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The Buoyancy Forcing and Dynamical Response of the Red Sea

by

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M.Sc.

A Thesis submitted in Partial Fulfillment of the
Requirements for the Degree of

DOCTOR OF PHILOSOPHY
IN THE SCHOOL OF EARTH AND OCEAN SCIENCES

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Abstract

The buoyancy forcing of the Red Sea and its dynamical response are examined. Buoyancy transports through the Strait of Bab el Mandab, the major oceanic exchange point of the Red Sea with the open ocean, provide a strong constraint on the surface buoyancy fluxes. Hydrographic data and current records at the Strait require the annual mean surface heat flux to be $-8 \pm 2 \, \text{W m}^{-2}$. For the annual mean freshwater fluxes the conservation of volume and salt give the net evaporation rate as $1.60 \pm 0.35 \, \text{m y}^{-1}$.

The surface fluxes estimated from the heat and freshwater transports at the Strait are compared to the annual mean surface fluxes estimated from standard meteorological data sets and formulae used on a global scale as in the revised Comprehensive Ocean-Atmosphere Data Set (UWM/COADS). The difference between the surface heat fluxes and that implied by the exchange through the Strait is large and close to $100 \, \text{W m}^{-2}$. A large portion of this difference is explained by the overestimated solar irradiance due to the neglect of spatial and seasonal variations of aerosol concentration, and misapplication of a standard formula for insolation. Another portion of the difference comes from the underestimated longwave radiation due to the use of a bulk formula which is adequate for the open ocean but inappropriate for the Red Sea. The evaporative losses are also found to be underestimated, probably because of underestimated wind speeds. The net evaporation is the main contributor to the annual mean buoyancy loss approximately of $2 \times 10^{-8} \, \text{m}^2 \text{s}^{-3}$.

The annual mean surface buoyancy flux, which is compatible with the oceanic buoyancy transport, is used with Phillips’ similarity model to investigate
the buoyancy driven flow of the upper 140 m of the Red Sea. The observed stratification of the Red Sea can be achieved only with a very large eddy viscosity in the return flow. It is possible that this high vertical viscosity could be a proxy for processes neglected by this model such as bottom friction on the sloping boundaries. The effect of wind stress is small, but a southward wind combined with the bottom friction of a modified model with depth-dependent basin width could account for the viscous force required by a model.

The effectiveness of the bottom friction in retarding the flow depends on the magnitude of the lateral diffusion of momentum. To explore the possibility of measuring the horizontal momentum fluxes above a sloping boundary in a channel, we performed an experiment in the Strait of Georgia with two Acoustic Doppler Current Profilers. Although further investigation of such measurements is required and several issues remain to be resolved, it is shown that an estimate of the horizontal eddy viscosity acting on the tidal currents is possible with this method and gives about 50 m$^2$s$^{-1}$.

Overall, the dynamics of the Red Sea appears to be determined mainly by the surface buoyancy fluxes and internal and lateral frictional forces. Accurate modelling of the Red Sea requires improved knowledge of the forcing and of an appropriate parameterization of the friction.
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Chapter 1

Introduction

1.1 Motivation and thesis outline

Improving our understanding of the oceanography of small, semi-enclosed basins is important for the countries surrounding them, but may also be of considerable scientific interest, especially if the processes encountered there have global relevance. The Red Sea is an example of such a water body. Because of its small size and simple geometry, the Red Sea provides an ideal natural laboratory for the study of problems of general interest. These problems include: (i) The buoyancy driven circulation of an inverse estuary with turbulent convection. (ii) The evaluation of meteorological datasets and bulk parameterizations for the surface buoyancy fluxes that are used on a global scale, exploiting the semi-enclosed nature of the Red Sea. (iii) Problems related to the hydraulic control exerted by a strait and a sill and its importance on the circulation and stratification of a semi-enclosed basin. (iv) The role of horizontal eddy viscosity above sloping topography on the dynamics of the flow in a channel.

Apart from these problems of broader relevance, the Red Sea is an interesting basin in its own right because of the unique phenomena taking place there. The exceptionally strong evaporation rate of the Red Sea (of almost 2 m year\(^{-1}\)) is the cause for the very high surface salinity of more than 40 at the northern end – the highest surface salinity in the world ocean apart from very small lagoons. This surface buoyancy loss combined with the geometry of the basin leads to the formation of deep waters with the highest temperature and salinity in the world. Salty
intermediate and deep water produced in the Red Sea flows into the Indian ocean and could possibly affect its thermohaline circulation; Red Sea water can be traced as far as the southern hemisphere. Finally, the Red Sea is an example of the first stage of oceanic basin formation due to the spreading of tectonic plates.

Despite the fascinating oceanography of the Red Sea, it is a very poorly surveyed basin. The main reason for the lack of synoptic observations is that the Red Sea is surrounded by developing or third world countries which are in hostile relations. These countries face severe problems from lack of freshwater and the examination of the hydrological cycle of the area, with the Red Sea being an integral part of it, could provide valuable information towards solutions to the problem. For the western world, the study of the Red Sea is interesting because of its geographic location; the Red Sea is in an area of high oil production and it provides an important route between the Mediterranean and the Indian Ocean.

In this thesis, some introductory information about the geography, forcing, hydrography and circulation patterns of the Red Sea will be provided in Chapter 1. In Chapter 2, a detailed review of the observations of the heat and freshwater fluxes through the Strait of Bab el Mandab is presented. These fluxes provide a strong constraint for the surface buoyancy fluxes estimated from meteorological parameters and bulk formulae, examined in Chapter 3. The dynamical response of the Red Sea to the buoyancy forcing using a simple similarity model is investigated in Chapter 4. Questions about the dynamical balance in the Red Sea lead to the examination of horizontal Reynolds stress in Chapter 5, where an attempt to determine horizontal Reynolds stress from ADCP records is presented. General conclusions from this thesis are presented in Chapter 6.
1.2 Geography

The Red Sea is a deep and narrow semi-enclosed basin that separates Africa from Asia (Figure 1.1). In the south, it is connected to the Gulf of Aden and the open ocean through the Strait of Bab el Mandab. The Suez Canal at the north provides a navigational connection of the Red Sea to the Mediterranean, but is not considered an important oceanographic exchange route. The Red Sea has a channel-like shape with a length of 1932 km and an average width of 280 km (Morcos 1970). The Strait of Bab el Mandab is the narrowest point with a width of 26 km at Perrim Narrows, but the shallowest depth (137 m) of the mouth occurs about 150 km northwest of Bab el Mandab at the Hanish Sill. At the north end, the Red Sea bifurcates into two narrow gulfs: the shallow Gulf of Suez (of about 40 m depth) without a sill at the mouth, and the deep Gulf of Aqaba (with a depth of more than 1800 m) which is separated from the Red Sea by a sill of less than 300 m (Laughton 1970).

The average depth of the Red Sea is 491 m (Morcos 1970), but abyssal depths of more than 2500 m have been recorded along the axial trough of the basin (Tomczak and Godfrey 1994). Topographic sections across the Red Sea (Allan 1970) show very wide shallow shelves (less than 50 m deep) in the southern portion occupying more than two thirds of the basin width and there are broad banks of submerged reefs that form a complicated bathymetry (Ross 1983). The width of the shelves becomes narrower towards the central and northern parts. Large island systems exist close to the southern end, off the coasts of Eritrea and Yemen.

The Red Sea is considered a model for sea-floor spreading as a mechanism for the formation of an oceanic basin (Ross 1983). The rich geothermal and geomagnetic activity on the bottom of the Red Sea has been associated with the
Fig. 1.1. Bathymetric chart of the Red Sea.
1. Introduction

sea-floor spreading, and it is the first region in the world where hot brines were discovered on the sea floor (Miller 1964).

1.3 Climate conditions

The climate of the Red Sea area is extremely arid. Evaporation rates are exceptionally high and of the order of 2 meters per year (Yegorov 1950; Neumann 1952; Privett 1959). Precipitation is very small and never exceeds 20 cm per year even in the convergence zone (Pedgley 1974). There is no river runoff in the Red Sea.

Regarding the heat flux in the Red Sea, it has long been recognized that there is a small annual overall surface heat loss of less than 10 W m\(^{-2}\) (Yegorov 1950), and that the oceanic heat transport through the Strait provides a strong constraint on the surface fluxes (Bunker et al. 1982). A review of the available measurements of the oceanic heat transport will be given in Chapter 2 and a detailed examination of each component of the surface heat and freshwater budgets will be presented in Chapter 3. For example, it has been speculated that the extensive desert areas surrounding the Red Sea are sources of aerosol particles, such as sand and dust, that could be responsible for the depletion of incoming solar radiation (Neumann 1952), and this will be examined in detail.

The wind stress climatology is influenced by the high mountains and plateaux that border the Red Sea, while the monsoons are responsible for the seasonal changes in the wind direction. As shown in Figure 1.2, monthly mean winds are, on average, directed along the main axis of the basin; the data are obtained from the revised version of the Comprehensive Ocean-Atmosphere Data Set (COADS) by da Silva et al. (1994) at the Department of Geoscience of the University of Wisconsin-Milwaukee (henceforth referred to as UWM/COADS). North of about
Fig. 1.2. Climatological monthly mean wind stress (45-year averages from 1945 to 1989 from UWM/COADS).
1. Introduction

19°N the winds are from the north-northwest throughout the year. South of 19°N the winds change direction under the influence of the monsoon system. From October through December the convergence zone of the winds is at around 19°N, but after December the convergence zone moves southward, so that by June and until September the winds are from the northwest over the entire basin. Overall, winds are stronger during winter months.

1.4 Hydrographic structure

The Red Sea is very poorly surveyed. Most of the water property measurements are taken along the main axis of the basin, whereas very few synoptic cruises contain any cross sections. From all the surveys carried out in the Red Sea during the last 50 years only 4 cruises contain extensive cross-section measurements. These are the cruises of the R.V. *Atlantis* in May 1958, the *Cdt. R. Giraud* during January-February 1963, the R.V. *Sonne* in January-March 1984, and more recently, the NAVOCEANO survey in June 1993 and August 1994. The north-south hydrographic structure is thus relatively better surveyed, while there is very little confidence in the cross-axis property distribution.

Climatological monthly mean values for the sea surface temperature (SST), salinity (SSS), and potential density ($\sigma_\theta$) in January, April, July, and October are presented in Figure 1.3 (the data for temperature and salinity have been obtained from the UWM/COADS climatology). The SST (Figure 1.3a) increases from the northern end towards maximum values in the southern Red Sea and decreases again close to the Strait of Bab el Mandab. The zone of maximum temperature, which in late summer occurs in the south at around 14°N (with SST higher than 31°C), moves northward up to 19°N in the winter (with SST higher than 26°C). This phenomenon is attributed to the reversal of the winds that pile up water towards
Fig. 1.3. Monthly mean climatology of the Red Sea for January, April, July and October (data from UWM/COADS). (a) Sea surface temperature in °C, (b) Sea surface salinity, and (c) $\sigma_q$ in kg m$^{-3}$.
1. Introduction

the central zone of the Red Sea during winter. In summer, the north-northwestern winds dominate throughout the basin causing the southward transport of the temperature maximum (Morcos 1970). The seasonal variation of the monthly mean SST varies with position and is higher at the northern part (where the seasonal change is about 8°C) and smaller in the area of Bab el Mandab (where the seasonal change is about 5°C).

The distribution of SSS (Figure 1.3b) reveals the most striking feature of the Red Sea: the highest surface salinity in the world oceans. This occurs as a result of the strong evaporation rate. Sea-surface salinity increases from about 36.5 near Bab el Mandab Strait to more than 40 at the northern end of the basin. Seasonal variations show that the SSS is higher in fall than in spring at all latitudes. At the northern end, salinity ranges from 40 to 40.5 in winter, but in summer it reaches values of more than 41. The SSS near Bab el Mandab Strait increases slightly during summer months as the north-northwestern winds carry salty Red Sea surface water into the Gulf of Aden.

Figure 1.3c shows that throughout the year there is a strong north-south potential density difference of up to 4 kg m\(^{-3}\). Maximum surface densities are observed during winter and the north end density reaches \(\sigma_\theta = 28\) kg m\(^{-3}\) or more.

The vertical distribution of the hydrographic properties in the Red Sea is shown in Figures 1.4 and 1.5. These are hydrographic sections along the main axis of the basin for winter and summer from the cruises of Cdt. R. Giraud in January-February 1963 and the R.V. Atlantis in August 1963, respectively. It is evident that most of the stratification occurs in the top 150 m above the sill depth.

During winter (Figure 1.4), a 50 m thick surface inflow of warm and low-salinity water (\(T \approx 25^\circ\text{C}, S \approx 36.5, \sigma_\theta \approx 25\) kg m\(^{-3}\)) through the Strait of Bab el Mandab is observed. This surface layer becomes saltier as it advances through the
Fig. 1.4. Hydrographic sections along the axis of the Red Sea from the Gulf of Suez to the Strait of Bab el Mandab using data for the potential temperature (upper panel), salinity (middle panel), and $\sigma_{\theta}$ (lower panel) from the cruise of Cdt. R. Giraud during January-February 1963.
Red Sea until it reaches a maximum salinity of more than 40 at the northern part of the Red Sea. Water of higher density is thus produced due to evaporation. It sinks and flows out of the basin in a lower layer of salinity between 37 and 40. It is likely that the outflowing intermediate Red Sea water is produced throughout the basin on the shallow continental shelves of the Red Sea and not only at the far north end (Maillard 1974). The outflowing salty Red Sea intermediate water spreads into the Indian Ocean, much as Mediterranean water spreads into the Atlantic, and significantly affects the structure of the northwest Indian Ocean (Shapiro and Meschanov 1991).

During summer the wind changes direction and blows out of the Red Sea into the Gulf of Aden (Figure 1.5), causing an outflow of the top 20 m of warm and salty Red Sea water ($T \approx 30^\circ C$, $S \approx 37.5$, $\sigma_\theta \approx 23$ kg m$^{-3}$). Below this shallow outflow, a cool low-salinity inflow from the Gulf of Aden ($T \approx 18^\circ C$, $S \approx 36$) between 30 and 80 m depth is observed (Maillard and Soliman 1986). This intrusion propagates into the Red Sea at a speed of 0.06 m s$^{-1}$ (Smeed 1988) and is detected as far north as 19°N until October when it vanishes. Salty Red Sea water continues to outflow into the Gulf of Aden in the bottom layer, but the volume flux is much less (Murray and Johns 1997).

Below the top 200 m the Red Sea is weakly stratified and the water properties are relatively uniform down to the bottom, with $T \approx 21.5^\circ C$, $S \approx 40.6$, and $\sigma_\theta \approx 28.6$ kg m$^{-3}$. Siedler (1968), however, found that the Red Sea deep water undergoes notable seasonal variations and appears colder by 0.1°C and saltier by 0.05 psu in summer than in winter. Finally, it is worth mentioning that, as a result of the basin geometry and the local buoyancy forcing, the deep water of the Red Sea has a higher temperature than any other part of the world ocean at this depth (Morcos 1970).
Fig. 1.5. Hydrographic sections along the axis of the Red Sea using data for the potential temperature (upper panel), salinity (middle panel), and $\sigma_\theta$ (lower panel) from the cruise of R.V. *Atlantis* during July-August 1963.
1.5 Circulation

1.5.1 Shallow circulation

Current measurements in the Red Sea are sparse and the few available are in the vicinity of the Strait of Bab el Mandab (Vercelli 1927; Siedler 1968; Maillard and Soliman 1986; Murray and Johns 1997), or at the mouth of Gulf of Aqaba (Murray et al. 1984). Thus our present knowledge of the currents of the Red Sea proper is based on geostrophic current estimates (Maillard 1974; Quadfasel and Baudner 1993), on data compiled from ship drift observations (Patzert 1974a), on satellite altimetry data (Eshel et al. 1995), and on matching OGCM results to hydrographic observations (Eshel and Naik 1997; Clifford et al. 1997).

The near-surface current system above the sill depth of about 150 m, is believed to be the result of the combined effects of wind and buoyancy forcing, while the effects of the bottom topography are rarely mentioned. A large-scale, annual-mean picture of the shallow circulation of the Red Sea is that of an inverse estuary where the water mass deficit due to evaporation causes a near-surface inflow of fresher water, from the Gulf of Aden, which undergoes an increase in salinity and becomes denser as it flows towards the northern end of the basin. This causes vertical convection into a lower layer which leaves the basin as an undercurrent over the sill.

This first-order buoyancy-driven circulation of the Red Sea was first examined by Phillips (1966) who used the simple geometry of the basin to introduce an elegant similarity model based on scaling arguments. Recently, Maxworthy (1997) proposed a modified similarity scaling for the two-layer circulation of a buoyancy-driven basin and attempted to include the effects of a contraction at the entrance. Solutions of Phillips’ model for the Red Sea were calculated by Tragou and Garrett
(1997) and will be discussed and compared with Maxworthy’s solutions in detail in Chapter 4.

The wind also influences the near-surface circulation particularly in the southern end of the basin; according to Patzert (1974b) the wind in the northern part of the Red Sea has a southward direction that opposes the buoyancy-driven circulation during the entire year, but affects the circulation only during the summer (Siedler 1969). In the southern part the wind changes direction seasonally (Figure 1.2) enhancing the buoyancy-driven circulation in the winter (northward wind) and opposing it in the summer (southward wind). Thus the two-layer system of inflow-outflow at the Strait of Bab el Mandab becomes a weaker three-layer system during the summer months from June to September (Maillard and Soliman 1986) with a shallow surface outflow above an intermediate inflow and deep outflow. The wind clearly affects the circulation in the Red Sea but it seems that the buoyancy-driven circulation dominates the large-scale flow pattern throughout the year.

Eshel et al. (1994) also suggested that the thermohaline driven circulation is more important than the wind driven. They estimated the circulation from hydrographic and $^3$He data with an inverse calculation which allows for advective circulation without mixing. This is essentially a calculation of the mass and tracer budgets; the relative dynamical importance of thermohaline and wind driven circulation cannot be determined from their model.

Two OGCM studies of the dynamics of the Red Sea (Eshel and Naik 1997; Clifford et al. 1997) show a complex circulation pattern, seasonally variable, composed of a series of eddies, jets and subgyres. Apart from the cyclonic motion confined to the far north end of the basin (which is assumed to be associated with
deep-water formation), there are no consistent patterns between these model results. Nevertheless, both studies agree that the eddy circulation of the Red Sea is mainly due to wind stirring and the the eddy activity is more important when the winds are cross-axis. Current speeds of the eddies reach 0.5 m s\(^{-1}\) or higher, whereas there are no estimates for the climatological-mean speed. Eddy motion with current speeds of up to 0.4 m s\(^{-1}\) were found from geostrophic currents estimated by Maillard (1974) and Quadfasel and Baudner (1993). The latter also suggested that the eddies are wind driven, but topographically trapped at the wider areas of the basin. The forcing and the results of the two OGCM studies will be discussed in Chapter 4.

1.5.2 Deep circulation: formation and renewal time

Since there are no direct deep current observations our knowledge of the deep water circulation is based on tracer budgets. The deep water temperature and salinity are nearly homogeneous, but the strongly structured distribution of nutrients, dissolved oxygen, and geochemical tracers are used to estimate the deep circulation.

It is generally believed (Cember 1988; Eshel et al. 1994; Woelk and Quadfasel 1996) that deep water formation occurs at the far north end of the basin in two processes: continental shelf formation in the shallow Gulf of Suez, and open-ocean deep convection at the northern end of the Red Sea. Some contribution also comes from the outflow of the Gulf of Aqaba into the Red Sea, although this is considered small compared to the first two sources (Cember 1988).

While there is general agreement on the formation site, there is a variety of estimates for the annual mean production rates that range from 0.04 to 0.16 Sv (Wyrtki 1974; Maillard 1974; Cember 1988; Eshel et al. 1994), and for the renewal
time from 36 to 200 years. These production rates suggest a rather weak deep circulation compared to the shallow thermohaline cell of 0.3 Sv (Siedler 1969) of the top 150 m. Moreover, the deep circulation is considered to be rather independent of the shallow circulation (Wyrski 1974) and it is possible that the deep water flows out through the Strait of Bab el Mandab independently of the upper thermohaline circulation.

It is believed that all the activity in the deep basin is limited to the winter months (Wyrski 1974; Woelk and Quadfasel 1996) when deep water is produced. During summer, higher temperatures and lower evaporation rates prevent deep-water formation and the deep basin is considered stagnant. Woelk and Quadfasel (1996) also emphasize the importance of interannual variability in the deep-water formation rates.
The Strait of Bab el Mandab

2.1 Introduction

The existence of a contraction (in the form of a sill and/or narrows) at the point connecting a semi-enclosed basin to the open ocean can be crucial for the oceanography of the basin. First, it provides a convenient measuring point for the surface heat and freshwater budgets of the basin, and second, the density differences between the inflow and outflow, as well as the depth of their interface allows for conclusions on the dynamics in the interior of the basin.

In a pioneering study, Bryden and Stommel (1984) showed how the volume and property exchange rates at the strait may influence the stratification and circulation of a buoyancy-driven semi-enclosed inverse estuary such as the Mediterranean. Garrett et al. (1990) emphasized the importance of resolving the question whether the exchange at the strait is maximal or submaximal in order to determine the overmixed or not-overmixed state of the interior, and to assess the role of the strait as an appropriate monitoring point for changes in the interior.

Recent studies of the Strait of Bab el Mandab suggest that the exchange there is submaximal (Maxworthy 1997). The outflow occurs in a thin but hydraulically controlled layer as there is strong upstream/downstream asymmetry of the flow. Pratt et al. (1998), however, based on current measurements at the sill and narrows, found that hydraulic control may not exist throughout the year, especially at the narrows. At any rate, the strong seasonal signal in the outflow
2. The Strait of Bab el Mandab

rates (Murray and Johns 1997) strongly implies that the exchange at the Strait is submaximal and the Red Sea is not overmixed.

Submaximal exchange at the Strait suggests that the exchange is not controlled by the Strait alone, but processes in the interior of the Red Sea may be important. The submaximal nature of the exchange also indicates that there is a much more rapid response of the Strait to conditions in the Red Sea, so that it is a convenient monitoring point for changes in the interior.

In fact, even standard records of water properties and currents at the Strait of Bab el Mandab may provide valuable information about the interior of the basin. A useful starting point involves estimates of heat and freshwater budgets which can be used as constraints for the surface buoyancy fluxes of the basin. In the following section we will examine the capability of measurements at the Strait of Bab el Mandab to provide constraints for the heat and freshwater fluxes and present up-to-date estimates of these constraints.

2.2 Oceanic heat and freshwater transports through the Strait of Bab el Mandab

2.2.1 Review of annual estimates

Heat and freshwater transports through a section at the entrance of the Red Sea may be calculated from direct in situ measurements of temperature and salinity with simultaneous observations of the currents, but the results must be compatible with conservation of volume and salt (the Knudsen formulae). For the Red Sea these are

\[-F_1 + F_2 - F_3 = \bar{E}_{net}A \quad (2.1)\]

\[-\rho_1S_1F_1 + \rho_2S_2F_2 - \rho_3S_3F_3 = 0, \quad (2.2)\]
where $F_i \ (i = 1, 2, 3)$ are the annual mean volume fluxes (in $\text{Sv} = 10^6 \text{m}^3\text{s}^{-1}$) of the surface outflow $F_1$ (present during summer months only), the intermediate (summer only) or surface inflow $F_2$, and the bottom outflow $F_3$. $A$ is the surface area of the Red Sea ($0.45 \times 10^{12} \text{m}^2$), and $E_{\text{net}} = E - P$, where $E$ is the evaporation and $P$ the precipitation (both in $\text{m} \text{s}^{-1}$); there is no runoff in the Red Sea. $\rho_i$ and $S_i$ are the average densities and salinities of the corresponding layers.

Monthly volume fluxes for the near surface layer (i.e. $F_1$ or $F_2$) have been estimated by Patzert (1974a) using ship drift observations [from the Koninklijk Nederlands Meteorologisch Instituut (KNMI) Atlas (1949)], and summertime volume fluxes for the intermediate layer ($F_2$) are available from measurements by Maillard and Soliman (1986). Monthly mean temperature and salinity profiles are taken from Levitus et al. (1994) at the grid point closest to the Strait ($15.5^\circ\text{N}, 41.5^\circ\text{E}$), but 250 km to the northwest of Bab el Mandab where the depth is about 400 m. Since the sill at Hanish Island is about 150 km northwest of Bab el Mandab narrows, the hydrographic data are about 100 km away from the sill. The sensitivity of our results to this effect was checked by allowing for a time lag of one month for the inflowing water to reach that point, which is the speed of propagation of the inflowing water according to Smeed (1988). No significant differences were found.

