# TECHNIQUES FOR ASSESSING THE EFFECTS OF PEST HAZARDS ON LONG-RUN TIMBER SUPPLY

By

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# TECHNIQUES FOR ASSESSING THE EFFECTS OF PEST HAZARDS ON LONG-RUN TIMBER SUPPLY

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## 1. Introduction

Canada's forest resource is characterized by large stocks of mature and over-mature timber which are particularly susceptible to infestation by pests of various types. As increased pressure is placed upon the resource base there is growing concern as to the effects of these infestations on forest output. In view of this, various pests have been studied to determine their population dynamics; the factors which contribute to epidemic levels; and the nature of the damage caused. These studies tend to focus on the short term aspects of the pest problem in that they concentrate on the characteristics of a single infestation. Less appears to be known of the long term aspects -- for example how pest populations interact with the forest over the long term and what are the long term impacts on forest production.

Pests are varied in their behaviour and in how they affect their hosts. For some pests, losses might be indicated through an adjustment of a yield curve (we term these "increment" losses) to represent the slowing of growth of individual trees (by, for example an insect defoliator which does not actually kill its host, such as western hemlock looper (_Lambdina fiscellaria lugubrosa_)) or where several, but not all, trees in a stand are killed (such as endemic infestations of Douglas fir bark beetle (_Dendroctonus pseudotsugae_). Where large-scale epidemics occur such that whole stands are killed (we call these "catastrophic" losses) the losses cannot be properly represented through a yield curve adjustment. These catastrophic losses are similar in some respects to those of fire. Pest populations behave in a varied manner as well. Some pests are ever-present or endemic and their population levels fluctuate little from year to year indefinitely (such as dwarf mistletoe (_Arceuthobium_). Some pest populations fluctuate very widely, increasing to epidemic levels for several years and then almost dying out and remaining at very low levels for as long as several decades (such as mountain pine bark beetle (Dendroctonus ponderosae). In addition, population dynamics are often complicated depending on a combination of factors such as climate, and the age, condition, size, and extent of host trees.

All of these factors make modelling the effects of pests on forest output a difficult problem. Indeed, except for one recent exception (Johnson and Stuart, 1985) harvest scheduling models have not explicitly addressed the problem of losses due to pests. In consequence little is known about the long term effects of pest infestation on forest yeild. In this paper we present two timber supply models one at the stand-level and one at the forest-level, which allow the incorporation of rates of infestation, in order assess the impact on wood supply. The work herein follows from previous work on the effects of fire on output from the forest (Reed and Errico, 1985, 1986).

The following section details the stand-level approach with examples from the Quesnel Timber Supply Area (TSA) of British Columbia. Section 3 gives a forest-level approach with examples from the same area. Our main difficulty in implementing these approaches lies in our inability to predict pest behaviour in the long-term. Section 4, therefore, gives some indication of how good our approximations of pest infestation rates might be by simulating the repeated use of the forest-level approach over time.

## 2. Stand-level Analysis

In this section we consider the long-run yield from a forest site on which an even-aged stand grows. We shall assume that the growing stand is subject both to the risk of infestation through pests and to the risk of catastrophic destruction through fire. The analysis is based on an extension to the method developed for the case of fire risk alone described in Reed and Errico (1985) and Reed (1984).

It will be assumed that fires on the site occur in an age independent fashion with a constant hazard (instantaneous fire probability) $\lambda$. Pest infestations on the other hand will be assumed to be age-dependent, with a <u>hazard function</u> (see Reed and Errico, 1985) $h(x)$, ($h(x)$ represents the instantaneous probability rate of infestation in a stand of age $x$, given that it has not suffered infestation up until that age.)

Since the analysis is at the stand-level, cutting decisions will depend only on conditions endogenous to the stand. Specifically let us suppose a cutting decision is made on the basis of:

(a) the age of the stand,

(b) the state of the stand (infested or not), and

(c) the time since infestation, if such has occurred.

If it is assumed that the stand ceases to grow after an infestation, then clearly an optimal stand-level harvesting policy will involve the immediate clearing and re-establishment of the site after an infestation has occurred. If this is the case a harvesting policy can be specified by a single cutting (or rotation) age $T$ at which a non-infested stand is cut.

Under these assumptions a stand can be destroyed (and the site re-established) in three ways. They are:

(a) the cutting of a non-infested stand at age $T$,

(b) the cutting of a stand immediately following a pest infestation, at some age $x < T$, or

(c) destruction by fire at some age $x < T$.

The combined hazard function $h_c(x)$ for destruction through any one of these causes is

$$[1] \qquad h_c(x) = \begin{cases} \lambda + h(x) & x < T \\ \infty & x = T \end{cases}$$

It follows (see Appendix) that the lifetime of the stand is a random variable, $X$ with cumulative distribution function (c.d.f.)

$$[2] \qquad F_c(x) = \begin{cases} 1 - e^{-\lambda x} S(x), & x < T \\ 1 \qquad\qquad , & x \geq T \end{cases}$$

where

$$[3] \qquad S(x) = \exp\left\{- \int_0^t h(t)dt\right\}$$

Now, let $Y$ be a random variable denoting the volume (or value) yield from the stand at the time of its destruction. If we let $V(x)$ be a volume- (or value-) age relationship (VAC) for non-infested stands, and suppose that if a stand is infested at age $x$ then the volume (or value) of usable timber in it is reduced from $V(x)$ to $\tilde{V}(x)$, then it is clear that the random variable $Y$ can assume values $V(T)$, $\tilde{V}(x)$ $(0 < x < T)$ and zero corresponding to the three causes of destruction (a), (b) and (c) above.

