Curvature Gouge Detection and Prevention in 5-axis CNC Machining

by

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B.Eng., Shenyang Poly-technique University, 1982
M.A.Sc., Northeast University, 1987

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Abstract

Five-axis CNC machining presents high efficiency and unparalleled flexibility in the machining of complex curved surfaces. However, generation of gouge-free CNC tool path and cutter orientation in 5-axis CNC machining remains a challenge due to the complex nature of the geometry problem encountered and the wide variations of surface geometry. In particular, curvature gouge is the biggest obstacle that hinders the advantages of 5-axis CNC machining. At present to avoid curvature gouge, a ball mill cutter with simple cutter geometry is mostly used in machining complex curved surfaces, although this either leads to lengthy machining time or poor surface finish with larger cusps which require extensive amount of hand polishing later on. An end and/or torus mill cutters with better cutter-surface curvature match can considerably improve the efficiency of the machining and the quality of the machined surface. But generation of appropriate tool paths and orientations for the more complex cutter geometry in gouge-free 5-axis CNC machining of curved surface requires a better understanding and a rigorous model of cutter-surface interaction, which do not exist at present.

In this work, a rigorous mathematical model of cutter-surface geometry that facilitate better understanding on the interactions between various mill cutters, including ball, torus and end mills, and curved surface is introduced. The model is based on the new Euler-Meusnier Sphere (EMS) concept from a generic mathematical and geometric model of
the cutter and surface geometry. The EMS model determines the curvature gouge constraints with varying cutter size and maximum cutter tilting angle for any given surfaces. A generic, global curvature gouge detection and avoidance method for the 5-axis CNC machining of concave, curved surfaces has been introduced. The method also improves curvature match between the cutter and the machined surface by facilitating the use of torus and end mill cutters.

Computer simulation tests and real part machining have been carried out to assess the effectiveness of the new theory and newly introduced curvature gouge detection and avoidance criteria. Five-axis CNC machining experiments on curved surface, e.g. ellipse and exponent surfaces, are carried out. The machined surfaces following different tool path and cutter orientation strategies are measured using a CMM to appraise the real benefit of the introduced approach. The method has been applied to all three types of commonly used mill cutters: end, torus and sphere, for concave curved surfaces with limited curvature variation. The machining experiments have demonstrated the superior capability of the new method in providing guaranteed gouge elimination, better surface quality, and simple implementation, in comparing with existing 5-axis tool path and cutter orientation planning methods.

The new EMS concept and curvature gouge detection/elimination method form the foundation for generating highly efficient, high quality surface producing 5-axis CNC tool path, and cutter orientation planning, programming and optimization for machining curved surfaces.
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Thanks to Dr. Sanjeev Bedi and Mr. Armando Roman of University of Waterloo for their advices and support to use their 5-axis CNC machine to carry out my machining experiments.

Thanks to Minh Ly for his constant support.
Dedication

To my dear wife, Honglu Zhang, in memory of our pilgrim life in West Canada
Chapter 1  Introduction

Five-axis CNC machining presents high efficiency and unparalleled flexibility in the machining of complex curved surfaces. However, generation of gouge-free CNC tool path and cutter orientation in 5-axis CNC machining remains a challenge due to the complex nature of the geometry problem encountered and the wide variations of surface geometry. In particular, curvature gouge is the biggest obstacle that hinders the advantages of 5-axis CNC machining. An end and/or torus mill cutters with better cutter-surface curvature match can considerably improve the efficiency of the machining and the quality of the machined surface. But generation of appropriate tool paths and orientations for the more complex cutter geometry in gouge-free 5-axis CNC machining of curved surface requires a better understanding and a rigorous model of cutter-surface interaction, which do not exist at present.

In this work, a rigorous mathematical model of cutter-surface geometry that facilitate better understanding on the interactions between various mill cutters, including ball, torus and end mills, and curved surface is introduced. The model is based on the new Euler-Meusnier Sphere (EMS) concept from a generic mathematical and geometric model of the cutter and surface geometry. The EMS model determines the curvature gouge constraints with varying cutter size and maximum cutter tilting angle for any given surfaces. A generic, global curvature gouge detection and avoidance method for the 5-axis CNC machining of concave, curved surfaces has been introduced. The method also improves curvature match between the cutter and the machined surface by facilitating the use of torus and end mill cutters.

Computer simulation tests and real part machining have been carried out to assess the effectiveness of the new theory and newly introduced curvature gouge detection and avoidance criteria. Five-axis CNC machining experiments on curved surface, e.g. ellipse and exponent surfaces, are carried out. The machined surfaces following different tool path and cutter orientation strategies are measured using a CMM to appraise the real benefit of the introduced approach. The method has been applied to all three types of
commonly used mill cutters: end, torus and sphere, for concave curved surfaces with limited curvature variation.

1.1 Studies on 5-Axis CNC Machining

Mechanical parts with sculpture surfaces possess special aesthetic appeal and often a superior function. Today these parts have found wide applications in many areas, including aeronautical, automotive, marine, and consumer electronics products. CNC machining, especially 5-axis CNC machining, is the most efficient way to produce parts with sculpture surface, or the dies and moulds for making these parts. Numerous researches have been dedicated to improve the productivity and quality of the machined parts. At present active research areas in sculptured part CNC machining includes: tool path planning, real time tool path interpolation, feed rate adaptation, error modeling and compensation, gouge and interference detection and prevention, etc.

1.1.1 Tool Path Planning

Tool path planning is aimed at automatically generating CNC tool paths to produce good quality surface using short machining time.

Conventional tool path planning strategies include Iso-Parametric method (Broomhead, et al. 1986 and Choi, et al. 1988), Iso-Planar method (Huang, et al. 1994), and Iso-Cusp method (Lin, et al. 1996, Sarma, et al. 1997, Suresh, et al. 1994). These methods are simple and easy to implement, but they have some significant drawbacks. For instance, the Iso-parametric and Iso-Planar tool paths use fixed step size; as a result, the machined part has varying height cusps over the finished surface, which makes the surface very rough. For the Iso-Cusp tool path, the cutter feed direction depends on the generated tool path trend, which may not be in the optimal direction for the whole machined surface, so that machining efficiency can be compromised. For example, the Iso-Cusp tool path distribution shown in Fig.1.1 is much denser in the area where surface curvature is large; and tool path distribution is much sparser where surface curvature is small.
Chen et al. (2004) presents the Steepest Directed Iso-Cusp (SDIC) tool path planning method that combines the steepest directed tool path with the Iso-Cusp tool path, so that it has the joined advantages of both tool paths. It is characterized with both the high machining efficiency of steepest directed tool path and the ensured surface quality of the Iso-Cusp tool path. This balance of efficiency and quality is achieved in the way that the patch-wise quality surfaces produced from the Iso-Cusp tool path are optimally distributed over the workpiece surface domain with the guiding of the steepest directed tool path frames as shown in Fig.1.2. However, the application of the SDIC tool path planning on concave surface machining is yet to be developed.
Other advanced tool path planning strategies include the minimum distance algorithm (Masood, et al., 2002) and the machining potential field (MPF) algorithm (Chiou, et al., 2002).

1.1.2 Real Time Tool Path Interpolation

Tool path interpolation is the process that converts a designed tool path into machine axes motion driving commands. In sculpture surface machining, tool paths are designed in workpiece related coordinate system $R_w$; while the cutter movements are defined in the machine axes related coordinate system $R_m$. Tool path interpolation connects these two coordinate systems in order that machine commands are created to drive the cutter along the designed workpiece surface.

Conventional Tool Path Interpolation

Fundamentally in conventional tool path interpolation, tool paths are not expressed either implicitly or explicitly in relating coordinate system. The tool paths are usually approximated with discrete points; the cutter is driven by motion commands to visit these discrete points.

In conventional tool path interpolation, line segments or circular curves are used to join discrete points distributed over the workpiece surface, to approximate the tool path curves. Tool path interpolation includes two stages. Stage one is an off-line approximation process performed in the CAD software where the workpiece surface is created. The generated tool paths, such as the G-Code, are machine driving codes that connect the discrete points into piecewise linear or circular segments. Stage two involves the CNC machine controller translating the G-Code into sequential cutter motion commands to visit each discrete point.

Although simple and easy to use, conventional tool path interpolation methods have some disadvantages, including: the significant contour and orientation errors between the approximation line segments and the designed tool path curves, the considerable accelerations and decelerations in executing the line segment, the heavy
loading of communication and large memory requirements for the CNC controller due to the large number of piecewise segments and G-Codes commands.

**Real Time Tool Path Interpolation**

Real time tool path interpolation is a one-stage process; CNC machine control systems are installed with built-in free-form curve interpolators. Tool path geometry parameters are directly input to the CNC machine interpolating system; CNC machine controller interpolates the input tool path curve and generates machine axes driving commands on site. Accordingly machined surface accuracy and quality are greatly improved; machining time is significantly reduced. Also, with the elimination of large quantity of line or circular segments and related G-Codes, communication and computer memory space is drastically reduced, Fig.1.3.

![Designed Tool Path](image1)

![Real Time Interpolation](image2)

**Figure 1.3 Tool Path Interpolation**

Among the earliest real time tool path interpolation schemes are the PH (Pythagorean Hodograph) curve based methods. Farouki *et al.* (1998) and Tsai *et al.* (2001) present real time tool path interpolation algorithms by interpolating tool paths through the PH curves. The PH curve serves as the bridge that connects the tool path defining space $R_w$ to the machine coordinate system $R_m$. Thus interpolated tool paths possess many dynamic advantages such as smooth acceleration and deceleration, as well as improved surface quality. The single parameter of time $t$ in the PH curve also facilitates the time dependent feedrate interpolating algorithms in the real time interpolation. Variable feedrate adaptation strategy can be performed in the PH curve.
based real time tool path interpolation algorithm (Farouki et al. 1998). Thus, constant material removal rate and stabilized machining conditions are realized.

Omirou (2003, 2004) presents real time tool path interpolation methods based on the Danielson step selection principles. The interpolated tool paths are implicitly expressed in the Cartesian coordinate system as locus, or boundaries and intersections. The Danielson’s rule is used to guide the tool path interpolation step sizes, so that the tolerance of the interpolated tool path is maintained within accuracy.

Cheng et al. (2002), Tikhon et al. (2004), Tsai et al. (2003) and Xua et al. (2003) present some other real time tool path interpolation algorithms, such as Iso-photos and NURBS tool paths. These real time tool path interpolation methods also explore the capability of feedrate adaptation in real time tool path interpolations. A common feature of these methods is that they focus the feedrate adaptation on the cutter contact point (CCP), rather than on the cutter location point (CLP). So, the feedrate adaptation strategy fits the actual machining conditions.

A fundamental characteristic in the real time tool path interpolation algorithms is that the tool path curve has to be explicitly defined in the machine coordinate system. The using of PH curve, Iso-phote curve, or intersections between surfaces, all serves to this purpose.

1.1.3 Feedrate Adaptation

Feedrate affects both machining efficiency and machined surface quality. High feedrate leads to shorter machining time, thus higher machining efficiency; however, high feedrate often induces unstable machining conditions because the fast moving tool can incite enhanced resistance. The optimal feedrate produces suitable balance between machining efficiency and machined surface quality.

Constant feedrate in surface machining induces variable material removal rate, which in turn induces varying machining forces. Thus, constant feedrate is not optimal in machining. Many researchers have noticed this fact and worked on developing variable feedrate adaptation methods.
Timar et al. (2004) presents a time optimal feedrate adaptation algorithm that pursues shortest traversal time along a designed tool path curve. This algorithm is based on the so called “bang-bang” control theory for driving rockets where the time-optimal solution of moving an object is achieved by driving the object at the limit of one of the motive-force constraints at each instant during the traversal. This feedrate scheduling method leads to high machining efficiency.

Fleisig et al. (2001) presents a feedrate scheduling method that maintains constant feedrate and reduced angular acceleration for moving a cutter along tool path curves. This algorithm is developed to deal with the cutter axis orientation variation induced problems in 5-axis CNC machining. The orientation variation of cutter axis introduces inertia forces that affect machining stability. To stabilize machining situations, cutter axis angular acceleration is investigated, and adaptation algorithm is introduced. Xu (2003) also investigates feedrate adaptation in respect of liner and angular motion of the cutter. Relations between linear and angular feedrate are discovered, and hybrid feedrate scheduling algorithms, such as constant linear feedrate with constrained angular feedrate, or constant angular feedrate with constrained linear federate, are presented.

Ko et al. (2003) presents a feedrate optimization algorithm to maintain stable machining forces. This feedrate adaptation scheme is based on an off-line model usually used in virtual machining simulation practice.

The development trend in feedrate adaptation study is to relate the feedrate adaptation with the real time tool path interpolation. This is reasonable since the real time tool path interpolation often depends on the single parameter of time \( t \) that also affects the cutter feed.

### 1.1.4 Machining Error Compensation

Machining errors in sculpture surface production are usually represented by shape deformation of the machined parts; the shape deformation is the result of many error contributing factors, including CC point position errors, cutter axis orientation errors, cusp height variations, and thermal effect related errors. The machining error
compensation deals mainly with the CC point position errors. A usual practice is to predicate the CC point position errors through a machining system error model, then, use the predication to modify cutter locations so that the predicated errors can be compensated. The basis for machining error compensation sets on the construction of a machining system error synthesis model that can reasonably approximate the actual machining condition. In order to verify the constructed error synthesis model, dependable error-measuring devices are also required.

Yuan et al. (1998) presents some general descriptions of machining error compensation techniques, such as formulation of error synthesis models, mapping of machine errors, optimal modeling, thermal error mode analysis, optimization of sensor location, cutting force induced error compensation, etc.

Bohez (2002) analyzes systematic errors of 5-axis CNC machining process, and presents an error compensation method based on kinematics transformation techniques. Systematic errors are compensated either directly through inverse kinematics equations or through total differentials of the inverse kinematics relations. Lo (2002) also presents real time machining error compensation method based on kinematics transformation techniques. The cutter contact point deviation error, cutter axis orientation error and cutting point track-lag error are directly eliminated through inverse kinematics transformation.

As has been stated, machining error is the combined result of many sources; CC point position error is only one of them. Other important error contributing factors, such as the cutter axis orientation, should not be overlooked. The cutter axis orientation determines the actual cutting profile of the cutter, thus the generated surface shape. Unfortunately, these factors have not been studied in many of the current error compensation schemes.

1.1.5 Gouge Detection and Avoidance

5-axis CNC machining, with its additional two rotations for tool axis orientation, has incomparable advantages over 3-axis CNC machining. The advantages include extended accessibility, improved surface quality and enhanced machining efficiency.
Along with the benefits from additional axis rotations, comes the complexity of orientating cutter axis for the 5-axis CNC machine. Unsuitable cutter axis orientation will either prevent the full usage of 5-axis machine or induce gouge and interference between cutter and workpiece surface.

Vickers et al. (1989) introduce the concept of effective cutter radius to represent the projected cutting edge curve on the plane perpendicular to feed direction for analyzing machining situations of end mill cutter. Yu et al. (1996) and Lee et al. (1997) apply this concept of effective cutting shape in estimating cutter surface curvature on the normal plane perpendicular to feed direction for gouge detection. The effective cutting edge is the projection of end mill cutter end face on a plane perpendicular to feed direction. By comparing curvatures of the effective cutting edge with the curvature of the workpiece surface on the projection plane, curvature gouge problem is estimated. Similarly, Choi et al. (1999) and Park et al. (2003) present profile-matching methods for curvature gouge detections on the plane perpendicular to feed direction; instead of using the projected effective cutting edge, they use the active cutting surface curvature on this plane.

These kinds of curvature gouge detection schemes can be classified as one-dimension method; and they have the same drawbacks of confining gouge detection on a single plane, leaving potential gouge problem elsewhere unattended.

Rao et al. (1997) examine the principal axis method (PAM), which is an application of the active cutting edge method. PAM, based on the curvature match concept proposed by C. G. Jensen et al. (1993), can provide curvature gouge avoidance solutions for certain machining set up. PAM, pursuing curvature match to enhance material removal in machining, is not exactly a strategy for curvature gouge detection and prevention. However, in certain conditions, such as when the machined surface has uniform principal curvatures and the curvature match is realized on the normal plane on which the machined surface maximal principal curvature exists, PAM can lead to curvature gouge avoidance. But, when curvature match happens on a plane on which the curvature is not the maximum, curvature gouge can happen. More interestingly, when the workpiece surface has non-uniform principal curvatures,
whether PAM leads to curvature gouge avoidance is uncertain. Generally, PAM cannot be used as a curvature gouge detection method. Besides, as a method based on the active cutting edge conception, PAM is not apposite to use for end mill cutter of which the cutter principal curvatures are not applicable.

Yoon (2003) presents the Dupin's indicatrix method for curvature gouge detection. The Dupin's indicatrix embraces the normal curvature information for the vicinity of given point on surface. By comparing the Dupin's indicatrices of cutter and workpiece surface, normal curvature gouges problems about the CC point could be identified. Similarly, Yoon (2003) and Li et al. (1995) present gouge detection methods, for the vicinity of given point, by checking intersection curves of cutter end plane with the workpiece surface. These curvature gouge detection algorithms study gouge problems on a plane about the CC point, they are classified as two-dimensional method for curvature gouge detection. Curvature gouge problems, however, are issues in the three-dimensional space.