The evaporation rates for the Red Sea have been estimated by Yegorov (1950) ($2.30 \text{ m} \text{y}^{-1}$), Neumann (1952) ($2.15 \text{ m} \text{y}^{-1}$), and Privett (1959) ($1.83 \text{ m} \text{y}^{-1}$) using a bulk formula for the water vapor transfer, as described by Morcos (1970) in his thorough review of the Red Sea. There are considerable differences among these estimates not only for the annual rate but also in the seasonality; Privett reports a higher evaporation rate during winter, while Yegorov and Neumann found maximum evaporation rates during summer.
2. The Strait of Bab el Mandab

To provide an estimate for the evaporation rate that would satisfy the Knudsen formulae, we consider both $E_{\text{net}}$ and $F_3$ as unknowns and solve the system of equations (2.1) and (2.2). This gives the annual mean $E_{\text{net}} = 1.60 \pm 0.35 \text{ m y}^{-1}$, smaller than previously claimed. The overall uncertainty is estimated from the $\pm 20\%$ uncertainty in the volume flux and $\pm 10\%$ uncertainty in the salinity differences between inflow and outflow. The latter is estimated from the average deviation of the salinity difference from the smoothed annual cycle of the salinity difference with a 5th degree polynomial fit. A similar estimate of $E_{\text{net}}$ using the same climatological salinities and updated volume fluxes for the lower layer $F_3$ from Murray and Johns (1997), and considering $F_2$ as unknown, also gave $E_{\text{net}} = 1.60 \text{ m y}^{-1}$.

The estimated monthly $E_{\text{net}}$ (assumed constant) balances the monthly volume budget (2.1) to provide the unknown volume transport $F_3$ (seasonal changes in the evaporation rate and volume changes due to sea level changes are neglected), so that we may estimate the annual mean heat transport $F_T$. This is expressed as the equivalent flux across the surface of the Red Sea and is estimated from the formula

$$F_T = \frac{1}{A} \rho c_p \left( \frac{1}{T_1} \frac{F_1}{T_1} + \frac{1}{T_2} \frac{F_2}{T_2} - \frac{1}{T_3} F_3 - A E_{\text{net}} T_s \right)$$  \hspace{1cm} (2.3)

where $\rho$ is the mean water density (1025 kg m$^{-3}$), $c_p$ is the heat capacity of water (3986 J K$^{-1}$ kg$^{-1}$), and $T_i$ are the mean temperatures of the inflowing and outflowing layers. The last term in equation (2.3) represents the heat transport due to the volume that leaves the basin through its surface and it should be included when $\sum F_i \neq 0$; otherwise the net heat transport at the Strait depends on the choice of temperature scale (i.e. Celsius or Kelvin). The temperature of the evaporated volume of water is the sea surface temperature, therefore we have used the monthly mean temperature climatology from UWM/COADS for $T_s$ combined with
the annual evaporation rate \( E_{\text{net,A}} \) of 0.023 Sv to estimate the net heat transport \( \overline{F}_T \) as approximately \( 8 \pm 2 \text{ W m}^{-2} \) (uncertainty in the temperature differences between inflow and outflow is 10\%). This compares well with the original estimate by Patzert (1974) of 7 W m\(^{-2}\), who used historical monthly temperature profiles from the National Oceanographic Data Center (NODC) combined with the aforementioned monthly volume transports from KNMI. Our monthly estimates for the heat transport are presented in Figure 2.1a. We note that, in principle, the last term in equation (2.3) should be \(-EAT_s + PAT_p\) where \( P \) is the rate and \( T_p \) the temperature of the precipitation, but the effect of this is small (less than 0.5 W m\(^{-2}\)). The monthly heat transport estimated from equation (2.3) is rather insensitive to uncertainty and seasonal changes in \( E_{\text{net}} \) so that use of the annual average \( E_{\text{net}} \) in (2.3) is adequate.

Our estimates for the monthly salt transports are shown in Figure 2.1b. While we have assumed a monthly constant net volume flux, the salt transport from \( F_s = -\rho_1S_1F_1 + \rho_2S_2F_2 - \rho_3S_3F_3 \) allows for a monthly salt transport. The annual mean \( \overline{F}_s \) is zero. We note that in our estimates for the heat and salt transports we have not taken into account the correlation terms between the volume flux and the temperature and salinity fluctuations in time scales shorter than one month as these are not possible to estimate with the present data sets.

A second estimate of the annual heat exchange through the Strait was provided by Ahmad and Sultan (1989), who used the same hydrological dataset as Patzert, but unfortunately, the source of their volume flux data was not clearly stated in their paper which makes comparison with Patzert's results difficult. The most striking difference between the two analyses is that, according to Ahmad and Sultan, the Red Sea gains heat through the Strait during the summer instead of losing it as suggested by Patzert's original results (-17 W m\(^{-2}\)). Ahmad and Sultan
Fig. 2.1. Estimates of heat and salt advected through the Strait of Bab el Mandab. Crosses mark results using Patzert's volume transport, circles mark Vercelli's results, diamonds mark Maillard and Soliman's, and squares mark Siedler's. The annual mean heat transport is equivalent to a surface heating of the Red Sea of $8 \text{ W m}^{-2}$. The annual mean salt transport is zero.
attributed this discrepancy to Patzert's neglect of the third bottom layer carrying water out of the Red Sea, although an estimate of the summer heat transport we carried out using Maillard and Soliman's volume fluxes, allowing for three layers, also results in a net loss of heat of $-10 \text{ W m}^{-2}$ averaged over the summer months.

Ahmad and Sultan (1989) estimated the annual average input of heat through the Strait as equivalent to $19 \text{ W m}^{-2}$ using the formula

$$F_T = \frac{1}{A} \rho c_p (F_{in}T_{in} - F_{out}T_{out}). \quad (2.4)$$

The above formula is commonly used for heat transport calculations, although it is inappropriate when $F_{in} \neq F_{out}$ (the result is sensitive to the temperature scale). As mentioned previously, the problem can be avoided if we take into account the heat transport due to the water leaving the sea from its surface. If this amount of around $6 \text{ W m}^{-2}$ is deducted from Ahmad and Sultan's estimate, their annual heat input is reduced to $13 \text{ W m}^{-2}$, closer to our result.

2.2.2 Review of seasonal estimates

Other estimates of the volume transport through Bab el Mandab have been carried out using limited time series of simultaneous current data and water properties. These include:

(a) Vercelli's volume transport estimates, based on current measurements from 15 days during March of 1924 (Arimondi cruise). Observations during this cruise for volume flux, temperature, and salinity (Vercelli 1927) are presented in Table 2.1. As pointed out by Morcos (1970) and Patzert (1974a), the difference of $0.09 \text{ Sv}$ between the volume transports of the surface and subsurface layers is unlikely to be due to evaporation and sea level rise; the annual mean evaporation rate is estimated around $0.023 \text{ Sv}$, and the maximum seasonal sea level change,
according to Patzert's analysis, can be as high as 0.30 m leading to an equivalent volume transport of 0.01 Sv. A volume transport compatible with the measured evaporation rate is possible, if we assume that either the transport of the inflowing or outflowing layer is correct and adjust the other so that \( F_{in} - F_{out} = E_{net}A = 0.023 \text{ Sv} \).

The results of these adjustments are presented in Table 2.1, and we will assume that the value of heat and salt transport lies somewhere between the two results. The calculation of the heat flux due to evaporated water was done using the mean sea surface temperature during March given by UWM/COADS. We have ignored the volume difference caused by the sea level rise during winter, but this is expected to introduce a small error compared to the uncertainties of the volume transports at the Strait.

(b) Maillard and Soliman (1986) estimated the volume fluxes through Bab el Mandab using current-meter observations obtained during the summer (July to September) of 1982 (*Marion Dufresne cruise*). Although a problem with the instrumentation allowed for measurements only of the intermediate and bottom layer properties, one may still get an estimate of the heat transport using monthly mean temperatures at the surface layer from Levitus climatology for August and September. No salinity records are available for August. The volume fluxes for the surface and bottom layers in their study are estimated from the volume budget assuming \( E_{net}A = 0.03 \text{ Sv} \), and that there is a proportionality between the volume transports and the corresponding areas \( A_1 \) and \( A_2 \) of the cross sections such that \( F_1/F_3 = A_1/A_3 \). The data from this cruise along with the salt transport analyzed by Souvermezoglou *et al.* (1989), and our estimates of the heat transport are presented in Table 2.2.
Mean volume transports, temperatures and salinities of the two layer exchange system during March 1924 (Vercelli 1927), and the resulting heat and salt transports, adjusting the volume transport of the outflowing layer (upper) and inflowing layer (lower).

<table>
<thead>
<tr>
<th></th>
<th>( F_i ) (Sv)</th>
<th>( T_i ) (°C)</th>
<th>( S_i ) (psu)</th>
<th>( F_T ) (W m(^{-2}))</th>
<th>( F_S ) (10(^6) kg s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflowing layer</td>
<td>0.58 (observed)</td>
<td>26.3</td>
<td>36.56</td>
<td>+135</td>
<td>+21</td>
</tr>
<tr>
<td>Outflowing layer</td>
<td>-0.56 (adjusted)</td>
<td>25.3</td>
<td>37.43</td>
<td>-125</td>
<td>-21</td>
</tr>
<tr>
<td>Evaporated water</td>
<td>-0.02</td>
<td>25.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat and salt transports</td>
<td></td>
<td></td>
<td></td>
<td>+5</td>
<td>0</td>
</tr>
</tbody>
</table>

Inflowing layer 0.51 (adjusted) 26.3 36.56 +119 +19
Outflowing layer -0.49 (observed) 25.3 37.43 -110 -18
Evaporated water -0.02 25.3 -5
Heat and salt transports +4 +1

(c) Siedler (1968) used a very short time series of observations during two and a half days of November 1964 (Meteor cruise) to estimate the volume transports through the Strait. His estimates for volume transport are presented in Table 2.3. The difference between the inflow and outflow is 0.16 Sv, five times larger than the measured difference due to evaporation! An adjustment of the volume transports to match the observed evaporation rate, similar to that applied on Vercelli’s measurements, has been used in order to estimate the heat and salt transports implied by Siedler’s volume transports, and the results are presented in Table 2.3. Climatological data (Levitus et al. 1994) for the temperature and salinity of the two layers were used in our calculations.
Table 2.2.
Volume transports, temperatures and salinities of the three layer exchange system during summer 1982 [Maillard and Soliman (1986); Souvermezoglou et al. (1989)], and the estimated heat and salt transports.

<table>
<thead>
<tr>
<th></th>
<th>$F_i$ (Sv)</th>
<th>$T_i$ (°C)</th>
<th>$S_i$ (psu)</th>
<th>$F_T$ (W m$^{-2}$)</th>
<th>$F_S$ ($10^6$ kg s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface outflow</td>
<td>-0.16</td>
<td>23.66</td>
<td>36.55</td>
<td>-34</td>
<td>-5.8</td>
</tr>
<tr>
<td>Subsurface inflow</td>
<td>+0.25</td>
<td>18.59</td>
<td>36.28</td>
<td>+41</td>
<td>+9.1</td>
</tr>
<tr>
<td>Bottom outflow</td>
<td>-0.06</td>
<td>25.31</td>
<td>38.85</td>
<td>-13</td>
<td>-2.3</td>
</tr>
<tr>
<td>Evaporated water</td>
<td>-0.03</td>
<td>30.25</td>
<td></td>
<td>-8</td>
<td></td>
</tr>
<tr>
<td>Heat and salt transports</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-14 +1.0</td>
</tr>
<tr>
<td>August</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface outflow</td>
<td>-0.25</td>
<td>30.90</td>
<td></td>
<td>-70</td>
<td></td>
</tr>
<tr>
<td>Subsurface inflow</td>
<td>+0.33</td>
<td>16.79</td>
<td></td>
<td>+49</td>
<td></td>
</tr>
<tr>
<td>Bottom outflow</td>
<td>-0.05</td>
<td>15.90</td>
<td></td>
<td>-7</td>
<td></td>
</tr>
<tr>
<td>Evaporated water</td>
<td>-0.03</td>
<td>30.79</td>
<td></td>
<td>-9</td>
<td></td>
</tr>
<tr>
<td>Heat transport</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-37</td>
</tr>
<tr>
<td>September</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface outflow</td>
<td>-0.14</td>
<td>31.20</td>
<td>36.57</td>
<td>-40</td>
<td>-5.1</td>
</tr>
<tr>
<td>Subsurface inflow</td>
<td>+0.20</td>
<td>17.14</td>
<td>35.92</td>
<td>+30</td>
<td>+7.2</td>
</tr>
<tr>
<td>Bottom outflow</td>
<td>-0.03</td>
<td>15.98</td>
<td>38.57</td>
<td>-4</td>
<td>-1.2</td>
</tr>
<tr>
<td>Evaporated water</td>
<td>-0.03</td>
<td>30.58</td>
<td></td>
<td>-9</td>
<td></td>
</tr>
<tr>
<td>Heat and salt transports</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-23 +0.9</td>
</tr>
</tbody>
</table>
Table 2.3.
Mean volume transports during November 1964 from Siedler (1968), and estimates of heat and salt transport using climatological temperatures and salinities (Levitus et al. 1994), adjusting the volume transport of the outflowing layer (upper) and inflowing layer (lower).

<table>
<thead>
<tr>
<th>Layer Type</th>
<th>$F_i$ (Sv)</th>
<th>$T_i$ (°C)</th>
<th>$S_i$ (psu)</th>
<th>$F_T$ (W m$^{-2}$)</th>
<th>$F_S$ $(10^6$ kg s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflowing layer</td>
<td>0.58 (observed)</td>
<td>26.6</td>
<td>38.02</td>
<td>+136</td>
<td>+22</td>
</tr>
<tr>
<td>Outflowing layer</td>
<td>-0.56 (adjusted)</td>
<td>20.6</td>
<td>40.33</td>
<td>-102</td>
<td>-23</td>
</tr>
<tr>
<td>Evaporated water</td>
<td>-0.02</td>
<td>28.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Heat and salt transports

<table>
<thead>
<tr>
<th>Layer Type</th>
<th>$F_i$ (Sv)</th>
<th>$T_i$ (°C)</th>
<th>$S_i$ (psu)</th>
<th>$F_T$ (W m$^{-2}$)</th>
<th>$F_S$ $(10^6$ kg s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflowing layer</td>
<td>0.44 (adjusted)</td>
<td>26.6</td>
<td>38.02</td>
<td>+103</td>
<td>+17</td>
</tr>
<tr>
<td>Outflowing layer</td>
<td>-0.42 (observed)</td>
<td>20.6</td>
<td>40.33</td>
<td>-76</td>
<td>-17</td>
</tr>
<tr>
<td>Evaporated water</td>
<td>-0.02</td>
<td>28.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Heat and salt transports

---

Figure 2.1a shows all the above results for the heat transport and Figure 2.1b the salt transport, and how they compare to each other. The monthly heat transport using volume transport from Patzert has an annual average of approximately 8W m$^{-2}$ and compares well with the short-time observations except during late summer; the heat loss in August and September is stronger with Maillard and Soliman's records. The seasonal cycle shows that the maximum transport of heat to the Red Sea occurs during the fall, while there is heat loss during the summer months. The annual average salt transport is zero and its seasonal cycle shows that there is gain of salt during the summer and loss during the winter,
in agreement with the estimates of Souvermezoglou et al. (1989) with an inverse model.

Observations of temperature, salinity and currents at Bab el Mandab were also presented by Jones et al. (1974), and Van Aken and Otto (1974) for two experiments conducted during September 1971 and March 1967 respectively. Calculations of the heat and freshwater transport were not provided by either of the two studies. Jones et al. (1974) argue that during their three-week experiment the currents show high variability, and, although the current meters suggest a two-layer exchange system, the vertical distributions of temperature and salinity show a three-layer system. Still, an estimate of the heat exchange would have been possible if the bottom topography at the current meter station was known. Estimation of the heat and freshwater transport using the one-day data of Van Aken and Otto is not possible as their current meter station was located at a shallow area of the Strait and only one layer of the exchange system is captured, suggesting outflow at all depths.

It is evident that the exchange at Bab el Mandab is not accurately measured and that more simultaneous in situ records of currents and water properties in the vicinity of Bab el Mandab are needed to estimate the annual heat and salt transports of the Red Sea. Some results of such a project were presented by Murray and Johns (1997) and the complete analysis of their data should shed more light on the physics of the Strait. At any rate, if there is a small temperature difference between the inflow and the colder outflow [of about 2°C (Smeed, 1997)], a typical outflow of 0.4 Sv would give an equivalent surface heat loss of about 7 W m⁻². It is unlikely, therefore, that updated measurements will dramatically upset the present picture of the heat budget of the Red Sea. For the freshwater budget, however, updated simultaneous records for the salinity and volume fluxes
are indeed a necessity and will hopefully narrow the large error bounds of the currently known values.

For the moment, based on the above analysis, we will consider that the volume exchange through Bab el Mandab suggests that the Sea is experiencing net cooling through its surface at a small rate (of approximately $8 \pm 2 \text{ W m}^{-2}$) while the freshwater transport is equivalent to an annual average net evaporation rate of $1.60 \pm 0.35 \text{ m y}^{-1}$. We have not considered the interannual variability of these fluxes, though this may be significant.
3. Surface Buoyancy Fluxes of the Red Sea

3.1 Introduction

At present there are two methods of estimating the air-sea heat fluxes: (i) bulk formulae in conjunction with long-term marine meteorological and hydrographic observations, and (ii) satellite observations. Because of the duration of data sets from voluntary observing ships, estimates from bulk formulae are still more widely used; they will be examined in this chapter for their implications for the heat and freshwater budget of the Red Sea. It is well known, however, that there are systematic errors in the formulae, or in the data, causing uncertainties even in the sign of the heat and freshwater budget of certain areas of the world ocean (e.g. Garrett et al. 1993; da Silva et al. 1994). Uncertainties exist in all four components of the surface heat flux, although the latent heat is considered to have a higher uncertainty than the other components (Weare 1989). Estimating the air-sea fluxes of the Red Sea is useful for the study of the buoyancy driven circulation of the Sea itself, but the semi-enclosed nature of the Red Sea can also be utilized to provide a check on the datasets and formulae that are used on a global scale, as has been done for the Mediterranean by Bunker et al. (1982, henceforth BCG), by Garrett et al. (1993), and by Gilman and Garrett (1994).

Patzert (1974a) estimated a small net heat transport (of about 7 W m$^{-2}$) into the Red Sea from temperature and current measurements at the Strait of Bab el Mandab, which implies that an annual heat loss from the surface of 7 W m$^{-2}$ must occur if the heat content of the basin is conserved. Exchanges between the
3. Surface Buoyancy Fluxes of the Red Sea

Red Sea and the Mediterranean through the Suez Canal are negligible (about 0.0002 Sv, Bétoux and Gentili 1997, personal communication), so heat and freshwater exchanges at Bab el Mandab provide a strong constraint on the Red Sea heat and freshwater surface fluxes. However, in the first estimate of the heat fluxes of the Red Sea using data from voluntary observing ships (VOS) and bulk formulae, BCG estimated a significant surface net heat gain by the Red Sea instead of a loss. They suggested that this discrepancy might be due to an overestimation of the solar irradiance, and, more significantly, due to an underestimation of the latent heat flux because of systematic errors in the observations.

In this chapter we reexamine the estimates of individual heat flux components, using the revised Comprehensive Ocean Atmosphere Data Set of da Silva et al. (1994, henceforth referred to as UWM/COADS), and currently accepted bulk formulae. We find a total net surface heat input, significantly higher than the original estimate of BCG, rather than a loss. We suggest that possible explanations for this discrepancy include the role of regionally high aerosol concentration in attenuating the solar irradiance, and we explore this using optical thickness data from satellites. There also appears to have been an underestimation of the longwave back radiation, and underestimation of the latent heat flux, possibly due to an underestimation of wind speed.

3.2 Surface heat flux estimates

BCG first used a dataset of meteorological observations, made from VOS for the years 1941-1972, to calculate the surface heat fluxes for the Mediterranean and the Red Sea. Their initial results using bulk formulae for the heat flux components of the Red Sea were
\[ \overline{Q}_s - \overline{Q}_b - \overline{Q}_e - \overline{Q}_h = \overline{Q}_t \]

\[ 263 \quad -76 \quad -135 \quad -5 \quad = \quad +47 \text{ W m}^{-2}. \]

where \( Q_s \) is the solar irradiance, \( Q_b \) the net longwave radiation, \( Q_e \) the latent heat flux, \( Q_h \) the sensible heat flux, and \( Q_t \) the total heat flux. An overbar denotes the annual and basin average. BCG recognized that there may be considerable errors in the radiative fluxes and pointed out the increased aerosol concentration of the Red Sea region as a likely cause for reducing the possibly overestimated insolation. They preferred, however, to attribute the large difference between the estimated \( \overline{Q}_t \) and the expected small heat loss to an underestimated latent heat flux caused by the fact that they used monthly mean winds and temperature and humidity differences rather than their monthly mean products, as well as systematic errors in observations of meteorological parameters (wind, temperature and humidity); they mentioned that such errors may include ship's flow distortion and avoidance of high winds, although the latter seems implausible for the narrow Red Sea. Nevertheless, BCG tried to account for these biases by significantly increasing the exchange coefficients for latent and sensible heat fluxes. This gave

\[ 263 \quad -76 \quad -183 \quad -3 \quad = \quad +1 \text{ W m}^{-2}, \]

in better agreement with the oceanic heat flux through the Strait. Apart from the known biases on the wind speed scale (e.g. Kaufeld 1981), however, there has been no confirmation for such an increase of the value of the exchange coefficients. In addition, as we will discuss later, such an arbitrary increase of the latent heat flux may not be consistent with the water budget of the basin, as was pointed out by Garrett et al. (1993) for the Mediterranean. We also note that the original estimate of BCG was carried out with exchange coefficients which depend on the wind speed and the difference between the air and sea surface temperature. In their second estimate, however, the larger coefficients were held constant for all
wind speeds and temperature differences resulting in larger $Q_e$ but smaller $Q_h$,
probably because of increased warming when the air temperature is warmer.

Ahmad and Sultan (1989) used meteorological data from three coastal stations and four near-coast stations on the east coast of the Red Sea, taken during the period 1970 to 1985, to obtain

$$210 \ -66 \ -169 \ +3 = -22 W m^{-2}.$$  

The negative $Q_e$ is in quantitative agreement with the observed advected heat flux, although the net heat loss through the sea surface is higher than the expected $-8 W m^{-2}$. This is partly due to the relatively low insolation (obtained from inland stations where the effect of aerosols is expected to significantly attenuate the solar radiation), and partly because the latent and sensible heat fluxes were estimated with the increased exchange coefficients suggested by BCG. Again, the small positive $Q_h$ may be because of the constant large exchange coefficient applied in conditions of stable inversion.

An up-to-date estimate of the total heat flux is also possible by extracting the estimates for the four heat flux components over the Red Sea from the global data set UWM/COADS of da Silva et al. (1994, henceforth dSYL). This data set covers the 45-year period from January 1945 to December 1989 and comes from the initial COADS release (Woodruff et al. 1987) objectively analyzed on a $1^\circ \times 1^\circ$ global grid. The data set includes corrections to the wind speed, cloud cover, and Present Weather observations, as well as global estimates for the four components of the heat budget evaluated using standard bulk formulae, and estimates for the freshwater and buoyancy fluxes. From the UWM/COADS the heat fluxes averaged over 45 years and over the Red Sea give

$$272 \ -55 \ -129 \ -5 = +83 W m^{-2}.$$
which is very far from the expected small negative $\overline{Q}_f$. The same data set contains data for the constrained net heat and freshwater fluxes that are compatible with meridional global balances at 65°S (0 PW for the heat and 0.06 Sv for the freshwater transports). Unfortunately, even the constrained net heat flux yields

$$250 - 57 - 146 - 5 = +42 \text{ W m}^{-2}.$$  

for the Red Sea.

In the following sections we have re-examined each component of the heat budget separately, using meteorological parameters from the recent UWM/COADS and commonly used bulk formulae, in an attempt to find an explanation for the discrepancy between the heat flux at the Strait and the surface heat budget.

### 3.2.1 Solar irradiance

The insolation $Q_s$ at the sea surface was calculated by dSYL using the commonly used formula

$$Q_s = Q_{CS}(1 - c_n n + 0.0019 k)(1 - \alpha). \quad (3.1)$$

In this formula the clear sky irradiance $Q_{CS}$ comes from the sum of direct and scattered solar radiation (List 1958) as given by Rosati and Miyakoda (1988). This is

$$Q_{CS} = \frac{1}{2} Q_0 (T r^{1/\cos z} + (1 - A)), \quad (3.2)$$

where $Q_0$ is the incident radiation at the top of the atmosphere ($Q_0 = S_0 \cos z$, with $S_0 = 1370 \text{ W m}^{-2}$ the solar constant, and $z$ the solar zenith angle), $Tr$ is the transmission factor for a clear atmosphere (held constant at 0.7), and $A = 0.09$ is the absorption factor due to ozone and water vapor. In equation (3.1) the reduction due to clouds is parameterized, following Reed (1977), by the product of the cloud
fraction \( n \) and a coefficient \( c_n = 0.62 \). \( h \) is the noon solar altitude in degrees and \( \alpha \) is the albedo of the sea surface taken from Payne (1972).

