Planting Decisions in the Face of Uncertainty. II. Statistical and numerical aspects of assessing the future financial returns from a given planting action.

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EXECUTIVE SUMMARY

Given the desirability of sustainable forestry, a question of considerable importance to the managers of public forest land, and to the owners of private forest land, is how a given site should be replanted after a clear-cut harvest. What species should be planted? at what density? and with what follow-up treatment? Or is it better to take no restocking action and expect the site to regenerate naturally?

Such decisions are necessarily made in the face of much uncertainty, since the economic benefits from silvicultural investments are long delayed. Uncertainty in the future demand and supply for timber products, and hence in price, as well as uncertainty in biological growth and the possibility of unpredictable catastrophic events such as fire, tempest and pest infestation, all combine to make the restocking decision an extremely chancy one.

In a previous report (Reed [1991]) a method of calculating the distribution of net present values of returns from a given planting option was outlined along with a method for considering the mean and variance in net present values for planting decisions involving partitioning the available area into sub-areas using different planting options. However in that paper only the theory was discussed. Statistical and numerical aspects of the problem were not addressed. This paper deals with these issues along with a comparison of various harvesting rules and an assessment of the performance of predictions of expected values and variances of net present values. The main purpose of the research is to find an acceptably simple and adequate procedure for determining the characteristics of the probability distribution of financial returns

for a given planting option.

Inputs to the procedure are:

(1) time series data of timber product prices for the given planting option, up to the current time,

(2) volume growth information for the planting option from test plots.

The first step is to fit statistical models to these data. Of course standard statistical safeguards, including model checking and validation are necessary at this stage. The output from this step consists of probability distributions (and point estimates) for the parameters of the models.

With these estimates it is possible to make predictions of the expectation and variance of the net present value (N.P.V.) of returns from employing the particular planting option.

To assess the adequacy of these predictions and to compare the performance of various harvest rules, along with assessing the importance of improving the precision of the parameter estimates through obtaining more data, many simulations of the process were run. Two harvest procedures, the myopic-look-ahead (M.L.A.) rule, which cuts the stand when its net value is growing in expectation at the discount rate, and the fixed harvest age certainty-equivalence (C.E.) rule were used. It turns out that *expected* NPV is slightly higher using the MLA rule, while the *median* of NPV is higher for the CE rule. In both cases the probability distribution of the NPV is highly skewed with a long tail to the right, indicating occasional very rich returns. Overall there is very little to choose, in terms of financial return, between the two rules. Because it is simpler to compute there is much to be said for use of the CE rule.

In Reed [1991] formulas were derived for the mean and variance of expected NPV using a fixed-age harvest rule. It turns out that these provide reasonably good predictions of the mean and variance, even when there is uncertainty in parameter values. It would therefore appear that provided there are adequate data available on price and growth, fairly reliable predictions of the mean and variance of net present value of returns for a particular planting option, can be obtained using these formulas.

Another conclusion which can be drawn from the simulations is that there appears to be very little to be gained from considering the fact that parameter estimates can be updated as more data (especially price data) become available, in the determination of the net present value of returns from a particular planting option. This conclusion may not hold however in other situations where the original data, which are the inputs to the procedure, are of poorer quality or quantity than those used here.

1. Introduction

An important component of sustainable forestry is the replanting of forest sites which have been clear-cut harvested. Across Canada, but especially in British Columbia, there is a considerable back log of sites which have been clear cut but which have not been re-stocked. An important question facing the managers of public forest land, and the owners of private forest land is how a particular site should be re-planted. What species should be planted? at what density? or is it better to take no restocking action and allow the site to perhaps regenerate naturally.

Such decisions are necessarily made in the face of much uncertainty since the benefits from silvicultural investments are long delayed. Uncertainty in the long-run future demand for timber products, and hence in their price, as well as uncertainty in biological growth and the possibility of unpredictable catastrophic events such as fire, windstorms and pest infestation all combine to make the planting decision an extremely chancy one.

In a previous report (Reed [1991]), a scheme for assisting decision makers was outlined. Stochastic models for the price process and the biological growth process were outlined, and it was indicated how the valuation (in terms of expected net present value) of a particular planting option could be carried out. In this paper these methods are applied to assess the potential payoffs of a particular planting option, using historical time series data on sawlog prices and growth information from sample test plots. The species chosen was loblolly pine grown at 12 foot spacing in the south-eastern U.S.A. This choice was made because data on prices and growth were readily available. The methodology would be similar for planting options available in B.C. and Canada, provided similar data were available.

For a given planting option, the methodology in this paper allows the calculation of a probability distribution of net present values. Of course from this probability distribution summary statistics such as the expected net present value (NPV) and the standard deviation of NPV can be calculated. The decision maker would face the problem of choosing among probability distributions in making the planting decision. For this some standard procedure could be used, such as selecting the option with the maximum expected utility (assuming the decision maker can specify the appropriate utility function to reflect the relevant attitude toward risk), or alternatively, a more subjective choice could be made.

Another possibility is the partitioning of the site into two or more areas with different planting choices. This is discussed in Reed [1991], where it is shown how quadratic programming can be used to obtain the partition with minimum variance in NPV, for a given level of expected NPV, and in consequence how mean-variance trade-off charts could be constructed. This is not pursued in this paper. However a crucial step in the above procedure is to obtain estimates of the expected NPV and variance in NPV for a given planting option. This aspect is addressed in this paper.

2. *The Price Data*

Quarterly prices net of harvest costs in dollars per thousand board feet, for loblolly pine for the period 1977-1988 were available.[1] The logarithm of the price is plotted as a time series in Fig. 1. Under the assumption that price, P, follows (Stratonovich) *geometric Brownian motion*

(1) $$dP = bPdt + \underline{\sigma} P dw(t)$$

(see Reed [1991]), the first differences in the logarithms of monthly prices

(2) $$y_{i+1} = \ell n(P_{i+1}) - \ell n(P_i) \quad i = 1,\cdots$$

where P_i is the price in month i, should be independent identically distribution (i.i.d.) normal random variables with mean[2] $\frac{b}{4}$ and variance $\frac{\sigma^2}{4}$ *i.e.*

(3) $$y_i \sim NID\left[\frac{b}{4},\frac{\sigma^2}{4}\right]$$

(see Reed [1991, equation (42)]). Fig. 2 gives a plot of the first differences in ℓn (price) and they do indeed appear to be consistent with this model. Tests for lack of normality or independence were not significant. The maximum likelihood (ML) estimates of the parameters b and σ^2 are (see Reed [1991, equations (47) and (49)])

(4) $$\hat{b} = 4\bar{y} = -0.01188$$

(5) $$\hat{\sigma}^2 = 4s^2 = 0.07281$$

where $s^2 = \frac{1}{n}\sum_1^n (y_i - \bar{y})^2$ is the sample variance. The standard errors of these estimates are respectively 0.04023 and 2.356×10^{-4}.

3. *The Growth Data*

Figure 3 exhibits volumes (in board feet (international) per acre) at five-year intervals for test plantations of loblolly pine in Georgia, with a 12 × 12 ft. spacing. For convenience in separating visually the four plots, the points for each have been joined by straight line segments.

