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PLASTIC SHRINKAGE CRACKING PREDICTION IN CEMENT-BASED MATERIALS USING FACTORIAL DESIGN

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Abstract

Shrinkage cracking is a major issue that affects the durability of concrete structures. Plastic shrinkage of cementitious materials can lead to cracking within 24hrs of casting and sets the stage for premature deterioration. Even though test techniques exist that can be used to evaluate the plastic shrinkage cracking potential of cement-based materials, mathematical models that predict the influence of various parameters such as w/c, aggregate to cement ratio and effect of fibers on cracking are not available. This paper presents a model that can be used to predict plastic shrinkage cracking in cement-based materials. The model is developed using factorial design and utilizes representative data generated using a technique developed by the authors. The effectiveness and limitations of the model in predicting crack areas and width are discussed. Effect of variables such as w/c, sand/cement ratio, fiber dosage and the interaction between these variables is also presented in this paper.

Keywords: Restrained plastic shrinkage cracking, factorial design, cement-based materials, fiber reinforcement

Introduction

Concrete experiences various types of volumetric changes including shrinkage at early ages and also over long-term. In addition to standardized test methods to quantify drying shrinkage of concrete, methods are now also available to measure plastic shrinkage. However, as far as prediction of shrinkage is concerned, numerous proposed models only focus on predicting the drying shrinkage of cementitious materials. Goel et al. (2007) summarize and compare various existing models: ACI-209R-82 model, the B3 model, the CEB-FIP model code 1990, and the GL2000 (Gardner and Lockman 2001), and Muller model (Muller et al. 1999). Expressions for some of these important models to predict creep and shrinkage as described by Goel et al. (2007) are presented below.

ACI 209 Code Provisions

ACI-209R-82 (2005) recommends the following expression for shrinkage:

$$\varepsilon_{sh}(t,t_c) = \frac{(t-t_c)}{T_c + (t-t_c)}\varepsilon_{shu} \qquad \text{Equation 1}$$

where, t_c= 7 days for moist cured concrete, and 1-3 days for steam cured concrete, ε_{shu}= ultimate shrinkage strain, 780 for standard conditions, t is age of concrete in days, Tc = 35 days for moist cured concrete and 55 days for steam cured concrete,

CEB-FIP Model Code 1990

According to this model, total shrinkage/swelling is calculated based on the following equation:

$$\varepsilon_{sh}(t,t_c) = [160 + 10\beta_{sc}(9 - 0.1 f_{cm})] \times 10^{-6}\, \beta_{RH} \sqrt{\frac{\{t - t_c\}}{\{350(\frac{2A_c}{100\mu})^2 + (t - t_c)\}}} \quad \text{Equation 2}$$

β_{sc} is constant, depends on type of cement, f_{cm} is the concrete mean compressive strength at 28 days in MPa, β_{RH} is a constant that depends on relative humidity, μ perimeter of the member in contact with atmosphere (mm), A_c cross-sectional area (mm^2), and other variables as defined before.

B3 Model

Mean shrinkage strain in the cross section is given by:

$$\varepsilon_{sh}(t,t_c) = -\varepsilon_{shu} k_h S(t) \qquad \text{Equation 3}$$

where, k_h is the humidity dependence, $S(t)$ is the time curve, and other variables as defined before.

The ultimate shrinkage strain is given by:

$$\varepsilon_{shu} = \alpha_1 \alpha_2 (0.091 w^{2.1} (f_{cm})^{-0.28} + 270) \frac{E_c(7 + 600)}{E_c(t_c + \tau_{sh})} \qquad \text{Equation 4}$$

where, w is the water content in kg/m^3, α_1 and α_2 are constants related to the cement type and curing condition, E_c is the modulus of elasticity of concrete at the age of 28 days (MPa), τ_{sh} is the shrinkage half-time (days), and other variables as defined before.

The time function is given by:

$$S(t) = \tanh \sqrt{\frac{t - t_c}{\tau_{sh}}} \qquad \text{Equation 5}$$

where, t and t_c are the age of concrete and the age drying commenced, end of moist curing in days, respectively, τ_{sh} is the shrinkage half-time as given in Equation 6, h is the relative humidity of the environment at ambient temperature (in decimal), and other variables as defined before.

$$\tau_{sh} = 0.085 t_c^{-0.08} (f_{cm})^{-0.25} (k_s 2\{V/S\})^2 \qquad \text{Equation 6}$$

where, k_s is the cross-section shape correction factor, and V/S is the volume-surface ratio in mm.