The Red Sea average of \( \bar{Q}_s \), calculated from dSYL with equation (3.1) and the cloud cover fraction from UWM/COADS, is 272 W m\(^{-2}\). However, when the cloud cover fraction \( n \) is smaller than about 0.2 the calculated solar radiation \( Q_s \) is larger than the clear sky irradiance \( Q_{CS} \) and an overestimation of \( Q_s \) is introduced (Gilman and Garrett 1994). Moreover, the cloudiness from UWM/COADS includes some small negative values, mainly during the period 1967 to 1975, presumably stemming from extrapolations in the objective analysis. To correct for these effects we apply the Reed cloud reduction only when it gives \( Q_s < Q_{CS} \), and otherwise set \( Q_s = Q_{CS} \). This correction gives \( \bar{Q}_s = 264 \) W m\(^{-2}\) which is a 3% (8 W m\(^{-2}\)) reduction in \( \bar{Q}_s \) for the Red Sea.

The spatial distribution of the corrected annual mean insolation \( \bar{Q}_s \) is shown in Figure 3.1, and the seasonal variability of the spatial mean \( Q_s \) is shown in Figure 3.5a. It should be noted that there is a clear north-south gradient in \( \bar{Q}_s \) (changing by 7%) with higher \( \bar{Q}_s \) values at the southern end.

dSYL also carried out an inverse calculation for the global heat fluxes, to match certain global constraints. They provided tuning parameters for the heat flux components, though they provided no physical justification. The correcting factors for the insolation formula are 0.92 (on the clear-sky radiation) and 1.04 (on the cloudiness coefficient), so that equation (3.1) becomes

\[
Q_{s\text{dSYL}} = 0.92Q_{CS}(1 - 1.04c_n n + 0.0019h)(1 - \alpha).
\] (3.3)

The mean Red Sea insolation obtained using the above formula is 250 W m\(^{-2}\).

Another independent estimate for the solar irradiance of the Red Sea comes from the data set of Bishop et al. (1997). They have calculated the global \( Q_s \) for
Fig. 3.1. Long-term averaged net shortwave solar radiation \( Q_s \) estimated from equation (3.1) (with the cloud correction) and cloudiness from UWM/COADS.

an eight-year period (July 1983 to June 1991) using data from the International Satellite Cloud Climatology Project (ISCCP) and a revised algorithm of Bishop and Rossow (1991). The surface solar irradiance from this data set (denoted \( Q_{sBR2} \)) gives an average of 273 W m\(^{-2}\) over the Red Sea. All of the above \( Q_s \) estimates are listed in Table 3.1 (the last entry will be discussed shortly).

For the Mediterranean insolation Gilman and Garrett (1994) showed that there is a systematic error in the parameterization of atmospheric attenuation in the formula for the solar irradiance. Indeed, the transmission coefficient in formula (3.1) is assumed spatially and temporally constant and a value of 0.7 is commonly used to parameterize the extinction due to absorption and scattering by aerosols and other atmospheric constituents such as water vapor, ozone, \( \text{CO}_2 \), etc. However, concentrations of natural and anthropogenic aerosols may vary both in space
Table 3.1.
List of estimates for the mean surface solar radiation of the Red Sea.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Method</th>
<th>Data Set</th>
<th>$Q_s$ (W m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>da Silva et al. (1994)</td>
<td>bulk formula (3.1)</td>
<td>UWM/COADS</td>
<td>272</td>
</tr>
<tr>
<td>this study</td>
<td>bulk formula (3.1) with cloud correction</td>
<td>UWM/COADS</td>
<td>264</td>
</tr>
<tr>
<td>da Silva et al. (1994)</td>
<td>bulk formula (3.3)</td>
<td>UWM/COADS</td>
<td>250</td>
</tr>
<tr>
<td>this study</td>
<td>bulk formula (3.1) with cloud and aerosol correction</td>
<td>UWM/COADS</td>
<td>236</td>
</tr>
</tbody>
</table>

and time so that the parameterization of aerosol effects as constant can lead to significant errors in estimating the insolation. In particular, the Red Sea appears to be within a dusty environment due to anthropogenic and, particularly, mineral aerosols. This is evident from the global distributions of the index of aerosol optical thickness detected by polar orbiting satellites (AVHRR data) over the oceans presented by Husar et al. (1997), who also showed that there is significant seasonal variability of the aerosol concentration in the atmosphere. Unfortunately, the currently available index of aerosol optical thickness cannot be simply related to the transmission coefficient $T_r$ of clear sky insolation, because the data do not provide information about aerosol properties (i.e. size, shape, and composition), and lack the required calibration (Lacis and Mishchenko 1995).

Nevertheless, monthly data for insolation from coastal stations can be compared to $Q_s$, obtained from the bulk formula (3.1), with the cloud correction, to estimate a factor which will be assumed to represent the attenuation due to aerosols. The results of this comparison can be used in an attempt to calibrate satellite data.
so that we can obtain a picture of the spatial variability of the effect of aerosols throughout the Red Sea.

Observations of surface solar irradiance from the Global Energy Balance Archives (GEBA) are available for two coastal stations on the west coast of the Red Sea (Ohmura and Gligor 1991). These include monthly mean measurements of \( Q_s \) from July 1983 to August 1985 for the locations marked GEBA-1 and GEBA-2 in Figure 3.1. Ground-truth observations are also available for a station at Jeddah, Saudi Arabia (Figure 3.1), on the central east coast of the Red Sea, in the form of monthly averages of \( Q_s \) for the period 1980 to 1985 (Ahmad et al. 1989).

The monthly solar irradiance at the three stations according to the ground-truth measurements and the estimates from equation (3.1) are presented in Figure 3.2. The observed values (solid circles) are, in general, lower than the estimated \( Q_s \) values (crosses), and the minimum insolation is observed at Jeddah. We note, however, that the ground measurements at GEBA-1 are very irregular and include values higher than the \( Q_s \) estimates.

Bishop et al. (1997) compared the globally estimated \( Q_{sBR2} \) using their revised algorithm and data from the ISCCP, with ground truth data from globally scattered GEBA stations \( Q_{sGEBA} \). In this algorithm the effect of aerosols was held constant in space and time. Therefore, in order to bring the calculated surface solar irradiance into agreement with surface observations, they have estimated the optical extinction anomaly \( \tau^* \) defined from

\[
\frac{Q_{sGEBA}}{Q_{sBR2}} = \exp(-\tau^*/\cos z^*),
\]

where \( Q_{sGEBA} \) is the monthly averaged solar irradiance measured at the ground stations GEBA, \( Q_{sBR2} \) is the monthly solar irradiance from Bishop et al. (1997) averaged over the eight year period July 1983 to June 1991 (the resolution is
Fig. 3.2. Net shortwave radiation at three stations. Crosses mark the estimated $Q_s$ from equation (3.1) (with the cloud correction), solid circles mark the observed $Q_s$ and open circles mark the corrected $Q_s$ using the transmission anomaly $Tr^*$ from satellite data calibrated to ground stations.
2.5° × 2.5° and has been interpolated to the position of the ground stations), and
\(\cos z^*\) is the flux-weighted cosine of solar zenith angle.

Alternatively, equation (3.4) also defines a transmission coefficient anomaly
\(T r^*\) due to the neglect (or underestimation) of the effect of aerosols in equation
(3.2) by using a spatially and temporally constant \(T r\). The transmission coefficient
anomaly \(T r^*\) is defined by comparing the ground measurements \(Q_{sg}\) with the
insolation estimated with equation (3.1) for the same period of time

\[
\frac{Q_{sg}}{Q_s} = T r^*,
\]

where \(Q_{sg}\) is the monthly mean insolation at the two stations from GEBA and
the one at Jeddah. The resulting transmission anomaly coefficient for the three
stations in the central Red Sea is shown in Figure 3.3a along with the mean of \(T r^*\)
from GEBA and Jeddah and a smoothed version of this. It should be noted that
there is a strong seasonal signal in the anomaly, with a 25% difference between
summer and winter months.

The transmission factor \(T r^*\) calculated with the above procedure can be
used to calibrate satellite data for the index of optical thickness in order to obtain
the spatial variability of \(T r^*\). These data include monthly observations for the
aerosol optical thickness index \(\tau^A_s\) derived for the ocean from NOAA/AVHRR
data on a 1° × 1° resolution for the period from July 1989 to June 1991 (Stowe
et al. 1997) and they have been averaged to obtain the seasonal cycle. In the
method of derivation of \(\tau^A_s\), aerosols are assumed to be purely scattering. We
assume that the satellite data for the index \(\tau^A_s\) define a transmission coefficient
\(T r^A_s = \exp(-\tau^A_s/\cos z)\) due to aerosol scattering. We also introduce a coefficient
\(f_c\) for the attenuation of \(Q_0\) due to absorption and scattering by atmospheric
Fig. 3.3. (a) Transmission anomaly coefficient estimated from comparison of the insolation at ground-truth stations with the calculated \(Q_s\) (with the cloud correction) during the same period of time \((Tr^* = Q_{sG}/Q_s)\). The dashed line is the mean \(Tr^*\) from the GEBA stations, the dashed-dotted line is \(Tr^*\) from the station at Jeddah, the solid line is the average between the results at GEBA and Jeddah (assuming 50% weights to GEBA and Jeddah, i.e. 25% to each GEBA station and 50% to Jeddah). The thick solid line is the smoothed average \(Tr^*\). The smoothing is obtained with a 10th order polynomial fit on three repetitive annual cycles of the mean \(Tr^*\) and keeping the second annual cycle. (b) Calibration factor for the satellite transmission coefficient index estimated from comparison of the insolation from satellite data with the ground-truth records; the line-style corresponds to that of (a).
3. Surface Buoyancy Fluxes of the Red Sea

molecules and absorption by aerosols, so that the clear-sky insolation at the sea surface is given by \( Q_0 f_c T_{r_s}^A \).

Combining equations (3.1), (3.2) and (3.5), and assuming that the transmission anomaly coefficient \( T_{r_s}^* \) is solely due to errors in the estimation of \( Q_{CS} \), the clear-sky insolation at the coastal stations is given by \( Q_0 f T_{r_s}^* \), where \( f = \frac{1}{2}(T_{r_s}^{1/\cos z} + (1 - A)) \). To calibrate the clear-sky insolation from the satellite data to the measured clear-sky insolation, the coefficient \( f_c \) has to be adjusted so that

\[ f_c = f \frac{T_{r_s}^*}{T_{r_s}^A}. \]

At GEBA and Jeddah stations the calibration factor \( f_c \) is evaluated using the average monthly values of \( T_{r_s}^* \), the monthly aerosol transmission coefficient \( T_{r_s}^A \) (at grid points nearest to the coastal stations), and the monthly values of the parameter \( f \). The seasonal cycle of \( f_c \) for GEBA, and Jeddah, the mean \( f_c \), and the smoothed \( f_c \) are shown in Figure 3.3b. We note that the calibration factor \( f_c \) includes all the corrections required to match \( Q_{CS} \) from satellite data to the measured quantity at ground stations. These include the attenuation of \( Q_0 \) due to absorption and scattering by atmospheric molecules, aerosol absorption, as well as errors in \( T_{r_s}^A \).

We have used this seasonally varying but spatially constant calibration factor \( f_c \) and the seasonal value of \( f \) to estimate the transmission coefficient anomaly \( T_{r_s}^* = \frac{f_c}{f} T_{r_s}^A \), using the satellite aerosol transmission coefficient \( T_{r_s}^A \) everywhere in the Red Sea. Figure 3.4a shows that there is strong seasonality of the basin-averaged \( T_{r_s}^* \) with a seasonal change of around 25%. Figure 3.4a also suggests that there is some underestimation of the insolation during winter months \( (T_{r_s}^* > 1) \), while there is significant overestimation during summer. From the spatial distribution of the annual mean \( T_{r_s}^* \) (Figure 3.4b) it is evident that there is a north-south
3. Surface Buoyancy Fluxes of the Red Sea

1.1. Transmission anomaly coefficient $T_r^*$ for the Red Sea from the optical extinction index calibrated to ground observations, (a) seasonal cycle, and (b) geographical distribution.

Fig. 3.4. Transmission anomaly coefficient $T_r^*$ is overestimated towards the southern end of the Red Sea.

This spatially and seasonally varying transmission coefficient $T_r^*$ is multiplied by the surface solar irradiance obtained with equation (3.1) including the cloud correction. The resulting corrected $Q_{sc} = T_r^*Q_s$ is presented in Figure 3.5. Figure 3.5a shows the seasonal cycle of the 45 year average of $Q_{sc}$. We note that there is a reduction of 28 W m$^{-2}$ (or 11%) from the initial $Q_s = 264$ W m$^{-2}$, presumably because of the effect of aerosols on the insolation. Figure 3.5b shows the spatial distribution of the mean $Q_{sc}$. Comparison of Figure 3.5b with Figure 3.1 shows that the north-south gradient is now reversed and the higher insolation occurs towards the northern end of the Red Sea because of the higher aerosol.
Fig. 3.5. Insolation corrected for the attenuation due to aerosols. (a) Seasonal cycle, and (b) spatial distribution.

concentration at the southern end. The reversal of the geographical gradient in insolation cannot be verified by ground stations as the available data come from the central Red Sea area.

The basin-averaged 45-year mean $\overline{Q}_{sc}$ is listed in Table 3.1 and the monthly values of the corrected $Q_{sc}$ at the ground stations are plotted in Figure 3.2.

3.2.2 Longwave radiation

The net longwave radiation at the sea surface is the result of emission, absorption and scattering in the atmospheric column, and so depends on the concentrations of atmospheric constituents and the vertical distribution of temperature, humidity and cloud properties. Since direct measurements of the net longwave radiation at the ocean surface are sparse, it has become common to derive $Q_b$ with
the aid of various bulk formulae or, more recently, with a radiative transfer model (Zhang et al. 1995).

The bulk formulae assume that the surface properties of temperature and humidity represent those of the atmospheric column, and they introduce various parameterization schemes with constants derived from regression fitting to certain sets of observations. In general, only the cloud fraction is taken into consideration and the cloud type (high, medium or low) is ignored, thus increasing the uncertainty of the calculations (Fung et al. 1984). Thus one cannot expect very accurate estimates from the bulk formulae. Bulk formulae are still widely used, however, because they allow the computation of $Q_b$ from routinely measured parameters such as sea surface temperature, air temperature, the water vapor pressure near the sea surface, and cloudiness. The direct effect of aerosols on longwave radiation is small because the opacity of aerosols decreases at longer wavelength. Moreover, aerosols close to the surface do not affect the longwave radiation as they radiate at a temperature similar to the surface temperature. Thus their effect will be neglected.

In Table 3.2 we present estimates of the 45 year mean net longwave radiation calculated using three different bulk formulae and data for sea temperature, air temperature, vapor pressure and cloud fraction from UWM/COADS. The formula of Berliand and Berliand (1952) was used by dSYL in their calculation of $Q_b$. However, a comparison of the mean $\overline{Q}_b$ for the Mediterranean carried out by Gilman and Garrett (1994) suggested that this formula significantly underestimates $Q_b$ there. We have calculated $\overline{Q}_b$ using the formula introduced by Bunker and Goldsmith (1979) which is the only bulk formula that allows for the effect of a seasonally prescribed mix of low, medium and high cloud type. Data for the cloud
type in the Red Sea were provided by R. Goldsmith. The mean $\overline{Q}_b$ we found using the Bunker and Goldsmith formula is 73 W m$^{-2}$, while BCG estimated 76 W m$^{-2}$.

Table 3.2.
List of methods used to calculate net longwave radiation, and the annual Red Sea estimates using various data sets.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Method</th>
<th>Data Set</th>
<th>$\overline{Q}_b$ (W m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>da Silva et al. (1994)*</td>
<td>$\varepsilon\sigma T_s^4(0.39 - 0.05\varepsilon^{1/2})(1 - \chi c^2)$ + $4\varepsilon\sigma T_s^3(T_s - T_a)$</td>
<td>UWM/COADS</td>
<td>55</td>
</tr>
<tr>
<td>Bunker and Goldsmith (1979)</td>
<td>Bunker (1976) formula with seasonal mix of cloud type</td>
<td>UWM/COADS</td>
<td>73</td>
</tr>
<tr>
<td>Bignami et al. (1995)</td>
<td>$\varepsilon\sigma T_s^4 - \sigma T_a^4(0.653 + 0.00535\varepsilon)$ \times (1 + 0.1762c^2)</td>
<td>UWM/COADS</td>
<td>76</td>
</tr>
<tr>
<td>Rossow and Zhang (1995)</td>
<td>Radiative Transfer Model ISCCP</td>
<td></td>
<td>104</td>
</tr>
</tbody>
</table>

*based on Berliand and Berliand (1952).

$\varepsilon = 0.98$ is the sea surface emissivity,
$\sigma = 5.6697 \times 10^{-8}$ W m$^{-2}$K$^{-4}$ is the Stefan-Boltzmann constant,
$T_s, T_a$ are the sea surface and air temperatures,
$\varepsilon$ is the near sea-surface water vapor pressure in hPa,
$c$ is the cloud cover fraction,
$\chi$ is a coefficient varying with latitude.

More recently, Bignami et al. (1995) proposed another formula with constants calculated from regression to measurements of longwave radiation in the western Mediterranean. This formula is the first strictly derived from marine data. They compared the observations in the Mediterranean with results from previous formulae and found that the latter systematically underestimate longwave radiation by approximately 30 W m$^{-2}$. Josey et al. (1998) found that the Bignami et al. formula performs best in semi-enclosed seas rather than open ocean conditions.
where other formulae (such as Clark et al. (1974)) are more appropriate. We have tested the Bignami et al. formula in the Red Sea (assuming that the relations between surface parameters and atmospheric profiles are similar to those of the Mediterranean) and found a value of \(76 \text{ W m}^{-2}\), close to the result using the formula of Bunker and Goldsmith (1979).

We have also compared the results of the bulk formulae with \(Q_b\) estimates extracted from a global dataset for radiative fluxes, evaluated using a radiative transfer model and data from ISCCP (during the period April 1985 to January 1989) by Rossow and Zhang (1995). The mean value of \(\bar{Q}_b\) over the Red Sea area is \(104 \text{ W m}^{-2}\) though uncertainties in the calculation reach \(15 \text{ W m}^{-2}\). The source of these uncertainties in subtropical regions is equally distributed between uncertainties in cloud properties, and atmospheric and surface water properties. It should be emphasized that \(\bar{Q}_b\) from the radiative transfer model is significantly larger than that from the bulk formulae and the uncertainty of the former cannot account for the difference. We note, however, that the resolution of the Rossow and Zhang (1995) dataset is \(2.5^\circ \times 2.5^\circ\) which is very coarse for the narrow Red Sea and it is very likely that the extracted annual mean \(Q_b\) is more representative of the surrounding land rather than the Red Sea itself. If the humidity profile over the Red Sea is typical of arid regions, not of the marine environment, then the \(\bar{Q}_b\) estimated from this dataset could be more appropriate. However, the available meteorological fields for the humidity profile have inadequate resolution (usually \(2.5^\circ \times 2.5^\circ\)) and this hypothesis cannot be explored. Therefore we will consider the \(\bar{Q}_b = 76 \text{ W m}^{-2}\) found with the Bignami et al. (1995) formula as our best estimate for the longwave radiation.
3.2.3 Latent and sensible heat fluxes

Latent and sensible heat fluxes are, in general, given by the formulae

\[ Q_e = \rho c_p L C_e (q_s - q) W \]  
\[ Q_h = \rho c_p C_h (T_s - T_a) W \]

with \( \rho \) the air density determined from atmospheric and air temperatures, \( c_p \) the specific heat of air, \( L \) the latent heat of evaporation. Also, \( q_s \) is the saturation humidity at the sea surface temperature, \( q \) the observed specific humidity, \( T_s \) and \( T_a \) are the sea surface and air temperatures, and \( W \) the wind speed. \( C_e \) and \( C_h \) are exchange coefficients.

As mentioned previously, BCG used large values for \( C_e \) and \( C_h \) (\( C_e = C_h = 2.1 \times 10^{-3} \)), compared to those deduced from measurements, to account for the discrepancy of the total heat budget. Such an increase could compensate for the fact that they used monthly mean winds and temperature and humidity differences rather than their monthly mean products, as well as for substantial systematic uncertainties in the observations, particularly the wind speed. The same high coefficients were used by Ahmad and Sultan (1989).

dSYL estimated \( C_e \) and \( C_h \) following the method of Large and Pond (1982) which gives numbers very close to those suggested by Smith (1988, 1989). According to this method, both of the exchange coefficients are functions of wind speed, air temperature and its difference from sea surface temperature, and the difference of saturation humidity at the sea surface from the specific humidity. In neutral conditions \( C_e \) has a value of \( 1.2 \times 10^{-3} \), and \( C_h \) is around \( 1.2 \times 10^{-3} \) for unstable stratification and \( 0.75 \times 10^{-3} \) for stable stratification. The basin-averaged 45-year...
mean $\overline{Q}_e$ and $\overline{Q}_h$ extracted from dSYL's data set are 129 W m$^{-2}$ and 5 W m$^{-2}$ respectively.

The mean $\overline{Q}_e$ and $\overline{Q}_h$ from the above studies are listed in Table 3.3 along with the exchange coefficients in neutral conditions. The last entry will be discussed shortly. Very large differences (up to 40%) exist between the quoted $\overline{Q}_e$ values, which is indicative of the uncertainty of this heat flux component. However, $\overline{Q}_e$ can be constrained by the water budget of the basin which is examined next.

Table 3.3.
List of exchange coefficients (in neutral conditions) used to calculate latent and sensible heat fluxes and their mean values in the Red Sea.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$C_e \times 10^{-3}$</th>
<th>$C_h \times 10^{-3}$</th>
<th>$\overline{Q}_e$ (W m$^{-2}$)</th>
<th>$\overline{Q}_h$ (W m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunker et al. (1982)</td>
<td>2.1</td>
<td>2.1</td>
<td>183</td>
<td>3</td>
</tr>
<tr>
<td>Ahmad and Sultan (1989)</td>
<td>2.1</td>
<td>2.1</td>
<td>169</td>
<td>-3</td>
</tr>
<tr>
<td>da Silva et al. (1994)</td>
<td>1.2</td>
<td>1.2</td>
<td>129</td>
<td>5</td>
</tr>
<tr>
<td>da Silva et al. (1994) corrected</td>
<td>1.3</td>
<td>1.2</td>
<td>146</td>
<td>5</td>
</tr>
</tbody>
</table>

3.3 Implications from the water budget

In Chapter 2 we provided an estimate for the net annual mean evaporation rate of $\overline{E}_{net} = 1.60 \pm 0.35 \text{my}^{-1}$, using the constraint of zero salt flux through the Strait. From this estimate we may infer the annual mean evaporation $\overline{E} = 1.75 \pm 0.35 \text{my}^{-1}$ assuming an annual mean precipitation rate of $\overline{P} = 0.15 \text{my}^{-1}$.
3. Surface Buoyancy Fluxes of the Red Sea

from the UWM/COADS climatology. Since the evaporation rate is related to the latent heat flux as \( E = Q_e/\rho L \), the estimated annual mean \( \bar{E} \) corresponds to an annual mean \( \bar{Q}_e = 140 \pm 28 \text{ W m}^{-2} \). The \( \bar{Q}_e \) values estimated by BCG and Ahmad and Sultan (1989), listed in Table 3.3, are too large to satisfy the water budget constraint, as a consequence of using an excessively high exchange coefficient \( C_e \). The \( \bar{Q}_e \) value from dSYL falls within the limits of the constraint, although it is closer to the lower end.

For the calculation of the latent heat flux dSYL used corrected data for the wind speed. This correction was applied on a global basis to reduce the bias associated with an erroneous Beaufort equivalent scale that causes underestimated climatological wind speeds. This effect appears as a trend in the long-term time series as the proportionality of wind speeds estimated from sea state versus speeds measured from anemometers changes. The correction, based on a regression formula, has been applied to those data flagged as estimated, and according to dSYL has reduced the trend.

