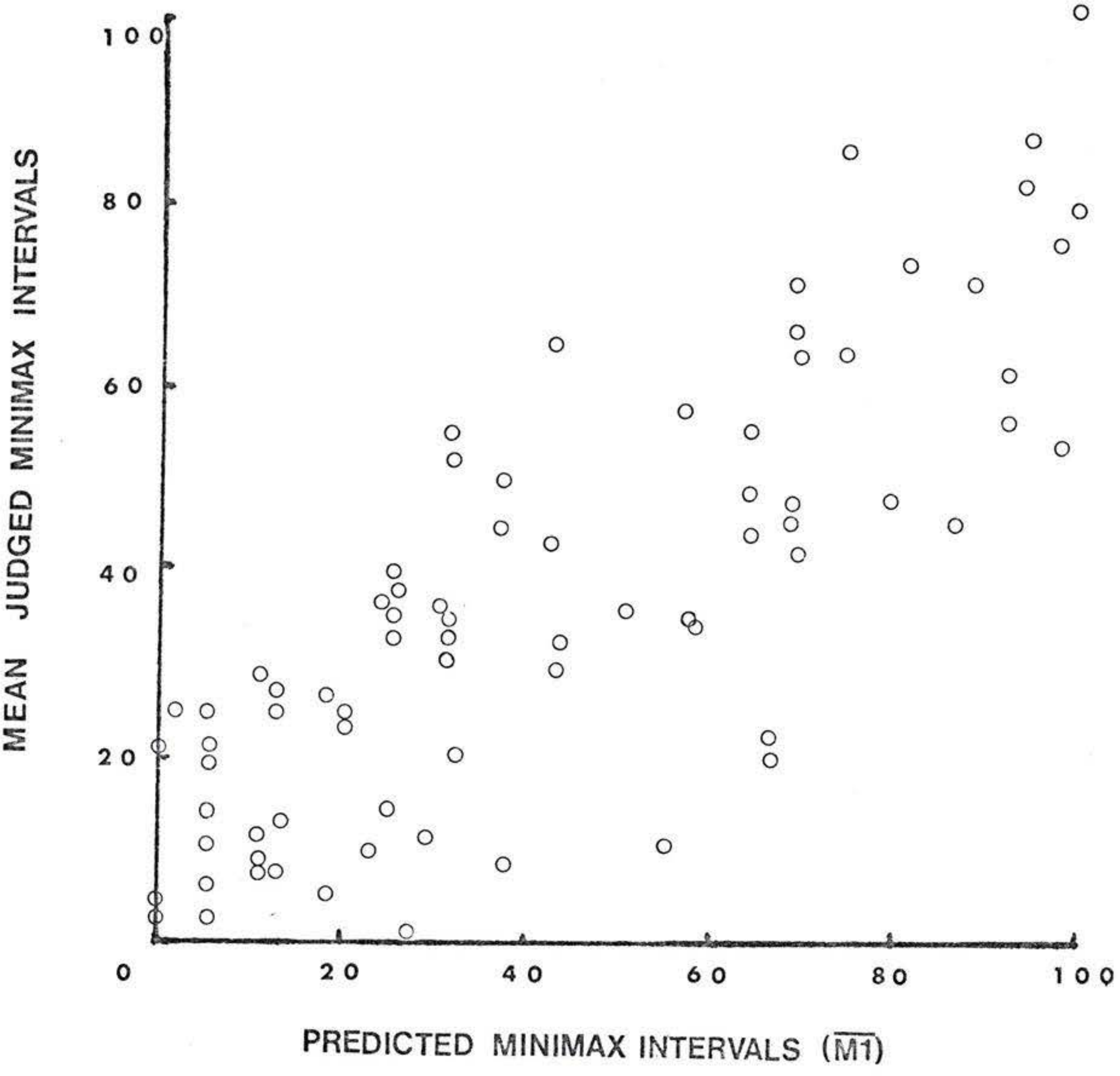

Figure 14. Intervals between the obtained minimax judgments as a function of the intervals between the minimax judgments predicted by the weighted-means model ($\overline{M1}$) for Experiment 1.

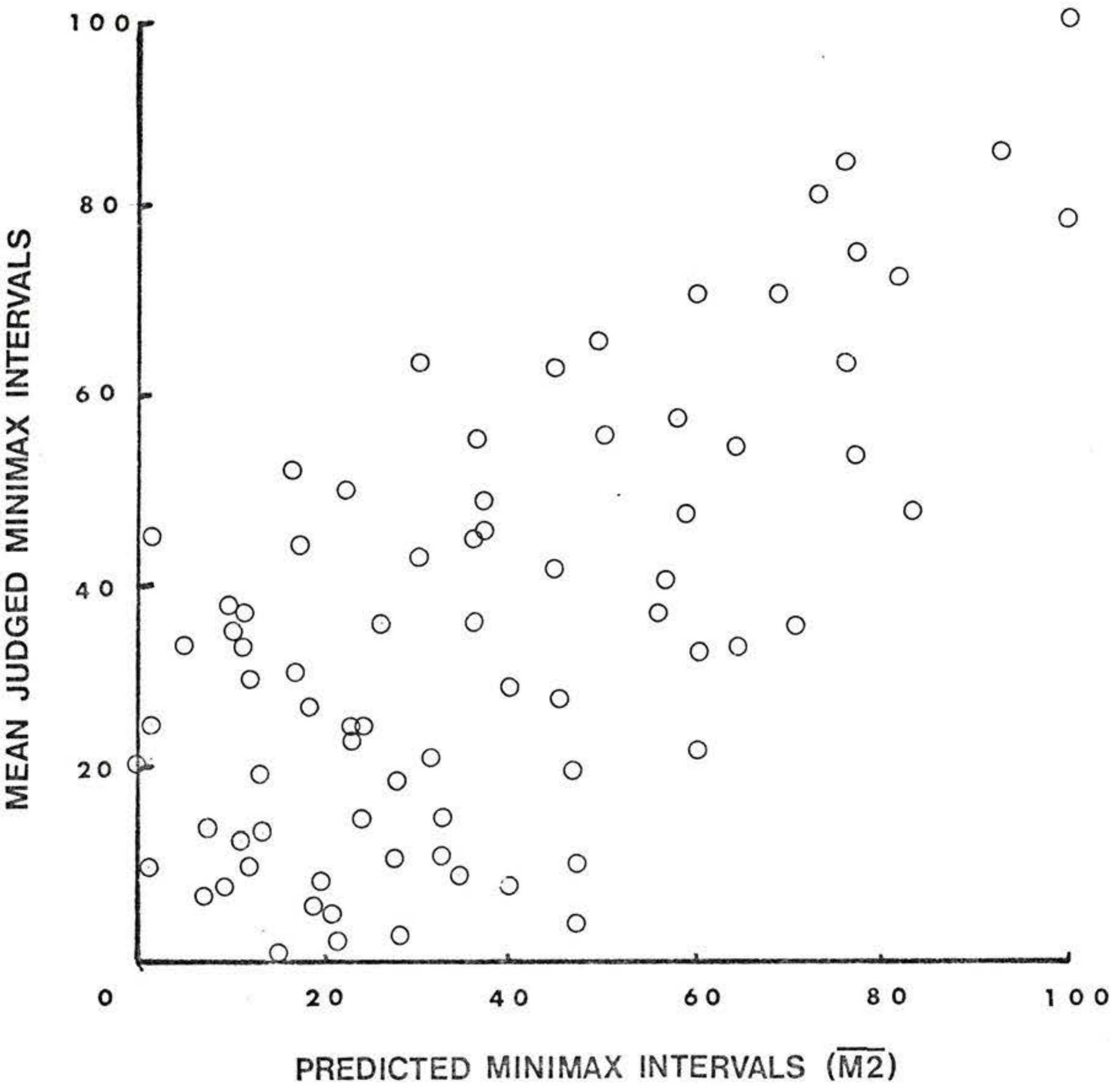

Figure 15. Intervals between the obtained minimax judgments as a function of the intervals between the minimax judgments predicted by the unweighted-means model ($\overline{M2}$) for Experiment 1.

