A SPECIAL INSTANCE OF "PERCEPTUAL WORK"

AND ITS QUANTIFICATION

by

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Abstract.

The attempt was made to quantify the relative amount of "perceptual work" involved in placing items of different degrees of difference together. A 5 x 5 grid was considered as a structure in which the parts differ only with respect to location. A numbering system was proposed as a method of describing the locational property of each part relative to the structure. Permutations were adopted as special mathematical transformations which were applicable to the description of certain locations. The permutational relationship between locations reflected the relative amount of perceptual work necessary to transform one location into another. The general hypothesis was that those locations would be perceived as most similar which required least perceptual work to make them identical. This was tested in two experiments. In the first experiment 24 subjects chose between two locations the one which they perceived as being most similar to a target location. A binomial test of the responses was significant ($p < .006$). In the second experiment 12 subjects were required to categorize 8 locations with no restrictions on the number of categories that they could make. Following this, half of the subjects were required to place the stimuli in 3 categories, half in 5 categories. Two measures were recorded: the time it took

to make each category and the number and type of locations included in each category. A one tail t-test of the time difference was significant ($p < .0005$). A binomial test ($p < .02$) indicated that the most frequent groupings were those based on minimizing perceptual work.

[redacted]

Alex Bavelas, Ph.D.

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D.R. Walker, Ph.D.

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This thesis at its best may be considered as a footnote to Professor A. Bavelas's ideas and teachings. For this reason I owe a great debt of gratitude to him.

I am indebted to Dr. G. Miller, Dr. W. Gordon, and Dr. D. Walker for their helpful suggestions. I am grateful to Mr. E. Lee and J. MacGregor for many helpful discussions and painstaking work of correcting the english to make the text readable. I thank my wife, Susan, for her comments and assistance on various parts of the thesis.

I. INTRODUCTION.

Max Wertheimer (1925) in a paper on Gestalt theory expressed the "formula" of Gestalt theory in this way:

> There are wholes, the behaviour of which is not determined by that of their individual elements, but the Part-Processes are themselves determined by the intrinsic nature of the whole. It is the hope of Gestalt theory to determine the nature of such wholes.

Gestalt psychologists set forth to study 'wholes' as opposed to the traditional concerns of elementism psychology. These studies led to the discovery of the principles of perceptual organization and, later, to the development of a dynamic field theory. In this theory, the visual field is a dynamic distribution whose parts are interdependent through their participation in the whole. Two sets of organizing forces were hypothesized, internal and external. As Koffka (1935) puts it:

> . . . we have two kinds of forces, those which exist within the process in distribution itself and which will tend to impress on this distribution the simplest possible shape, and those between this distribution and the stimulus pattern, which constrain this stress toward simplification (p. 138).

The strength of "internal organizing forces" varies as a function of the principle under which the visual field is organized. For example, the greater the similarity between processes, the greater the internal organizing force between them.

The concept of "perceptual work" derives from the notion of "internal organizing forces." Koffka (1935) writes,

> Demonstrations of the effectiveness of the internal organizing forces . . . occur at practically every moment of our lives. We are surrounded by rectangular things which look to us rectangular. Even when we disregard the fact of perspective distortion, each one of these cases is a point in hand: for what real rectangle is a mathematically exact rectangle? The deviations . . . are there, and yet we see perfect rectangles . . . the fact that we see rectangles everywhere is due to the fact that the true rectangle is a better organized figure than the slightly inaccurate one would be, and that only a very slight dislocation is necessary to change the latter to the former (pp. 140-141).

These "dislocations" which are required to make one thing into another, i.e. a non-rectangle into a rectangle, are what is meant by "perceptual work." The amount of perceptual work increases as a function of the extent to which two percepts are "dislocated" with respect to each other. That is, as more differences appear between two "non-rectangles" it becomes more difficult to see them as the same. Since the energy for perceptual work stems from the "internal organizing forces," increases in differences between two "non-rectangles" correspond to a decrease in available energy for perceptual work. For example, the amount of available energy for seeing a rectangle and a circle as the same is rather small compared to the greater amount of energy available to see a square and a rectangle as similar. To summarize, the amount of perceptual work is proportional to the extent to which two percepts differ from one another and it is inversely related to the strength of "internal organizing forces" between the two percepts.

The scarcity of attempts at quantification of Gestalt principles is characteristic of the phenomenological approach. Boring (1950) writes:

> In general the phenomenologist seeks to find an experimentum crucis, the convincing single demonstration of some observed generality. . . . Since phenomenology deals with immediate experience its conclusions are instantaneous. They emerge at once and need not wait upon the results of calculations derived from measurements. Nor does a phenomenologist use statistics, since a frequency does not occur at a given instant and can not be immediately observed. For these reasons many of Hering's elaborate pieces of apparatus for the study of color are more appropriately used for demonstration than for experimentation. In the same way one finds in the writings of the modern Gestalt psychologists many neat demonstrational diagrams printed on the page, experimenta crucis designed to make a phenomenologist of the reader, letting him have at once the immediate experience which constitutes the evidence. (p. 602)

A rare example is Orbison (1939) who made a quasi-quantitative attempt in which the strength of field forces were estimated as a function of spatial separation in the visual field. Distortion of geometric shapes was predicted and experimentally verified when they were super-imposed on geometrical fields in which the strength and direction of fields forces were specified.

Neither Orbison nor other quasi-quantitative attempts achieve a strictly quantitative definition of the perceptual work involved. The purpose of this thesis is to measure the relative amount of perceptual work involved in placing items of different degrees of difference together i.e. in the same category. Specifically, the subject is asked which two of the three items are the most similar. For example, the subject is shown three grids. On each grid one location has been specified by being filled in.

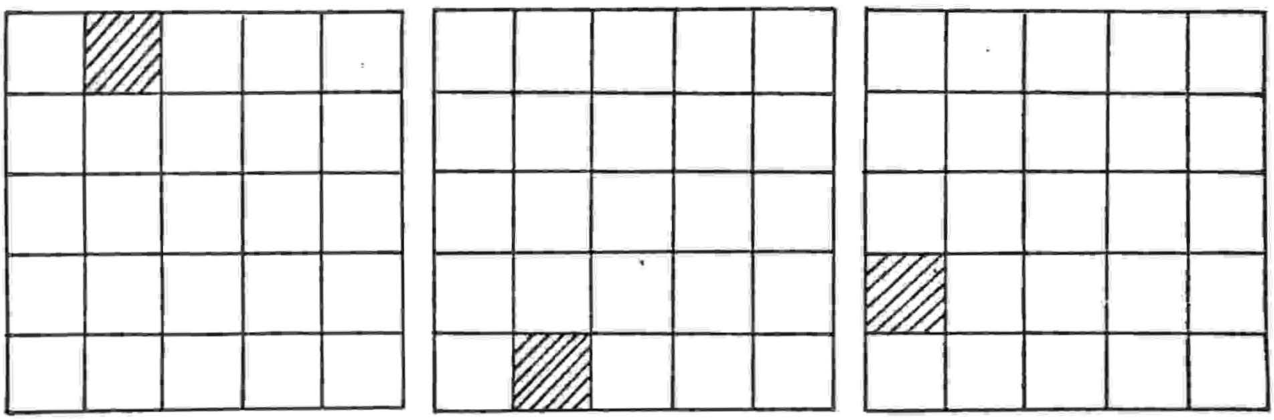


Fig. 1

Clearly these three items are identical in all respects except for their location in the square "field".

In the attempt to quantify perceptual work, it is assumed that the differences between two items can be measured by an assessment of how readily one location can be transformed into another. Here, transformation refers to the mapping of one location onto another. For example, rotations of 180° from one location to another with respect to a point, line, or a plane in geometry are operations which can transform one location into another. On the basis of this assumption, the relative amount of perceptual work is proportional to the extent to which one location must be transformed in order to match another on the pertinent parameter. The way in which the locational property will be treated is given in the following section.

II. NUMBERING SYSTEM.

Consider a 5 x 5 grid. This structure is composed of 25 identical squares. The general problem is to describe each part or square relative to the rest of the structure, field, or grid. Since all squares are identical with respect to size, form, colour, etc., the only relationship of each part to the rest of the structure is that of location.

To study the perception of similarity among the squares, it is necessary to describe systematically the relative location of each square in a way which reflects the way such locations are perceived. With this consideration, polar and cartesian co-ordinate systems, for example, are inappropriate as descriptive systems of locational relationships. Both of these co-ordinate systems have a structure of their own and impose a structure, which may not be present phenomenally, upon another structure.

Before the proposed method can be described, two problems must be considered; connectivity, and unitary organization.

Connectivity: there are two ways in which two squares of a grid could be perceived as connected to each other, a side connection in which two squares share a common side, or a corner connection in which two squares share a common corner. (see Figure 2)

Fig. 2

The question is whether one of these connections is perceptually dominant over the other. To answer this question consider the condition in which both connections are present. (See Figure 3)

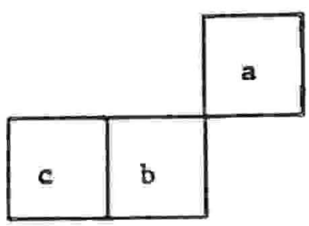


Fig. 3

It is easier to see (b and c) as a unit than it is to see (a and b) as a unit. As the number of connections is increased, perception of a group of squares connected only by side connections becomes even more dominant. Increasing the number of corner connections does not affect the perception of the group with side connections. (See Figure 4)

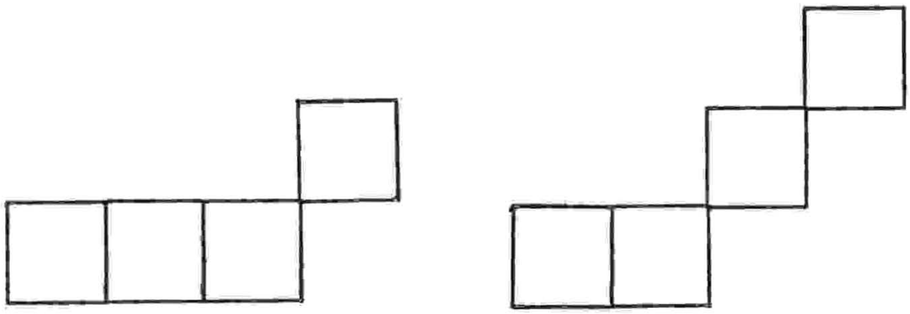


Fig. 4

Another way of observing the same phenomena is to randomly select a square in a 5 x 5 grid, and attempt to view it as the intersection of the appropriate row and column, and/or intersection of appropriate diagonals. Because the side connection is dominant the proposed method of describing the location of each square relative to the grid will be concerned primarily with such connections.