Yu et al. (1996) and Lee (1997) present multi-point method (MPM) for curvature gouge avoidance. In MPM, curvature gouge detection is first carried out at an initial position, i.e. current CCP; then the cutter is re-orientated until it finds another contact point with the workpiece surface without gouge and interference.

Gray et al. (2004) present the rolling ball method (RBM), and later its improved version, for curvature gouge avoidance. A rolling ball is an arbitrary virtual ball that can freely roll over the workpiece surface and kept closest contact with the surface. This rolling ball can be used to confine a physical cutter, so that the cutter would not gouge the workpiece surface. An optimal gouge free cutter size and its orientation are determined when the cutter is confined in the rolling ball. Although a rather straightforward method, the determination and defining of the rolling ball demands further analysis and details. Besides, the principal theory background of the rolling ball and its relation with surface curvatures need to be explained.

Other methods dealing with curvature gouge problems include: the configuration space method presented by Jun (2003). The configuration space is composed of cutter orientation parameters; within this space, tool orientations are ensured to be
gouge free. The bounding box and octree method presented by Ding et al. (2004), and the Minkowski method presented by Tangelder et al. (1998). These methods use spatial research concepts for curvature gouge detection.

1.2 Research works on Curvature Gouge Detection and Prevention

5-axis CNC machining for producing parts with sculptured surface has become more and more popular because of its high efficiency and flexibility. In general, 5-axis CNC machining provides greater accessibility for the cutter to reach its material removing positions and offers better chances for gouge avoidance in using more productive cutting tools, such as the end and torus mill cutters; so that efficient material removal and high quality surface finish can be achieved. Meanwhile, the addition of the two extra degrees of freedom to the traditional 3-axis CNC machining has also introduced considerable new challenges. In order to take the full advantages that a 5-axis CNC machine can offer in machining sculpture parts with complex shape, the optimal orientation of the cutter at each instance of the cutting process needs to be determined, in addition to the optimal trajectory of the cutter – the conventional 3-axis tool path.

Automated generation of 5-axis CNC tool paths for machining sculpture parts is both an important area in manufacturing due to the extensive applications of sculptured surface in various consumer products, and a major technological challenge due to the complex nature of related technical problems. In CNC machining, tool path is the controlled trajectory of cutter motion. The focus of automated CNC tool path generation is to create the most efficient cutter motion trajectory to machine high quality surfaces derived from the CAD model of the machined part. The meaning of CNC tool path has been extended in 5-axis CNC machining, to include both cutter reference point locations and cutter axis orientations at each instance of the cutting process. Determination of control parameters for cutter axis orientations is the most challenging task in 5-axis CNC tool path programming due to the lacking of knowledge and generic guidelines. To facilitate automated tool path generation, the introduction of generic principles that guide the determination for cutter orientation during 5-axis CNC machining becomes critical.
The planning and programming of CNC tool path and cutter orientation in the finish machining of a sculpture part involve different strategies and considerations based on the geometry of the surface and the cutter to be used. A generic sculptured surface can be classified into convex, concave and saddle shapes, as well as surfaces with uniform or non-uniform curvatures. In recent years, extensive efforts have been devoted by the author’s group to generate optimized CNC tool paths for various convex surfaces, including the Steepest-Directed (SD) Tool Path (Chen et al., 2003 and Chen et al., 2004), the Integrated Steepest-Directed and Iso-Cusped (SDIC) Tool Path (Chen et al., 2003), and their extension from 3-axis CNC milling to 3 ½ ½ axis and 5 axis CNC machining (Chen et al., 2003). In the latter case, both tool path and cutter orientation are considered to achieve maximum material removal rate while maintaining high and uniform surface quality. However, these strategies, although work well with convex surfaces, may cause curvature gouge problems in the machining of concave and saddle surfaces. To improve productivity, highly efficient, large diameter flat and torus mill cutters are used. Due to the miss match between the curvatures of the cutter and the machined surface, these methods may often lead to curvature gouge and/or collision, like most of other existing tool path generation algorithms.

One of the major considerations in determining cutter axis orientation in machining concave or saddle surfaces is to avoid gouge between cutter and the workpiece surface. Gouge indicates that cutter’s cutting surface curvature conflicts with the desired shape of workpiece surface so that the cutter over-cuts workpiece surface beyond the desired shape. The traditional, quick and easy approach to avoid curvature gouges for curved surface machining is to use a small diameter ball mill cutter for an arbitrary surface. However, the setbacks of this easy approach include reduced rigidity of machining structure and high cusps over the machined surface. These, in turn, lead to low productivity and poor surface quality. Prolonged machining time is also a significant phenomenon for using ball mill cutter because of the small side steps. Nevertheless, as a solution to no solution, this approach is widely used in industry at present.

Curvature gouge problem can be avoided by properly orienting the cutter axis without the need of using a small diameter ball mill cutter. In recent years, extensive studies have
been carried out on the development of new methods for determining the gouge-free cutter orientation parameters in 5-axis CNC machining; no generic solution, however, has been established. A new method that can effectively and accurately identifies curvature gouge in all occasions and facilitates the search of optimal tool path/orientation for efficient and curvature gouge-free machining sculptured surface is demanded.

Researches on curvature gouge detection and avoidance have been carrying out for some decades. Due to the continuous efforts of numerous researchers, some commendable methods for curvature gouge detection and prevention have been found. These traditional approaches can be classified as one-dimensional (1D) and two-dimensional (2D) solution methods based on the main variable(s) being considered.

At the early stage of curvature gouge investigations, studies mainly focus on one dimension or one-control variable solutions, mostly concerning with the cutter feed direction. These 1D solution methods do not provide satisfactory resolutions for curvature gouge-free machining because of potential gouge problems that are not dealt with. More advanced techniques in dealing with curvature gouge problems, the 2D solution methods, are thus introduced. The 2D solution methods investigate curvature gouge problems on the tangent plane of the machining surface at the CC point, leading to more rigorous resolutions for curvature gouge detection and elimination. However, limitations still exist in the 2D approaches since curvature gouge problem is actually an issue in the three-dimensional space.

1.2.1 1D Solution Methods for Curvature Gouge Detection

The 1D solution methods for curvature gouge detection and prevention study curvatures of cutter and workpiece surface on one or two normal planes. Mostly, these methods analyze related curvatures on the plane perpendicular to cutter feed direction; curvature gouge avoidance solution is made only for the concerned direction. Representatives of the 1D solution method include the effective cutting shape method, the active cutting edge method, and the principal axis method (PAM).

The concept of effective cutting shape is among the first apparatuses to deal with curvature gouge problems. Effective cutting shape represents the projection of cutter
end face on the plane perpendicular to feed direction at the CC point (Vickers et al. 1989). Yu et al. (1996) and Lee et al. (1997) use effective cutting shape in estimating cutter surface curvature on the normal plane perpendicular to feed direction for gouge detection. Effective cutting shape method of curvature gouge detection works well for end mill cutter when tool path are straight lines where the cutter orientation is fixed. However, for curved tool path where cutter orientation varies with surface curvatures, this method of effective cutting shape induces dubiousness because of its using the projection of cutting edge to replace the actual cutting edge in gouge detection. Analytical basis for the hypothetical comparison of surface geometries locating on different planes in curvature gouge detection demands verifying. Li et al. (2005) analyze the errors resulted from using the effective cutting shape method when tool path is curved. Furthermore, the effective cutting shape method does not provide curvature gouge detection solutions for torus mill cutter of which the cutter end face is not relevant to the material removal.

Choi et al. (1999) and Park et al. (2003) propose to use the active cutting edge, i.e. the intersection of cutter surface with workpiece normal plane perpendicular to the feed direction at the CC point, for curvature gouge detection. Lee (1997) also use this concept of active cutting edge on two planes, i.e. perpendicular to and along with cutter feed direction, for curvature gouge detection. Active cutting edge method is reasonable to use for machining situations where cutter and workpiece surface share the same tangent plane at the CC point; its gouge detection solutions are still limited on the one or two selected normal planes. Moreover, when cutter and workpiece surface do not share a tangent plane at the CC point, such as when using an end mill cutter, the active cutting edge method can not be used; otherwise, it induces errors in detecting curvature gouge problems due to its overlooking at the trailing effect of the cutter end face.

Generally, the 1D solution methods of curvature gouge detection provide limited resolutions for curvature gouge avoidance on one or two normal plane(s) at the CC point. These solution methods do not provide ensured curvature gouge-free machining conditions.
1.2.2 2D Solution Methods for Curvature Gouge Detection

The 2D solution methods for curvature gouge detection examine normal curvatures represented on the common tangent plane shared by the cutter and workpiece surface. These solution methods provide inclusive normal curvature gouge avoidance resolutions. The basis of the 2D solution method is the Dupin’s indicatrix that is the 2D expression of the Euler theorem from differential geometry.

Yoon (2003) presents the Dupin’s indicatrix method for curvature gouge detection. The Dupin’s indicatrix exhibits comprehensive normal curvature distribution on the tangent plane at given point. By checking the interference between Dupin’s indicatrices of cutter and the machined surfaces, normal curvature gouge problems can be identified. To simplify the calculation of Dupin’s indicatrix for workpiece surface, Yoon et al. (2003) propose an approximating method to determine the Dupin’s indicatrix for arbitrary surfaces, though it unavoidably introduces added inaccuracies.

Dupin’s Indicatrix

Dupin’s indicatrix is derived from the Euler equation, Eq.1.

\[ k_\delta = k_{\text{max}} \cos^2 \delta + k_{\text{min}} \sin^2 \delta \]  

(1.1)

where \( k_\delta \) represents normal curvature in the direction of angle \( \delta \) to the maximal principal curvature \( k_{\text{max}} \); \( k_{\text{max}} \) and \( k_{\text{min}} \) are principal curvatures.

When substituting \( x \) and \( y \), Eq.1.2, to equation (1.1), the Dupin’s Indicatrix is derived as follows, Eq.1.3 and Fig.1.4.

\[
\begin{align*}
  x &= \frac{\cos \delta}{\sqrt{k_\delta}} \\
  y &= \frac{\sin \delta}{\sqrt{k_\delta}} \\
  \frac{x^2}{r_{\text{max}}^2} + \frac{y^2}{r_{\text{min}}^2} &= 1
\end{align*}
\]  

(1.2)  

(1.3)
Dupin’s indicatrix, Eq. 1.3, depicts the local normal curvature $k_\delta (= \pm \frac{1}{\rho_\delta} = \pm \frac{1}{r^2})$ at a given point of surface, with respect to angle $\delta$ between the principal normal curvature $k_{max}$ and the normal curvature in the concerned direction. As a quadratic curve, Dupin’s indicatrix may take the forms of elliptic, hyperbolic and parallel lines. For concave surface, such as torus mill cutter, the Dupin’s indicatrix is an ellipse. When Dupin’s indicatrices of workpiece surface and cutter are known, they can be used to detect normal curvature gouges. The interference between these Dupin’s indicatrices indicates curvature gouges, Fig. 1.5.

Dupin’s indicatrix is a good device for normal curvature gouge detection. However, to obtain the Dupin’s indicatrix is not an easy task because of the need to calculate curvature radii of $\rho_{\text{min}}$, $\rho_{\text{max}}$, and $\rho_n$ over the surface. Farin (1988) and Yoon et al. (2003) suggest the following approximation method for determination of the Dupin’s indicatrix.

First, approximate the workpiece surface by second order Taylor’s expansion:
\[ \mathbf{r} = \mathbf{r}_e + \frac{d\mathbf{r}}{dt}t + \frac{1}{2} \frac{d^2\mathbf{r}}{dt^2}t^2 \]  

(1.4)

Using a plane, parallel to the tangent plane at a distance of \( d \) to given CC point, to intersect the approximated workpiece surface. The curve obtained is treated as the Dupin’s indicatrix, to a scale factor \( m \), i.e. the obtained intersection curve’s polar radius \( r' \) is a scaled \( r \), Eq. 1.5.

\[ r' = r \cdot m = \sqrt{2d \cdot r} = \sqrt{2d \rho_s} \]  

(1.5)

where \( m = \sqrt{2d} \) is the scale factor.

The Dupin’s indicatrix obtained this way can be very rough. First, surface is approximated with the second order Taylor’s expansion. The function (Eq.1.5) introduces further approximation as shown in Fig.1.6 and Eq.1.6.

![Figure 1.6 Calculation of Dupin’s Indicatrix](image)

In Fig.1.6, the normal curvature radius \( \rho_n \) of a given curve segment can be calculated as:

\[ \rho_n = \frac{1}{2} \frac{\Delta x^2}{\mathbf{n} \cdot \Delta \mathbf{x}} \]  

(1.6)

where \( \Delta \mathbf{x} \) represents the chord length vector; \( \mathbf{n} \) represents the normal curvature direction.
For a parabolic curve (Fig. 1.7), Eq. 1.6 can be approximated by replacing \( \Delta x \) with \( \delta \). Here, \( \delta \) is the half chord length of the parabola curve at the distance \( d \). Therefore, in obtaining the approximated Dupin’s indicatrix radius, Eq. 1.6, the half chord length \( \delta \) is used for \( \Delta x \) and get the following expression, Eq. 1.7

\[
\rho_n = \frac{1}{2} \frac{\delta^2}{d}
\]

(1.7)

where \( \delta \) represents half chord length of parabola curve; \( d \) is the distance from intersection plane to the curve’s tangent plane at CC point.

![Parabola Curve Normal Curvature](image)

**Figure 1.7 Parabola Curve Normal Curvature**

In Eq. 1.7, let \( d = \frac{1}{2m^2} \), to get:

\[
\rho_n = (m\delta)^2 = r^{12}
\]

(1.8)

or equivalently:

\[
r' = \sqrt{\rho_n}
\]

(1.9)

Eq. 1.8 and 1.9 demonstrate that the proposed intersection curve’s half chord length \( \delta \) has the nature of Dupin’s indicatrix polar radius \( r \). It possesses the similar feature to the normal curvature radius \( \rho_n \). The only difference between \( \delta \) and \( r \) is a scale factor \( m (= \sqrt{2d} \) ). So, by multiplying the factor \( m \) to the intersection curve’s half chord length \( \delta \), the Dupin’s indicatrix is approximated. However, thus obtained Dupin’s indicatrix introduces inaccuracies resulting from the approximations, both in the Taylor’s second order approximation for the surface and the approximated
curvature $\rho_n$, from using $\delta$ for $\Delta x$, as well as using parabola to represent the curvature circle.

Dupin's indicatrix, Fig.1.4 and Eq. 1.3, is a comprehensive representation of surface normal curvatures on the tangent plane at the concerned point $C$.

The basis of the 2D solution methods rests in the Dupin's indicatrix that represents normal curvature distributions on the tangent plane at the CC point; these solution methods are capable of identifying and eliminating normal curvature gouges in the vicinity of the CC point. However, curvature gouge problems are caused not only by the normal curvature conflicts but also by the osculating curvature interference. So, the 2D solution methods do not guarantee to provide curvature gouge-free machining even though they do provide normal curvature gouge-free solutions. When normal curvature gouge exists, Dupin's indicatrix method can reveal it, when osculating curvature gouge exists, but normal curvature conflicts do not, Dupin's indicatrix will miss it. In other words, when Dupin's indicatrix predicts gouge problems, there might be gouges; but when Dupin's indicatrix do not predict gouge problems, gouge could still exist. Experiment from this research has identified situations where the normal curvature gouge problem is eliminated, but the osculating curvature gouges exist as shown in Fig.1.5.

![Figure 1.8 Dupin’s Indicatrix for Torus Mill Cutter](image)

Figure 1.8 Dupin's Indicatrix for Torus Mill Cutter

In Figure 1.8 (b), x-axis represents the direction of the minimal principal curvature ($k_{min} = 0$) of workpiece surface. The parallel lines represent the Dupin's Indicatrix of the workpiece surface. Axes of $x_i$ and $y_i$ represent the minimal and
maximal principal curvature directions of the torus mill cutter. The ellipse represents the Dupin's Indicatrix of the cutter. The Dupin's Indicatrices of cutter and workpiece surface are tangent at point C so that the optimal normal curvature match is accomplished, that is, on the normal plane represented by line $L_{oo}$, normal curvatures of cutter and workpiece surface are equal. Cutter feed is in the direction perpendicular to the curvature match normal plane which is not coincides with any of the principal curvature directions. $\sqrt{R_{xx}}$ and $\sqrt{R_{yy}}$ are typical points corresponding to the minimal and maximal principal curvatures, on the Dupin's Indicatrix of the workpiece. $\sqrt{R_{x}}$ and $\sqrt{R_{y}}$ are typical points corresponding to the minimal and maximal principal curvatures, on the Dupin's Indicatrix of the cutter.

In Fig.1.8 (a) shows the cutter over-cuts the surface resulting from the osculating curvature conflicts. Normal curvature match of cutter and the workpiece surface is perfect which is shown by the matching Dupin's indicatrix of involving parts. This example demonstrates that the osculating curvature conflicts can exit independently to the normal curvature conditions.