scatter in the unweighted scores. Second, a subject's average deviation between predicted and obtained judgments for the 13 partitions should be smaller for the model that corresponds closest at the interval level. As discussed in the results section of Experiment 1, there was no significant difference between the models. Third, the deviation between <u>averaged</u> predicted and obtained minimax judgments should be minimal if there is correspondence at the interval level. However, as discussed in the results section to Experiment 1, both models deviated a small, but significant, amount from the obtained judgments. And fourth, the deviation between <u>individual</u> predicted and obtained minimax judgments for the 13 partitions should be minimal if there is correspondence at the interval level. This prediction was tested in two parallel repeated measures analyses of variance, each with two within-groups factors. The first factor was minimax, that is, predicted and obtained minimax judgments for each of the eight subjects; the second factor was partitions with 13 levels. Thus, for each subject there were 13 predicted scores and 13 obtained scores. The two analyses, one for each model, are summarized in Tables 16 and 17. There was a significant minimax by partitions interaction in both analyses indicating a significant departure of predicted from obtained judgments for some but not all of the partitions. Tests of simple main effects (see Table 16) showed that the weighted predictions deviated

Table 17

ANOVA of the Difference Between Predicted (M1) and

Obtained Minimax Judgments in Experiment 1

Source	df	MS	F
1. Subjects	7	824.48	
2. Minimax	1	1,296.00	1.97
3. Minimax X Subjects	7	658.30	
4. Partitions	12	14,495.02	47.81***
5. Partitions X Subjects	84	303.16	
6. Minimax X Partitions	12	543.36	3.04**
7. Minimax X Partitions X Subjects	84	178.60	
8. Minimax at Partition 1	1	167.03	
2	1	40.06	
3	1	175.25	
4	1	664.55	
5	1	73.55	
6	1	441.00	
7	1	1,226.68	4.76*
8	1	1,363.38	5.29*
9	1	190.16	
10	1	1,258.55	4.89*
11	1	1,930.88	7.50**
12	1	286.42	
13	1	.33	
9. Error (3+5+7)	175	257.58	

* $p < .05$
** $p < .01$
*** $p < .00001$

Table 18

ANOVA of the Difference Between Predicted (M2) and Obtained Minimax Judgments in Experiment 1

Source		df	MS	F
1. Subjects		7	860.32	
2. Minimax		1	840.42	1.33
3. Minimax X Subjects		7	630.13	
4. Partitions		12	12,436.72	44.32***
5. Partitions X Subjects		84	280.63	
6. Minimax X Partitions		12	851.52	4.68***
7. Minimax X Partitions X Subjects		84	181.75	
8. Minimax at Partition	1	1	170.43	
	2	1	12.33	
	3	1	657.16	
	4	1	664.55	
	5	1	64.87	
	6	1	756.25	
	7	1	3,641.88	14.74**
	8	1	515.47	
	9	1	885.99	
	10	1	3,433.96	13.89**
	11	1	50.75	
	12	1	217.34	
	13	1	0.0	
9. Error (3+5+7)		175	247.15	

** p < .01
*** p < .00001

significantly from the obtained judgments for four partitions--7, 8, 10, and 11. Similarly, the unweighted predictions deviated significantly from the obtained judgments for two partitions--7 and 10. Thus, by this analysis the unweighted model was a slightly better predictor of the individual obtained judgments. It should be noted, however, that comparisons between models are not judtified in this type of analysis because both model predictions are based on the same sample of similarity data.

On the basis of the preceding four analyses, there is no clear superiority of one model over another at the interval level of measurement.

APPENDIX D

Pilot Study 1

In this study the stimuli were the 13 possible partitions (excluding division into four groups or one group) of the four stimulus objects RC, RS, BT, and GT. The hypothetical pairwise similarities listed on page 14 were used to generate predicted rankings of the 13 partitions on the minimax dimension for both the weighted and the unweighted models (see Table 1). Five subjects ranked the 13 partitions on the minimax dimension according to the instruction "Rank order the cards in terms of the degree to which the similarity within groups is maximized and, simultaneously, the similarity between is minimized." The judged ranks for each partition, averaged across subjects, are shown in Table 14. The rank order correlation of .90 between theoretical ranking (either by $\overline{M1}$ or $\overline{M2}$) and mean minimax judgments is obviously significant. The corresponding Pearson r of .88 is also high. The models account for a considerable proportion of the variation in minimax judgments (r^2 = .77), but the models are indistinguishable in terms of their predictions in this situation.

Subsequent to the partition scaling, two of the original subjects scaled the six possible pairs of the objects on similarity. The scaling method employed was a modified version of that described for Experiment 1 in the section

Table 14

Predicted and Obtained Rankings of Partitions on the Minimax Dimension in Pilot Study 1

Partition Description[a]	Mean Ranked Minimax	Predicted Ranking	Postdicted Ranking	Postdicted Judgments
(GT,BT),(RS),(RC)	1.0	1	1	.25
(RC,RS),(GT,BT)	2.8	2	2	.27
(RC,RS),(GT),(BT)	4.4	3	3	.48
(RS,GT,BT),(RC)	5.2	4.5	4	.63
(GT,BT,RC),(RS)	6.6	4.5	9	1.34
(RS,GT),(BT),(RC)	6.7	9.5	5.5	1.02
(BT,RS),(GT),(RC)	7.1	9.5	5.5	1.02
(GT,RC,RS),(BT)	8.2	6.5	7.5	1.08
(RC,GT),(BT),(RS)	8.8	9.5	12.5	1.78
(BT,RC,RS),(GT)	9.4	6.5	7.5	1.08
(RC,BT),(GT),(RS)	9.6	9.5	12.5	1.78
(BT,RS),(RC,GT)	10.0	12.5	10.5	1.35
(BT,RC),(RS,GT)	11.2	12.5	10.5	1.35

[a] A partition description consists of an enumeration of all objects plus a specification of how the objects are divided into groups. Objects enclosed within parentheses are in the same group; objects separated by parentheses are in different groups. G = green, R = red, B = black, C = circle, T = triangle, S = square.

on Similarity Scaling. Converted to a common modulus (see Appendix B), the two scales appeared very similar and were averaged:

Object Pair	Mean Similarity S_{ij}
GT-BT	1.00
RC-RS	1.85
RS-GT	3.55
RS-BT	3.55
RC-GT	5.50
RC-BT	5.50

Postdicted rankings of the partitions on minimax were generated using the empirical pairwise similarities (see Table 14). Again the postdicted rankings did not differ for the two models. The correlation of <u>rho</u> = .87 between the postdicted ranking based on empirical similarities and mean minimax judgments was again highly significant. The corresponding product-moment correlation was <u>r</u> = .89.

Pilot Study 2

Pilot Study 2 was designed to answer three questions: first, whether the correlation between predicted and obtained minimax judgments would be higher for the "intuitive" or the "non-intuitive" version of the minimax instructions; second, whether the correlation between predicted and obtained judgments would be lower when a reduced, rather than a total, set of pairwise stimuli was employed in similarity scaling; and third, to test the predictive power of the weighted and unweighted models. For one-half of the subjects, the minimax instructions in the intuitive condition were identical to those for Experiment 1 (see Appendix A). For the other half of the subjects, those in the non-intuitive condition, the fourth paragraph was deleted from the Experiment 1 instructions. For one-half of the subjects, the stimulus set employed in similarity scaling consisted of the 120 possible pairs of the 16 objects. For the other half of the subjects, the set of 120 stimulus pairs was reduced to 48 by eliminating "logically equivalent" stimulus pairs. For example, GS-YH and GH-YS are equivalent pairs on the assumption that the stimulus dimensions do not interact in their effect on the perception of similarity. Also, pairs such as RH-RC, BH-BC, YH-YC, and GH-GC are all equivalent in the sense that the two objects in a given pair differ in exactly the same way across all four pairs. In all other respects the experimental procedure, instructions, stimuli, etc. were the same as those employed in Experiment 1, but with partitions number 3 and 11 removed. The eight subjects were not randomly assigned to experimental conditions.