The Red Sea averaged time series for the wind speed (Figure 3.6a), however, shows that there is still a remarkable trend in the wind, particularly during the last two decades, which is transferred to the estimated latent heat flux (Figure 3.6b). This may be caused by limitations of the dSYL method [such as the assumption that all anemometer measurements are made at 20 m height, or due to falsely flagged data (Kent and Taylor 1997)]. If the trend in the wind speed is unrealistic and results in an underestimation of the winds in the Red Sea, it may provide an explanation for the relatively low latent heat flux compared to the expected value from the water budget constraint.

We note that in their inverse calculation, dSYL suggested a correction factor of 1.13 for \( C_e \) and 1.02 for \( C_h \). They ascribed this correction to the uncertainties
in the calculation of the exchange coefficients, implying even higher $C_z$ and $C_h$ than those suggested by Large and Pond (1982). We suggest that for the Red Sea the correction to the exchange coefficients could be related to underestimated winds rather than underestimated exchange coefficients. Using dSYL’s correction parameters the mean latent heat flux is $\overline{Q}_e = 146 \, \text{W m}^{-2}$, very close to the value inferred from the water budget. The sensible heat flux is again $\overline{Q}_h = 5 \, \text{W m}^{-2}$ (Table 3.3).

3.4 Total heat flux

The heat flux components averaged over the Red Sea according to the initial calculations of dSYL in the world ocean, gave
We have found an overestimate of $\overline{Q}_s$, due to two separate processes, amounting to $36 \text{ W m}^{-2}$. The net longwave radiation was re-estimated with a new formula from Bignami et al. (1995), which gave $\overline{Q}_b = 76 \text{ W m}^{-2}$. The latent heat flux appears low compared to the value of $140 \text{ W m}^{-2}$ suggested by the water budget, although the uncertainty is very large (up to $39 \text{ W m}^{-2}$). Further increase of the coefficient $C_e$ seems physically implausible, but the persistent trend in the wind speeds may be the reason for the low $\overline{Q}_e$ values. With the above corrections, the total heat fluxes are

\[
\overline{Q}_s - \overline{Q}_b - \overline{Q}_e - \overline{Q}_h = \overline{Q}_t
\]

\[
272 - 55 - 129 - 5 = +83 \text{ W m}^{-2}
\]

so that the original discrepancy from the expected negative $\overline{Q}_t$ of $-8 \pm 2 \text{ W m}^{-2}$ is reduced by 75%. The remaining difference could be due to an overestimation of $\overline{Q}_s$ or, more likely, due to an underestimation of $\overline{Q}_b$ since a large further increase of $\overline{Q}_e$ would result in inconsistency with the currently known water budget.

It should be emphasized, however, that considerable uncertainties exist in all four heat flux components. Since the number of observations used for the estimate of the long-term averages is $O(3 \times 10^5)$, random errors become insignificant and the total uncertainty for each heat flux estimate is only due to systematic errors in the observations and the bulk formulae parameterization. Following the error analysis of Gilman and Garrett (1994) the uncertainties for the heat flux components are: $\Delta \overline{Q}_s = \pm 10 \text{ W m}^{-2}$ (allowing for an extra $\pm 5 \text{ W m}^{-2}$ uncertainty introduced by the transmission anomaly coefficient), $\Delta \overline{Q}_b = \pm 10 \text{ W m}^{-2}$, $\Delta \overline{Q}_e = \pm 15 \text{ W m}^{-2}$, and $\Delta \overline{Q}_h = \pm 4 \text{ W m}^{-2}$. The uncertainty in the total heat flux is $\pm 21 \text{ W m}^{-2}$. If we consider a change in each individual component only 50% of its uncertainty we
obtain our best estimate

$$231 - 81 - 148 - 7 = -5 \text{ W m}^{-2}.$$  

The seasonal cycle of the four heat flux components is shown in Figure 3.7a. We note that seasonality is mostly determined by the insolation and latent heat flux, and that the total heat flux becomes negative during the winter. Figure 3.7b shows the long-term average geographical distribution of the total heat flux. There is a remarkable gradient in the heat flux of the basin which loses heat in its northern end and gains heat in the southern end. The 45-year time series (filtered for the seasonal cycle) of the four heat flux components and the total are shown in Figure 3.8. The total heat flux mainly follows the interannual variability of the latent heat flux, whereas $Q_s$ and $Q_b$ appear to be relatively stable. We also note that there is a trend in $Q_s$ similar to the trend of $Q_t$ and that $Q_t$ is mostly positive before 1977 but becomes negative from 1977 to 1989.

### 3.5 Buoyancy flux

Finally, we provide an estimate for the surface buoyancy flux, as this is the important quantity for driving the thermohaline circulation of the Red Sea. The surface buoyancy flux is given by

$$B_0 = -c_w^{-1}g\alpha Q_t + \rho_0^{-1}g\beta S(E - P)$$  

$$= B_{0T} + B_{0S}$$

where $c_w$ is the specific heat of water, $g$ the acceleration due to gravity, $\alpha = -\rho^{-1}(\partial \rho / \partial T)_{p,S}$ the coefficient of expansion of water at fixed pressure and salinity,
Fig. 3.7. (a) Seasonal cycle of the heat flux components; the dashed line is the insolation, the dash-dotted line is the net longwave radiation, the thin solid line is the latent heat flux, the dotted line is the sensible heat flux, and the thick solid line is the total heat flux. (b) Spatial distribution of the total heat flux $Q_t$.

and $\beta S = \rho^{-1} S (\partial \rho / \partial S)_{p,T}$. Positive $B_0$ values correspond to buoyancy loss by the ocean, and $B_{OT}$ and $B_{OS}$ are the thermal and haline contributions.

The long-term average thermal buoyancy flux is estimated from

$$
\overline{B_{OT}} = -c_w^{-1} g (\overline{\alpha Q_t} + \alpha' \overline{Q_t'})
$$

(3.10)

$$
= (6.0 - 4.1) \times 10^{-6} \text{kgm}^{-1} \text{s}^{-3}
$$

$$
= 1.9 \times 10^{-6} \text{kgm}^{-1} \text{s}^{-3}
$$

using the adjusted total heat flux ($\overline{Q_t} = -8 \text{ Wm}^{-2}$) that matches the heat budget at the Strait. The nonlinearity of the equation of state gives rise to the second
term of equation (3.10) involving seasonal perturbations of $\alpha$ and $Q_i$. This correlation term is comparable to the first term, and is compensated by cabling or densification on mixing (Garrett et al. 1993). It should be emphasized, however, that forcing a model, that uses buoyancy as a variable, with the total thermal buoyancy flux, including the second term of equation (3.10), would produce a false buoyancy input (Zahariev and Garrett 1997).

The long-term average haline buoyancy flux is

$$\bar{B}_{0S} = \rho_0^{-1} g \beta S (\bar{E} - \bar{P})$$  \hspace{1cm} (3.11)

$$= (15.8 - 1.3) \times 10^{-6} \text{kgm}^{-1}\text{s}^{-3}$$

$$= 14.5 \times 10^{-6} \text{kgm}^{-1}\text{s}^{-3}$$
where $\overline{E} = 1.75\text{my}^{-1}$ is the evaporation rate estimated in Section 3.3 and $\overline{P} = 0.15\text{my}^{-1}$ from the UWM/COADS climatology.

We conclude that the total mean surface buoyancy flux of the Red Sea is $\overline{B}_0 = 20.5 \times 10^{-6}\text{kgm}^{-1}\text{s}^{-3}$, ignoring the correlation term $(-c_w^{-1}g\alpha'Q'_t)$ that would produce spurious buoyancy flux. This value is in accord with our estimate of the buoyancy flux through the Strait using the water properties, as was done in Chapter 2 for the heat and freshwater fluxes. The total surface buoyancy flux is dominated by the haline term and in particular the evaporation; precipitation is very small and including it changes the buoyancy budget by less than 7%. Cooling plays a secondary role in the long-term mean buoyancy flux (due to its small annual mean value), although it is dominant on seasonal time scales. We also note that the interannual variability of $B_0$ is determined by the thermal term, which in turn is determined by the evaporation via the latent heat flux (Figure 3.8). There is buoyancy loss during most of the 45-year period with a few, short, periods of small buoyancy gain which are unlikely to upset the thermohaline cell of the Red Sea. The spatial distribution of the mean total buoyancy flux and its thermal and haline components are shown on Figure 3.9. Spatial changes in the total buoyancy flux are controlled by the thermal component, while the haline component determines the basin-averaged value.

3.6 Summary and discussion

Our interest in the heat and freshwater budgets of the Red Sea is twofold: first, they allow discussion of the reliability of parameterization schemes and data sets (such as the UWM/COADS) used globally to determine surface fluxes, and second, because they provide information about the main driving force of the basin.
Fig. 3.9. Spatial distribution of the mean surface buoyancy flux. (a) Thermal buoyancy flux. (b) Haline buoyancy flux. (c) Total surface buoyancy flux.
3. Surface Buoyancy Fluxes of the Red Sea

The total heat and freshwater transports through the Strait of Bab el Mandab provide a strong constraint for the surface fluxes. Our review of the existing observations in Chapter 2 showed that there is an annual heat transport equivalent to a surface heat flux of $-8 \pm 2 \text{ W m}^{-2}$, and the conservation of volume and salt allowed for a new estimate of the net evaporation rate of $\overline{E - P} = 1.60 \pm 0.35 \text{ m y}^{-1}$. When combined with the UWM/COADS estimated $\overline{P} = 0.15 \text{ m y}^{-1}$, the evaporation rate is $\overline{E} = 1.75 \text{ m y}^{-1}$, lower than previously considered.

UWM/COADS contains measured and derived quantities for the air-sea exchanges which are very important for climate studies and climate change. For the Red Sea, however, the 45-year average total surface heat flux (estimated using standard formulae) appears to be in error, compared to the heat exchange at the Strait of Bab el Mandab, by almost 100 W m$^{-2}$. We suggest that a significant part of the discrepancy comes from the overestimated insolation. The remaining difference appears to be due to the underestimated net longwave radiation and latent heat flux; the sensible heat flux plays a minor role.

More specifically, the bulk formula used globally to calculate the insolation does not appear to perform well in the Red Sea as indicated by ground-truth measurements; spatial and seasonal variations of the transmission coefficient may become significant in areas of high aerosol load. We have applied a correction for this effect using satellite data calibrated to ground-truth records. We note, however, that the inverse estimate of dSYL gave a considerable reduction for $Q_s$, close to our estimate for the dust-laden area of the Red Sea. This implies that insolation has been reduced uniformly even in areas under "clear" atmospheric conditions, which may introduce an error in the heat flux estimates for those areas.

Several uncertainties can be identified in our correction to $Q_s$: The insolation has been multiplied by a transmission anomaly estimated using observations
from the period 1980-1985 and the monthly means were used to calibrate the transmission coefficient calculated from satellite data during just the three years 1989-1991. The monthly means from this calibration were used to correct the long-term insolation. These approximations were introduced because of the paucity of ground-truth records for the insolation available in the Red Sea, but the weak interannual variability of $Q_s$ justifies our approach. At any rate, these observations show that there is significant overestimation of $Q_s$ by the bulk formula widely used.

For the estimation of $Q_b$, it seems that application of one single bulk formula on a global scale is a source of errors. Since the available bulk formulae are derived for certain atmospheric conditions it is unlikely that they can produce reliable results everywhere in the world ocean. We have used the formula from Bignami et al. (1995) which was derived from marine data in the Mediterranean. It is not certain, however, that the same conditions in the atmospheric column apply in the Red Sea, but there are no marine observations for the longwave radiation in the Red Sea which would allow comparison with the results of bulk formulae or the radiative transfer model.

Regarding the estimation of $Q_e$, we have pointed out that it has to be compatible with the water budget. The latent heat flux obtained from UWM/COADS appears to be underestimated compared to the evaporative loss implied by the water budget, although the uncertainties of the latter reach 28 W m$^{-2}$. A likely cause for the underestimation is the remaining trend in the wind speed, particularly during the last two decades. However, Kent and Taylor (1996) found that dSYL’s Beaufort equivalent scale is satisfactory for the global data set. In addition, winds higher than average and drier conditions during the last decade are in qualitative agreement with the trend in the North Atlantic Oscillation index analysed by Hurrell (1995), but there is no similarity in the previous decades. Another
limitation of the available evaporative losses comes from the coarse horizontal res-
olution which does not resolve the possibly large spatial gradients in $Q_e$ and $Q_h$
neal land, because of the stronger turbulent heat loss close to the coast (Simon
Josey, personal communication).

By applying certain corrections to $Q_s$, $Q_b$ and $Q_e$ we have reduced the
originally estimated error in $Q_t$ by 75%. Still there is a difference of 23 W m$^{-2}$
from the expected value of $\overline{Q}_t = -8 \pm 2$ W m$^{-2}$, but this can be justified by only
50% of the uncertainty of each heat flux component. We emphasize that the
total uncertainty in $Q_t$ was inadequate to account for the original discrepancy of
100 W m$^{-2}$.

Updated records and further measurements for the volume fluxes at the
Strait combined with the requirement for conservation of salt may allow for a more
accurate estimate of the evaporation rate, which is very important for constraining
the evaporative cooling of the basin and revealing the problematic heat flux
components. The oceanic heat budget at the Strait seems to provide a strong con-
straint to the surface heat fluxes and is rather insensitive to volume flux changes.
Overall, from this study we conclude that the thermohaline circulation of the Red
Sea is forced mainly by the evaporative losses; the thermal forcing is of secondary
importance on long time scales.
4. The Buoyancy Driven Circulation of the Red Sea

Chapter 4

The Buoyancy Driven Circulation of the Red Sea

4.1 Introduction

To investigate the dynamical response of the Red Sea circulation to the buoyancy forcing various methods has been used. These include simple analytical models (e.g. Phillips 1966; Maxworthy 1997), Ocean General Circulation Models (OGCMs, e.g. Eshel and Naik 1997; Clifford et al. 1997), while some conclusions about the circulation of the Red Sea have been attempted through analogies to laboratory tank experiments (e.g. Grimm and Maxworthy 1996; Sturman and Ivey 1998).

The channel-like shape of the Red Sea allows for a simple, two-dimensional, first-order approach to the study of the shallow buoyancy-driven circulation of the top 140 m above the sill depth, which is significantly stronger than the thermohaline circulation of the deeper water. Such an approach was first followed by Phillips who introduced a similarity, steady-state model, based on dimensional analysis. Tragou and Garrett (1997) pursued actual solutions of this model to match the Red Sea stratification; these results will be discussed at length shortly.

The problem of matching the Red Sea circulation in the interior to the Strait of Bab el Mandab has been investigated by Maxworthy (1997) who proposed a modified scaling, but considered a three-layer model rather than Phillips' continuously stratified structure. Maxworthy assumed a balance only between buoyancy and frictional forces including wind stress, interfacial friction between the two layers in motion and a stagnant bottom layer, as well as side wall friction.
Regarding numerical studies of the Red Sea circulation, Eshel and Naik (1997) used a 3-dimensional primitive equation OGCM with a realistic coastline, but flat bottom topography fixed at 1500 m depth even at the Strait, to explore the wintertime water mass formation at the northern portion of the basin. Surface forcing to this model was produced with a planetary boundary layer model, but this appears to produce too strong heat losses with an annual mean surface heat flux of about $-100 \text{ W m}^{-2}$ compared to $-8 \text{ W m}^{-2}$ estimated from oceanic heat transport through the Strait. This remarkable discrepancy is probably due to the relatively strong evaporation rates of about $-180 \text{ W m}^{-2}$, which, at least for the water budget, are compensated by the unrealistically high precipitation rates of more than $0.7 \text{ m yr}^{-1}$ in a very arid region. This spurious forcing has lead the authors to the conclusion that the haline contribution to the buoyancy budget is small, although, as was shown in Chapter 3, observations support the opposite.

The study by Clifford et al. (1997) also involves a 3-dimensional primitive equation OGCM that includes a turbulent closure submodel, with more realistic bathymetry and is forced by recently obtained data from meteorological forecasts, remote sensing, and in situ hydrographic measurements. However, no observations of humidity were available for this study and the authors have resorted to a method for calculating their own. This method produced a net evaporation rate of $1.1 \text{ m yr}^{-1}$, lower than the $1.6 \text{ m yr}^{-1}$ estimated in Chapter 3. No information is provided on the heat forcing of this model. Their study focuses on the role of short time scale effects of the winds on the eddy activity of the Red Sea, so it is difficult to assess the influence of the wind-driven circulation in the climatological mean Red Sea circulation examined in this chapter.

Finally, observations from tank experiments by Sturman and Ivey (1998) and Grimm and Maxworthy (1996) have provided some useful indications about
the important processes of the Red Sea, both for the steady-state and the time-dependent response of the basin. Sturm and Ivey (1998) have examined the dynamical response of the flow in a cavity to buoyancy forcing, and provided evidence for Phillips' scaling of the flow in periods of buoyancy loss and gain. Grimm and Maxworthy (1996) attempted an estimate of the frictional forces from a tank experiment, although the differences between the model and the Red Sea allowed only for qualitative results. Results from all of the above studies will be compared and discussed further in this chapter, but first it is useful to obtain some idea about the strength of the circulation and the dynamical balances of the Red Sea.

Clearly, the northward increase of density near the surface of the Red Sea must cause an axial pressure gradient at some depths. We may assume that all of the pressure gradient occurs in the upper layer so that the interface between the near-surface inflow and the lower outflow is flat. Similarly, we could have assumed that the sea surface is flat so that the interface slopes up towards the north. If the former case is true, and the density increases from \( \rho \) at the entrance to \( \rho + \delta \rho \) at the north, then

\[
\rho h = (\rho + \delta \rho)(h - \delta h)
\]

and the surface will slope down towards the north by \( \delta h = h - \frac{\delta \rho}{\rho} \). If the flow were inviscid, application of Bernoulli's theorem would lead to a current of

\[
u = \left(2g\delta h\right)^{1/2} = \left(2gh\frac{\delta \rho}{\rho}\right)^{1/2}
\]

which, combined with the buoyancy equation for the upper inflowing layer of thickness \( h \)

\[
u h \frac{g \delta \rho}{\rho L} = B_0,
\]

(4.1)
4. The Buoyancy Driven Circulation of the Red Sea

gives that the flow in the upper layer is

\[ u = (2B_0L)^{1/3} \]

and scales with \( (B_0L)^{1/3} \) with a constant of proportionality of \( O(1) \).

For \( \delta \rho/\rho \approx 3 \times 10^{-3} \) (as for the observed change in salinity from 36 to 40) and \( h \approx 70 \text{ m} \) (half the thickness of the upper thermohaline circulation), \( \delta h \approx 0.2 \text{ m} \), so that equation (4.1) gives an inviscid current of \( u \approx 2 \text{ m s}^{-1} \). However, the buoyancy equation (4.2) for the same change of density over a distance of \( L \approx 2 \times 10^8 \text{ m} \), under a buoyancy loss of \( B_0 \approx 2 \times 10^{-8} \text{ m}^2 \text{s}^{-3} \), corresponds to a speed of \( 0.02 \text{ m s}^{-1} \). This mismatch of the salinity budget with the metric for an inviscid flow leads to the conclusion that the observed conditions of the Red Sea require significant frictional forces to slow the flow.

Thus any model used to investigate the circulation and properties of the Red Sea must include frictional as well as inertial and buoyancy forces and will, indeed, be sensitive to the details of the frictional forces. A slab layer model seems to be inappropriate as the vertical buoyancy profiles of the Red Sea show no sharp gradients, implying the existence of a continuous downflow into the lower layer throughout the basin; there is no evidence that convection of intermediate Red Sea water takes place only at the far north end of the basin.

A full OGCM forced with reasonable buoyancy fluxes, that match the observed oceanic buoyancy transport through the Strait of Bab el Mandab, and includes the realistic complicated bathymetry of the Red Sea will eventually be useful. Before using an OGCM, however, a rational first step would be to examine the important dynamical balances of the Red Sea with a simple model. At this stage, the similarity model of Phillips is considered to be an instructive starting point.
4.2 A simple model of the Red Sea

4.2.1 Governing equations and similarity forms

Phillips introduced a model for the circulation in the vertical plane of a narrow sea of length $L$ and constant width. The semi-enclosed basin in this model is driven by a constant and spatially-uniform surface buoyancy loss rate $B_0$ and is separated from the exterior ocean by a sill of depth $h$ (Figure 4.1), but with no contraction of the width. With the Boussinesq approximation and retaining only vertical mixing, the governing equations are (Phillips 1966)

\[ u \cdot \nabla b = \frac{\partial}{\partial z} \left( \kappa \frac{\partial b}{\partial z} \right) \tag{4.3} \]

\[ u \cdot \nabla u + \frac{1}{\rho_0} \frac{\partial p'}{\partial x} = \frac{\partial}{\partial z} \left( \nu \frac{\partial u}{\partial z} \right) \tag{4.4} \]

\[ \frac{1}{\rho_0} \frac{\partial p'}{\partial z} = b \tag{4.5} \]

\[ \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{4.6} \]

where the buoyancy $b = -g(\rho - \rho_0)/\rho_0$ is referenced to the density $\rho_0$ of the assumed stagnant reservoir at $z < 0$, the perturbation pressure $p'$ is referenced to the pressure in a fluid at rest with density $\rho_0$, and we have assumed a hydrostatic balance in the vertical (as appropriate for $h \ll L$).

Phillips showed on dimensional grounds that a solution independent of $L$ is given by the similarity forms

\[ b = (B_0 x)^{2/3} h^{-1} g(\eta) \tag{4.7} \]

\[ u = (B_0 x)^{1/3} f(\eta) \tag{4.8} \]
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where \( \eta = z/h \). The exponents of \( x \) are chosen to satisfy the buoyancy integral

\[
\int_0^h \eta b \, dz = -B_0 x \tag{4.9}
\]

and to give the same \( x \)-dependence of the advective and buoyancy torque terms in the vorticity equation

\[
\frac{\partial}{\partial z} \left( u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} \right) = \frac{\partial^2}{\partial z^2} \left( \nu \frac{\partial u}{\partial z} \right) - \frac{\partial b}{\partial x}. \tag{4.10}
\]

We introduce a streamfunction \( \psi(\eta) \) such that \( f = d\psi/d\eta \), and may then write the vertical velocity as

\[
w = -\frac{1}{3} (B_0/x^2)^{1/3} h \psi(\eta) \tag{4.11}
\]

showing an integrable singularity at \( x = 0 \).
For the diffusive and viscous terms in (4.3) and (4.10) to have the same 
\( x \)-dependence as the other terms, we require the eddy coefficients to be of the form

\[ \kappa = \frac{1}{3}(B_0/x^2)^{1/3}h^2\mathcal{K}(\eta) \]  

\[ \nu = \frac{1}{3}(B_0/x^2)^{1/3}h^2\mathcal{N}(\eta) \]

also increasing towards the head of the sea. \( \mathcal{K} \) and \( \mathcal{N} \) are the similarity functions of eddy diffusivity and eddy viscosity. The buoyancy and velocity equations then become the nonlinear coupled ordinary differential equations

\[ 2\psi'g - \psi g' = (\mathcal{K}g')' \]

\[ (\psi'^2 - \psi \psi'')' = (\mathcal{N}\psi'')'' - 2g \]

where the prime denotes \( d/d\eta \). These may be solved numerically by decomposition into the six first-order differential equations

\[ \psi' = q_1 \]

\[ g' = q_2/\mathcal{K} \]

\[ q_1' = q_3/\mathcal{N} \]

\[ q_2' = 2q_1g - \psi q_2/\mathcal{K} \]

\[ q_3' = q_4 + q_1^2 - \psi q_3/\mathcal{N} \]

\[ q_4' = 2g \]

where (4.16, 4.17, 4.18, 4.20) define four new functions \( q_1, q_2, q_3, q_4 \), respectively.
Two boundary conditions at the top ($\eta = 1$) and bottom ($\eta = 0$) are

$$\psi = 0 \quad \text{at} \quad \eta = 0, 1$$ (4.22)

and zero stress at the top and bottom gives two more as

$$q_3 = 0 \quad \text{at} \quad \eta = 0, 1.$$ (4.23)

The surface buoyancy flux condition $-\kappa \partial b/\partial z = B_0$ at $z = h$ becomes

$$q_2 = -3 \quad \text{at} \quad \eta = 1.$$ (4.24)

Only one more boundary condition is allowable, though for the problem as posed we would like to have both $b = 0$ at $z = 0$ so that $g(0) = 0$, and zero buoyancy flux $Kg' = 0$ at $\eta = 0$ so that the flow is driven only at the surface. We can achieve this by choosing $K(0) = 0$ without an extra condition on $g'(0)$. In fact, if we assume that both $\psi$ and $g$ are proportional to $\eta$ as $\eta \to 0$, then (4.14) can only be in balance as $\eta \to 0$ if $Kg' \propto \eta^2$ which implies that $K$ has to be proportional to $\eta^2$. This is required by the nature of the mathematical problem, but cannot be justified by any physical process of the basin. Also, allowing $N$ to tend to 0 as $\eta \to 0$ would make the problem degenerate, so for the moment $N$ is kept finite which implies the existence of a drag at the bottom boundary. Subject to these constraints we may solve for $\psi(\eta)$ and $g(\eta)$ for arbitrary $K(\eta)$ and $N(\eta)$.