Following the procedure set out in Reed [1991], it will be assumed that volume growth follows the (Stratonovich) stochastic differential equation

(6) $$dx = g(t)\,x\,dt + \tilde{\sigma}\,x\,d\bar{w}(t)$$

where $x(t)$ is the volume at time t (again with the time unit being a year). If this is the case then the volume x_j at the end of the jth five yearly interval should satisfy:

(7) $$x_j = x_{j-1}\exp\left\{\int_{5(j-1)}^{5j} g(s)\,ds + \tilde{\sigma}(w(5j) - w(5(j-1)))\right\}$$

where $\{w(t)\}$ is a standard Weiner process. Thus the first-differences in logarithms of volume

(8) $$x_j = \ell n\, x_j - \ell n\, x_{j-1}$$

should satisfy

(9) $$x_j = \int_{5(j-1)}^{5j} g(s)\,dx + \epsilon_j$$

where the ϵ_j are i.i.d. normal random variables

(10) $$\epsilon_i \sim N(0, 5\tilde{\sigma}^2).$$

Thus if some parametric form for the mean age-dependent growth rate function $g(t)$ can be assumed, then its parameters can be estimated (using maximum likelihood) by non-linear least squares. The simplest form is a linear one ($g(t) = \alpha - \beta t$). This was suggested in Reed [1991]. It turns out however that fitting this curve leads to unsatisfactory residual plots, indicating that the model is not valid. An alternative form is thus required. The *von Bertalannty* growth curve which has[3]

(11)
$$g(t) = \frac{3\alpha e^{-\alpha(t-\beta)}}{1 - e^{-\alpha(5(j-1)-\beta)}}$$

was tried next. Integrating $g(t)$ leads to (in (9))

(12)
$$x_j = 3\,\ell n \left[\frac{1 - e^{-\alpha(5j-\beta)}}{1 - e^{-\alpha(5(j-1)-\beta)}}\right] + \epsilon_j.$$

Non-linear least squares was then used to estimate the parameters α and β. The residual plots for this fit were satisfactory and a test for lack of fit, by partitioning the error sum of squares proved to be non-significant. The model was thus assumed to be adequate. The ML estimates of the parameters were (with asymptotic standard errors in brackets)

$$\hat{\alpha} = 0.17071 \quad (0.02226)$$

and

$$\hat{\beta} = 10.6258 \quad (0.38399).$$

The residual mean square which provides an estimate of $5\tilde{\sigma}^2$ was 0.010644 (*i.e.* $\hat{\tilde{\sigma}}^2 = 3.288 \times 10^{-3}$) while the asymptotic correlation between $\hat{\alpha}$ and $\hat{\beta}$ was 0.91994.

In order to completely solve equation (6) an initial condition is required. For each test plot the earliest age with nonzero volume is 15 years, and the volume at that age is different for each of the sample plots. If growth really does follow something close to the model (6), then the volume at age 15 years should have a distribution close to a log normal distribution. We have thus assumed that the four volumes at age 15 years are i.i.d. log normal random variables with unknown mean and variance. This mean and variance are easily estimated using the sample mean and sample variance of the logarithms of the four points. These values are

$$M = 7.91044$$

$$V = 0.13815$$

Thus the starting (age 15) volume for any stand will be assumed to be generated from a log normal distribution. Specifically it will be a realization of the random variable

(13)
$$X_0 = \exp\{m + \sqrt{v}\, Z\}$$

where Z is a standard normal deviate. Note that this has mean value $\bar{X}_0 = \exp\left\{m + \frac{1}{2}v\right\} = 2{,}920.52$ board feet. Using this as a starting value we are now able to construct a "mean-value growth curve" as (see Reed [1991], equation (10))

(14)
$$\bar{X}(t) = \bar{X}_0 \left[\frac{1 - e^{-\tilde{\alpha}(t-\tilde{\beta})}}{1 - e^{-\tilde{\alpha}(15-\tilde{\beta})}}\right]^3 \exp\left[\frac{1}{2}\tilde{\sigma}^2(t-15)\right]$$

for $t \geq 15$. This is plotted superimposed on the sample plot volumes in Fig. 4.

4. *Evaluating a Given Planting Option*

To determine the maximum expected present value, *assuming all parameter values are known with certainty*, involves solving a difficult optimal stopping problem (see Reed [1991, Section 3, p. 13]). In principle this problem could be solved in discrete time using dynamic programming, and indeed the problem with no growth uncertainty (only price uncertainty) has been so solved (see Haight & Holmes, 1991). However if one recognizes that parameter values are not known with certainty (*i.e.* that there is sampling error present in the estimates) then the problem becomes much more difficult. Possibly one could consider an array of values of the parameter (within say plus or minus two standard errors for each estimate) and for each compute the optimal solution. This however would become extremely cumbersome computationally and interpretation of the results would become extremely difficult.

Another difficulty with the dynamic programming approach is that it permits the determination only of the optimal *expected* net present value; it does not provide further information on the *probability distribution* of the NPV corresponding to the optimal policy.

Another approach which shall follow here is to use a procedure which should lead to something close to the optimal solution *i.e.* the plausible heuristic harvest rule known as the *myopic look ahead* (MLA) rule (see Reed [1991], p. 14). For this feedback rule one allows the stand to continue growing until the expected rate of growth in the net revenue drops to, or below, the rate of discount. In Reed (1991, p. 14) it is shown that this rule implies no harvesting when

$$\left[\delta - g(t) - b - \frac{1}{2}\sigma^2\right] P(t)X(t) + \left[\delta - b_L - \frac{1}{2}\sigma_L^2\right] L(t) < \delta C \tag{15}$$

where $\sigma^2 = \left(\underline{\sigma}^2 + \tilde{\sigma}^2\right)^{\frac{1}{2}}$; $L(t)$ is the land expectation value (assumed to follow a geometric Brownian motion, correlated with the price process and with parameters b_L and σ_L^2) and C is the cost of re-establishment. In using this rule we have simulated sample paths of the price, volume growth and land expectation value processes, and continued until the condition (15) is violated. The observed values of $P(t)$, $X(t)$ and $L(t)$ at the time of harvest permits the determination of the net present value for that particular simulation. This has been repeated many times to obtain a distribution of net present values and harvest times. The mean of the net present value provides the 'value' of the particular planting option to a risk-neutral forest owner.

In generating the sample paths the *sampling error in the parameter values has been taken into account*. Specifically assuming that the sampling distributions of the parameter estimates are approximately normally distributed (which asymptotically they will be) we have generated for each run, for each parameter, a value of a normal deviate with mean equal to the point estimate of the parameter and variance equal to the square of the standard error of the estimate. When parameter estimates are correlated (*e.g.* $\hat{\alpha}$ and $\hat{\beta}$ for the growth model) points have been generated from a bivariate normal distribution with the appropriate covariance. In doing this we are in essence adopting something like a Bayesian point of view with a non-informative prior distribution (see *e.g.* Box & Tiao, 1973) in that we are assuming that each parameter itself has a distribution determined by the likelihood function. It is the fact that asymptotically, near the maximum, the log likelihood is quadratic, which offers justification for the use of a normal distribution. Once parameter values have been determined, sample paths of $P(t)$, $X(t)$ and $L(t)$ are generated by means of discretizing to a fine mesh and generating serially uncorrelated normal deviates to obtain sample path solutions to (1) and (6), using as starting value for price the latest value ($121.2 in the first quarter of 1988) and as starting value (at age 15) for volume, a log-normal deviate of the form (13). Since no historical data were available for land values it was assumed that the mean growth rate b_L was equal to the mean growth rate for timber price and that the variance in growth rate, σ_L^2, was equal to σ_P^2, and further that the noise in the land value process was correlated with the noise in the price process with a correlation equal to 0.70. The initial land value was set at $500 per acre, which is about the current value for bare land in rural locations in N. Carolina (R. Haight, *pers. comm.*). These values are rather *ad hoc* in nature, but it seems reasonable that land values should grow, on average, at the same rate as timber price, and that they should be strongly correlated with timber prices.

The cost parameter c was set at $300 per acre. This is about the current cost for site preparation and planting (R. Haight, *pers. comm.*). (Note the stumpage price P_t is net of harvest costs.) The discount rate was set at $\delta = 0.05$ per annum.

In performing the simulations four different harvest procedures were adopted. The first two used the MLA procedure described above, while the latter two used a *certainty-equivalence* (C.E.) procedure (see Reed 1991, p. 11) in which a harvest rule based on a deterministic analysis using expected values of all random variables involved. This leads to open-loop rules based on a fixed harvest age.

For both the MLA procedure and the CE procedure two different scenarios were used. The first assumed that the actual parameter values used in a particular run were known, and the harvest rule was based upon these parameter values. The second, in contrast, assumed that the actual parameters used were not known exactly, and the harvest rules were based on the point estimates of the parameter values from the historical data. The first scenario represents a situation in which *more* information is available than would be in practice, while the second represents one in which *less* information than that available in practice is known.

In practice one would start at the time of planting with information on parameter values based solely on the historical data (as in the second case above). However as time passed more observations on the price time series would become available and parameter estimates could be updated. Similarly estimates of the parameters of the growth and land-value processes would be improved. If observations were to continue indefinitely the updated parameter estimates would converge to the true parameter values in operation. However in practice observations would only continue for a finite time and therefore parameter estimates, although based on more information than the historical data, would still not coincide exactly with the actual parameters. Thus the former case above corresponds to a situation with more information available than there would be in practice. One would expect then that performance of a harvesting rule under this scenario would be better than that of the corresponding rule under the second scenario, in which the prior parameter estimates were used. Furthermore one should expect that the performance of a real-life rule in which parameter estimates were sequentially updated in a Bayesian fashion, should be bounded above and below by the performances of these two rules.