GL2000 Model

This model proposes the following equation:

$$\varepsilon_{sh}(t,t_c) = \varepsilon_{shu}(1-1.18h^4)\sqrt{[\frac{t-t_c}{t-t_c+0.15(V/S)^2}]} \qquad \text{Equation 7}$$

where, variables as described before.

$$\varepsilon_{shu} = 1000K\sqrt{\frac{30}{f_{cm}}} \times 10^{-6} \qquad \text{Equation 8}$$

where, K is a shrinkage constant that depends on the cement type and other variables as described before.

Above-mentioned models were used for predicting creep and shrinkage of various grades of concrete by Goel et al. (2007). They found that the predictions from the GL2000 model were the closest to the experimental results. Shrinkage estimated using the fib 2000 model has been found to be about 75% of the measured shrinkage (Charron et al. 2003). ACI 209 (2005) reports predictions from the four models described above were compared to that of the results in the RILEM data bank. GL2000 and B3 models provided the best fit for shrinkage strain and the CEB model underestimated the strain. Coefficient of variation was calculated to compare model predictions with test data. The variation was 35% and 36% for GL2000 and B3 respectively. Variation for CEB was 37% and finally ACI model resulted in a variation of 45%. It was also noted that the use of more input data (test results) in the models improved the predictions of all models accept for the ACI model.

Many other microstructural models have been proposed to predict behavior and shrinkage of cement-based materials. These models are based on the cement hydration process and are described by Charron J. –P., Marchand J., Bissonnette B., and Pigeon M. in the report edited by Bentur A (Charron et al. 2003). The model “CEMHYD3D” was developed at the National Institute of Standards and Technology in the US. This model has been modified over the past few years and one version of this model is described by Bentz D.P. (Bentz 2006a,b,c). “HYMOSTRUC” (Hydration Morphology and Structural Development) model was developed at Delft University of Technology in the Netherlands. The details and application of this model to study formation of microstructure of cement and concrete is reported by Ye G. et al. (2003) and Princigallo A. et al. (2003). Some other models such as “DuCON” (Ishida et al. 1998) developed at the Tokyo University and “ENPC” (Hua et al. 1995; Hua et al. 1997) that was developed in France, have been used to predict shrinkage of cement based materials. These models are based on the assumption that capillary effects such as the formation of capillary depression due to tension in the liquid phase are responsible for autogenous shrinkage. Hua et al. (1995) reports good agreement between predicted values with measured values for a w/c ratio of 0.42. At the age of two days, 60 μm was measured when compared to a strain of 57.6μm from the model.

There exist some other empirical models that were developed to predict behavior of concrete at early ages including the effect of basic creep and autogenous shrinkage of concrete. Description of models such as “CESAR’s model,” “LeRoy’s model,” “Granger’s model,” “Bazant-Baweja’s model,” and “DeShutter and Taerwe’s model” and a comparison of their predicted results is reported by Charron, J.-P, et al. (2001a,b). Charron J. –P., Marchand J., Bissonnette B., Pigeon M, and Gerard B have commented on the effectiveness of the above-mentioned empirical models in chapter 5.3 of the report edited by Bentur A (Charron et al. 2003). According to this report, for the mixes investigated during their study, except for the CESAR’s model, all other models did not result in very reliable predictions.

Most models described above have been developed to predict creep and shrinkage in concrete in terms of the strain development. Limited work has been done to predict early-age behavior (plastic shrinkage) of concrete, especially in terms of evaluating the shrinkage cracking of fiber-reinforced cement-based composites containing polypropylene fibers. In this paper, the authors present a factorial design based model to predict early-age plastic shrinkage cracking. This model is based on a test technique developed by the authors.