Unitary Organization: the concept of unitary organization refers to the tendency for a structure composed of a number of parts to be perceived as a 'whole' rather than as a number of parts. Since parts are perceived as wholes, it is imperative that the location of each part be described relative to the 'whole' structure of which it is a part, and not to some other structure. For example, the structure in Figure 5 tends to be perceived as two parts; a 3 x 3 grid and a 5 x 5 grid. Consequently, a description of location (a) should be made relative to the 5 x 5 grid and not to the whole structure.

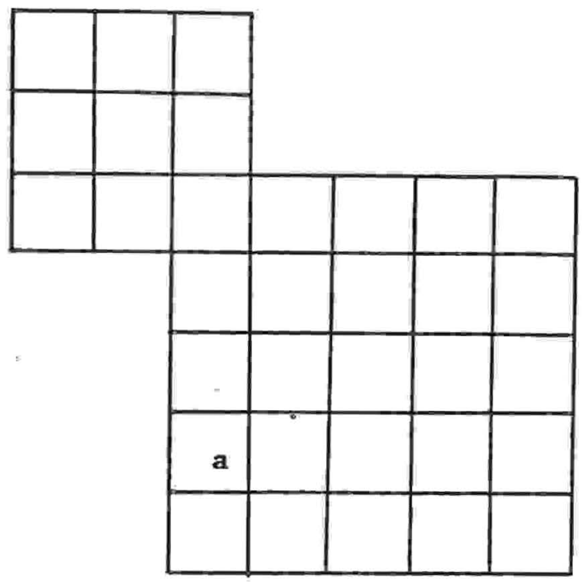


Fig. 5

Thus, this method must describe the locational relationship of a part relative to the rest of the structure as a function of side connections, and organized wholes. Since a 5 x 5 grid has unitary organization, the problem of describing any location relative to the grid consists in specifying the surroundings of that location. A location can be described as the intersection of the appropriate column and row in the grid.

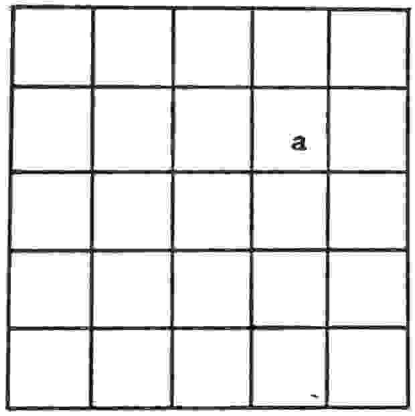


Fig. 6

In Figure 6, location (a) is surrounded by one square on top, one square on the right, three squares below, and three squares on the left. That is to say, the location of any square in the grid can be given by specifying the number of adjoining squares on its perimeter. By convention let the four numbers be read in clockwise order, starting from the top or north. This description is called the orientational set and is written as $\{n_N, n_E, n_S, n_W\}$. For example, location (a) from Figure 6 is represented as $\{1, 1, 3, 3\}$ in orientational notation. Every square of a 5 x 5 grid could be described similarly. (See Figure 7)

0440	0341	0242	0143	0044
1430	1331	1232	1133	1034
2420	2321	2222	2123	2024
3410	3311	3212	3113	3014
4400	4301	4202	4103	4004

Fig. 7

Let a <u>normalized set</u> of numbers be defined as the orientational set which is written from the smallest number to the largest. In other words, the order of top, right, bottom, and left is taken out of the system. Under these conditions, there are only six unique normalized sets in a 5 x 5 grid. Namely; $\{0, 0, 4, 4\}$, $\{0, 1, 3, 4\}$, $\{0, 2, 2, 4\}$, $\{1, 1, 3, 3\}$, $\{1, 2, 2, 3\}$, and $\{2, 2, 2, 2\}$. The normalized set of numbers may be considered as representing the type of relationship that a location may have with respect to the grid without specifying its precise location. That is to say, the normalized set of numbers is similar to the way people may talk about a corner location without specifying which corner.

How do people talk about a corner location? What are the characteristics of a corner location? Consider the correspondence between the way people may talk about their perception of a location relative to the structure and the numbering system. People may say, for example, that corner locations have two open or unconnected sides, and two sides that are connected to the grid and equidistant from its boundaries. "Two open or unconnected sides" can be translated as or represented by two zeros in the normalized set $\{0, 0, 4, 4\}$, the "two sides that are connected to the grid and equidistant from its boundaries" can be represented in the normalized set by a pair of equal numbers. If asked about the magnitude of the distance from the boundaries, they may count the 'natural' units of measurement i.e. the number of squares that occupy the space between the corner and the boundaries of the grid. In the preceding example this corresponds to the presence of fours in the normalized set $\{0, 0, 4, 4\}$. Similarly, the center is that location which is most balanced or equidistant from the boundaries of the structure. The description given to the center location by the numbering system is $\{2, 2, 2, 2\}$ for a 5 x 5 grid.

To consider a slightly different example, it is worth noting that an edge region (all the squares on the perimeter of the grid) is characterized by two relations: a) all the members of the region have at least one side open, and b) at least one other side which is maximally distant from the opposite boundary of the grid. In the numbering system this corresponds to the presence of a zero and a four in all the members of the edge region. (See Figure 7)

Since a normalized set has been defined, it is clear that the order of the four numbers refers to the orientation of a location in the grid. In this sense a 5 x 5 grid is composed of six different normalized sets which are repeated with different orientations. When orientation is disregarded, the orientational sets collapse into one or another of the normalized sets. Therefore, the locations of a 5 x 5 grid may differ from one another in two ways: 1) different orientational sets for a given normalized set, and 2) different normalized sets.

This thesis will be mainly concerned with a 5 x 5 grid. However, the method described is applicable as a general method for systematically describing the relationship between the cells of a structure and the structure itself as a 'whole'. An infinite number of structures can be constructed by connecting variable number of squares together in different ways. The method is not limited to structures composed of squares only, it is equally applicable to structures composed of hexagons.

In the next section permutations are considered as a special type of transformation among locations with identical normalized sets. The relative amount of perceptual work is evaluated with respect to the permutational relationship between two locations.

III. PERMUTATIONS AND PERCEPTUAL WORK.

Consider a given normalized set. There may be several orientational sets corresponding to it. The possible transformations between these orientational sets involve a systematic change from one order of the four numbers to another. The general problem of transforming two orientational sets (having the same normalized set) into one another has been dealt with in abstract algebra. Such transformations are called Permutations and have served as a key concept in development of the theory of groups. Most of the definitions given below are taken from a text in abstract algebra by J. Fraleigh (1967), and they will become clearer in the example which will follow.

1. A collection of particular things are said to be a set, such as the set of numbers that describe a location.

2. Function or mapping from set A into set B is a rule which assigns to each element a of A exactly one element b of B.

3. A function from a set A into a set B is one to one if each element of B has at most one element of A mapped into it, and is onto B if each element of B has at least one element of A mapped into it.

4. A Permutation of a set A is a function from A into A which is both one to one and onto. In other words, a Permutation of A is a one to one function from A onto A.

5. If A is a finite set $\{1, 2, \ldots n\}$ then the group of all Permutations of A is the symmetric group on the n elements

of the set A. This group has n! permutations.

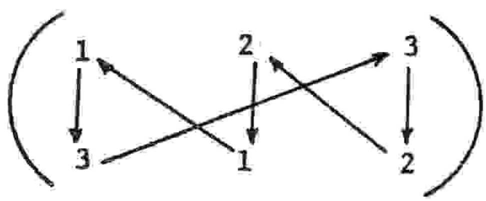
<u>example</u>: let set A = $\{1, 2, 3\}$. There are 3! or six members in the symmetric group; $\{1, 2, 3\}$, $\{2, 3, 1\}$, $\{3, 1, 2\}$, $\{1, 3, 2\}$, $\{3, 2, 1\}$, $\{2, 1, 3\}$. Suppose set A is to be permuted to one of the other sets, say set $\{3, 1, 2\}$. The permutation between these two sets can be represented as $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ which consists of the following mappings:

1 ⟶ 3

2 ⟶ 1

3 ⟶ 2

or



The mappings read as 3 replacing 1, 1 replacing 2, and 2 replacing 3.

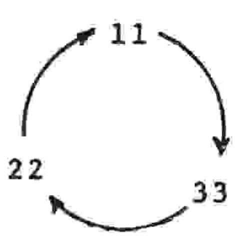
Another notation for representing permutations is by cycles. That is to say the mappings are represented in cyclic form.