5. Results

In order to run the simulations the continuous processes for price, volume growth and land value (equations (1) and (6) and equation (12) of Reed (1991)) had to be discretized. The time step used was 0.02 years, corresponding approximately to one week. Two thousand runs were made. In each, sample paths of the $P(t)$, $X(t)$ and $L(t)$ processes were generated using parameter values generated from the sampling distributions as described in Section 4 above. For each run the four harvest policies described above (MLA rule with actual parameter values and with prior estimates of parameters, and CE rule with actual parameter values and with prior estimates of parameter values) were employed and the present value of the resulting harvest observed. Also the cutting ages for the two MLA rules were observed. The results are presented graphically in Figs. 5-10.

Figs. 5 and 6 display the frequency histograms of the cutting ages using the MLA rule under the two scenarios. When actual parameter values were used the mean and standard deviation of the MLA cutting age were 28.62 yrs. and 6.04 yrs. respectively (see Table 1).

If we regard the mean cutting age as an estimate of the mean age over all infinity of runs, it would have a standard error of 0.135 yrs. When the prior estimates of parameter values were used the mean and standard deviation of the cutting age were 30.47 yrs. and 3.75 yrs. respectively. The standard error of the mean as estimator is 0.084 yrs. Thus there appears to be a slightly higher mean cutting age with smaller variance, when the prior estimates of parameter values are used in calculating the MLA rule.

Figs. 7 and 8 display frequency histograms of the net present value of returns using the MLA rule under the two scenarios. It can be seen that in both cases the distribution is considerably skewed with a long tail to the right. This reflects the skewness of the lognormal distribution which both price and volume would be expected to follow. The mean and standard deviation in net present value using actual parameter values are \$1071.51 and \$2889.46 while the corresponding figures when prior estimates of the parameters are used are \$1099.49 and \$3087.32 (see Table 2). Thus rather surprisingly, the mean net present value corresponding to the MLA rule using prior estimates is slightly larger than that using actual parameter values. However this difference is not statistically significant ($P = 0.76$) nor is it significant in any practical sense. Since, as discussed in the previous sections, these two mean present values should provide bounds on the mean present value in the case where parameter estimates are updated period by period, it would appear that in terms of expected present value, not much is to be gained through the updating procedure. One characteristic evident in both cases (and presumably in the case where parameter estimates are updated recursively) is the large variance in net present value. This is in large part due to the long tail to the right in the frequency distributions reflecting the possibilities of extremely rich payoffs albeit with low probability. Because of the skewness of the distribution, possibly a better method of characterizing the central value is to use the median rather than the mean. The medians are respectively \$349.55 and \$404.46 in the cases when actual parameter values are known, and when prior estimates are used. Again there is no evidence of improvement through knowledge of actual parameter values, and by extension little to be gained from updating parameter values.

Figs. 8 and 9 show histograms of the net present value using the certainty equivalence (CE) procedures. These involve cutting at a fixed predetermined age. Again highly skewed distributions are in evidence. The mean and standard deviation in net present value when parameter values are assumed known are \$1028.40 and \$2330.03 and when prior estimates are used they are \$995.13 and \$2000.87. Again the difference is not statistically significant (P = 0.63).

Although there is a difference between the pooled mean NPV for the two MLA procedures (\$1085.50) and the pooled mean for the two CE procedures (\$1011.77) it is not statistically significant (P = 0.26). The median NPV's for the two CE procedures are \$450.14 and \$476.55. These are both higher than the corresponding medians using the MLA procedures.

When one compares the standard deviation in NPV of the CE procedures with those of the MLA procedures it can be seen that the latter are smaller, the ratios being about .80 and .65 in the two cases. This difference, and the fact that the expected NPV's are higher for the MLA procedure while the median NPV's are higher for the CE procedures is probably due to the fact that there is a greater probability of an extremely high NPV using an MLA procedure than with a CE procedure. For the two MLA procedures the maximum NPV's were \$28,353 and \$28,311 while for the CE procedures the corresponding maxima were \$19,965 and \$14,678.

In Reed [1991] formulas for the expected value and variance in NPV were given, assuming that a fixed age harvest policy was used (equations (19) and (20)). The formulas were in terms of the model parameters -- thus the only uncertainty considered was in the growth and price stochastic processes. Using the optimal fixed age policy with parameter values estimated from the historical data, estimates of the expected value and standard deviation in NPV using these formulas were respectively \$1148.50 and \$2431.42. Thus it appears that the estimate of expected NPV, is somewhat on the high side, for the expected NPV when uncertainty in parameter values is also included. However the error appears to be fairly small. In fact as an estimate of the expected NPV of returns using the MLA rule it appears, somewhat coincidentally, to be quite good. In fact the observed MLA means are not significantly different from this estimate ($P = 0.23$ and 0.48 respectively).

However as an estimate of the variance using the MLA rules, the calculated value seems too small. In contrast for the CE rules, the situation is reversed with the variance estimates appearing to be reasonably good, while the estimate of the means is significantly too high in both cases. Of course if there were greater uncertainty in parameter estimates then the discrepancies between the predicted and actual mean and variance of NPV could be much greater.

6. Conclusions

From the results presented in Section 5 the following conclusions can be drawn.

(a) It appears that there is very little to be gained in terms of net present value through updating parameter estimates. Regardless of whether an MLA or a CE procedure is used the mean and variance in NPV do not change greatly when exact parameter values are assumed known, rather than simply estimated from the historical data. Indeed in terms of *median* NPV both procedures using the prior estimates of parameters do somewhat better than the corresponding procedures using exact parameter values.

(b) There appears to be little advantage in using an MLA procedure over a CE procedure. While the MLA procedures do slightly better than the CE procedures in terms of *expected* NPV, the situation is reversed when one looks at *median* NPV. Furthermore the standard deviations in NPV are smaller using the CE procedures. It seems that the MLA procedures allow for the occasional extremely high NPV, to a greater extent than do the CE procedures.

(c) In terms of evaluating the potential returns of a particular planting option it appears that a calculation based on a fixed harvest age is reasonably adequate for assessing the mean and standard deviation of NPV. This is of some practical use because the formulas for these descriptors are available (Reed [1991, equations (19) and (20)]). Of course for other planting options exhibiting more variability in growth or price, or for which there is greater uncertainty in parameter values (due to poor or inadequate data etc.) these conclusions may not hold, or at least be somewhat diluted. The degree to which the conclusions established here would hold in such cases could only be determined by further analysis of the type carried out in this paper. One thing is clear however and that is that without adequate data on forest growth or on timber price time series, it will be extremely difficult to make realistic predictions on the potential payoffs from various planting options.

Prediction is always a risky business. No one knows the future. Here we have assumed, in a probabilistic sense that the future will be like the past *i.e.* that timber prices will follow a stochastic process similar to that observed in the past; and that growth of stands of trees will be similar to that observed on test plots on similar sites in the past. However things could change drastically. For example technological change could drastically decrease the price of sawlogs relative to pulpwood, or global climate change could severly change growth patterns. These aspects have been ignored here, simply because they do not lend themselves easily to quantification and modelling. This does not mean, of course, that they should be ignored by the decision maker. The methods outlined in this paper should be considered only as one input in the decision making process. They are based on the assumption that probabilistically the future will be like the past. The decision maker should give weight to the prediction methods described herein to the degree that he or she believes in this assumption.

FOOTNOTES

1. The price and growth data used in this report were kindly made available by Dr. Robert Haight of the USDA Forst Service.

2. We are assuming that the time unit in (1) and elsewhere is the year. This is the reason why the factor $\frac{1}{4}$ occurs for the mean and variance of the first differences in *quarterly* price.

3. In a deterministic setting, integrating the differential equation $\dot{X} = Xg(t)$ leads to the familiar von Bertalannty form $X(t) = X_{\infty}(1 - e^{-\alpha(t-\beta)})^3$.

	Using actual parameter values	Using prior estimates of parameter values
Mean	28.62	30.5
Standard Deviation	6.04	3.75
Median	27.00	29.26

Table 1. Summary statistics (in years) for the observed distribution of cutting ages using the myopic-look-ahead (MLA) cutting rule. In the first column the actual parameter values (generated from the distribution computed from the historical data) were used to determine the MLA rule. In the second column the actual parameters were assumed unknown and the MLA rule was calculated using point estimates of parameter values from historical data.