4.2.2 Do observations support the similarity scaling?

Phillips analysed hydrographic data obtained in May and June 1958 (Figure 4.2a), during a period of weak winds. He provided a first indication that
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Fig. 4.2. (a) Symbols mark the stations of the R.V. *Atlantis* along the main axis of the Red Sea in May and June 1958. (b) (Surface buoyancy)^{3/2} as a function of horizontal distance. The intersection with the horizontal axis is the virtual origin of the basin. (c) Buoyancy profiles. (d) Buoyancy similarity profiles according to Phillips’ scaling, i.e. \( g(\eta) = bh(B_0z)^{-2/3} \) using \( h = 140 \text{ m} \) and \( B_0 \approx 2 \times 10^{-8} \text{ m}^2\text{s}^{-3} \). The solid line is the mean profile and the non-dimensional vertical axis \( \eta \) is in scale with the dimensional axis of (c), i.e. \( \eta = 0 \) at \( z = 140 \text{ m} \).
the similarity scaling is valid by plotting the 3/2 power of the surface buoyancy \((b(0)^{3/2})\) as a function of distance from the origin (Figure 4.2b). A least squares fit shows that the values from all the stations along the basin fall reasonably well along a straight line. The intersection of this with the horizontal axis gives the "virtual origin" of the basin (the starting point of the similarity solution). Phillips provided stronger evidence for the validity of the similarity scaling using the buoyancy profiles along the main axis of the basin (Figure 4.2c). When \(b\) is scaled according to equation (4.7) as \(g = \frac{bh}{(B_0x)^{2/3}}\) the profiles do indeed collapse reasonably well into a single curve (Figure 4.2d). We have chosen \(b = 140 \text{ m}\) (instead of 120 m used by Phillips) because this seems to include most of the strongly stratified upper layer of the Red Sea. It is also approximately the maximum depth of the shallower region of the sill, but includes some allowance for outflowing water to be sucked up from slightly below sill depth, as in any stratified withdrawal problem (Turner 1973).

This scaling assumes a spatially, and temporally, uniform surface buoyancy loss rate \(B_0\). Phillips used an annual average \(B_0 \approx 4.4 \times 10^{-8} \text{ m}^2\text{s}^{-3}\), as available at that time from Defant (1961). Since then buoyancy flux budgets have been further examined and, as presented in Chapter 3, a value of \(B_0 \approx 2 \times 10^{-8} \text{ m}^2\text{s}^{-3}\) is estimated from the advective flux constraint at the Strait of Bab el Mandab. This reduction of the annual buoyancy flux from that used by Phillips has a significant influence on the dimensionless surface buoyancy, giving \(g(1) \approx 70\) compared to \(g(1) \approx 35\) estimated by Phillips. The annual average buoyancy flux is higher in the north than in the south. This is neglected here as it is not possible to find an appropriate scaling for \(B_0\) decreasing with \(x\), though it will have to be taken into account in later models.
Some indication for the validity of Phillips’ similarity scaling for the currents \( u \sim (B_0 x)^{1/3} \) of a buoyancy driven flow has been provided from a laboratory experiment by Sturman and Ivey (1998). In this experiment, steady and spatially uniform buoyancy loss resulted in \( u = 0.2(B_0 x)^{1/3} \), with a parameter that corresponds to the similarity function \( \psi'(1) = 0.2 \). The fact that Phillips’ scaling holds for both uniform \( B_0 \) (tank experiment) and spatially variable \( B_0 \) (Red Sea observations) perhaps indicates that spatial variability of the forcing is not a key factor, but this requires further investigation.

By considering stations from the early summer of 1958, Phillips may have avoided the effect of wind forcing (not taken into account into Phillips’ model) but the profiles do suggest the influence of summertime heating and resulting stable stratification close to the surface. We suggest that it is more relevant to the buoyancy driven circulation model to infer the similarity function of buoyancy from hydrographic data during the buoyancy loss season.

Such profiles are available from the cruise of the Cdt. Robert Giraud during January and February of 1963 (Figure 4.3a), which is the only synoptic hydrographic survey during winter available to us that spans the full length of the basin. The same analysis was followed as for the summer observations. The virtual origin was obtained by plotting \( b(0)^{3/2} \) against distance from Suez (Figure 4.3b) and the buoyancy profiles along the basin (Figure 4.3c) give a similarity profile close to a single curve (Figure 4.3d) near the surface, though the winter profiles do not seem to collapse as well into a similarity function as the summer profiles, possibly due to convergent or divergent motions caused by the winds. Stations 14, 15 and 16 are omitted from the scaled buoyancy profiles (Figure 4.3d) because they are close to the head of the basin where the scaling is sensitive to the choice of the virtual origin.
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Fig. 4.3. The same as Figure 4.2 for stations of the Cdt. *Robert Giraud* in January and February 1963. (a) Stations along the main axis of the Red Sea. (b) (Surface buoyancy)$^{3/2}$ as a function of distance from Suez. (c) Buoyancy profiles. (d) Scaled buoyancy profiles.
In any case, the $g$ profile in winter (Figure 4.3d) shows that the interior circulation extends to a depth of about 140 m, the mixed layer extends to a little less than half of this, and the surface value of the scaled buoyancy function is quite large ($g(1) \simeq 50$).

Maxworthy (1994) drew attention to the large value of $g(1)$ compared with the $O(1)$ value that one might expect for a similarity solution based on dimensional arguments, as shown in Section 4.1. This is perhaps an indication that the thermohaline circulation is rather slower than might have been expected, but interpretation requires the determination of the profiles $K(\eta)$ and $\mathcal{N}(\eta)$ of diffusivity and viscosity that lead to a model solution that reproduces the observations.

4.3 Particular solutions

The system of equations (4.16)–(4.21) is solved for the six functions $\psi$, $g$, $q_1$, $q_2$, $q_3$, $q_4$ as an ordinary two-point boundary value problem. This is done by iterating with the Newton-Raphson method, which implements a fourth-order Runge-Kutta scheme (Press et al. 1986). For the solution of the system we also need to supply a choice of $K$ and $\mathcal{N}$ profiles.

A strongly-mixed surface layer (as expected for surface buoyancy loss) overlying a much less diffusive and viscous lower region can be obtained for $K(\eta)$ and $\mathcal{N}(\eta)$ profiles of the general form

$$K(\eta) = \eta^2 \left[ \frac{1}{2}(K_0 + K_1) + \frac{1}{2}(K_0 - K_1) \tanh(\eta - H)/\varepsilon \right], \quad (4.25)$$

$$\mathcal{N}(\eta) = \frac{1}{2}(\mathcal{N}_0 + \mathcal{N}_1) + \frac{1}{2}(\mathcal{N}_0 - \mathcal{N}_1) \tanh(\eta - H)/\varepsilon + \gamma(\eta - \frac{1}{2}H)\mathcal{N}_1, \quad (4.26)$$

where large $K_0$, $\mathcal{N}_0$ apply in the upper layer, and small $K_1$, $\mathcal{N}_1$ in the lower layer. $H$ is the mid-depth of the transition layer between the two regions, and the thickness...
of the transition layer is defined by $\varepsilon$. The parameter $\gamma$ changes the $N$ profile in the lower layer. The $\eta^2$ factor in $K(\eta)$ is required as $\eta \to 0$ in order to satisfy boundary conditions as described in Section 4.2.1. The free parameters can be modified in order to find the sensitivity to them of the buoyancy function $g$ and streamfunction $\psi$. In particular, we will check the surface buoyancy $g(1)$ against the observed value of about 50, but we will also check the shape of $g(\eta)$.

A model solution of the near surface circulation above the sill depth ($h = 140\,\text{m}$) for $K_0 = 10$, $K_1 = 0.1$, $N_0 = 100$, $N_1 = 25$, $H = 0.7$, $\varepsilon = 0.1$ and $\gamma = 0$ is presented in Figure 4.4. The particular combination of parameters for this “default” solution is chosen so that the buoyancy similarity function (Figure 4.4c) resembles the observations (Figure 4.3d). Note that the buoyancy similarity function has a well-mixed surface layer extending to approximately half the depth of the domain. The circulation (Figure 4.4e) shows that light water enters the basin in the upper layer, gradually becomes heavier due to the surface forcing, sinks and leaves the basin in the lower layer with little mixing across the deep isopycnals.

For this particular solution, the profiles of each term of the vorticity (4.15) and buoyancy (4.14) balances are presented in Figure 4.5. The dynamical balance of the vorticity equation is predominantly between the viscous and the buoyancy terms at all depths; the inertial terms are negligible everywhere (Figure 4.5a) perhaps showing that they are not required in the basic Phillips scaling. For the buoyancy equation in the upper layer, there is only horizontal advection of buoyancy that balances the strong diffusion, while downward advection of buoyancy becomes important only in the lower return flow (Figure 4.5b).

The solution for the interesting special case of a well-mixed surface layer above a perfect fluid (with zero eddy viscosity and eddy diffusivity) has been presented in Garrett et al. (1995). In this case, the problem becomes degenerate.
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Fig. 4.4. Solution of the model for profiles of (a) $\kappa$ and (b) $\mathcal{N}$ given by equations (4.25) and (4.26). The similarity functions of buoyancy and velocity are shown in (c) and (d). The actual scales of the dimensional variables at the middle of the basin ($x = 10^6$ m) are given at the top of each diagram ($\kappa$ and $\nu$ are given in m$^2$s$^{-1}$, buoyancy in m s$^{-2}$, and velocity in m s$^{-1}$), and the non-dimensional vertical axis corresponds to a depth range of 0 – 140 m which is the sill depth. The streamlines (continuous) and the isopycnals (dashed) are shown in (e).
Fig. 4.5. (a) Profiles of the vorticity equation (4.34) terms show a balance between viscous and buoyancy terms at all depths. (b) The buoyancy equation (4.35) terms show a depth-dependent balance between the terms.
with a single infinity of solutions for any choice of the streamfunction $\psi_H$ at the base of the well-mixed layer. The value of $\psi_H$ cannot become too small (to correspond to the small $\psi \simeq 0.04$ of the default solution) without a significant increase of $\psi$ at greater depths. Figure 4.6 is an example of a solution for $\psi_H = 0.25$ (six times larger than the $\psi$ of the default solution), corresponding to a slow surface inflow with a fairly flat sea surface. However, as expected from our discussion in Section 4.1, this solution shows strong northward flow just below the surface mixed layer and an associated density field that is hydrostatically unstable. This is a consequence of the lack of frictional terms necessary to balance the large buoyancy term which becomes even larger in absence of mixing. We conclude that a realistic circulation pattern that matches the observations must be associated with significant frictional forces.

4.3.1 Sensitivity to mixing coefficients

For sensitivity tests we distinguish between the parameters that change the values of the eddy viscosity and eddy diffusivity ($\mathcal{N}_1$, $\mathcal{K}_1$, $\mathcal{N}_0$, and $\mathcal{K}_0$), and those determining the form of the profiles ($\mathcal{H}$, $\varepsilon$, and $\gamma$). Testing the model to the first group of parameters shows that there is only weak dependence of $g(1)$ on the surface values $\mathcal{N}_0$ and $\mathcal{K}_0$, although both of them need to be high (at least of order 10) to avoid large static instability in the upper layer. However, $g(1)$ does depend on the mixing coefficients $\mathcal{N}_1$ and $\mathcal{K}_1$ of the return flow layer as presented in Figure 4.7 where contours of $g(1)$ in $(\mathcal{N}_1, \mathcal{K}_1)$ space are plotted. The stratification increases for increasing viscosity and mixing in the lower layer.

The parameters of the default solution (Figure 4.4) are such that $\mathcal{N}_1 \gg \mathcal{K}_1$ since model solutions with $g(1) \simeq 50$ and smaller $\mathcal{N}_1/\mathcal{K}_1$ give an unrealistically deep mixed layer. An estimate of the actual mean eddy viscosity and eddy diffusivity
Fig. 4.6. Solution for the special case of a well-mixed layer (large vertical eddy viscosity and diffusivity) above a perfect fluid ($\mathcal{K}_1 = \mathcal{N}_1 = 0$). (a) Diffusivity profile, (b) Streamfunction, (c) Similarity profile of the velocity, and (d) Buoyancy similarity profile. (e) Streamlines (solid lines) and isopycnals (dashed lines).
in the return flow layer can be obtained for the default value of $\kappa_1$, $\mathcal{N}_1$, using equations (4.12) and (4.13). At half the distance from the head of the basin ($x = 1 \times 10^6$ m), and for $B_0 = 2 \times 10^{-8} \text{m}^2\text{s}^{-3}$ and $h = 140$ m we find that the average eddy diffusivity and eddy viscosity in the lower layer (between 74 m and 140 m depth) are $\kappa \approx 1.6 \times 10^{-5} \text{m}^2\text{s}^{-1}$ and $\nu \approx 4.4 \times 10^{-2} \text{m}^2\text{s}^{-1}$. Thus, the default solution (which is close to observations) is obtained only when the mixing coefficients represent a very viscous return flow in a weakly mixed lower layer. We cannot reduce $\nu$ by more than a factor 2 or so without a significant mismatch in $g(\eta)$, though $\kappa$ could be up to a factor 10 or so higher.

One final parameter of interest is the Richardson number. As shown by Phillips, it is independent of horizontal position and is given as a function of depth by $Ri = \frac{g'}{\psi \nu^2}$. A large eddy viscosity might be plausible if $Ri$ were small in the
stratified return flow. We therefore check the dependence of $Ri$ on various shapes of the $\mathcal{N}$ profile determined by the second set of parameters ($H$, $\varepsilon$, and $\gamma$), which, incidentally, do not significantly affect $g(1)$. The three panels of Figure 4.8 show $Ri$ as a function of $\mathcal{N}$ for various choices of $H$, $\varepsilon$, and $\gamma$. The solid line in each figure corresponds to the default solution. Either increasing or decreasing the depth $H$ of the transition layer still gives a large value of $Ri$ in the return flow (which has the smaller, but still large, value of $\mathcal{N}$, Figure 4.8a). Similarly, a decrease of the thickness of the transition layer $\varepsilon$ to 0.01 does not reduce $Ri$ in the low viscosity flow and an increase of $\varepsilon$ to 0.5 gives an even larger $Ri$ (Figure 4.8b). $Ri$ remains large in the return flow even when we taper off $\mathcal{N}$ to a small value near $\eta = 0$ by changing the parameter $\gamma$ (Figure 4.8c). These tests imply that no matter how we change the shape of the eddy viscosity profile we cannot obtain reasonable $g(1)$ and $g(\eta)$ without a very high viscosity in a high $Ri$ return flow.

This is characteristic of this model probably produced by the choice for $\mathcal{K}(0) = 0$ at $\eta = 0$ in order to achieve zero buoyancy flux at the bottom of the domain. The very small diffusivity is associated with large stratification and
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Fig. 4.9. (a-c) $\sigma_t$ sections from the cruise of Cdt. R. Giraud in January and February of 1963 (d). (e) The geostrophic velocity for section C is calculated from the average relative velocity between each successive pair of stations assuming that the level of no motion is the deepest measurement of each pair.

therefore large $Ri$. Nonetheless, the high viscosity results in a very slow circulation which is examined next.

4.3.2 Circulation for the default solution and comparison with observations

Using the streamfunction from the default solution we may calculate the horizontal velocity and compare it with geostrophic currents estimated from sections across the Red Sea such as those available from the winter (January and
February) cruise of Cdt. R. Giraud in 1963 (Figure 4.9d). Here we present the $\sigma$ sections across the southern part of the Red Sea where the wind is weak during the winter. Figures 4.9a, b and c show that the tilt of the isopycnals is small (less than $3 \times 10^{-4}$) without any dominant feature. This is also evident from the map of geostrophic currents of Maillard (1974) who first analyzed these data and attempted to estimate the longitudinal water transport. She found that the horizontal circulation of the Red Sea includes mesoscale eddies, as also suggested by altimetry data and OGCM results reported by Eshel et al. (1995), and by hydrographic surveys analyzed by Quadfasel and Baudner (1993).

Although we recognize that the hydrographic sections cannot provide reliable geostrophic speeds (because they may include tidal and other high frequency signals), we have calculated the width-averaged geostrophic velocity for the three sections of Figure 4.9, in order to estimate the order of magnitude of the currents in the Red Sea. The currents are weak (less than 0.05 m/s) with different patterns for each section. We show only the width-averaged geostrophic velocity profile for section C (Figure 4.9e). This shows $u_y \approx 0.03$ m/s at the surface and a return flow of about 0.02 m/s centred at about 120 m. From equation (4.8) the surface velocity of the default solution, taking $B_0 = 2 \times 10^{-8}$ m$^2$s$^{-1}$, the distance of the section from the head of the basin $x \approx 1.35 \times 10^6$ m and $\frac{\partial \psi}{\partial \eta} \bigg|_t = -0.1$, is also 0.03 m/s.

It is also possible to obtain a buoyancy cross section from the default solution if we take $\partial b/\partial z$ from the solution and $\frac{\partial b}{\partial y} = -f \frac{\partial u}{\partial z}$ as for thermal wind. Figure 4.10 shows the buoyancy sections at three different distances from Suez. The tilt of the isopycnals is, as expected, small in the stratified layer below 70 m and the surface layer is relatively homogeneous.

Figures 4.9 and 4.10 show that, even when the Rossby radius of deformation is smaller than the width of the basin (for the circulation in the upper 140 m the
Rossby radius is $O(30\,\text{km})$, the very weak along basin velocities give a small cross sea isopycnal depth change. This result is a consequence of a simple application of the geostrophic balance across the basin, which shows that the fractional depth change of the isopycnals across a channel $\Delta h/\bar{h}$ (where $\bar{h}$ is the average depth) depends not only on the ratio of the width of the basin $W$ to the Rossby radius $R$, but also on the Froude number with $\Delta h/\bar{h} = FW/R$ (Bormans and Garrett 1989a). If the Froude number is small (slow flow) the change in isopycnal depth can be small over a distance much greater than the Rossby radius. It is possible that the axial flow could still be confined to boundary layers, on both sides of the sea, with a width of the order of the internal Rossby radius. A full investigation
of this is beyond the scope of this chapter, but we note that in that case the axial pressure gradient in the centre of the basin would have to be balanced by a cross-basin geostrophic flow. For our default solution the total sea level drop from south to north is 0.3m, which, for $f = 5 \times 10^{-5} \text{s}^{-1}$, would give a cross-basin geostrophic flow of 0.03m$^{-1}$, comparable with the axial flow. The associated strong cross-basin advection of mass would seem to be incompatible with the observed density unless opposed by lateral mixing, which would also, presumably, distribute the axial flow laterally! Nonetheless, as a preliminary step we next investigate whether a cross-channel flow in mid-channel, regardless of how it is matched to the side boundaries, can significantly affect the above solution.

4.4 The effect of rotation and cross-channel flow

We assume that the flow field is independent of y, that the Coriolis term is included only in the x-momentum equation and that the y-momentum equation represents a geostrophic balance without viscous forces. Similarly to equations (3.9)-(4.6) the governing equations become

\begin{align}
\frac{u}{\partial x} + v \frac{\partial b}{\partial y} + w \frac{\partial b}{\partial z} = \frac{\partial}{\partial z} \left( \kappa \frac{\partial b}{\partial z} \right) \quad (4.27) \\
-fv + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} + \frac{1}{\rho_0} \frac{\partial p'}{\partial x} = \frac{\partial}{\partial z} \left( \nu \frac{\partial u}{\partial z} \right) \quad (4.28) \\
fu + \frac{1}{\rho_0} \frac{\partial p'}{\partial y} = 0 \quad (4.29) \\
\frac{1}{\rho_0} \frac{\partial p'}{\partial z} = b \quad (4.30) \\
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0. \quad (4.31)
\end{align}
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The similarity forms for the $u$, $v$ and $b$ are as in equations (4.7) and (4.8). The thermal wind equation, from (4.29) and (4.30), is $f \frac{\partial u}{\partial z} + \frac{\partial b}{\partial y} = 0$, so that appropriate similarity forms for $\partial b/\partial y$ and $v$ are

$$\frac{\partial b}{\partial y} = -f(B_0 x)^{1/3} h^{-1} \psi''(\eta) \quad (4.32)$$

$$v = f^{-1}(B_0^2 / x)^{1/3} r(\eta), \quad (4.33)$$

leading to dimensionless equations of buoyancy and vorticity

$$2\psi'g - 3r\psi'' - \psi g' = (Kg')' \quad (4.34)$$

$$-r' + (\psi^2 - \psi \psi'')' = (N\psi'')'' - 2g. \quad (4.35)$$

Solutions to equations (4.34)–(4.35) can be obtained for an arbitrary profile of the similarity profile $r(\eta)$. The ratio $v/u$ is $f^{-1}(B_0 / x^2)^{1/3}(r/\psi')$, or about $5 \times 10^{-3} r/\psi'$ in the middle of the Red Sea. If $v/u$ is no more than 0.1 and for $|\psi'| \simeq 0.1$ then $r$ will be $O(2)$. If $v/u \simeq O(1)$, then $r \simeq O(20)$. In any case, $r$ is clearly much less than $2g$ which is $O(10^2)$, in (4.35), and also has a minimal effect in (4.34). Experiments with linear and sinusoidal forms of $r(\eta)$ (conserving mass) showed that neither the stratification nor the circulation changes significantly with the modified system of equations. The maximum change in $g$, even for $v/u = 1$, is less than 25%.

A simple calculation can also be carried out to justify the neglect of the $\beta$-effect in the above model. The $\beta$-effect would produce a vorticity tendency of order $-\beta U$, where $U$ is a measure of the northward flow, and, to be balanced, would require frictional forces to be greater on the western side of the basin than on the eastern side by an amount $\delta F \simeq W \beta U$, where $W$ is the basin width. But the
average frictional force approximately matches the axial pressure gradient which, at the surface for example is \( g\delta \zeta /L \) (where \( \delta \zeta \) is the free surface elevation). Thus \( \delta F/F \approx LW\beta U/g\delta \zeta \), which, for \( L = 2 \times 10^8 \text{ m} \), \( W = 2 \times 10^5 \text{ m} \), \( \beta = 2 \times 10^{-11} \text{ m}^{-1}\text{s}^{-1} \), \( U = 0.03 \text{ ms}^{-1} \) and \( \delta \zeta = 0.3 \text{ m} \), is less than 10%. Thus only a small lateral gradient in viscous force, and hence in the flow, is needed to balance the \( \beta \)-effect.

Another contribution to the vorticity of the upper layer would come from vortex stretching induced by the vertical velocity. This tendency is of opposite sign to the \( \beta \)-effect and of comparable magnitude, and so does not change our conclusion that vorticity tendencies are easily balanced by small gradients in the flow and frictional forces.

4.5 Is bottom friction responsible for the apparent high viscosity?

Our solutions so far have suggested the need for high viscosity in the stratified return flow in order to account for the magnitude of the observed surface buoyancy change and the associated slow circulation. It is possible that this high viscosity is an artifact resulting from the omission from the model of bottom friction on the sloping sides of the basin. To examine this, we consider a basin of depth averaged width \( W \approx 100 \text{ km} \) and bottom slope \( s \approx 1.3 \times 10^{-3} \) as appropriate for the southern part of Red Sea (Figure 4.11). The bottom slope is certainly not uniform along the Red Sea. It is quite small in the southern part of the basin but for latitudes north of 20\(^\circ\)N the slope becomes larger. For our comparisons we will consider the slope of the southern basin (where there are estimates of the observed geostrophic currents), and emphasize that the following calculations should be regarded as an approximate scaling rather than a precise comparison.
The bottom friction per unit depth is then (allowing for two sides to the channel)

$$\tau_b \simeq 2s^{-1}C_d u^2. \quad (4.36)$$

Using the value of the current for the default solution \(u = 0.024\text{ms}^{-1}\), estimated at \(x = 1.35 \times 10^6\text{m}\) as the average speed of the return flow) and a drag coefficient \(C_d = 2 \times 10^{-3}\), this bottom friction is 0.002\text{m}^2\text{s}^{-2}. On the other hand, the viscous force integrated across the Sea can be calculated from the similarity solution as

$$W \frac{\partial}{\partial z} \left( \nu \frac{\partial u}{\partial z} \right) = \frac{1}{3} W \left( B_0^2 / x \right)^{1/3} \frac{d}{d \eta} \left( \mathcal{N} \frac{d^2 \psi}{d \eta^2} \right). \quad (4.37)$$

For the model solution we have \(\frac{d}{d \eta} \left( \mathcal{N} \frac{d^2 \psi}{d \eta^2} \right) \simeq 20\) (average over the return flow), so that \((4.37)\) gives at least 0.05\text{m}^2\text{s}^{-2}, 25 times more than sidewall friction can apparently provide.