From Reed (1984) it is known that the long-run average yield (LRAY) from the site, $\bar{Y}(T)$, using a cutting age $T$ is given by

$$[4] \qquad \bar{Y}(T) = E(Y)/E(X)$$

Conceptually we can think of the LRAY as the long-run average yield (per annum) from a single site over many rotations or alternatively as the average equilibrium yield (per site) in any year over many sites all with similar growth characteristics and similar, although independent fire and pest hazards (see Reed and Errico, 1985).

A convenient way of representing the dependence of the LRAY on the chosen rotation age $T$ is through the use of a "pest and fire adjusted volume rotation curve" (VRC) (Reed and Errico, 1985). Specifically the adjusted VRC, $U(T)$ would be given by

$$U(T) = T\bar{Y}(T) \tag{5}$$

If future revenues from harvests are discounted at the per annum instantaneous rate $\delta$, then the expected present value, $J(T)$ of the stream of future revenues from the forest using the cutting age $T$ is (Reed, 1984)

$$J(T) = \frac{E(e^{-\delta X}\;Y)}{1 - E(e^{-\delta X})} \tag{6}$$

Costs of re-establishment different after a fire, an infestation and a regular harvest can be included in the net revenue $Y$ in each cycle. Details are given in Appendix 1, where it is also shown how expressions [4], [5] and [6] for the LRAY, the VRC and the expected present value can be evaluated.

As an example we have considered the volume-age relationship given in Table 1. This corresponds to pure lodgepole pine (Pinus contorta Dougl.) growing on sites of site index 19-24m (base age 100) in the Quesnel Timber supply area of central British Columbia. This is an area currently experiencing infestation by mountain pine beetle (Dendroctonus ponderosae Hopk.).

Major outbreaks of mountain pine beetle occur at irregular intervals with intervals ranging from one to several decades. Causes of outbreaks include climate, tree diameter and tree age as well as the amount and distribution of stands having tree diameters and ages conducive to beetle population growth. Because of the unpredictability of the timing and size of beetle outbreaks, we have chosen to assume that outbreaks occur independently of one another in a time-homogeneous, though age-dependent fashion. (If more detailed knowledge of the dynamics of beetle outbreaks were available, this could possibly be incorporated into a probabilistic model. Unfortunately such information does not appear to be available). Specifically we have assumed a piece-wise hazard function with zero rate below 50 years of age and various constant annual rates $\nu$ above age 50 ($\nu = 0, .001, .005, .01$). As a benchmark with which to compare the results one may consider the average annual infestation rates for the Quesnel TSA for the last three years. These have been estimated to be .0011 for timber aged between 30 and 50 years; .0054 for timber between aged between 50 and 60 years; and .0106 for timber aged between 60 and 90 years[3]. It should be noted however that these are rates experienced during a beetle outbreak. The annual rates averaged over the longer term would be much lower.

Since we are dealing with long-term effects, one must think of the hazard rates $\nu$ used, in this sense. For example a hazard of $\nu = 0.01$ per annum could result in say 80 percent of the area being destroyed in a decade-long infestation, with such infestations occurring on average every 80 years; or alternatively it could result in a 40 percent destruction in a decade long infestation with such infestations occurring on average every 40 years etc.

We have also assumed that

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[3] R. Murta. B.C.F.S. Cariboo Region. Personal communication.

$$\bar{V}(x) = \theta V(x)$$

i.e. that infestation results in a fixed proportional reduction in usable timber. Figs. 1 and 2 show the pest-adjusted VRC's, corresponding to $\theta = .75$ and $\theta = .25$ respectively, for the four average annual infestation rates. In all cases the fire hazard $\lambda$ was set to zero. In consequence for the top curve $(\nu = 0)$ there is neither pest nor fire hazard and it is simply the ordinary volume-age curve as given in Table 1. It can be seen from Figs. 1 and 2 that the reduction in long-run average yield (given by the slope of the tangent to the VRC that passes through the origin) caused by the presence of the pest hazard is not great. For example the reduction in LRAY caused by the presence of the highest hazard $(\nu = .01)$ when $\theta = .75$ is only about 9 percent (from 2.35 $m^3$/ha/yr to 2.15 $m^3$/ha/yr). The reduction in LRAY due to the presence of fire is often much higher than this (Reed and Errico, 1985). There are two reasons for this. Firstly trees of all ages are usually vulnerable to destruction by fire whereas in the above examples we have assumed that only trees of age 50 years or above are vulnerable to pest infestation. Secondly the rate of salvage after a pest infestation is likely to be larger than after a fire.

To investigate the first of these aspects the VRC has been computed for various values of the parameter, a, which denotes the age of onset of vulnerability to infestation; a = 50, 30, 10 and 0, using a constant hazard $\nu = .01$ for vulnerable ages. The salvage parameter $\theta$ was again set at $\theta = .75$. The results are shown in Fig. 3. The maximum reduction in LRAY, when all ages are susceptible $(a = 0)$ is about 20% (from 2.35 $m^3$/ha/yr to 1.88 $m^3$/ha/yr).

The effect of the salvage parameter $\theta$ can be seen by comparing Figs. 1 and 2. For example when the pest hazard is $\nu = .01$ for ages 50 and above, the reduction in LRAY with $\theta = .25$ is about 17 percent (from $2.35\ m^3/ha/yr$ to $1.95\ m^3/ha/yr$) compared with 9 percent when $\theta = .75$.