Factorial Design

Several models exist that predict early age characteristics of cement composites subjected to shrinkage (Mabrouk et al. 2004; van Zijl et al. 2001; Sanjuan and Moragues 1994; Toledo Filho an Sanjuan 1999; Radocea 1994). Mabrouk et al. (2004) have proposed a model to predict early age shrinkage and creep of concrete composite using a solidification model that uses microphysical information such as temperature, hydration ratio, porosity, saturation, etc. According to the authors the behavior of young age concrete composite can be predicted. Similarly, another model of plastic shrinkage is based on linking the change in capillary pressure in a saturated mixture, exposed to drying, to geometry of the spaces between the solid particles. However, both models (Mabrouk et al. 2004; Radocea 1994) only predict shrinkage strains or deformation and are not focussed at predicting cracking. Another numerical model focusses on predicting crack initiation and growth in cementitious materials (van Zijl et al. 2001). However, the focus of this model is to predict crack widths in structures made using cement-based materials as opposed to using the model to study the influence of mix design or addition of fibers on shrinkage cracking. In a study by Sanjuan and Moragues (1994), the authors study the influence of mix proportion and synthetic fibers on early-age plastic shrinkage. However, this technique too only involves measurement of shrinkage strain and not shrinkage cracking. Another study involved investigating influence of mix design including various fiber types on both free and restrained plastic shrinkage, however the factorial design based model only focussed on predicting free shrinkage strains (Toledo Filho and Sanjuan 1999).

To fill the knowledge gaps identified in the models above, a study was initiated by the authors, which is described here. Scientific experiments in the past are usually modeled by keeping all parameters constant and varying one variable at a time. This traditional method is very time consuming and does not incorporate the correlation between variables. In this paper, a statistical prediction model based on factorial design is used and is first introduced here. In this study, the limit for plastic shrinkage is assumed to be 24 hours. This is a structured method that provides an efficient way of studying properties of a material that depends on several factors (Sanjuan and Moragues 1994; Toledo Filho and Sanjuan 1999) as in the case of cement composites.

Properties such as w/c, s/c (sand-cement ratio), fiber content, etc. are considered as variables and their effect on restrained shrinkage cracking is investigated. Factorial design enables establishing correlation between different variables. In this investigation, multiple linear regression analysis was used to develop a mathematical model that predicts restrained shrinkage cracking of a representative mix. This study was aimed at understanding the effect of different factors/parameters on cracking of cement composites under restrained shrinking conditions. A factorial design analysis was conducted to determine the relative importance of different parameters and their interaction on the cracking area and the average crack width.

Definition of Factorial Design

Factorial design is a form of mathematical analysis which enables one to study the influence of several parameters with only a few tests. Contrary to "one factor at a time" approach, in which factors are varied one at a time, in the factorial design approach, much fewer tests need to be carried out to determine the effect of parameters studied, and to determine the interaction between them. The interaction between parameters cannot be evaluated using "one factor at a time" approach because we assume that the factors act additively, which is not always the case. In this study, three factors mentioned earlier are investigated with an upper "+" and lower level "-" for each factor. This method is described in detail by Box et al. (2005) and can be further explained using the following example.

Example: In order to describe this method, let's consider the effect of three factors: A, B and C on given variable "y". Let's also assume that each factor has two levels : - and +. The levels signify a selected lower and an upper bound value. If using factorial design, we need $2 \times 2 \times 2 = 2^3 = 8$ tests. On the contrary, based on "one factor at a time" approach, 8 tests for each factor would be required, resulting in a total of 24 tests. According to factorial design, the 8 tests are the different arrangements of the 3 factors, which are shown in Table 1.

Calculation of "Effect" and "Interaction"

Calculation of Main Effect: "Effect of a factor" in factorial design refers to the change in "y" as that factor changes from "-" to "+." The "-" and "+" refer to the two levels of each parameter "A", "B", and "C".

Calculation of the Main Effect of Variable B**:** The main effect of variable B is also known as "B main effect." In this example there are two tests for each arrangement of A and C: one with the lower level of B and one with the upper. Since the two tests differ only in the B factor, the B effect will be the difference between the + and the − level. Then, the B total effect is the average of the four pairs of tests.

Hence, B's main effect can be calculated as:

$$B_{Effect} = \frac{y3 + y4 + y7 + y8}{4} - \frac{y1 + y2 + y5 + y6}{4} \qquad \text{Equation 9}$$

As seen from Equation 9, the main effect of B is basically the difference between two averages: the average response for the + level and the average response for the − level.

$$B_{Effect} = \overline{y}_{+} - \overline{y}_{-} \qquad \text{Equation 10}$$

Calculation of B × C Interaction

This interaction is calculated by the difference between the average B effect at C's upper level and the average B effect at C's lower level (and vice versa). Half of the difference is termed as B x C interaction.