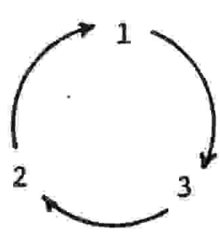
1 ⟶ 3

2 ⟶ 1

3 ⟶ 2

or



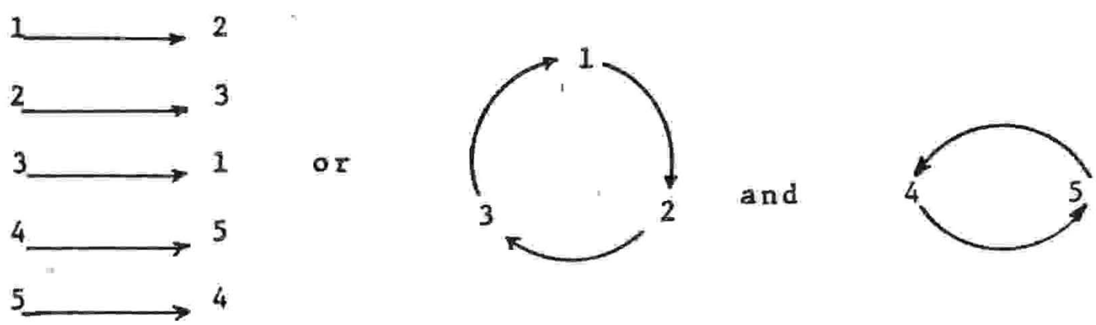


The cyclic notation can also be presented as a single row enclosed by parentheses (1 3 2). Since this particular cycle involves mappings of three numbers it is called a <u>cycle of length three</u>. Note that the same cycle can be written in the following orders; (2 1 3), (3 2 1). So long as the direction is preserved the starting position has no significance because the notation is cyclic.

If each element is associated with itself, the mapping is called Identity mapping $\begin{pmatrix}1 & 2 & 3\\1 & 2 & 3\end{pmatrix} = I$. If only some of the elements are associated with themselves they will not be presented in the cycle length. For example:

$\begin{pmatrix}1 & 2 & 3\\1 & 3 & 2\end{pmatrix} = (2\ 3)$

Suppose $A = \{1, 2, 3, 4, 5\}$ and is to be permuted to $A' = \{2, 3, 1, 5, 4\}$, the following mappings are involved:



This can be written as $\begin{pmatrix}1 & 2 & 3 & 4 & 5\\2 & 3 & 1 & 5 & 4\end{pmatrix} = (1\ 2\ 3)(4\ 5)$. Every permutation of a finite set can be written as a product of cycles with no common elements.

Permutations, which are applicable to locations with identical normalized sets, are a particular kind of transformation. To apply permutations, consider the next to the corner locations, the normalized set $\{0, 1, 3, 4\}$. This group of locations has the largest possible group of orientational sets for any given normalized set in a 5 x 5 grid.

	a		b	
h				c
g				d
	f		e	

Fig. 8

The orientational sets corresponding to each location are as follows:

$a = \{0, 3, 4, 1\}$ $\qquad$ $e = \{4, 1, 0, 3\}$

$b = \{0, 1, 4, 3\}$ $\qquad$ $f = \{4, 3, 0, 1\}$

$c = \{1, 0, 3, 4\}$ $\qquad$ $g = \{3, 4, 1, 0\}$

$d = \{3, 0, 1, 4\}$ $\qquad$ $h = \{1, 4, 3, 0\}$

All possible permutations between any two sets are given below.

$(a \,\&\, b) = \begin{matrix}0341\\0143\end{matrix} = (31)$ $\qquad$ $(c \,\&\, e) = \begin{matrix}1034\\4103\end{matrix} = (1430)$

$(a \,\&\, c) = \begin{matrix}0341\\1034\end{matrix} = (0143)$ $\qquad$ $(c \,\&\, f) = \begin{matrix}1034\\4301\end{matrix} = (14)(03)$

$(a \,\&\, d) = \begin{matrix}0341\\3014\end{matrix} = (03)(41)$ $\qquad$ $(c \,\&\, g) = \begin{matrix}1034\\3410\end{matrix} = (13)(04)$

$$(a \,\&\, e) = \begin{matrix}0341\\4103\end{matrix} = (04)(31)$$

$$(a \,\&\, f) = \begin{matrix}0341\\4301\end{matrix} = (04)$$

$$(a \,\&\, g) = \begin{matrix}0341\\3410\end{matrix} = (0341)$$

$$(a \,\&\, h) = \begin{matrix}0341\\1430\end{matrix} = (01)(34)$$

$$(b \,\&\, c) = \begin{matrix}0143\\1034\end{matrix} = (01)(43)$$

$$(b \,\&\, d) = \begin{matrix}0143\\3014\end{matrix} = (0341)$$

$$(b \,\&\, e) = \begin{matrix}0143\\4103\end{matrix} = (04)$$

$$(b \,\&\, f) = \begin{matrix}0143\\4310\end{matrix} = (04)(13)$$

$$(b \,\&\, g) = \begin{matrix}0143\\3410\end{matrix} = (03)(14)$$

$$(b \,\&\, h) = \begin{matrix}0143\\1430\end{matrix} = (0143)$$

$$(c \,\&\, d) = \begin{matrix}1034\\3014\end{matrix} = (13)$$

$$(c \,\&\, h) = \begin{matrix}1034\\1430\end{matrix} = (04)$$

$$(d \,\&\, e) = \begin{matrix}3014\\4103\end{matrix} = (34)(01)$$

$$(d \,\&\, f) = \begin{matrix}3014\\4301\end{matrix} = (3410)$$

$$(d \,\&\, g) = \begin{matrix}3014\\3410\end{matrix} = (04)$$

$$(d \,\&\, h) = \begin{matrix}3014\\1430\end{matrix} = (13)(04)$$

$$(e \,\&\, f) = \begin{matrix}4103\\4301\end{matrix} = (13)$$

$$(e \,\&\, g) = \begin{matrix}4103\\3410\end{matrix} = (4301)$$

$$(e \,\&\, h) = \begin{matrix}4103\\1430\end{matrix} = (41)(03)$$

$$(f \,\&\, g) = \begin{matrix}4301\\3410\end{matrix} = (43)(01)$$

$$(f \,\&\, h) = \begin{matrix}4301\\1430\end{matrix} = (4103)$$

$$(g \,\&\, h) = \begin{matrix}3410\\1430\end{matrix} = (31)$$

Note that on the basis of different cycle lengths three types of permutations are possible. Namely; one cycle of length two, two cycles of length two, and a cycle of length four.

Two problems must be considered in relation to different cycle lengths: the amount of perceptual work, and their geometric representations.

1) The relative amount of perceptual work and length of cycles: consider an example for each different cycle length.

I. One cycle of length two: let set $A = \{a, b, c, d\}$ and set $A_1 = \{a, d, c, b\}$

$$\begin{pmatrix} a & b & c & d \\ a & d & c & b \end{pmatrix} = (bd)$$

Mappings:

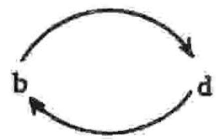
a ⟶ a

b ⟶ d

c ⟶ c

d ⟶ b

or



A cycle of length two means that two elements are mapped onto themselves, whereas the other two elements are mapped onto their opposite. Where opposite refers to a reversal in order of two things.

II. Two cycles of length two: let set $A = \{a, b, c, d\}$ and set $A_1 = \{d, c, b, a\}$.

$$\begin{pmatrix} a & b & c & d \\ d & c & b & a \end{pmatrix} = (ad)(bd)$$

Mappings:

a ⟶ d

b ⟶ c

c ⟶ b

d ⟶ a

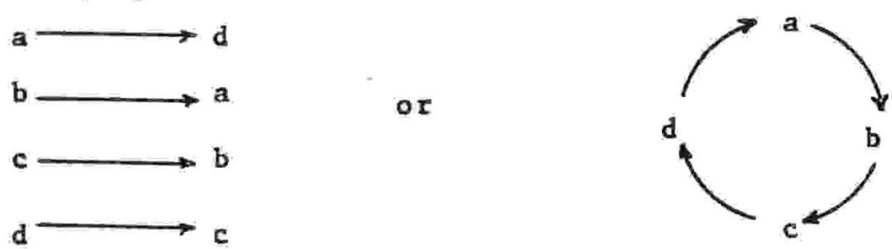
or

In this case each element is mapped onto a different element. However, the elements are grouped in pairs and each pair is mapped onto its opposite.

III. Cycle of Length four: let set $A = \{a, b, c, d\}$, and set $A_1 = \{d, a, b, c\}$.

$$\begin{pmatrix} a & b & c & d \\ d & a & b & c \end{pmatrix} = (adcb)$$

Mappings:



For a cycle of length four each element is mapped onto a different element. All elements of mappings are part of a single relationship.

Above considerations suggest that a cycle of length two involves the least amount of perceptual work possible except for identity permutation. A cycle of length two involves lesschange than other cycles. In other words, only two elements are mapped onto their opposite. For two cycles of length two and a cycle of length four each element is mapped onto a different element. To decide whether two cycles of a length two involve less, equal, or more perceptual work than a cycle of length four, the order in which the mappings are coded must be considered. Both permutations consist of an equal number of changes. That is, each element is mapped onto a different element. The complexity of the order in which the mappings are coded can be studied by constructing a set that will permute to another given set with specified cycle length.

For example, suppose set $A = \{a, b, c, d\}$ is given, the problem is to construct two other sets A_1 and A_2 in such a way that set A can be permuted to set A_1 with two cycles of length two, and to set A_2 with one cycle of length four.