If fire risk is present in addition to the pest infestation hazard then of course the reduction in LRAY is greater. Fig. 4 shows the VRC for a constant fire hazard, $\lambda = .001$ and a piecewise pest hazard equal to $\nu = 0, .001, .005, .01$ for stands of age 50 and older, with the salvage proportion $\theta = .25$. This should be compared with Fig. 2 which is similar except that $\lambda = 0$. The additional reduction in LRAY due to the presence of the low fire risk is about $.05\ m^3/ha/yr$ (from $1.95\ m^3/ha/yr$ to $1.90\ m^3/ha/yr$).

Figs. 5 and 6 show the expected present value as a function of cutting age with a per annum discount rate of 3 percent. Stumpage value of timber is assumed constant at $\$10/m^3$ and the cost of re-establishment following a harvest of non-infested timber is assumed to be $C_1 = \$25/ha$, while the cost of re-establishment following a harvest of infested timber is assumed to be $C_3 = \$50/ha$. The fire hazard $\lambda$ is zero, and the pest hazard is non-zero only for stands 50 years and older. Fig. 5 corresponds to the salvage parameter $\theta = .75$ and Fig. 6 is for $\theta = .25$. It can be seen that in neither case is the effect of the presence of the risk of pest infestation large either on the optimal rotation age or on the land expectation value (the maximum value of the expected present value). Note that in Fig. 5 the expected present value curves cross. The reason for this is that at very long rotation ages, one is better off to have a pest infestation destroy the stand earlier, and be able to salvage 75% of the volume, rather than wait for a harvest at the rotation age. Thus at long rotation ages the expected present value curve is higher for high infestation probabilities. This is not the case for shorter rotation ages.

From the results above it can be concluded that the presence of the risk of catastrophic pest infestation on long-run yield from a forest site is large only if:

(i) trees become vulnerable to pest infestation at a young age,

(ii) the probability of infestation is large, and

(iii) the drop in value of timber after an infestation is large.

The relative importance of these factors for a given timber type growing on a given site can be assessed by calculating volume-rotation curves and expected present value curves in the manner indicated in this section and in the Appendix.

## 3. Forest-level Analysis

In this section we consider an optimal harvesting model in which forest-level harvesting concerns such as harvest flows and forest-wide constraints may be incorporated.

We shall suppose that at any time the state of the forest can be described by a vector

$$\begin{bmatrix} \underset{\sim}{x}_t \\ \underset{\sim}{y}_t \end{bmatrix}$$

where $\underset{\sim}{x}_t' = (x_1^t, x_2^t, \ldots, x_k^t)$ represents the areas of uninfested timber in the various age-classes $1, 2, \ldots, k$ in period $t$ (see Reed and Errico, 1986) and $\underset{\sim}{y}_t' = (y_a^t, y_{a+1}^t, \ldots, y_k^t)$ where $y_i^t$ is the area of timber <u>which was infested at age $i$</u> and is still standing in period $t$. The age $a(\geq 1)$ is the youngest age at which pest infestation can occur. Note that the classes for infested timber correspond to the age at which infestation occurred, and not the current (period t) age of the timber as for the non-infested timber. This will be appropriate for the case when no growth occurs after the infestation has happened -- in other words for catastrophic infestation where stands are completely killed. Another model for non-catastrophic infestation will be described later.

We shall suppose that random proportions $r_a^t, r_{a+1}^t, \ldots, r_k^t$ of the areas of previously non-infested timber are struck by pests in period t, and move into the infested categories. Furthermore we shall allow for the possibility of catastrophic fire by assuming that random proportions $p_1^t, p_2^t, \ldots, p_k^t$ of the areas in the various age-classes of the non-infested timber, and random proportions $q_a^t, q_{a+1}^t, \ldots, q_k^t$ of the areas of infested timber are destroyed by fire in period t, and are returned to the first age-class of non-infested areas in period t + 1. Typically we might expect higher rates of fire among infested timber than among non-infested timber.

Under these assumptions the dynamics of the forest can be described by the following (stochastic) difference equation

$$[7] \quad \begin{bmatrix} x_1^{t+1} \\ x_2^{t+1} \\ \vdots \\ x_{a+1}^{t+1} \\ \vdots \\ x_k^{t} \\ \hline y_a^{t+1} \\ \vdots \\ y_k^{t+1} \end{bmatrix} = \left[\begin{array}{l} h_1^t+\ldots+h_k^t+p_1(x_1^t-h_1^t)+\ldots p_k(x_k^t-h_k^t)+(g_a^t+\ldots g_k^t)+q_a(y_a^t-g_a^t)+\ldots+q_k^t(y_i^t-y_k^t) \\ (1-p_1^t)(x_1^t-h_1^t) \\ \vdots \\ (1-p_a^t-r_a^t)(x_a^t-h_a^t) \\ \vdots \\ (1-p_{k-1}^t-r_{k-1}^t)(x_{k-1}^t-h_{k-1}^t)+(1-p_k^t-r_k^t)(x_k^t-h_k^t) \\ \hline r_a^t(x_a^t-h_a^t)+(1-q_a^t)(y_a^t-g_a^t) \\ \vdots \\ r_k^t(x_k^t-h_k^t)+(1-q_k^t)(y_k^t-g_k^t) \end{array}\right]$$

where $\underset{\sim}{h}_t' = (h_1^t, h_2^t, \ldots, h_k^t)$ and $\underset{\sim}{g}_t' = (g_a^t, \ldots, g_k^t)$ represent the areas in the various classes of uninfested timber respectively, that are harvested in period $t$.

Equation [8] can be written conveniently in vector notation as:

$$[9] \qquad \underset{\sim}{X}_{t+1} = R_t \underset{\sim}{X}_t - S_t \underset{\sim}{h}_t$$

where $\underset{\sim}{X}_t' = (\underset{\sim}{x}_t', \underset{\sim}{y}_t')$ and $R_t$ and $S_t$ are $(2k-a+1) \times (2k-a+1)$ square matrices as displayed in Tables 2 and 3.