The Pearson correlations between predicted and obtained minimax judgments are reported in Table 15. An analysis of variance with two between-groups and one within-groups factors were performed on the Fisher z-score equivalents of the Pearson correlations. There were no significant differences on any of the factors. Although the weighted model was not a significantly better predictor of the obtained minimax judgments than the unweighted model, six of the eight subjects were best predicted by the weighted model. One subject in each of the instructional conditions--intuitive and non-intuitive--were best predicted by the unweighted model. However, both models were very accurate predictors of the obtained minimax judgments; the average correlation (by the Fisher r to z transformation) between the obtained minimax judgments and those predicted by the weighted model was .81; the corresponding correlation between the obtained judgments and those predicted by the unweighted model was .79.

Although the instructional and stimulus set factors were not significant, these data indicated that the total set of pairwise stimuli and the non-intuitive minimax instructions produced the highest correlations between predicted and obtained judgments. In Experiment 2, the use

Table 15

Pearson Correlations Between Predicted and Obtained Minimax Judgments for Each Subject in Pilot Study 2

		Subject	EM1	EM2	M1M2
A_1	C_1	1	.86	.84	.92
		2	.76	.60	.88
	C_2	3	.76	.72	.85
		4	.30	.39	.83
A_2	C_1	5	.96	.89	.85
		6	.88	.98	.86
	C_2	7	.38	.31	.85
		8	.94	.88	.87

Note. All correlations based on N = 13 partitions. A correlation coefficient must exceed the critical value of .553 to be significant at the .05 level and .684 at the .01 level. E = individual minimax judgments; M1 and M2 = the predicted minimax judgments for the weighted and unweighted means models, respectively; A_1 = intuitive minimax instruction; A_2 = non-intuitive minimax instruction; C_1 = total set of stimuli in similarity scaling; and C_2 = reduced set of stimuli in similarity scaling.

of a version of the non-intuitive instructions and the total set of object pairs was partially justified on these grounds.

APPENDIX E

Tables

Table 13

Spearman Correlations Between Predicted and Obtained Minimax Judgments for Each Subject in Experiment 2

Subject	EM1	EM2	M1M2	$E\overline{M1}$	$E\overline{M2}$
1	.840	.755	.856	.861	.789
2	.761	.954	.794	.670	.755
3	.835	.830	.930	.808	.817
4	.748	.946	.729	.541	.640
5	.771	.918	.815	.791	.960
6	.958	.912	.898	.937	.926
7	.820	.879	.664	.665	.658
8	.420	.142	.801	.538	.313
Median *rho*	.796	.896	.808	.730	.772
Mean *rho*	.769	.792	.811	.726	.738

NOTE. All correlations based on $n = 16$ partitions. A correlation coefficient must exceed the critical value of .427 to be significant at the .05 level and .582 at the .01 level. E = individual minimax judgments; M1 and M2 = the predicted minimax judgments for the weighted and unweighted means models, respectively (both M1 and M2 are based on individual similarity judgments); $\overline{M1}$ and $\overline{M2}$ = predictions based on the average similarity judgments.

Table 16

Standard Deviations of the Averaged Similarity

Judgments of Object Pairs in Experiment 1

Colour	Shape						
	Same	S-H	C-H	T-S	C-S	T-H	C-T
Same	0	21.99	30.15	18.15	26.23	17.02	21.39
G-R	9.95	16.46	16.16	14.81	18.90	13.78	11.45
B-G	12.37	16.80	16.94	14.25	15.91	12.76	15.67
G-Y	17.76	16.41	18.26	16.14	16.96	15.43	17.09
R-Y	18.87	16.70	18.53	14.92	16.70	16.16	15.11
B-R	12.78	15.68	12.25	12.25	9.46	10.46	10.35
B-Y	21.66	16.70	13.87	14.31	17.59	11.99	5.05

NOTE. Each table entry is the standard deviation of the judgments of similarity across 8 subjects for any pair of objects with the shapes and colours specified in the column and row headings, respectively. The column entries under same shape, or the row entries in line with same colour, are each based on 32 scores--8 subjects by 4 possible comparisons. All other entries are each based on 16 scores--8 subjects by 2 possible comparisons. B = black, G = green, R = red, Y = yellow, C = circle, T = triangle, S = square, H = hexagon.

APPENDIX F

Partition Descriptions

The following notational system will be employed to describe all partitions:

(a) each object in the stimulus set is identified by a two-letter label;

(b) objects within a single group are enclosed within parentheses. Within a group the objects are separated by commas and listed in sequence as drawn from top to bottom on the stimulus card;

(c) objects in different groups are separated by parentheses, and the groups are listed in sequence from left to right as they occur on the stimulus card;

(d) and if the partition was never drawn on a stimulus card, then the sequencing of groups and of objects within groups is arbitrary.

Example: (RC,RS),(BT),(GT) represents a partition of 4 objects into 3 groups with a red circle and a red square in the first group, a black triangle in the second group, and a green triangle in the third group. The red square is drawn below the red circle on the stimulus card.

Experiment 1

Let B = black, G = green, R = red, Y = yellow, C = circle, T = triangle, S = square, and H = hexagon.

Partition Number	Partition Description
1	(RH,YH,BH,GH),(BT,RT,GT,YT),(GS,BS,YS,RS),(YC,BC,RC,GC)
2	(BS,YS),(GH,BH,RH,YH),(GT,YT),(GS,RS)
3	(BT,RT,YT,GT),(YC,BS),(YH,GH,RH,BH)
4	(RS,RH,RT),(GC,YC,BC)
5	(RS,RC,RH,RT),(BT,BH,BC,BS),(YH,YT,YS,YC),(GC,GS,GT,GH)
6	(GT,YS),(BC,YC,RC,GC),(RH,GH,BH,YH),(GS,YT)
7	(BS,GS),(RC,RH,RT,BC,RS),(YT,YH)
8	(GS,GC,GH,GT),(BC,YT)
9	(YC,YS),(GS,GC),(RC,BS,GT,YH),(BT,RT),(GH,YT)
10	(BT,BS),(GC,RH,YS,GT,YH,RS,RC)
11	(YS,BH,GT,RC),(RH,RS),(GH,BS,YC,RT)
12	(RS,YT),(GH,RT,YC,BS,RC,BH),(YS,GT,RH,GS,BC,BT),(YH,GC)
13	(BH,YT,RC,GS),(GC,RS,BT,YH),(RT,BC,GH,YS),(BS,RH,YC,GT)

Experiment 2

Let R = red, G = green, P = purple, O = orange, Y = yellow, C = circle, S = square, and H = hexagon.