	M.L.A. Rule		C.E. Rule	
	Using actual parameter values	Using prior estimates of parameter values	Using actual parameter values	Using prior estimates of parameter values
Mean	1,071.51	1,099.49	1,028.41	995.13
Standard Deviation	2,889.46	3,087.32	2,330.03	2,000.87
Median	349.55	404.46	450.14	476.55
Maximum	28,353.17	28,311.20	19,965.21	15,678.41

Table 2 Summary statistics (in dollars) for the net present value (NPV) using myopic-look-ahead (MLA) cutting rules and certainty-equivalence (CE) rules. In the first column for each rule the actual parameter values (generated from the distribution computed from the historical data) were used to determine the cutting rule, while in the second column point estimates of parameter values from the historical data were used to compute the cutting rule.

ACKNOWLEDGEMENT

I wish to thank Dr. Robert Haight of the USDA Forest Service for providing the price and growth data, and for many stimulating ideas expressed in conversation and correspondence.

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Reed, W.J. (1991). Planting dicisions in the fact of uncertainty. I. Theoretical results. *F.E.P.A. Working Paper 156*.

NATURAL LOGARITHM OF QUARTERLY SAWLOG PRICE VS. QUARTER.

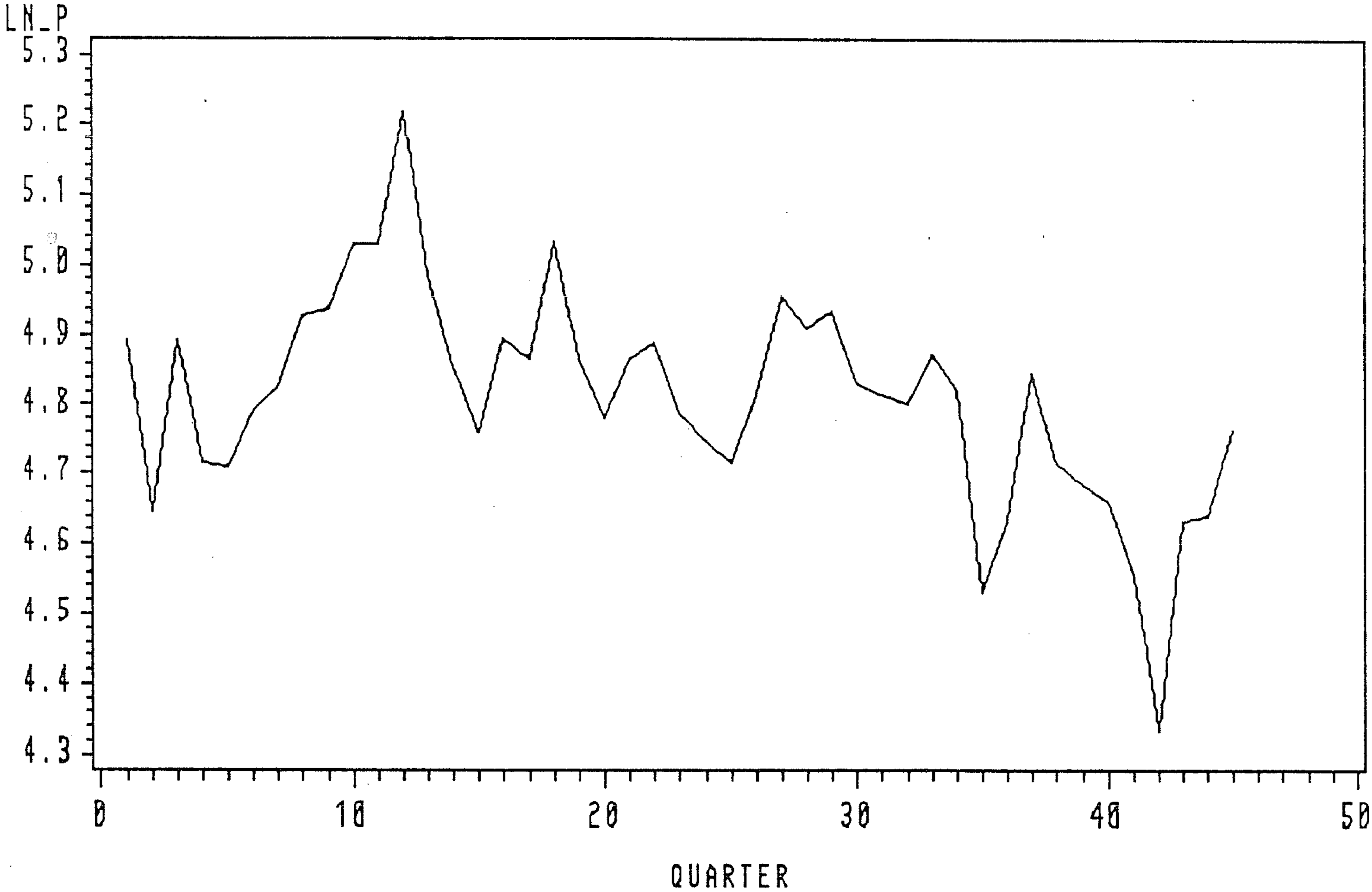


Figure 1

FIRST DIFFERENCE, Y, OF LOGARITHM OF SAWLOG PRICE VS. QUARTER.