The bottom friction could possibly be larger if the time-dependent tidal currents were taken into account for the Red Sea. In this case bottom friction per
unit width is given by

\[ \tau_b \approx 2s^{-1}4\pi^{-1}C_d u_{tide} u, \]  

(4.38)

where \( u_{tide} \) is the r.m.s. tidal current. Observations of tidal currents or application of the continuity equation for observed tidal elevations indicate that \( u_{tide} \) cannot be larger than 0.07m s\(^{-1}\), so that the bottom friction is no more than 0.007m\(^2\)s\(^{-2}\) (7 times less than the viscous force). A much larger tidal current of about 0.5m s\(^{-1}\) would be required to account for the large viscosity in the Red Sea. Flow of such strength could be associated with wind-driven gyres (Quadfasel and Baudner 1993) which cannot be resolved by the present model. We note that, even for a slope independent of \( x \), the \( x \)-dependence of \( u \), and hence of bottom friction, is not the same as that of the internal viscous force. Bottom friction becomes relatively more important near the southern end of the Red Sea.

4.6 Solution for a basin of depth-dependent width

We assume that there is no \( x \)-dependence of the width of the basin, but we will allow for change of the width with depth by introducing the dimensionless function \( S(\eta) \) of the width in the continuity equation, so that

\[ \frac{\partial}{\partial x} (S u) + \frac{\partial}{\partial z} (S w) = 0. \]  

(4.39)

The buoyancy and momentum equations become

\[ \frac{\partial}{\partial x} (S_{bu}) + \frac{\partial}{\partial z} (S_{bw}) = \frac{\partial}{\partial z} \left( S_k \frac{\partial b}{\partial z} \right) \]  

(4.40)

\[ \frac{\partial}{\partial x} (S_{uu}) + \frac{\partial}{\partial z} (S_{uw}) + \frac{S}{\rho_0} \frac{\partial p'}{\partial x} = \frac{\partial}{\partial z} \left( S_v \frac{\partial u}{\partial z} \right), \]  

(4.41)
4. The Buoyancy Driven Circulation of the Red Sea

or, if $U = Su$ and $W = Sw$,

$$U \frac{\partial b}{\partial x} + W \frac{\partial b}{\partial z} = \frac{\partial}{\partial z} \left( S \kappa \frac{\partial b}{\partial z} \right)$$

(4.42)

$$\frac{U}{S} \frac{\partial}{\partial x} \left( \frac{U}{S} \right) + \frac{W}{S} \frac{\partial}{\partial z} \left( \frac{U}{S} \right) + \frac{1}{\rho_0} \frac{\partial p'}{\partial x} = \frac{1}{S} \frac{\partial}{\partial z} \left( S \nu \frac{\partial}{\partial z} \left( \frac{U}{S} \right) \right).$$

(4.43)

The similarity forms of buoyancy and velocity in this case are

$$b = (B_0 x)^{2/3} h^{-1} g(\eta)$$

(4.44)

$$U = S(1)(B_0 x)^{1/3} \frac{d\psi(\eta)}{d\eta}$$

(4.45)

$$W = -\frac{1}{3} S(1)(B_0 / x^2)^{1/3} h \psi(\eta),$$

(4.46)

so that $U = u$ and $W = w$ at $\eta = 1$ and the buoyancy similarity form is same as before. Substitution of the similarity forms into the buoyancy and vorticity equations gives a system of two coupled ODE’s

$$2\psi'g - \psi g' = S(1)^{-1}(SK \psi')'$$

(4.47)

$$[S(1)^{-1} S^{-1}(SN(\psi' S^{-1})')' + (\psi S^{-1})(\psi' S^{-1})' - (\psi' S^{-1})^2]' = S(1)^{-2}2g,$$  

(4.48)

which can be solved numerically for a given function of the width $S(\eta)$ with the standard method described in detail in Subsection 4.2.1. For experiments with variable width we have chosen the linear function $S(\eta) = S_0 + (1 - S_0)\eta$, so that $S = S_0$ at $\eta = 0$ and $S = 1$ at $\eta = 1$. We assume that the width of the basin at the sill depth $h = 140m$ reduces to half its width at the surface so that $S_0 = 1/2$. The solution for a buoyancy profile that matches the observations ($g(1) \simeq 50$) is shown in Figure 4.12. Note that the similarity function for the horizontal velocity
4. The Buoyancy Driven Circulation of the Red Sea

Fig. 4.12. Solution of the model for depth-dependent width of the basin. The values for the lower layer of $\zeta$ and $\mathcal{N}$ profiles (first two panels) are $\zeta_1 = 0.5$ and $\mathcal{N}_1 = 12$ respectively.

has increased in the lower layer ($\psi' \approx 0.2$ compared to $\psi' \approx 0.1$ for the default solution), while the viscosity in the lower layer, required to match the solution to the observations, has decreased to $\mathcal{N}_1 = 12$. The ratio $\mathcal{N}_1/\zeta_1$ has decreased by an order of magnitude compared to the value of the default solution and the depth of the surface mixed layer has become slightly shallower, in better agreement with the observations.

Using equation (4.38) we estimate that the bottom friction for this experiment is $0.015\text{m}^2\text{s}^{-2}$ at $x = 1.35 \times 10^6\text{m}$, whereas the viscous force from the solution is $0.047\text{m}^2\text{s}^{-2}$. This brings the bottom friction up to about one third of the viscous force, although this ratio depends on the choice of the distance from the origin where this calculation is carried out (bottom friction becomes more important at the southern end of the Red Sea). It still seems that a significant internal viscous force is required, but this may not be the case if the background currents are larger than those associated with tides.
4.7 The effect of wind

A comparison of wind and buoyancy forcing requires a more elaborate model than the one dimensional model here, but the sensitivity of the circulation to the addition of wind can be investigated for a wind stress that scales with $x^{-1/3}$ so that the surface boundary condition is

$$\tau = \rho \nu \frac{\partial u}{\partial z} = \frac{1}{3} \rho (B_0^2/x)^{1/3} h N \psi''.$$  \hspace{1cm} (4.49)

From the UWM/COADS we find that maximum values of wind stress in the Red Sea are around 0.07 Pa. Assuming that such a value occurs in the middle of the basin ($x = 1 \times 10^6$ m) the boundary condition for the similarity model should be $N \psi'' \approx 20$ at $\eta = 1$. It should be stressed that equation (4.49) implies that the wind stress has a maximum close to the head of the basin and decreases with distance from Suez. This is not the case in the real data, so the surface boundary condition introduced in (4.49) is only an approximation which allows us to obtain a rough estimate of the sensitivity of the circulation to the wind stress but is hardly related to the real world.

In the numerical experiments presented in Figure 4.13 we have assumed both cases for the direction of the wind: southward wind stress, against the thermohaline circulation, and northward stress, in the same direction as the thermohaline circulation. Figure 4.13a shows that the stratification decreases in the case of a northward wind and, as expected, the circulation becomes faster (Figure 4.13b). The situation reverses in the case of southward wind so that a higher stratification is achieved. This implies that for a southward wind the observed $g$ profile of buoyancy can be obtained with a weaker viscosity in the lower layer than the viscosity of the default solution. In fact, a solution with $g(1) \approx 50$ is found with
4. The Buoyancy Driven Circulation of the Red Sea

Fig. 4.13. Solutions for (a) buoyancy, and (b) horizontal velocity, under the influence of wind stress. The continuous line represents the default solution, the dashed line is the solution for a northward wind, and the dot-dashed line is the solution for a southward wind.

\( \mathcal{N}_1 \approx 5 \) and \( \mathcal{K}_1 \approx 0.3 \) and the actual eddy coefficients in the return flow become \( \nu \approx 8.9 \times 10^{-3} \text{m}^2\text{s}^{-1} \) and \( \kappa \approx 2.4 \times 10^{-5} \text{m}^2\text{s}^{-1} \). The viscous force for this solution drops to 0.023m²s⁻² and the bottom friction becomes 0.010m²s⁻² so that bottom friction could account for nearly half the viscous force. The above calculations suggest that the inclusion of wind does not alter our conclusion that Phillips' model requires a large frictional force when matched to observations.

If, however, the effect of wind (southward, opposing the thermohaline circulation) is included in the model with a depth-dependent width, the viscosity and diffusivity for \( g(1) = 50 \) drop to \( \mathcal{N}_1 = 2.5 \) and \( \mathcal{K}_1 = 0.15 \) respectively. The comparison between the bottom friction and the required viscous force shows that the two terms are approximately of the same size. We note, though, that the wind stress parameterization we used in this experiment is rather unrealistic and that the comparisons above depend on the choice of the distance from the origin. It might be possible to explain the high viscous force required by Phillips' model by the combined effect of bottom friction on the sloping boundaries and the wind stress at the surface, but this is not a robust conclusion of this study. It certainly
seems that large frictional forces of some kind are required to account for the slow circulation of the Red Sea, but a more elaborate model and measurements of the fine-scale shear and the bottom friction in the Red Sea are needed to clarify whether this friction is internal, or at the lateral boundaries.

4.8 Matching the solution to the Strait of Bab el Mandab

The solution of Phillips' model in the interior of the Red Sea needs to be matched to the physics of the flow through the Strait of Bab el Mandab. This is a more general problem concerning a semi-enclosed sea forced by a surface buoyancy loss and connected to the open ocean by a narrow strait with a sill. Following Bryden and Stommel (1984) and Garrett et al. (1990) it has seemed that there are essentially three possible configurations, as sketched in Figure 4.14.

In state (a) the internal circulation in the sea is slow, so that the water mass formed by the surface buoyancy loss is much less buoyant than the inflow. The exchange through the strait is weak; the outflowing lower layer is hydraulically controlled but thin. In state (b) more rapid circulation, or stronger mixing, has driven the buoyancy difference between inflow and outflow down to the minimal "overmixed" limit with maximal exchange and hydraulic control of both inflow and outflow. State (c) shows what one might expect if the interior circulation is even more rapid, or the interior mixing even stronger, so that the interior flow does not provide even the minimum buoyancy difference required for maximal exchange in the strait. In this case the conditions in the strait remain those of state (b), with maximal exchange, but there must now be a buoyancy jump just inside the strait and recirculation within the sea in order for the interior flow to build up the buoyancy difference required by the outflow. (In Figure 4.14 we have sketched the
Fig. 4.14. Schematic of three possible states of circulation of a semi-enclosed sea forced by surface buoyancy loss. State (a): slow circulation, strongly stratified. State (b): rapid circulation at the overmixed limit. State (c): even more rapid flow, so that recirculation is required.
streamlines to suggest some withdrawal from below the top of the sill, though the extra depth may not be very great.)

Garrett et al. (1990) argued that the Mediterranean Sea is close to state (b) but that the exchange through the Strait of Gibraltar becomes submaximal for part of each year. The Red Sea may well be in state (a) due to the large density difference, as suggested by the analysis of Maxworthy (1997), and the interior of the basin is not overmixed. In this case, Maxworthy's analysis shows that the depth of the interface is low, if the outflow is hydraulically controlled. However, the depth of the interface also depends on the geometry of the Strait (Bormans and Garrett 1989b), therefore it is difficult to directly compare Maxworthy's depth estimates with observations at the strait, for example from Murray and Johns (1997). Further analysis of the dynamics of the strait is needed to take account of the geometry of the strait and the presence of a non-colocated contraction and sill.

At any rate, the solution from Phillips' model matched to the data also suggests that the basin is not overmixed, because the large stratification is associated with weak circulation. We note, though, from Figure 4.7, that increasing the mixing coefficient in the lower layer ($K_1$) results in an increase in the stratification as measured by $g(1)$. This result appears to contradict the expectation that stronger mixing should lead to weaker stratification in the interior and a tendency towards the overmixed limit! This seemingly surprising result may be explained using the buoyancy integral constraint (4.9). Increased mixing in the lower layer changes the buoyancy profile so that more buoyant water from the surface is mixed downwards into the return flow layer, as shown in Figure 4.15a where the similarity function $g^*$, normalized to 1 at the surface, is plotted for eddy diffusivity $K_1 = 0.1$ and $K_1 = 0.5$. In other words, stronger mixing results in higher buoyancy in the lower
layer, so that if $g(1)$ remained the same, the buoyancy difference $\Delta g$ between the inflow and the outflow would decrease. In order to satisfy the buoyancy integral (4.9) (in non-dimensional form $\int_0^1 (d\psi/d\eta) g \, d\eta = -1$), this decrease of $\Delta g$ in the lower layer would have to be associated with an increase of the circulation $d\psi/d\eta$ (Figure 4.15b). Of course, the constraint would also be satisfied if $g$ increased while $d\psi/d\eta$ remained the same. In fact, as the mixing gets stronger, both $g$ and $d\psi/d\eta$ increase (Figure 4.15b and c) since they are related through the vorticity equation (4.15) (approximated by the balance between the viscous and the buoyancy torque terms, $(N \psi'')'' = 2g$); for constant $N$ an increase in $g$ has to be followed by an increase in $d\psi/d\eta$ and vice versa. Here the similarity function for $u$ at the surface actually increases by 25% and $g(1)$ by 20%. In summary, solutions of Phillips' model contradict the expectation that stronger mixing should lead to weaker density difference from the bottom to the surface in the interior, although if we consider the mean buoyancy difference between the two layers then there is a tendency towards the overmixed limit.
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4.9 Comparison with laboratory experiments

Two laboratory tank experiments simulating turbulent convective flow in a channel have been conducted recently by Sturman and Ivey (1998) and Grimm and Maxworthy (1998). The dynamics of the flow is determined by the Rayleigh number and the geometry of the tanks. These are listed in Table 4.1 along with the characteristics of the Red Sea.

<table>
<thead>
<tr>
<th></th>
<th>$Ra$</th>
<th>$L$ [m]</th>
<th>$W$ [m]</th>
<th>$h$ [m]</th>
<th>$L/h$</th>
<th>$W/h$</th>
<th>$L/W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sturman and Ivey (1998)</td>
<td>$2.6 \times 10^{11}$</td>
<td>0.25</td>
<td>0.5</td>
<td>0.1</td>
<td>2.5</td>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
<td>Grimm and Maxworthy (1998)</td>
<td>$34 \times 10^{11}$</td>
<td>2.50</td>
<td>0.2</td>
<td>0.1</td>
<td>25</td>
<td>2</td>
<td>12.5</td>
</tr>
<tr>
<td>Red Sea</td>
<td>$1.7 \times 10^{23}$</td>
<td>$2 \times 10^{6}$</td>
<td>$2 \times 10^{5}$</td>
<td>500</td>
<td>4000</td>
<td>400</td>
<td>10</td>
</tr>
</tbody>
</table>

Both experiments were conducted at high Rayleigh numbers (greater than $10^{11}$) so that the flow was turbulent and insensitive to the Prandtl number which was $10^3$ in the Grimm and Maxworthy experiment which used salt for the buoyancy flux, but 7 in the Sturman and Ivey experiment using heat. The geometry of the tanks, however, was quite different, with an aspect ratio $L/h$ that was very small for the Sturman and Ivey experiment and an order of magnitude larger for the Grimm and Maxworthy experiment, though both values were very small compared to the aspect ratio of the Red Sea (if the average depth of the Red Sea is 500 m). Moreover, the ratio of the basin width over depth was very small compared to the dimensions of the Red Sea. Only the ratio of $L/W$ was close to that of the Red
Sea for the Grimm and Maxworthy experiment simulating the narrowness of the Red Sea. In principle, of course, the Phillips solution should be independent of these factors.

The dynamical response of the fluid in the two tanks can be described by the buoyancy difference between the inflow and the outflow (scaled as $g(1)$), and the surface current (expressed with the non-dimensional flow scale $\psi'(1)$). These are listed in Table 4.2 along with the actual internal eddy viscosity and diffusivity (halfway along the tanks) required by Phillips' model to match these similarity functions. In the Sturman and Ivey experiment the density difference was large ($g(1) \sim 26$) and solutions to Phillips' model that match the flow required a large internal viscosity of about $1.2 \times 10^{-3} \text{m}^2\text{s}^{-1}$. This implies that, similarly to the Red Sea, weak circulation was associated with strong stratification and high viscosity, much larger than the molecular viscosity. A comparison of the lateral friction $\nu_m \frac{\partial^2 u}{\partial y^2}$ (where $\nu_m$ is the molecular viscosity of $1.14 \times 10^{-6} \text{m}^2\text{s}^{-1}$) to the internal friction $\nu \frac{\partial^2 u}{\partial z^2}$ indicates that the former was small compared to the latter in this experiment, though it could be comparable if one should use the eddy viscosity also in the lateral friction.

Table 4.2.
List of the similarity solutions for the buoyancy and current, and the actual eddy coefficients halfway along the basins required by Phillips' model to match the observations from the two tank experiments and the Red Sea.

<table>
<thead>
<tr>
<th></th>
<th>$g(1)$</th>
<th>$\psi'(1)$</th>
<th>$\kappa [\text{m}^2\text{s}^{-1}]$</th>
<th>$\nu [\text{m}^2\text{s}^{-1}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sturman and Ivey (1998)</td>
<td>26</td>
<td>0.2</td>
<td>$1.2 \times 10^{-4}$</td>
<td>$1.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>Grimm and Maxworthy (1998)</td>
<td>3</td>
<td>$&gt; 0.5$</td>
<td>$&lt; 10^{-5}$</td>
<td>$&lt; 10^{-5}$</td>
</tr>
<tr>
<td>Red Sea</td>
<td>50</td>
<td>0.1</td>
<td>$1.6 \times 10^{-5}$</td>
<td>$4.4 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
4. The Buoyancy Driven Circulation of the Red Sea

In the Grimm and Maxworthy experiment, however, the buoyancy difference was very small \( g(1) \sim 3 \) and the circulation more rapid, contrary to the intuitive expectation that the narrowness of their tank would possibly cause stronger lateral friction, more sluggish circulation, and thus stronger stratification. Lateral friction was also found to be small compared to the small internal friction in this experiment if the molecular viscosity is used laterally.

Overall, the flow in the experiment performed by Sturman and Ivey appears to be more relevant to the Red Sea conditions, because of the large dimensionless density difference which is also observed in the Red Sea. The mismatch between the results of the two tank experiments is contrary to the expectations from the geometry of the tanks and the reason for the difference remains unclear.

4.10 Discussion

In this chapter, I have shown that the Phillips model, although limited in various aspects, provides a useful tool for examining the stratification and circulation of the Red Sea. The similarity scaling of buoyancy matches Red Sea observations with reasonable success, and the solution that is close to winter observations gives along basin velocities comparable to the observed geostrophic currents. Moreover, the very weak along basin velocities give a small cross sea isopycnal tilt and partially justify the use of a 2-dimensional model. The neglect of rotation can be further justified from the recognition that a geostrophic balance of the cross-basin flow with the along-basin pressure gradient would require excessively large cross-basin currents.

In this model the buoyancy integral relates the hydrographic observations in the basin and the flow with the surface buoyancy forcing, and provides insight for the expected state of the Red Sea. Thus, the strong buoyancy difference between
the inflow and outflow is associated with weak flow. The buoyancy integral clearly shows that modelling the Red Sea with false buoyancy fluxes, while attempting to match the buoyancy observations, will give spurious flow, as may be the case for the model of Eshel and Naik (1997); stronger buoyancy loss, matched to the observed densities resulted in stronger currents in the interior. Conversely, the model results of Clifford et al. (1997) gave weaker transport (0.2 Sv instead of more than 0.3 Sv measured at the Strait) possibly because of the weaker haline forcing, although no comparison is possible for the thermal forcing.

The basic model shows that the observed state of the Red Sea can be achieved only with a very viscous lower layer (\( \nu \approx 4 \times 10^{-2} \text{m}^2\text{s}^{-1} \) halfway along the Sea). The dynamical balance in this solution is mainly between the viscous and buoyancy forces with negligible inertial terms. The circulation in the interior is very slow and is probably associated with submaximal exchange through the Strait of Bab el Mandab. We note, though, from Figure 4.7, that increasing \( C \) results in an increase in the stratification.

A modified scaling for \( b \) was suggested by Maxworthy (1997) who assumed a linear dependence of \( b \) with distance from the origin as \( b = B_0 e^{x} h^{-4/3} k \), where \( k \) a constant of proportionality. This scaling, combined with a dynamical balance between buoyancy and frictional forces (equally distributed among wind, internal friction, and side wall friction), and matched to the Red Sea buoyancy observations gave a very large drag coefficient of 0.07. This corresponds to very strong eddy viscosity for the Red Sea of \( \nu \approx C_d U h / 2 \approx 0.4 \text{m}^2\text{s}^{-1} \). We note that, although the constant \( k \) estimated by Grimm and Maxworthy (1998) was \( O(1) \), as expected for a similarity scaling, the actual buoyancy difference also corresponded to a large viscosity for the Red Sea.
Two laboratory experiments by Grimm and Maxworthy (1998) and Sturman and Ivey (1998) gave different results about the dynamics of the flow. In the former experiment the density difference was very small and associated with rapid circulation, whereas in the latter the density difference was larger and the circulation slower, closer to the Red Sea conditions. In both experiments lateral friction was unimportant compared to the internal friction. The differences in the results from the two experiments must be due to the different tank geometries (although their aspect ratios are far from those of the Red Sea), but not for obvious reasons.

A comparison of the bottom friction on the sloping boundaries with the viscous force of the basic solution shows that the former can only account for a small portion of the latter. However, if the geometry of the model is modified so that the basin becomes narrower with depth, bottom friction might account for a significant fraction of the required viscous force, particularly at the southern end of the Red Sea. The effect of the wind stress is shown to be rather small, though the combined effect of a southward wind and of bottom friction might be responsible for the viscous force required by our model.

These rough estimates are subject to a series of uncertainties. (i) The $x$-dependence of the bottom friction is different from that of the viscous force so that the viscous force becomes less important near the southern end of the basin. (ii) The slope of the side walls is a function of the distance from the head of the basin and the value we have chosen does not apply to all areas of the Red Sea. (iii) Similarly, for the modified model with depth-dependent width of the basin we have considered an idealized geometry which is close to the real geometry of the Red Sea only in the southern part. (iv) The spatial distribution of the wind stress is unrealistic. (v) The wind stress may give rise to gyres strong enough to provide an effective lateral mixing mechanism for mean momentum and water properties.
This is not resolved by this simple model. In spite of the above inadequacies, the hierarchy of models we have considered allows for preliminary conclusions regarding the important parameters determining the magnitude of the circulation of the Red Sea.

Several other issues also need to be addressed. These include:

(i) Time dependence, particularly seasonal, in the surface buoyancy flux. We have matched solutions of Phillips' steady state model to winter hydrography, and suggest that Phillips' model, driven by the annual average surface buoyancy flux, is relevant since the circulation time is several years. It would be interesting to develop a time dependent model to allow for an annual cycle that may reverse the buoyancy flux briefly in summer. The important parameters of the time-dependent problem are: (a) the ratio of the seasonal change to the mean annual buoyancy flux, and (b) the ratio of the seasonal time scale (one year) to the circulation time.

We note that a change of sign of $B_0$ in the tank experiment of Sturman and Ivey (1998) resulted in a surface countercurrent. It is possible that the surface outflow of the Red Sea during summer months could be related to the buoyancy reversing sign as well as to the wind forcing. In the same experiment, the circulation was found to be an order of magnitude weaker during the stabilising forcing than during the destabilising forcing. This demonstrates the overall importance of the buoyancy loss period on the circulation of a semi-enclosed basin. Observations of the currents at the strait support this laboratory finding, showing weak currents during summer (e.g. Maillard and Soliman 1986; Murray and Johns 1997).

(ii) The need to match the interior solution to the exchange through the Strait of Bab el Mandab, where the mixing is larger and the flow is hydraulically controlled. Maxworthy's (1997) analysis was a first attempt to solve the problem, but his layered model cannot apply to the Red Sea because of the continuous
stratification observed there. It is also interesting to check whether the penetration of the surface buoyancy forcing below sill depth is associated with stratified withdrawal.

(iii) The influence of the deep water formation and flow on the upper thermohaline circulation. We have assumed that deep water is formed in the extreme north of the Red Sea, at a very slow rate compared to the surface circulation, and does not affect our solution except for the northernmost part where Phillips’ model is not expected to hold.