With this dynamic equation we seek to maximize the expected present value, $J$, of the flow of harvests from the resource subject to any harvest flow constraints and area constraints that may be imposed. We have

$$[10] \qquad J = E\left\{\sum_1^\infty \alpha^t [\underset{\sim}{v}'\underset{\sim}{h}_t + \underset{\sim}{w}_t'\underset{\sim}{g}_t]\right\}$$

where $\alpha$ is the per annum discount factor (related to $\delta$ of the previous section by $\alpha = e^{-\delta}$) and $\underset{\sim}{v}' = (v_1, v_2, \ldots, v_k)$ and $\underset{\sim}{w}' = (w_a, w_{a+1}, \ldots, w_k)$ are vectors representing the net stumpage values per hectare of timber in the various infested and non-infested classes. If there is suitable knowledge of future prices, $\underset{\sim}{v}'$ and $\underset{\sim}{w}'$ can be made to depend upon $t$ without introducing any additional difficulties.

An approximate solution to this stochastic control problem can be obtained by invoking the Certainty Equivalence Principle (e.g. Chow, 1975). This involves solving in each period the deterministic control problem obtained by replacing random variables by their expected values. Each of these control problems is linear in both their objectives and constraints, in the variables $\underset{\sim}{x}_t$, $\underset{\sim}{y}_t$ and $\underset{\sim}{h}_t$ and so can be solved by linear programming (see Reed and Errico, 1987 for an application to a similar problem).

One point about the dynamic equation [7] should be noted. While it is reasonable to assume that the random proportions burnt are statistically independent from period to period, this assumption may not be so realistic for the proportions infested by pests. Thus the stochastic process $\{X_t\}$ may not in fact be Markovian. If more were known about the (probabalistic) dynamics of pest infestation this could be incorporated into the dynamic model. For example if it were known that infestations occurred in a fairly regular cycle of known period the distributions of the random variables $r_i^t$ could be made to depend on $t$ to reflect this or alternatively could be given some autoregressive structure. However little seems to be known about the dynamics of infestations over the long-term for most pests, and the assumption of independent probabilities seems to be as good as any alternative. It should be noted that with this model and a skewed distribution for the $r_i^t$, the resulting sample paths would exhibit erratically occurring outbreaks of infestation.

The above model can be modified to allow for non-catastrophic infestation by allowing the vector $\underset{\sim}{y}_t$ to represent areas <u>by age</u> at time $t$ of infested timber and let the matrices $R_t$ and $S_t$ in equation [7] be as displayed in Tables 4 and 5. Note that the entries $r_a^t$, $r_{a+1}^t$, ... etc. and $(1-q_{a+1}^t)$, $(1-q_{a+2}^t)$ ... have been shifted down to a subdiagonal relative to their positions in Tables 2 and 3. This reflects the fact that aging of infested timber is now being taken into account. Also for this reason the youngest infested timber at the start of period $t$ is of age $a + 1$ -- it was infested at age $a$ during the previous period.

The vector $w' = (w_1, w_2, \ldots, w_k)$ now represents stumpage values (volumes) per hectare of infested timber in the various age classes.

A difference of this model to the previous one is that it allows timber to continue growing after an infestation. A drawback however is that the value (volume) of infested timber is assumed to depend only on its age and not specifically on how long it has been infested. This should be of little consequence provided that optimal policies as determined by the model involve harvesting infested timber very soon after infestation. This appears to be the case in all trial runs we have performed. The reason is that in allocating the harvest, the infested timber has priority for inclusion because it has slower growth potential than the non-infested timber. Thus infested timber is not allowed to remain in the infested category for long after an infestation.

As an example we have used the mountain pine beetle example of the previous section. The age-class distribution of the lodgepole pine areas in the Quesnel TSA for that example is given in Table 6. We are assuming in this case that these areas comprise their own forest, and as such the present simplification does not necessarily provide a realistic analysis for the Quesnel TSA, which contains various other sites and types, all managed as a single unit. Other assumptions were:

(i) period length was 10 years;

(ii) planning horizon was 25 periods;

(iii) harvest constraints were of the sequential flow type with the harvest of any period constrained to change by no more than ±10% from the harvest of the previous period;

(iv) all timber in an infested stand is destroyed; that is, there is no residual volume; $w_1 = w_2 = \ldots w_k = 0$. This is a "worst-case" assumption.

(v) no constraints for accessibility, cutting age, or ending inventory are imposed.

(vi) Discount rate is zero, i.e. objective is to maximize long-run average yield.

Figure 7 shows the harvest schedules using constant annual infestation rates (probabilities) of 0, .01, .03 and .05 for ages 50 and older. While these rates may be considered as unrealistically high, for mountain pine beetle infestation, they have been chosen here for demonstration of the sensitivity of results to changes over a wide range of infestation rates and for ease of illustration. Actual infestations are more likely to yield annual rates less than .01. The schedules are shown only for the first 20 periods, by which time the solutions have indicated that the forest would have reached equilibrium. While it is seen that the reductions in harvest levels are of an order of magnitude larger than their respective annual infestation rates, when one considers realistic annual rates of infestation the reductions in harvest levels are less impressive. Keeping in mind that a .01 annual rate of infestation is equivalent to a .1 infestation rate per decade; a .2 infestation rate (of duration 1 decade) every other decade; a .4 infestation rate every 4 decades;