Normal curvature can represent osculating curvature in given tangent direction; and such representation is to use a 2D circle to represent a 3D sphere. However, pursuing normal curvature match, and forgetting the original osculating curvature does not work since degenerating a sphere to a circle omits important spatial information; so that the solution is 2D.

Finally, the 2D solution methods are not suitable to use when cutter and workpiece surface do not share a tangent plane at the CC point, such as when using an end mill cutter, Fig.1.9.
Figure 1.9 Dupin’s Indicatrix for End Mill Cutter

In Fig.1.9 (a) shows an end mill cutter over-cuts the surface. This end mill cutter has a radius smaller than the cylindrical workpiece surface and feeds in the direction of the surface symmetrical axis. Using the Dupin’s indicatrix method, the gouge problem cannot be revealed. In Fig.1.9 (b), axes $x$ and $x_t$ represent the minimal principal curvature directions of cutter and workpiece surface. Since the minimal principal curvatures are all zeros, their reverses are infinity, so that axes $x$ and $x_t$ are perpendicular to $y$-axis, pointing to infinity. Dupin’s Indicatrices of the cutter and workpiece surface are represented by parallel lines. $\sqrt{R_{xx}} (\rightarrow \infty)$ and $\sqrt{R_{yy}}$ are typical points corresponding to the minimal and maximal principal curvatures of the Dupin’s Indicatrix on the workpiece surface. $\sqrt{R_{xx}} (\rightarrow \infty)$ and $\sqrt{R_{yy}}$ are typical points corresponding to the minimal and maximal principal curvatures of the Dupin’s Indicatrix on the cutter surface. When the cutter radius is smaller than that of the cylinder surface and their minimal principal curvature directions are coincident, there is no interference between the Dupin’s Indicatrices of cutter and the workpiece surface. However, gouge problem still exists as shown in the Fig.1.9 (a).

Li et al. (1995) and Yoon (2003) present a numerical simulation scheme for curvature gouge detection. This method is also viewed as 2D solution because it checks curvature gouge problems on a special plane, i.e. the cutting edge plane. The cutting edge plane contains end mill cutter’s end face; it intersects the workpiece surface to form an intersection curve. This intersection curve is expressed, approximately, by second order Taylor’s expansion. Curvature gouge problems are predicted when the cutting edge circle interferes with the intersection curve on the
cutting edge plane. A problem in applying this method is how to decide the orientation of the cutting edge plane; the orientation of the cutting edge plane can be vital in revealing gouge problems. Also, considerable calculations will be involved in applying this method both for the trial orientation of the cutting edge plane and the identifying of possible interference problems on the cutting edge plane. Finally, for torus mill cutter, the decision of the cutting edge plane will be more difficult.

1.3 Outline of the Thesis

To address the issues related to curvature gouge detection, this research introduces a new mathematical model and a three-dimensional solution method based on this model for accurately detecting various curvature gouges and for creating gouge-free CNC tool paths. This study focuses on the more challenging five-axis, gouge-free concave surface machining, since the cutter orientation issue in five-axis, gouge-free convex surface machining is relatively straightforward and is better understood. This work comprises seven chapters covering the following five main parts:

- Surface curvature geometry expression, the Euler-Meusnier Sphere
- Milling cutter curvature characteristics
- Curvature gouge-free criterion in surface machining
- Milling cutter orientation and realization in 5-axis CNC machine
- Experiment results and analysis.

Chapter one presents an overview of concurrent CNC machining studies. Curvature gouge detection and prevention techniques are detailed. Chapter two lays the theoretical preparation for curvature gouge analysis. Necessary mathematic functions and theorems are introduced. Chapter three analyzes milling cutter curvature characteristics and their performance in surface machining. Chapter four introduces curvature gouge-free criterion for surface machining. Chapter five defines milling cutter orientation parameters, analyses 5-aixs CNC machine transformation mechanisms. Chapter six
presents simulation and real machining experiment results and analysis. Chapter seven summarizes this work.
Chapter 2 Modeling of Surface Curvature Geometry

Curvature gouge is a problem happening in the three-dimensional (3D) space. A method that can effectively detect and eliminate curvature gouge in curved surface machining has to be a 3D solution. The 3D solution should be based on a full understanding of the surface curvature geometry in the 3D space. This leads to the introduction of the new concept and mathematical model of Euler-Meusnier sphere, which is the combination of Euler and Meusnier theorems from differential geometry.

2.1 Euler Theorem
Given a point on a curvature continuous \((G^2)\) surface, there exists a surface normal vector \(\vec{e}\) and a tangent plane \(P\), Fig.2.1. A normal plane that goes through the surface normal \(\vec{e}\) intersects the surface to form a normal curve (or normal section). For any given point on the surface, there exist unlimited numbers of normal curves (around 360\(^\circ\)). The normal curves with the maximum and the minimum curvatures form the principal curvatures of the surface at the point, represented by the maximal and the minimal principal curvatures \((k_{\text{max}} \text{ and } k_{\text{min}})\) or the corresponding curvature circle radii \((\rho_{\text{max}} \text{ and } \rho_{\text{min}})\). The Euler theorem primarily concerns with these surface properties, so that an arbitrary normal section curvature can be determined from the principal curvatures, Eq.2.1. Most of the existing curvature gouge detection methods focus on the relations between the normal curvatures of machined surface and cutter.

\[
k_\delta = k_{\text{max}} \cos^2 \delta + k_{\text{min}} \sin^2 \delta
\]  

(2.1)

where \(k_\delta\) represents the normal curvature in the direction of angle \(\delta\) to the maximal principal curvature \(k_{\text{max}}\); \(k_{\text{max}}\) and \(k_{\text{min}}\) are principal curvatures.
2.2 Meusnier Theorem

In differential geometry, the Meusnier theorem deals with the osculating curvatures. Given a point $P_o$ on a $G^2$ surface, there exists a normal vector $\vec{e}$. Passing through the normal vector $\vec{e}$, there exist unlimited numbers of normal planes, with each plane intersects the surface to form a normal section curve. Corresponding to each normal section curve, there exists a tangent direction $\vec{t}$, \textit{i.e.} the intersection of the normal plane with the tangent plane at the surface point $P_o$, Fig.2. Passing through the tangent direction $\vec{t}$, there exist also unlimited numbers of planes ($\pm 90^\circ$), named as the osculating plane. The osculating plane is determined by the osculating angle $\phi$ from the normal plane, Fig.2.2. Each osculating plane intersects the surface to form an osculating curve with curvature $k$. The Meusnier theorem describes the relation between the normal curvature $k_n$ and the osculating curvature $k$, Eq.2.2.

$$k \cdot \cos \phi = k_n$$  \hspace{1cm} (2.2)
where $\vec{k}$ is the osculating curvature on an osculating plane; $\vec{k}_n$ is the curvature on the normal plane through. Angle $\phi$ is the osculating angle between the normal plane and the osculating plane, Fig.2.2.

![Figure 2.2 Osculating Curvatures](image)

In Fig.2.2, a normal plane passing through the surface normal vector $\vec{e}$ intersects the surface with a normal curve. An osculating plane, passing through the tangent $\vec{t}$, intersects the surface with an osculating curve. Vector $\vec{n}$ locates on the osculating plane.

The normal curvature $k_n$ is common to all the osculating curvatures related to the tangent direction $\vec{t}$. The osculating curves are obtained by intersecting the concerned surface with an osculating plane which is at an angle $\phi$ to the normal plane. The curvature circle radius $\rho$ of the osculating curve is determined as:

$$\rho = \rho_n \cdot \cos \phi$$  \hspace{1cm} (2.3)

The osculating curves share the common normal curvature $\vec{k}_n$, and constitute a curve family. The curvature circles of this family curve form a sphere of radius $\rho_n(=\frac{1}{k_n})$. This sphere is named as the Meusnier sphere, Fig.2.2. The Meusnier sphere represents the family of osculating curvatures on the surface in given tangent direction $\vec{t}$. 
To better understand the Meusnier sphere, imagine placing a sphere at an arbitrary point $P_s$ on surface. Make the radius of this sphere to be the normal curvature circle radius $\rho_n$ corresponding to given tangent direction $\vec{t}$ on the surface. An arbitrary osculating plane defined by the tangent direction $\vec{t}$ and the osculating angle $\phi$ intersects the sphere to form a circle of radius $\rho$, i.e. the osculating curvature circle radius Eq.2.3.

Meusnier theorem offers a device for investigating curvature gouge problems from the viewpoint of osculating curvatures. For a given tangent direction $\vec{t}$, in order for all the related osculating curvatures of the machined surface to be smaller than that of the cutter, so that the osculating curvature interference can be avoided, it is necessary that the normal curvature of the cutter surface $k_{nc}$ be equal to or greater than that of the workpiece surface $k_{nw}$, Eq.2.54.

$$k_{nw} \leq k_{nc} \quad \text{or} \quad \rho_{nw} \geq \rho_{nc}$$  \hspace{1cm} (2.4)

The relation of Eq.2.4 indicates that the cutter’s Meusnier sphere should be smaller than the Meusnier sphere of the workpiece surface, so that it can be confined within the latter, Fig.2.3.

![Figure 2.3 Osculating Curvature Gouge Free](image)

When the condition of Eq.2.4 is satisfied, the cutter surface Meusnier sphere lies inside that of the workpiece surface, Fig.2.3. Any osculating curvature of the cutter
surface, corresponding to the given tangent direction $\vec{t}$, is greater than that of the workpiece surface; as a result, the osculating curvature conflict is eliminated.

Fig. 2.3 shows the situation of concave surface ($k_{nw} > 0$) where curvature gouge may happen. For convex and/or flat surface ($k_{nw} \leq 0$) curvature gouge problem does not exist. The Meusnier spheres of cutter and workpiece surface locate in opposite sides of the tangent plane, Fig.2.4.

![Figure 2.4 Convex Surface Meusnier Sphere](image)

The Meusnier theorem provides the guideline for osculating curvature gouge avoidance; however, this curvature gouge elimination solution is limited in certain tangent direction $\vec{t}$. When the tangent direction $\vec{t}$ changes, related Meusnier sphere fit conditions could vary, and osculating curvature gouge problem may exist.

### 2.3 Euler-Meusnier Sphere

Last section sees that the Meusnier theorem provides a device for investigating osculating curvature gouge problems. But, the Meusnier theorem is related to certain tangent direction $\vec{t}$. In other words, the Meusnier theorem only gives the solution for a single tangent direction $\vec{t}$. When the tangent direction $\vec{t}$ changes, the osculating curvature match situation from the workpiece surface and cutter may be destroyed. So, the
Meusnier theorem itself does not offer curvature gouge elimination solution for the concerned point.

To get a more comprehensive curvature gouge elimination solution about a given point on surface, both the Euler and Meusnier theorems should be concerned. The Euler theorem, Eq. 2.1 states that for any point on the surface, the normal curvatures are defined in a range of $[k_{\min}, k_{\max}]$. As the radius of the Meusnier sphere is the inverse of the normal curvature, the Euler theorem indicates that no matter how the tangent direction $\vec{t}$ varies, there always exist two extreme Meusnier spheres corresponding to the principal curvatures $k_{\max}$ and $k_{\min}$. One extreme Meusnier sphere has the largest radius; the other has the smallest one. Any other Meusnier sphere, resulted from the varying direction of tangent $\vec{t}$, is defined between these two extreme Meusnier spheres, Eq.2.5. These two extreme Meusnier spheres are named as the principal Meusnier spheres. Obviously, the principal Meusnier spheres determine the range of the curvature geometries for the concerned surface point.

$$\frac{1}{k_{\max}} \leq \rho \leq \frac{1}{k_{\min}}$$  \hspace{1cm} (2.5)

where $\rho$ represents the radius of an arbitrary Meusnier sphere; $k_{\max}$ and $k_{\min}$ are principal curvatures.

Combining the Euler and Meusnier theorems, a new concept of Euler-Meusnier sphere is derived. At any elliptical point of a $G^2$ surface, there exists a family of Meusnier spheres that portrays the overall curvature geometry of the point. Any Meusnier sphere in this family represents a family of osculating curvatures corresponding to a tangent direction $\vec{t}$. The complete set of the spheres is a collection of varying size Meusnier spheres resulting from the variation of tangent direction $\vec{t}$. The largest and smallest Meusnier spheres in this family of spheres are determined by the principal curvatures at the point; the size of any other Meusnier sphere in the set is determined from the Euler theorem Eq.2.1, relating to given tangent direction $\vec{t}$. This family of Meusnier spheres is called the Euler-Meusnier sphere, Fig.2.5.
The creation of the Euler-Meusnier sphere can be viewed as a plane intersects the surface while it separately rotates about two axes at the concerned point \( P_0 \). The two rotation axes are surface normal vector \( \vec{e} \) and tangent direction \( \vec{t} \). First, the concerned plane rotates about the surface normal vector \( \vec{e} \). At each location, this rotating plane intersects the surface tangent plane to form a tangent direction \( \vec{t} \), Fig.2.2. Secondly, the rotating plane rotates about the tangent direction \( \vec{t} \). At each position defined by the osculating angle \( \phi \), the rotating plane intersects the surface to form an osculating curve. The collection of the osculating curvature circles related to the tangent direction \( \vec{t} \) is a Meusnier sphere, Fig.2.6. The overall collection of all the osculating curvature circles resulted from the plane’s rotation about the two axes \( \vec{e} \) and \( \vec{t} \) is the set of Euler-Meusnier sphere, Fig.2.5. The Euler-Meusnier spheres are cotangent with each other at the concerned point, with the largest and the smallest Meusnier spheres determined by the principal curvatures of the point.
Figure 2.6 Meusnier Sphere

The introduction of the Euler-Meusnier sphere concept extends the surface curvature geometry representation to both osculating and normal curvatures; as a result, three-dimensional solution for curvature gouge detection and elimination can be derived. When the Euler-Meusnier sphere of workpiece surface completely envelope those of the cutter, so that there is no conflicts between the Euler-Meusnier sphere of cutter and the machined surface, curvature gouge problem related to both normal and osculating curvatures can be avoided.
Chapter 3 Modeling of Milling Cutter Geometry and Their Cutting Conditions

Curvature gouge problem in surface machining is resulted from the curvature conflicts between cutter and the machined surface. To seek for curvature gouge avoidance solutions, curvature characteristics of milling cutters should be understood. There are three types of milling cutters, i.e. the ball mill, end mill and torus mill, Fig.3.1. These milling cutters are inter-transformable with the torus mill cutter as a general model, Fig.3.2. When the corner radius \( r \) of the torus mill cutter is zero, the cutter is transformed to end mill cutter; when the centre radius \( R \) is zero, the cutter is transformed to ball mill cutter. In following curvature studies, the torus mill cutter is first analyzed. Curvature study results of the torus mill cutter are then transferred to ball mill and end mill cutters.

Figure 3.1 Mill Cutter

Figure 3.2 Torus Mill Cutter
3.1 Surface Curvatures of Torus Mill Cutter

The cutting surface of a torus mill cutter is a portion of torus. This torus surface is generated when a partial piece of circle rotates about an axis a distance $R (> 0)$ away from the circle centre, Fig.3.3. This piece of circle is called the generation circle. A plane that passes through the rotation axis is named as the orientation plane; the orientation plane is a normal plane for the torus surface. The radius of the generation circle locates on the orientation plane; the surface normal vector at any point along the generation circle coincides with the circle radius. So, the generation circle of the torus surface is a normal curve with a constant curvature radius $r$. When the orientation plane rotates about the circle radius $r$ to an angle $\delta$, another normal plane $P_\delta$ of the torus surface is obtained, Fig.3.4. Normal curve curvature of the torus surface is the largest when the rotation angle $\delta$ is zero. So, the orientation plane is the maximal principal normal plane $P_{\max}$ for the torus surface, the surface generation circle is the maximal principal curvature curve with curvature $k_{\max}$ to be the inverse of the circle radius $r$, Eq.3.1. On any other normal plane $P_\delta$, the generation circle is projected/stretched to have a smaller curvature. When the angle $\delta$ reaches $90^\circ$, the transformed generation circle possesses the minimal curvature.

![Figure 3.3 Generation of Torus](image)

$$k_{\max} = \frac{1}{r}$$ (3.1)

According to the Euler theorem from differential geometry, the minimal principal normal plane $P_{\min}$ of the torus surface is the normal plane when $\delta$ is $90^\circ$ to the orientation plane, i.e. the maximal principal normal plane. The torus surface curvature at any point
can be determined with the Euler and Meusnier theorems, Eq.2.1 and Eq.2.2, when the principal curvatures are defined.