Partition Number	Partition Description
1	(YC,GC,PC,OC,RC),(GS,YS,RS,PS,OS),(PH,OH,RH,GH,YH)
2	(YC,YS,YH),(GH,GC,GS),(PH,PC,PS),(OC,OS,OH),(RS,RH,RC)
3	(YS,RH,OC),(OS,PC,YH),(GH,RS,YC),(PH,RC,GS),(OH,GC,PS)
4	(RC,GC,PC,OC,YC),(GS,OH),(RS,PS,OS,YS)

Partition Number	Partition Description
5	(OC,OS,OH),(YC,RS,PH),(GC,PS,RH),(RC,YS,GH)
6	(RS,GC),(OH,GH,YH,PH,RH),(YS,RC),(OC,GS)
7	(PC,OC),(YS,GS,RS)
8	(GH,GS),(YC,OC,RC,PC)
9	(RH,RC),(GH,PH,OH,YH)
10	(OH,OC),(YS,RS,PS,GS)
11	(YC,PS),(PH,GH,RH,OH,YH)
12	(PC,GH),(OS,YS,GS,PS,RS)
13	(GC,OS),(RH,YH,PH)
14	(RS,RH),(OC,PS,YH)
15	(GH,OH),(RH,YS,PC)
16	(GS,RS,OS,YS,PS),(RC,OH)
17	(YS,GH),(RC,GC,PC,OC,YC)
18	(GS,RH,YH,PC,OC,RS),(PH,GH)
19	(RS,YS),(RC,OS,PH,GC,GS,YH)
20	(YC,GC),(YH,OC,OS,RC,GH,GS)
21	(PC,PS),(GH,OC,RC)
22	(RH,OC,GS),(YH,YC)
23	(GS,PC,YH,RC,OS,GH),(RH,RS)
24	(GS,GC),(OC,RC,YS,OH,PS,PH)
25	(RS,GS,PS,OS,YS),(GC,YH),(RH,GH,PH,OH)
26	(RC,GC,PC,OC),(YC,GS),(RS,PS,OS,YS)
27	(RC,GC,PC,OC),(YC,PH),(RS,PS,OS,YS)
28	(GC,PC,OC,YC),(RC,OS),(RH,GH,OH,YH)

Partition Number	Partition Description
29	(RC,GC,PC,OC,YC),(OH,YS),(RS,GS,PS,OS)
30	(GS,GH),(PC,RH,OS),(RC,PS,OH)
31	(RS,RH),(GC,PH,YS),(PC,YH,GS)
32	(RS,GS,PS,YS),(OS,OH),(RH,GH,PH,YH)
33	(RC,PC,GC,OC),(GH,OH,PH,RH),(YC,YH)
34	(RC,GC,PC,OC,YC),(RH,OS),(PS,YH)
35	(RC,GC,PC,OC,YC),(RH,OS),(RS,OH)
36	(OS,YC,RH),(GC,GS),(PS,PH)
37	(RC,PS,OH),(OC,GS,PH),(PC,OS,RH),(YC,GC)
38	(RC,PS,OH),(OC,GS,PH),(PC,OS,RH),(YS,YH)
39	(RC,RS,RH),(PC,PS,PH),(YC,YS,YH),(OC,GS)
40	(RC,RS,RH),(PC,PS,PH),(YC,YS,YH),(GC,OH)
41	(RC,RS,RH),(PC,PS,PH),(YC,YS,YH),(GS,OS)
42	(GC,OC,YC),(GS,OS,YS),(GH,OH,YH),(RC,PH)
43	(GC,OC,YC),(GS,OS,YS),(GH,OH,YH),(PC,RS)
44	(GC,RH,PS),(GH,PC,RS),(YH,YC),(RC,GS,OH)
45	(GH,PC,RS),(GC,RH,PS),(RC,OH,GS),(PH,YH)
46	(PH,RC,GS),(OH,PC,YS),(RS,OS),(OC,YC)
47	(PH,RC,GS),(OH,PC,YS),(RS,RH),(GC,GH)
48	(PH,RC,GS),(OH,PC,YS),(YC,YH),(OC,OS)
49	(RS,GH),(RH,GS),(YC,OC,PC,GC,RC)
50	(GS,RS,PS,OS,YS),(YC,PH),(YH,PC)
51	(RC,OS),(OC,RS),(PH,OH,GH,YH,RH)
52	(OH,YH,GH,RH,PH),(PC,GS),(GC,PS)

Partition Number	Partition Description
53	(RC,GC,PC,OC,YC),(RS,GS),(PS,OS)
54	(RC,GC,PC,OC,YC),(RS,PS),(OS,YS)
55	(RS,GS,PS,OS,YS),(RC,PC),(OC,YC)
56	(RS,GS,PS,OS,YS),(RH,PH),(OH,YH)
57	(RH,GH,PH,OH,YH),(RC,YC),(GC,OC)
58	(RH,GH,PH,OH,YH),(RS,YS),(GS,OS)
59	(GC,GS),(GH,YH,PH,RH,OH),(OC,OS)
60	(YH,OS),(OH,YS),(RC,YC,PC,GC,OC)
61	(YS,GC),(RC,GS),(PH,YH,RH,OH,GH),(YC,OS),(OC,PS)
62	(RH,OH,PH,GH,YH),(PS,OS,YS),(RC,GC,PC),(GS,YC),(RS,OC)
63	(RS,GS,PS,OS,YS),(RC,GC,OC,YC),(PC,RH),(PH,OH),(GH,YH)
64	(RS,GS,PS,OS,YS),(RC,GC,OC),(PC,RH),(YC,GH),(PH,YH)
65	(YC,YS),(OC,GS,YH,RC,PS,GH),(RS,RH),(OH,OS),(PH,PC)
66	(RS,OS),(PS,GS,GH,RC,OC,YH),(RH,OH),(PC,GS),(YS,YC)
67	(GH,PH,RS,OC,OS,YC),(GS,GC),(PC,PS),(RH,RC),(YS,OH)
68	(OC,RS,PH,YC,OS,GH),(RC,RH),(GC,GS),(PS,OH),(YS,PH)
69	(RC,RS,RH),(GC,GS,GH),(PC,PS,PH),(OC,OS,YH),(YC,YS,OH)
70	(RS,GS,PS,OS,YS,PC),(RH,GH),(OC,YH)
71	(RH,GH,YH,PH,OH,GC),(RS,OC)
72	(YH,GC,OC,PC,RC,YC),(GH,PS)
73	(RS,GS,PS,OS),(YC,YS,YH),(GC,RH)
74	(PC,RH,RC,RS,PH,PS),(GC,OC),(GH,YH)
75	(GH,OC),(PH,PS,RC,RS,PC,RH),(GC,YH)
76	(OC,YS),(PH,GH,YH,RH,OH),(RS,GS)

Partition Number	Partition Description
77	(RH,GH,PH,OH,YH),(RS,GS),(OC,YC)
78	(RH,GH,PH,OH,YH),(PS,YS),(RC,PC)
79	(GC,OS),(OH,RH,YH,GH,PH),(YS,PS)
80	(RH,GH,PH,OH,YH),(RC,PS),(OC,GS)
81	(PC,RC),(OS,YS)
82	(PC,RC),(YH,GH)
83	(OS,YS),(PH,RH)
84	(YC,RC),(PS,OS)
85	(PS,PC),(GH,GC)
86	(GH,GS),(RC,RS)
87	(GS,GH),(RH,RC)
88	(GS,PS,RS),(OH,YH)
89	(GC,PC,YC),(RH,OH)
90	(GC,PC,OC,YC),(RS,RH)
91	(GC,GS),(RH,PH,YH,OH)
92	(RS,GS,PS,YS),(OC,OH)
93	(GS,GH),(OC,YC,RC,PC)
94	(YH,YC),(PS,OS,RS,GS)
95	(RC,GC,OC,YC),(PS,PH)
96	(GS,PS,OS,YS),(RC,RH)
97	(GC,RS),(RC,GS)
98	(PC,YH),(YC,PH)
99	(GS,OH),(GH,OS)
100	(RC,GS),(RS,GH),(GC,RH)