etc., it is difficult to envision that present levels of infestation in the Quesnel TSA are sufficiently high to cause large long-run impacts unless of course they were to persist for many years. We emphasize, however, that these results do not take away from the short-term impacts of maintaining continuity of timber supply for a local mill centre for the next 5 to 10 years. such impacts can be devastating. One aspect of the solutions illustrated in Figure 7 should be noted. Because the objective of the optimization problem is to maximize volume yield over the planning horizon, any solution will indicate that an infested stand be immediately converted to a newly established productive stand, regardless of the cost of rehabilitation. This is indeed the case in the examples presented, in that infested stands are shown to be re-established as age-class-one timber two decades following infestation (that is, areas shifted into the infested class in the first decade following infestation, and then back to the uninfested class in the second decade). This amounts to an average regeneration delay of 20 years, which was accepted as reasonable, given the nature of lodgepole pine silvics of this area. This result, though, was fortuitously convenient rather than explicitly controlled. Where such a result is not realistic due to the logistics or cost of artificial regeneration of infested stands, or where natural influences dictate otherwise, extra constraints should be imposed in the model to reflect this fact.

## 4. Examples of the Feedback Application of the Optimal Solution to the Deterministic Forest-Level Problem

In the previous sections we have suggested that in the absence of knowledge of the specific timing and intensity of mountain pine beetle infestations one might choose to use an estimated annual average annual rate of infestation in the procedures given here to produce an approximation of the yield from the forest. In this section, we attempt to demonstrate how good this approximation might be, using the forest-level approach of Section 3.

Because of the stochastic nature of the true forest-level harvest scheduling problem (due to factors which affect growth estimates, volume estimates, actual amounts harvested, etc.) it has always been an implicit understanding that deterministic approaches ranging from formula methods to more sophisticated simulators or optimization models would be re-solved for a particular area, from time to time, with an updating of information each time. In the deterministic procedures detailed in the present paper, we are introducing an additional element of randomness (that is, infestation due to pests) with the explicit understanding that a given problem be re-solved from time to time. Such a process of re-solving over time, with appropriate updating is known as a feedback procedure.

This feedback procedure was simulated for the Quesnel TSA problem, for selected intensities and frequencies of beetle infestations to demonstrate a range of conditions. Specifically, for each condition, the optimal deterministic solution was obtained, using an average annual infestation rate of .1 per decade as in the previous section (Figure 7). The starting age-class distribution was then updated for

(i) harvest (using the first period harvest prescription from the solution);

(ii) aging; and

(iii) "actual" insect infestations, if any;

according to the dynamics of equation [9]. This procedure was then repeated for several iterations, thus simulating the activities of harvest scheduling, implementation, and updating of results over the long-term. Insect infestations of step (iii) were applied deterministically (that is, at regular intervals and of equal intensity during each infestation) such that the average annual rate was .1 per decade for timber in age class 5 and above. Three frequency and intensity combinations were chosen, these being a rate of .6 every 6 decades; .4 every 4 decades; and .2 every 2 decades all for timber in age class 5 and above. In each case an infestation is imposed in the first decade, since large infestations are presently occurring in the area being modelled. While these feedback simulations are themselves deterministic they are mainly intended to give an indication of the effect of approximating the interval nature of pest infestations with a constant rate. Figures 8, 9, and 10 show the first 20 periods of the feedback solutions for each of the frequency and intensity combinations, superimposed on the optimal deterministic solution using the average annual rate of .01. In all cases the feedback schedules fall below the deterministic schedule for the first several periods, (this is due to the imposition of a pest infestation in the first period; had the first infestation been imposed at some later period, the feedback solution would lie above the deterministic solution until after that period), however, eventually the two schedules close together and the feedback schedule fluctuates around the deterministic schedule. The magnitude of the fluctuations, is dependent upon the intensity of the infestations. Thus, in the long-run, the deterministic solution using constant average annual infestation rate, provides an approximate average to the feedback solution. This approximation improves (that is, deviation of the feedback result from the deterministic result decreases) as the intensity of actual pest infestations diminishes. The decade infestation intensities here of .6 and .4 (for timber of age class 5 and above) are unrealistically high, based upon past records. Even the decade rate of .2 for timber of this age, for a large area is probably at the upper limit for this pest. For realistic situations, therefore, this approach should provide a good approximation.

## 5. Discussion

Modelling the effect of pest infestations on forest output is a difficult exercise due to the many ways in which pests may cause damage to occur; due to the random (for example, climatic) and non-random (for example, age of timber, geographic dispersion) causal factors; and due to lack of knowledge on the long-term behaviour of pests. However, if we are to truly evaluate long-term pest management programs we need to deal with these difficulties and increase our knowledge of pest behaviour. In this paper we have given stand-level and forest-level approaches to estimating the long-term effects of pests on forest yields and have provided examples using the mountain pine beetle in the Quesnel TSA of British Columbia. Because infestations of this pest occur in intervals and because of our inability to predict these intervals we have chosen to represent infestations using an average annual rate. Judging from present infestation levels in this area, the long term effects of mountain pine beetle on harvest flows is not very great. Short-term effects, however, are still very dramatic. The management implications of this is that attention should be focussed on short-term management problems, such as insect management, salvage, and rehabilitation and restocking.

The use of an average annual infestation rate in these methods yields an approximate solution. We show how good this approximation might be by simulating the application of the forest-level approach over time in a feedback environment. These simulations show that using an average annual infestation rate in this manner provides a reasonable approximation over time.