![Diagram of normal curvatures](image)

**Figure 3.4 Normal Curvatures**

The maximal principal curvature of the torus surface is the inverse of the surface generation circle radius \( r \). To determine the surface curvatures, the minimal principal curvature has to be learned. One way to determine the minimal principal curvature at given point \( P \) on the torus surface is to define the minimal normal curve, which passes through the point on the minimal principal plane \( P_{\text{min}} \), and then calculate its curvature. To do this, the coordinate transformation method is used. The torus surface Eq.3.2 in the global coordinate \( x-y-z \) is transformed to the local coordinate \( x_2-y_2-z_2 \), Fig.3.5. The local coordinate \( x_2-y_2-z_2 \) is originated at the centre of the surface generation circle. The \( x_2 \)-axis passes through the concerned point, and coincides with the surface normal vector \( \vec{n} \). This \( x_2 \)-axis is at an angle of \( \phi \) to the \( x \)-axis. The \( x_2 \), \( z_2 \)-axis and \( z \)-axis are on the same plane, *i.e.* the orientation plane. The \( y_2 \)-axis passes through the centre of the surface generation circle, and is parallel with the \( y \)-axis. The \( x_2-y_2 \) plane is the minimal principal plane.
Figure 3.5 Torus Surface

\[
\begin{align*}
  x &= (R + r \cos \theta) \times \cos \lambda \\
  y &= (R + r \cos \theta) \times \sin \lambda \\
  z &= -r \sin \theta \\
\end{align*}
\]

(3.2)

where point \( P' \) is an arbitrary point on the torus surface. Angle \( \theta \) and \( \lambda \) are parameters for the surface generation. Angle \( \theta \) is measured on the orientation plane, \((0 \leq \theta \leq 90^\circ)\). Angle \( \lambda \) is measured on the \( x-y \) plane, \((0 \leq \lambda \leq 360^\circ)\), Fig.3.5.

The transformation from coordinate \( x-y-z \) to \( x_2-y_2-z_2 \) involves a translation \( R_T \), Eq.3.3 that moves the coordinate \( x-y-z \) to \( x_1-y_1-z_1 \), and a rotation \( R_\phi \), Eq.3.4 that rotates coordinate \( x_1-y_1-z_1 \) to \( x_2-y_2-z_2 \), Fig.3.6.

Figure 3.6 Transformation
\[
R_r = \begin{bmatrix}
1 & 0 & 0 & -R \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(3.3)

\[
R_t = \begin{bmatrix}
\cos \phi & 0 & -\sin \phi \\
0 & 1 & 0 \\
\sin \phi & 0 & \cos \phi \\
\end{bmatrix}
\]

(3.4)

The torus surface expressed in the local coordinate \(x_2\)-\(y_2\)-\(z_2\) is:

\[
\begin{bmatrix}
x_2 \\
y_2 \\
z_2 \\
1
\end{bmatrix} = R_t R_r \begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = \begin{bmatrix}
(R + r \cos \theta) \cos \lambda \cos \phi + r \sin \theta \sin \phi - R \cos \phi \\
(R + r \cos \theta) \sin \lambda \\
(R + r \cos \theta) \cos \lambda \sin \phi - r \sin \theta \cos \phi - R \sin \phi \\
1
\end{bmatrix}
\]

(3.5)

On the minimal principal plane, i.e. the \(x_2\)-\(y_2\) plane, coordinate \(z_2\) is zero (= 0).

This plane intersects the torus surface Eq.3.5 to get the minimal principal curvature curve Eq.3.6:

\[
\begin{align*}
x_2 &= (R + r \cos \theta) \cos \lambda \cos \phi + r \sin \theta \sin \phi - R \cos \phi \\
y_2 &= (R + r \cos \theta) \sin \lambda \\
z_2 &= (R + r \cos \theta) \cos \lambda \sin \phi - r \sin \theta \cos \phi - R \sin \phi = 0
\end{align*}
\]

(3.6)

Equation 3.6 is further transformed to, Eq.3.7:

\[
y_2 = \pm \sqrt{r^2 - x_2^2 - 2R x_2 \cos \phi + 2R \sqrt{r^2 - x_2^2 \sin^2 \phi}} \quad (0 \leq \phi \leq \frac{\pi}{2})
\]

(3.7)

Using planar curve curvature calculation function, Eq.3.8:

\[
k = \frac{|y''|}{(\sqrt{1 + y'^2})^3}
\]

(3.8)

the minimal principal curvature of the torus surface for the concerned point \(P\) is determined as:

\[
\begin{align*}
k_{\min} &= \frac{\cos \phi}{R + r \cos \phi} \\
\rho_{\min} &= r + \frac{R}{\cos \phi}
\end{align*}
\]

(3.9)
Eq. 3.9 indicates that the minimal principal curvature of the torus mill cutter is not constant, but is bounded within the range of Eq.3.10,

$$0 \leq k_{\text{min}} \leq \frac{1}{R + r}$$ \hspace{1cm} (3.10)

When the concerned point is at the bottom edge of the torus surface, i.e. ($\phi = 90^\circ$) the minimal principal curvature is the least, $k_{\text{min}} = 0$; when the concerned point is on the top edge of the torus surface, i.e. ($\phi = 0^\circ$) the minimal principal curvature is the largest, $(k_{\text{min}} = \frac{1}{R + r})$.

The minimal principal curvature of a mill cutter is a critical factor for investigating curvature gouge problems in surface machining. Curvature gouge happens when milling cutter surface curvature is smaller than that of the machined surface. The minimal principal curvature of the cutter is the least over the cutter surface; so, it can be used to represent the cutter for curvature gouge detection.

Curvatures of the torus mill cutter is summarized as:

$$\begin{cases} 
    k_{\text{min}} = \frac{\cos \phi}{R + r \cos \phi} \\
    k_{\text{max}} = \frac{1}{r} \\
    k_\delta = \frac{1}{r} \cdot \cos^2 \delta + \frac{\cos \phi}{R + r \cos \phi} \cdot \sin^2 \delta
\end{cases}$$ \hspace{1cm} (3.11)

where $k_\delta$ represents the normal curvature on the torus surface; $\delta$ is the angle between $k_\delta$ and $k_{\text{max}}$; $k_{\text{max}}$ and $k_{\text{min}}$ are principal curvatures.

### 3.2 Ball Mill Cutter

A ball mill cutter is viewed as taking the centre radius $R$ as zero from a torus mill cutter. Ball mill cutter surface curvature calculation can thus be derived from that of the torus mill cutter, Eq.3.11, when substituting $R$ with zero, Eq.3.12.

$$k_\delta = k_{\text{min}} = k_{\text{max}} = \frac{1}{r}$$ \hspace{1cm} (3.12)
So, the ball mill cutter has a constant normal curvature all over the cutting surface. This is obvious because any normal plane that has an angle $\alpha$ to the orientation plane Fig.3.4 intersects the cutter surface with a circle of radius $r$.

### 3.3 End Mill Cutter

The end mill cutter is also a transformed torus mill cutter when taking the corner radius $r$ as zero. Substituting $r$ with zero in Eq. 3.11, the end mill cutter curvatures are derived as:

\[
\begin{align*}
    k_{\min} &= \frac{\cos \phi}{R} \\
    k_\delta &= k_{\max} \to \infty
\end{align*}
\]

(3.13)

The cutting edge of an end mill cutter is the shrunk torus surface that has a generation circle of zero radius. The orientation plane intersects the cutting edge with a point; the maximal principal curvature is infinite at this point. The normal plane $P_\delta$ also intersects the cutting edge with a point when $\delta$ is not 90°; when $\delta$ is 90°, the normal plane $P_\delta$ intersects the cutting edge with a circle of radius $R$.

To understand the minimal principal curvature $k_{\min}$ of the end mill cutter, the Meusnier theorem in differential geometry offers the help Fig.3.7.

![Figure 3.7 End Mill Cutter](image)

In Fig.3.7, the local coordinate $x_2$-$y_2$-$z_2$ is similarly defined as in Fig.3.5; the normal vector $\vec{n}$ represents the machined surface normal at the cutter contact (CC) point. The angle between the minimal principal normal plane, which is perpendicular to the
orientation plane passing through the cutter contact point, and the cutter end face is the osculating angle $\phi$. The cutting edge circle has a constant curvature $k$ that is the inverse of the cutter radius $R$. The cutter end face locates on the osculating plane corresponding to the normal plane $x_2-y_2$ that contains the normal vector $\vec{n}$ and is perpendicular to the orientation plane $z-x_2$. According to Meusnier theorem, the normal curvature $k_n$ on the minimal principal normal plane $x_2-y_2$ is thus determined as Eq.3.14:

$$k_n = k \cos \phi$$  \hspace{1cm} (3.14)

This is exactly what Eq.3.13 stipulates.

The end mill cutter minimal principal curvature defined in Eq. 3.13 and/or 3.14 is a pseudo factor. It transfers the cutting edge curvature onto the normal plane of the machined surface for curvature gouge analysis.

### 3.4 Milling Cutter Representative Sphere

Milling cutter surface curvatures have been defined. Applying the Euler-Meusnier sphere concept developed in previous chapter, a new concept of mill cutter representative sphere is established. As usual, the torus mill cutter is used as a model of mill cutter to introduce the representative sphere.

#### 3.4.1 Torus Mill Cutter

The concept of Euler-Meusnier sphere indicates that at any point on a $C^2$ surface, there exists the largest Meusnier sphere, corresponding to the minimal principal curvature at the point. This sphere enwraps any other Meusnier sphere related to the concerned point. A mill cutter surface curvature, at the given point, does not interfere with workpiece surface if this largest Meusnier sphere keeps away from the workpiece surface.

Moreover, the largest Meusnier at an arbitrary point on the torus mill cutter surface enwraps the cutter surface completely Fig.3.9.
Given a torus surface Fig.3.8, a plane that is normal to cutter axis and passes through a given point \(P\) is called the iso-curvature plane. The iso-curvature plane intersects the torus surface to get an iso-curvature circle. Any point on the iso-curvature circle has the same principal curvatures. So, the largest Meusnier sphere at any point on the iso-curvature circle has the same radius, \(\rho = l_{cp}\) Fig.3.8. Moreover, the torus surface normal vector at the concerned point \(P\) on the iso-curvature circle concentrate to the same point \(C\) locating on axis \(A\) of the cutter. Point \(C\) is the common center for any of the largest Meusnier sphere corresponding to the point on the iso-curvature circle. So, the largest Meusnier sphere at any point on the iso-curvature circle has the same radius \(\rho\), locates at the same center point \(C\). They constitute a single sphere. This sphere is tangent with the torus surface at each point along the iso-curvature circle. Thus, the iso-curvature circle is the contact curve along which the largest Meusnier sphere is tangent to the torus surface. Observation shows that this sphere can be tangent with the torus surface only once. At any contact point, in any direction around the point, this largest Meusnier sphere has the minimum curvature; the torus surface bends inwardly and is thus covered within the sphere locally around the contact point \(P\). As a result, the whole torus surface is enwrapped inside the sphere completely Fig.3.9, since the torus surface curvature is uniform signed.
The largest Meusnier sphere at a given point on the torus surface is named as the cutter representative sphere. Because this sphere enwraps the cutter surface completely, it can be used to represent the cutter for curvature gouge detection in surface machining. Curvature gouge problem cannot happen when the cutter representative sphere does not interfere with the designed surface. If any gouge problem exists, the cutter representative sphere must already interfere with the machined surface.

The size of the cutter representative sphere of a torus mill cutter depends on the position of cutter contact point. That is, the size of the cutter representative sphere changes when the cutting point moves along the cutter surface. Since the cutter-osculating angle $\phi$ also changes with the cutter contact point, the size of the cutter representative sphere can be adjusted through adjusting the cutter orientation angle $\phi$ Eq.3.11. The smallest representative sphere for a torus mill cutter has a radius of $\rho_{\text{min}}$:

$$\rho_{\text{min}} = R + r$$  \hspace{1cm} (3.15)

when the osculating angle is zero. The largest representative sphere for a torus mill cutter has a radius of infinity $\rho_{\text{max}} \rightarrow \infty$, when the osculating angle $\phi$ is 90° Fig.3.10.
Figure 3.10 Representative Spheres

In Fig.3.10 shows the torus mill cutter and representative sphere whith different inclination angle $\Gamma$. The inclination $\Gamma$ is the complement of the osculating angle $\phi$ Eq.3.16.

$$\Gamma = 90^\circ - \phi$$  (3.16)

### 3.4.2 End Mill Cutter

The cutting edge of an end mill cutter is the shrunk torus surface; so, the cutting edge itself is the iso-curvature circle of the end mill cutter Fig.3.11. The end face of the cutter is the iso-plane. Given the workpiece surface normal vector $\vec{n}$ at the cutter contact point, its relative position to the cutter iso-plane is determined. Any point on the cutting edge, when passing through the cutter contact point, keeps the same relation to the normal vector $\vec{n}$ and the cutter end face. Thus, the curvature definition Eq.3.14 for the whole cutting edge is uniform.
Given the workpiece normal vector $\vec{n}$, all points on the cutting edge (or iso-curvature circle) takes the same size largest Meusnier sphere; all these Meusnier spheres share the same center point $C$ on the cutter axis $A$. Thus, there exists only one largest Meusnier sphere that is tangent to the cutting edge and enwraps it. This Meusnier sphere is the representative sphere of the end mill cutter Fig.3.11.

Adjusting the osculating angle $\phi$ changes the size of the representative sphere for the end mill cutter Fig.3.13. The smallest representative sphere has a radius of $R$, i.e. the cutter radius; the largest representative sphere has a radius $\rho_{\text{max}} \to \infty$. 
Figure 3.13 Various Size Representative Sphere

The representative sphere for the end mill cutter can be used to represent the cutter for curvature gouge detection. The end mill cutter will not over cut the machined surface if the cutter representative does not interfere with the designed surface. If there is any curvature gouge problem in surface machining, the cutter representative must already interfere with the machined surface.

3.4.3 Ball Mill Cutter

Representative sphere of a ball mill cutter is the cutter surface itself. The whole surface of a ball mill cutter has a constant curvature, so that the surface itself is isocurvature. Changing orientation of the ball mill cutter does not affect the size of the cutter representative sphere. Curvature gouge avoidance solution for a ball mill cutter is that the inverse of the cutter radius is larger than the surface curvature.

3.5 Milling Cutter Comparison

A major difference among milling cutters, i.e. ball mill, torus mill and end mill, lies in the difference of their minimal principal curvature determination. The minimal principal curvature of a mill cutter determines the feature of the cutter representative sphere; so, from the comparison of milling cutters’ representative sphere, performances of milling cutters are evaluated. A ball mill cutter has an invariable representative sphere that has the same size as the cutter surface. A torus and/or end mill cutter has varying size
representative spheres depending on the cutter-osculating angle $\phi$. For the same size torus and end mill cutters, when cutter osculating angle $\phi$ takes the same value, the representative sphere of the torus mill cutter is smaller than that of the end mill cutter.

### 3.5.1 Ball Mill vs. End Mill

A ball mill cutter has a fixed-size representative sphere; cutter orientation angle, \textit{i.e.} the osculating angle $\phi$, does not affect the cutter surface curvature. In working, the cusp height left on the machined surface will not reduce when modifying the osculating angle $\phi$ Fig.3.14.

![Figure 3.14 Ball Mill vs. End Mill](image)

The cutting speed of a ball mill cutter varies over the cutter surface, with the tip point $P$ having zero cutting speed Fig.3.15. So, it is preferable for a ball mill cutter to have a nonzero inclination angle $\gamma$ at working, to avoid the tip point $P$ plowing over the designed surface.
An end mill cutter works with a constant cutting speed along its cutting edge. Besides, the cusp height left on the machined surface can be reduced when the inclination angle $\Gamma$ decreases, i.e. when the cutter representative sphere increases size.

Compared to the ball mill cutter, the end mill cutter has better dynamic and/or geometric characteristics for surface machining, due to its constant cutting speed along the cutting edge and adjustable representative sphere to match the machined surface, which results in the adjustable cusp heights. The end mill cutter produces high quality surfaces. However, a ball mill cutter can easily avoid curvature gouge problems.

Figure 3.16 shows a simulation test where the same size ball mill and end mill cutters are used to cut a flat surface. The side steps are equal. The end mill cutter is orientated with a nonzero inclination angle $\Gamma$. The cusps left on the surface are much smaller from the end mill cutter machining.
3.5.2 Torus Mill vs. End Mill Cutter

Radii of cutter representative sphere of the torus and end mill are listed as follows Eq.3.17.

\[
\begin{cases}
\rho_{\text{torus}} = \frac{R_T + r_T \cos \phi_T}{\cos \phi_T} \\
\rho_{\text{end}} = \frac{R_E}{\cos \phi_E}
\end{cases}
\]  

(3.17)

where \(\rho_{\text{end}}\) and \(\rho_{\text{torus}}\) are radii of the cutter representative sphere of the end and torus mill respectively. \(R_T\) and \(r_T\) represent the torus mill cutter center and corner radii; \(R_E\) is the radius of the end mill cutter.