Partition Number	Partition Description
101	(GH,YC,PH,GC,YS),(PS,PC,RC)
102	(OS,GC,YC,PH,YS),(YH,PC),(PS,GH),(RC,OC,GS)
103	(RH,OS,YH,OC),(GC,GH),(OH,RC,PC,RS),(YS,PS),(PH,YC,GS)
104	(RH,PC,YC),(RC,GS,OH,RS),(OS,PS,YS),(OC,YH),(GH,GC,PH)
105	(OC,GC),(PS,OS,RH,YH,GS,RS),(YS,PH,PC,YC),(GH,OH)
106	(GH,OH,RC,PS),(YS,RH),(OS,YH),(PC,OC)
107	(GS,RS),(RH,OC,PC),(YS,RC),(PC,OH,PS,YC)
108	(PC,RS,PS,YC),(GH,GS,PS),(OH,GC),(YH,RC),(RH,OC)
109	(OH,RH,PH,PS,YC,PC),(GS,OC,YS),(YH,RS,RC,OS,GH,GC)
110	(PH,GS,GC),(YC,PS),(GH,YS,RC,OH),(PC,OS),(OC,YH)
111	(GS,PH,OS,GH,OH,YC),(RS,GC,YH),(RH,PS),(YS,PC),(RC,OC)
112	(PH,YH,OS,PC),(GS,OH,RH),(RS,YC,PS,GH,RC),(OC,YS)
113	(PH,GS,RS,OS,PC,GH),(RH,YC,OH,YH,PS),(OC,RC),(YS,GC)
114	(YS,GS,GC,PH),(YH,OS,RC,GH,PS),(OC,OH,PC)
115	(RS,YC),(OC,PS)
116	(RC,PH,OC,PC,RH),(GC,RS,YS),(GH,OH)
117	(PS,GC,YC,YS),(OC,OS,RS,PC),(OH,GH,PH,RC)
118	(GS,PH,GC),(OH,GH)
119	(PS,GC,YC,YS,OC,OS),(RS,PC)
120	(YS,OC,PS,OH),(YH,RH),(GH,PH),(RC,GC,OS,GS,RS),(YC,PC)
121	(PH,GC,RH,YH),(PC,GH,OC,OS),(YC,RS,RC),(GS,PS),(OH,YS)
122	(RC,PC,OS),(OC,GS,YC)
123	(YH,RC),(GC,OC,GH)
124	(RC,YC,PH),(OC,PC,RS,GS),(OS,PS,RH,GC,YS)

Partition Number	Partition Description
125	(RS,PC),(RC,GS),(OC,OS,YS,OH)
126	(PC,RS,OH,OC,OS,YS),(RC,GS,YC,GC),(PH,YH,GH),(RH,PS)
127	(RC,PS,GH,OS),(PH,YH),(GS,YS),(YC,GC,RH,OH,PC)
128	(OC,RC,RS,YH,PH),(GS,GC,PC,GH,PS),(YS,OS,YC)
129	(PC,PS,YC),(GC,GH,RH,YS),(YH,GS,OH),(RS,PH,OS),(RC,OC)
130	(GC,PS,GS,GH,OC),(OS,YH)
131	(OS,OH,GC,YS,RH,YH),(OC,RC),(GS,RS,PH),(PS,PC)
132	(YC,OC,GH,PH,PC),(OH,RH,GC,YH,GS,OS)
133	(PH,PS,RS,YS),(RH,GS,YH,PC,OH,OC),(GH,RC,OS,PC,YC)
134	(PS,OH),(GC,RC),(YS,RH)
135	(PS,OH),(GH,YH,RS,PH)
136	(YS,RS),(GC,OH,PS,OS,RH,YC),(GS,OC),(RC,PC,YH),(GH,PH)
137	(YH,YS),(PH,PS,OS,OC,GS),(RC,GC,OH)
138	(GH,YS,RH),(PS,RC,OH,OS,YH,GS)
139	(RC,PC,YS,OC,RS,GC),(GH,PH,OH,PS,RH),(GS,OS,YC,YH)
140	(YS,PC),(PS,GC,GH,RC,RS,YC),(YH,PH),(OS,OH),(OC,GS)
141	(RC,PC,RH,OC,OS),(YS,PH,PS,YC,GC,RS)
142	(YH,PC,RH,GS,OC,OS),(YC,RS,OH,GH,YS,PS),(GC,RC)
143	(RS,GS,RC,PS,GH,YH),(PH,OC,OS,YS),(OH,PC,YC,GC,RH)
144	(YS,GS,PC),(GH,OH,RS,RH,PH),(RC,YC,OC)
145	(RH,GS,GH,PH,PS),(OS,YH,YC,OH,PC,RC)
146	(OC,PS),(YH,YS,PH),(GC,YC),(GH,RC,GS,OS,RS,PC),(RH,OH)
147	(RS,YC,OS,OH),(PS,PH,OC,YS,YH,PC),(GC,GS),(GH,RC)
148	(OH,YS,GH,PC,GC,OC),(OS,RH),(GS,PS,YC,YH),(RS,PH)

Partition Number	Partition Description
149	(GH,OH,RS,PC,OS),(PH,YC,YH)
150	(OS,YH,GH),(PS,GS,OH,RS,PC)
151	(PS,RC,YH,OH,GS,RH),(GH,PH,RS),(YS,YC),(OC,OS)
152	(GS,GC),(YH,OC,OS,YS,YC),(RH,PC,GH)
153	(YC,RH,OS,PH,RS),(GH,RC,GS,PS),(PC,YH),(OC,YS),(OH,GC)
154	(PH,OC),(RS,YC,OH),(RC,GS,PS,PC,YH)
155	(GH,RS,YS),(GS,GC,RH,OS,PS),(PH,OC,YC,RC,OH,PC)
156	(PH,RC,OH,GS,PC),(RH,YH,YC,YS),(GH,OS),(RS,OC),(GC,PS)
157	(PH,RH,OC,YS,RS,RC),(PC,OH),(GS,GH,PS,OS)
158	(RS,GC,GH,YS,PS,OC),(PC,OH),(GS,YC,OS),(PH,YH),(RH,RC)
159	(GH,GS),(YH,OH),(OC,YS)
160	(YS,GH),(YH,RS,OC,RC),(GC,YC),(PC,OS,PH,RH,GS)
161	(YC,PH,RC),(OC,GH),(YH,OH,GS)
162	(PS,YC,RS),(RC,OC)
163	(PH,YC,YH,GS,GH,PS),(OH,OC,RC,YS),(RS,RH,PC,OS,GC)
164	(PC,YS,YC,OC,YH),(RS,RH,GH,GS,RC),(PS,OH,GC)
165	(OH,PS,YC),(PH,GC,RS,PC,GS,OC),(RH,YH,RC,OS),(GH,YS)
166	(RH,GH,YS,RS,PS),(YC,OC,GS,PC)
167	(YH,OH,PH,GC,PS),(RC,YS),(RH,PC),(GH,OC,GS)
168	(OS,RC),(RS,PS,GH,PC,YC)
169	(RS,OH),(OS,YC,GC,YS,GS,PC),(RC,OC)
170	(GH,YC),(GS,OC,GC,PS,RC),(RS,RH),(PC,YH),(OS,YS)
171	(OC,GS,OS,PH,GC),(RC,PS,RS,YS,RH)
172	(YS,GH,YC),(YH,RS,PS,RC,RH),(PC,GC,OH),(OC,PH),(OS,GS)

Partition Number	Partition Description
173	(YS,PS,RC,OC),(OH,GC,RS),(PC,OS,YC),(GS,GH,YH,RH,PH)
174	(RC,RS,GS,OH,RH,YC),(PH,OC)
175	(OS,OC),(PC,RC,RS,YH,YS,OH),(RH,GS),(PS,GH),(PH,YC)
176	(GH,YS,OS,PC),(OC,GC,GS)
177	(RC,RH,OS,YC),(PS,RS,GS,GC,YS),(GH,PC),(OH,PH),(OC,YH)
178	(YS,PH,PC,OS,YH),(RC,OC,GS,GH),(RS,RH)
179	(OC,PS,GS,YH,OH),(OS,YS,PC),(GC,RH),(PH,RS),(RC,GH)
180	(RS,GC,YC),(OS,OH,YS),(GH,PH,OC,GS,RC),(PS,RH),(PC,YH)
206	(PH,GC),(OH,GS,GH)

Partitions Presented in Experiment 2

Subject	Partitions Presented
1	1,2,23,30,49,50,61,65,75,87,97,98,99,100,135,206
2	1,2,10,33,49,59,60,65,72,82,87,94,98,99,100,125
3	1,2,8,11,22,44,49,52,79,81,87,98,99,100,115,118
4	1,2,6,7,16,50,61,71,74,83,91,98,115,118,120,159
5	1,2,6,11,15,20,49,65,66,81,82,94,97,99,115,125
6	1,2,3,20,23,50,65,66,76,87,94,98,99,100,115,123
7	1,2,6,11,12,18,52,61,62,74,79,81,98,115,118,162
8	1,2,6,10,23,31,50,51,52,60,65,67,85,97,98,100

Raw Data

Experiment 2: Minimax Scaling of Partitions

In the following data: P = partition number; E = obtained minimax judgement, M1 and M2 = predicted minimax judgements for the weighted and unweighted models, respectively (based on the individual similarity judgements); $\overline{M1}$ and $\overline{M2}$ = predicted judgements based on the averaged similarity judgements.