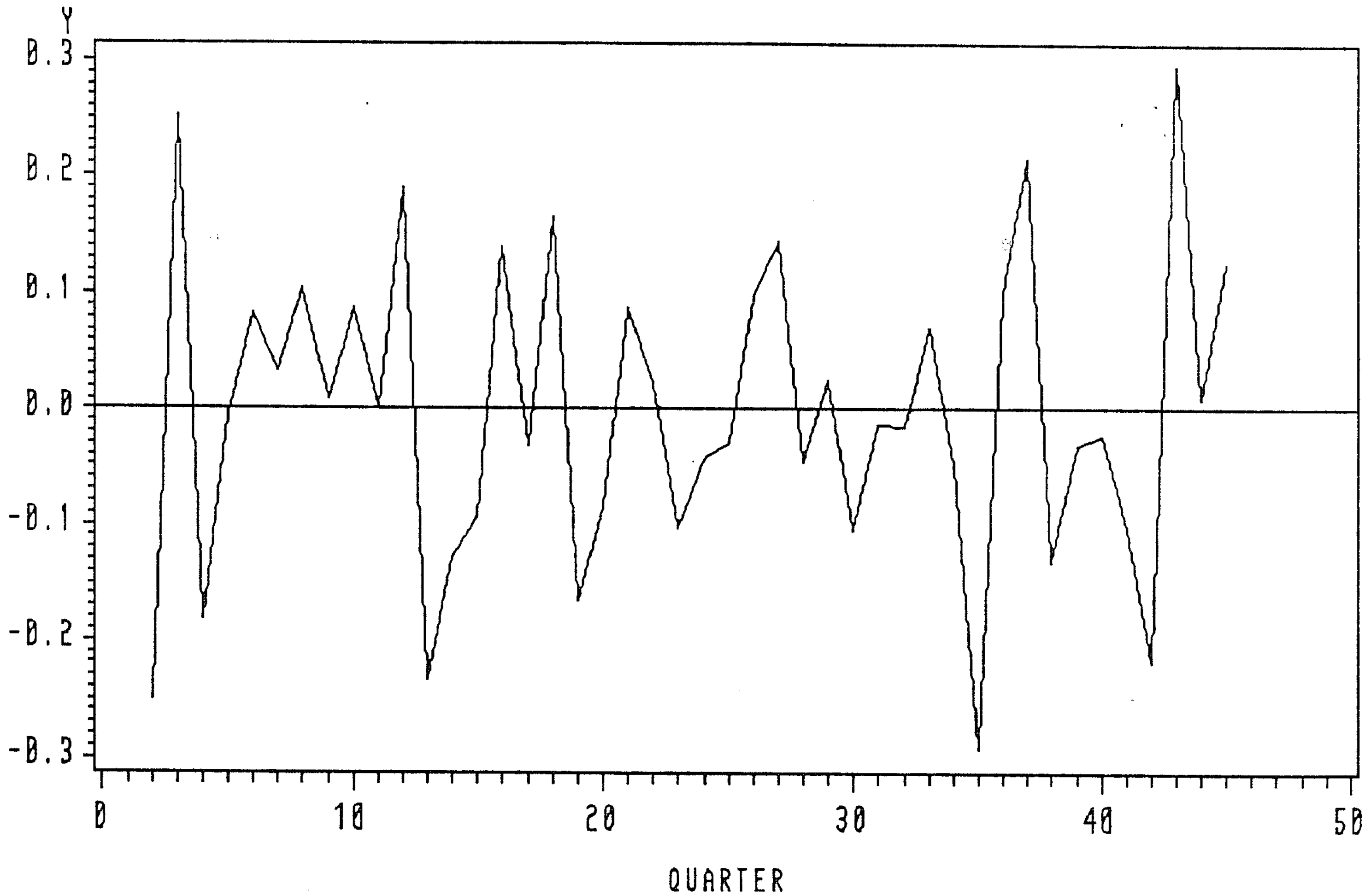


Figure 2

BOARD FEET OF LUMBER PER ACRE FOR TEST PLOTS OF LOBLOLLY PINE

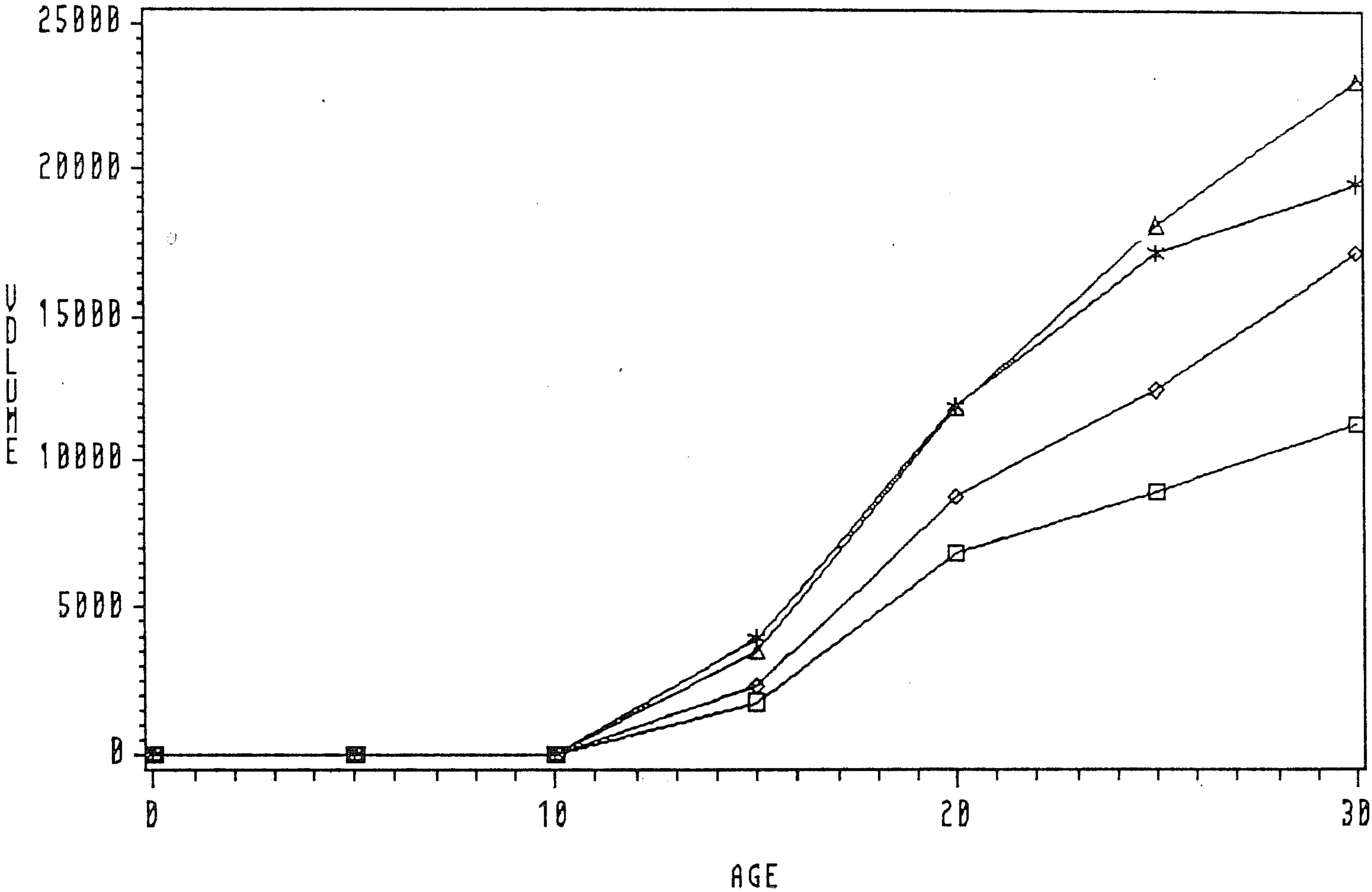


Figure 3

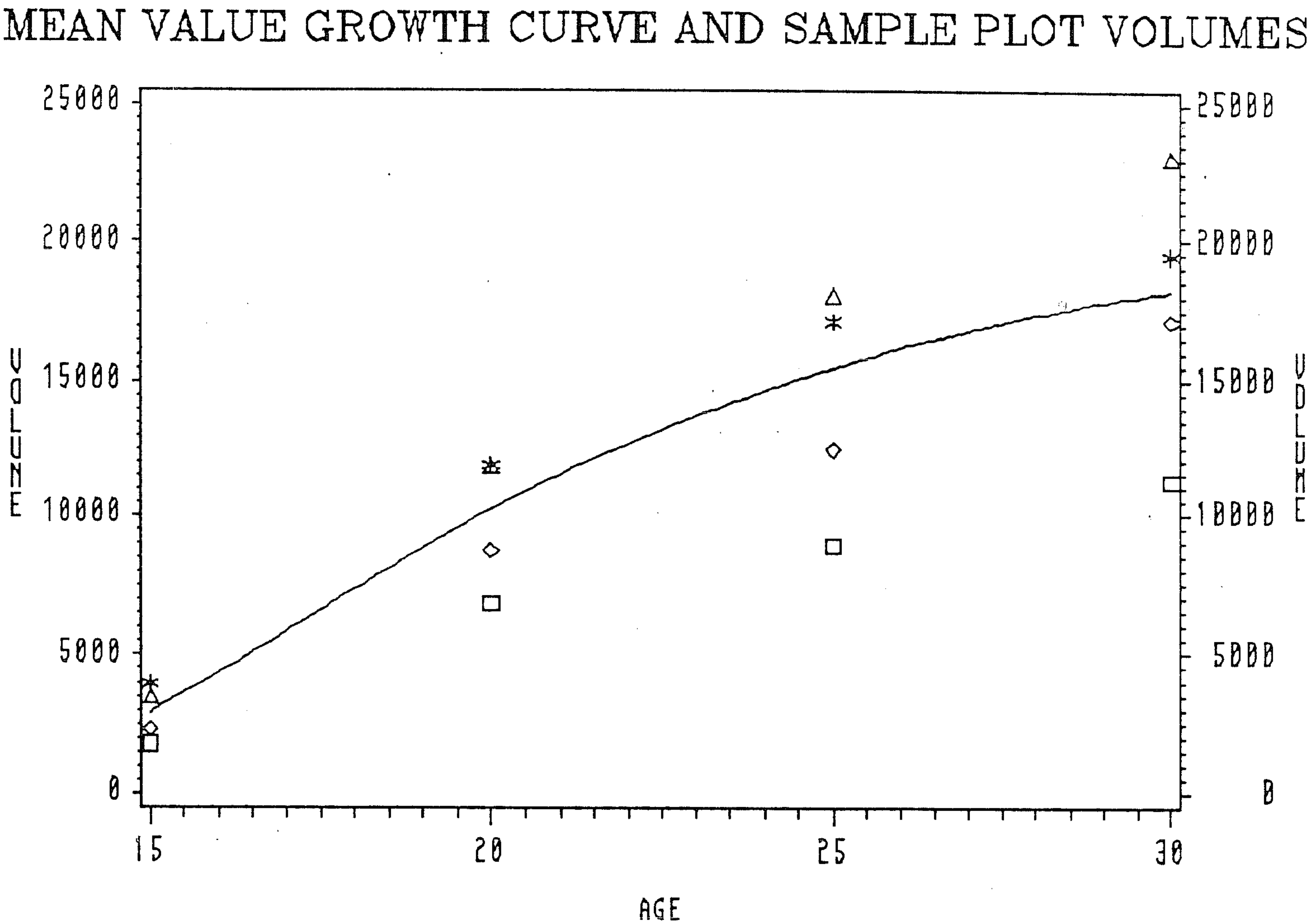
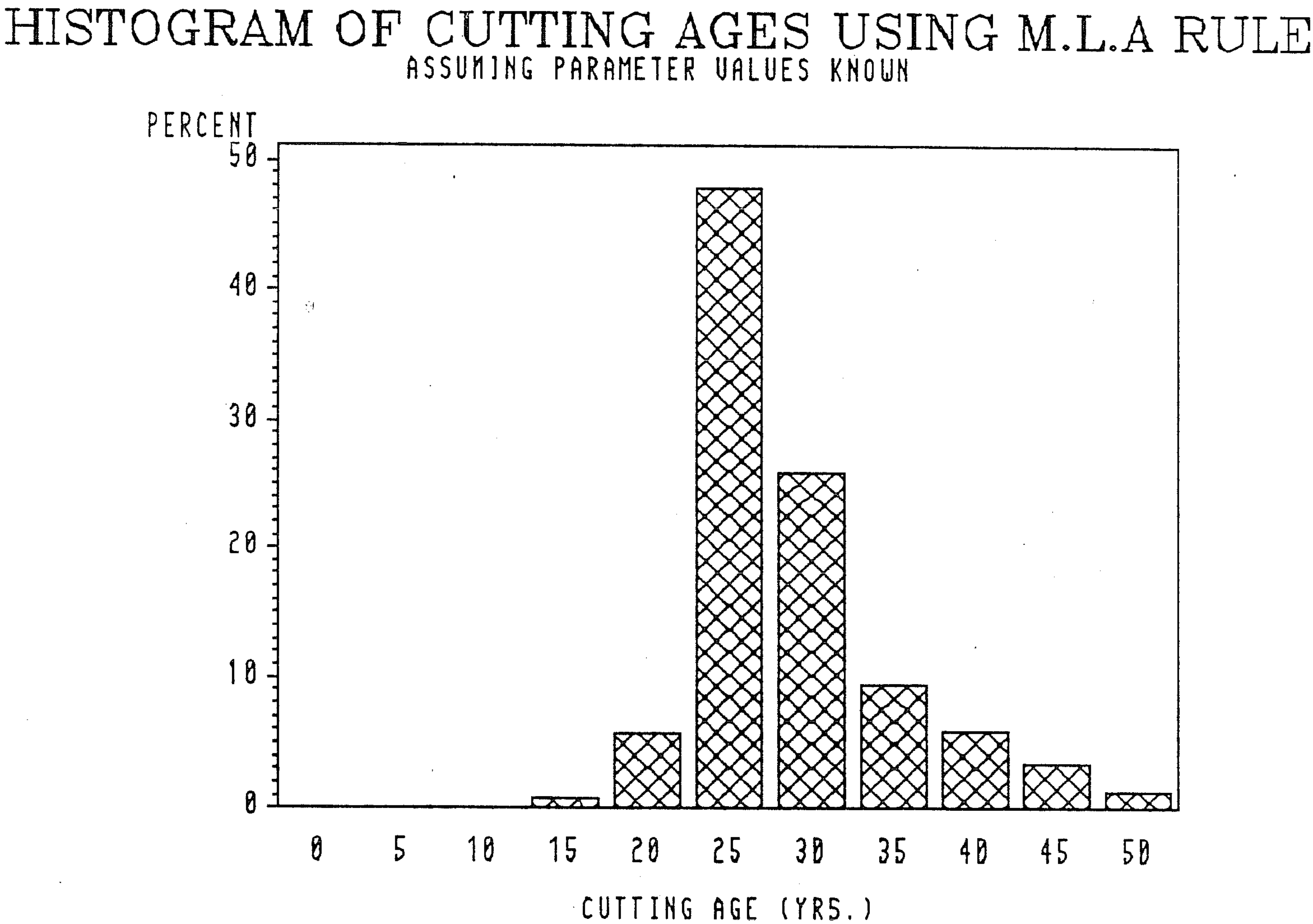

MEAN VALUE GROWTH CURVE AND SAMPLE PLOT VOLUMES
VOLUME
25000
20000
15000
10000
5000
0
15
20
25
30
AGE

Figure 4


HISTOGRAM OF CUTTING AGES USING M.L.A RULE
ASSUMING PARAMETER VALUES KNOWN