For comparison, assume the torus and end mill cutter have the same size Eq.3.18.

\[
R_E = R_T + r_T
\]  

(3.18)

Given the same osculating angle \(\phi\) Eq.3.19,

\[
\phi_T = \phi_E = \phi
\]  

(3.19)

the torus mill cutter will have a smaller representative sphere Eq.3.20,

\[
\rho_{\text{torus}} \leq \rho_{\text{end}}
\]  

(3.20)

So, the torus mill cutter is less possible to over cut the machined surface, though, the machined surface may have higher cusps from the torus mill cutter machining.
On the other hand, given a workpiece surface for which the size of a suitable cutter representative sphere radius \( \rho \) is determined Eq.3.21,

\[
\rho = \rho_{\text{torus}} = \rho_{\text{end}}
\]  

(3.21)

the torus mill cutter will take a greater osculating angle \( \phi_r \). This leads to a smaller inclination angle \( \Gamma \) Eq.3.22 for the torus mill cutter to be orientated. Inclination angle \( \Gamma \) is measured between cutter axis and the machined surface normal vector \( \vec{n} \) Fig.3.17. The smaller the inclination angle \( \Gamma \), the easier for a CNC machine to realize it. Experience tells that it is sometimes impossible to realize a given inclination angle \( \Gamma \) for certain CNC machines. For example, the DECKEL MAHO 80P CNC machine has an inclination angle limit of \(-45^\circ\) to \(30^\circ\). It is easier to orientate a torus mill cutter for curvature gouge avoidance due to the smaller inclination angle.

\[
\Gamma_{\text{torus}} \leq \Gamma_{\text{end}}
\]  

(3.22)

Figure 3.17 Inclination Angle

Generally, a torus mill cutter is easier to avoid curvature gouge problem than is an end mill cutter; however, given same inclination angle \( \Gamma \), an end mill cutter produces smoother surfaces than does a torus mill cutter.
Chapter 4 Curvature Gouge Detection and Elimination Solutions

Curvature gouge problem in surface machining is one of the major concerns in tool path planning. One contributor to curvature gouge problem, the milling cutter, is analyzed in previous chapter; this chapter deals with another factor, the workpiece surface. Generally, workpiece surface can be classified into uniform curvature surface and non-uniform curvature surface. The uniform curvature surface has constant principal curvatures all over the surface. This kind of surface includes cylindrical and spherical surfaces Fig.4.1. The non-uniform curvature surface has varied principal curvatures on the surface. Curvature gouge detection and avoidance solutions for different types of surfaces can be different; though, they are all based on the EMS principles. For the uniform curvature surface, the curvature gouge detection and elimination solution is simple and unique. For the non-uniform curvature surface, curvature gouge detection and avoidance solutions are diversified, including global safe, area safe and the feed-direction-optimal solutions.

![Figure 4.1 Uniform Curvature Surface](image)

4.1 Curvature Gouge-Free Solution for Uniform Curvature Surface

The uniform curvature surfaces, i.e. cylinder and sphere, have continuous curvatures, i.e. $G^2$. Applying EMS concept, surface curvature geometries can be presented Fig.4.2. For the spherical surface, all the Meusnier spheres of the EMS have the same radius, so that they combine to a single sphere coincident with the concerned sphere surface. For the
cylindrical surface, the EMS is a collection of spheres with the smallest one having a radius equal to the cylinder surface and the largest sphere having a radius of infinity. Moreover, the EMS of the uniform curvature surface is identical everywhere over the surface. Any set of EMS at an arbitrary point of the surface can represent the whole surface curvature geometries.

![Figure 4.2 EMS for Uniform Curvature Surface](image)

Curvature gouge avoidance solutions for the uniform curvature surfaces are directly derived from the EMS of cutter and workpiece surface. Locally, at a CC point, curvature interference between the cutter and workpiece surface can be avoided if the smallest Meusnier sphere of workpiece surface envelops the largest Meusnier sphere of the cutter. When this condition is met, there is no interference between the EMSs of cutter and the workpiece surface Fig.4.3.

![Figure 4.3 Gouge-Free Condition](image)
In Fig.4.3 shows the EMS relation of cutter and workpiece surface at a CC point. Workpiece surface curvatures exist in the outer space defined by the largest and the intermediate spheres. Cutter surface curvatures exist in the middle space defined by the intermediate and the smallest spheres. The largest sphere in the assembly represents the largest Meusnier sphere of workpiece surface. The intermediate sphere represents both the smallest Meusnier sphere of workpiece surface and the largest Meusnier sphere of cutter. The smallest sphere in the assembly represents the smallest Meusnier sphere of cutter. The largest Meusnier sphere of the cutter matches the smallest Meusnier sphere of the workpiece surface at the intermediate sphere. There is no interference among surface curvatures from the cutter and workpiece surface because there is no overlapping among their EMSs. Such nesting of the EMSs from the cutter and workpiece surface leads to the elimination of curvature conflicts at the concerned CC point.

Interestingly, the local curvature gouge-free situation for the uniform curvature surface can result in global curvature gouge elimination consequence, due to the uniformity of principal curvatures over the surface.

First, the local EMS match at a CC point leads to the curvature gouge elimination along the maximal principal curvature curve through the CC point. The maximal principal curvature curve is the intersection between the surface and its smallest Meusnier sphere at the concerned point; or equally, it is the intersection between the largest Meusnier sphere of cutter and the machined surface. The uniformity of principal curvatures on the surface means that all points along any maximal principal curvature curve form a circular curve. The maximal principal curvature curve is an iso-curvature curve; any point on the maximal principal curvature curve has the same curvature radius. In addition, the surface normal vectors from any point on the maximal principal curvature curve passes through the same point locating on the surface axis. Thus, all the points on the maximal principal curvature curve belong to the same circle.

The local curvature gouge-free condition demands that the representative sphere of cutter matches the smallest Meusnier sphere of workpiece surface at the CC point; thus, the representative sphere of the cutter has the same radius as that of the maximal
principal curvature circle. This makes the representative sphere of cutter tangent to the workpiece surface along the maximal principal curvature curve Fig.4.4.

![Cutter Representative Sphere](image)

**Figure 4.4 The Maximal Principal Curvature Curve**

All points locating on the maximal principal curvature curve share this circular curve, as well as the corresponding Meusnier sphere; in other words, all the smallest Meusnier spheres at points along the maximal principal curvature curve converge to one. One single Meusnier sphere represents the maximal principal curvatures of every point on the same maximal principal curvature curve. The fact that the cutting edge fits to the local Meusnier sphere at the CC point leads to its global fitting to all the Meusnier spheres belonging to the points along the same maximal principal curvature curve. Therefore, the local curvature gouge-free situation at the CC point entails a global curvature gouge-free consequence along the maximal principal curvature curve.

Second, the curvature gouge-free situation along the maximal principal curvature curve leads to the global curvature gouge-free consequence over the surface. This is obvious for a spherical surface since the maximal principal curvature curve is actually extends all over the surface, i.e. the smallest Meusnier sphere at any point on the surface is tangent to the surface everywhere. The cutter representative sphere conforms to the concerned surface.

For a cylindrical surface, the maximal principal curvature circle is the only place where the cutter interacts with the workpiece surface because the cutter representative sphere is tangent to the cylinder surface. The cutter cannot over cut the cylinder surface
since it has no other contact with the surface than the maximal principal curvature curve. So, the local curvature gouge-free situation along the maximal principal curvature curve results in the global curvature gouge-free consequence over the surface.

The maximal principal curvature curve is the intersection between cutter representative sphere and the machined surface. The cutter is confined to the sphere and the maximal principal curvature curve. This is the situation all over the surface.

The EMS based curvature gouge elimination solution for the uniform curvature surface is exclusive. It is both necessary and sufficient. Observance to the solution, curvature gouge is eliminated; no matter how the cutter feed direction is set. Violation of the curvature gouge-free solution leads to curvature gouges. Violation to this curvature gouge-free criterion, leads to the cutter surface interfere with the maximal principal curvature curve, and the surface itself.

The criterion for curvature gouge-free machining of the uniform curvature surface is: matching the cutter representative sphere with the smallest Meusnier sphere of workpiece surface at the CC point. This criterion offers the unique global curvature gouge-free solution for uniform curvature surface machining. Cutter feed direction does not affect the curvature gouge-free solution.

Previous chapter shows that the milling cutter has an iso-curvature circle related to given CC point. The plane that contains the iso-curvature circle is named as the cutting edge plane. For an end mill cutter, the cutting edge plane is the cutter end face. The cutting edge plane is the osculating plane corresponding to a normal plane at the CC point Fig.4.5. For any milling cutter, the iso-curvature circle is the working portion that acts on the designed surface. Nothing else of the cutter surface touches the designed surface. Now, the workpiece surface has the maximal principal curvature curve, on which the cutter and the workpiece surface interacts, and the cutter has the iso-curvature circle, through which the cutter contacts the designed surface, therefore, the real machining is through the interaction between the maximal principal curvature curve and the iso-curvature circle.
Two circular curves, the maximal principal curvature curve on workpiece surface and the iso-curvature circle on the cutter, involve in the uniform curvature surface machining. These two circles can have at most two contact points Fig.4.6. These two contact points rest on both the iso-curvature circle and the maximal principal curvature curve of workpiece surface. One of these contact points is the CC point. Except in the extreme situation, where cutter orientation plane, \textit{i.e.} the plane defined by cutter axis and workpiece surface normal vector, is along the minimal principal curvature direction of workpiece surface, these two contact points are separate. In the extreme situation as above described, these two contact points converge to one Fig.4.7. At these two contact points the cutter meets the designed surface.
In machining the uniform curvature surface, when the local curvature gouge-free criterion is met, the mill cutter interacts with the designed workpiece surface at two points. These two contact points locate both at the maximal principal curvature circle and the cutter iso-curvature circle. Nothing else on the cutter surface touches the designed workpiece surface; so that, there is no over cutting. This vision of two contact points in surface machining leads to the best practical solution for solving non-uniform curvature surface gouge problem.

4.2 Non-Uniform Curvature Surface
The non-uniform curvature surface has varied principal curvatures over the surface. The local curvature gouge-free situation does not necessarily result in the global curvature gouge avoidance consequence. This can be seen from the example where the smallest Meusnier sphere at a CC point interferes the concerned surface globally Fig.4.8.
Figure 4.8 Smallest Meusnier Sphere Interfere the Surface

Global curvature gouge-free solutions for the non-uniform curvature surface can be very different depending on the factors including application easiness and the machined surface quality. In this study, the global curvature gouge-free solutions for the non-uniform curvature surface are classified as the global safe solution, the area safe solution, the feed-direction-optimal solution, and the best-possible solution.

4.2.1 Global Safe Solution

Global safe solution for the non-uniform curvature surface provides the easiest to use method for curvature gouge avoidance in surface machining. It is derived from the critical point of the non-uniform curvature surface and is applicable all over the surface. This solution has nothing to do with the cutter feed direction. However, this solution is not necessarily an optimal one for surface machining.

To understand the global safe solution for the non-uniform curvature surface machining, curvature match conditions of the uniform curvature surface should be reviewed Fig.4.9.
When the EMS based curvature gouge-free criterion is met for the uniform curvature surface, the smallest Meusnier sphere at a CC point is tangent to the concerned surface along the maximal principal curvature curve. Now, let the surface curvature change to form a non-uniform curvature surface. When the surface curvature increases at some points, a portion of the surface will bend into the smallest Meusnier sphere Fig.4.10 (a), that is, the smallest Meusnier sphere interferes the surface. Contrarily, if the surface curvature decreases, the deformed surface portion will bend away from the sphere; nothing of the surface exists inside the smallest Meusnier sphere, so that there is no interference. This phenomenon indicates that for the smallest Meusnier sphere at a CC point to interfere the surface, there must be curvature increases at some points on the surface. Suppose another smallest Meusnier sphere at the curvature-increasing portion is drawn, this newly drawn Meusnier sphere must be totally enwrapped by the previous Meusnier sphere at the CC point Fig.4.10 (b). This is because the curvature-increasing portion locates on or inside the original Meusnier sphere, the newly drawn Meusnier sphere has a smaller radius, the surface curvature is continuous and uniformly concave; and outside of the smallest sphere, surface curvatures are all smaller, so that the corresponding spheres are all large. So, an indicator for the smallest Meusnier sphere at the CC point to
interfere the surface is that it enwraps some other Meusnier spheres. In other words, if there is no any other Meusnier sphere existing completely inside the concerned Meusnier sphere, this sphere will not interfere the surface.

![Diagram showing interference area and smallest spheres.](image)

**Figure 4.10 Non-Uniform Surface**

If there exists a sphere that does not interfere the surface, no matter where it is positioned, this sphere must be the smallest Meusnier sphere over the surface, so that no any other Meusnier sphere can be put inside it. Using this sphere to define a mill cutter orientation, the cutter will not over cut the surface.

The desired sphere can be defined from the surface overall maximal principal curvature because thus defined sphere has the smallest radius compared to any other Meusnier sphere of the surface. No any other Meusnier sphere can stay inside it. Put it anywhere, this sphere does not interfere the surface. At the critical point where the surface maximal principal curvature locates, this sphere does not interfere the surface since the surface curvatures around the critical point are all smaller. No curvature is larger; no surface portion bends inside this sphere. Put this smallest Meusnier sphere at other positions, it cannot enclose any other Meusnier sphere because it is the smallest one. Placing this smallest sphere an infinitesimal distance \( \Delta \) away from the critical point, it will not contain the critical point any more Fig.4.11 (b); otherwise these two points locate on one sphere. So, moving away from the critical point, the smallest Meusnier sphere does not interfere the surface of the critical point. Other portions of the surface have curvatures either equal to or smaller than that of the
critical point; they will either remain on the smallest Meusnier sphere or bend further away from it. This is just like using a ball mill cutter to machine the surface. As long as the curvature of the cutter is larger than the overall maximal principal curvature of the surface, the cutter will not over cut the surface.

![Diagram of two locations](image)

**Figure 4.11 Two Locations**

To summarize, the global safe solution is: on a curvature continuous $G^2$ concave surface, there exists a critical point at which the maximal principal curvature is the largest over the surface. Using the maximal principal curvature radius to define the representative sphere of a mill cutter, the cutter will not over cut the surface at any point. Thus defined cutter orientation is globally safe in machining a non-uniform curvature surface. Since the cutter orientation has nothing to do with cutter feed direction, the cutter can feed in any direction according to machining requirements. This global safe solution is, however, not necessarily an optimal one because the cutter may be confined in a too small sphere, especially when the surface curvature changes drastically. The global safe solution can lose potential curvature match potentials.

### 4.2.2 Cutter Active Area Safe Solution

The global safe solution can ensure curvature gouge-free situation for machining non-uniform curvature surfaces; however, the obtained cutter orientation may be too tight,
and lose potential curvature match advantages. To avoid losing usable curvature match allowance, the cutter axis orientation should be determined according to local curvature geometries.

The cutter active area safe solution for cutter axis orientation considers curvature geometries in the vicinity of the CC point, so as to improve curvature usages. Cutter orientation is originally determined from the smallest Meusnier sphere at the CC point. If the smallest Meusnier sphere at the CC point interferes with the workpiece surface, then, searching from the smallest Meusnier sphere enclosed portion of the surface to find the point that is closest to the center of the sphere. From this closest point and the CC point, a new smaller sphere that has center point locating on the surface normal vector can be determined. Check to see if there is any portion of the surface enclosed inside the newly defined sphere. If so, repeating the search process with the newly obtained sphere until there is nothing from the surface enclosed. Then, use the final non-interference sphere to match cutter representative sphere for cutter axis orientation. The resulting cutter orientation can ensure that the cutter does not over cut the surface when machining the CC point Fig.4.12 (b), and can improve curvature match situation between cutter and the machined surface. Obviously, the obtained cutter orientation has nothing to do with cutter feed direction.

![Diagram](image)

**Figure 4.12 Local Min. Sphere**

According to the EMS theory, the smallest Meusnier sphere at a CC point represents the maximal principal curvature at the concerned point. This smallest Meusnier sphere at the CC point usually does not interfere the surface in the vicinity
of the CC point. For non-uniform curvature surface, the interference between the
closest Meusnier sphere and the workpiece surface mostly occurs somewhere in a
distance from the CC point. So, in searching for a new sphere, the closest point
usually lies far away from the CC point. And the newly obtained sphere is always
smaller than previous ones.

To summarize, the cutter active area safe solution is: given a CC point on a
concave curvature continuous $G^2$ surface, there exists the largest sphere that has two
contact points with the surface, and has its center locating on the surface normal
vector through the CC point. One of the contact points is the CC point; the other is
the closest point to the center of the sphere. This locally largest sphere does not
interfere the surface when positioned at the CC point; however, any increase of its
size induces interference with the surface. Using this sphere to define a mill cutter
axis orientation, the cutter will not over cut the surface. Since thus determined cutter
axis orientation considers the surface local curvature geometries, it can improve
curvature match situation. Moreover, the cutter axis orientation is independent to
cutter feed direction.

**Algorithm:** MaxBallRadius ($\rho$)

*Input.* Surface data and cutter radius $r$

*Output.* The radius of the largest non-interference sphere

1. Determine the smallest Meusnier sphere.
2. while there is interference between the sphere and workpiece surface
   Shrink the sphere $\rho$
end
3. return $\rho$

### 4.2.3 Feed-Direction-Optimal Solution

The cutter active area safe solution improves the utilization of surface curvature
allowance, but it is still not an optimal solution. The cutter active area safe solution
overlooks the influence of cutter inclination and/or feed direction in determining cutter
axis orientation; therefore, it can lose potential curvature match advantages.