I. Construction of set A_1: the opposite of any pair of the four elements can be selected from set A and placed in set A_1. For example; say the selected elements are a & c, thus, set $A_1 = \{c, -, a, -\}$. The other pair of elements b & d are determined, and their opposite completes set $A_1 = \{c, d, a, b\}$. Thus, set A permutes to set A_1 such that two cycles of length two are produced.

$$\begin{pmatrix} a & b & c & d \\ c & d & a & b \end{pmatrix} = (ac)(bd)$$

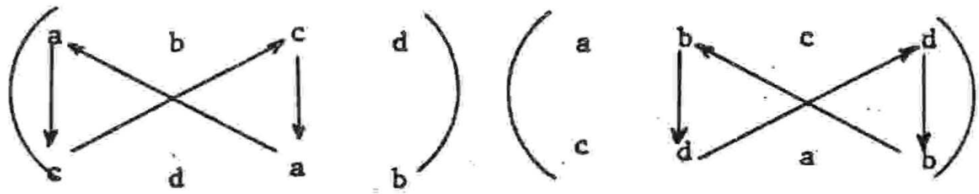
II. Construction of set A_2: any element from set A may be selected as one of the elements in set A_2 provided this element does not occur in its original position. For example, select b from A and let it correspond to d in set A_2, thus, $A_2 = \{-, d, -, -\}$. Then, from the remaining elements (a, b, c) in A, an element is chosen to correspond to position d in set A. Since selection of b would result in a cycle of length two, let c be the second element in A_2, which corresponds to d in set A, thus $A_2 = \{-, d, -, c\}$. To select the third element to correspond to c in set A, the element a is the only possibility since if b was chosen it would result in one cycle of length three. The fourth element is determined. Thus, $A_2 = \{b, d, a, c\}$ permutes to set A such that one cycle of length four is produced.

$$\begin{pmatrix} a & b & c & d \\ b & d & a & c \end{pmatrix} = (a\ b\ d\ c)$$

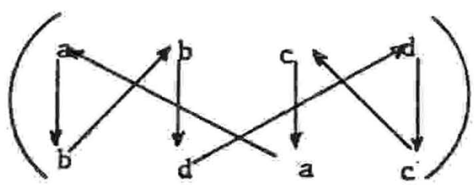
In comparing the process of construction of set A_1 and set A_2 the following points may be noted; firstly, construction of set A_1 requires less steps than that of A_2. Secondly, construction of set A_1 is based on direct rules, whereas construction of set A_2 is based on a trial and error method in which development of cycles of length two and three are avoided. Thirdly, it is more difficult to follow the development of set A_2 than set A_1. In general, construction of set A_2 requires more information than construction of set A_1. If a computer were to construct each set, it would require more information or instructions and computer time to construct set A_2 than set A_1. This suggests that a cycle of length four involves more perceptual work than two cycles of length two.

Another way of observing that a cycle of length four involves more perceptual work than two cycles of length two is by viewing the diagrams of the mappings corresponding to each permutation.

Two cycles of length two:



One cycle of length four:



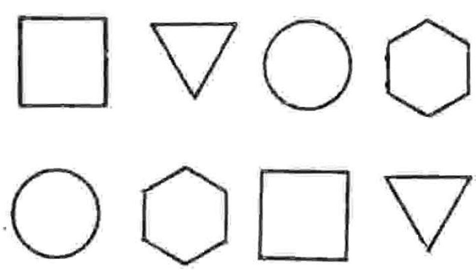
The mappings may be represented as graphs in the following way:

Two cycles of two

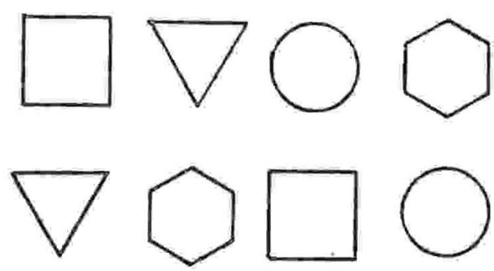
One cycle of length four

The graphs corresponding to two cycles of length two can be memorized easier and faster than the pattern presented in the graph of one cycle of length four.

A third way of illustrating that a cycle of length four involves more perceptual work than two cycles of length two is by the following hypothetical experiment. A subject could be presented with 8 objects in a particular order. His task would be to put similar objects in the same column as quickly as possible. There are two conditions corresponding to permutations of two cycles of length two and cycle of length four.



two cycles of length two



cycle of length four

It is obvious that the two cycles of length two will be completed sooner. The reason for this is that in two cycles of length two, two changes take place in a single step.

2) Geometric relations and operations corresponding to cycles of different length:

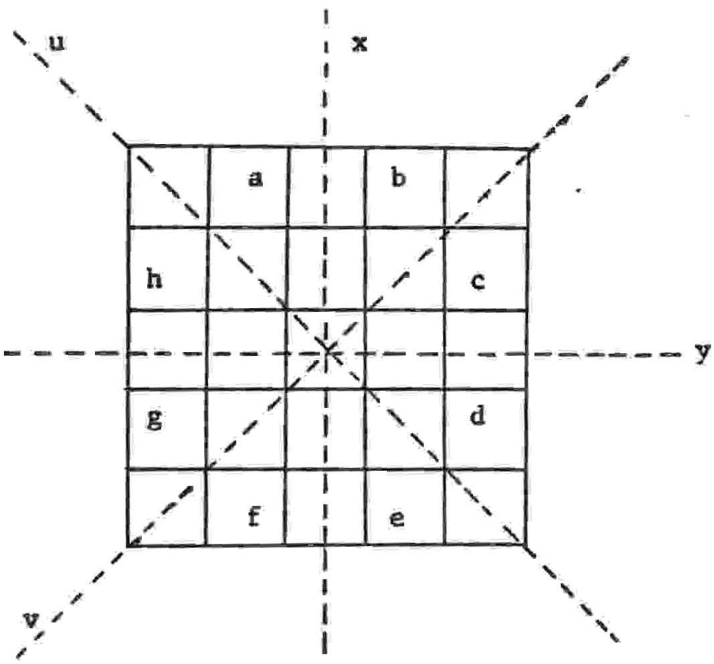


Fig. 9

The following table classifies all possible pairs of eight next to the corner locations on the basis of cycle length.

The locations that are mirror images of one another with respect to x or y axis correspond to a cycle of length two (e.g.: (a & b), (c & h) ...). So, if the grid is rotated 180° in the third plane about the x-axis, all the locations of mirror images with respect to that axis will exchange their positions; for example, $a \longleftrightarrow b$, $d \longleftrightarrow g$.

A cycle of length four corresponds with the geometric operation of rotation. For example, if (a) is to be transformed to c, or d to f, the grid must be rotated 90° clockwise about its center.

There are twelve permutations of two cycles of length two whereas there are only eight permutations of each of the other cycle lengths. Geometrically, there are two

TABLE 1.

Classification of Cycles.

	Cycle of length two	Cycle of length four	Two Cycles of length two
1.	(a & b)	(a & c)	(a & d)
2.	(a & f)	(a & g)	(a & e)
3.	(b & e)	(b & d)	(a & h)
4.	(c & d)	(b & h)	(b & c)
5.	(c & h)	(c & e)	(b & f)
6.	(d & g)	(d & f)	(b & g)
7.	(e & f)	(e & g)	(c & f)
8.	(g & h)	(f & h)	(c & g)
9.			(d & e)
10.			(d & h)
11.			(e & h)
12.			(f & g)

different transformations which correspond to two cycles of length two. One is flipping the grid about the diagonal axis u or v. (Figure 9) The operation of diagonal flip accounts for eight of the twelve permutations of two cycles of length two. The second operation which accounts for the other four members of the group is a 180° rotation of the grid about its center. An example of diagonal flip is the relationship between (a & d), and an example of a 180° rotation or two diagonal flips is the relationship between (a & e).

Consider the permutations of set (a) to set (d) in comparison to the permutation of set (a) to set (e).

$$a = \{0, 3, 4, 1\}$$
$$d = \{3, 0, 1, 4\}$$
$$e = \{4, 1, 0, 3\}$$

$$(a \ \& \ d) = \begin{pmatrix} 0341 \\ 3014 \end{pmatrix} = (03)(41)$$
$$(a \ \& \ e) = \begin{pmatrix} 0341 \\ 4103 \end{pmatrix} = (04)(31)$$

One difference between these two permutations is the particular numbers that are permuted to one another. For example; in (a & d), 0 and 3 replace each other, whereas in (a & e), 0 and 4 exchange positions.

Since each cycle contains information about how one location can be permuted to another location, one may ask how much information does each cycle contribute to the whole permutation. More specifically, the question can be asked in the context of the following problem:

Location a = $\{0, 3, 4, 1\}$, find location x which is permutable to location (a) under the following conditions:

(1) when one cycle of permutation is (03)

(2) when one cycle of permutation is (04)

Solution: location x must be one of the 8 next to the corner locations, since it is permutable to location a.

(1) cycle (03) implies two possible solutions

$$x_1 = \{3, 0, 4, 1\}$$
$$x_2 = \{3, 0, 1, 4\}$$

or

$$(a \ \& \ x_1) = \begin{pmatrix}0341\\3041\end{pmatrix} = (03)$$
$$(a \ \& \ x_2) = \begin{pmatrix}0341\\3014\end{pmatrix} = (30)(14)$$

Since location x_1 does not exist, the only unique solution is location x_2 = d = $\{3, 0, 1, 4\}$.

(2) Cycle (04) implies two solutions

$$x_1 = \{4, 3, 0, 1\}$$
$$x_2 = \{4, 1, 0, 3\}$$

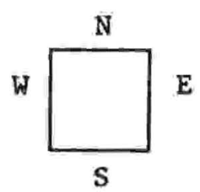
or

$$(a \ \& \ x_1) = \begin{pmatrix}0341\\4301\end{pmatrix} = (04)$$
$$(a \ \& \ x_2) = \begin{pmatrix}0341\\4103\end{pmatrix} = (04)(31)$$

Both solutions exist, x_1 corresponds to f and x_2 to location e.