PERCENT
50
40
30
20
10
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0
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10
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45
50
CUTTING AGE (YRS.)

Figure 5

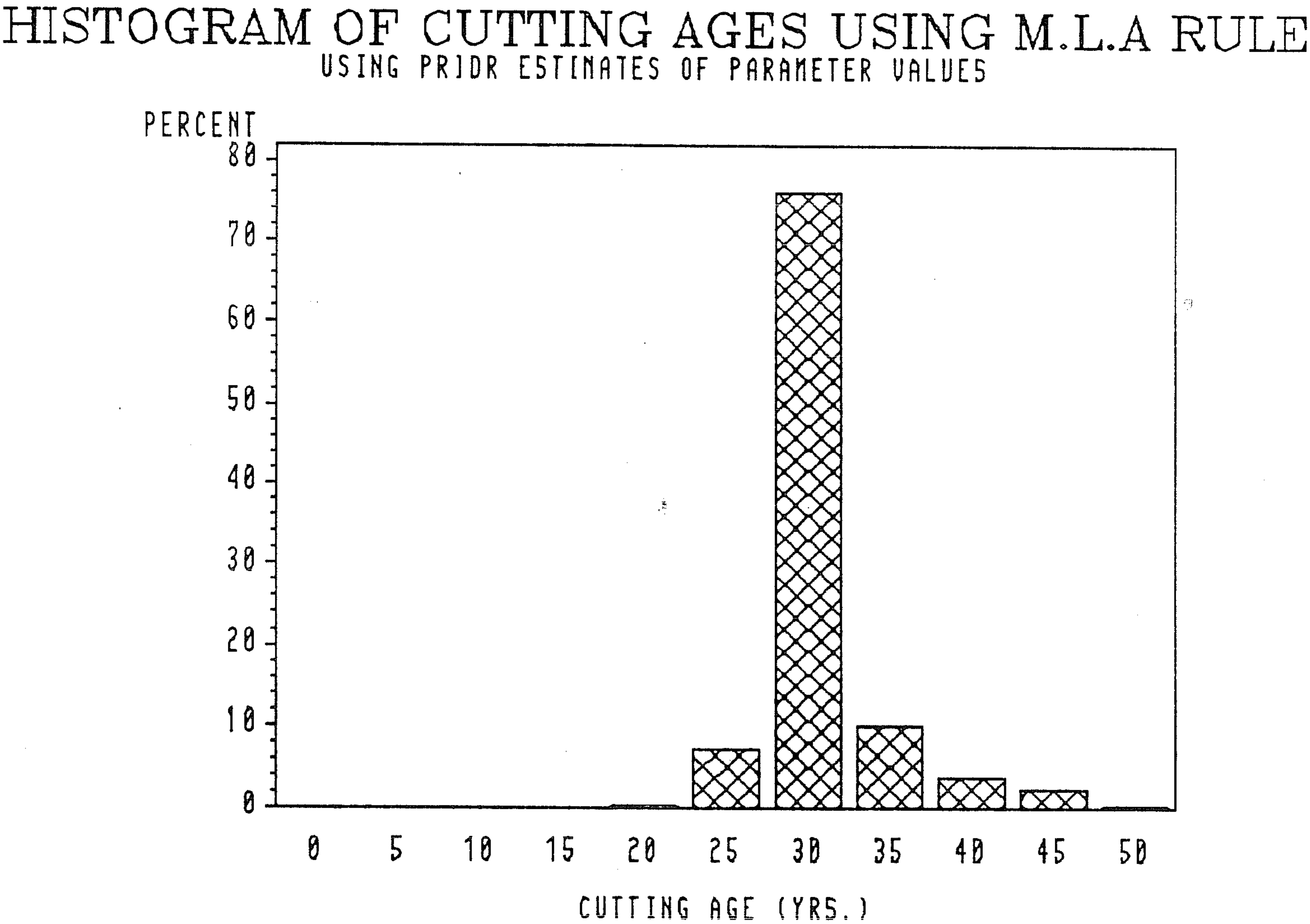
HISTOGRAM OF CUTTING AGES USING M.L.A RULE
USING PRIOR ESTIMATES OF PARAMETER VALUES
PERCENT
80
70
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35
40
45
50
CUTTING AGE (YRS.)