All data has been transformed to the common range of 100. To obtain the original (pre-transformed) empirical minimax judgements in terms of distance in cm from the lower end of the scale, the E values must be multiplied by the following constants:

SUBJECT	CONSTANT
1	2.3375
2	4.5900
3	4.5900
4	4.2500
5	1.8275
6	4.3350
7	3.3788
8	2.5075

For example, subject 1 placed partition 49 a distance of 5.5(2.3375) = 12.86 cm to the right of partition 1.

	P	E	M1	M2	$\overline{M1}$	$\overline{M2}$
S	1	0.0	3.4	3.4	9.1	9.1
U	49	5.5	7.7	22.8	16.3	41.3
B	50	12.7	6.7	17.5	13.9	33.7
J	87	18.2	0.0	0.0	0.0	0.0
E	2	21.8	0.6	0.6	8.7	8.7
C	23	36.4	18.9	10.5	38.5	24.0
T	30	38.2	12.5	9.0	37.0	29.8
	75	49.1	4.7	12.2	20.2	29.8
1	206	50.9	18.3	25.0	33.2	46.6
	65	52.7	14.9	4.1	34.1	14.9
	61	76.4	9.2	20.3	26.0	50.0
	135	78.2	21.5	26.6	47.1	53.4
	100	80.8	41.7	41.7	72.1	72.1
	98	98.2	100.0	100.0	100.0	100.0
	97	100.0	91.3	91.3	94.2	94.2
	99	100.0	64.9	64.9	91.8	91.8

	P	E	M1	M2	$\overline{M1}$	$\overline{M2}$
S	87	0.0	0.0	0.0	0.0	0.0
U	2	8.3	5.5	5.5	8.7	8.7
B	65	15.3	43.6	15.5	34.1	14.9
J	82	21.3	18.8	18.8	0.5	0.5
E	59	37.0	57.5	33.1	14.4	18.7
C	10	38.9	63.5	37.0	7.2	2.9
T	94	38.9	54.7	32.0	4.3	0.5
	125	44.0	32.0	56.4	28.8	47.1
2	60	46.8	72.9	60.2	16.3	32.2
	33	50.5	63.0	45.9	16.8	13.0
	49	63.9	57.5	79.0	16.3	41.3
	1	77.8	56.4	56.4	9.1	9.1
	72	81.9	50.8	68.5	15.4	29.3
	98	85.2	100.0	100.0	100.0	100.0
	99	92.1	83.4	83.4	91.8	91.8
	100	100.0	96.1	96.1	72.1	72.1

	P	E	M1	M2	$\overline{M1}$	$\overline{M2}$
S	81	0.0	0.0	0.0	0.0	0.0
U	8	13.0	9.3	11.1	6.6	6.6
B	79	17.6	25.9	40.7	17.4	29.6
J	1	29.6	19.0	19.0	11.3	11.3
E	87	36.1	5.1	5.1	2.3	2.3
C	118	44.4	68.1	57.4	53.1	42.7
T	22	48.1	44.0	29.2	39.4	27.2
	11	50.9	30.6	57.4	18.3	38.5
3	49	53.7	26.4	50.0	18.3	42.7
	52	58.3	33.8	66.7	23.0	54.0
	2	64.8	21.3	21.3	10.8	10.8
	44	71.3	53.2	43.5	49.8	40.8
	100	80.6	72.2	72.2	72.8	72.8
	99	88.0	91.2	91.2	92.0	92.0
	98	90.7	100.0	100.0	100.0	100.0
	115	100.0	78.2	78.2	66.7	66.7

	P	E	M1	M2	$\overline{M1}$	$\overline{M2}$
S	1	0.0	2.8	2.8	8.7	8.7
U	7	11.0	0.0	0.0	4.3	3.9
B	83	15.0	10.7	10.7	0.0	0.0
J	74	23.0	46.3	19.8	15.9	10.6
E	16	32.0	7.3	27.1	8.7	13.5
C	50	34.0	12.4	42.4	13.5	33.3
T	120	44.0	51.4	27.7	33.3	28.0
	118	51.0	74.0	53.1	51.7	41.1
4	91	58.0	17.5	48.0	7.2	10.1
	159	62.0	53.7	53.7	29.0	29.0
	71	71.0	29.9	67.2	20.3	33.8
	6	79.0	27.1	73.4	24.6	53.6
	61	79.0	31.6	77.4	25.6	49.8
	115	91.0	92.7	92.7	65.7	65.7
	98	93.0	100.0	100.0	100.0	100.0
	2	100.0	62.1	62.1	8.2	8.2

	P	E	M1	M2	$\overline{M1}$	$\overline{M2}$		P	E	M1	M2	$\overline{M1}$	$\overline{M2}$
S	2	0.0	15.3	15.3	11.4	11.4	S	87	0.0	0.0	0.0	0.0	0.0
U	94	7.0	7.3	6.6	7.0	3.0	U	94	2.9	10.3	5.2	4.3	0.5
B	81	9.3	0.0	0.0	0.0	0.0	B	1	6.9	15.8	15.8	9.1	9.1
J	82	16.3	3.8	3.8	3.0	3.0	J	2	22.5	1.3	1.3	8.7	8.7
E	1	39.5	12.2	12.2	11.9	11.9	E	76	24.5	18.4	23.2	15.4	26.9
C	65	41.9	32.6	20.5	37.8	17.9	C	65	31.4	25.8	7.7	34.1	14.9
T	66	44.2	30.6	14.6	37.8	20.4	T	66	51.0	29.0	15.8	34.1	17.3
	20	48.8	36.8	24.7	45.8	29.4		50	53.9	21.6	41.3	13.9	33.7
5	49	58.1	18.7	39.9	19.4	45.3	6	20	59.8	39.4	28.7	41.8	26.0
	15	60.5	47.6	36.8	46.8	36.3		123	62.7	29.0	38.7	32.7	43.3
	11	65.1	16.0	31.2	19.4	40.8		23	66.7	32.3	19.0	38.5	24.0
	125	74.4	28.1	44.4	32.3	51.2		3	70.6	43.9	43.9	53.4	53.4
	6	79.1	24.0	47.6	28.4	58.2		99	77.4	84.8	84.8	91.8	91.8
	99	95.3	100.0	100.0	97.5	97.5		100	86.3	69.0	69.0	72.1	72.1
	97	97.7	83.3	83.3	100.0	100.0		115	93.1	47.7	47.7	65.9	65.9
	115	100.0	63.9	63.9	70.6	70.6		98	100.0	100.0	100.0	100.0	100.0
S	81	0.0	0.0	0.0	0.0	0.0	S	1	0.0	23.9	23.9	4.5	6.4
U	1	6.3	4.3	4.3	11.3	11.3	U	50	16.9	26.9	39.3	9.6	31.7
B	12	18.9	3.7	11.8	10.8	22.1	B	10	16.9	12.4	2.5	2.5	0.0
J	11	21.4	15.0	52.4	18.3	38.5	J	60	16.9	31.3	46.8	12.1	30.2
E	79	22.6	11.2	30.5	17.4	29.6	E	51	18.6	36.3	56.7	18.2	44.6
C	62	22.6	15.0	35.8	17.8	28.2	C	52	18.6	34.3	54.2	17.2	51.5
T	74	30.8	48.7	22.5	18.3	13.1	T	6	31.4	34.8	50.2	21.2	52.5
	118	42.8	70.1	50.3	53.1	42.7		23	44.1	42.3	21.9	35.4	21.8
7	162	51.6	40.6	27.3	36.2	26.3	8	31	52.5	42.3	31.3	39.9	31.7
	18	55.3	64.7	43.9	43.2	35.2		65	66.9	32.3	6.0	30.8	12.4
	52	71.1	21.9	67.4	18.3	54.0		67	66.9	33.3	17.4	29.3	18.3
	6	74.8	27.3	72.2	26.8	54.9		85	80.5	0.0	0.0	0.0	2.0
	61	76.1	31.0	74.3	27.7	51.2		100	83.1	70.1	70.1	70.7	71.3
	2	88.0	56.1	56.1	10.8	10.8		2	96.6	0.5	0.5	4.0	5.9
	98	98.1	100.0	100.0	100.0	100.0		98	98.3	100.0	100.0	100.0	100.0
	115	100.0	90.4	90.4	66.7	66.7		97	100.0	90.5	90.5	93.9	94.1