Note that cycle (03) contains all the information necessary to locate d as the only unique solution, whereas location e does not become the only unique solution unless the second cycle of permutation is given. Thus, in the permutation of (a & d) one of the two cycles is redundant relative to the other cycle, whereas in the permutation of (a & e) both cycles contain information.

To understand this difference consider all possible ways that a cycle of length two can be constructed in terms of direction.



(NS), (NE), (NW), (ES), (EW), (SW)

These cycles can be classified into two groups on the basis of whether they are single directional cycles or two directional. A single directional transformation is a replacement which takes place along the NS or EW axis, whereas two directional transformation takes place along two axes N & E, S & W, E & W, or N & W.

Single directional	Two directional
(NS)	(NE)
(EW)	(ES)
	(SW)
	(NW)

Single directional cycles suggest a relationship that exists between two squares in a grid. A single directional cycle of length two corresponds to two locations that are mirror images of one another. However, a two directional cycle by itself does not imply the existence of a relationship, unless the product of two such cycles are present in the permutation. A (NE) cycle cannot exist by itself without implying (NE) (SW) because a square is considered as one unit and no change can take place on the NE half of a square unless (SW) is included. Geometrically a flip suggests a permutation which includes orientational changes of all numbers in two cycles. The above table can be rewritten as:

Single directional	Two directional
(NS)	(NE) (SW)
(EW)	(NW) (ES)

Two cycles of length two can be defined as redundant cycles when presence of one cycle implies presence of the other, and as non-redundant cycles when presence of one cycle does not imply presence of the other. Two non-redundant cycles of length two are intimately related with cycle of length two. That is, two sets of changes take place, each being one cycle of length two. Whereas, two redundant cycles of length two are more similar to a cycle of length four. That is, one of the cycles implies mapping of the four numbers. This suggests that two non-redundant cycles of length two involve less perceptual work than two redundant cycles of length two.

Geometrically, redundant cycles correspond to single flips of the field about one of the diagonal axes. Non-redundant cycles correspond to one of the following operations; a product of two mirror images or, two 90° rotations or, two flips of the field once about each of the diagonal axes.

To summarize, the amount of perceptual work involved between any two of the eight next to the corner locations follows the order given below, starting from minimum perceptual work:

1. cycle of length two
2. two non-redundant cycles of length two
3. two redundant cycles of length two
4. cycle of length four

TABLE 2.

Classification of Cycles.

	Cycle of length two	Two cycles of length two: Non-redundant	Two cycles of length two: Redundant	Cycle of length four
1.	(a & b)	(a & e)	(a & d)	(a & c)
2.	(a & f)	(b & f)	(a & h)	(a & g)
3.	(b & e)	(c & g)	(b & c)	(b & d)
4.	(c & d)	(d & h)	(b & g)	(b & h)
5.	(c & h)		(c & f)	(c & e)
6.	(d & g)		(d & e)	(d & f)
7.	(e & f)		(e & h)	(e & g)
8.	(g & h)		(f & g)	(f & h)

IV. DEVELOPMENT OF THE HYPOTHESES.

In the context of the numbering system the implication was made that locations with identical normalized sets will be perceived as more similar to one another than other possible locations. Since it is also possible that some of the elements in two or more normalized sets are identical (for example, only two numbers from one normalized set are identical to two numbers of another set) the following hypothesis may be stated:

Hypothesis I: (a) The probability of perceiving two locations as being relatively more similar to one another increases directly as a function of the number of identical elements (numbers) in the normalized sets corresponding to those locations. (b) For locations with two identical elements in their normalized sets, the relative similarity of two locations will depend on the way in which those elements are positioned in their respective orientational sets.

For locations with identical normalized sets, the theory states that, the relative amount of perceptual work is proportional to how readily one location can be permuted into another. It follows that a decrease in the amount of perceptual work corresponds to an increase in perceived similarity between two locations.

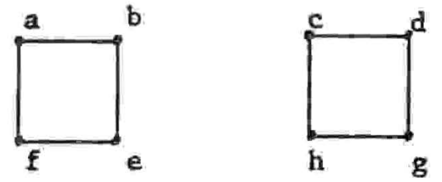
Hypothesis II: The probability of perceiving two locations with identical normalized sets as being relatively more similar to one another varies as an inverse function with the amount of perceptual work required to permute one location into another. (The following are the list of cycles starting from minimum perceptual work:)

1. cycle of length two
2. two non-redundant cycles of length two
3. two redundant cycles of length two
4. cycle of length four

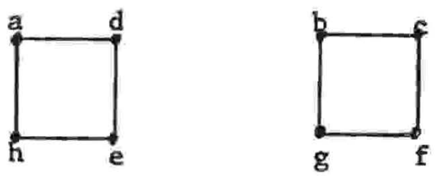
In the table of classifications of possible permutations between the eight next to the corner locations, it was noted that there are eight pairs of locations in each of the different length cycles, and four pairs in the non-redundant two cycles of length two. Since formation of sub-groups is possible for the next to the corner locations, it could be asked whether there are sub-groups among these locations which are dictated by the structure of the grid and reflected in the classification table of possible pairs on the basis of different cycle lengths.

To find these sub-groups, the connectivity among different pairs within each kind of cycle can be examined by a graphic representation. More specifically, one pair of locations can be selected randomly from one of the categories in the classification table. (See Table 2) The two locations are represented as two points that are connected with a line. All other pairs of that category are examined, and the pairs that include one of the original locations are added to the graph by more points and lines which connect them. The process is carried out until no other pairs can be included for that particular category. The following are graphs for each cycle length:

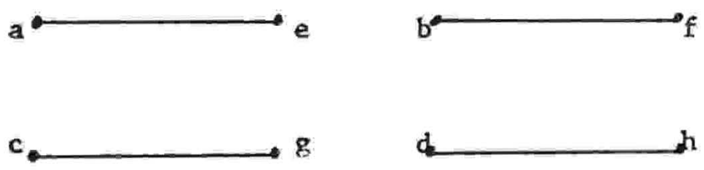
1. Sub-groups for cycles of length two:



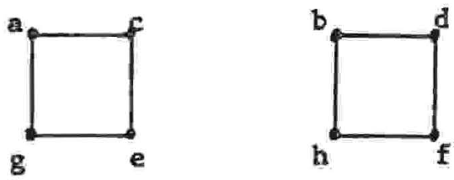
2. Two redundant cycles of length two:



3. Two non-redundant cycles of length two:



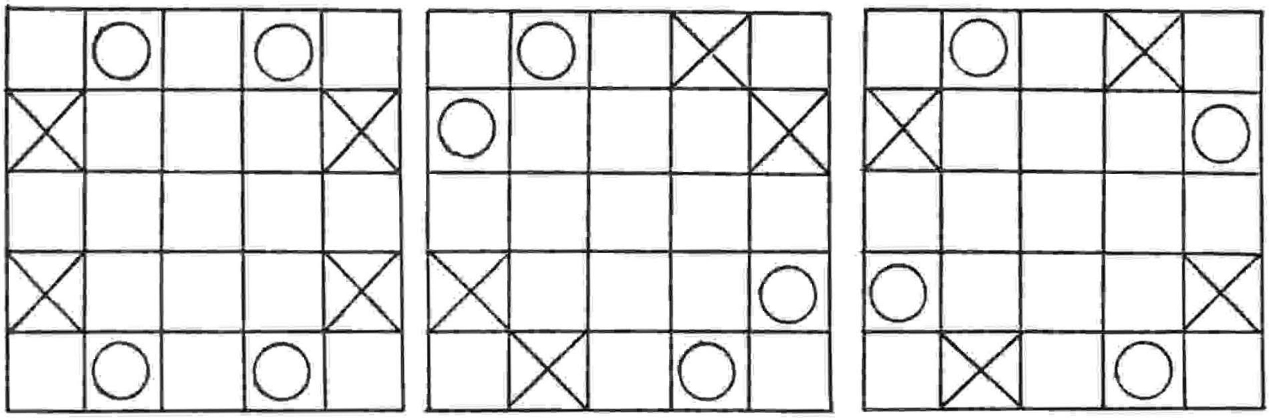
4. Sub-groups for a cycle of length four:



Note that except for two non-redundant cycles of length two the graphs suggest formation of two sub-groups within the eight next to the corner locations. If the formation of two sub-groups is inherent in the structure, it will be expected that the eight next to the corner locations will more frequently decompose into two sub-groups rather, than into any other theoretically possible number of sub-groups such as 3, 5, 6, or 7 sub-groups. Furthermore, it will be expected that the task of finding two sub-groups will be simpler phenomenally and less time consuming than that for any other possible number of sub-groups.

Hypothesis III: The probability of decomposition of the eight next to the corner locations into two equal sized sub-groups is higher than the probability of other possible decompositions. The task of finding two sub-groups will be perceived as relatively simple and less time consuming than finding more than two sub-groups.

On the basis of the graph analysis, there are three possible sub-groups of size four. (See Figure 10)



cycle of two   two cycles of two   cycle of four

Fig. 10

Since the graph for these sub-groups become complete with a particular order of four locations in the field, it will be expected that these sub-groups will be 'complete' and 'orderly' phenomenally relative to other theoretically possible sub-groups of size four. According to Gestalt theory, there is a tendency to minimize perceptual work. It follows that, the frequency of occurrence of these cycles will vary as a function of the amount of perceptual work i.e. the particular cycle length from which the sub-groups were derived.

Hypothesis IV: The probability of formation of sub-groups derived from graph analysis are in general higher than the probability of formation of other possible sub-groups. Specifically, the frequency of their formation varies as an inverse function of the amount of perceptual work involved in cycle lengths from which the sub-groups were derived.