The approaches discussed in this paper, while applied to the very specific case of the mountain pine beetle, are general enough to cover a variety of pests. The key elements which pest managers need to supply for their use pertain to long-term behaviour of pests; that is, the continuity of infestations (Are they of an interval or ongoing nature?) and the rate of damage (What propostion of area is infested during an infestation?). Only with these items quantified can we begin to evaluate the effect of pests on forest outputs.

| Age (years) | Volume ($m^3$/ha) |
|---|---|
| 0 | 0 |
| 10 | 0 |
| 20 | 1 |
| 30 | 12 |
| 40 | 39 |
| 50 | 78 |
| 60 | 119 |
| 70 | 156 |
| 80 | 185 |
| 90 | 207 |
| 100 | 222 |
| 110 | 233 |
| 120 | 240 |
| 130 | 245 |
| 140 | 248 |
| 150 | 250 |

Table 1. Volume-age relationship for lodgepole pine (*Pinus contorta* Dougl) on sites-of-site index 19-24 in (base age 100) in the Quesnel TSA of British Columbia.

$$\left[\begin{array}{ccccccc|ccc}
p_1^t & \cdot\ \cdot\ \cdot & p_{a-1}^t, & p_a^t, & & & p_k^t & q_a^t & \cdot\ \cdot\ \cdot\ \cdot & q_k^t \\
(1-p_1^t) & & & & & & & & & \\
 & \ddots & & & & & & & & \\
 & & (1-p_{a-1}^t), & & & & & & & \\
 & & & (1-p_a^t-r_a^t), & & & & & & \\
 & & & & \ddots & & & & & \\
 & & & & & (1-p_{k-1}^t-r_{k-1}^t), & (1-p_k^t-r_k^t) & & & \\
\hline
 & & & r_a^t & & & & (1-q_a^t) & & \\
 & & & & r_{a+1}^t & & & & & \\
 & & & & \cdot & \cdot\qquad\cdot & & & \ddots & \\
 & & & & & & r_k^t & & & (1-q_k^t)
\end{array}\right]$$

<u>Table 2</u>. The matrix $R_t$ in equation [7]. Only non-zero entries are displayed.

$$\left[\begin{array}{ccccccc|ccc}
-(1-p_1^t), & \ldots & -(1-p_{a-1}^t), & -(1-p_a^t), & \ldots & \ldots & -(1-p_k^t) & -(1-q_a^t) & \ldots & -(1-q_k^t) \\
(1-p_1^t), & & & & & & & & & \\
 & \ddots & & & & & & & & \\
 & & (1-p_{a-1}^t) & & & & & & & \\
 & & & (1-p_a^t-r_a^t) & & & & & & \\
 & & & & \ddots & & & & & \\
 & & & & & (1-p_{k-1}^t-r_{k-1}^t), & (1-p_k^t-r_k^t) & & & \\
\hline
 & & & r_a^t & & & & (1-q_a^t) & & \\
 & & & & \ddots & & & & \ddots & \\
 & & & & & & r_k^t & & & (1-q_k^t)
\end{array}\right]$$

<u>Table 3</u>. The matrix $S_t$ in equation [7]. Only non-zero entries are displayed.

$$\left[\begin{array}{ccccccccc|cccc}
p_1^t, & p_2^t & & p_{a-1}^t, & p_a^t & \cdots & p_{k-1}^t, & \cdots & p_k^t & q_{a+1}^t & \cdots & q_{k-1}^t, & q_k^t \\
(1-p_1^t) & & & & & & & & & & & & \\
 & (1-p_2^t) & & & & & & & & & & & \\
 & & \ddots & & & & & & & & & & \\
 & & & (1-p_{a-1}^t) & & & & & & & & & \\
 & & & & (1-p_a^t-r_a^t) & & & & & & & & \\
 & & & & & \ddots & & & & & & & \\
 & & & & & & (1-p_{k-1}^t-r_{k-1}^t), & & (1-p_k^t-r_k^t) & & & & \\
\hline
 & & & & r_a^t & & & & & & & & \\
 & & & & & r_{a+1}^t & & & & (1-q_{a+1}^t) & & & \\
 & & & & & & \ddots & & & & (1-q_{a+2}^t) & & \\
 & & & & & & & \ddots & & & & \ddots & \\
 & & & & & & r_{k-1}^t, & & r_k^t & & & (1-q_{k-1}^t) & (1-q_k^t)
\end{array}\right.$$

Table 4. The matrix $R_t$ for the model for non-catastrophic infestation. Only non-zero entries are displayed.

$$\left[\begin{array}{cccccccc|cccc}
-(1-p_1^t), & (1-p_2^t) & \ldots & -(1-p_{a-1}^t), & -(1-p_a^t) & \ldots & \ldots & -(1-p_k^t) & -(1-q_{a+1}^t) & & & (1-q_k^t) \\
(1-p_1^t) & & & & & & & & & & & \\
 & (1-p_2^t) & & & & & & & & & & \\
 & & \ddots & & & & & & & & & \\
 & & & (1-p_{a-1}^t) & & & & & & & & \\
 & & & & (1-p_a^t-r_a^t) & & & & & & & \\
 & & & & & \ddots & & & & & & \\
 & & & & & & (1-p_{k-1}^t-r_{k-1}^t), & (1-p_k^t-r_k^t) & & & & \\
\hline
 & & & & r_a^t & & & & & & & \\
 & & & & & r_{a+1}^t & & & (1-q_{a+1}^t) & & & \\
 & & & & & & \ddots & & & \ddots & & \\
 & & & & & & r_{k-1}^t, & r_k^t & & & (1-q_{k-1}^t) & (1-q_k^t)
\end{array}\right]$$

<u>Table 5</u>. The matrix $S_t$ for the model for non-catastrophic infestation. Only non-zero entries are displayed.