Experiment 1: Minimax Scaling of Partitions

The transformed minimax judgements (E) for the partitions in Experiment 1 are given in Table 3. To obtain the original (pre-transformed) empirical minimax judgements in terms of distance in cm from the lower end of the scale, the E values must be multiplied by the following constants:

SUBJECT	CONSTANT
1	3.0600
2	4.5900
3	3.6550
4	3.0600
5	1.1985
6	0.7225
7	4.1650
8	3.3575

Subject	Partition Number												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	61.1	61.9	53.9	28.3	0.0	72.6	27.4	13.3	77.9	49.6	100.0	82.3	99.1
2	0.0	6.4	8.3	17.4	81.7	7.3	72.5	80.7	83.5	77.1	100.0	81.7	98.2
3	29.3	36.2	31.9	6.9	0.0	41.4	22.4	20.7	62.9	42.2	97.0	73.3	100.0
4	0.0	1.5	6.7	5.2	19.4	9.7	31.3	41.0	70.9	35.1	94.0	78.4	100.0
5	0.0	3.3	8.2	10.7	27.9	12.3	40.2	43.4	73.0	29.5	100.0	78.7	99.2
6	0.0	3.1	6.2	24.7	43.4	12.4	37.1	51.5	76.3	60.8	96.9	79.4	100.0
7	0.0	5.8	8.0	1.4	21.7	8.7	36.2	30.4	66.7	50.0	100.0	71.7	99.0
8	0.0	2.0	5.4	2.7	14.8	11.4	28.9	26.2	64.4	41.6	96.0	71.1	100.0

Experiment 1. Predicted minimax judgments based on individual similarity judgments for weighted model (M1).

Subject	Partition Number												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	61.6	63.4	53.6	28.6	0.0	99.1	24.1	46.4	52.7	26.8	73.2	90.2	100.0
2	0.0	7.5	33.6	17.8	83.2	31.8	50.5	96.3	74.8	71.0	94.4	85.0	100.0
3	29.3	38.0	48.3	6.9	0.0	60.3	13.8	63.8	36.2	25.0	68.1	75.0	100.0
4	0.0	3.0	29.8	5.2	19.4	32.1	17.9	78.4	45.5	27.6	75.4	73.9	100.0
5	0.0	4.2	33.9	10.7	28.1	38.0	21.5	80.2	48.8	26.4	85.1	72.7	100.0
6	0.0	5.2	25.8	24.7	43.4	37.1	28.9	66.0	57.7	52.6	85.6	84.5	100.0
7	0.0	7.4	31.6	1.5	22.1	33.1	19.9	61.0	45.6	33.8	83.1	75.7	100.0
8	0.0	2.7	27.5	2.7	14.8	38.3	18.1	53.7	40.3	30.2	71.8	68.5	100.0

Predicted minimax judgments based on individual similarity judgments for unweighted model (M2), Experiment 1.

Experiment 1: Similarity Judgements of Object Pairs

The following abbreviations are employed for the description of stimuli: B = black, G = green, R = red, Y = yellow, C = circle, T = triangle, S = square, and H = hexagon.

All data has been transformed to the common range of 100. To obtain the original (pre-transformed) empirical similarity judgements in terms of distance in cm from the lower end of the scale, the table entries must be multiplied by the appropriate constant:

SUBJECT	CONSTANT
1	3.6975
2	4.9300
3	4.6962
4	3.9100
5	3.1662
6	0.1275
7	2.6350
8	3.7400

SUBJECT 1

Colour/Shape	SAME	S-H	C-H	T-S	C-S	T-H	C-T
SAME	.0	5.7	4.6	13.8	9.2	24.1	19.5
G-R	31.0	65.5	74.7	67.8	42.5	70.1	82.8
B-G	40.2	62.1	65.5	74.7	72.4	79.3	65.5
G-Y	55.2	80.5	69.0	93.1	78.2	88.5	94.3
R-Y	60.9	77.0	71.3	83.9	73.6	90.8	83.9
B-R	49.4	58.6	79.3	78.2	73.6	82.8	69.0
B-Y	77.0	80.5	83.9	88.5	44.8	100.0	87.4

SUBJECT 2

Colour/Shape	SAME	S-H	C-H	T-S	C-S	T-H	C-T
SAME	0.0	56.9	84.5	58.6	85.3	59.5	86.2
G-R	16.4	62.9	89.7	66.4	93.1	70.7	98.3
B-G	18.1	63.8	90.5	67.2	94.0	71.6	99.1
G-Y	15.5	62.1	88.8	65.5	92.2	69.8	97.4
R-Y	15.5	62.1	88.8	65.5	92.2	69.8	97.4
B-R	17.2	62.9	89.7	66.4	93.1	70.7	98.3
B-Y	19.0	64.7	91.4	68.1	94.8	72.4	100.0

SUBJECT 3

Colour/Shape	SAME	S-H	C-H	T-S	C-S	T-H	C-T
SAME	0.0	10.3	12.7	24.4	15.9	19.8	31.2
G-R	36.1	47.2	50.7	98.0	81.4	91.0	100.0
B-G	33.8	45.1	49.4	53.1	48.9	55.7	58.3
G-Y	38.9	62.7	57.7	65.2	60.8	65.6	67.0
R-Y	36.2	55.5	53.8	69.7	59.5	62.1	82.0
B-R	31.2	66.5	68.6	76.0	71.5	72.4	82.0
B-Y	42.1	77.0	71.5	87.8	78.3	86.0	92.7

SUBJECT 4

Colour/Shape	SAME	S-H	C-H	T-S	C-S	T-H	C-T
SAME	0.0	27.2	7.6	33.7	30.4	35.9	40.2
G-R	15.2	76.1	75.0	78.3	76.6	79.9	81.0
B-G	12.0	67.9	67.9	70.1	68.5	71.7	73.4
G-Y	8.7	54.9	54.3	57.1	55.4	58.7	60.3
R-Y	11.4	66.8	66.3	69.0	67.4	70.6	71.7
B-R	17.4	84.8	84.2	87.5	85.9	89.1	90.2
B-Y	18.5	95.1	94.6	97.8	95.6	99.5	100.0

SUBJECT 5

Colour/Shape	SAME	S-H	C-H	T-S	C-S	T-H	C-T
SAME	0.0	32.2	26.8	38.1	34.9	29.5	40.3
G-R	13.4	59.6	57.7	66.6	63.1	58.7	70.5
B-G	18.8	72.5	70.3	79.2	76.1	71.1	84.6
G-Y	10.7	54.1	51.0	59.1	57.0	53.1	62.3
R-Y	9.4	49.7	47.0	51.5	50.5	48.3	54.5
B-R	20.1	79.2	76.5	85.9	82.6	77.4	89.9
B-Y	24.2	89.9	87.2	95.3	92.6	88.6	100.0