Observation of the interference between the smallest Meusnier sphere and
workpiece surface reveals that the smallest Meusnier sphere at a CC point on non-
uniform curvature surfaces does not interfere the surface peripherally Fig.4.13. The smallest Meusnier sphere usually interferes the surface on a small portion, and keeps away from the surface at a large portion in the opposite side.

![Figure 4.13 Best Practical Solution](image)

In Fig.4.13 shows the smallest Meusnier sphere at a CC point interacting with a non-uniform curvature surface. This sphere interferes the surface at positions where the surface curvature gets larger. At other places of the surface where the surface curvature gets smaller, there is no interference. This brings up the idea that the curvature gouge avoidance solution should be related to cutter inclination direction. The cutter inclination direction is represented by the angle $\delta$ on the tangent plane at the CC point Fig.4.14. In Fig.4.14, the axes of $x$ and $y$ are the principal curvature directions at the CC point. When the cutter inclination direction makes the cutter uphill to the surface topography, so that the cutter axis is toward the curvature increase portion of the surface Fig.4.13 (a), the cutting edge locates in the area where curvatures are smaller; and there is no gouge when cutter is orientated within its representative sphere. When the cutter inclination direction makes the cutter downhill to the surface topography, so that the cutter axis is toward the curvature decrease portion of the surface Fig.4.13 (b), the cutting edge locates in the interference area. Then, there is curvature gouge if the cutter representative sphere matches the smallest Meusnier sphere of the CC point. So, the cutter inclination direction affects the cutter orientation decision for curvature gouge avoidance in the non-uniform curvature surface machining.
Figure 4.14 Cutter Orientation

Furthermore, when the cutter inclination is uphill to the surface topography Fig.4.13 (a), the cutting edge does not have to be confined in the smallest Meusnier sphere at the CC point Fig.4.15. The cutting edge can locate outside the smallest Meusnier sphere without over cutting the workpiece surface. Therefore, the cutter representative sphere can be larger. This means that the cutter can take smaller curvature than the smallest Meusnier at the CC point.

Taking the cutter inclination direction into consideration for determining the cutter axis orientation can make the curvature match situation even better. As cutter inclination direction is usually coincident with cutter feed direction, this solution of curvature gouge avoidance is named as the feed-direction-optimal solution.

Figure 4.15 Orientation Modification
To determine the feed-direction-optimal cutter axis orientation, the smallest Meusnier sphere at the CC point is used to define the initial orientation of cutter axis. Given cutter inclination direction angle $\delta$ and the smallest Meusnier sphere at the CC point, the initial location of the cutting edge can be determined. Searching on workpiece surface that locates within the range of the cutting edge to see if there is any point interfering the cutting edge plane Fig.4.16. The cutting edge plane contains the cutting edge at the CC point. If any portion of the workpiece surface interferes the cutting edge plane, reduce the osculating angle $\phi$, to modify the cutter and cutting edge plane orientation. If there is no portion of the workpiece surface interfering the cutting edge plane, increase the osculating angle $\phi$, until there is another point, beside the CC point, locating on the cutting edge plane. Mostly, this point locates on the cutting edge circle, or the iso-curvature circle for a torus mill cutter. Then, the cutter orientation relating to given inclination direction is determined, in terms of angle $\delta$ and $\phi$. In this process of modifying the osculating angle $\phi$, the cutter representative sphere is also re-defined.

![Image](image.png)

**Figure 4.16 Cutter Orientation**

To summarize, the feed-direction-optimal solution for cutter axis orientation is: given a CC point and cutter inclination direction angle $\delta$, on a concave curvature continuous $G^2$ surface, the cutter axis orientation can be determined through adjusting the osculating angle $\phi$, to the extent that no point of the workpiece surface locates above the cutting edge plane along the direction of cutter axis. The derived cutter
axis orientation with angle $\delta$ and $\phi$ is the feed-direction-optimal solution for given machining situations; curvature allowance of the machined surface is fully utilized. Any violation to the determined cutter axis orientation incurs curvature gouges. This feed-direction-optimal solution, though, restricts the cutter feed direction.

**Algorithm:** Feed-Direction-OptimalCutterOrientation ($\varphi, \delta$)

*Input.* Surface data, cutter radius $r$ and inclination direction $\delta$

*Output.* Osculating angle $\varphi$ for cutter axis orientation

1. Determine the smallest Meusnier sphere and original osculating angle $\varphi$.
2. Determine cutting edge plane.
3. Determine desired tolerance $\Delta$.
4. **while** calculated interference is larger than $\Delta$
   - reduce osculating angle $\varphi$
**end**
5. **return** $\varphi$

### 4.2.4 Best Possible Solution

Investigation on the interference between the smallest Meusnier sphere at the CC point and workpiece surface Fig.4.13 reveals that the interference area happens in the direction of the maximal principal curvature of the workpiece surface Fig.4.17, and in the side where curvature gets tighter.

On non-uniform curvature surface, there usually exists a region where the smallest Meusnier sphere at the CC point intersects the workpiece surface. When cutter’s cutting edge falls into this interference area, the cutter representative sphere has to be smaller than the smallest Meusnier sphere, so as to avoid the cutter over cutting the surface. When cutter’s cutting edge locates outside of this interference area, the cutter representative sphere can be larger than the smallest Meusnier sphere, without cutter over cutting the surface.

The overall optimal situation for the machining is that the cutter feeds in the maximal principal curvature direction and is aligned uphill Fig. 4.18 (a). This way, the cutter representative sphere can get its largest possible size, and the osculating angle $\phi$ gets its largest value. The cutting edge gets its largest possible contact with the workpiece surface without gouges. This is the best possible orientation for cutter axis.
When cutter feeds in the maximal principal curvature direction and is aligned downhill, its representative sphere can be the smallest and the cutter axis orientation is the poorest Fig.4.18 (b).

The best possible orientation of cutter axis leads to the largest cutter representative sphere for the machining, thus, the optimal machining situation is achieved. However, this best possible orientation of cutter axis is not always achievable in real machining. The major obstacles to achieve the best possible cutter
axis orientation come from the drastic changes either of workpiece surface principal curvature directions or of workpiece surface topography.

On the ellipse surface shown in Fig.4.19, the principal curvature direction changes at point C. At point C, the principal curvatures are equal. On the left side of point C, the maximal principal curvature direction is parallel to $P_{\text{max(Left)}}$; but, on the right side of point C, the maximal principal curvature direction is parallel to $P_{\text{max(Right)}}$. So, when cutter gets to point C in machining, it can either change its feed direction 90° or lost direction guidance to follow.

![Figure 4.19 Principal Curvature Direction Switch](image)

For the surface shown in Fig.4.20, when cutter feed direction follows the maximal principal curvatures, the cutter inclination characteristics change. At the start point, the cutter is uphill to the surface topography; but at the exit point, the cutter is downhill to the surface topography.

![Figure 4.20 Topography Change](image)
So, to realize the best possible orientation of cutter axis in real machining, the tool path might have to be split into patches.

Cutter feed and inclination direction should avoid the minimal principal curvature direction of workpiece surface because in such alignment the cutting edge is closet to the interference area when machining non-uniform principal curvature surfaces. No matter how to incline the cutter, i.e. downhill or uphill, there could be curvature gouges Fig. 4.21 if cutter feed direction is in the minimal principal curvature direction. Thus, when cutter feeds in the minimal principal curvature direction of workpiece surface, its representative sphere will always be smaller than the smallest Meusnier sphere at the CC point.

![Figure 4.21 Poor Alignment of Cutter Orientation](image)

**4.2.5 Summary**

The curvature gouge avoidance solutions can be summarized with Fig.4.22. The largest and intermediate circles in Fig.4.22 represent the largest and the smallest Meusnier spheres at the CC point. The smallest circle represents the overall smallest Meusnier sphere for the workpiece surface.

The periphery of the intermediate circle, i.e. the smallest Meusnier sphere at CC point, represents the unique curvature gouge-free solution for uniform curvature surfaces. That is, the cutter representative sphere should coincide with this circle.
when machining uniform curvature surfaces to avoid curvature gouges and to get desired machining productivity.

When machining non-uniform curvature surfaces, the periphery of the smallest circle is for the global safe solution; the area between this circle and the intermediate circle is for the cutter-active-area-safe solution and the feed-direction-optimal solution. The cutter representative sphere is always smaller than the smallest Meusnier sphere for these solutions.

The area between the largest and the intermediate circles is for the feed-direction-optimal solution and the best-possible solution. The cutter representative sphere is larger than the smallest Meusnier sphere for these solutions, and machining productivity can be enhanced. The best solution, if the machining condition permits, is the largest circle. Whatever solution it is, the cutter representative sphere cannot be outside of this largest circle.

Figure 4.22 Curvature Avoidance Solutions
Chapter 5  Identification of Proper Mill Cutter Orientation

Mill cutter orientation in 5-axis CNC machining affects productivity and machined surface quality when the cutter is not a ball mill. Particularly, the mill cutter orientation is the decisive factor for curvature gouge avoidance when the cutter is end or torus mill. Previous chapters investigate how to determine mill cutter orientations for curvature gouge avoidance. This chapter studies how to identify the proper mill cutter orientation for the planned tool paths during 5-axis CNC machining. To facilitate the study, parameters that define mill cutter orientation are first introduced.

5.1 Mill Cutter Orientation Parameters

Mill cutter orientation specifies the cutter axis direction in a given coordinate system. The coordinate system to define the mill cutter orientation is usually a local frame with workpiece surface normal $\hat{n}$ as the z-axis. The x-axis is the feed direction $\vec{f}$ on the tangent plane at the CC point; and the y-axis is derived from the x and z axes Fig.5.1.

![Local Coordinate Frame](image)

Figure 5.1 Local Coordinate Frame
Mill cutter orientation is defined by cutter inclination angle $\Gamma$ and the inclination direction angle $\delta$. Fig. 5.2.

![Figure 5.2 Cutter Orientation](image)

The inclination angle $\Gamma$ is between cutter axis $A$ and workpiece surface normal $\vec{n}$ at the CC point. This angle is measured on the orientation plane that is represented by cutter axis and workpiece surface normal $\vec{n}$. The inclination angle $\Gamma$ is the complement of the osculating angle $\phi$. Eq. 3.16.

The inclination direction angle $\delta$ is between the inclination direction and the feed direction. It is measured on the tangent plane at the CC point. The inclination direction is the intersection of the orientation plane and the tangent plane.

For uniform curvature surface machining, the inclination angle $\Gamma$ itself decides the curvature gouge avoidance solution; the inclination direction angle $\delta$ has no affects to curvature gouge avoidance. For non-uniform curvature surface, however, both the inclination angle $\Gamma$ and the inclination direction angle $\delta$ determine the curvature gouge avoidance situations.

Although the inclination direction angle $\delta$ has nothing to do with the curvature gouge avoidance solution for uniform curvature surface, it affects the curvature match situations between cutter and workpiece surface. For better curvature match, the inclination
direction should coincide with the minimal principal curvature direction of workpiece surface when the principal curvatures are constant.

5.2 Mill Cutter Orientation Realization in Tool Path Programming

In tool path programming for 5-axis CNC machining, cutter orientation is specified by the lead and tilt angles. The lead angle $\beta$ is a complement of the angle between cutter axis and the feed direction; it is measured on the plane defined by cutter axis and the feed direction Fig.5.2. The tilt angle is the same as the inclination direction angle $\delta$.

The difference between the inclination angle $\Gamma$ and the lead angle $\beta$ comes from their defining planes. The inclination angle $\Gamma$ is defined on the orientation plane. Only when the tilt angle $\delta$ is zero, the orientation plane coincides with the lead angle-defining plane.

![Figure 5.3 Cutter Orientation Angle](image)

Fig.5.3 shows the relation between cutter inclination angle $\Gamma$ and the lead angle $\beta$. The length of cutter axis $OA$ is represented by $a$; the length of line $OB$ is represented by $b$, which is the projection of $OA$ on the feed direction. The length of line $OC$ is represented by $e$, which is the projection of $OA$ on the inclination direction. The length
of line $BC$ is represented by $c$; the length of line $CA$ is represented by $d$; the length of line $BA$ is represented by $f$.

From the triangle $\triangle OBC$:

$$e^2 = b^2 + c^2 + 2b \cdot c \cdot \cos \Delta$$  \hfill (5.1)

where angle $\Delta$ is between $OB$ and $OC$.

From the triangles of $\triangle OAB$ and $\triangle ABC$

$$e^2 = a^2 - d^2$$
$$= b^2 + f^2 - d^2$$
$$= b^2 + c^2 - d^2$$
$$= b^2 + c^2$$

So,

$$\cos \Delta = 0$$

or

$$\Delta = 90^\circ$$  \hfill (5.3)

This leads to:

$$\begin{align*}
\sin \beta &= \sin \Gamma \cdot \cos \delta \\
\sin \Gamma &= \frac{\sin \beta}{\cos \delta} \leq 1; \\
\cos \delta &\geq \sin \beta \\
\delta &\leq \cos^{-1}(\sin \beta)
\end{align*}$$  \hfill (5.4)

Eq.5.4 indicates that the tilt angle $\delta$ affects the inclination angle $\Gamma$ for a given lead angle $\beta$. When the tilt angle $\delta$ is zero, the inclination angle equals the lead angle. The bigger the tilt angle, the bigger is the inclination angle when the lead angle is fixed. Therefore, the tilt angle helps to increase cutter inclination angle $\Gamma$ for a given lead angle $\beta$. In case a certain lead angle $\beta$ does not meet the curvature gouge avoidance solution, input some tilt angle $\delta$ based on Eq. 5.4 can help to increase the inclination angle $\Gamma$. However, the value of tilt angle $\delta$ is limited within the complement of lead angle; the bigger the lead angle, the smaller the allowed tilt angle.
5.3 Mill Cutter Orientation Realization on 5-Axis CNC Machine

Most of present 5-axis CNC machines are tilt-rotary type. Cutter axis rotation is realized through a system of two connected rotations, i.e. the body-fixed frame. One of these rotations, the tilt rotation, is the primary rotation that orients the axis of the secondary rotation, the rotary rotation. The tilt rotation belongs to a space-fixed frame as its rotation axis is fixed to the machine coordinate system; the rotary rotation belongs to a body-fixed frame since its rotation axis moves. The rotary rotation does not affect the tilt movement. Usually, the tilt rotation is limited; for example, on the DECKEL MAHO 80 P CNC machine, the tilt rotation is from $+45^\circ$ to $-30^\circ$ about the X-axis of the machine. The rotary rotation, however, can be $\pm 360^\circ$. The tilt rotation realizes the inclination angle for cutter axis orientation; the rotary rotation forms the inclination direction.

Cutter and workpiece relative orientations can be obtained either by moving the workpiece, e.g. on the tilt-rotary table Fig.5.4, or by cutter movements, e.g. on the routers CNC machine Fig.5.5.

![5-Axis CNC Machine](image1)

![Tilt-Rotary Table](image2)

**Figure 5.4 5-Axis CNC & Tilt-Rotary Table**
The monotonous configuration of 5-axis CNC machines, i.e. the tilt-rotary structure, has its reason. The tilt-rotary system makes the combination of two rotating motions rotation-sequence-independent. That is, the order of the two rotations does not affect their joined results, no matter which motion goes first, or whether they are moving simultaneously.

Take the tilt-rotary table Fig. 5.6 as an example, the rotation sequence of $A$ and $C$ does not affect the final orientation of $Z_T$-axis.
Figure 5.6 Tilt-Rotary Table

The primary tilt rotation $A$ is about the machine $X_W$ axis; the secondary rotation $C$ is about the body-fixed $Z_T$ axis. Use matrix $R_A(\alpha)$ to represent the primary rotation, and matrix $R_C(\gamma)$ to represent the secondary rotation, the final orientation of the table $Z_T$-axis is obtained through the joined rotation matrix $R_T$, Eq.5.5.

\[ Z_T = R_T \cdot Z_W \]  \hspace{1cm} (5.5)

Rotation matrix $R_T$ is:

\[ R_T = R_C(\gamma) \cdot R_A(\alpha) \]  \hspace{1cm} (5.6)

No matter which rotation moves first, or whether the two rotations move simultaneously, $R_T$ is the same as in Eq.5.6.

The reason that the rotation sequence does not affect the final result for the tilt-rotary table lies in the unique structure of the system. The tilt-rotary structure is an interchangeable body- and space-fixed system. When the primary rotation $R_A$ goes first, the tilt-rotary table is a body-fixed system because the secondary rotation axis changes orientations. However, when the secondary rotation $R_C$ goes first, the tilt-rotary table becomes a space-fixed system because the primary rotation axis does not change orientations with the rotation $R_C$.