The amount perceptual work involved in each sub-group follows the order given below, starting from minimum perceptual work:

1. sub-groups of size four based on a cycle of length two
2. sub-groups of size four based on two cycles of length two
3. sub-groups of size four based on a cycle of length four.

V. EXPERIMENT 1.

This experiment was carried out to test the first two hypotheses. The task consisted of fourteen forced choice comparisons for similarity among different locations.

	a		b	
h				c
	m	o		
g			n	d
	f		e	

Fig. 11

To test Part (a) of the first hypothesis two sets of comparisons were selected. One in which the target and the comparisons either had no common element in their normalized sets or had two identical elements in their normalized sets. Location b was selected as target and locations o and m as comparison items (b to o vs. b to m). In the other forced choice situation, the target and comparisons either had four identical elements in their normalized sets or had two identical elements in their normalized sets (f to m vs. f to d). Comparison between (b to n vs. b to m) was selected to test Part (b) of the first hypothesis. These comparisons had two identical elements in their normalized sets.

However, in case of (b to n) the common elements occupied similar positions in their orientational sets.

To test the second hypothesis the following examples of the locations with different permutational relationships were selected:

1. Two comparisons between a cycle of length two and two non-redundant cycles of length two (a to b vs. a to e) and (d to h vs. d to g).

2. Two comparisons between a cycle of length two and two redundant cycles of length two (c to b vs. c to d) and (a to f vs. a to d).

3. One comparison between a cycle of length two and a cycle of length four (c to a vs. c to h).

4. Two comparisons between two non-redundant cycles of length two and two redundant cycles of length two (c to b vs. c to g) and (a to e vs. a to d).

5. One comparison between two non-redundant cycles of length two and a cycle of length four (h to d vs. h to b).

6. Two comparisons of two redundant cycles of length two to a cycle of length four (a to h vs. a to c) and (f to h vs. f to c).

7. One comparison between a cycle of length four and a cycle of length four (b to h vs. b to d) as a control for possible right or left bias.

Method

Subjects

Twenty-four students from an undergraduate psychology class.

Procedure

The subjects were tested together in a classroom. Each subject was presented with a booklet. On each page three 5 x 5 grids of size (8.5 cm x 8.5 cm) were drawn. One of the three grids was located in the middle of the upper half of the page, and the other two grids were drawn symmetrically in the lower half of the page. One location was filled in on each grid. There were 14 pages in each booklet. Each booklet contained the same pages put together in a random order. The following instructions were read to the subjects:

> On each page of your booklet three grids of equal size are drawn. As you can see, the grids are identical except for the location of the square which is filled-in on each grid. What I want you to do is to compare the location of the filled-in square of the grid on the upper half of the page to the location of each of the filled-in squares in the grids on the lower half of the same page. Choose the lower grid which you see as having the filled-in square most similar to the filled-in square in the top grid. Draw a line between the grid on top to one of the grids below it to indicate your choice.
>
> Do you have any questions?
>
> After you have indicated your choice by drawing a line between the grid on top and one of the grids below it, write one sentence on the bottom of the page indicating the way in which you see the location of the filled-in squares as being similar to one another.

Results

The written statements of the 24 subjects from experiment one may be classified into three mutually exclusive categories according to the type of similarity noted between two locations by the subject. One type of statement was a description of the geometric similarities between two locations (e.g. both locations are next to the corner and in the same column). This type of statement is called descriptive. A second type of statement was a judgement of the relative similarity of two locations based on the application of rotational operations (e.g. two locations are the same if the top grid was rotated 90° clockwise, or two locations are the same if the top grid was rotated 180° in the third plane). This type of statement is called rotational. The third type of statement was based on the proximity of two locations (e.g. two locations are closer to each other). This type of statement is called proximal.

From the total of 346 forced choice responses about 50% had descriptive statements, 45% rotational, and 5% proximal. Table 3 shows the distribution of the forced choice responses on each comparison item. The predicted choices are indicated by an asterisk (*). The column after each comparison item gives the associated one-tail probabilities under the null hypothesis for the one-sample binomial test.

The first three comparisons in Table 3, labelled "Comparison among non-identical normalized sets," were significant ($P < .001$) in the predicted direction. That is, locations with more identical elements in their normalized

TABLE 3.

EXPERIMENT I. DISTRIBUTION OF RESPONSES ON EACH COMPARISON ITEM.

	Comparison Among NON-IDENTICAL Normalized Sets									Comparison Among Cycle of length two with other cycles														
	1			2			3			4			5			6			7			8		
T	b			b			f			a			d			c			a			c		
C	*m	o	p	m	n*	p	m	d*	p	b*	e	p	h	g*	p	b	d*	p	*f	d	p	a	h	p
D	20	0	.001	0	23	.001	1	10	.006	14	0	.001	0	13	.001	0	14	.001	15	0	.001	0	11	.001
R	2	0		0	1		0	13		4	6		7	4		2	7		9	0		9	4	
P	0	2		0	0		0	0		0	0		0	0		1	0		0	0		0	0	
To	22	2	.001	0	24	.001	1	23	.001	18	6	.01	7	17	.032	3	21	.001	24	0	.001	9	15	.154

T = target
C = choice
D = descriptive responses
R = rotational responses
P = proximal responses
To = total of all responses
* = predicted choice
p = one tail probability--binomial test

	a		b	
h				c
	m	o		
g			n	d
	f		e	

TABLE 3.

	COMPARISON OF TWO CYCLES OF LENGTH TWO WITH EACH OTHER AND WITH CYCLE OF LENGTH FOUR															Control		
T	9 c			10 a			11 h			12 a			13 f			14 b		
C	b	g*	p	e*	d	p	*d	b	p	h*	c	p	h*	c	p	h	d	p
D	0	10	.001	11	0	.001	10	0	.001	9	0	.002	0	8	.004	6	7	
R	2	9		10	3		5	6		3	10		12	0		9	10	
P	3	0		0	0		0	3		2	0		4	0		1	1	
TO	5	19	.003	21	3	.001	15	9	.154	14	10	.271	16	8	.076	16	18	.50

	a		b	
h				c
g				d
	f		e	

sets were consistently selected as more similar to one another. For the case in which the target and both comparisons had two identical elements in their normalized sets, the identical elements of the selected comparison were oriented in a similar way to that of the target in their respective orientational sets.

Items 4-8 labelled "Comparison of cycle of length two with other cycles," and items 9-13 labelled "Comparison of two cycles of length two with each other and with cycle of length four" were selected to test the second hypothesis.

For theoretical reasons that will be elaborated in the discussion section, the forced choice responses were analyzed according to the type of written statements made by the subjects, and as total (sum of descriptive, rotational, and proximal) responses.

1. Descriptive: All comparisons were significant ($p < .006$) in the predicted direction. That is, the permutational relationship of the selected pairs of locations followed the order given below; starting from maximum similarity, (a) cycle of length two, (b) two non-redundant cycles of length two (c) two redundant cycles of length two, (d) cycle of length four.

2. Rotational: Most comparisons were not consistent between subjects except for items number 7 and 13. The responses on item 7 (comparison between cycles of length two and two redundant cycles of length two) was in the predicted direction. The responses on item 13 (comparison between

two redundant cycles of length two and a cycle of length four) were not in the predicted direction. In general, rotational responses did not support the second hypothesis.

3. Total: Table 4 gives the predicted percent of total responses on possible comparisons among cycles of different length (items 4-13). Total responses on comparison between cycles of length two and two non-redundant cycles of length two (items 4 and 5) were significant ($p < .05$). Total responses on comparisons between cycles of length two and two redundant cycles of length two (items 6 and 7), and comparisons between two non-redundant cycles of length two and two redundant cycles of length two (items 9 and 10) were significant ($p < .01$). Items number 8, 11, 12, and 13 were not significant. Those four items had 62.5% or less of their total responses in the predicted direction (see Table 4). Responses to the control item were distributed randomly. That is, there was no left or right bias in the selections made by the subjects when asked to compare one cycle of length four to another cycle of length four.

To summarize, 50% of the total responses were descriptive and they supported the second hypothesis. That is, the probability of perceiving two locations as being relatively more similar to one another varies as an inverse function of the amount of perceptual work required to permute one location into another.

TABLE 4.

omparison among different cycles	Corresponding Item No. in Table 3	Predicted % of Total Responses
Cycle of length two and two redundant cycles of length two	6 and 7	93.6
Two non-redundant cycles of length two and two redundant cycles of length two	9 and 10	83.4
Cycle of length two and two non-redundant cycles of length two	4 and 5	73
Cycle of length two and a cycle of length four	8	62.5
Two non-redundant cycles of length two and cycle of length four	11 and 12	60.5
Two redundant cycles of length two and cycle of length four	13	33.3

Supplementary Experiment

This experiment was carried out on the five items that were not significant at $p < .01$ as total responses. Those items were: (d to h vs. d to g), (c to a vs. c to h), (h to d vs. h to b), (a to h vs. a to c), and (f to h vs. f to c). The attempt was made to increase the probability of descriptive responses by asking the subjects to describe the location of each filled in square prior to their forced choice responses.

Method

Twelve subjects were tested under the procedure of Experiment I with the following alterations:

1. Each subject was tested individually.
2. There were five pages in the booklet.
3. Subjects were asked to describe where the location of each filled-in square was prior to each of their forced choice responses.

Results

Table 5 shows the responses of the twelve subjects, and associated one-tail probabilities under the null hypothesis for the one sample binomial test. From the total of 60 responses 77% were descriptive and 23% rotational. As a result of the increase in descriptive responses, the total responses on 4 of the 5 items reached significance ($p < .02$) in the predicted direction. The non-significant comparison

SUPPLEMENTARY EXPERIMENT. DISTRIBUTION OF RESPONSES ON EACH COMPARISON ITEM.