SUBJECT 6

Colour/Shape	SAME	S-H	C-H	T-S	C-S	T-H	C-T
SAME	0.0	66.7	66.7	66.7	66.7	66.7	66.7
G-R	33.3	100.0	100.0	100.0	100.0	100.0	100.0
B-G	33.3	100.0	100.0	100.0	100.0	100.0	100.0
G-Y	33.3	100.0	100.0	100.0	100.0	100.0	100.0
R-Y	33.3	100.0	100.0	100.0	100.0	100.0	100.0
B-R	33.3	100.0	100.0	100.0	100.0	100.0	100.0
B-Y	33.3	100.0	100.0	100.0	100.0	100.0	100.0

SUBJECT 7

Colour/Shape	SAME	S-H	C-H	T-S	C-S	T-H	C-T
SAME	0.0	25.8	30.6	24.2	32.3	27.4	33.9
G-R	12.9	51.6	77.4	61.3	87.1	69.4	96.8
B-G	9.7	48.4	74.2	58.1	83.9	66.1	93.5
G-Y	12.9	51.6	77.4	61.3	87.1	69.4	96.8
R-Y	11.3	50.0	75.8	59.7	85.5	67.7	95.2
B-R	12.9	51.6	77.4	61.3	87.1	69.4	96.8
B-Y	14.5	53.2	79.0	62.9	88.7	71.0	100.0

SUBJECT 8

Colour/Shape	SAME	S-H	C-H	T-S	C-S	T-H	C-T
SAME	0.0	12.5	5.7	27.3	19.3	31.8	39.8
G-R	14.8	71.6	64.8	80.7	93.2	88.6	100.0
B-G	8.0	62.5	54.5	70.5	83.0	78.4	90.0
G-Y	5.7	61.4	53.4	69.3	81.8	77.3	88.6
R-Y	6.8	60.2	52.3	68.2	80.7	76.1	87.5
B-R	12.5	68.2	60.2	76.1	88.6	84.1	95.5
B-Y	10.2	64.8	56.8	72.7	85.2	80.7	92.0

Experiment 2: Similarity Judgements of Object Pairs

The following abbreviations are employed for the description of stimuli: C = circle, H = hexagon, S = square, G = gold, P = purple, R = red, O = orange, and Y = yellow.

All data has been transformed to the common range of 100. To obtain the original (pre-transformed) empirical similarity judgements in terms of distance in cm from the lower end of the scale, the table entries must be multiplied by the appropriate constant:

SUBJECT	CONSTANT
1	2.8900
2	3.7400
3	4.0588
4	2.3375
5	3.4425
6	3.5700
7	4.4838
8	1.3600

SUBJECT 1

Description of Object Pair	Same Shape	C-H	S-H	C-S
Same Colour	0.0	2.9	4.4	5.9
O-Y	8.8	45.6	47.1	48.5
G-Y	14.7	52.9	54.4	55.9
R-O	7.4	38.2	39.7	41.2
R-P	5.9	30.9	32.4	33.8
G-O	22.1	61.8	60.3	63.2
P-O	11.8	75.0	76.5	77.9
P-Y	16.2	67.6	69.1	70.6
R-Y	19.1	82.3	83.8	85.3
G-P	23.5	89.7	91.2	92.6
R-G	25.0	97.1	98.5	100.0

SUBJECT 2

Description of Object Pair	Same Shape	C-H	S-H	C-S
Same Colour	0.0	4.5	12.5	15.9
O-Y	26.1	28.4	30.7	33.0
G-Y	10.2	19.3	21.0	22.7
R-O	46.0	47.2	50.0	53.4
R-P	59.7	61.4	62.5	64.2
G-O	36.4	38.1	39.2	42.0
P-O	54.5	55.7	56.8	58.0
P-Y	64.8	65.9	67.0	68.2
R-Y	75.0	76.1	77.8	80.1
G-P	84.1	85.2	86.9	89.2
R-G	94.3	95.4	97.7	100.0

SUBJECT 3

Description of Object Pair	Same Shape	C-H	S-H	C-S
Same Colour	0.0	4.2	27.7	51.3
O-Y	14.7	25.1	51.3	64.9
G-Y	25.1	35.6	56.0	73.3
R-O	9.4	19.9	46.1	60.7
R-P	4.2	14.7	40.8	60.7
G-O	30.4	40.8	60.7	79.6
P-O	19.9	30.4	56.0	69.1
P-Y	40.8	51.3	69.1	89.0
R-Y	35.6	46.1	64.9	83.8
G-P	46.1	56.0	73.3	95.3
R-G	51.3	56.0	75.4	100.0

SUBJECT 4

Description of Object Pair	Same Shape	C-H	S-H	C-S
Same Colour	0.0	61.8	30.9	96.4
O-Y	12.7	70.9	40.0	100.0
G-Y	12.7	70.9	40.0	100.0
R-O	12.7	70.9	40.0	100.0
R-P	12.7	70.9	40.0	100.0
G-O	12.7	70.9	40.0	100.0
P-O	12.7	70.9	40.0	100.0
P-Y	12.7	70.9	40.0	100.0
R-Y	12.7	70.9	40.0	100.0
G-P	12.7	70.9	40.0	100.0
R-G	12.7	70.9	40.0	100.0

SUBJECT 5

Description of Object Pair	Same Shape	C-H	S-H	C-S
Same Colour	0.0	21.6	33.3	40.7
O-Y	6.2	30.9	42.6	67.9
G-Y	18.5	50.6	69.1	75.3
R-O	7.4	42.0	46.3	74.7
R-P	12.3	44.4	81.5	84.6
G-O	24.7	54.3	86.4	90.7
P-O	37.0	46.9	85.2	85.8
P-Y	55.6	72.8	96.3	98.1
R-Y	60.5	77.2	97.5	100.0
G-P	32.1	66.7	88.9	92.6
R-G	38.3	79.6	91.4	98.8

SUBJECT 6

Description of Object Pair	Same Shape	C-H	S-H	C-S
Same Colour	0.0	13.1	14.3	15.5
O-Y	26.2	65.5	65.5	65.5
G-Y	26.2	57.1	57.1	57.1
R-O	32.1	61.9	61.9	61.9
R-P	38.1	65.5	65.5	65.5
G-O	40.5	69.0	69.0	69.0
P-O	42.9	78.6	78.6	78.6
P-Y	48.8	95.2	95.2	95.2
R-Y	50.0	100.0	100.0	100.0
G-P	46.4	89.3	89.3	89.3
R-G	45.2	85.7	85.7	85.7

SUBJECT 7

Description of Object Pair	Same Shape	C-H	S-H	C-S
Same Colour	0.0	25.1	60.7	82.0
O-Y	7.6	28.9	68.7	89.6
G-Y	4.1	28.9	64.0	85.3
R-O	7.6	28.9	64.9	85.8
R-P	15.2	29.2	65.9	86.2
G-O	11.4	33.2	69.2	90.0
P-O	16.4	33.4	70.6	94.3
P-Y	16.8	41.7	77.2	99.5
R-Y	11.4	32.7	73.0	93.8
G-P	19.0	42.2	78.2	100.0
R-G	18.0	37.3	73.5	94.8

SUBJECT 8

Description of Object Pair	Same Shape	C-H	S-H	C-S
Same Colour	0.0	6.2	18.8	42.2
O-Y	56.2	69.8	83.6	96.9
G-Y	56.2	69.8	83.6	96.9
R-O	56.2	71.9	84.4	98.4
R-P	58.6	75.0	85.9	100.0
G-O	56.2	69.8	83.6	96.9
P-O	58.6	75.0	85.9	100.0
P-Y	58.6	75.0	85.9	100.0
R-Y	56.2	71.9	84.4	98.4
G-P	58.6	75.0	85.9	100.0
R-G	56.2	71.9	84.4	98.4

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