(iv) The actual value of the Red Sea buoyancy flux. The recent analysis by Tragou et al. (1998) (presented in Chapter 3) gives more reliable values than previous ones, since at least the heat budget is compatible with the advective heat transport through the Strait, but there are still uncertainties that need to be resolved. Nevertheless, these results showed that there is spatial variation in the buoyancy flux along the basin which cannot be ignored in future modelling efforts.

Further work on the relative importance of wind and buoyancy forcing, and on the effect of topography, requires a more elaborate model, but Phillips’ model draws attention to the key dynamical question. The main conclusion is that the buoyancy driven circulation of the Red Sea seems to be very viscous. It is possible that the high viscosity required in the model is a proxy for neglected processes such as bottom friction on the sloping boundaries. Data on the currents and drag at the sloping sea floor would clarify the relative importance of internal friction and bottom friction in retarding the flow, particularly in the southern part of the Red Sea.
Chapter 5

Measurements of Horizontal Eddy Momentum Fluxes

5.1 Motivation and introduction

Examination of the dynamical balance of the thermohaline circulation of the Red Sea with Phillips’ similarity model showed that frictional forces are important, but the exact origin of the friction is unclear. The flow could be retarded mostly because of friction away from the boundaries (internal friction), or because of bottom friction on the sloping boundaries of the basin. The magnitude of the latter depends on the magnitude of the lateral diffusion of momentum. Dominance of each process results in different distributions of the flow across the basin for the same width-averaged volume transport.

An idea about the flow distribution in each case can be obtained from the solution of the standard problem of viscous flow (laminar or turbulent) in a channel of width $L$. The flow is considered to be two-dimensional and “fully developed”, i.e. the velocity profile does not change in the direction of the flow, therefore, from continuity, $\frac{\partial V}{\partial y} = 0$. From the boundary condition $V = 0$ at $y = \pm L/2$ follows that $V = 0$ everywhere. Thus the momentum equation in the $x$-direction is a balance between the viscous term and the externally imposed constant pressure

$$A_h \frac{d^2 U}{dy^2} = \frac{1}{\rho} \frac{dP}{dx} = -G$$  \hspace{1cm} (5.1)
where $A_H$ is the horizontal eddy viscosity parameterizing the lateral diffusion of momentum. The solution for $U(y)$ is a parabola, shown in Figure 5.1, which is given by

$$U_I = \frac{G}{2A_H}((L/2)^2 - y^2).$$

If some sort of internal friction exists in the flow, the momentum equation becomes

$$A_H \frac{d^2 U}{dy^2} - rU = -G$$

where $r$ is a drag coefficient, and the solution for $U$ takes the more complicated form of

$$U_{II} = \frac{G}{r} \left[ 1 - \frac{\cosh((r/A_H)^{1/2}y)}{\cosh((r/A_H)^{1/2}L/2)} \right]$$

also shown in Figure 5.1. In this case most of the flow retardation comes from the internal friction and a smaller amount of momentum is left to be diffused close to the boundaries. This solution takes the parabolic shape of the previous case when $(r/A_H)^{1/2}L \ll 1$, i.e. when the horizontal viscosity is very large compared to the internal friction. The volume transport for case II is

$$Q_{II} = \frac{G}{r} \left[ L - 2(r/A_H)^{-1/2} \tanh \left((r/A_H)^{1/2}L/2\right) \right],$$

which is a function of $G$, $A_H$ and $r$, and at the limit of $(r/A_H)^{1/2}L \ll 1$, $Q_{II}$ becomes the volume transport of case I, i.e. $Q_I = \frac{2}{3}(G/A_H)(L/2)^3$.

Phillips' model allows only for vertical mixing and requires an unusually high vertical eddy viscosity for the solutions to match the observations. If the lateral eddy divergence of momentum is significant in the Red Sea, the large eddy viscosity could be a proxy for the neglect of horizontal eddy viscosity. To resolve the ambiguity we need to know the relative importance of the lateral compared to the vertical momentum flux divergence in the momentum equation. The question explored in this chapter is whether we can directly measure horizontal eddy
viscosity on the sloping boundaries of a channel. We have considered the tidal channel of the Strait of Georgia to test our ability to measure horizontal momentum fluxes from Acoustic Doppler Current Profiler (ADCP) records on a smooth sloping boundary that is uniform alongshore.

Sloping regions of ocean basins are of more general interest because much of the diapycnal transport of properties may occur near the sloping boundaries rather than in the interior. However, to my knowledge, measurements of lateral eddy transfer of momentum on a sloping boundary have not been reported before. Measurements of $\overline{u'v'}$ have been carried out in the open ocean (e.g. the Mid-Ocean Dynamics Experiment, the MODE group (1978), and the POLYMODE experiment) or in regional basins (e.g. Stacey et al. 1991), but always away from the sloping boundaries. More specifically, Stacey et al. (1991) used current meter records from the middle of the Strait of Georgia to measure low-frequency correlations $\overline{u'v'}$. Recently, ADCP records have been used to measure vertical Reynolds
5. Measurements of Horizontal Eddy Momentum Fluxes

stress $\overline{u'w'}$ (e.g. Lu and Lueck 1998a; Stacey et al. 1998) in tidal channels. In this chapter I explore the possibility of measuring $\overline{u'v'}$ from ADCPs, and, in particular, the role of higher frequency eddies in extracting energy from tidal currents.

5.2 Strait of Georgia: momentum balance and scaling

The Strait of Georgia is an estuarine channel located between Vancouver Island and the mainland of British Columbia. The southern-central Strait of Georgia (between 48.5° and 50°N) is 100 km long and 20 km wide (Figure 5.2). The coastline is fairly uniform on the western boundary towards the Gulf Islands, but more complicated at the eastern boundary of mainland British Columbia. At the west coast the sloping bottom is rather uniform and the major axis of the tidal constituents is assumed to be almost parallel to the main axis of the Strait. The long-term mean circulation of the Strait of Georgia is caused primarily by freshwater runoff from the Fraser River, while tidal and wind forcing play an important role in higher frequency flow (Thomson 1981). The main tidal constituents are $M_2$ (12.42 h period) and $K_1$ (23.93 h period) and the tidal current amplitude is generally about 0.2 to 0.3 m s$^{-1}$ LeBlond (1983). This study will focus on short-time circulation, therefore in the following equations the mean current will refer to the mean tidal current.

To examine the momentum balance in the western part of the Strait of Georgia, close to the eastern side of Galiano Island, the east-north coordinate system has been rotated clockwise by 35° so that the main axis of the Strait is approximately parallel to the x-direction (positive towards Haro Strait). The x-momentum equation along the main axis is

$$\frac{\partial U}{\partial t} + U \cdot \nabla U - fV = -\frac{1}{\rho_0} \frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left(A_H \frac{\partial U}{\partial x}\right) + \frac{\partial}{\partial y} \left(A_H \frac{\partial U}{\partial y}\right) + \frac{\partial}{\partial z} \left(A_V \frac{\partial U}{\partial z}\right)$$

(5.3)
5. Measurements of Horizontal Eddy Momentum Fluxes

Fig. 5.2. Map of the southern end of Vancouver Island (upper panel). The area of the experiment is marked with a border and expanded in the lower panel, which also shows the mooring locations and the direction of the tidal currents at the two locations.
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where $U$ and $V$ are the horizontal components of the mean tidal current as the long-term mean currents are smaller by almost an order of magnitude compared to the tidal currents, $f$ is the Coriolis parameter, $P$ the pressure, $\rho_0$ the reference water density, and $A_H$ and $A_V$ the horizontal and vertical eddy viscosity (defined as the appropriate Reynolds stress divided by minus the shear), respectively.

The relative importance of each term of equation (5.3), for a wedge-shaped volume near the coast (Figure 5.3), can be determined by integrating the $x$-momentum equation over the width of the wedge (from $y = 0$ to $L = 5$ km from the coast) and with depth ($h \approx 50$ m at $y = L$). Assuming that $U$ increases linearly with distance from the coast, the first term becomes $\frac{1}{2}Lh\overline{U}_t$ (overline denotes depth average). For a semidiurnal tidal current of $O(0.3 \text{ m s}^{-1})$ this term is $3.5 \text{ m}^3 \text{s}^{-2}$. The largest component of the second term is $U\partial U/\partial x$ which after integration becomes $\frac{1}{4}hL(\overline{U}^2)_x$ and cannot be greater than $0.05 \text{ m}^3 \text{s}^{-2}$. For a small
5. Measurements of Horizontal Eddy Momentum Fluxes

cross-channel flow of $O(0.03 \text{ m s}^{-1})$ (one order of magnitude less than the along-channel component) the Coriolis force is $\frac{1}{2} L h f \bar{V} \simeq 0.3 \text{ m}^3\text{s}^{-2}$. The contribution of the pressure term is estimated from $\frac{1}{2} h L g \eta_x$, using a tidal change of the sea surface level of approximately 0.20 m (Thomson 1981) over a horizontal distance of 100 km. This term is found to be approximately $2.5 \text{ m}^3\text{s}^{-2}$. The horizontal eddy momentum flux term in the direction of the flow is $\frac{1}{2} h L (A_H \bar{U}_x)_x$ and is very small (less than $0.001 \text{ m}^3\text{s}^{-2}$) for eddy viscosity $A_H \simeq 300 \text{ m}^2\text{s}^{-1}$ (estimated as the tidal speed times a mixing length of 1 km). However, for the same $A_H$ the lateral momentum flux term $h A_H \bar{U}_y$ can be as large as $1 \text{ m}^3\text{s}^{-2}$ and may play an important role in the momentum budget. Finally, the vertical eddy momentum flux term is given by $L C_D U^2$, where $C_D \simeq 2 \times 10^{-3}$ is the bottom drag coefficient, and is as high as $0.9 \text{ m}^3\text{s}^{-2}$.

From the above scaling, we conclude that the advective terms, the Coriolis term, and the turbulent flux divergence in the direction of the flow are small compared to the rest of the terms, and the momentum balance is mainly between the acceleration, the pressure term and turbulent flux divergence of momentum laterally and vertically giving

$$\frac{\partial U}{\partial t} + \frac{1}{\rho_0} \frac{\partial P}{\partial x} = \frac{\partial}{\partial y} \left( A_H \frac{\partial U}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_V \frac{\partial U}{\partial z} \right).$$

Away from the boundaries the flow is not expected to be significantly affected by the turbulence terms, therefore equation (5.4) may be reduced to

$$\frac{\partial U_c}{\partial t} + \frac{1}{\rho_0} \frac{\partial P}{\partial x} = 0$$

where $U_c$ represents the tidal current in the centre of the channel. Pressure change in the $y$-direction is expected to be insignificant, therefore there is no significant
change of $\partial P/\partial x$ in the $y$-direction and the pressure term in (5.4) and (5.5) is the same, so that equation (5.4) becomes

$$\frac{\partial}{\partial t}(U - U_e) = \frac{\partial}{\partial y} \left( A_H \frac{\partial U'}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_V \frac{\partial U'}{\partial z} \right). \quad (5.6)$$

To estimate the flux of $x$-momentum into a wedge-shaped volume of water on the sloping side of the channel (Figure 5.3), we integrate (5.6) in the $y$- and $z$-directions. Integration of equation (5.6) with distance from the coast gives

$$\frac{\partial}{\partial t} \int_0^L (U - U_e) \, dy = \left( A_H \frac{\partial U}{\partial y} \right)_L + \int_0^L \frac{\partial}{\partial z} \left( A_V \frac{\partial U}{\partial z} \right) \, dy \quad (5.7)$$

and integration with depth gives

$$\frac{\partial}{\partial t} \int_{-h}^0 \int_0^L (U - U_e) \, dy \, dz = \int_{-h}^0 \left( A_H \frac{\partial U}{\partial y} \right)_L \, dz - \int_0^L C_D U_b^2 \, dy \quad (5.8)$$

where $C_D$ is the bottom drag coefficient and $U_b$ the bottom current at distance $L$ from the coast. The effect of the wind stress has been ignored.

The first term on the right-hand side of equation (5.8) is the Reynolds stress parameterization of horizontal eddy momentum fluxes according to

$$A_H \frac{\partial U}{\partial y} = -\bar{u}' \bar{v}'. \quad (5.9)$$

where $u'$ and $v'$ are the small-scale fluctuations from the mean tidal flow defined from

$$u = \bar{U} + U + u' \quad v = \bar{V} + V + v', \quad (5.10)$$

where $u$ and $v$ are the recorded components of the current, $\bar{U}$ and $\bar{V}$ the long-term mean current components, and $U$ and $V$ the low frequency components of the tidal
current. As I shall show later, $\overline{U}$ and $\overline{V}$ are very small close to the side walls so that the recorded components can be approximated by $u \approx U + u'$ and $v \approx V + v$. Thus, in this chapter, I present the results of a pilot experiment exploring the role of eddies of higher frequency (formed around mean tidal currents) in transferring momentum across tidal currents. Using current records of $u$ and $v$ from ADCPs and removing the tidal part, I attempt an estimate of $-\overline{u'v'}$ and determine the relative importance of the horizontal eddy momentum flux compared to the left-hand side of equation (5.8). Furthermore, I examine whether the deployment of a second ADCP can allow for the estimate of $A_H$, i.e. the horizontal eddy viscosity used in a tidal model, from measurements of the Reynolds stress $-\overline{u'v'}$ and the horizontal shear $\partial U/\partial y$ between the two ADCPs in a tidal channel.

5.3 The experiment

Two 300kHz broad-band ADCPs, manufactured by RD Instruments, were mounted in metal brackets and moored for 14 days (from December 5 to 19, 1996) on a sloping bottom along the east coast of Galiano Island at the southern part of the Strait of Georgia (Figure 5.2). The mooring site was selected for the smooth bottom slope and the nearly straight coastline to minimize the effects of complicated bathymetry and boundary inhomogeneities. The first ADCP (GALI-1) was located at 48°57.3' N, 123°28.8' W, approximately 3 km away from the coast at a depth of about 37 m, and the second (GALI-2) at 48°57.9' N, 123°28.5' W, 8.5 km off the coast at depth of about 80 m.

The recorded velocity profiles were divided into uniform segments (depth cells or bins) of 1 and 2 m for the shallow and deep ADCPs, respectively. Useful bins at the two moorings ranged from 3 to 33 m and 6 to 74 m below the sea surface; the first bin was recorded 3 m above the shallow and 4 m above the deep
Fig. 5.4. $\sigma_z$ profiles estimated from six CTD casts taken 14 days apart near the ADCP moorings. Each panel shows the $\sigma_z$ profile on the day of deployment and recovery.

ADCP, but was discarded as it commonly gives bad data (Schott 1988), whereas the top 6% of the total depth range was considered contaminated from the strong scattering at the sea surface. The ADCP at GALI-1 pinged every 0.41 s and the recorded velocities in beam coordinates (parallel to each beam) were the ensemble averages of 35 pings; beam velocities were recorded every 15 s. The ADCP at GALI-2 pinged at a lower frequency (every 0.48 s) so that the recorded velocities (every 15 s) are the ensemble averages of 30 pings.

Six CTD profiles were also recorded near the ADCP moorings on the days of deployment and recovery of the ADCPs (locations are shown in Figure 5.2). The density profiles at these stations are shown in Figure 5.4. There was strong stratification, close to the shallow station, which became stronger on the day of recovery. Close to the deep mooring and below 25 m the water column appeared to be less stratified on the deployment day and more stratified on recovery. The water density was reduced through the water column probably due to advection of fresh water from the Fraser River and/or precipitation during the 14 days.
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5.4 Data analysis

5.4.1 Derivation of velocity components and data quality

The ADCPs were set to record the ensemble averages of the velocity data in beam coordinates (i.e. four along-beam velocities), as well as the pitch, roll and heading of the instrument. The ADCPs also recorded “percent good” data which give the fraction of pings per ensemble that passed two tests. The first test was a comparison of the correlation level of the echoes of each beam with a threshold value of 64 counts. If the correlation was less than the threshold, the record at this beam was marked bad. The second test was a comparison of the error velocity (i.e. the difference between the vertical velocities calculated from different beams) with a threshold value of 1 m s$^{-1}$. If the error velocity was higher, all the beams were marked bad. If more than 75% of the pings of one ensemble were marked bad, this ensemble was discarded.

The measured velocities contained uncertainties due to internal factors of the ADCPs (e.g. frequency, bin size, number of pings per ensemble) and external factors (e.g. turbulence). The noise level of the beam velocity for a single ping, specified by the manufacturer, was 0.02 m s$^{-1}$ for GALI-1, and 0.01 m s$^{-1}$ for GALI-2 (the difference is due to the different bin size at the two moorings). For an ensemble average of $N$ pings the noise level was reduced by $N^{1/2}$ to about 0.003 m s$^{-1}$ and 0.002 m s$^{-1}$ for each ADCP. The uncertainty due to turbulence is usually higher than the Doppler noise. A measure for the random error due to turbulence is given from the standard deviation of the error velocity, because random errors are independent from beam to beam and because the error velocity is scaled by the ADCP to give the magnitude of horizontal velocity random errors. This error was about 0.025 m s$^{-1}$ for the shallow ADCP and was slightly smaller (0.022 ms$^{-1}$) for the deep ADCP.
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To transfer the velocity components into earth coordinates we have taken into account corrections for the rotation (pitch, roll and heading) and the beam inclination angles. The trigonometric calculations and the bin mapping were carried out with a routine developed by Michael Ott and are similar to those derived by Lohrmann et al. (1990), or Lu and Lueck (1998b). The only difference is that here the inclination angles of the beams were considered separately for each beam instead of using the same angle for all beams. As in every ADCP recording for the derivation of the velocity components in earth coordinates, it is assumed that the flow is homogeneous in the horizontal plane over the distance separating the beams.

Figures 5.5 and 5.6 show examples of five-day time series from approximately the middle of the 14-day record. The derived velocity components are in earth coordinates, extracted from one bin at mid-depth of the water column at each mooring (20 m below the sea surface for the shallow mooring and 40 m below surface for the deep mooring). The flow is mainly tidal with peak currents of about 0.3 m s\(^{-1}\) and rms turbulent fluctuations of \(O(0.1)\) m s\(^{-1}\).

Strong vertical velocities of the order of 0.1 m s\(^{-1}\) or more are observed at GALI-1 periodically (once a day at around 16:00 PST), while at the same time there are spikes on the time series of the horizontal components. A similar, but weaker periodic activity is observed at GALI-2. This is an interesting feature of this record which requires further investigation and could possibly be due to daily vertical migration of scatterers, such as zooplankton, in the water column. A detailed examination of this feature is beyond the scope of this thesis.
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Fig. 5.5. An example of the three velocity components in earth coordinates, derived from the ADCP records in beam coordinates. These time series were extracted from a bin at 20 m below the sea surface at the shallow mooring.

5.4.2 Long-term mean currents

Averaging the 14-day records of the horizontal velocity components at each bin depth gives a measure of the low-frequency flow in the area of the experiment. This long-term mean flow is more likely driven by the thermohaline and wind forcing.

The direction of the mean flow at the two moorings from the 14-day mean $u$ and $v$ is shown in Figure 5.7 for the useful bins from 3 to 33 m, and 8 to 74 m for the two ADCPs respectively. At the shallow mooring the mean current between depths 9 and 21 m is directed $-38^\circ$ with respect to the east-west axis and flows
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Fig. 5.6. Same as in Figure 5.5, but for a bin at 40 m below the sea surface for the deep mooring.

towards Haro Strait. At the deep mooring the direction is towards $-40^\circ$ between 30 and 58 m depth. Near the surface the current flows in the opposite direction and is probably wind driven. Close to the bottom boundary, the current veers anticlockwise as expected for a bottom Ekman layer. We note here that in order to estimate $A_H$ as the eddy viscosity of the long-term mean flow (not of the tidal flow which is examined in this chapter), an estimate for the mean current cross-channel gradient $\frac{dU}{dy}$ would have been required. However, this gradient would have been impossible to determine at the same depths for both moorings without using mean currents from the surface or the bottom boundary layers.
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Fig. 5.7. Direction of the mean current estimated from the full 14-day record at the shallow (left panel) and deep mooring (right panel). Arrows in the left panel indicate the mean-current direction at each 1-m bin, and in the right panel at each 2-m bin.

After rotating the long-term mean records from both moorings to the same angle of $-40^\circ$, the profiles of the mean current components are plotted in Figure 5.8. The mean along-channel component at the shallow mooring is very weak, of the order of a few cm s$^{-1}$, but at the deep mooring it reaches 0.1 m s$^{-1}$. In the area of the shallow mooring there is upwelling of about 0.01 m s$^{-1}$ throughout the water column whereas, as expected, vertical velocities are much weaker at the deep mooring.

5.4.3 Harmonic analysis

To remove low-frequency motion due to tides, harmonic analysis was performed on the time series of the current velocities at each bin depth using Foreman’s (1978) method. This tidal current analysis calculates the tidal ellipse parameters (major and minor semi-axis and inclination) and phase lags using a least squares
Fig. 5.8. Depth dependence of the mean current components estimated from the full 14-day record at the shallow (upper panels) and deep (lower panels) moorings. The frame of reference has been rotated $-40^\circ$ with respect to the east-west axis.
method with nodal modulation for the constituents that can be resolved over the length of the record. The tidal current output data are given hourly.

It is well known that the shorter a time series is, the smaller the number of constituents which may be accurately determined. A commonly used criterion for the choice of the constituents, due to Rayleigh, requires that only constituents separated by at least one complete period from their neighbouring constituents over the length of data available should be included. If, for example, \( F_1 \) and \( F_2 \) are the frequencies of two neighbouring constituents in cycles per time unit, then they can be independently determined only if \( |F_1 - F_2|T > 1 \), where \( T \) is the duration of the recorded data. In other words, to determine \( M_2 \) and \( S_2 \) independently in an analysis requires \((1/12 - 1/12.42)^{-1} \geq 354.48 \) hours = 14.77 days. Thus for our time series of 14 days the main tidal constituents that satisfy the Rayleigh criterion are \( O_1 \), \( K_1 \), \( M_2 \), \( M_3 \), and six more shallow water constituents (\( M_4 \), \( 2MK_5 \), \( 2SK_5 \), \( M_6 \), \( 3MK_7 \) and \( M_8 \)). However, in the Strait of Georgia the solar semidiurnal constituent \( S_2 \) is also important. To include this and other shallow water constituents we lowered the Rayleigh criterion to 0.55 which resulted in a tidal fit including the fortnightly MSF, the diurnal \( O_1 \), \( K_1 \), the semidiurnal \( M_2 \), \( S_2 \), the terdiurnal \( M_3 \) and eleven more shallow water constituents.

In this harmonic analysis, when the length of the time series is such that important constituents are not included (even with a lower Rayleigh criterion), these can be inferred from neighbouring constituents. In our analysis inferred constituents include \( P_1 \), \( N_2 \) and \( K_2 \) which would normally require a record of 182.6 days for \( P_1 \) and \( K_2 \), and 27.6 days for \( N_2 \). The major and minor semi-axis lengths and phase lags of the inferred constituents are estimated from their neighbouring constituents (\( P_1 \) is inferred from \( K_1 \), \( K_2 \) from \( S_2 \), and \( N_2 \) from \( M_2 \)), provided that the amplitude ratios and phase differences with the known constituents are
empirically obtained from previous analyses of longer time records at surrounding stations. For our calculations we used amplitude ratios and phase differences from longer current records from the eastern end of Juan de Fuca Strait.

Examples of the tidal fit from the above constituents to the horizontal components of the recorded current are shown in Figure 5.9. The time series were taken from each mooring at bin depths 20 m and 40 m. The horizontal components of the recorded data were low-pass filtered to fluctuations of period longer than one hour and plotted along with their harmonic fit. The harmonic fit captures several details of the recorded data such as the double high and low tidal currents caused by higher harmonics. However, there are several misfits, particularly during the last four days of the record at the shallow mooring, and throughout the whole record at the deep mooring.

Figures 5.10 and 5.11 show the amplitude of the 21 tidal constituents at various bin depths as a result of the tidal analysis of data from the shallow and deep moorings. For both moorings $M_2$ has maximum amplitude at most depths, but near the surface low frequency motions become important, presumably because of wind effects, and appear as the fortnightly MSF. Away from the surface boundary, the most important constituents are the semidiurnal principal lunar $M_2$, the diurnal luni-solar $K_1$, the principal lunar $O_1$, and the semidiurnal solar $S_2$.