From the dynamics theory, the rotation matrix formation order for two rotations $R_A$ and $R_C$ is exactly reversed in terms of body-fixed and space-fixed structure. If in a body-
fixed system, there exist two rotations in sequence with \( R_A \) going first, the matrix formation order is:

\[
R_T = R_C \cdot R_A
\]  

(5.7)

But for a space-fixed system of the same structure and rotation sequence, the matrix formation order will be:

\[
R_T = R_A \cdot R_C
\]  

(5.8)

Suppose there exists rotation sequence between the primary and secondary rotations on the tilt-rotary table, when the primary rotation \( R_A \) goes first, from the body-fixed system mechanism, the combination \( R_T \) is given as in Eq.5.6. However, if the secondary rotation \( R_C \) goes first, the tilt-rotary table becomes a space-fixed system, and the matrix formation order changes in the way that the first rotation \( R_C \) is put to the left of the following rotation \( R_A \), so that the result of \( R_T \) is the same as given in Eq.5.6. No matter which movement goes first, or whether they are moving simultaneously, the final result is always \( R_T \) as shown in Eq.5.6.

Rotation matrix \( R_T \) is:

\[
R_T = R_C(\gamma) \cdot R_A(\alpha) = \begin{bmatrix}
\cos \gamma & -\sin \gamma & 0 \\
\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & -\sin \alpha \\
0 & \sin \alpha & \cos \alpha
\end{bmatrix}
\]

(5.9)

which gives the normalized \( Z_T \)-axis:

\[
Z_T = \begin{bmatrix}
\sin \alpha \sin \gamma \\
-\sin \alpha \cos \gamma \\
\cos \alpha
\end{bmatrix}
\]

(5.10)

So,

\[
\begin{aligned}
\cos \Gamma &= \cos \alpha \\
\tan \delta &= -\frac{\cos \gamma}{\sin \gamma}
\end{aligned}
\]

(5.11)
or equally:

\[
\begin{align*}
\Gamma &= \alpha \\
\delta &= \gamma + 90^\circ \\
\delta &= \gamma - 90^\circ
\end{align*}
\] (5.12)

Angle \( \Gamma \) in Fig.5.6 is between \( Z_W \) and \( Z_T \) axes; it is the same inclination angle as in Fig.5.2. Similarly, angle \( \delta \) is the same inclination direction angle as in Fig.5.2, and is between \( X_W \) and \( Z_W-Z_T \) plane. It is measured on the \( X_W-Y_W \) plane. Angle \( \alpha \) represents the tilt-rotation; angle \( \gamma \) represents the rotary rotation.

From Eq.5.12, the inclination angle \( \Gamma \) is solely determined from the tilt-rotation angle \( \alpha \); the rotary rotation angle \( \gamma \) does not affect the inclination angle. This is obvious considering that the tilt-rotation axis orientation is space-fixed in the machine coordinate system, and does not change with the rotary motion. Besides, the rotary rotation direction can be reversed, that is, a self-rotation of \( 180^\circ \) of the rotary does not affect the inclination angle \( \Gamma \) when the tilt rotation is also reversed. So, in case one rotary motion \( \gamma \) does not meet the machining requirement, the rotary can rotate the supplement of the angle \( \gamma \) in the reversed direction to seek help. This feature has the advantage of extending the CNC machine’s tilt rotation range. For example, the DECKEL MAHO 80 P CNC machine’s tilt rotation \( \alpha \) has limitations of \( +45^\circ \) to \( -30^\circ \). If the tilt rotation reaches its \( -30^\circ \) limitation without satisfying the machining requirement, the rotary can make \( 180^\circ \) turn to bring the \( +45^\circ \) rotation range to the needed direction. However, this self-adjusting rotation can also bring disasters in real machining. The rotary’s self-adjusting rotation usually works without warning and lifting the cutter, thus workpiece surface is damaged from the motion.
Chapter 6  5-Axis CNC Machining Experiments

The first part of this chapter presents 5-axis CNC machining experiments to evaluate the Euler-Meusnier Sphere (EMS) based curvature gouge elimination solutions. The second part of this chapter presents a real machining practice with the application of the curvature gouge avoidance solution method.

6.1 5-Axis CNC Machining Experiment

Two surface models are used for the machining experiments, Fig.6.1. Surface model #1 is a piece of ellipsoid surface with axes of $100 \times 75 \times 50$ mm. This surface model is originally designed in imperial unit, and the axes are $4 \times 3 \times 2$ inches, Eq.6.1. Surface model #2 is an exponent surface, Eq.6.1. Machining is done on the concave side of both surface models. Mill cutters used in the machining are 25.4 mm (1” Dia.) end mill and ball mill. The ball mill cutter is used to produce benchmark samples for comparison.

\begin{align}
\text{Model } #1: \frac{x^2}{4^2} + \frac{y^2}{3^2} + \frac{z^2}{2^2} &= 1 \\
\text{Model } #2: z &= 50 - 40 \cdot \left(\frac{x}{60}\right) \cdot e^{-\left[\left(\frac{x}{60}\right)^2 + \left(\frac{y}{30}\right)^2\right]} \tag{6.1}
\end{align}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{surface_models.png}
\caption{Surface Models}
\end{figure}

Tool path data for the machining experiments are prepared according to the EMS based curvature gouge-free principles introduced in the early sections. The calculated
data, including the CC points, surface normal vectors and cutter orientation components at the CC points, are sent to the University of Waterloo to generate G-Code for the CNC machine. Machining experiments are performed at the University of Waterloo with their DECKEL MAHO 80 P 5-axis CNC machine Fig.6.2. Thanks to the University of Waterloo for their kindness of offering this 5-axis CNC machine and human power.

Figure 6.2 DECKEL MAHO 80 P CNC Machine

Cutter axis orientation rotations on the DECKEL MAHO 80 P 5-axis CNC machine are realized with a tilt-rotary table that supports the workpiece. Through rotating the workpiece, cutter and workpiece relative orientation angle is accomplished. The tilt-rotary table performs two rotations, i.e. the tilt-rotation about A-axis and the rotary-rotation about C-axis. The tilt-rotation axis A is the X-axis in the machine coordinate system; the rotary-rotation axis C changes orientations with the tilt-rotation. The tilt-rotation is the primary rotation that works in the space-fixed frame, i.e. the machine coordinate system. This rotation is limited in the range from $-30^\circ$ to $+45^\circ$. The rotary-rotation is the secondary rotation that works in the body-fixed coordinate system moving with the tilt-rotation. The rotary-rotation can be from $-360^\circ$ to $+360^\circ$. 
The tilt-rotation limit (-30° to +45°) imposes a problem for machining the ellipse surface where big rotation angles are required but cannot be fulfilled in the CNC machine. As a consequence, the ellipse surface piece cannot be completely machined as planned.

In Fig. 6.3 shows the ellipse surface model. The Z-vectors are parallel to the CNC machine spindle at tip points A, B and C. N- and T-vectors represent the surface normal and cutter axis orientation at corresponding points. The angle between N and T is the inclination angle for the cutter to avoid curvature gouge at each point. Taking one of the machining setups (setup 5) as an example, the cutter inclination angle $\Gamma$ is 30°. The angle between Z and T is the angle for the tilt-rotation to realize the required inclination angle $\Gamma$. To accomplish the angle between Z and T, the tilt-rotary table first performs a rotary-move to bring T and Z vectors in the same plane perpendicular to the tilt-rotation axis-A. Then, the tilt-rotary table rotates about axis-A to align T with Z.

At point A, the tilt-rotation is 30° to align T with Z. At point B and C, the tilt-rotation angle is 60°, which is over the tilt-rotation limit of +45°. So, areas close to tip points B and C cannot be machined on this CNC machine.

In this machining experiment, tool path segments that demand over 40° tilt-rotation angle are deleted to facilitate the machining.
6.2 Machining Setups

The tool paths are planned for the cutter to visit all the specified grid points on the surface. The grid points, i.e. the CC points, are calculated from squarely distributed reference points on the surface parameter plane Fig. 6.4. The intermittences between the reference points on the parameter plane are $3.81 \times 3.81$ mm, or $0.15 \times 0.15$ inch. Therefore, the step sizes along and across cutter feed direction are both $3.81$ mm on the parameter plane, i.e. $30\%$ of the cutter radius.

![Figure 6.4 Tool Path Grid](image)

<table>
<thead>
<tr>
<th>Surface</th>
<th>Setup</th>
<th>Feed</th>
<th>Grid</th>
<th>Length</th>
<th>Time</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellipse</td>
<td>#1</td>
<td>X</td>
<td>282</td>
<td>1056.48</td>
<td>N/A</td>
<td>feed-direction-optimal solution</td>
</tr>
<tr>
<td>#2</td>
<td>Y</td>
<td>314</td>
<td>1220.44</td>
<td>N/A</td>
<td></td>
<td>feed-direction-optimal solution</td>
</tr>
<tr>
<td>#3</td>
<td>45°</td>
<td>324</td>
<td>1254.72</td>
<td>N/A</td>
<td></td>
<td>feed-direction-optimal solution</td>
</tr>
<tr>
<td>#4</td>
<td>$P_{\text{max}}$</td>
<td>308</td>
<td>1109.87</td>
<td>N/A</td>
<td></td>
<td>best-possible solution</td>
</tr>
<tr>
<td>#5</td>
<td>Y</td>
<td>332</td>
<td>1291.38</td>
<td>N/A</td>
<td></td>
<td>global safe solution</td>
</tr>
<tr>
<td>#6</td>
<td>Y</td>
<td>314</td>
<td>1220.44</td>
<td>N/A</td>
<td></td>
<td>ball mill machining</td>
</tr>
<tr>
<td>Exponent</td>
<td>#7</td>
<td>Y</td>
<td>288</td>
<td>1092.1</td>
<td>7&quot;20&quot;</td>
<td>feed-direction-optimal solution</td>
</tr>
<tr>
<td>#8</td>
<td>Y</td>
<td>288</td>
<td>1092.1</td>
<td>6&quot;20&quot;</td>
<td></td>
<td>global safe solution</td>
</tr>
<tr>
<td>#9</td>
<td>Y</td>
<td>288</td>
<td>1092.1</td>
<td>6&quot;20&quot;</td>
<td></td>
<td>ball mill machining</td>
</tr>
</tbody>
</table>

* The maximal principal curvature direction
Nine machining setups are designed in this experiment to test different curvature
gouge avoidance strategies. Determination factors in designing the machining setups
include curvature gouge avoidance methods, feed directions, etc., Table 6.1.

Machining time has been recorded for cutting surface model #2. For surface model
#1, real machining time is not recorded. However, the CC points and their adding up
length for the tool paths are calculated and listed in Table 6.1. According to the data
listed, the designed tool paths are about the same length, so that the cutting times for each
setup should be comparable.

6.3 Machined Parts
Overview photos of the machined parts are shown in Fig.6.5.

The machined parts have demonstrated the validity of the tested curvature gouge
avoidance solutions. Machined surface quality comparisons reveal the effectiveness of
various curvature gouge-free solution methods.

6.4 Machined Surface Measurement
The main purpose in this machining experiment is to verify the effectives of the EMS
based curvature gouge avoidance solutions and to evaluate different solution methods
based on machined surface qualities. Checking for gouges and comparing the cusp
heights on the machined surfaces serve this purpose. Surface cusp heights and areas are
evaluated through the measured data from a Mitutoyo CMM machine Fig.6.6; this CMM
machine has a measurement tolerance of 0.001mm.

Surface measurements and quality comparisons are made for setups 2, 4, 5, and 6 of
surface model #1, and setups 7, 8 and 9 of surface model #2. These setups have the same
feed direction along Y-axis; though setup 4 has differed but close to Y-axis feed direction
from.
Figure 6.5 Machined Parts
Figure 6.6 CMM Measuring

The CMM machine measurements are made along the direction perpendicular to cutter feed *i.e.* along X-axis of the surface parameter plane. Step-size in the measurement is 0.5 mm to secure the reflection of surface variations. On surface model #1, four segments are measured; on surface model #2, three segments are measured Fig.6.7. The measured segment positions are detailed in Table 6.2.

![Image of measured segments](image)

Figure 6.7 Measured Segments

Table 6.2 CMM Measured Segments

<table>
<thead>
<tr>
<th>Segment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unit (mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model #1</td>
<td>X_{start}</td>
<td>10</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>X_{end}</td>
<td>24</td>
<td>64</td>
<td>24</td>
</tr>
<tr>
<td>Model #2</td>
<td>Y</td>
<td>10</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>X_{start}</td>
<td>40</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>X_{end}</td>
<td>54</td>
<td>84</td>
<td>84</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>20</td>
<td>20</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

For each measured segment, data graph is drawn, typical cusp height is measured; average value of cusp heights and areas are calculated with Eq.6.2.

\[
\begin{align*}
\bar{H} &= \frac{\sum_{i=1}^{N} h_i}{N} \\
\bar{A} &= \sum_{i=1}^{N} \tau \cdot \frac{(h_i + h_j)}{2}
\end{align*}
\]  

(6.2)

where $\bar{H}$ and $\bar{A}$ are average cusp heights and areas for each segment; $\tau$ is the measurement step size (0.5 mm); $h_i$ is the measured height value at each point; $N$ is the total data number measured on each segment.

**6.4.1 Set-up #2 CMM Measurement**

Fig.6.8 and table 6.3 present the measured CMM data and calculated average cusp height and area for setup #2.

**Table 6.3 Setup 2 Data Measurement**

<table>
<thead>
<tr>
<th>Segment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (h)</td>
<td>0.1032</td>
<td>0.1333</td>
<td>0.1290</td>
<td>0.4505</td>
<td></td>
</tr>
<tr>
<td>Average Height ($\bar{H}$)</td>
<td>0.0456</td>
<td>0.0633</td>
<td>0.0541</td>
<td>0.1900</td>
<td>0.0882</td>
</tr>
<tr>
<td>Area ($\bar{A}$)</td>
<td>0.6350</td>
<td>0.8842</td>
<td>0.7515</td>
<td>2.6366</td>
<td>1.2268</td>
</tr>
</tbody>
</table>
6.4.2 Set-up #4 CMM Measurement

Fig. 6.9 and table 6.4 present the measured CMM data and calculated average cusp height and area for setup # 4.

Table 6.4 Setup 4 Data Measurement

<table>
<thead>
<tr>
<th>Segment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (h)</td>
<td>0.0468</td>
<td>0.0702</td>
<td>0.1277</td>
<td>0.1474</td>
<td></td>
</tr>
<tr>
<td>Average Height ($\bar{H}$)</td>
<td>0.0201</td>
<td>0.0160</td>
<td>0.0271</td>
<td>0.0556</td>
<td>0.0297</td>
</tr>
<tr>
<td>Area ($\bar{A}$)</td>
<td>0.2784</td>
<td>0.2050</td>
<td>0.3488</td>
<td>0.7802</td>
<td>0.4031</td>
</tr>
</tbody>
</table>
6.4.3 Set-up #5 CMM Measurement

Fig. 6.10 and table 6.5 present the measured CMM data and calculated average cusp height and area for setup # 5.

<table>
<thead>
<tr>
<th>Segment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (h)</td>
<td>0.1149</td>
<td>0.1527</td>
<td>0.1720</td>
<td>0.9511</td>
<td></td>
</tr>
<tr>
<td>Average Height ((\bar{H}))</td>
<td>0.0527</td>
<td>0.0722</td>
<td>0.0767</td>
<td>0.1989</td>
<td>0.1002</td>
</tr>
<tr>
<td>Area ((\bar{A}))</td>
<td>0.7345</td>
<td>1.0059</td>
<td>1.0656</td>
<td>2.6408</td>
<td>1.3617</td>
</tr>
</tbody>
</table>
6.4.4 Set-up #6 CMM Measurement

Fig.6.11 and table 6.6 present the measured CMM data and calculated average cusp height and area for setup # 6. Only segment 1 is presented.

Table 6. 6 Setup 6 Data Measurement

<table>
<thead>
<tr>
<th>Segment</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (h)</td>
<td>0.2218</td>
</tr>
<tr>
<td>Average Height ($\bar{H}$)</td>
<td>0.0947</td>
</tr>
<tr>
<td>Area ($\bar{A}$)</td>
<td>1.315</td>
</tr>
</tbody>
</table>
6.4.5 Set-up #7 CMM Measurement

Fig. 6.12 and table 6.7 present the measured CMM data and calculated average cusp height and area for setup # 7.

<table>
<thead>
<tr>
<th>Segment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (h)</td>
<td>0.0649</td>
<td>0.1054</td>
<td>0.0489</td>
<td></td>
</tr>
<tr>
<td>Average Height ($\bar{H}$)</td>
<td>0.0313</td>
<td>0.0507</td>
<td>0.0222</td>
<td>0.0347</td>
</tr>
<tr>
<td>Area ($\bar{A}$)</td>
<td>0.4355</td>
<td>0.7094</td>
<td>0.3103</td>
<td>0.4851</td>
</tr>
</tbody>
</table>
6.4.6 Set-up #8 CMM Measurement

Fig 6.13 and table 6.8 present the measured CMM data and calculated average cusp height and area for setup # 8.

<table>
<thead>
<tr>
<th>Segment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (h)</td>
<td>0.0957</td>
<td>0.1255</td>
<td>0.1021</td>
<td></td>
</tr>
<tr>
<td>Average Height ((\bar{H}))</td>
<td>0.0526</td>
<td>0.0627</td>
<td>0.0469</td>
<td>0.0541</td>
</tr>
<tr>
<td>Area ((\bar{A}))</td>
<td>0.7374</td>
<td>0.8780</td>
<td>0.6562</td>
<td>0.7572</td>
</tr>
</tbody>
</table>
6.4.7 Set-up #9 CMM Measurement

Fig. 6.14 and table 6.9 present the measured CMM data and calculated average cusp height and area for setup #9. Only segment 1 is presented.