From Table 2,

$$B \times C = \frac{y1 + y2 + y7 + y8}{4} - \frac{y3 + y4 + y5 + y6}{4} \qquad \text{Equation 11}$$

Calculation of A × B × C Interaction

Two values of the B × C interaction can be calculated for the experiment, one for each level of A. Half the difference between these two values is defined as the three factor interaction (A × B × C interaction) and given by Equation 12.

$$A \times B \times C = \frac{y2 + y3 + y5 + y8}{4} - \frac{y1 + y4 + y6 + y7}{4} \qquad \text{Equation 12}$$

Geometric Representation

The 2^3 factorial design can be represented as a cube in which each corner is a test. The three factors A, B and C form the axes of the space (Figure 1). As an example, these three factors (A, B and C) mentioned in Equations 9-12 and Figures 1 and 2 could correspond to physical test variables in the restrained plastic shrinkage test such as w/c, s/c, and fiber dosage Vf. The corners of the cubes would then correspond to the result (crack area or crack width) of a unique combination of A, B and C. Also, as an example the effect of B alone can be studied by comparing the change from one face of the cube to another as shown in Figure 2(a).

Previously described effects and interactions are graphically shown in Figure 2. Main effects may be viewed as a contrast between observations on parallel faces of the cube; the interaction is a contrast between results on two diagonal planes and the three factor interaction is a contrast between the two tetrahedrals.

Calculation of Standard Error

Standard error is calculated as described by Box et al. (2005). First of all, the variance is calculated for all tests using the following equation:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\sigma_i^{\,2} \qquad \text{Equation 13}$$

Where, σ_i^2 is the variance for one test and is defined by Equation 14

$$\sigma_i^{\,2} = \frac{1}{m-1}\sum_{i}^{m}(y_i - \bar{y})^2 \qquad \text{Equation 14}$$

where, m is the number of runs for one test

Standard error is then calculated using Equation 15

$$e = \frac{2}{\sqrt{n}}\sigma \qquad \text{Equation 15}$$

where n is the total number of runs

According to the technique, three runs (m=3) are made for each test, hence in this case for the 8 tests, n = 24.

APPLICATION OF FACTORIAL DESIGN TO REPRESENTATIVE TEST RESULTS

The factorial design technique described above was applied to some typical test results (Table 3) generated using a technique developed by the authors. The technique developed by the authors involves placing the material being tested as an overlay above a substrate with standard roughness. Figure 3 (a) shows a typical overlay cast over a substrate. The inset shows the protrusions on a typical substrate. The substrate provides realistic restraint to the overlay and this assembly is placed in an environmental chamber. In the environmental chamber a temperature of 50°C is chosen, which results in a relative humidity of less than 5% and this produces an evaporation rate of approximately 1.0 kg/m^2/h from the specimen surface. After 24hrs, cracking in the overlay (Figure 3(b)) is measured and the average crack width and total crack area is determined using a minimum of three specimens. Further details about this test technique has been previously published by the authors (Banthia and Gupta 2006; Banthia and Gupta 2007; Banthia and Gupta 2009) and is beyond the scope of this paper. In Table 3, eight different mixes are included along with the test data (crack area, average and maximum crack width). The mixes in Table 3 are mixes surrounding a control mix, which is not included in Table 3 to maintain clarity. This control mix had a w/c ratio and s/c ratio both equal to 0.5 with no fibers. This mix had a crack area of 305mm^2, average crack width of 2.17, and maximum crack width of 2.77mm. Along with the test data the standard deviation in percentage calculated using equation 14 is also included in Table 3. It is evident from the test results that w/c, s/c, and addition of fibers have an important effect on cracking. Factorial design was used to quantify this effect and determine the interaction between the factors affecting shrinkage cracking. Commercially available micro polypropylene fiber (PP) with a fiber length of 20 mm was selected for this study and commonly used range of 0 to 0.066% by volume of fibers was considered. A volume of 0.066% of polypropylene would translate into a dosage of 0.6kg/m^3. The parameters, their levels and the resulting mixes/tests forming the corners of a 2^3 factorial cube are given in Table 4.

Main and interaction effects were calculated for the parameters and are presented graphically in Figure 4. The figure shows the effect and interaction of parameters on crack area and crack width. In this case, the "effect" of a parameter is defined as the difference in crack area or width for lower and upper values of the given parameter.