Table 6. Age class distribution for stands of puse lodgepole pint (*Pinus contorta* Dougl.) on sites of site index 19-24 in (base age 100) in the Quesnel TSA of British Columbia.

| Age (years) | Area ha |
|---|---|
| 10 | 8059 |
| 30 | 1446 |
| 50 | 37576 |
| 70 | 26655 |
| 90 | 64724 |
| 110 | 68511 |
| 130 | 24531 |
| 150+ | 54414 |

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## Appendix

## Evaluation of LRAY and Expected Present Value for a Single Stand

Let $h(x)$ denote the pest hazard function i.e.

[A1.1] $$h(x) = \lim_{\Delta\to 0}[P(\text{stand infested between ages } x \text{ and } x+\Delta \mid \text{stand not infested at age } x)/\Delta]$$

and let $\lambda$ be the constant fire hazard. It follows that, if the cutting age for non-infested stands is set at $T$, and if infested stands are cut immediately upon infestation, then the combined hazard for the destruction of the stand, $h_c(x)$ is

[A1.2] $$h_c(x) = \begin{cases} \lambda + h(x) & x < T \\ \infty & x = T \end{cases}$$

The survivor function (see e.g. Kalbfleisch, 1979), $S_c(x)$, which gives the probability that the stand survives until age $x$, is

[A1.3] $$S_c(x) = \exp\left\{- \int_0^x h_c(t)dt\right\}$$

[A1.4] $$= \begin{cases} e^{-\lambda x} S(x) & x < T \\ 0 & x \geq T \end{cases}$$

where

[A1.5] $$S(x) = \exp\left\{-\int_0^x h(t)dt\right\}$$

is the "survivor function" for pest infestation alone. From [A1.4] it follows that the life-time of the stand is a random variable, $X$ with cumulative distribution function (c.d.f.),

[A1.6] $$F_c(x) = \begin{cases} 1 - e^{-\lambda x} S(x), & x < T \\ 1 \qquad , & x \geq T \end{cases}$$

For values of $x < T$, the distribution of $X$ is absolutely continuous with probability density function (p.d.f.)

[A1.7] $$f_c(x) = [\lambda + h(x)]e^{-\lambda x} S(x)$$

The expected lifetime of the stand is

[A1.8] $$\begin{aligned} E(X) &= \int_0^\infty x \, dF_c(x) \\ &= \int_0^T x(\lambda + h(x))e^{-\lambda x} S(x)dx \\ &\quad + Te^{-\lambda T} S(T) \end{aligned}$$

To evaluate the expected value of the reward $Y$ earned in a given cycle requires conditional expectation. Specifically

[A1.9]
$$E(Y) = \int_0^T E(Y|\text{no infestation nor fire in } [0,t) \cap \text{infestation at } t]h(t)dt$$
$$+ \int_0^T E(Y|\text{no infestation nor fire in } [0,t) \cap \text{fire at } t]\lambda\, dt$$
$$+ E(Y|\text{no infestation nor fire in } [0,T])S_c(T)$$

Since the second term is zero, this gives

[A1.10]
$$E(Y) = \int_0^T \tilde{V}(t)S_c(t)h(t)dt + V(T)S_c(T)$$

i.e.

[A1.11]
$$E(Y) = \int_0^T \tilde{V}(t)e^{-\lambda t} S(t)h(t)dt + V(T)e^{-\lambda T} S(T)$$

The expression [6] for the present value $J$ of the site involves terms $E(e^{-\delta X})$ and $E(e^{-\delta X}Y)$. The first is easily evaluated as

[A1.12]
$$\begin{aligned} E(e^{-\delta X}) &= \int_0^\infty e^{-\delta x}\, dF_c(x) \\ &= \int_0^T e^{-\delta x}(\lambda+h(x))e^{-\lambda x} S(x)dx \\ &\qquad + e^{-\delta T} e^{-\lambda T} S(T), \end{aligned}$$

whilst evaluation of the second involves conditional expectation as above. It can be shown that

$$[A1.13] \qquad E(e^{-\delta X}Y) = \int_0^T \tilde{V}(t)e^{-(\delta+\lambda)t}\, S(t)h(t)dt + V(T)e^{-(\delta+\lambda)T}\, S(T).$$

If costs of re-establishment are included [A1.13] must be modified slightly. Specifically suppose that the costs are respectively $C_1$, $C_2$ and $C_3$ (\$) for re-establishment after a harvest of uninfested timber, after a fire and after a harvest of infested timber. In this case it can be shown that

$$[A1.14] \qquad E(e^{-\delta X}Y) = \int_0^T [\tilde{V}(t)-C_3]e^{-(\delta+\lambda)t}\, h(t)S(t)dt + \int_0^T (-C_2)e^{-(\lambda+\delta)t}\, \lambda\, S(t)dt + [V(T)-C_1]e^{-(\lambda+\delta)T}\, S(T)$$

To evaluate [A1.8], [A1.11], [A1.12], [A1.13] and [A1.14] further requires specification of the pest hazard function $h(x)$ and of the functions $V(x)$ and $\tilde{V}(x)$. A number of possible models could be used. For example the Weibull model

$$[A1.15] \qquad h(x) = \gamma\beta x^{\beta-1}$$

with $\beta > 1$, might be appropriate for the pest hazard, and the Chapman-Richards growth curve

$$[A1.16] \qquad V(x) = b_1(1-e^{-b_2 x})^{b_3}$$

could be used for the VAC. However for many pests it seems that very young trees are completely invulnerable. In such cases a piece-wise defined hazard function of the form

$$[\text{A1.17}] \qquad h(x) = \begin{cases} 0, & x < a \\ \nu, & x \geq a \end{cases}$$