	COMPARISON AMONG CYCLES OF DIFFERENT LENGTH														
	5			8			11			12			13		
Target	d			c			h			a			f		
Choice	h	g*	p	a	h*	p	*d	b	p	h*	c	p	h	c*	p
Descriptive	0	9	.002	0	9	.002	9	0	.002	10	0	.001	1	8	.02
Rotational	2	1		3	0		2	1		0	2		3	0	
Total	2	10	.01	3	9	.01	11	1	.003	10	2	.01	4	8	.19

	a		b	
h				c
g				d
	f		e	

was between two redundant cycles of length two and a cycle of length four (f to h vs. f to c). All descriptive responses were significant ($p < .02$) in the predicted direction.

Discussion

The written statements of the subjects indicated that the descriptive responses were made on a different basis than the rotational responses. In the descriptive responses subjects basically described the similarities, or noted a relationship, between two locations. For example, the most frequent statement on a cycle of length two was "they are both in the same row or column"; on two non-redundant cycles of length two the most frequent statement was "inverse or opposite of one another", and two redundant cycles of length two, were most frequently described as "diagonal." Since a cycle of length four was rarely selected, there were virtually no descriptions for a cycle of this type. All the noted similarities and relationships were directly present in the comparisons. That is, they did not involve any maniputations of the field, except perhaps for that of superimposing one grid on top of the other mentally to make their relationship more perceptible.

In the rotational responses subjects basically rotated one field to match the other and the relative ease or difficulty of this operation was often used to determine the relative similarity of two locations. However, the application of rotational operations in most cases led to inconsistent responses between subjects. For example, the responses to the comparisons of two non-redundant cycles of length two (180^{o} rotation) with a cycle of length four (90^{o} rotation) was a chance distribution. The subjects who selected a cycle of length four wrote that 90^{o} rotation is easier, whereas the subjects who selected two non-redundant cycles of length two made the point that in this case 180^{o} rotation is easier. That is, on the one hand there is the logic that 90^{o} rotation is easier than 180^{o} because it involves less rotation, and on the other hand locations with 180^{o} rotational relationship are perceptually more similar to one another than those of 90^{o} rotation. Similarly, in the comparison of a cycle of length two (flip of the field about its vertical or horizontal axis) with a cycle of length four (90^{o} rotation), the operation of 90^{o} rotation is easier than a flip. However, the two locations involved in a cycle of length two (mirror-images) are perceptually more similar to one another. It is this kind of conflict which led to inconsistent responses among subjects. Which of these responses, the descriptive or the rotational, is to be considered as the 'true' response? The descriptive response has the advantage of being naive, it is no more than the subject's statement of what he sees. The rotational response, on the other hand, has a number of associated problems:

(1) since the measure of similarity between two locations for most cases is calculated from the relative ease of rotational operations, it is conceivable that a rotational

operation could be made to look easier to the subject by varying the physical constraints of the situation. For example, the operations of flip may be facilitated by drawing the grids on both sides of transparent material and hanging them from a string attached to the middle of the upper edge of the grid. Alternatively, the same effect might be achieved by emphasizing the axes of symmetry.

(2) Perception of similarity among locations takes place independently of education in geometrical operations. That is, everyone notes the similarity and symmetry between two mirror-image locations but not everyone knows the rotational operations required in order to match one with the other.

(3) If it were true that locations corresponding to a cycle of length four (90^o rotation) were more similar to one another than locations corresponding to a cycle of length two (flip about the horizontal or vertical axis), then it should be that in the second experiment some groupings would be made on the basis of cycle of length four. There were no instances of such groupings.

There were two instances in which there were consistent rotational responses between subjects. First, the subjects agreed that the operation of a flip about one of the horizontal or vertical axes was easier than a flip about one of the diagonal axes (cycle of length two vs. two redundant cycles of length two). Second, the operation of 90^o rotation was consistently reported as being easier than a flip about one of the diagonal axes (cycle of length four vs. two redundant cycles of length two).

In the supplementary experiment the most typical description of each location was given in terms of the intersection of a row and column. This supports the numbering system which described the relationship of each location to the field on a similar basis. One possible reason for the increase in descriptive responses in this supplementary experiment is that, because subjects were asked to describe each location, the perceived similarities were influenced accordingly. In other words, under these conditions it was easier to note "in the same column" type of relationships as opposed to rotational relationships between two locations.

VI. EXPERIMENT 2.

This experiment was carried out to test the third and the fourth hypothesis.

Method

Subjects

Twelve students.

Procedure

Each subject was tested individually. The subject was presented with eight square cards of size (10.5 cm x 10.5 cm). On each card there was a (5 x 5) grid of size (8.5 cm x 8.5 cm) drawn with one of the eight next to the corner locations filled in, and N, E, S, W indicating orientation. There are two parts to this experiment. In the first part the following instructions were read to the subjects:

> Here are eight cards. As you can see, the same grid has been drawn on each card. The only difference between these cards is the location of the square that is filled in. Note that each filled-in location is one of the eight next to the corner locations.
>
> (cards will be collected and turned over)
>
> What I want you to do is to categorize or group these cards by putting together the cards that seem to go together. You may make as many or as few categories as you like. Do you have any questions?
>
> While you will be categorizing the cards I will be timing you with a stopwatch. However, this does not mean that you should try to do it as fast as you can. I would like to know how long it takes when you are relaxed. Whenever you feel ready

> turn the cards over and start. Let me know when you are finished.

Part II

The same subject was presented with the same eight cards. He was instructed to categorize the cards on the basis of putting together the cards that seemed to go together into either three categories or five. The following instructions were read to the subject:

> On this part of the experiment I want you to categorize the same cards by putting together the cards that seem to go together into three (or five) categories. Like the first part whenever you feel ready turn the cards over and I will start the stop watch. Let me know when you are finished.

After the completion of Part II each subject was asked which task was easier to do.

Results

Table 6 summarizes the responses of twelve subjects. Under free-groupings, eleven of the twelve subjects made four sub-groups and one subject two sub-groups. When the eleven subjects, who made four sub-groups, were asked to reduce the number of their sub-groups, they combined the four sub-groups into two sub-groups. All subjects reported that the tasks of free groupings and reducing the number of sub-groups were easier to do than the tasks of making either three or five sub-groups. This difference is also reflected in the time it took to complete each task. The task of making either three or five sub-groups required much more

TABLE 6.

	a		b	
h				c
g				d
	f		e	

ts	Free-Groupings	Time In Seconds	Reduced-Groupings	3 or 5 Groupings	Time In Seconds
	(ab)(cd)(ef)(gh)	25	(abef)(cdgh)	(cdgh)(ab)(fe)	53
	(ab)(cd)(ef)(gh)	40	(abéf)(cdgh)	(ahgf)(bc)(de)	170
	(ab)(cd)(ef)(gh)	42	(abef)(cdgh)	(af)(bc)(cdgh)	70
	(ab)(cd)(ef)(gh)	32	(abef)(cdgh)	No sensible Answer on 5	150
	(ab)(cd)(ef)(gh)	22	(abef)(cdgh)	(ab)(c)(h)(gh)(de)	152
	(ab)(cd)(ef)(gh)	40	(abef)(cdgh)	(cd)(af)(gh)(b)(e)	225
	(ab)(cd)(ef)(gh)	35	(abef)(cdgh)	(gh)(af)(be)(c)(d)	246
	(ab)(cd)(ef)(gh)	30	(abef)(cdgh)	(gh)(cd)(ab)(f)(e)	167
	(ah)(gf)(bc)(de)	32	(ahde)(bcgf)	(bcd)(ahf)(ge)	58
	(ah)(gh)(bc)(de)	36	(ahde)(bcgf)	Can't think of a rationale	234
	(ah)(gf)(bc)(de)	28	(ahgf)(bcde)	(ahgf)(bc)(de)	63
	(bcde)(ahgf)	15		No valid reason	420

Mean = 31.4
Variance = 64.2

Mean = 167.3
Variance = 11247.8

$t = -4.42$ critical value of $t \leq -3.792$

time to complete ($t=-4.42$, $p < .0005$). These results support the third hypothesis on the claim that formation of two sub-groups will be simpler and less time consuming than formation of three or five sub-groups. However, it does not support the prediction that, originally, subjects will make two sub-groups.

Eight of the eleven subjects who originally made four sub-groups chose pairs of locations that were permutable to one another by a cycle of length two. These same subjects reduced the four sub-groups into two sub-groups; (abef)(cdgh) which was predicted as the most frequent sub-group of size four. The other three subjects who originally made four sub-groups selected pairs of locations that were permutable to one another by two cycles of length two. Two of these subjects reduced their four sub-groups into two sub-groups; (ahde)(bcgh) which was predicted as the second most frequent sub-groups of size four. The other subject reduced the four sub-groups into two sub-groups; (abch)(defg) which was the same kind of sub-group selected by the subject who made two sub-groups under free groupings.

The statistical significance of the results on the formation of sub-groups of size four may be demonstrated in two ways. First, the data were classified into two categories: those groupings predicted by theory and those that were not predicted. A one sample binomial test for 10 out of 12 responses in the predicted category was significant as $p < .02$.

That is, ten of the sub-groups (eight (abef)(cdgh) and two (ahde)(bcgh)) were predicted on the basis of the graphic analysis. A second alternative was to consider all possible sub-groups of size four. Appendix I presents a method of describing sub-groups of different size, and rank ordering them on the amount of perceptual work required to form each sub-group. Seven unique descriptions summarize the array of 70 possible sub-groups of size four. Application of the KOLMOGOROV-SMIRNOV test on the ranked sub-groups of size four resulted in a maximum difference in distribution of .66 which was significant at $p < .01$.