<table>
<thead>
<tr>
<th>Table 6.9 Setup 9 Data Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>Height (h)</td>
</tr>
<tr>
<td>Average Height ($\bar{H}$)</td>
</tr>
<tr>
<td>Area ($\bar{A}$)</td>
</tr>
</tbody>
</table>
6.5 Experiment Results Analysis

CMM measurements of the machined surfaces reflect characteristics of corresponding tool path. Put the measurements from different setups together, the quality and/or roughness of the machined surfaces are reviewed; also reviewed are the effectiveness of different tool path planning strategies.

6.5.1 Data Analysis

Plot the measured data against corresponding surface curves Fig.6.15, curvature gouge elimination result is illustrated. The fact that all the curves from the measured data locate above the green surface curve indicates that curvature gouge problem or undercut does not exist.

In Fig.6.15, the green bottom curves are the plotted surface curve, which are calculated from the surface guiding functions, Eq. 6.1. All the other curves represent the CMM measured data from different setups.
Figure 6. 15 Measured Data vs. Surface Model

The distance from the measured data curve to the surface model curve results from many factors including machining system errors, but mostly they come from the tool path interpolation mechanism error. Tool path uses line segments to interpolate
or approximate the surface curves. The line segments meet the surface model only at the planned grid points on surface; the grid points are positioned with equal intermittence of 3.81mm in both \(X\) and \(Y\) directions. Because the measured segments do not pass through the grid points, they do not meet the surface model. Tool path interpolation mechanism error is shown by \(\Delta h\) in Fig. 6.16. The interpolation mechanism error has nothing to do with curvature avoidance strategy evaluation; so, this mechanism error is compensated in latter analysis.

The distances between each measured data curve represent the quality of different curvature gouge avoidance strategies.

![Figure 6.16 Mechanism Error](image)

In the following data analysis, the mechanism error and other machining system errors are compensated by putting the analyzed data curve at the common zero position. The effectiveness of different curvature gouge avoidance solutions are exposed without the disturbance from tool path interpolation schemes.

### 6.5.2 Different Cutter

End mill and ball mill cutter have different effects on the machined surface, Fig.6.17.
Figure 6.17 Segment 1 Data

In Fig.6.17, CMM data from different setups are plotted on the same graph for each surface model; surface roughness can be evaluated from the variation of the curves. In model #1, the best surface quality is produced in set up 4, represented by the red area in Fig.6.17 (a); the poorest surface quality is produced from the ball mill cutter represented by the black area. Global safe solution, the purple area, is better than ball mill cutter machining, but poorer than the feed-direction-optimal solution which is represented by the blue area. The feed-direction-optimal solution is not as good as the best-possible solution setup 4. Further analysis, table 6.10, reveals that the cusp area of setup 4 is about 5 times less than the ball mill machined surface; the
cusp areas are 2 times less on setup 2 and 5 compared with the ball mill machined surface.

The same results can be found on model #2, Fig.6.17 (b). Blue area represents the feed-direction-optimal solution, which is the best one; the global safe solution, the purple area, is poorer than the feed-direction-optimal solution, but better than ball mill machined surface, the black area. Data analysis, table 6.10, reveals that the feed-direction-optimal solution has 3 times less cusp area than ball mill machined surface, the global safe solution has 2 times less cusp area than the ball mill machined surface.

**Table 6.10 Data Analysis**

<table>
<thead>
<tr>
<th>Model # 1</th>
<th>Setup</th>
<th>Peak</th>
<th>Av. Height</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.1032</td>
<td>1/2.15</td>
<td>0.0456</td>
<td>1/2.08</td>
</tr>
<tr>
<td>4</td>
<td>0.0468</td>
<td>1/2.74</td>
<td>0.0201</td>
<td>1/4.71</td>
</tr>
<tr>
<td>5</td>
<td>0.1149</td>
<td>1/1.93</td>
<td>0.0527</td>
<td>1/1.8</td>
</tr>
<tr>
<td>6</td>
<td>0.2218</td>
<td>1</td>
<td>0.0947</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model # 2</th>
<th>Setup</th>
<th>Peak</th>
<th>Av. Height</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.0649</td>
<td>1/4.08</td>
<td>0.0313</td>
<td>1/3.13</td>
</tr>
<tr>
<td>8</td>
<td>0.0957</td>
<td>1/2.77</td>
<td>0.0526</td>
<td>1/1.86</td>
</tr>
<tr>
<td>9</td>
<td>0.2647</td>
<td>1</td>
<td>0.0980</td>
<td>1</td>
</tr>
</tbody>
</table>

In summary, the end mill cutter, with proper gouge avoidance strategies, will produce much better surface than the ball mill cutter. Surface quality can be improved 2 to 5 times corresponding to different curvature gouge avoidance schemes, or equally, machining time can be reduces to the same scale.

**6.5.3 Different Gouge Avoidance Strategies**

Different curvature gouge avoidance strategies produce different quality surfaces, Fig. 6.18. Generally, the best-possible solution, of which the cutter feed in the maximal principal curvature direction, produces the best quality surface finish. This is shown in Fig.6.18 (a), the red area. Cusp areas produced in this tool path are overwhelmingly smaller; cusp heights are generally low. These factors indicate that the machined
surface is more close to the designed model. Further analysis, Table 6.11, shows that the average-cusp-area from the best-possible-solution tool path is 3 times smaller than the average-cusp-areas from both global safe and feed-direction-optimal solution tool paths, even though the so-called best-possible solution is not the best possible one considering that the cutter inclination is not always up-hill to the surface topography in the tool path.

**Table 6.11 CMM Data Analysis**

<table>
<thead>
<tr>
<th>Segment</th>
<th>Setup</th>
<th>Peak</th>
<th>Av. Height</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.1032</td>
<td>1.11</td>
<td>0.0456</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0468</td>
<td>2.46</td>
<td>0.0201</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.1149</td>
<td>1</td>
<td>0.0527</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.1333</td>
<td>1.15</td>
<td>0.0633</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0702</td>
<td>2.18</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.1527</td>
<td>1</td>
<td>0.0722</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.129</td>
<td>1.33</td>
<td>0.0541</td>
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<td></td>
<td>4</td>
<td>0.1277</td>
<td>1.35</td>
<td>0.0271</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.172</td>
<td>1</td>
<td>0.0767</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.4505</td>
<td>2.11</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.1474</td>
<td>6.45</td>
<td>0.0556</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.9511</td>
<td>1</td>
<td>0.1989</td>
</tr>
<tr>
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Figure 6. 18 CMM Data of Machined Surface

From table 11, the feed-direction-optimal solution produces better surface quality than the global-safe solution, with an average cusp-area reduced about 10% to 56%; the best-possible solution further reduces the cusp area by more than 300% times.
6.6 Real Industry Machining Practice

Presented here is a real machining production example for the application of the proposed curvature gouge avoidance method. The machining is done at a company in Fort Walton, Florida, USA. The machined part Fig.6.19 is used on helicopter tail adjustment component. The top surface of this part is to be machined on a 5-axis CNC machine. The surface is cylindrical and traditionally machined with a 1” ball mill cutter at side step of 0.005 inches. Applying the EMS based curvature gouge-free solution, esp. the global safe solution, a 1-inch end mill cutter is used at side step of 0.100 inch. The machined surface shows superior finish quality than previously made; and the tool path length is cut down about 20 times due to the wide side steps.

![Surface to be Machined](image)

**Figure 6.19 Machined Part**

In Fig.6.20 shows the tool path comparison from the end mill and ball cutters. On the right side shows two adjacent 1-inch ball mill cutters apart at 0.020-inch distance (it is difficult to draw them at 0.005-inch distance). On the left side shows two adjacent 1-inch end mill cutters apart at 0.100-inch distance. Corresponding tool paths for each type of cutters are also drawn for observation. Tool path (1) is for the end mill cutter; tool path (2) is for the ball mill cutter.

Since the machined surface has a cylindrical shape, PAM solution can also be used. But careful consideration reveals that the PAM solution would demand the cutter move in the minimal principal curvature direction on the machined surface. And this would result in a tool path of which the length in each segment is too short, and the cutter has to frequently change feed directions, as shown in Fig.6.21. Such tool path is impractical to use.
6.7 Summary

To summarize, the 5-axis CNC machining experiments and real production example have verified the validity of the EMS based curvature gouge-free solution methods. Comparisons from the machined surface qualities have revealed the effects of different solution methods on surface finishes. Further analysis of the tested tool path shows the advantages of the EMS based curvature gouge-free solution methods over conventional solution methods, such the PAM, in dealing with curvature gouge problems.
6.7.1 Effectiveness of the Curvature Gouge-Free Solutions

The effectiveness of the EMS based curvature gouge-free solutions are demonstrated both through computer simulations and the real machining experiments. Computer simulation tests are first performed with Pro/Engineering (CAD/CAM) software Fig.6.20. Then, prior to real machining, the G-Code of the tested tool paths are simulated at the University of Waterloo, using the program developed there. All the tool paths are proven gouge-free from the simulation tests before they are uploaded to work on the CNC machine. Checking on the machined surfaces also proves the elimination of curvature gouge problems and any other irregularities Fig.6.5.

![Surface Model #1](image1.png) ![Surface Model #2](image2.png)

Figure 6.22 Pro/Engineering Simulation

From both the real machining and computer simulation tests, it is observed that the specified cutter axis orientations are vital; no further modification is acceptable. Any amendment for the cutter orientation angle can either incur curvature gouges or reduce the curvature match quality. So, the tested tool paths can be considered benchmarks for evaluating other solution methods.

6.7.2 Surface Quality from Various Solution Methods

The CMM measurement results in table 6.10 and 6.11 demonstrate that different solution methods produce different quality surfaces. The end mill cutter generally produces better surface finishes than does the ball mill cutter. The cusp areas from the end mill cutter are 2 to 5 times less than that from the ball mill cutter. These
observations conform to visual and feel checks on the machined parts. Even for the global safe solution, setups 5 and 8, the end mill cutter produces much smoother surfaces than does the ball mill cutter.

When machining with an end mill cutter, the feed-direction-optimal solutions, setups 2 and 7, produces smoother surfaces than does the global safe solution of setups 5 and 8; the cusp area can be reduced by 10-56%. Surface quality is even better with the best-possible solution; the cusp area is further reduced to 338%.

### 6.7.3 Advantages of the EMS Based Solution Methods Over the PAM

The EMS based curvature gouge-free solution methods demand that the milling cutter be confined in its representative sphere. The size of the cutter representative sphere depends on the cutter axis orientation angle. So, from the cutter orientation angle, the cutter representative sphere at each cutting position can be determined. Studying the relations between the radius of the cutter representative sphere and the radius of the workpiece surface maximal principal curvature, the advantages of the EMS based curvature gouge-free solution methods are revealed, Fig.6.23.

In Fig.6.23 shows the comparisons of the radius of the end mill cutter representative sphere and the radius of the maximal principal curvature of the machined surface. The comparisons are made on setups 1, 2, 3, 4, and 7. Setups 5 and 8 are not included in the comparison because the radius of cutter representative sphere in these setups is constant. Setups 6 and 9 are for ball mill cutter, and are not included either. The red surfaces represent the radius of cutter representative sphere; the green ones represent the radius of workpiece surface maximal principal curvature.

The radius of cutter representative sphere in setup1, 2, and 3 is generally smaller than the radius of corresponding maximal principal curvature of the machined surface; this is observed from the red surfaces laying underneath the green ones in Fig.6.23. However, for setups 4 and 7, this situation changes. The red surfaces and the green ones are close to each other; the read and green surfaces intersect with the red surfaces coming above the green ones in some areas. This phenomenon indicates that when cutter feeds in certain directions, the radius of the cutter representative
sphere can be as much as the radius of the workpiece surface maximal principal curvatures, and at some locations, the radius of the cutter representative sphere can even be larger than that of the maximal principal curvature of the machined surfaces. The phenomena is understandable on setup 4 since the cutter feed direction is along the maximal principal curvature; and at certain areas, the cutter is inclined uphill to the surface topography. The reason that cutter representative sphere radius is larger than the surface maximal principal curvature for setup 7 is that the surface has saddle areas where the principal curvatures have different signs. At saddle areas, curvature gouge possibility is less, so the machining condition is better.

Observations from Fig.6.23 indicate that the feed direction has effects on cutter orientations, which further affects the machined surface qualities. When the cutter feeds in the maximal principal curvature direction, such as setups 4, it can take even smaller curvatures than the maximal principal curvature of the machined surface. As a result, the radius of the cutter representative sphere is close to or larger than that of the maximal principal curvature of the surface. This is advantageous to surface machining because both the material removal and the surface smoothness can be improved with a larger curvature radius of the cutter surface. When cutter feed direction is set other ways, the radius of the cutter representative sphere will be smaller than that of the maximal principal curvature of workpiece surface, which is disadvantageous to surface machining.

Observations also reveal that local maximal principal curvature radius of the machined surface generally cannot be used to guide cutter axis orientation for curvature gouge avoidance purpose when the surface possesses non-constant principal curvatures. Using the maximal principal curvature radius to define the orientation of the cutter can either incur curvature gouges for machining setups of 1, 2 and 3, where the radius of the cutter representative sphere is generally smaller than that of the maximal principal curvature radius of the surface, or lead to less optimal curvature matches for machining setups of 4 and 7, where the radius of cutter representative sphere can be larger than that of the maximal principal curvature of the surface.
Figure 6.23 Radiuses Comparisons

Applying PAM for the tested machining setups will either induce curvature gouge problem or result in less optimal curvature match. The feed direction in setup1 is close to PAM solution where cutter feed in the surface minimal principal curvature direction. In setup1, the radius of cutter representative sphere is normally smaller than that of the maximal principal curvature of the workpiece surface. This means
that PAM solution will incur curvature gouge if it is used to machine this surface because it takes the maximal principal curvature radius to define the cutter orientation. On the other hand, at areas where the cutter representative sphere radius can be larger than the surface maximal principal curvature radius (setup 4), PAM result in less curvature match.
Chapter 7 Conclusions

This study introduces some fundamental principles for solving curvature gouge problems in sculptured surface machining. The concepts of Euler-Meusnier sphere and mill cutter representative sphere, as well as the related curvature gouge elimination methods have proven to be successful applied to curved surface machining. These works lay foundations for further studies.

7.1 Summary

Curvature gouge problem involves the interaction of curvature geometries from cutter and workpiece surfaces. A deep understanding and correct modeling of surface curvature geometry are vital to solve curvature gouge problems. This work introduces a 3D generic mathematical model that portrays surface curvature geometries of curvature continuous (G²) surface. This curvature geometry model is named as the Euler-Meusnier sphere (EMS). Based on the concept of EMS, some successful curvature gouge elimination solution methods are presented.

Chapter 2 introduces the concept of EMS. The EMS model is a collection of co-tangent spheres. The creation of the EMS model comes from the combination of the Euler and Meusnier theorems from the differential geometry.

Curvature gouge problem is the result of cutter and workpiece surface interaction. Surface curvature geometry model, i.e. the EMS, is applicable to both workpiece and cutter surfaces. Chapter 3 presents the application of the EMS to mill cutters, which leads to the concept of mill cutter representative sphere. With the help of mill cutter representative sphere, investigation of curvature gouge problems is simplified.

Workpiece surface contributes the most complicated factor to curvature gouge problems in sculptured surface machining. The diversity of workpiece surfaces determines that the methods for solving curvature gouge problems are miscellaneous. Chapter 4 presents curvature gouge-free criteria based on the characteristics of workpiece surface curvature geometries.
To apply the established curvature gouge-free solutions for sculptured surface machining, related parameters in tool path programming and CNC machine transmission are to be clarified. Chapter 5 defines cutter orientation parameters in terms of tool path programming and CNC machine transmission.

Chapter 6 presents machining experiment results and data analysis from a DECKEL MAHO 80 P 5-axis CNC machine and a Mitutoyo CMM machine.

### 7.2 Research Contribution

- A rigorous generic mathematical model portraying surface curvature geometries, *i.e.* the Euler-Meusnier sphere (EMS), is established.
- This 3D curvature model serves as a constraint to define cutter size and orientation angle in tool path planning and optimization.
- Based on the 3D curvature model, the concept of cutter representative sphere is introduced.
- 3D curvature gouge-free criterion is established.
- Application of the curvature gouge-free solutions on end and/or torus mill cutters is implemented.
- The introduced curvature model and the gouge-free criterion lay foundations for further studies.

### 7.3 Future Work

The presented curvature gouge-free criteria are important breakthroughs in sculptured surface machining. They have great application opportunities in manufacturing industry. Present commercial CAD/CAM software, such as the Pro/Engineering, UGS, and MasterCAM, *etc.*, are incapable of dealing with curvature gouge problems, which hinders the application of the end and/or torus mill cutters for curved surface machining. Converting the research results to industry practice is the next step for this study.
References