In Figure 4, the first two bars represent average values of crack width and crack area of the data represented in the 2^3 factorial cube. The last two bars represent the standard error in the test data for crack width and crack area, which indicate the reliability of the presented test results. The other bars in the figure indicate the effect of parameters on crack area and crack width. The influence of the parameters on both crack area and width is similar except in the case of s/c. This means that crack area is generally proportional to the crack width. Keeping the average and standard error in mind, w/c followed by fiber dosage (%PP) are the most important factors that affect cracking; the range of s/c (sand-cement ratio) studied did not affect the results considerably. A low fiber dosage of 0.066% reduces crack area by 129 mm^2. As far as the interactions are concerned, s/c − %PP and w/c − %PP were the only two significant interactions considering the standard error associated with the data; fibers interacted with both w/c and s/c to reduce crack area and width.

MATHEMATICAL MODEL

Factorial design method is further used to predict plastic shrinkage cracking in cementitious composites using a multi-exponential regression model (as shown in Equation 16)

$$y = b \times m_1^{s/c} \times m_2^{w/c} \times m_3^{\%PP} \qquad \text{Equation 16}$$

where, y= “Crack Area” or “Crack Width,”

b, m_1, m_2, and m_3 are all coefficients (typically chosen notations in factorial design) and their values are presented in Table 5,

s/c is the sand to cement ratio,

w/c is the water to cement ratio, and

%PP is the percentage of the volume fraction of polypropylene fiber

The coefficients in Equation 16 were evaluated by least-square regression analysis based on the data presented in Table 3 and are presented in Table 5. Note that values have been rounded off to three decimal places.

To assess the effectiveness of the model in predicting crack area and crack width for mixes not used in formulating the equation, test results of additional mixes with other combinations of w/c, s/c, and Vf were used. Some of the mixes reinforced with polypropylene fibers were used, as this proposed model was developed for synthetic fiber only. These mixes are highlighted in grey in Table 6. Test data for these mixes along with results predicted by the model are presented in Figures 5 and 6 and are summarized in Table 6. Standard error was calculated according to Equation 15.

Figure 5 and 6 compare the laboratory test results with the predictions from the regression model for crack area and width respectively. Standard error for each test is also included in the plots. The predicted crack area and width values corroborated well with test data for a wide range of mixes both fiber reinforced and unreinforced mixes except for mix 6-4, where the predicted crack area and width exceeded the test results. The calculated percentage error in the predicted values compared to test result error is presented in Table 7.

The average percentage error in crack area and crack width was 108% and 62% respectively, which indicates good corroboration with test results considering that only a few mixes resulted in high error because the crack area and width values were very low absolute values. Fiber reinforced mixes 4-6-6 and 5-5-10 developed very small crack area and width, hence the percentage error was higher than 200% for both area and width; average percentage error neglecting these values dropped to 46% and 36% respectively for crack area and crack width. The average percentage error further dropped to 34% and 26% for crack area and width respectively by considering data for mix 6-4 as an outlier.

DISCUSSION OF RESULTS

As was discussed earlier, the main effect of w/c ratio and fibers was dominant on the crack characteristics and hence the effect of these factors on crack area and width evaluated using the proposed model was compared to actual test data. The effect of w/c ratio alone was evaluated by averaging the results of mixes with the same w/c but different s/c (except for w/c = 0.6, where predicted crack area and width for mix 6-4 were very large and were ignored). Similarly, effect of fiber was studied for mixes by fixing the w/c and s/c ratios. These results are presented in Figures 7 and 8. It is clear that the model is quite effective in predicting both the crack area and width for cement-based composites and can be useful in predicting the effect of w/c and fibers on crack characteristics.

CONCLUDING REMARKS

Factorial design analysis confirmed that w/c ratio is the most significant factor that affects crack areas and widths. Increase in w/c ratio increased both crack area and crack width. A small range of s/c ratio studied, did not affect the crack characteristics significantly. The effect of polypropylene fibers was studied that indicated that addition of fibers clearly reduced cracking. The proposed model predicts crack area and widths and the results corroborate well with the actual test data. Plastic shrinkage induced cracking is major issue in slabs-on-grade, thin overlays used in new and repair applications. The model proposed in this paper has practical implications as the model shows the influence of w/c and s/c on both crack area and crack widths. Moreover, this model also predicts the influence of low volume of polypropylene fibers that can be useful in developing ‘crack-free’ materials for real applications. Future scope of research in this area needs to focus on studying the applicability of this model to variation in mix design, type of binder/supplementary cementing materials, types and dosage of fibers, and inclusion of coarse aggregates.