Figure 6

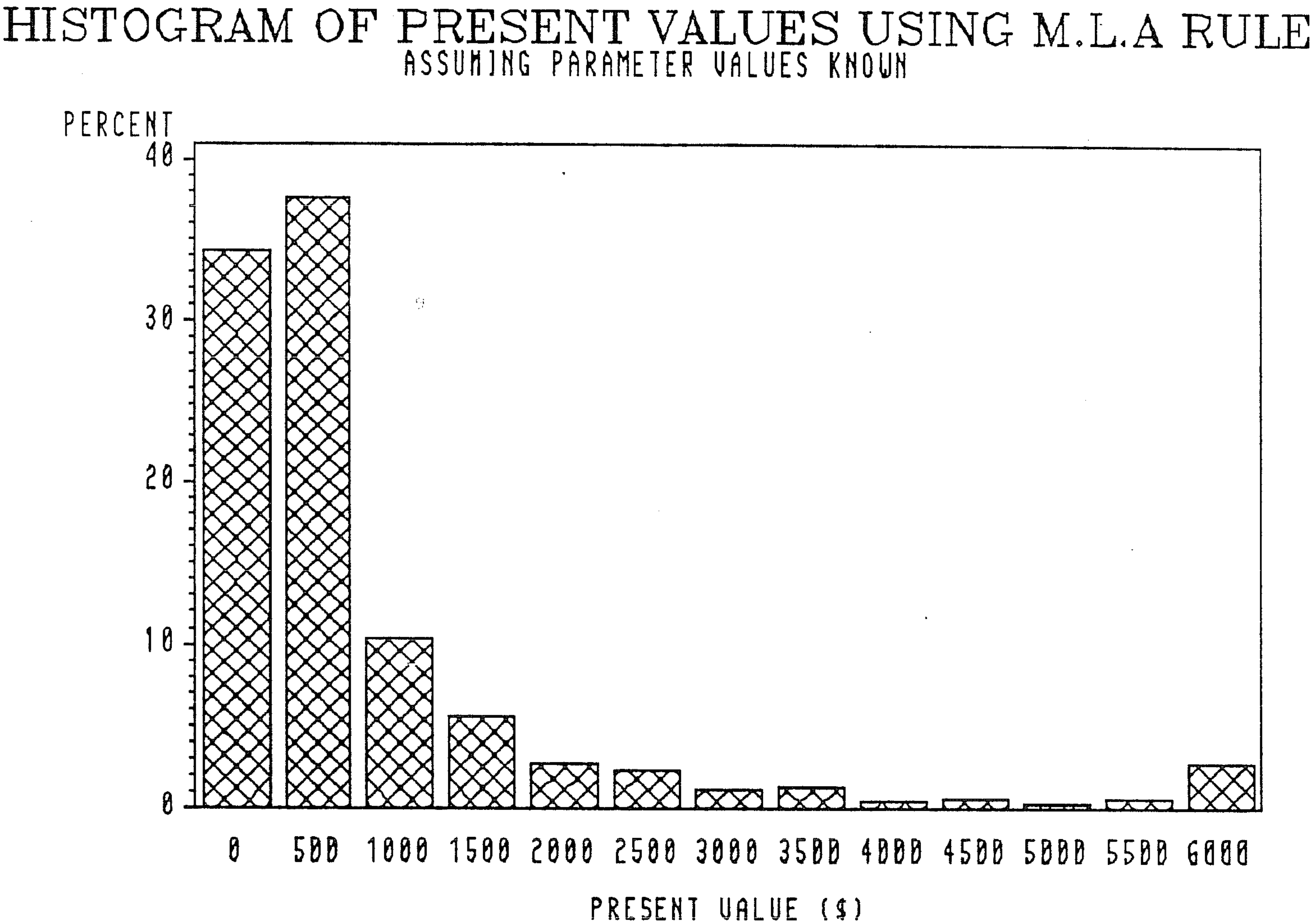
HISTOGRAM OF PRESENT VALUES USING M.L.A RULE
ASSUMING PARAMETER VALUES KNOWN
PERCENT
40
30
20
10
0
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
5500
6000
PRESENT VALUE ($)

Figure 7

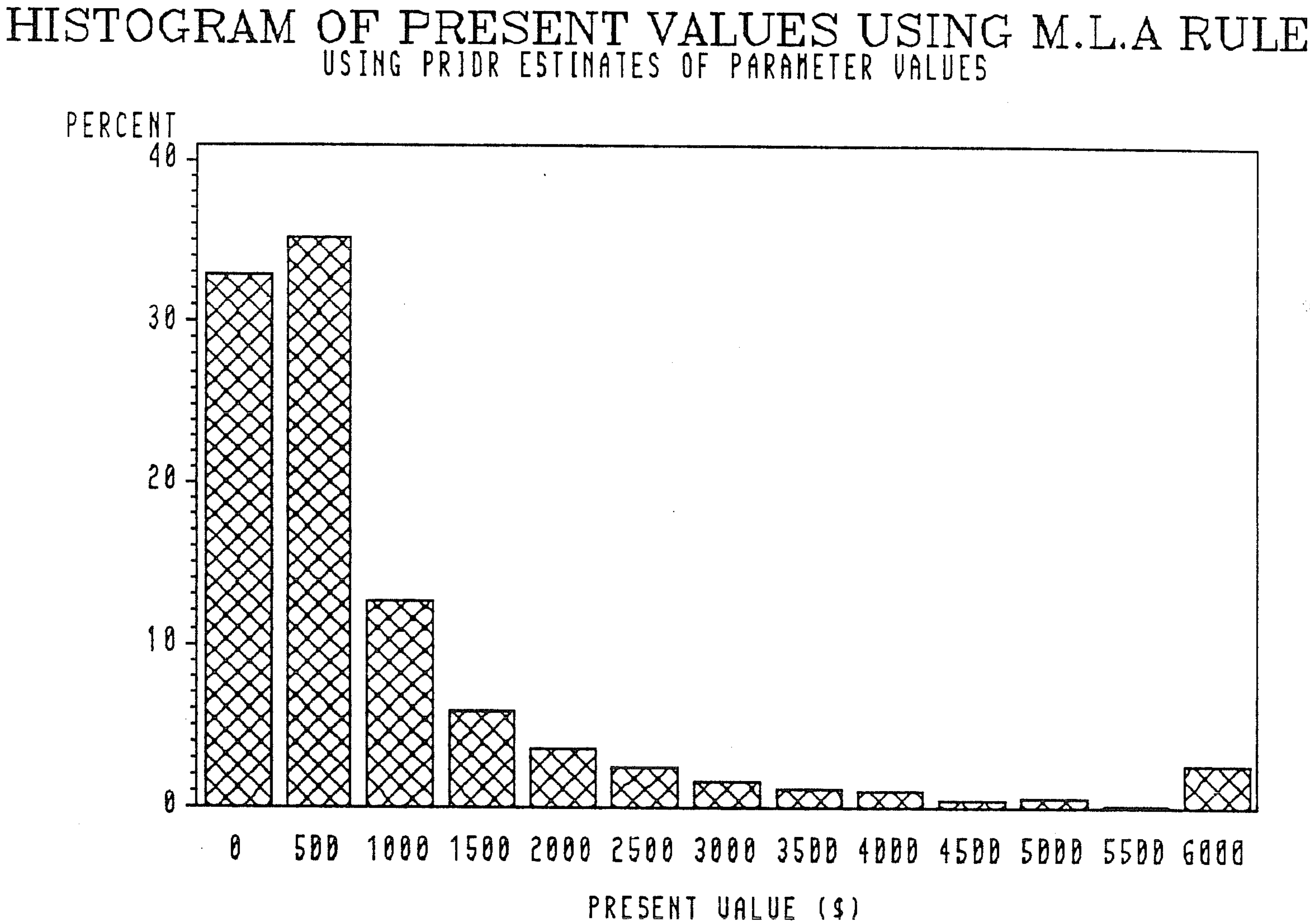
HISTOGRAM OF PRESENT VALUES USING M.L.A RULE
USING PRIOR ESTIMATES OF PARAMETER VALUES
PERCENT
40
30
20
10
0
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
5500
6000
PRESENT VALUE ($)