Discussion

The third hypothesis had predicted that subjects would construct two sub-groups whereas, in fact, most subjects made four sub-groups. On reflection, it becomes apparent that the graph analysis does not yield pure groupings. Each of the two sub-groups of size four contain two types of cyclic relationship among the pairs. For example, four locations which form a sub-group on the basis of a cycle of length two contain two pairs of locations which are related to one another by two non-redundant cycles of length two. Since they were given eight locations which were highly similar to one another, most subjects initially minimized perceptual work by making four sub-groups which contain only one cyclic relationship i.e. cycle of length two. However, formation of two sub-groups is the only other number under which perceptual work may be minimized. This was indicated by the fact that all subjects reduced four sub-groups into two, their verbal reports indicated task of making 3 or 5 sub-groups were more difficult than two, and time measurements reflected the difference. Time measurements were not possible when the subjects were asked to reduce the number of their sub-groups due to the immediacy of the response. However, the difference in time is so great that even adding 10 seconds to the time of making four sub-groups would still result in a significant difference.

There were two subjects who made two sub-groups of (ahgf)(bcde) which were not predicted on the basis of the graph analysis. However, these groups are also understandable under the hypothesis of minimizing perceptual work. The computations in Appendix I places this grouping among those requiring least perceptual work.

The most difficult aspect of making three or five groups were breaking up the dominant groupings. Most subjects express dis-satisfaction with their groupings by saying, it is not "right", or "lacks rationale", or not "sensible."

The results of this experiment clearly supports the hypothesis that the most frequent groupings involve minimum amount of perceptual work.

VII. GENERAL DISCUSSION.

The purpose of this thesis was to gather experimental evidence for the theoretical Gestalt notion of a tendency to minimize perceptual work in placing items of different degrees of difference together. Data supported the general hypothesis in two ways: first, the descriptive responses of Experiment I clearly indicated that the similarity of two locations is an inverse function of the amount of perceptual work required in order to permute one location into another. Second, the most frequent groupings in Experiment II involved those in which the amount of perceptual work was minimized. The only apparently non-supporting data were the rotational responses. On reflection, these responses do not contradict the hypothesis since the subjects were minimizing perceptual work by a different method. That is, the locations which involved least rotation were more frequently selected as being most similar to one another.

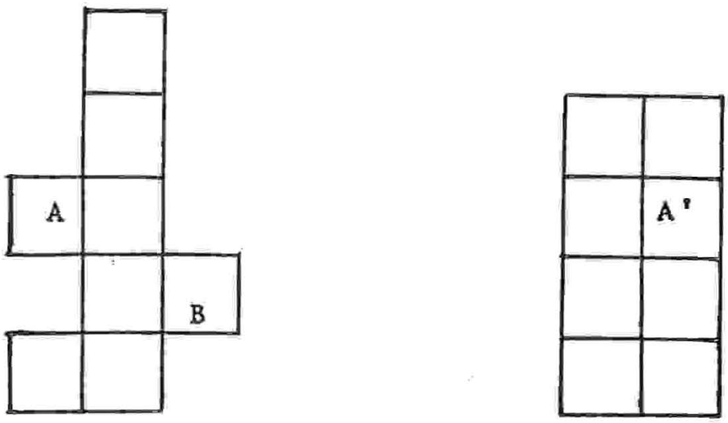
This support of quantification leads to additional questions for future research:

1. The most immediate follow up would be to experimentally test the amount of perceptual work under the computational method of Appendix I for sub-groups of different size. One possible way of doing this would be to present different sub-groups (indicated on a single grid) on a display. The subject could control the amount of time that each presentation stays on the display and his task would be to

memorize the pattern of the sub-group as fast as he can. Later the subject would be asked to reproduce the sub-groups. The computed amount of perceptual work and the amount of time required to memorize each sub-group should be correlated. Also, the number of errors in reproduction should be inversely related to the amount of perceptual work in each sub-group.

2. The method is extendable to other structures in two ways: (a) to symmetrical structures that are composed of hexagons. Such structures would have six numbers in their orientational sets. Therefore, they would yield more cyclic relationships and so allow for more discrimination among different cycles. The following is the list of theoretically possible cycle lengths for structures composed of hexagons: cycle of length two, two cycles of length two, three cycles of length two, cycle of length three, cycle of length two and a cycle of length three, two cycles of length three, cycle of length four, cycle of length two and a cycle of length four, cycle of length five, and a cycle of length six, (b) to asymmetrical structures that are composed of either squares or hexagons. The cells of these structures would be described by the numbering system relative to those portions of the structure in which a cell is most readily a part, since it would be difficult to rotate asymmetrical structures, the probability of rotational responses in Experiment I could be decreased if not entirely eliminated by using asymmetrical structures.

3. The permutational relationship between two locations in one structure can be used to infer the particular sub-part of another structure in which similar relationship between two locations could be found. For example, the subject would be presented with the following two structures:



(a) (b)

Fig. 12

The subject's task would be to find location B' in structure (b) such that the relationship between B' and A' would be similar to that of the relationship between B and A in structure (a). i.e. A is to B as A' is to what?

$$A = \{0, 1, 0, 0\}$$

$$B = \{0, 0, 0, 1\}$$

$$A \ \& \ B = \begin{pmatrix} 0100 \\ 0001 \end{pmatrix} = (10)$$

In order to find B' the subject must consider a sub-part in structure (b) such that the relationship between A' and B' = (10). Figure 13 shows the only sub-part in structure (b) which meets the criteria.

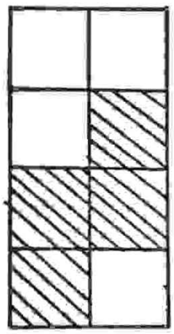


Fig. 13

Relative to the structure $A' = \{1, 0, 2, 1\}$, and $B' = \{3, 1, 0, 0\}$. However, relative to the sub-part $A' = \{0, 0, 1, 0\}$ and $B' = \{1, 0, 0, 0\}$. Thus yielding a (10) cyclic relationship between A' and B'.

The present method is restricted to locations with identical normalized sets and, as a further elaboration, it would be of great interest to develop the mathematical relationship between any two locations within the structure, and not necessarily those having the same number. This more general development would allow the study of the theoretical construct of minimizing perceptual work when a structure is decomposed into sub-parts. The principle under which each sub-part is organized could be specified and the relationship between different sub-parts as well as the relationship of each sub-part to the structure as a whole placed in a common mathematical language.

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APPENDIX I.

Description of Sub-Groups

The relationship of any two locations with identical normalized sets can be described by the number and the length of cycles that permute one location into another. For only group of size two with identical normalized sets the following has been theorized.

$$X_4 > X_3 > X_2 > X_1$$

Where $>$ means greater than in amount of perceptual work.

And X_4 = cycle of length four.

X_3 = two redundant cycles of length two.

X_2 = two non-redundant cycles of length two.

X_1 = cycle of length two.

Any group of location with identical normalized sets may be described as the sum of all possible pairs of permutational relationships. That is an equation with the following general form:

$$G = aX_1 + bX_2 + cX_3 + dX_4$$

Where a, b, c and d are constants that correspond to the number of a particular permutational relationship.

Example: Description of group acdh will be as follows:

$$ac = X_4$$
$$ad = X_3$$
$$ah = X_3$$
$$cd = X_1$$
$$dh = X_2$$
$$acdh = 2X_1 + X_2 + 2X_3 + X_4$$

The number of possible sub-groups of size four for the eight next to the corner locations can be computed by

$\frac{n!}{r!\ (n-r)!}$ where n = number of locations, and r = size of the group.

$$N = \frac{8!}{4!(8-4)!} = 70$$

Seven unique equations which describe the seventy possible groups are given in the following table.

The extent to which each group is symmetrical or redundant is reflected in two ways in the equation. (a) by fewer unique cycles and (b) by a common co-efficient.

To discriminate among these equations on a rank order continuum from minimum to maximum perceptual work. Let $X_1 = 1$, $X_2 = 2$, $X_3 = 3$ and $X_4 = 4$. Furthermore, let the common coefficient = 1 to eliminate the redundancy. Under

$=2(X_1+X_3+X_4)$	$G_3=2X_1+X_2+2X_3+X_4$	$G_1=X_1+X_2+2X_3+2X_4$	$G2=2X_1+X_2+X_3+2X_4$
abcd	abcf	abcg	abce
abch	abde	abdh	abdf
abdg	abeh	acde	abeg
abgh	abfg	aceh	abfh
acdf	acdh	acfg	acdg
acfh	adef	adeg	acef
adfg	adgh	adfh	acgh
afgh	aefh	aegh	aefg
bcde	bcdg	bcdf	bdef
bcdh	bcef	bceg	bdgh
bceh	bcgh	bcfh	befh
bdeg	befg	bdeh	cdeg
begh	cdeh	bdfg	cdfh
cdef	cdfg	bfgh	cegh
cefh	cfgh	cefg	dfgh
defg	degh	defh	
efgh			

$=2(X_2+2X_4)$	$G_6=2(X_2+2X_3)$	$G_7=2(2X_1+X_2)$
aceg	adeh	abef
bdfh	bcfg	cdgh

	a		b	
h				c
g				d
	f		e	

these conditions:

$$G_1 = X_1 + X_2 + 2X_3 + 2X_4 = 17$$
$$G_2 = 2X_1 + X_2 + X_3 + 2X_4 = 15$$
$$G_3 = 2X_1 + X_2 + 2X_3 + X_4 = 14$$
$$G_4 = 2(X_2 + 2X_4) = 10$$
$$G_5 = 2(X_1 + X_3 + X_4) = 8$$
$$G_6 = 2(X_1 + 2X_3) = 8$$
$$G_7 = 2(2X_1 + X_2) = 4$$

$$\therefore \quad G_1 > G_2 > G_3 > G_4 > G_5 = G_6 > G_7$$

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