Below the surface boundary of 5 and 10 m for moorings 1 and 2 respectively, the amplitude and inclination of the tidal ellipse for each of the four major tidal constituents are rather uniform with depth (Figure 5.12). There is a general tendency for decreasing inclination closer to the bottom for both moorings, but not for all the constituents; $O_1$ and $S_2$ inclinations become larger close to the bottom at the deep mooring. We emphasize that there is a notable difference between the
Fig. 5.9. Examples of the tidal fit (red line) compared to the low-pass filtered recorded components (blue line) during the 14-day record. The two upper panels correspond to the time series at 20 m depth for the shallow mooring, and the two lower panels to the time series at 40 m depth for the deep mooring.
Fig. 5.10. The contribution of each tidal constituent to the amplitude of the tidal current at various depths for the shallow mooring, as estimated by the harmonic analysis. The vertical scale is correct only for the deepest bin (33 m) and has been offset by a multiple of 0.2 m s\(^{-1}\) for the shallower depths.
Fig. 5.11. Same as Figure 5.10, but for the deep mooring.
depth-averaged (below the surface boundary) tidal inclinations of the two moorings: at the shallow mooring the depth-averaged tidal inclination is about 20° compared to 40° at the deep mooring. This difference may well be the result of topographic steering as the angle of inclination coincides with the direction of the isobaths in the vicinity of the two moorings. It could also reflect differences in the stratification and mixing at the two locations (e.g. Visser et al. (1994) found that in well mixed conditions tidal currents are parallel to the coast, whereas stratification causes significant cross-shore tidal currents), but this cannot be explored with the data available.

One final comment on Figure 5.12 is that the inclination of each constituent at the deep mooring is more variable with depth than that at the shallow mooring with changes of more than 20° compared to changes of less than 10° at the shallow mooring, perhaps because of variations in the stratification due to Fraser River runoff and internal tides. We also note that, although the inclinations of the tidal constituents in the shallow mooring are very similar to each other, at the deep mooring there are large differences exceeding 20°.

5.4.4 Reynolds stress estimates

Any attempt to measure Reynolds stress requires careful consideration of the direction of the mean flow as well as of the averaging period. Reynolds stress is defined as the transport of momentum across mean streamlines by the correlated velocity fluctuations around the mean (e.g. Kundu 1990). In this experiment the mean streamlines are those of the slowly veering tidal current over a tidal period. Thus to estimate the turbulent momentum transport across the tidal streamlines we need to determine the time-dependent direction of the hourly tidal current and rotate the coordinate system in the direction of the tidal current vector. A similar
Fig. 5.12. Depth dependence of the tidal current amplitude (left panels) and the inclination of the tidal ellipse (right panels) for the four most important tidal constituents ($M_2$: solid line, $O_1$: dashed line, $K_1$: dot-dashed line and $S_2$: dotted line) for the shallow (upper panels) and deep (lower panels) moorings.
approach was followed by Gross and Nowell (1983) to determine vertical Reynolds stress in a tidal channel. In particular, if we want to estimate the horizontal Reynolds stress across tidal streamlines that would be simulated by the horizontal eddy viscosity in a barotropic tidal model, it would be more appropriate to rotate the coordinate system in the direction of the depth-mean tidal current.

The depth-mean tidal current direction is computed from the arctan(\(\langle V \rangle / \langle U \rangle\)), where \(\langle U \rangle\) and \(\langle V \rangle\) are the depth-mean tidal components interpolated to a 15-s time step. Depth-averaging includes tidal components from depths at 5–33 m at GALI-1 and 12–74 m at GALI-2 so that the less reliable tidal fit near the surface is not considered. Figure 5.2 shows the direction of the depth-mean tidal current with respect to the earth coordinates. The tidal current at the shallow mooring is directed approximately to a \(-20^\circ\) compared to \(-40^\circ\) direction at the deep mooring. Such a discrepancy in the frame of reference between the two moorings introduces problems in the estimate of the eddy viscosity using the cross-channel gradient \(dU/dy\) of the tidal current and will be discussed later. At each bin depth the records are transferred to the depth-mean time-dependent frame of reference.

To estimate the Reynolds stress it is also very important to choose an appropriate averaging period. This period should be short enough so that there are no significant changes in the slowly veering tidal current, yet long enough to obtain a confident estimate of the fluctuating turbulent components. In the cospectrum of horizontal velocity components, the existence of a "spectral gap", between the slowly-varying mean tidal current and the rapidly varying fluctuations, can provide an ideal averaging period. In practice, however, this frequency band is often dominated by an inertial or tidal frequency which can mask the spectral gap (van Haren et al. 1994). In the absence of a spectral gap, the Reynolds stress could be measured from the ensemble average of the turbulent fluctuations during short
blocks of time at the same tidal phase of successive tidal cycles, as was carried out, for example, by Heathershaw and Simpson (1978).

### 5.4.4.1 Ensemble averages of $-u'v'$

To calculate the Reynolds stress from ensemble averages of $-u'v'$, I proceed as follows. At each bin depth both the interpolated tidal components and the recorded horizontal velocity components are transferred to the time-dependent depth-mean direction of the tidal current vector so that the cross-channel component for the tidal current is minimized. In the new time-dependent frame of reference, the fluctuating parts $u'$ and $v'$ are found by subtracting the tidal and the long-term means from the recorded current component (i.e. $u' = u - U - \bar{U}$ and $v' = v - V - \bar{V}$ respectively). Note that in the new frame of reference the cross-channel tidal component $V$ is very close to zero, but not exactly equal to zero because of the small differences in the direction of the depth-mean tidal current from the tidal current direction at each bin.

The Reynolds stress is calculated from the averaged product of the residuals $u'v'$ over 1-h periods; the choice of the averaging period will be discussed shortly. These 1-h short blocks of data are extracted from the same tidal phase (accelerating, maximum and decelerating), and are centred around a chosen tidal current that has the same direction (positive or negative). Thus, we end up with an ensemble of several short time blocks from successive tidal cycles to estimate $\bar{u'}v'$. The number of these blocks is up to 28 for positive tidal current (because of the small semidiurnal maximum), and up to 14 blocks for negative tidal current.

The expected sign of $\bar{u'}v'$ depends on the sign of the cross-channel gradient $dU/dy$ during the 1-h interval of the extracted time block. For positive tidal current and positive cross-channel gradient $\partial U/\partial y$ (i.e. decreasing current towards the
boundary), \( x \)-momentum should tend to flow towards the negative \( y \)-direction as the turbulence tends to diffuse the gradients and decrease \( \partial U / \partial y \). If, however, the cross-channel gradient is negative, the \( x \)-momentum flux changes direction, i.e. \( \bar{u}'v' > 0 \). Therefore, positive horizontal shear is expected to be associated with negative \( \bar{u}'v' \), and negative shear with positive \( \bar{u}'v' \).

Figures 5.13 and 5.14 show the estimates of the ensemble averages for the Reynolds stress at the two ADCP moorings. The same figures also show the 68\% confidence limits calculated from the standard error \( \sigma_{uu}/N^{1/2} \), where \( \sigma_{uu} \) is the standard deviation and \( N \) the number of blocks of data available for each calculation. These plots contain profiles of \( \langle u'v' \rangle \) (the brackets represent ensemble averaging) extracted around positive accelerating tidal speeds from 0.05 m s\(^{-1} \) up to 0.20 m s\(^{-1} \), of 1-h blocks around the maximum tidal speed of each tidal cycle, and around decelerating tidal speeds from 0.20 m s\(^{-1} \) down to 0.05 m s\(^{-1} \). The \( \langle u'v' \rangle \) profiles around negative tidal speeds, for accelerating, minimum, and decelerating tidal phases are plotted in the lower panel of each figure.

For each short time block, the \( u' \) and \( v' \) components are found to have quasi-normal distributions, but their product \( u'v' \) is generally non-Gaussian. According to Heathershaw and Simpson (1978), such distributions suggest intermittent processes which arise from correlated variables.

There are statistically meaningful estimates of \( \langle u'v' \rangle \) (i.e. values greater than their confidence intervals) and certain features which will be discussed next. For the shallow mooring the highest positive or negative Reynolds stresses are estimated between 5 and 20 m depth with magnitude of about 0.002 m\(^2\) s\(^{-2} \), while for the deep mooring \( \langle u'v' \rangle \) values of the same magnitude are found between 10 and 40 m depth.
Fig. 5.13. Estimates of Reynolds stress with depth at the shallow mooring from ensemble averages of the correlation $\overline{u'v'}$ averaged over short blocks of data which have been extracted around the same tidal speed (indicated in m s$^{-1}$ at the top of each profile) and during the same tidal phase (accelerating, maximum, and decelerating indicated at the top of each diagram). The horizontal scale is correct for the first profile and has been offset by a multiple of 0.005 m$^2$ s$^{-2}$ for the subsequent profiles. The dashed lines indicate 68% confidence intervals.
Fig. 5.14. Same as Figure 5.13, but for the deep mooring. The horizontal scale is correct for the first profile and has been offset by 0.01 m$^2$ s$^{-2}$ for the subsequent profiles.
To compare the sign of the estimated ensemble averages against the expected sign we also need to consider the cross-channel gradient of the tidal current. For this comparison we may use the tidal current at the deep mooring and estimate the difference $\Delta U = U_2 - U_1$ (where $U_1$ and $U_2$ are the along-channel components of the tidal current at the two moorings) at each depth. However, difficulties for this comparison exist, first because of the different frame of reference used for each mooring, and second because the highest $\langle u'v' \rangle$ values at GALI-1 are estimated at depths where there is no satisfactory tidal fit at GALI-2 due to surface boundary layer processes. Also, the depth-by-depth difference cannot be calculated at the deep mooring for depths greater than 33 m. For the frame of reference difference no solution is readily available. To overcome the problem of the non-overlapping levels of tidal currents, a possibility is to estimate the difference of the depth-mean tidal currents at GALI-1 and GALI-2.

The cross-channel difference between the depth-mean tidal currents at the two moorings is shown in Figure 5.15 for one day of the record. The tidal current obtains its maximum first at the deep and then at the shallow mooring with a phase lag of about 1.5 h. Their difference $\Delta U$ oscillates around zero and, in general, is positive for accelerating positive $U_1$, negative for decelerating positive $U_1$ and vice versa for negative $U_1$.

The sign of the estimated $\langle u'v' \rangle$ at the two moorings is in general agreement with the expected, if we consider that $dU/dy$ reverses sign during the decelerating tidal phase. Negative ensemble averages are obtained around positive accelerating tidal currents, and positive around negative and accelerating tidal currents. During the decelerating phase positive $\langle u'v' \rangle$ are found around positive tidal currents, and negative around negative tidal currents, in accord with the expected.
Fig. 5.15. Depth-mean tidal current at the shallow mooring (solid line), the deep mooring (dashed line) and their difference (dot-dashed line) for one day of the recorded time series.

Close to the bottom, in a layer about 10 m thick, estimates of $\langle u'v' \rangle$ obtain greater values at both moorings. The direction of the $x$-momentum flux is mainly positive around maximum positive tidal currents and negative around minimum negative tidal currents, indicating that closer to the bottom the tide at the shallow mooring leads the tide at the deep mooring.

In this analysis we have chosen an averaging period of 1 hour. As mentioned previously, the length of the averaging period is chosen in the hope that the averaging gives statistically meaningful results while the mean current has not changed significantly. Figure 5.16 shows how the normalised values of the ensemble averages depend on the increasing averaging period for blocks of data extracted around the maximum tidal speed. The sensitivity of the ensemble average Reynolds stress to the averaging period is found to be up to a factor of two, with larger Reynolds
Fig. 5.16. Reynolds stress estimates from ensemble averages around maximum tidal speeds as a function of the averaging period. Each line represents the Reynolds stress at a certain depth. Reynolds stress values have been normalised to the mean value for all averaging periods.

stress for shorter periods of averaging. However, the standard error increases for shorter averaging periods, thus the 1-h period is considered an optimum averaging period that balances these effects. At each tidal phase (accelerating, maximum or decelerating), changes in the averaging period have similar effects on the magnitude of the Reynolds stress. This implies that the size of the eddies transferring momentum is similar for all the tidal phases. The existence of a spectral gap in the cospectrum of the fluctuating horizontal components can provide an ideal averaging period and this will be examined next.
5.4.4.2 Ensemble average cospectra

In this attempt to measure the horizontal eddy momentum flux so far, we have averaged the product of the rapidly fluctuating horizontal components over an arbitrary period of time. This subjective choice could have been avoided if there was a spectral gap in the cross-spectrum of the fluctuating parts separating the low frequency motions from turbulent smaller scale motions. The existence of such a gap would allow averaging over a time scale corresponding to the frequency of the gap, and thus we could obtain reliable statistical estimates of small-scale fluctuations before the mean tidal momentum flux has changed significantly.

To examine the existence of a gap in the horizontally divergent components of the eddy momentum, we have computed the cross spectrum $C_{u',v'}$ of the fluctuations around the mean tidal components. This was done at each bin depth of the two moorings for a series of short blocks of data around the same tidal speed and phase, extracted in the same way as was described in the previous section. The velocity components were transferred to the same time-dependent frame. In this case we extracted 2-h long blocks of data and estimated the ensemble average of up to 28 cospectra.

The cospectrum of each 2-h block is the real part of the cross-spectrum, which was computed with an FFT length of 2 h. The cospectra of successive blocks were averaged to decrease the statistical uncertainty of the spectral estimates. Figure 5.17a shows an example of the ensemble average of the variance preserving cospectra at various bin depths for 2-h blocks of data around a positive and accelerating tidal current of 0.1 m s$^{-1}$. Before computing the cospectra, the fluctuating components $u'$ and $v'$ were low-pass filtered to frequencies lower than 0.01 Hz. The cospectra around this and other tidal currents (not presented here) do not show any systematic conspicuous peak at a particular frequency which would
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imply that eddies around these frequencies are transferring momentum across the mean tidal current. Peaks appear in the cospectra occasionally, such as those of frequencies $6 \times 10^{-4}$ Hz and $1.5$ to $2.5 \times 10^{-3}$ Hz between 20 and 30 m for the decelerating negative tidal current of $-0.1$ m s$^{-1}$ (Figure 5.17b), but cannot provide enough evidence for the existence of a reliable separation between these and the tidal frequencies.

Similar results were found from the cospectra at various bin depths from the deep mooring. In Figure 5.18a and b we present two examples of ensemble average cospectra estimates around an accelerating positive tidal current of $0.1$ m s$^{-1}$, and decelerating negative tidal current of $-0.1$ m s$^{-1}$. Again, some peaks appear at certain depths around frequencies of $2$ to $3 \times 10^{-3}$ Hz. We conclude that, although there are some sporadic peaks in the turbulent momentum cospectra, these cannot define a robust frequency that would allow integration of turbulent motions of higher frequencies. However, horizontal Reynolds stress estimates are statistically possible from the ensemble averages presented in the previous section, and those will be used next to calculate the horizontal eddy viscosity.

5.4.5 Estimate of the horizontal eddy viscosity

An estimate for the horizontal eddy viscosity $A_H$ of the tidal flow, given by the ratio $A_H = -\overline{u'v'/(dU/dy)}$, can be attempted using the statistical estimates of the Reynolds stress and the cross-channel gradient of the tidal current. To reduce uncertainties we estimate the depth-mean values of the ensemble averages of the Reynolds stress around various tidal speeds and phases. These are plotted against the horizontal shear of the depth-mean tidal current at the two moorings and are shown in Figure 5.19. The scatter diagram from the shallow mooring estimates shows that there is a reasonable straight-line fit through the points of the diagram.
Fig. 5.17. Ensemble averages of variance preserving cospectra of the fluctuating parts $u'$ and $v'$ around the mean tidal current at various depths for the shallow mooring. Data have been extracted from successive tidal cycles in 2-h blocks (a) around a positive tidal current of 0.1 m s$^{-1}$ at the accelerating tidal phase and (b) around a negative tidal current of $-0.1$ m s$^{-1}$ at the decelerating tidal phase. The vertical scale has been offset by $-0.5 \times 10^{-3}$ m$^2$ s$^{-2}$. 

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Fig. 5.18. Same as Figure 5.17, but for the deep mooring. The vertical scale has been offset by $-0.3 \times 10^{-3} \text{ m}^2 \text{s}^{-2}$. 
Fig. 5.19. Scatter plot of the depth-mean estimates of the ensemble averages \( \langle u'v' \rangle \) around various tidal speeds and phases, versus the horizontal shear \( dU/dy \) of the depth-mean tidal current. The panel on the left corresponds to the estimates at the shallow mooring, and the panel on the right to the deep mooring.

with a slope of about \(-50 \, \text{m}^2 \, \text{s}^{-1}\) which corresponds to the tidal eddy viscosity. However, there is also an offset of about \(5 \times 10^{-4} \, \text{m}^2 \, \text{s}^{-2}\) which cannot be easily explained. A scatter plot with estimates from the deep mooring does not give a satisfactory fit through the points, perhaps because \(dU/dy\) is uncertain for the deep mooring. The slope of the linear fit is much smaller but of the same sign as for the shallow mooring.

This estimate for \(A_H\) can be used to evaluate the dimensionless parameter \((r/A_H)^{1/2}L\), introduced in Section 5.1 for the idealized fluid mechanics problem, to draw some conclusion about the expected cross-channel flow. If the internal friction coefficient is associated with bottom friction, then this is given by \(r = CDU_bh^{-1}\). For a drag coefficient \(CD \approx 2 \times 10^{-3}\), bottom current \(U_b \approx 0.1 \, \text{m} \, \text{s}^{-1}\), and depth \(h \approx 50 \, \text{m}\), \(r\) is about \(4 \times 10^{-6} \, \text{s}^{-1}\). The dimensionless parameter at 5 km from the coast is about 1.4, indicating that there the bottom frictional force is of the same order and somewhat greater than the lateral friction. This conclusion is
also confirmed by comparing the last two terms of equation (5.8), i.e. the depth integral of the estimated Reynolds stress with the bottom friction, assuming a drag coefficient of $C_D \simeq 2 \times 10^{-3}$.

5.5 Conclusions

In this chapter the possibility of measuring horizontal Reynolds stress from ADCPs above the sloping bottom of a tidal channel has been explored. The main conclusions from this experiment are:

(i) Measurements of the horizontal Reynolds stress from ADCPs in a tidal channel are possible. The estimates are based on the statistical ensemble averages of short blocks of data extracted around certain tidal speeds and phases. The direction of the turbulent momentum flux depends on the sign of the cross-channel gradient of the tidal current, and appears to be in agreement with the expected results.

(ii) No spectral gap was found in the cospectra of the fluctuating parts of the horizontal components around the tidal mean current. This means that it is not possible to identify an averaging period that would confidently allow integration of higher frequency motions before the mean tidal current has changed significantly. A 1-h averaging period was considered an optimum choice and was used for the ensemble averages that gave our Reynolds stress estimates, so that there were estimates larger than the error bounds.

(iii) An attempt to estimate the horizontal eddy viscosity in a tidal channel using Reynolds stress measurements and the cross-channel gradient of the depth-mean tidal current between the two stations gave $A_H \simeq 50$ m$^2$ s$^{-1}$. This is an approximate value for the horizontal eddy viscosity estimated from a
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scatter plot of the depth-mean Reynolds stress ensemble averages extracted around different tidal currents and their phases for the shallow mooring; the estimate from the deep mooring is less reliable.

The above estimates are plagued by the following questions and uncertainties:

- The need to specify one single frame of reference to rotate the coordinates to the along-channel direction at both moorings. This was not possible as the direction of the tidal currents appeared to be influenced by the direction of the isobaths at each location. In this study $dU/dy$ was estimated from the depth-averaged tidal current at each mooring which was rotated to a different angle at each station. This gave an approximate value for $dU/dy$, but a different approach needs to be followed for a robust estimate.

- The horizontal shear of the tidal current shows that the offshore tide leads the onshore one with a lag of about 1.5 h, contrary to the expected boundary layer behaviour where the tide close to the boundary leads. This may be particular to the Strait of Georgia system, but more careful examination of the boundary layer processes is required.

- Further analysis is required on the statistics of the Reynolds stress estimates. The 68% confidence limit provided for these estimates requires normal distribution of the ensemble. For these ensembles this is not usually true. It would also be useful to examine whether the estimates are statistically different from zero (i.e. whether the estimated Reynolds stress is not the correlation of a finite sample of two unrelated random variables), with a method similar to that described by Fleury and Lueck (1994).
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Many questions remain on the processes of the horizontal loss of momentum above a sloping bottom in a tidal channel. These could be answered in a future experiment which could be performed on a uniform slope where isobaths follow the same direction at both moorings. This could perhaps allow for a single frame or reference at both moorings. Also, performing the same experiment in a tidal channel with stronger tides and more vigorous tidal mixing could possibly allow for better estimates of eddy fluxes.
Concluding Remarks

Investigation of the physical oceanography of a small regional basin such as the Red Sea is of considerable scientific interest and practical value. First, a small basin with simple geometry can be considered a test basin to explore problems of global applicability, including climate change, both through modelling and monitoring. Second, the surrounding countries often face important environmental and societal problems associated with the management of the natural resources.

Ocean general circulation models are necessary for the simulation and prediction of climate, biological processes and waste disposal. Model results, however, depend on the boundary forcing, and on the appropriate parameterization of sub-grid scale processes. From the examination of the Red Sea buoyancy exchanges with the open ocean and with the atmosphere we have seen that global data sets for the surface buoyancy fluxes, such as the UWM/COADS, which are often used to force models for climate or shorter time scale studies, can give significantly incorrect buoyancy budgets.

More specifically, the heat and freshwater transports at the Strait have been compared to the annual mean surface fluxes as estimated from the UWM/COADS using standard meteorological parameters and bulk formulae. The difference between the annual mean surface buoyancy flux and that implied by the exchange through the Strait was found to be very large and close to 100 W m$^{-2}$. A large portion of this difference is explained by the overestimated solar irradiance due to the misuse of a standard formula and the neglect of spatial and seasonal variations of aerosol concentration. The effect of these has been determined from ground
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stations and satellite data on the optical thickness index. Another portion of the difference came from the underestimated longwave radiation due to the use of a bulk formula which is adequate for the open ocean, but inappropriate for the Red Sea. The evaporative losses were also found to be underestimated, probably because of underestimated wind speeds. These results for the aerosol depletion of insolation, and the underestimation of the evaporative fluxes due to the trends in the wind speeds, may have some global relevance.

From a different perspective, the Red Sea can be considered a test basin for the examination of basic processes of the thermohaline circulation including turbulent convection. This is essentially a paradigm of the thermohaline circulation of the world ocean. In this study a first order approach has been followed to investigate the buoyancy driven flow of the basin, although a more elaborate numerical model of the Red Sea is now required to explore the remaining questions. Still, Phillips' simple similarity model has been very useful in identifying the important dynamical balance between the buoyancy and frictional forces, and revealing a key question about the Red Sea. This involves the origin of the strong friction (internal and lateral) which slows down the circulation of the Red Sea.

To explore the possibility of identifying the relative importance of internal friction and lateral friction and provide appropriate sub-grid scale parameterizations, an attempt was made to measure horizontal Reynolds stress above a sloping bottom in a channel with a configuration similar to that of the Red Sea. The method was tested in the Strait of Georgia, although the main currents in this case were tidal. From the measurements of the horizontal Reynolds stress and the horizontal shear we have seen that it is possible, from ADCP records, to estimate the lateral eddy viscosity acting on the tidal currents. A similar method could be applied to the Red Sea to determine the role of friction on the sloping boundaries.
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From this thesis it is evident that more effort is required to improve our knowledge of the air-sea exchanges both by improving the observations and the bulk parameterizations. We have seen how effective simple constraints are in revealing misfits in our data and in focusing research. In this respect, the haline forces of the Red Sea are found to play the major role in the buoyancy forcing, thus implying that it is crucial to accurately determine the evaporation rates. We have also seen that the combination of satellite data with ground-truth observations, although primitive in this study, holds great promise for the monitoring of the ocean in a global scale.

Updated methods for measuring the currents and drag at the sloping sea floor would provide valuable information on the important processes and improve sub-grid scale parameterization. With deeper understanding of the small-scale processes and better data sets for the surface forcing we may increase our confidence on model results since models of the ocean are the essential next step for the investigation of the oceans.
References


References


Fung, I. Y., D. E. Harrison, and A. A. Lacis, 1984: On the variability of the net longwave 

Garrett, C., M. Bormans, and K. Thompson, 1990: Is the exchange through the Strait 
of Gibraltar maximal or submaximal?, in *The Physical Oceanography of Sea Straits*, 

mation and the surface buoyancy flux, with application to Phillips’ Red Sea model, 

Garrett, C., R. Outerbridge, and K. Thompson, 1993: Interannual variability in Mediter­

Sea: The role of atmospheric aerosols and constraints from the water budget, *J. 

Grimm, T., and T. Maxworthy, 1996: Convectively induced mean flow in a long channel, 

Grimm, T., and T. Maxworthy, 1998: Buoyancy-driven meanflow in a long channel with 

Gross, T. F., and A. R. M. Nowell, 1983: Mean flow and turbulence scaling in a tidal 


stress and its relation to boundary shear stress and drag coefficient measurements, 


aerosols over the oceans with the NOAA advanced very high resolution radiometer 
References


References


References