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Table 1 - Data from a 2^3 Factorial Design

Test	Factors			Variable
	A	B	C	y
1	-	-	-	y1
2	+	-	-	y2
3	-	+	-	y3
4	+	+	-	y4
5	-	-	+	y5
6	+	-	+	y6
7	-	+	+	y7
8	+	+	+	y8

Table 2 - Effect of Variable B or "B Main Effect"

Difference between + and − levels of B	A	C
y3-y1	-	-
y4-y2	+	-
y7-y5	-	+
y8-y6	+	+

Table 3 - Crack Analysis Results

Mix Designation	w/c	s/c	V_f (%)	Crack Area (mm^2)	Average Crack Width (mm)	Maximum Crack Width (mm)	Standard Deviation (%)		
							Crack Area	Average Crack Width	Maximum Crack Width
4-4	0.4	0.4	0	285	1.21	1.93	28.9	34.8	21.8
4-6	0.4	0.6	0	260	1.20	2.15	31.1	5.3	14.0
6-4	0.6	0.4	0	499	1.11	2.20	6.5	18.7	3.7
6-6	0.6	0.6	0	732	1.79	4.03	25.4	30.2	23.0
4-4-6	0.4	0.4	0.066	111	0.64	1.07	10.2	28.6	23.1
4-6-6	0.4	0.6	0.066	9	0.09	0.12	173.2	173.2	173.2
6-4-6	0.6	0.4	0.066	619	1.92	3.62	18.3	19.0	20.8
6-6-6	0.6	0.6	0.066	519	1.48	2.75	27.6	36.7	3.4

Table 4 - Mixes for 2^3 Factorial Design

Mix Designation	Parameters			Factorial Levels
	w/c	s/c	V_f (%)	(w/c, s/c, V_f)
4-4	0.4	0.4	0	(-, -, -)
4-6	0.4	0.6	0	(-, +, -)
6-4	0.6	0.4	0	(+, -, -)
6-6	0.6	0.6	0	(+, +, -)
4-4-6	0.4	0.4	0.066	(-, -, +)
4-6-6	0.4	0.6	0.066	(-, +, +)
6-4-6	0.6	0.4	0.066	(+, -, +)
6-6-6	0.6	0.6	0.066	(+, +, +)

Table 5 - Coefficients from Regression Analysis

Coefficient	b	m_1	m_2	m_3
Crack Area	18.079	0.053	9550.356	$5.84x10^{-8}$
Crack Width	0.2809	0.117	182.905	$2.266x10^{-5}$

Table 6 - Comparison of Predicted Data to Test Results

Mix Designation	w/c	s/c	V_f (%)	Test Results		Prediction Model	
				Crack Area (Standard Error) in mm²	Average Crack Width (Standard Error) in mm	Crack Area (mm^2)	Crack Width (mm)
4-4	0.4	0.4	0	285 (83)	1.21 (0.42)	218	0.96
4-6	0.4	0.6	0	260 (81)	1.20 (0.06)	121	0.62
6-4	0.6	0.4	0	499 (33)	1.11 (0.21)	1361	2.71
6-6	0.6	0.6	0	732 (186)	1.79 (0.54)	756	1.77
4-4-6	0.4	0.4	0.066	111 (11)	0.64 (0.18)	73	0.47
4-6-6	0.4	0.6	0.066	9 (16)	0.09 (0.16)	40	0.31
6-4-6	0.6	0.4	0.066	619 (113)	1.92 (0.37)	453	1.34
6-6-6	0.6	0.6	0.066	519 (143)	1.48 (0.54)	252	0.87
5-4	0.5	0.4	0	628 (96)	1.64 (0.33)	544	1.61
5-6	0.5	0.6	0	450 (148)	1.62 (0.16)	302	1.05
5-5-6	0.5	0.5	0.066	212 (35)	0.82 (0.06)	135	0.64
5-5-3	0.5	0.5	0.033	157 (74)	1.21 (0.34)	234	0.91
5-5-10	0.5	0.5	0.1	10 (18)	0.14 (0.25)	77	0.45
5-5	0.5	0.5	0	264 (33)	2.18 (0.85)	406	1.30

Table 7 - Comparison of Predicted Values and Test Results

Mix Designation	Error (%)	
	Crack Area	Crack Width
4-4	24	21
4-6	53	48
6-4	173	145
6-6	3	1
5-4	13	2
5-5	54	40
5-6	33	35
4-4-6	34	26
4-6-6	330	229
6-4-6	27	30
6-6-6	52	41
5-5-3	49	24
5-5-6	36	22
5-5-10	633	209