Figure 8

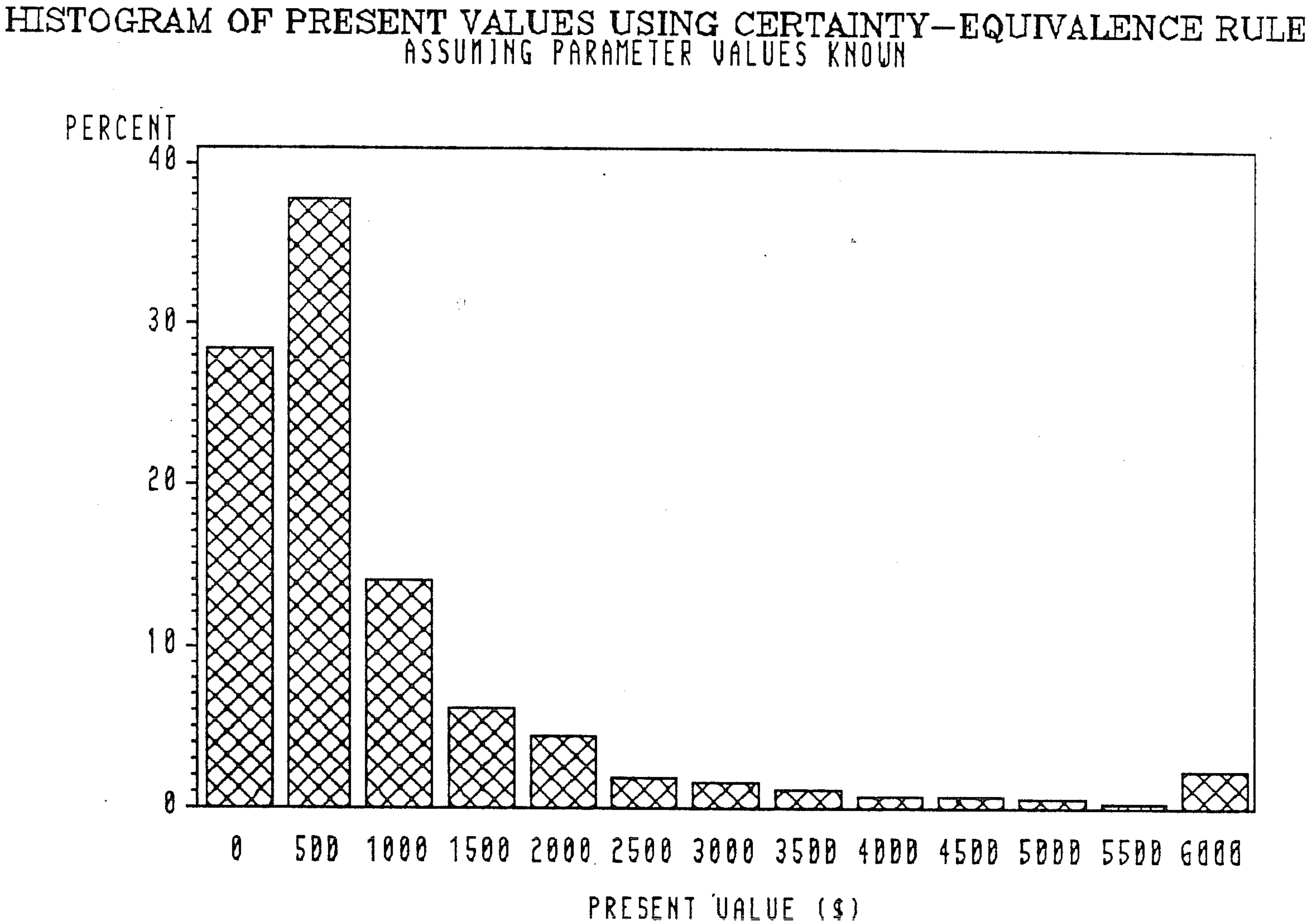
HISTOGRAM OF PRESENT VALUES USING CERTAINTY–EQUIVALENCE RULE
ASSUMING PARAMETER VALUES KNOWN
PERCENT
40
30
20
10
0
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
5500
6000
PRESENT VALUE ($)

Figure 9

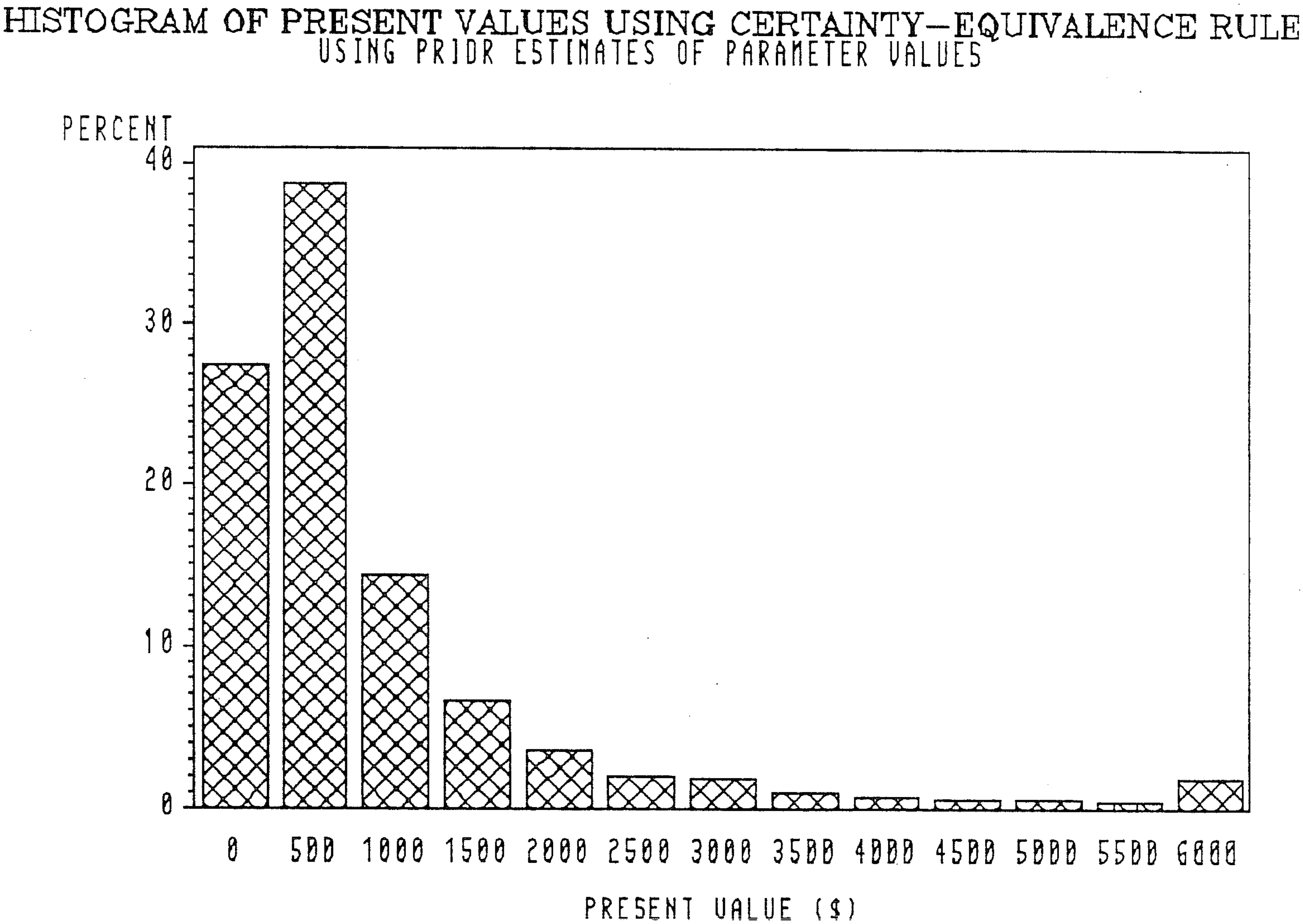
HISTOGRAM OF PRESENT VALUES USING CERTAINTY–EQUIVALENCE RULE
USING PRIOR ESTIMATES OF PARAMETER VALUES
PERCENT
40
30
20
10
0
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
5500
6000
PRESENT VALUE ($)

Figure 10