might be more appropriate. For this hazard function

$$[\text{A1.18}] \qquad E(X) = \begin{cases} (1-e^{-\lambda a})/\lambda + e^{-\lambda a}(1-e^{-(\lambda+\nu)(T-a)})/(\lambda+\nu), & T > a \\ (1-e^{-\lambda T}), & T \leq a \end{cases}$$

and

$$[\text{A1.19}] \qquad E(e^{-\delta X}) = \begin{cases} \dfrac{\lambda}{\lambda+\delta} + \left[\dfrac{\lambda+\nu}{\lambda+\delta+\nu} - \dfrac{\lambda}{\lambda+\delta}\right]e^{-(\lambda+\delta)a} + \dfrac{\delta e^{\nu a}}{\lambda+\delta+\nu}\, e^{-(\lambda+\nu+\delta)T}, & T > a \\ \dfrac{\lambda+\delta e^{-(\lambda+\delta)T}}{\lambda+\delta}, & T \leq a \end{cases}$$

Also assuming $\tilde{V}(x) = \theta V(x)$, where $\theta$ is a constant $(0 \leq \theta \leq 1)$ whose complement $1 - \theta$ reflects the proportional loss in volume (value) of the stand due to pest infestation, it can be shown that

$$[\text{A1.20}] \qquad E(Y) = \begin{cases} \nu e^{-\lambda a}\displaystyle\int_a^T \theta V(t)e^{-(\lambda+\nu)(t-a)}dt + e^{-\lambda a}\, V(T)e^{-(\lambda+\nu)(T-a)}, & T > a \\ V(T)e^{-\lambda T}, & T \leq a \end{cases}$$

and

$$[\mathrm{A1.21}] \quad E(e^{-\delta x}Y) = \begin{cases} \displaystyle\int_a^T [\theta V(t)-C_3]e^{-(\lambda+\delta)t}\,\nu e^{-\nu(t-a)}dt \\ \displaystyle\quad + \int_a^T (-C_2)e^{-(\lambda+\delta)t}(1+e^{-\nu(t-a)})\lambda\,dt \\ \quad + e^{(-\lambda+\delta)T}\,e^{-\nu(T-a)}[V(T)-C_1], & T > a \\ \displaystyle\int_0^T (-C_2)e^{-(\lambda+\delta)t}\,\lambda\,dt + e^{-(\lambda+\delta)T}[V(T)-C_1], & T \le a. \end{cases}$$

For any reasonable form of the function $V(x)$, numerical methods are required for further evaluation of [A1.20] and [A1.21]. Once these expressions have been evaluated the LRAY $\bar{Y}(T)$ and the present value $J(T)$ corresponding to a rotation age $T$ can easily be evaluated using [4] and [5].

## FIGURE CAPTIONS

Figure 1. Pest-adjusted volume-rotation curves (VRC) with a piecewise defined pest hazard function which is zero for trees aged 50 years or less and is $\nu = 0$ (curve (a)), $\nu = .001$ (curve (b)), $\nu = .005$ (curve (c)) and $\nu = .01$ (curve (d)) for older trees. The fire hazard $\lambda$ is zero, and the salvage parameter is $\theta = 0.75$. Curve (a) is equivalent to the ordinary VAC.

Figure 2. Pest-adjusted volume-rotation curves (VRC) with a piecewise defined pest hazard function which is zero for trees ages 50 years or less and is $\nu = 0$ (curve (a)), $\nu = .001$ (curve (b)), $\nu = .005$ (curve (c)) and $\nu = .01$ (curve (d)) for older trees. The fire hazard $\lambda$ is zero, and the salvage parameter is $\theta = 0.25$. Curve (a) is equivalent to the ordinary VAC.

Figure 3. Pest-adjusted volume-rotation curves (VRC) with a piecewise defined hazard function which is $\nu = 0.01$ for older trees. The age of onset of infestation vulnerability is, $a = 50$ (curve (b)), $a = 30$ (curve (c)), $a = 10$ (curve (d)) and $a = 0$ (curve (e)). The salvage parameter is $\theta = .75$. Also shown is the VAC (curve (a)).

Figure 4. Fire and pest adjusted volume-rotation curves (VRC) with a constant fire hazard ($\lambda = .001$) and a piecewise defined pest-hazard which is zero for trees aged 50 or less and is $\nu = 0$ (curve (b)), $\nu = 0.001$ (curve (c)), $\nu = 0.005$ (curve (d)) and $\nu = 0.01$ (curve (e)) for older trees. The salvage parameter $\theta$ is 0.25. Also shown is the VAC (curve (a)).

Figure 5. Expected present value as a function of rotation age, when there is a piecewise defined pest hazard which is zero for trees of age 50 or less and is $\nu = 0$ (curve (a)), $\nu = .001$ (curve (b)), $\nu = .005$ (curve (c)) and $\nu = .01$ (curve (d)) for older trees, and zero fire hazard. The salvage parameter is $\theta = 0.75$. Costs and stumpage values are as defined in text.

Figure 6. Same as Figure 5 except that the salvage parameter is $\theta = 0.25$.

Figure 7. Harvest schedules using constant annual infestation rates (probabilities) of (a) 0, (b) .01, (c) .03, (d) .05 for stands of ages 50 and older. Other specifications are as discussed in text.

Figure 8. First twenty periods of a simulated harvest sequence using the feedback procedure described in Section 4. In the simulation 20% of timber of age 50 years or older, was infested every second decade (path (a)). Also shown is the deterministic harvest schedule based upon a 10% infestation rate per decade (path (b)).

Figure 9. As in Figure 8, only with the simulation using 40% infestation rates every fourth decade.

Figure 10. As in Figure 8, only with the simulation using 60% infestation rates every sixth decade.